An economic analysis of the effects of production risk on the use and management of common-pool rangelands

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[Table of Contents](#)

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Table of Contents

2 The effects of risk in production under joint-maximization vs. individual optimization

2.1 Relevant literature

There have been few studies of the effect of risk on the use of common-pool resources. As noted above, most of the research on risk has focused on explaining the existence and resilience of even poorly managed common-pool pastures because of the resource's value as a means of insurance and/or risk mitigation through spatial mobility. In this paper, however, we will focus on the impact of risk on exploitation rates when the size of the common-pool pasture is fixed. The most active area of research for risk and common property in the context of a fixed resource has been in the area of fisheries management. Sandler and Sterbenz (1990) show that harvest uncertainty in a fisheries model will result in lower exploitation rates than under the corresponding certainty case, using general functional forms for both expected utility and production. This leads to them to conclude that "the tragedy of the commons is therefore mitigated ... in the face of harvest uncertainty".

More generally, it is posited that the greater the variability of an activity, the less resources will be devoted to that activity when producers are risk-averse. Sadoulet & de Janvry (1996) note that risk-averse producers will reduce output supplied as risk increases—in a single commodity model—as long as agents are not "too" risk-averse. That is to say, for very risk-averse agents, it is possible that they will dedicate more resources to the risky activity, to increase the chances that realized output reaches a sufficient level. Though this is theoretically plausible, empirical verification of such a high degree of risk aversion is nonetheless lacking. However, another interesting aspect to the problem occurs when one incorporates risky production into a household model, which allows for the fact that a household can be both a producer and primary consumer of its own output. Here Sadoulet & de Janvry note that it is more likely that the household will produce more as risk increases if the household is a net buyer of the commodity. The model developed below is a pure producer model, and as such does not permit interactions between production and consumption activities of households. Nonetheless, it will be important to bear this point in mind when discussing the more general applicability of the model.

2.2 Development of the theoretical model

There are many different ways in which a group can manage its resources, even in the simple model developed below. However, we follow the typical analysis and begin by examining the two extreme cases—joint-maximization and non-cooperation (c.f. Dasgupta & Heal, 1979). Joint maximization implies that a group can "perfectly" manage its common resources (in the sense that all negative externalities are internalized, and costs to this management are zero — an assumption that will be relaxed in section 3). Conversely, non-cooperation implies that each individual is concerned only with his/her own profit-maximization problem, and we use a non-cooperative game framework to arrive at the equilibrium outcome. Furthermore, the model developed below consists of a single-period; we do not consider either inter-temporal externalities or possible outcomes that are supportable under a repeated game structure. Overstocking occurs if the stocking level chosen under the non-cooperative game is higher than the level associated with joint-maximization. Finally, we use the mean-variance approximation for expected utility obtained using a second-order Taylor series expansion (Hirschleifer and Reilly, 1995).

Initially, players are assumed to be homogeneous in terms of marginal costs and risk preferences. We do this as a base case and show that, as in Sandler & Sterbenz (1991), the total number of cattle stocked under risk is less than the corresponding case under certainty. Furthermore, we establish that profits are actually higher, but expected utility lower, when there is riskiness in production and a non-cooperative game is played. Under joint-maximization, stock levels, expected utility and profits are all lower when there is risk vis-à-vis the riskless scenario. Note that for joint-maximization, a further assumption must be imposed on the model with respect to individual stocking rates, which are otherwise not identified. It is assumed that each herder is allocated rights to stock 1/n-th of the optimum. While this is an intuitively plausible assumption when producers are homogeneous, its justification under producer heterogeneity is more complicated. Thus in this section, we examine comparative statics results for the non-cooperative game when agents are heterogeneous, but defer a discussion of the effects of heterogeneity on joint-maximization until the third section.

2.2.1 Joint-maximization vs. non-cooperation; risk vs. no risk in production

The profit-maximization equations are given below for the following scenarios: 1) joint maximization without risk in production, 2) a non-cooperative game without risk in production, 3) joint maximization with risk in production, and 4) a non-cooperative game with risk in production. Immediately following are the respective first-order conditions.

<u>Scenario 1:</u>

Joint Maximization, No risk in production

$$\max_{L1,L2} EU\left(\pi^{SO}\right) \equiv \pi^{SO} = P_I\left[L_1 f\left(L_1 + L_2; \alpha, \beta\right) + L_2 f\left(L_1 + L_2\right)\right] - cL_1 - cL_2 \qquad [1]$$

<u>Scenario 2:</u>

Non-Cooperative Game, 2-Players, No risk in production

$$\max_{L_1} EU\left(\pi_1^{CN}\right) \equiv \pi_1^{CN} = P_I L_1 f\left(L_1 + L_2; \alpha, \beta\right) - cL_1 \qquad [2a]$$

$$\max_{L_2} EU\left(\pi_2^{CN}\right) \equiv \pi_2^{CN} = P_I L_2 f\left(L_1 + L_2; \alpha, \beta\right) - cL_2 \qquad [2b]$$

<u>Scenario 3:</u>

Joint Maximization, Risk in Production

$$\max_{L1,L2} \mathrm{E}U\left(\pi^{SO}\right) = \left[P_I L_1 * f\left(L_1 + L_2; \alpha, \beta\right) - cL_1 - ½\, \sigma_\theta^2 \phi_A \left(P_I L_1 * f\left(L_1 + L_2; \alpha, \beta\right)\right)^2\right]$$
$$+ \left[P_I L_2 * f\left(L_1 + L_2; \alpha, \beta\right) - cL_2 - ½\, \sigma_\theta^2 \phi_A \left(P_I L_2 * f\left(L_1 + L_2; \alpha, \beta\right)\right)^2\right] \qquad [\ 3]$$

$$s.t.\ \ L_1 = L_2$$

<u>Scenario 4:</u>

Non-Cooperative Game, 2-Players, Risk in Production

$$\max_{L_1} \mathrm{E}U\left(\pi^{CN}\right) = \left[P_I L_1 * f\left(L_1 + L_2; \alpha, \beta\right) - cL_1 - ½\, \sigma_\theta^2 \phi_A \left(P_I L_1 * f\left(L_1 + L_2; \alpha, \beta\right)\right)^2\right] \qquad [4a]$$

$$\max_{L_2} \; EU(\pi^{CN}) = \left[P_L L_2 * f(L_1 + L_2; \alpha, \beta) - cL_2 - \tfrac{1}{2} \sigma_\theta^2 \phi_A (P_L L_2 * f(L_1 + L_2; \alpha, \beta))^2 \right]$$

[4b]

Parameters:

$EU(\pi^{JM,CN})$= Expected utility of profits accruing under joint maximization, and under Cournot-Nash solution, respectively

P_L= price of livestock output

$f(\;)$= average product function

L_i = number of cattle stocked by players, i=1,2

$\alpha,\ \beta$= forage productivity parameters, where $\partial f/\partial\alpha > 0;\quad \partial f/\partial\beta < 0$.

c= constant marginal cost of livestock.

σ_θ^2= variance in rainfall

ϕ_A = coefficient of absolute risk aversion

2.2.2 First-order conditions and model propositions

To simplify notation when writing the first-order conditions, the following definitions will be used:

$f(\bullet)$= average product function, $f(\bullet) = f(L_i + L_j; \alpha, \beta)$

R = variance * coefficient of absolute risk aversion, ($\sigma_\theta^2 * \phi_A$)

<u>Scenario 1:</u>

Joint Maximization, No risk in production

$$2P_L * [f(\bullet) + 2 * L * f'(\bullet)] - 2c = 0$$

or

$$2[P_L * [f(\bullet) + 2 * L * f'(\bullet)] - c] = 0$$ [5]

s.t. $L_i = L_j = L$.

<u>Scenario 2:</u>

Non-Cooperative Game, 2-Players, No risk in production

$$P_L * [f(\bullet) + L_i * f'(\bullet)] - c = 0$$ [6]

For *i*=1,2

<u>Scenario 3:</u>

Joint Maximization, Risk in Production

$$2P_L*[f(\bullet)+2L*f'(\bullet)-R_i*L*f(\bullet)*[2P_L*(f(\bullet)+2L_i*f'(\bullet))]]-2c=0$$

or equivalently,

$$2[P_L*[f(\bullet)+2L*f'(\bullet)-R_i*L*f(\bullet)*[2P_L*(f(\bullet)+2L_i*f'(\bullet))]]-c]=0$$
[7]

<u>Scenario 4:</u>

Non-Cooperative Game, 2-Players, Risk in Production

$$P_L*[f(\bullet)+L_i*f'(\bullet)-R_i*L_i*f(\bullet)*[P_L*(f(\bullet)+L_i*f'(\bullet))]]-c=0$$
[8]

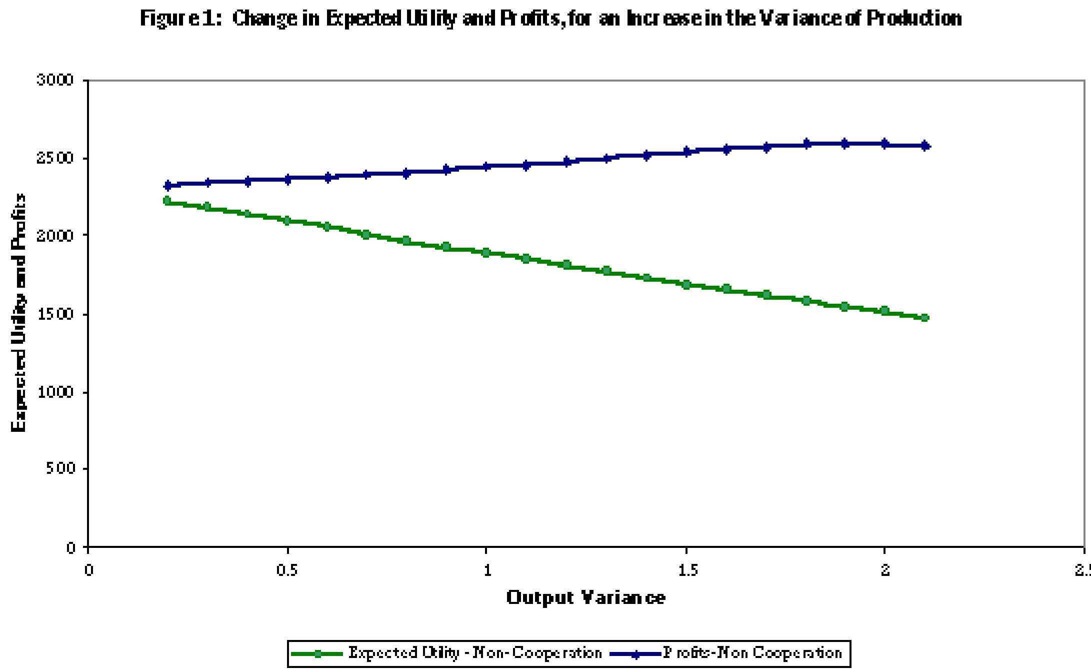
For i=1,2

<u>Proposition 1:</u> Comparing the first-order conditions between the risk and no-risk scenarios, we see that a) under non-cooperation, exploitation levels are lower when there is riskiness in production (which is easily verified by comparing equations 6 & 8), and b) under joint-maximization, exploitation levels are lower when there is production risk, (compare equations 5 & 7).

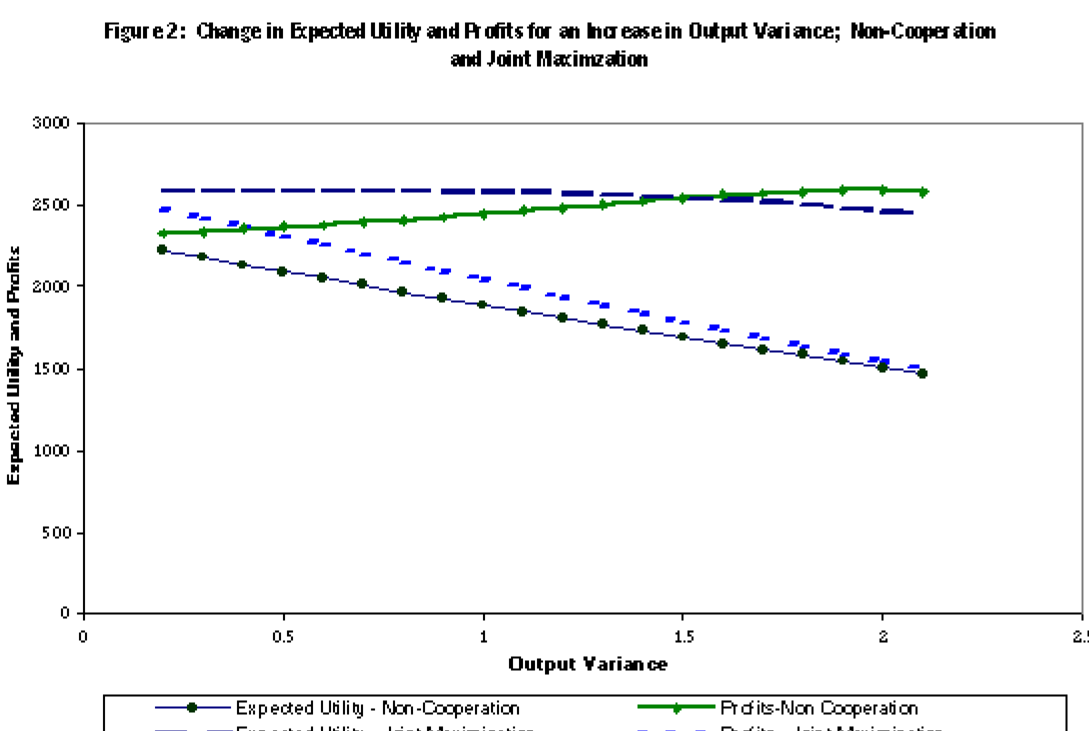
<u>Proposition 2:</u> Given riskiness in production, total stock levels are higher under non-cooperation than under joint maximization, the proof of which is provided in Appendix 1.

<u>Proposition 3</u>: Stock levels under non-cooperation and production risk may be lower than the levels under riskless, joint-maximization [proof provided in Appendix 1].

We can now compare these results to those obtained in the Sandler & Sterbenz model. As noted above, they conclude that "overstocking" is reduced as risk increases, and conclude that risk therefore "mitigates the tragedy of the commons". However, it is fitting to note that to obtain this result, we must make a comparison across two different types of management regimes as well as across two different levels of risk. That is to say, this result depends on using the joint-maximization solution in the absence of risk as the basis for calculating the degree of overstocking. If instead we compared the non-cooperative outcome under risk to the joint-maximization solution under risk, it is not necessary that overstocking—defined here as the difference between the joint-maximization and non-cooperative solutions—decreases with increases in risk. Nonetheless, a comparison to the riskless situation is appropriate if we consider that this reflects the socially efficient outcome. To make this point more forcefully, we can examine the profits accruing under both scenarios as the level of risk is increased. As stated in proposition 1, stocking rates are lower under both regimes when there is production risk. However, profits accruing to the individual are actually higher under the non-cooperative game for a wide range of values for risk, a result depicted graphically in Figure 1.



Thus, starting from a point of no risk, as risk increases, the stock level declines, and profits under non-cooperation will increase until the point where stock levels coincide with the optimal stock levels for the riskless joint-maximization solution. At this point, further increases in risk will reduce both profits and expected utility. Note that for the joint-maximization case, increases in risk will always reduce both profits and expected utility. In Figure 2, we illustrate the case where profits are actually lower under joint-maximization vs. non-cooperation; however, it should be stressed that expected utility will always be lower under non-cooperation. Thus, where producers are risk-averse, we may very well observe stocking rates which produce profits that coincide with joint maximization.



The danger here is in interpreting these profits as indicative of producer welfare, or even worse still, as using this proxy of profits as indicating that the group is actually managing its resources in a socially optimal way. Consider a policy option that will reduce output variance faced by the producer. If the initial assumption is that the group is cooperating (based on profitability), then a reduction in risk should lead to increased producer profits as well as expected utility, without increasing stocking rates beyond the socially efficient level (the riskless joint-maximization level). However, if the group is not cooperating, a reduction in output risk may very well lead to decreased profits and increased overstocking - though expected utility will still be higher. Nonetheless, valuing such an intervention will crucially depend on the situation *ex-ante*; a point we will return to in the third section of the paper where we examine the ability of the group to sustain cooperation in the face of exogenous parameter changes.

In the next section, we give the comparative static results for the non-cooperative game, first assuming that producers are homogeneous, and second, assuming that there is heterogeneity among producers either in marginal costs or in risk preferences. As is made clear in Section 3, heterogeneity among agents significantly complicates the joint-maximization problem; we thus defer the analysis until that section.

2.2.3 Comparative static results

Optimal stock levels are derived from the simultaneous solution of each player's respective first-order condition as given in Equation [8]. Thus, to the derive comparative statics, we totally differentiate the first order conditions and compute the comparative static matrix. However, though the problem looks similar to the single-agent problem with two choice variables, in fact, second-order sufficient conditions cannot be used to sign the Jacobian to the problem, as discussed in Caputo (1996). Dixit (1986) uses an ad-hoc dynamic adjustment process to arrive at the result that this matrix must be negative semi-definate, by appealing to necessary and sufficient conditions for local asymptotic stability. Instead of appealing to the ad-hoc adjustment process, we will instead make an assumption that fits well with our particular empirical focus, which is that $\frac{\partial f}{\partial L_1} = \frac{\partial f}{\partial L_2}$, $\frac{\partial^2 f}{\partial L_1^2} = \frac{\partial^2 f}{\partial L_2^2}$, and $\frac{\partial^2 f}{\partial L_1 \partial L_2} = \frac{\partial^2 f}{\partial L_2 \partial L_1}$ at the equilibrium. This assumption has been widely made in the theoretical literature (c.f. Dasgupta & Heal (1979); Sandler and Sterbenz, (1990)), and posits that the "inputs", in our case cattle, are equally productive across herders *in terms of their ability to convert forage to meat, milk and/or draft power* so that each producer's share of total output is equal to their share of variable inputs applied. While we allow for heterogeneity among herders in terms of costs or risk preferences, we still maintain that the animals are of the same productivity, an assumption that fits well with the empirical observation that herders in extensive and semi-extensive production systems generally stock the same type of cattle, usually indigenous breeds. This assumption would be somewhat dubious if, for instance, we were considering a herder who held indigenous cattle and who shared common pastures with another herder who held high-growth stock (with the assumption that both types of animals are equally adapted to their environment, equally capable of handling environmental stress, disease risks, etc). In the latter case, the high-growth stock would be more efficient at converting a given amount of forage into meat or milk than the indigenous breed, *ceteris paribus*. Given our empirical focus on animals held by herders in semi-arid Africa, this particular complication is not likely to arise. As shown in Appendix 2, this assumption assures that the Jacobian is negative semi-definite, allowing us to compute the following comparative statics results (proofs of which are provided in Appendix 3):

Under agent homogeneity

Proposition 4. Stock levels are decreasing in marginal costs. Stock levels may or may not be increasing in the productivity of the resource or with output prices. If agents are not "too risk-averse" and/or the output variance is fairly low, then stocking rates will increase with increases in forage productivity and output prices, though the response will be dampened vis-à-vis the certainty case.

There is nothing very surprising about the direction of these results. Though it is theoretically possible to get "perverse" responses, i.e. that stocking rates actually decline with an increase in output price, it is highly unlikely. However, as in other studies, responses will be dampened vis-à-vis the certainty case, since higher profits lead to a greater variance in income and raise the cost of risk, thereby leading to smaller increases in inputs. We now examine the case where there is heterogeneity among producers, where the results are more complicated but more interesting.

Heterogeneity in risk preferences

Let herder 1's coefficient of absolute risk aversion be greater than herder 2's. While all of the comparative statics are derived in Appendix 3, it is instructive to examine one of the results when heterogeneity is introduced into the problem. Below is the equation for the change in the i-th person's stock level given an overall positive change in forage productivity.

$$\frac{\partial L_i}{\partial \alpha} = \frac{-1}{|J|} * \Big\{ \overbrace{\Big[\Big[P_L\left(f_\alpha + L_i f_{i\alpha}\right)\Big]\Big[1 - P_L R_i L_i f\Big] - P_L R_i L_i f_\alpha \Big[P_L\left(f + L_i f_i\right)\Big]\Big]}^{A} * \frac{\partial^2 \pi_j}{\partial L_j^2}$$

$$- \overbrace{\Big[P_L\left(f_\alpha + L_j f_{i\alpha}\right)\Big]\Big[1 - P_L R_j L_j f\Big] - P_L R_j L_j f_\alpha \Big[P_L\left(f + L_j f_j\right)\Big]}^{B} * \frac{\partial^2 \pi_i}{\partial L_i \, \partial L_j} \Big\}$$

As with many of the following comparative static results, the signing of this term depends not only on whether or not both herders are not "too" risk averse, but also on the absolute difference between herders with respect to stocking levels, $[L_i - L_j]$, and the term representing the coefficient of absolute risk aversion times the variance, R_i. From Appendix 2, we know that $\left|\frac{\partial^2 \pi_j}{\partial L_j^2}\right| - \left|\frac{\partial^2 \pi_i}{\partial L_i \partial L_j}\right| < 0$. If $|A| \geq |B|$ in the equation above, then stock levels will increase with increases in forage productivity. *A* is always greater than *B* whenever $R_i L_i \leq R_j L_j$. For $R_i L_i >> R_j L_j$, however, it is possible for $\frac{\partial L_i}{\partial \alpha}$ to be negative. That is to say, if the i-th herder is sufficiently more risk averse than the j-th herder, then it is possible that the i-th herder will stock less animals on more productive land. The intuition is that the less risk averse herder will respond to changes in parameters relatively more than will the more risk averse herder. As captured in the comparative statics expression above, there are two effects of an increase in forage productivity, the first being the positive direct effect, and the second the effect stemming from the other herder's response to the same parameter change. Caputo (1996) calls this second effect the strategic effect, and we will use this term as well. It is possible for the strategic effect, which is negative, to dominate the direct effect for one herder — especially if that herder faces much higher marginal costs and/or is much more risk averse than the other herder. It must be the case that the direct effect dominates for the lower cost or less risk averse herder, however. Note that the possibility of a dominant strategic effect also holds in the absence of risk — if at the initial equilibrium, Lj>>Li, then it is also possible that the overall effect of an increase in forage productivity will be to reduce stock levels for the i-th herder.

Finally, however, note that optimal number of livestock, L_i is an inverse function of the cost of risk, R_i. That is to say, *ceteris paribus*, a relatively high L_i will be associated with a relatively low R_i. Thus, we expect that, except for large differences in costs or in risk preferences, higher forage productivity will induce a positive response by both players. Nonetheless, starting from an initial difference in risk preferences and hence stock levels, the more risk-averse individual will stock fewer animals in response to positive changes in exogenous variables than will the less risk-averse individual — and hence, distribution of livestock assets will widen even when both players respond positively.

Proposition 5. Given that agents are not "too risk" averse, nor "too" differentiated in terms of risk preferences, individual stock levels will increase with increases in output price and forage productivity. Any changes in exogenous parameters that positively affect profits will lead to a widening of the distribution of livestock holdings; conversely, any negative changes will lead to a narrowing of that distribution.

Proposition 6. A decrease in the i'th herder's marginal costs will lead to an unambiguous increase in that herder's stock, and to an unambiguous decrease in the other herder's stock. The overall effect on total stock levels is ambiguous, and will depend on whether it is the low-cost or high-cost herder's costs that are increasing.

Proposition 7. An increase in the coefficient of absolute risk aversion for the i-th player will result in lower stock levels for that herder, and to an increase in the other herder's stock. The effect on overall stock levels is ambiguous.

Summary of comparative statics results:				
A. Herders not "too" risk averse, nor "too" differentiated in terms of risk preferences				
Exogenous	Case 1: Homogeneity	Case 2: Herder 1 with lower coefficient of absolute risk aversion		
Parameters	Herder 1 & 2	Herder 1	Herder 2	**Total herd size**
Increase in:				
Pasture productivity	+	+	+	+
Output prices	+	+	+	+
Decrease in:				
Pasture fragility	+	+	+	+
Marginal costs				
For both players, Case 1	+			
For Herder 1, Case 2		+	–	+/–
Coefficient of absolute risk aversion				
For both players, Case 1	+			
For Herder 1, Case 2		+	–	+/–
B. Herders not "too" risk averse, but are sufficiently differentiated in terms of risk preferences				
Exogenous		Case 2: Herder 1 with lower coefficient of absolute risk aversion		
Parameters		Herder 1	Herder 2	**Total herd size**
Increase in:				
Pasture productivity		+	–	+/–
Output prices		+	–	+/–
Decrease in:				
Pasture fragility		+	–	

Marginal costs				
For Herder 1		+	–	+/–
Coefficient of absolute risk aversion				
For Herder 1		+	–	+/–

2.3 Summary

In this section, we have shown that, under joint-maximization, herders are better off in terms of welfare and profits as production risk decreases, and that their stock levels increase as production risk declines. However, under non-cooperation, though stock levels will also increase with decreases in production risk, profits may in fact decline — though herders are better off in terms of welfare when this risk is lower. Furthermore, we have derived the comparative statics for the case of two herders. When herders are sufficiently homogenous in terms of risk preferences and/or marginal costs, then changes in exogenous parameters that positively affect profits for both herders will increase stock levels. However, even in this case, as long as herders exhibit some degree of heterogeneity beforehand, then the distribution of livestock holdings will widen in response to these changes.

Where herders are sufficiently heterogeneous, it is possible for the more risk-averse or higher cost player to reduce his livestock holdings. Many other analyses have pointed to a widening distribution of assets when there is heterogeneity initially; however, in the case of non-cooperatively exploited common property, these differences will be exacerbated, because of the added "strategic" effect. Policy changes that affect direct producer incentives for all resource users must adequately account for both these effects, lest the resulting distribution be far larger than anticipated.

3 Incentives to cooperate, incentives to deviate and the scope for collective action

In Section 2, we focused on the two extreme cases of either no cooperation or perfect cooperation. Nonetheless, there is usually a large set of possible outcomes that would be pareto superior to the non-cooperative outcome, and thus there is no reason to arbitrarily restrict our attention to only these extremes. Thus in this section, we develop a model of a centralized local management institution who can choose any stocking level that leads to a pareto improvement for all players, subject to costs of cooperation. As noted in the introduction, we do not consider decentralized solutions, i.e. outcomes that can be supported under a repeated game structure. Instead, we posit that the ability of the group to make and enforce use-rules for the management of common pastures will be a function of the one-period incentives to cooperate, as well as incentives to deviate from any specified level of cooperation. What is unique to this model, then, is that although the group does attempt to jointly maximize the sum of members' utility, costs of doing so are a function of incentives to deviate from any agreements that are calculated from the non-cooperative game. The main question to be addressed is: If the group does attempt to cooperate, how does risk affect the different incentives to engage in cooperation, and how do differences in risk preferences affect the range of possible levels at which the group may decide to cooperate?

3.1 Relevant literature

There is now a vast literature on the use of common property resources by a well-defined group of users, as well as much empirical research that addresses the ability of groups to manage common property resources (Ostrom, 1990; Seabright, 1993; McKean, 1992; Stevenson, 1991; Bromley, 1992; Bardhan, 1993). Although the case-study and socio-anthropological literature has attempted to identify factors associated with successful management of common property resources, a rigorous theoretical framework has yet to be developed, so that the effect of changes in exogenous variables on exploitation rates and the functioning of a management institution are still not well understood. Nonetheless, many researchers with extensive field experiences have noted two distinct phenomena: 1) there generally exists some type of centralized management institution and/or regulatory body over resource use; or alternatively, lack of centralized management is usually associated with over-exploitation as predicted by the non-cooperative model (Ostrom, 1990; Balland & Platteau, 1996, McCarthy *et.al*., 1998), and 2) that groups undertake cooperation to the extent that the benefits from cooperation outweigh the costs of making and enforcing agreements (Ostrom, 1990; Thompson & Wilson, 1994), or stated somewhat differently, that partial cooperation is often observed in reality (Ostrom, 1990, Balland & Platteau, 1996).

Why is the first phenomena interesting? The answer lies in the way economists generally approach the problem of the commons vs. other disciplines, particularly sociologists, anthropologists, and even range ecologists (Behnke *et.al*., 1993; McKean, 1992; Berkes, 1989). Economic models based on game-theory hold that cooperation cannot be sustained in a one-period game, and conversely, that an infinite number of outcomes may be sustained by a group of users if the game is repeated and there is discounting of the future or uncertainty over when the game will end. These outcomes are sustained by credible threats to dissolve cooperation if any cheating is observed, either for ever or for some specific number of periods (Kreps, Chpt. 14, 1990). However, because these self-enforcing strategies are undertaken solely on the basis of individual actions, there is no economic reason for the group to form an institution to manage the commons, i.e. there is no need for group cooperation, at least with respect to managing externalities (Balland & Platteau, 1996). If groups actually do form to manage the commons, then this type of game-theoretic analysis cannot aid in explaining either the existence or the functioning of institutions to manage the commons. Balland & Platteau (1996) discuss a number of reasons why "collective regulation through a central authority may be desirable", including 1) where there are multiple equilibria, group-level regulation may aid in reaching the Pareto-optimal outcome, and 2) where information is not perfect, decentralized punishment strategies may be very unstable. These are plausible explanations, but they cannot address the second observation, which is that cooperation is often partial. That is to say, if the purpose of a centralized management institution is really only to act as a clearing house for information and for coordinating activities, we should not observe levels of cooperation that are below pareto optimal levels.

Several authors mention that there will be costs and benefits associated with cooperation, so that the group will weigh these costs and benefits when choosing a level of cooperation (Bromley, 1992; Ostrom, 1990, 1992; Wilson & Thompson, 1993). At the same time, a number of authors note that it is not likely that groups will be able to enforce use-rates that are socially optimal, and that cooperation is likely to be partial (Ostrom, 1992; Balland & Platteau, 1996). In fact, Oakerson (1992) states that "some degree of sub-optimal use may actually be efficient when costs of obtaining collective action are taken into account". On the other hand, Seabright (1993) cogently argues that as long as a group can cooperate, there is no reason why they would not pick the best possible outcome to cooperate over. And, as noted above, Balland & Platteau (1996) argue that arriving at the Pareto-optimal level of cooperation when there are multiple equilibria is likely to be a reason for the existence of a centralized regulatory body.

The main problem with the discussion of costs of cooperation thus far is that there has been little attention paid to the actual form of these costs, though much of the discussion seems to imply that they are fixed costs. Transaction costs of cooperating may be increasing in the number of members, but in many cases, the number of members is not the choice variable. The use-rate — in the case of grazing land, the number of livestock to graze, or the number of livestock per some time period — is generally the choice variable under the greatest direct control of the users, either as individuals or as members of the group. A group may face some given level of transactions costs, and they might have some given stock of "social capital" that reduces the costs of cooperation, but it is unclear from the literature why these costs of cooperation are themselves a function of use-rates, i.e. stocking rates, amount of fish to harvest, timber to fell, etc. To summarize, benefits are greatest at the joint-maximization solution, and if costs are fixed, then there is no reason to observe partial cooperation. Below, we argue that costs are in fact a function of the agreed-upon stocking level, thereby allowing for partial cooperation.[1]

> 1. Because the model is one-period with perfect information (perfect monitoring), I will not review the literature on repeated games and the possibility of partial cooperation/collusion where observability of actions is not perfect. For the oligopoly case see Green and Porter, 1984; for public goods provision, see Bendor & Mookherjee, 1988.

Finally, as in Section 2, we are concerned with the effects of heterogeneity on the ability of the group to cooperate. Perhaps the most relevant strand of literature to the model developed below has emerged from oligopoly theory, specifically the work on sustainability of collusion when firms are heterogeneous, though it is generally assumed that agents are risk neutral. There is a direct corollary between the non-cooperative game framework for explaining the exploitation of common property and optimal quantities to produce in an oligopoly. Perfect collusion in oligopoly is equivalent to perfect cooperation over a common property resource. Though much of the literature focuses on trigger strategies and mainly ignores explicit group collusion, there has been work establishing the individual participation constraints that will bound the set of feasible solutions, especially when there is heterogeneity among producers. For example, where marginal costs differ among firms, the optimal "collusive" outcome may not be individually rational for certain firms to participate in — i.e. it may entail output levels that cause some firms to shut down, and in the absence of side-payments, these firms would not enter into such agreements (Harrington (1991), Schmalensee (1987)). Johnson and Libecap (1982) note that this problem is likely to be further exacerbated if the allocation of grazing rights (or fishing quotas in their example) must be allocated equally (i.e. for socio-political reasons (equity), or administrative feasibility). Equity considerations may in fact be very important in the case of common property resources; the degree to which existing differences in wealth and/or efficiency can be institutionalized may very well be limited (though c.f. McKean, (1992) for the case of Japanese grazing lands).

3.2 Modelling incentives to cooperate and incentives to deviate

In what follows, we develop a model to determine: 1) whether it is worthwhile, in terms of marginal costs and marginal benefits, for the group to engage in cooperation, and 2) If so, at what level will they cooperate, and how will levels of cooperation change in response to changes in exogenous parameters. While there may be a whole host of socio-cultural factors that affect a group's ability to cooperate, in the analysis which follows, we focus only on the pure economic incentives to cooperate and to deviate from agreements. Furthermore, we rely heavily on a graphical analyses. For more rigorous mathematical treatment of the incentives to cooperate and to deviate, see McCarthy *et.al.* (1996); for a more rigorous treatment of individual participation constraints under agent heterogeneity, see Schmalensee (1989), and Harrington (1991).

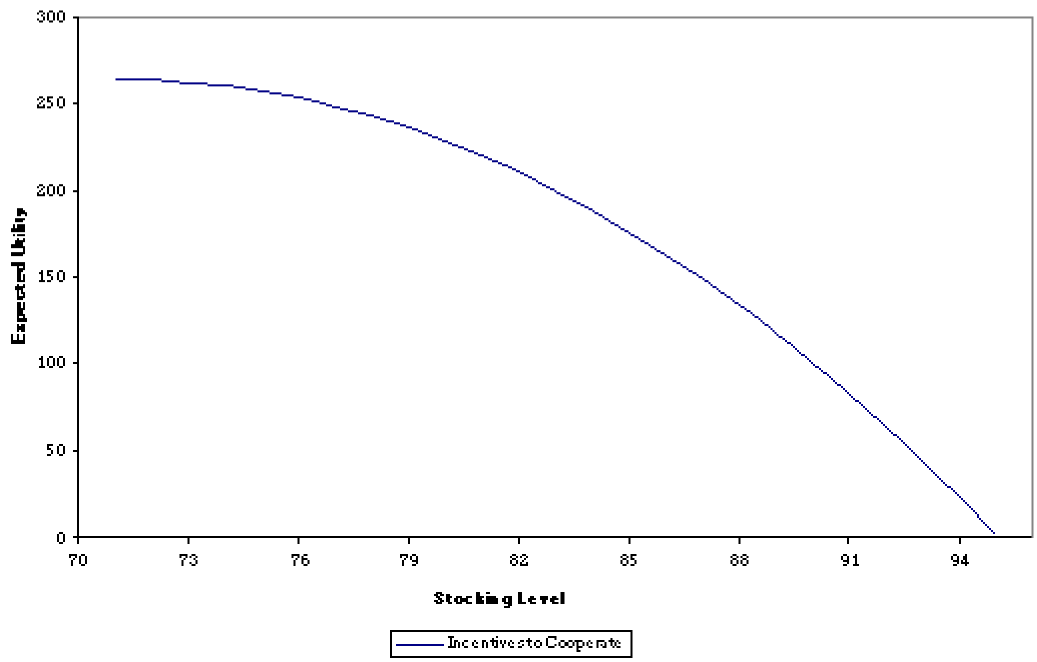
To graphically illustrate the model, we must give a functional form to the average product function, as well as to parameterize the model. In the analysis that follows, we use a linear-quadratic value function for livestock production. The coefficient of absolute risk aversion is chosen so as to yield a coefficient of relative risk aversion of .65 in the base scenario; a figure that implies mid-level risk aversion.[2] Given these parameter values — and even within wide ranges of all these values — the non-exceptional comparative statics results hold. That is to say, we have not reproduced the results where agents are too risk-averse, or too differentiated in terms of marginal costs or risk preferences, as the former case is not likely to be of importance in empirical applications, and the latter case is dealt with in more detail below.

2. A coefficient greater than 1 is considered highly risk averse.

Let us first discuss incentives to enter into agreements as well as to defect from them. First, consider the net gains to the individual from entering into an agreement. These are defined as the profits associated with cooperation, Π_i^{Cc}, minus profits from the initial position of non-cooperation (i.e. the Nash non-cooperative solution), Π_i^{NCnc}. In the analysis that follows, superscript notation will have the following meaning: capital letters will denote the actions taken by the player 1, which can either be cooperate at an agreed upon level (C), or to optimally deviate from this agreement (NC); similarly lower-case letters will refer to the actions of player 2 (c, nc).

The gains to cooperation are plotted in Figure 3; where gains from cooperation for the individual in terms of expected utility are plotted against stock levels, note that gains are achieved by moving from *right to left*, that is, as the group de-stocks. Thus, the benefits from destocking can be calculated over the entire interval from the non-cooperation outcome to the joint-maximization levels.

Figure 3: Gains of Cooperation at Varying Stock Levels



Next, consider the logic of the prisoner's dilemma game. There are two elements of the game which lock the players into the non-cooperative outcome. Below is a typical example of a Prisoners Dilemma game.

		Player 2: *Cooperate*	Player 2: *Not Cooperate*
Player 1	*Cooperate*	10, 10	0, 15
	Not Cooperate	15, 0	5, 5

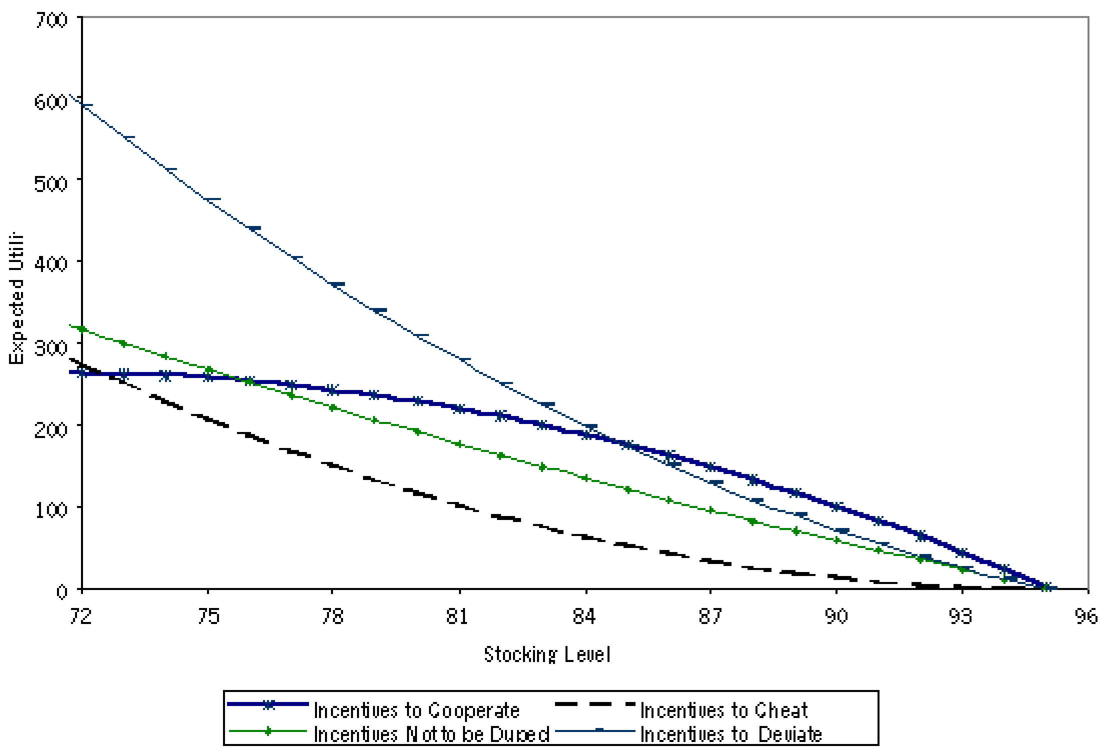
Consider player 1. In the first instance, he must choose the optimal decision to make given that player 2 cooperates. Clearly his best response is to cheat, and to gain 15 instead of 10. We define incentives to cheat as the difference between the profits acquired by optimally deviating when the other player abides by a cooperatively-agreed upon stocking level minus the profits associated with cooperation, $\Pi_i^{NCnc} - \Pi_i^{Cc}$. Next, he decides what is the best response when player 2 does not cooperate, clearly this is to not cooperate as well. In this case, the player is choosing not to be cheated on, or not to be duped. Thus, we define the incentives not to be duped as being the difference between both playing non-cooperatively, and player 1 being duped while player 2 plays his optimal deviation strategy, $\Pi_i^{NCnc} - \Pi_i^{Cnc}$.

This is the "relentless" logic of the prisoner's dilemma; and, even though there is no dominant strategy in the non-cooperative game,[3] at each possible point of cooperation, there are incentives to cheat and incentives to deviate, as well as the incentives to cooperate.

3. The non-cooperative game we have represented above is not a Prisoner's Dilemma, since there is not a dominant strategy (hence the reaction curves are curves and not a point, cf. Dasgupta and Heal (1979).

Hereafter, we refer to the combination of incentives to cheat and incentives not to be duped as incentives to deviate. In figure 4, we plot all four incentives as a function of stock levels. As we can see, again moving right to left, the gains from cooperation are increasing at a decreasing rate, but incentives to cheat and to not be duped are increasing at an increasing rate. In figure 4, the Nash non-cooperative outcome is to stock 96 animals apiece; whereas the joint-maximization solution is to stock 72 animals apiece. At the stock level of 96, all incentives are zero - if the group agrees to allow each to stock 96, then there are no gains to this agreement vis-à-vis the situation where none cooperated, and clearly incentives to deviate and not be duped our zero, which is why this level is the solution to the non-cooperative game.

Figure 4: Incentives to Cooperate and Deviate at Different Stock Levels



Next, consider the incentives for the group to stock 93 animals each. At this point gains to cooperating are quite large at the margin, whereas incentives to deviate are quite low. Now, consider a stock level of 73, just one above the joint-maximization solution. Here the gains from cooperating are very small, in fact quite close to zero. But marginal incentives to deviate are at their highest. We hypothesize that cost of monitoring and enforcing agreements is a function of the incentives to deviate. The following equation gives the maximization problem for the group:

$$\max W = \sum_i \left[\pi_i^{Cr} - \pi_i^{NCnc} \right] - g\left(\gamma^{Cheat} \sum_i I_i^{Cheat} - \gamma^{NotDuped} \sum_i I_i^{NotDupedt}, Z^C \right)$$

Cooperation costs are a function of both incentives to deviate as well as variables that may shift the cost function, Z^C . These variables may be thought of as representing extra-economic characteristics of the community that enable group members to achieve any level of cooperation at lower cost. We also note that though we have a general functional form for cooperation costs, this form must preserve the shape of the incentives, so that costs are increasing at an increasing rate as we move toward the joint-maximization solution. Given this specification, marginal benefits to cooperation will be decreasing as the number of animals is reduced whereas marginal costs are increasing; therefore, there will be some level that equates marginal costs and marginal benefits. In the absence of variables that may shift the cost function (i.e. the stock of socio-cultural capital), the solution to this equation will always lead to a group-determined and enforced stock level which lies between the joint-maximization and non-cooperative solutions - that is to say, we will observe a situation that appears to be partial cooperation.

Next, we consider that the group has reached some level of cooperation, given the associated incentives to deviate. What will happen for a given change in parameters? For all parameter perturbations, both the incentives to cooperate and the incentives to deviate will move in the same direction[4]. For instance, if the price of livestock output increases—-so that the livestock activity becomes more profitable — then the gains to cooperation will increase, but so will the incentives to deviate. Thus, we are concerned with relative changes. In Figures 5–8, we have plotted the change in expected utility from cooperating as well as the change in incentives to deviate. What is clear from all of the graphs is that, when groups are cooperating at relatively high levels *ex-ante*, then incentives to deviate will increase more rapidly than incentives to cooperate. Now facing higher relative incentives to deviate, it is likely that the group will not be able to enforce the same level of cooperation, and overstocking is likely to increase relative to the *ex-ante* situation. On the other hand, if there were very little cooperation *ex-ante*, then incentives to cooperate will increase more rapidly and may lead to less overstocking *ex-post*.

4. Note that this assertion relies on the assumption that all of the comparative static results of the previous section hold as stated above.

Figure 5: Change in Expected Utility; a = 82 to a 90

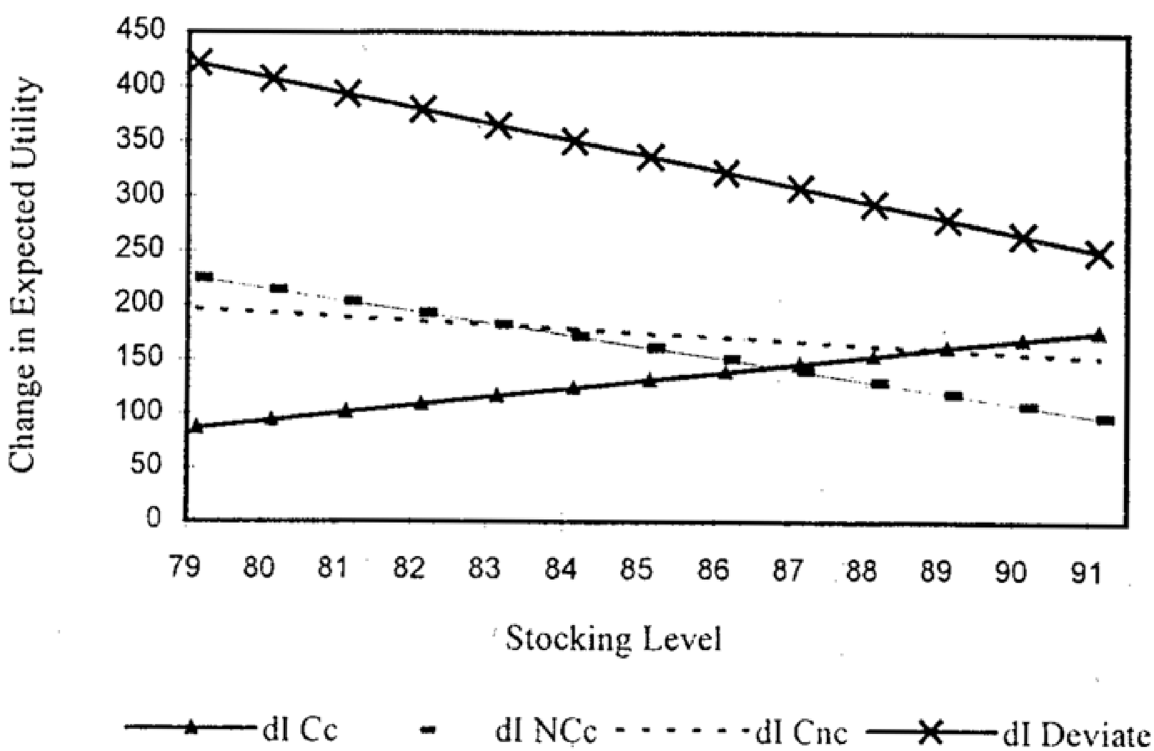


Figure 6: Change in Expected Utility; from c = 13 to c = 10

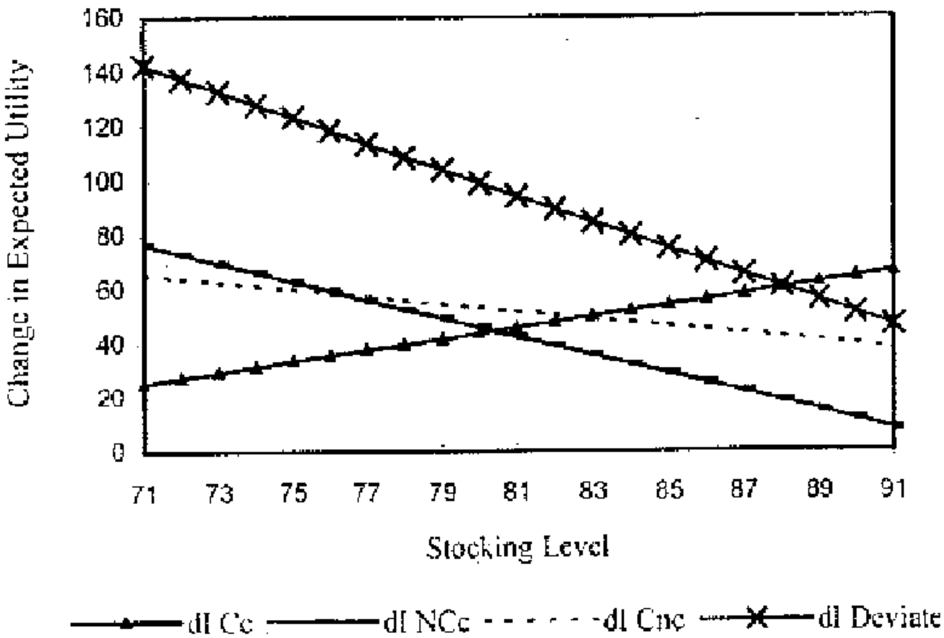


Figure 7: Change in Expected Utility; ß = 0.33 to ß = 0.25

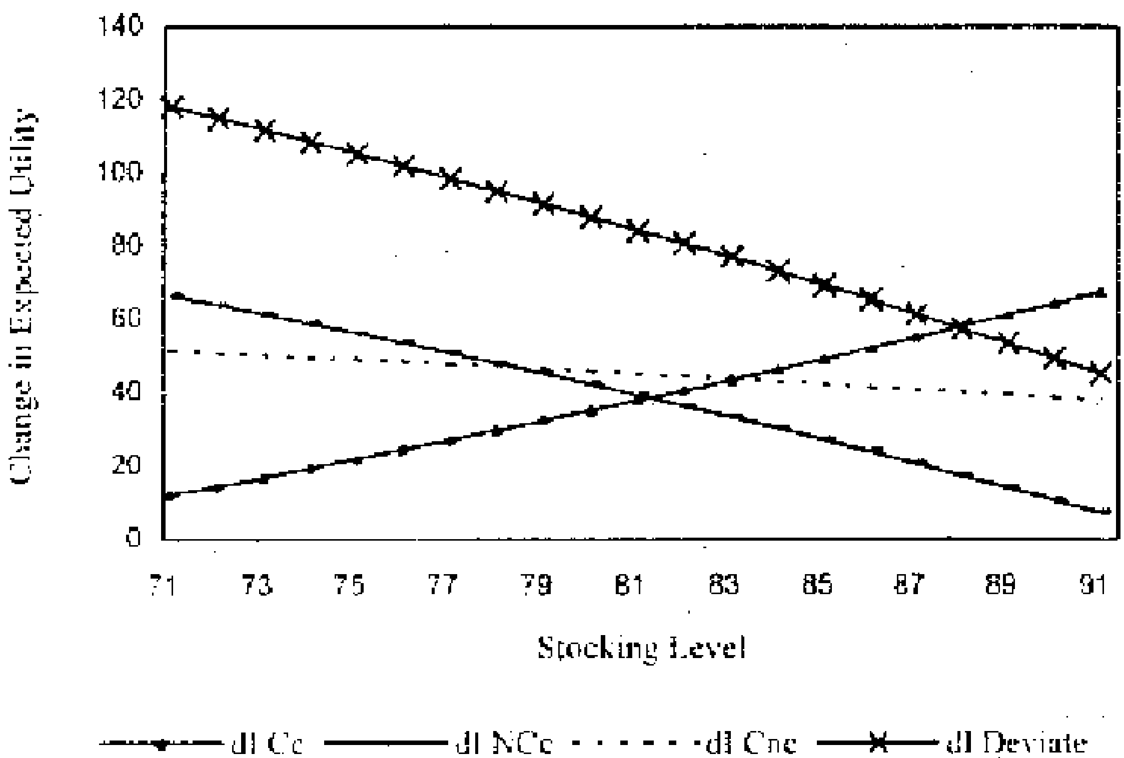
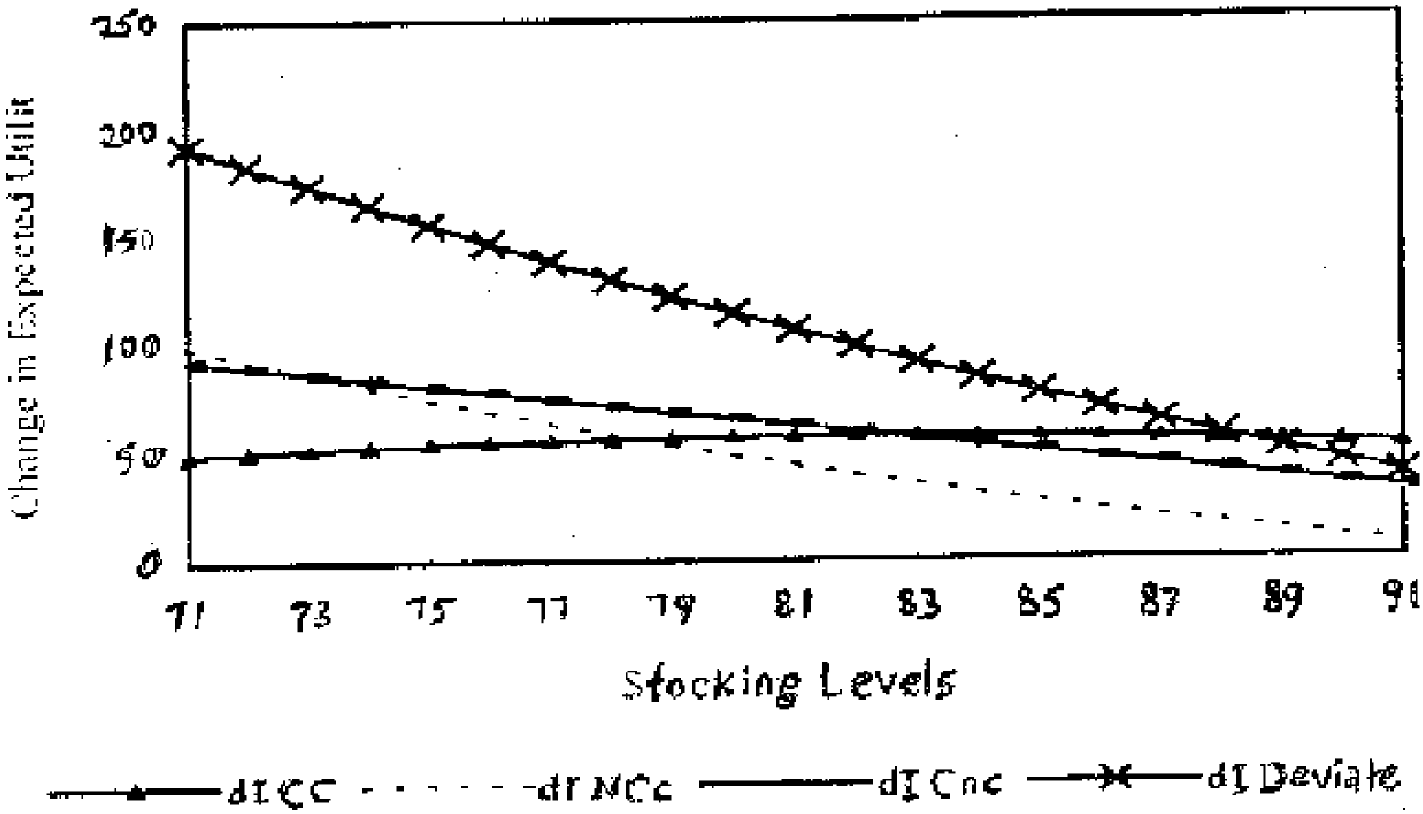


Figure 8: Change in Expected Utility; Φ = 0.0003 to Φ = 0.0001



Because of shift variables, we do not have any theoretical reason to know where any particular group will be *ex ante*. For a concrete example however, suppose that there are no shift variables, so that unit costs of enforcing agreements are equal to the sum of the incentives to deviate. In Figures 9 & 10, we have plotted the marginal incentive curves, in Figure 9, where the coefficient of absolute risk aversion is equal to .0001, and in Figure 10, where it is equal to .0005. In this case, an increase in the coefficient of risk aversion shifts all marginal incentives down; both the optimal number of animals to stock in the non-cooperative game as well as under joint maximization decrease. However, the difference between the optimal amount of animals to stock under the costless, joint maximization solution and the costly group-cooperation solution decreases with increases in risk aversion, and the rate of overexploitation goes down as well — from 33.33% to 32.97%. Thus, the more risk-averse are group members, the lower will be overgrazing.

Figure 9: Marginal Incentives, Coefficient of Absolute Risk Aversion = .0005

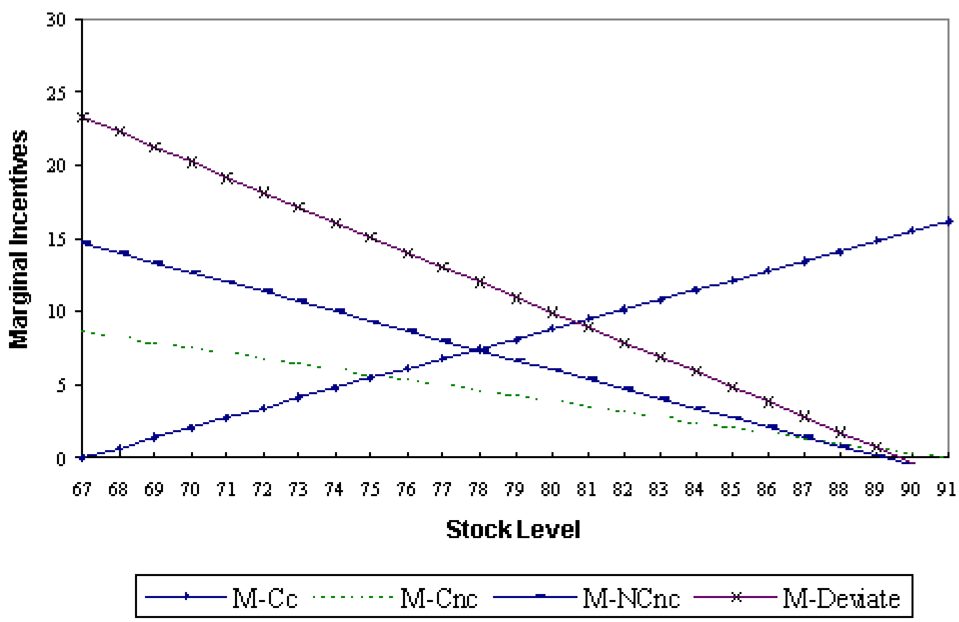
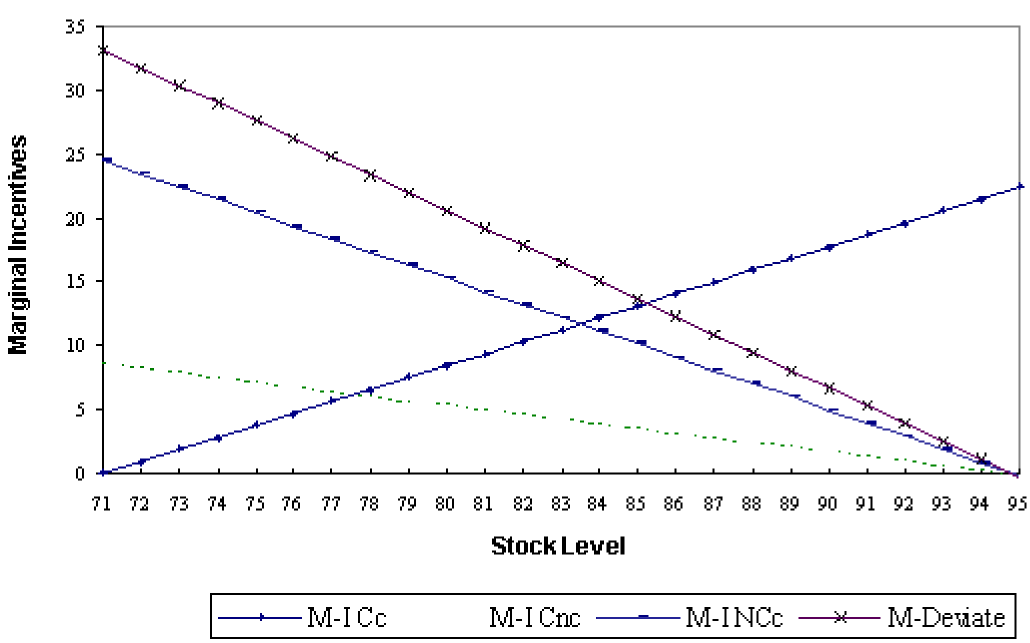


Figure 10: Marginal Incentives, Coefficient of Absolute Risk Aversion = .0001



To summarize, it is clear that both gains from cooperation as well as gains from optimally deviating increase with changes in all parameters that positively affect expected utility. And, by examining the relative changes in incentives, we note that the overall effect on cooperation will be ambiguous. Nonetheless, higher levels of cooperation should become more difficult to sustain, whereas low levels should become easier to sustain, and the ultimate response will depend on the shift parameters.

3.3 Effects of heterogeneity on incentives

Next, we consider the effect of heterogeneity among players, in terms of either marginal costs or in terms of risk aversion. In the case where players are homogeneous, gains from cooperation are positive for both players over the entire range from the non-cooperation to the joint maximization outcomes, where rights are allocated equally among players. Let us reiterate that to get a unique solution for both players under the joint maximization problem, we required the additional assumption of how total stock levels will be split among group members. In the case of homogeneity, equal allocation of rights seems a very plausible assumption. In the case of heterogeneity, however, such an assumption becomes more difficult to justify. For instance, in the case of different (linear) marginal costs, total expected utility would be maximized by allocating all rights to the low cost producer. Obviously, in the absence of side-payments, such an allocation would not be supported by the high-cost producer.

Typical reaction functions for two players with the same levels of risk aversion are illustrated in figure 11. The iso-profit curves are drawn for each player corresponding to the profit attained at the non-cooperative outcome. The area bounded by the two iso-profit curves represents pareto-improving allocations of stocking rights across individual producers, we will hereafter refer to this area as the scope for cooperation.

Figure 11: Reaction Functions and Iso-Utility; Homogeneous Herders

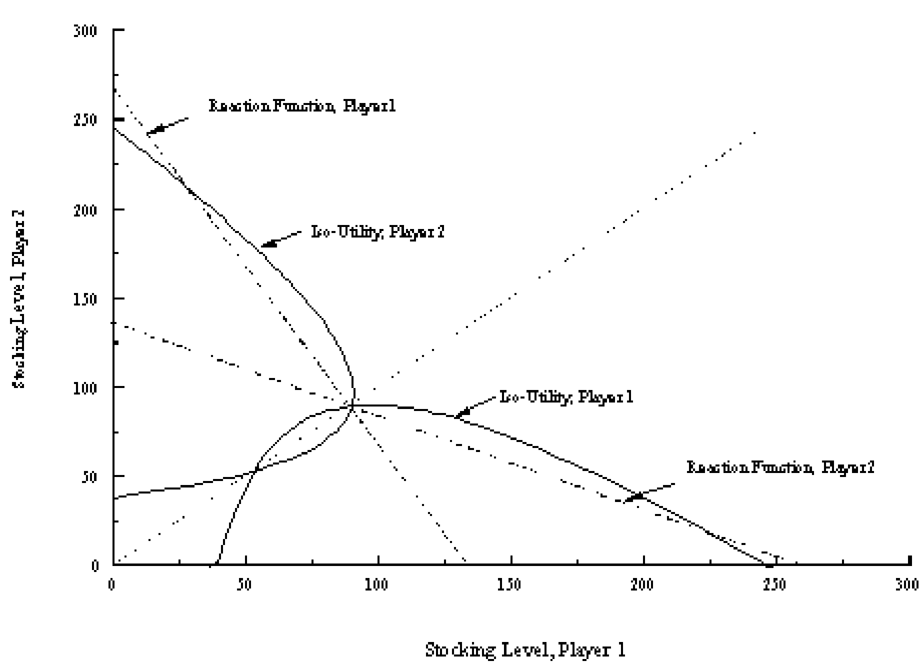
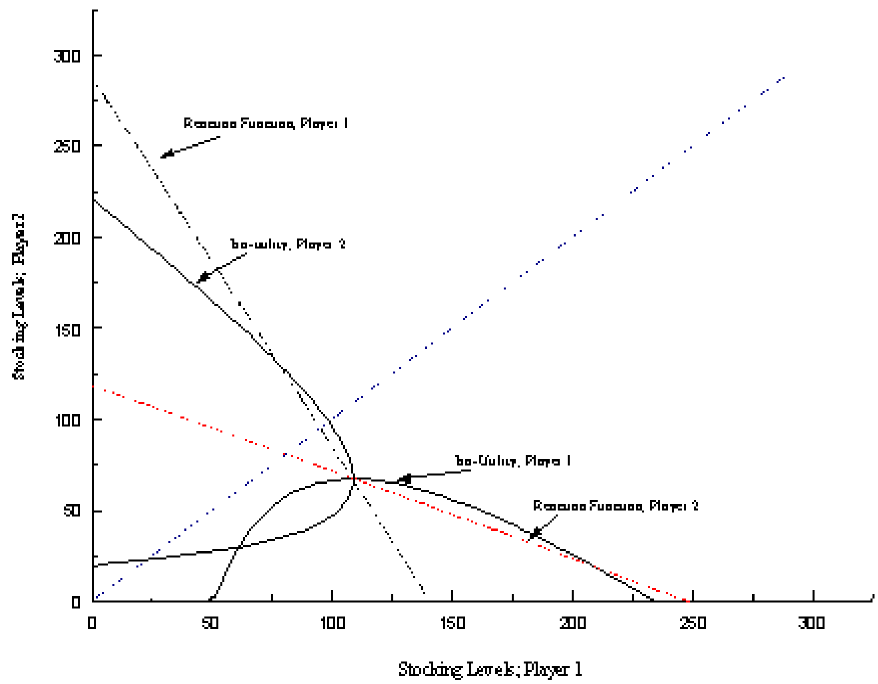


Figure 12 shows the same graph, except here, player 1 has a lower coefficient of absolute risk aversion. Note that in this case, there is no allocation of stock levels which falls on the 45 degree line; that is to say, an equal allocation of rights will not be supported by the low-cost producer, because profits for this herder are greater at the non-cooperative solution than for any allocation that falls on the 45 degree line. We are concerned with equal allocations belonging to the set of pareto improving allocations, because much of the empirical literature supports the notion that under most circumstances where there are use rules, these rules apply equally to all members (Johnson & Libecap, 1982; Ostrom, 1990; McCarthy, 1996). In the example given above, the coefficient of absolute risk aversion for player 2 has to be 5.5 times greater than that for player 1 for there to be no scope for cooperation, if stocking rights are allocated equally. It is worth noting that if player's wealth levels were different because of other income/assets (in addition to the differential income arising from livestock activity), then the coefficient of relative risk aversion (wealth times the coefficient of absolute risk aversion), would only need to be approximately 3.6 times greater. In the case of marginal costs, the difference between the two players need be much smaller in order for there to be no scope for cooperation; player 2 need only have costs approximately 41% greater. It is worth noting that all of these figures are based on a single set of parameter values, and though the direction of the responses are thus far invariant to parameter changes, the weight of such responses differs more significantly. Thus, obtaining actual parameter values would be of critical importance in analyzing policies in any particular area.

Figure 12: Reaction Functions and Iso-Utility; Heterogeneous Herders



3.4 Summary

In this section, we have developed a model that incorporates incentives to deviate from agreements into the cost function for a group-maximization problem over the use of a common rangeland. In section 3.2, where herders are homogeneous, we have shown that: 1) given incentives to deviate from agreements, optimal stock levels for the group are likely to lie between the non-cooperative and the costless, joint-maximization outcomes, and 2) where community-level shift variables are zero, significant reductions in risk increase overstocking, but only slightly. One of the more important testable hypotheses of the model is that overstocking itself should respond only very slightly to large changes in exogenous parameters as long as the group can cooperate, because of the offsetting effects of incentives to cooperate and incentives to deviate. In section 3.3, however, we see that we may observe large and discrete jumps to non-cooperation when a group solution is no longer feasible because of increased heterogeneity among herders. Thus, where only a fraction of herders gains access to outside income sources, or for any reason becomes less risk-averse or more efficient producers, we may observe discrete breakdowns in cooperation.

4 Discussion and policy implications

Essentially, the above analyses attempt to answer two distinct questions: 1) What happens to the non-cooperative game when risk is introduced, and how do results differ both vis-à-vis the riskless situation, as well as vis-à-vis the joint-maximization, or perfect cooperation, solution, and 2) If the group does attempt to cooperate, how does risk affect the different incentives to engage in cooperation, and how do differences in risk preferences affect the range of possible levels at which the group may decide to cooperate. The results suggest caution regarding the possible effects of risk reduction. Decreased risk may in fact result in lower incomes in the case of non-cooperation, and if producer's are differentiated either in terms of marginal costs or risk preferences themselves, then decreased risk will widen the distribution of livestock assets. In fact, any change in exogenous parameters that positively affect profitability will lead to a increase in this distribution, given some initial degree of heterogeneity.

Furthermore, if there were some type of cooperative arrangement in place before a decrease in income variability (risk), then cooperation will likely become more difficult to sustain at high levels of cooperation, and easier to sustain at lower levels of cooperation. There is no theoretical basis to assume, *a priori*, the functional form of these costs, and thus the comparative statics are indeterminate. However, if the cost function is a linear transformation of the sum of incentives to deviate, then a decrease in risk will lead to slightly lower levels of cooperation in terms of overstocking, as illustrated in Fig. 9 & 10.[5] Unlike a decrease in risk, however, an increase in producer prices, an increase in pasture productivity, and a decrease in production costs will all lead to slightly higher levels of cooperation. These latter results run counter to the commonly held - though not universal — belief that increases in parameters which positively increase profitability will lead to a lower levels of cooperation. Nonetheless, given the parameter values chosen (and over a wide range of parameter values), we observe only very small changes in the level of overgrazing due to most parameter changes, and this is because both incentives to cooperate as well as incentives to deviate move in the same direction. Thus, we hypothesize that communities which can cooperate, will not be adversely affected by policies which decrease risk or increase the profitability of livestock production.

5. Though note that the increase in overstocking is very slight compared to the large decrease in the coefficient of absolute risk aversion.

However, the models developed above can explain discrete jumps to non-cooperation for increases in profitability. As just noted, as long as we have an interior solution for the group maximization problem, increases in profitability will lead to greater gains from cooperation. However, if there are initial differences between the herders, then these will be exacerbated by increases in profitability. At some point, the differences may become sufficiently large as to cause a discrete jump to non-cooperation. Alternatively, consider that the productivity of the range is decreasing each year. As the resource degrades, differences between herders will diminish, and at some point, we may observe a discrete jump to cooperation. Finally, if non-economic variables that shift costs of cooperation change, we may also see a discrete jump from non-cooperation to group cooperation, or vice versa. Overall, the effect of increased profitability is ambiguous and depends on the degree of heterogeneity among herders, so that it is necessary to know how heterogeneous the community in question is, as well as the strength of the socio-cultural shift variables, before a prediction can be made about changes in cooperation for changes in exogenous variables.

These results indicate that precaution should be taken when undertaking development projects and policies that either alter the riskiness of livestock production itself, or of any exogenous parameters that improve profitability directly, since resulting outcomes may not be those desired — either decreased incomes and increased overstocking in the case of non-cooperation, or, a discrete jump to non-cooperation from a cooperative starting point. The analysis also points to the problem of using income as an indicator of well-being when livestock production is risky; overall utility increases with decreases in output variability, but income may in fact decline.

Finally, we can combine the results of this analysis with those analyses that examine the benefits of spatial mobility in terms of risk reduction (Van den Brink *et.al*., 1995; Wilson & Thomspon, 1993). Clearly, for most of the world's livestock owning population, access to common, or even open-access, pastures is of utmost importance in reducing the riskiness associated with climatic variability. Access to land, then, serves two very important functions — it is the source of an essential input, forage, and it reduces risk. In fact, spatial variability seems to be the single most important determinant of the resilience of common property grazing lands. Nonetheless, when the commons are not well-managed, there will be a trade-off between leaving lands in common versus privatization; namely, there will be increases in herder welfare due to a larger amount of land over which to spread the riskiness in production, but profits will be lower as stock levels per unit area are higher due to reduced riskiness. We hypothesize, then, that more land will be appropriated privately (or by ever smaller sub-groups), the lower is the ability to cooperate. Adding the results of the above models to the Van den Brink *et.al*. work — which considers that use rates are socially optimal and thus problems of non-cooperation and overgrazing are abstracted from — allows us to better identify factors associated with cooperation and hence identify policies that will increase the welfare of herders and their ability to harness benefits both cooperation, as well as to more accurately identify areas of potential conflict and policy measures needed to resolve conflict.

5 Future research

Dynamic considerations are absent from the model; and though we believe many of the hypotheses from the model will remain intact, a rigorous dynamic framework should be developed, perhaps with the express intent of capturing cyclical behaviour. The exogenous "shift" variables in the model of incentives need to be elaborated, and the added complexity of multiple users (with multiple interactions in other spheres) also needs to be addressed in a more systematic fashion. Finally, a simulation model should be developed to formally incorporate not only the spatial variability argument proposed by Van den Brink *et.al,* but also to capture the multiple co-variate risks and crop-livestock interactions faced by agro-pastoralists.

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Appendix 1: Proof that the stocking level under non-cooperation is greater than the stocking level under joint-maximization

In the following, we let $R = \sigma_{\theta}^{2}\phi_{A}$.

The first-order conditions for the n-player game and joint maximization solutions are as follows:

Joint-Maximization:

$$NP_I[f + LNf' - R_i Lf[NP_I(f + Lf')]] \quad - \quad N*c = 0$$

$$[f + NLf' - R_i Lf[NP_I(f + Lf')]] = \frac{c}{P_I}$$

[1]

Non-Cooperative Game:

$$P_I[f + L_i f' - R_i L_i f[P_I(f + L_i f')]] - c = 0$$

$$[f + L_i f' - R_i L_i f[P_I(f + L_i f')]] = \frac{c}{P_I}$$

[2]

In equilibrium, both first order conditions must be equal to $\frac{c}{P_I}$. Thus by establishing the sign of equation [3] below, we can determine under what conditions the stocking rate under non-cooperation is greater than under joint maximization.

$$[f + Lf']*[1 - RLf] \lessgtr [f + Lf' + (N-1)Lf']*[1 - RLf - (N-1)LRf] \qquad [3]$$

We immediately note that:

$$[f + Lf'] > [f + Lf' + (N-1)Lf']$$

and

$$[1 - RLf] > [1 - RLf - (N-1)LRf]$$

The left-hand side of the equation (non-cooperation) is greater than the right-hand side (joint maximization) at the same stocking rate, therefore, in equilibrium, the stock level must be greater under non-cooperation than under joint-maximization.

Similarly, we can examine the first order conditions for the risky, non-cooperative case versus the riskless joint-maximization case, the left-hand side of the equation below is the first-order condition for the risky, non-cooperative case, the right-hand side is the first-order condition for the riskless, joint-maximization case.

$$[f + Lf']*[1 - RLf] \lessgtr [f + NLf']$$

$$[f + Lf' - Rlf(f + Lf')] < [f + NLf']$$

$$[f - Rlf(f + Lf')] < [f + (N-1)Lf']$$

Since $(N-1)Lf' < 0$, the above expression holds true whenever $Rlf(f + Lf') > |(N-1)Lf'|$, or whenever the cost of risk (either in terms of high output variance or high coefficient of absolute risk aversion) is sufficiently high.

Appendix 2: Comparative statics matrix for the 2-player non-cooperative game with multiplicative risk in production

First, as described in Section II, we assume that, in equilibrium, $\frac{\partial f}{\partial L_1} = \frac{\partial f}{\partial L_2}$, $\frac{\partial^2 f}{\partial L_1^2} = \frac{\partial^2 f}{\partial L_2^2}$, and $\frac{\partial^2 f}{\partial L_1 \partial L_2} = \frac{\partial^2 f}{\partial L_2 \partial L_1}$. Essentially, this assumes that the "inputs", i.e. cattle, are equally productive across individuals (i.e. the conversion of forage to meat, milk, or draft power is the same across the TYPES of animals held across individuals).

In the derivation of the comparative statics below, we allow individuals to differ in terms of marginal costs and in terms of risk preferences. Though output variance is assumed to be the same for both individuals, the coefficient of absolute risk aversion may differ. Following Appendix 1, we let $R_i = \sigma_\theta^2 \phi_{Ai}$.

The original maximization problem for i=1,2 is as follows:

$$\max_{L_i} \; \mathrm{E}\left(U\left(\pi_i^{CN}\right)\right) = \left[P_I * L_i * f\left(L_i + L_j;\, \alpha, \beta\right) - cL_i - \tfrac{1}{2} R_i \left(P_I * L_i * f\left(L_i + L_j;\, \alpha, \beta\right)\right)^2\right]$$

with FOC:

$$P_I\left[f + L_i f' - R_i L_i f\left[P_I\left(f + L_i f'\right)\right]\right] - c = 0$$

where $f = f(L_i + L_j;\, \alpha, \beta)$.

$$\frac{\partial^2 EU_i^{CN}}{\partial L_i^2} = \left[P_I\left(2f' + L_i f''\right)\right] * \left[1 - P_I R_i L_i f\right] - R_i\left[P_I\left(f + L_i f'\right)\right]^2$$

Remembering that $f' < 0$ and $f'' < 0$, we see that the first term is clearly negative, since the first component is negative, and the second component is positive from the first-order conditions. The second term is positive, so that the entire term is negative, as required.

$$\frac{\partial^2 EU_i^{CN}}{\partial L_i \partial L_j} = \left[P_I * \left(f' + L_i f''\right)\right] * \left[1 - P_I R_i L_i f\right] - P_I R_i L_i f' \left[P_I\left(f + L_i f'\right)\right]$$

This sign of this term is indeterminate. The first term is clearly negative, but the second term is also negative. Note that, in the absence of risk, the term would be negative.

The Jacobian is thus:

$$\begin{vmatrix} \frac{\partial^2 EU_i^{CN}}{\partial L_i^{\,2}} & \frac{\partial^2 EU_i^{CN}}{\partial L_i \partial L_j} \\ \frac{\partial^2 EU_j^{CN}}{\partial L_j \partial L_i} & \frac{\partial^2 EU_j^{CN}}{\partial L_j^{\,2}} \end{vmatrix} \lessgtr 0$$

To establish the sign of | J |, we initially assume that player's are homogeneous so that $L_1 = L_2$ in equilibrium. Given these assumptions, we note that $\frac{\partial^2 EU_i^{CN}}{\partial L_i^{\,2}} = \frac{\partial^2 EU_j^{CN}}{\partial L_j^{\,2}}$ and also that $\frac{\partial^2 EU_i^{CN}}{\partial L_i \partial L_j} = \frac{\partial^2 EU_j^{CN}}{\partial L_j \partial L_i}$. Solving the determinant, then is equivalent to solving $A^2 - C^2$, where $A^2 = \left(\frac{\partial^2 EU_i^{CN}}{\partial L^2}\right)^2$ and $C^2 = \left(\frac{\partial^2 EU_i^{CN}}{\partial L_i \partial L_j}\right)^2$.

Next, $A^2 - C^2 = (A - C)(A + C)$, so to establish the sign of the Jacobian, we need to establish the signs of *(A-C)* and *(A+C)*:

(A-C) =

$$[P_L(2f' + Lf'')]*[1 - P_L R_i L f] - R_i[P_L(f + Lf')]*[P_L(f + Lf')]$$
$$-\{[P_L*(f' + Lf'')]*[1 - P_L R_i L f] + P_L R_i L f'*[P_L(f + Lf')]\}$$

Which equals:

$$\left\{[1 - P_L R_i L f]* P_L f' - P_L R_i f*[P_L(f + Lf')]\right\} < 0.$$

The first term is clearly negative, whereas the second is positive (the bracketed term is positive by the first-order conditions), thus the entire term is negative.

(A+C) =

$$[P_L(2f' + Lf'')]*[1 - P_L R_i L f] - R_i[P_L(f + Lf')]*[P_L(f + Lf')]$$
$$+\{[P_L*(f' + Lf'')]*[1 - P_L R_i L f] - P_L R_i L f'*[P_L(f + Lf')]\}$$

Which equals:

$$\left\{[1 - P_L R_i L f]* P_L(3f' + 2Lf'') - P_L R_i[P_L*(f + L_i f')(f + 2Lf')]\right\} \qquad [1]$$

In this form, the sign of the term is indeterminate. Though the first term is negative, the second term may also be negative. In fact, in the absence of costs of production, the second term would be negative, as $(f + 2Lf')$ would equal zero at the joint maximization stock level, and would be negative at the non-cooperative equilibrium.

In what follows, we show that whenever the first order condition, $(1 - P_L R_i L f) > 0$, is met, then Equation [1] must be negative.

Expanding Equation [1] gives the following:

$$P_1\left\{[1-P_1R_iLf]2Lf''+3f'-3P_1R_iLff'-R_iP_1\left[f^2+3Lff'+2L^2f'^2\right]\right\}$$

$$\Rightarrow\quad P_1\left\{[1-P_1R_iLf]2Lf''+3f'-P_1R_i\left[f^2+6Lff'+2L^2f'^2\right]\right\} \qquad [2]$$

A sufficient condition for Equation [2] to be negative, is that $3f'-P_1R_i\left[f^2+6Lff'+2L^2f'^2\right]<0$.

Since both terms are negative, this condition will hold when:

$|3f'|>\left|P_1R_i\left[f^2+6Lff'+2L^2f'^2\right]\right|$, or alternatively, when:

$$R<\frac{3f'}{P_I\left[f^2+6Lff'+2L^2f'^2\right]}.$$

However, as shown below, this condition will always hold whenever the first-order condition holds.

$$\frac{1}{P_ILf}<\frac{3f'}{P_I\left[f^2+6Lff'+2L^2f'^2\right]}\text{ when}$$

$$\frac{\left[f^2+6Lff'+2L^2f'^2\right]}{3Lff'}<1$$

$$\frac{\left[f^2+2L^2f'^2\right]}{3Lff'}+2<1$$

$$\left[f^2+2L^2f'^2\right]+3Lff'<0 \qquad [3]$$

Equation [3] will always hold whenever $(f+2Lf')$ is negative, the case in which we are interested. Rearranging the terms of equation [3] yields the following:

$$(f+Lf')(f+2Lf')<0.$$

Thus, we have just shown that $|A|=\left(\frac{\partial^2 EU_i^{CN}}{\partial L^2}\right)^2>|C|=\left(\frac{\partial^2 EU_i^{CN}}{\partial L_i\,\partial L_j}\right)^2$, and consequently, that the Jacobian is negative semi-definite.

Next, we would like to establish the sign of | J | when $L_1\neq L_2$, say, when $c_1\neq c_2$, or when $R_1\neq R_2$. In this case, we can let the following represent

| J |:

$$\begin{vmatrix} A & C \\ D & B \end{vmatrix} \lessgtr 0, \text{ or equivalently } AB - CD \lessgtr 0.$$

First note that if |A| > |C|, and |B| > |D|, then clearly AB > CD. But, we have just shown that |A| > |C|, and symmetrically, |B| > |D|. Therefore, the determinant is always positive. *QED*.

Appendix 3: Comparative statics

Differentiation with respect to each of the parameters give the following:

$$\frac{\partial \pi_i^{*NC}}{\partial L_i\,\partial \alpha} = \left[P_L\left(f_\alpha + L_i f_{i\alpha}\right)\right]\left[1 - P_L R_i L_i f\right] - P_L R_i L_i f_\alpha\left[P_L\left(f + L_i f_i\right)\right]$$

This term is positive, when R_i is not "too" high.

$$\frac{\partial \pi_i^{*NC}}{\partial L_i\,\partial \beta} = \left[P_L\left(f_\beta + L_i f_{i\beta}\right)\right]\left[1 - P_L R_i L_i f\right] - P_L R_i L_i f_\beta\left[P_L\left(f + L_i f_i\right)\right]$$

> 0, when R_i is not too high.

$$\frac{\partial \pi_i^{*NC}}{\partial L_i\,\partial P} = \left(f + L_i f_i\right)\left[1 - 2P_L R_i L_i f\right]$$

> 0, when R_i is not too high.

$$\frac{\partial \pi_i^{*NC}}{\partial L_i\,\partial c_i} = -1$$

$$\frac{\partial \pi_i^{*NC}}{\partial L_i\,\partial c_j} = 0$$

$$\frac{\partial \pi_i^{*NC}}{\partial L_i\,\partial R_i} = -P_L L_i f\left[P_L\left(f + L_i f_i\right)\right]$$

< 0

$$\frac{\partial \pi_i^{*NC}}{\partial L_i\,\partial R_j} = 0$$

And symmetrically for player j.

$$\frac{\partial L_i}{\partial \alpha} = \frac{-1}{|J|} * \frac{\partial \pi_i}{\partial L_i\,\partial \alpha}\left[\frac{\partial^2 \pi_j}{\partial L_j^{\,2}} - \frac{\partial \pi_i}{\partial L_i\,\partial L_j}\right]$$

> 0,

when $\frac{\partial \pi_i}{\partial L_i\,\partial \alpha}$ > 0 (players are not too risk averse), and when players are homogeneous.

$$\frac{\partial L_i}{\partial \alpha} = \frac{-1}{|J|} * \Big\{\Big[\overbrace{\left[P_L\left(f_\alpha + L_i f_{i\alpha}\right)\right]\left[1 - P_L R_i L_i f\right] - P_L R_i L_i f_\alpha\left[P_L\left(f + L_i f_i\right)\right]}^{A}\Big] * \frac{\partial^2 \pi_j}{\partial L_j^{\,2}}$$

$$-\overbrace{\left[P_L\left(f_\alpha + L_j f_{i\alpha}\right)\right]\left[1 - P_L R_j L_j f\right] - P_L R_j L_j f_\alpha\left[P_L\left(f + L_j f_j\right)\right]}^{B} * \frac{\partial^2 \pi_i}{\partial L_i\,\partial L_j}\Big\}$$

As we will see in many of the following comparative static results, the signing of this term depends not only on whether or not both players are not "too" risk averse, but also on the absolute difference between players with respect to stocking levels, Li, and coefficient of absolute risk aversion, Ri. From Appendix 3, we know that $\frac{\partial^2 \pi_j}{\partial L_j^2} - \frac{\partial^2 \pi_i}{\partial L_i \partial L_j}$ <0. If $A \geq B$ in the equation above, then the result will certainly be positive. $A \geq B$ when $R_i L_i \leq R_j L_j$. For $R_i L_i >> R_j L_j$, however, it is possible for this expression to be negative. That is to say, if the i-th player is sufficiently differentiated in terms of costs or risk preferences, then it is possible for the sign of this term to be negative.

$$\frac{dL_i}{d\beta} = \frac{-1}{|J|} * \Big\{ \Big[\big[P_L \big(f_\beta + L_i f_{i\beta} \big) \big] \big[1 - P_L R_i L_i f \big] - P_L R_i L_i f_\beta \big[P_L \big(f + L_i f_i \big) \big] \Big] * \frac{\partial^2 \pi_j}{\partial L_j^2}$$

$$- \Big[P_L \big(f_\beta + L_j f_{i\beta} \big) \Big] \big[1 - P_L R_j L_j f \big] - P_L R_j L_j f_\beta \Big[P_L \big(f + L_j f_j \big) \Big] * \frac{\partial^2 \pi_i}{\partial L_i \partial L_j} \Big\}$$

This term is also indeterminate, but in this case, the sign will be negative, as long as players' are not too risk averse, nor too differentiated in terms of livestock holdings or risk preferences. And, again, if player's are differentiated, then the less risk and/or lower cost individuals will increasing stocking rates by more.

$$\frac{dL_i}{dP_L} = \frac{-1}{|J|} * \left[\big[f + L_i f_i \big] \big[1 - 2 P_L R_i L_i f \big] \frac{\partial^2 \pi_j}{\partial L_j^2} - \big[f + L_j f_j \big] \big[1 - 2 P_L R_j L_j f \big] \frac{\partial^2 \pi_i}{\partial L_i \partial L_j} \right] \gtrless 0$$

As with the productivity parameters, changes in output price will have an ambiguous effect on stock levels, though at least one player will unambiguously increase her stock levels. And, again, starting from an initial point of inequality in risk preferences or stockholdings, the distribution among players will widen in response to changes in output price.

$$\frac{dL_i}{dc_i} = \frac{-1}{|J|} * -1 * \frac{\partial^2 \pi_j}{\partial L_j^2} < 0$$

$$\frac{dL_i}{dc_j} = \frac{-1}{|J|} * 1 * \frac{\partial^2 \pi_i}{\partial L_i \partial L_j} > 0$$

$$\frac{dL_i}{dc} = \frac{-1}{|J|} * \left[-1 * \frac{\partial^2 \pi_j}{\partial L_j^2} - \frac{\partial^2 \pi_i}{\partial L_i \partial L_j} \right] < 0$$

$$\frac{dL_i}{dR_i} = \frac{-1}{|J|} * - \Big[P_L L_i f \big[P_L \big(f + L_i f_i \big) \big] \Big] * \frac{\partial^2 \pi_j}{\partial L_j^2} < 0$$

$$\frac{dL_i}{dR_j} = \frac{-1}{|J|} * \Big[P_L L_j f \big[P_L \big(f + L_j f_j \big) \big] \Big] * \frac{\partial^2 \pi_i}{\partial L_i \partial L_j} > 0$$

$$\frac{\partial L_i}{\partial R} = \frac{-1}{|J|} * \left[- P_L L_i f \left[P_L \left(f + L_i f_i \right) \right] * \left[\frac{\partial^2 \pi_j}{\partial L_j^{\,2}} - \frac{\partial^2 \pi_i}{\partial L_i \partial L_j} \right] \right] < 0$$