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Sensitivity and uncertainty propagation in coupled models for assessing smallholder farmer food security in the Olifants River Basin, South Africa

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ABSTRACT

Using family balance (i.e., combined net farm and non-farm incomes less family expenses), an output from an integrated model, which couples water resource, agronomic and socio-economic models, its sensitivity and uncertainty are evaluated for five smallholder farming groups (A–E) in the Olifants Basin. The crop management practiced included conventional rainfed, untied ridges, planting basins and supplemental irrigation. Scatter plots inferred the most sensitive variables affecting family balance, while the Monte Carlo method, using random sampling, was used to propagate the uncertainty in the model inputs to produce family balance probability distributions. A non-linear correlation between in-season rainfall and family balance arises from several factors that affect crop yield, indicating the complexity of farm family finance resource-base in relation to climate, crop management practices and environmental resources of soil and water. Stronger relationships between family balance and evapotranspiration than with in-season rainfall were obtained. Sensitivity analysis results suggest more targeted investment effort in data monitoring of yield, in-season rainfall, supplemental irrigation and maize price to reduce family balance uncertainty that varied from 42% to 54% at 90% confidence level. While supplemental irrigation offers the most marginal increase in yields, its wide adoption is limited by availability of water and infrastructure cost.

1. Introduction

Smallholder farming systems, especially rainfed systems, continue to receive increasing attention due to their relatively low costs but high importance as a livelihood strategy for food security in arid and semi-arid areas of sub-Sahara Africa (CAWMA, 2007). However, these farming systems face major challenges of high variability of rainfall, low soil fertility and market perturbations, which are likely to be worsened by climate change (Ludi, 2009; Magombeyi and Taigbenu, 2008; Gilmour et al., 2005). These challenges require an integrated approach to management of soil, water, nutrients and market incentives to build resilience to shocks such as droughts, floods and market price perturbations.

In agricultural regions, future trajectories of food prices, food security, and crop land expansion are linked to future average crop yields, energy availability and population. Crop yield improvements can be achieved with some agricultural water management practices that involve rainwater harvesting, such as ridges, planting basins and supplemental irrigation from small storage systems in comparison to conventional rainfed practices. However, a fundamental constraint to the rapid assessment of the viability of these farming systems for food security is: which data is critical and requires more effort to monitor or to collect. Uncertainty in growing season weather, crop yield and market perturbations affect food security. Thus, model tools such as physically-based ones, which incorporate integrated approaches (Brown et al., 2005; Refsgaard et al., 2007; Mezic and Runolfsson, 2008), help to improve our knowledge of the viability of rural farming systems and consequently household food security under several shocks, such as rainfall variability, perturbations of market prices of basic agricultural commodities and inputs, and availability of market infrastructure and credit.

Depending on the field of study, uncertainty can be viewed as worst-case perturbation in the space of output measures (Helton et al., 2006) or departure of an observation from its true value (Shirmohammadi et al., 2006) or the degree of confidence in decision-making about possible outcomes from a model analysis

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(Refsgaard et al., 2007). The contributions of the uncertainty in individual model inputs to its predictions/outcomes give the sensitivity analysis (Helton et al., 2006; Xu and Gertner, 2007), which can be evaluated by local and global sensitivity analysis methods (Iooss and Ribatet, 2008; Saltelli et al., 2008).

In real life, errors and uncertainty in model outcomes never exist in an absolute sense due to model assumptions and the complexity of the system being modelled (Nilsen and Aven, 2002; Brown et al., 2005). An error, in any phase of modelling that is not due to lack of knowledge, is considered acceptable if the requirements of the analysis are met or the computational cost to correct it is prohibitive (Oberkampf et al., 2002). Uncertainty can either be bounded, in which possible outcomes are assumed known or unbounded, in which possible outcomes are not known (Oberkampf et al., 2002; Walker et al., 2003; Helton and Oberkampf, 2004; Mezic and Runolfsson, 2008).

According to van Asselt and Rotmans (2002), uncertainty treatment relevant for the decision-making process involves its identification, characterisation, communication and interpretation to interested parties. Some of the uncertainty assessment methods include data uncertainty engine (DUE), error propagation equations, multiple model simulation, numeral, unit, spread, assessment and pedigree (NUSAP), expert elicitation, stakeholder involvement (Vogel et al., 2007), inverse modelling (Refsgaard et al., 2006), scenario analysis (Brown et al., 2005) and Monte Carlo simulation with its improvements (Shirmohammadi et al., 2006; Storlie and Helton, 2008; Espinosa-Paredes et al., 2012a, b). Uncertainty application examples include missile flight (Oberkampf et al., 2002), nuclear diffusion (Espinosa-Paredes et al., 2012a, b), water policy and land use (Gilmour et al., 2005), wildlife conservation (Fieberg and Jenkins, 2005), climate change (van Asselt and Rotmans, 2002) and crop yield (Wang et al., 2005).

To our knowledge, a study of the sensitivity and uncertainty analyses of an integrated model of a community livelihood outcome, family balance (net farm income plus non-farm income less family expenses) impacted by crop management practices and market perturbations had not been undertaken, likely because of the challenges in coupling of biophysical and socio-economic models and often poorly understood error propagation in these models. The integrated models are prone to aggregated uncertainties as they try to capture an entire set of cause–effect relations involved in a specific problem (Walker et al., 2003). In this study, we applied an integrated model, Innovative Coupling of Hydrological and Socio-Economic Aspects (ICHSEA), developed by Magombeyi and Taigbenu (2011), that couples hydrology, agronomy and socio-economic models to predict the streamflow and livelihood impacts (food security and income) of crop yield, market perturbations and crop-water management practices. The family food security and income are directly related to family balance or savings (in monetary terms). The main objective of this paper is to illustrate the sensitivity and Monte Carlo uncertainty analyses (Espinosa-Paredes et al., 2012a, b; Schlüter and Rüger, 2007) of the ICHSEA output (family balance) to support decision-making on improved productivity of the rainfed maize crop to meet family food security, while satisfying downstream water requirements for both humans and the environment. This study was done in semi-arid B72A catchment of the Olifants Basin in South Africa.

2. Study site

The study area, with an estimated population of 56,000 people (Statistics South Africa, 2001) is located in B72A quaternary catchment (534 km^2) in Ga-Sekororo area in the Olifants River Basin of South Africa (Fig. 1). The mean annual rainfall and potential evapotranspiration rates are 603 mm and 1500 mm, respectively (Magombeyi and Taigbenu, 2008). The area experiences drought and dry spells every 2 years on the average, putting the majority of the resource-constrained smallholder farmers at high risk of food insecurity. The detailed social-economic description of the study area is reported in Magombeyi and Taigbenu (2008).

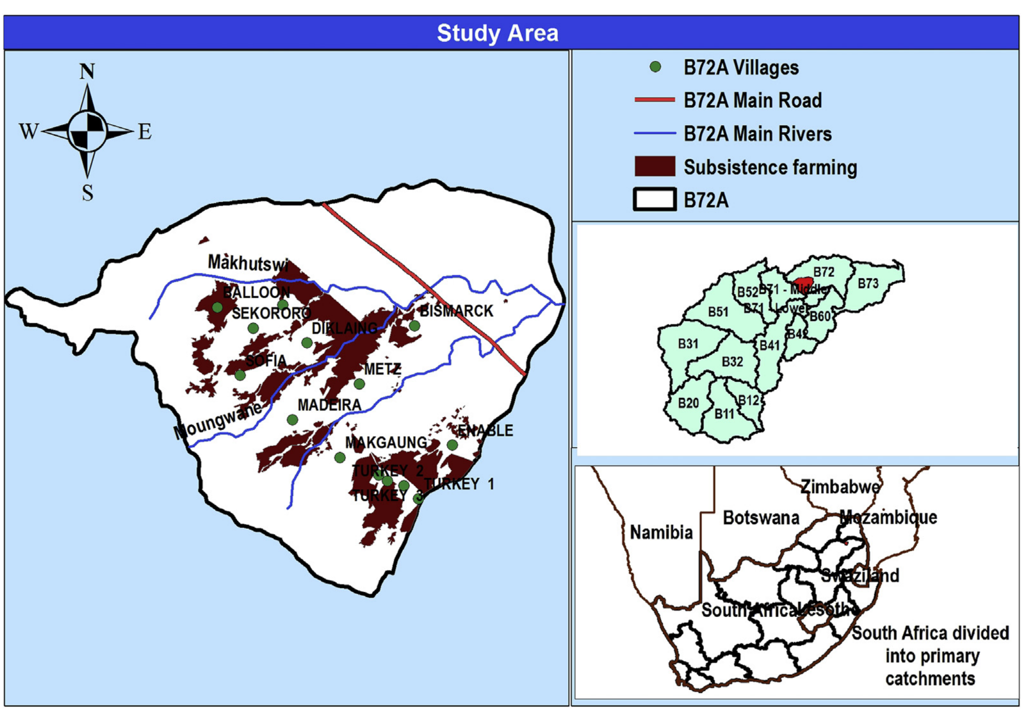

Fig. 1. Location of the B72A quaternary catchment with village names and streams in the Olifants River Basin of South Africa.

3. Methodology

The methodology uses an output, family balance from an integrated model, Innovative Coupling of Hydrological and Socio-Economic Aspects (ICHSEA) developed by Magombeyi and Taigbenu (2011). This integrated model is restricted to simulating farm production decisions that are seasonal and captures both farm and non-farm incomes and family expenses.

The methodology is based on the following four steps: (I) use of five identified farm types A–E in the study area by Magombeyi et al. (2012) and briefly described here – **Type A**: *Subsistence farmers with external jobs*; **Type B**: *Resource-constrained rainfed and irrigation farmers*; **Type C**: *Social grant supported rainfed farmers*; **Type D**: *Intensive and diversified irrigation farmers*; **Type E**: *Rich, salaried entrepreneurs – very extensive farmers*. The proportions of the population of the five farm types in the study area are 6% (A), 15% (B), 52% (C), 25% (D) and 2% (E). Although all the five farm types were studied, only results on types B and D are mostly reported, due to their huge resource variation and prevalence in the catchment. (II) An extended performance evaluation of ICHSEA outputs of grain yield, family balance and catchment stream outflows. This enabled identification of viable farm types that ensure family food security, income and savings. Family balance is expressed as

$$F = G - V - U + T - S \tag{1}$$

where F is the family balance; G is the gross farm income, V refers to the variable production costs, U is the fixed farm costs, T is the non-farm income, and S is the family expenses. The family balance depends on the crop yield, Y, the market price, P, and rainfall, R, while G varies linearly with Y, V reflects input costs, U is an accrued cost irrespective of production, and T and S generally behave inversely in relation to Y. With a poor harvest, typically from low rainfall, farmers engage more in non-farm income generating activities such as hawking, beer brewing and selling of craftwork to meet the shortfall in family food security, whereas in good harvest years they engage more in on-farm activities which include tillage, planting, nutrient application, weeding, harvesting and securing of fields from birds, wild and stray animals. This generally results in increased farm labour. A similar trend is also observed with family expenditures in relation to crop yield. High family expenditures attend years of poor harvest as substantial part of family budget is used to procure food, whereas in good harvest years most of the food and non-food needs are met from farm production and income from crop sales (Statistics South Africa, 2012). Thus we postulate that, because of the complex social, economic and agronomic practices of smallholder farmers in the catchment, T and S are related to the yield by the non-linear relationships $T = \alpha_1 Y^{\beta_1}$ and $S = \alpha_2 Y^{\beta_2}$ so that Equation (1) becomes

$$F = F(Y,\ R,\ P) = \lambda Y - V - U + \alpha_1 Y^{\beta_1} - \alpha_2 Y^{\beta_2} \tag{2}$$

where λ, α_1, β_1, α_2 and β_2 are constants. It should be pointed that due to limited resources and time (3 years) for field work in gathering socio-economic data, the modelling could not assess the relative contributions of each term to the family balance. (III) The integrated model, ICHSEA was validated against observed streamflows, maize crop yields and local socio-economic conditions with the support of inputs from local stakeholders – farmers, extension officers and others (Sojda, 2007). (IV) Sensitivity and uncertainty analyses on the family balance (outcome) and its predictors of rainfall, supplemental irrigation water, maize crop yield and market price were carried out for each of the five farm types.

3.1. Description of ICHSEA

The ICHSEA model (Magombeyi, 2010; Magombeyi and Taigbenu, 2011) innovatively couples SWAT (hydrology – Neitsch et al., 2001; Arnold et al., 1993), PARCHED-THIRST (crop growth – Young et al., 2002) and OLYMPE (socio-economics – Penot and Deheuvels, 2007). The ICHSEA interface was developed in Avenues script language in ArcView 3.3 to take advantage of the mapping capability of ArcView. Its modelling time-step is seasonal, but each model runs on a temporal resolution appropriate for the processes being modelled, ranging from daily to seasonal. The model simulation period was 20 years, from 2005 to 2024, and rainfall distribution used for the simulation reflected current trends in rainfall variability without superimposing climate change. Detailed descriptions of these individual models are presented in Magombeyi and Taigbenu (2011). The inputs of ICHSEA model include climatic data (rainfall, temperature, humidity, solar radiation and wind) at daily time-step, topography, land use, soil type, crop inputs, crop yield, farm family food and non-food expenditures, while the outputs are streamflows, sediment yield, and family balance. ICHSEA was validated using two data sources. Firstly, historic data were used to validate each model component prior to coupling by assessing the Nash–Sutcliffe efficiency coefficient of the observed data and the model results (Magombeyi and Taigbenu, 2011). Secondly, soft validation, using socio-economic data, was carried out for the integrated system through participatory interactions with farmers, extension officers, local agricultural managers and non-governmental organisation field officers. The use of soft validation constrained the several degrees of freedom inherent in the integrated model that are not captured when model validity is tested against a single or even multiple historical time series (Letcher et al., 2006; Sojda, 2007).

3.2. Assumptions

One of the assumptions made relates to the model structure, which is considered to closely reflect the current farming operations, and as such the uncertainty analysis only captures how input uncertainty propagates through the model. The hydrological model, which predicts the water resources, was considered as the driver of the integrated model, ICHSEA. In addition, prerequisites for any uncertainty and sensitivity analyses are the assumptions that statistical distributions for the input values are correct and that the model sufficiently captures the critical processes currently taking place in the farming and market systems (Loucks and Van Beek, 2005). However, these assumptions are rarely satisfied. Hence, a uniform distribution, according to widely accepted rule of thumb, was assumed for the in-season rainfall and grain yield inputs in the uncertainty analysis (Ju, 2009).

3.3. Sensitivity tests and uncertainty analysis

Scatter plots, which involved plotting of family balance (output from ICHSEA) against variables that affect it, one at a time, were used to evaluate the most sensitive variables. From derived relationships between family balance, available water (in-season rainfall, streamflow), maize price and crop yield, Monte Carlo method, using random sampling, was used to produce probability distributions of family balance predictions (Shirmohammadi et al., 2006; Refsgaard et al., 2007; Storlie and Helton, 2008). The error propagation equation for non-linear multiplication functions by Refsgaard et al. (2007) in Equation (3) was used to determine the family balance standard error, as the following conditions were met: (1) the crop yield and family balance were normally distributed; (2) the standard errors were relatively small and are less than 0.3 (family balance and in-season rainfall ranged from 0.001 to 0.2); and (3) the uncertainties had no significant covariance.

$$\frac{\sigma_z}{z} = \sqrt{\left(\frac{\sigma_x}{x}\right)^2 + \left(\frac{\sigma_y}{y}\right)^2 + \ldots} \tag{3}$$

where σ_z = standard deviation of predicted family balance, σ_x, σ_y, … are the standard deviation of the other inputs (in-season rainfall, streamflow, crop yield and market price variations), and z, x, y… are their mean values.

3.4. Scenarios analysed

Five crop management scenarios were analysed. The first is rainfed (serving as baseline reference for assessment of other scenarios) and the remaining four are improved techniques for upgrading rainfed agriculture in arid and semi-arid areas (CAWMA, 2007). The sixth scenario takes into account projected high maize price variations from NAMC (2008) of South Africa.

3.4.1. Scenario (i): current rainfed management practices in the catchment

This current rainfed practice involves the conventional ploughing and planting in rows on a flat surface.

3.4.2. Scenario (ii): untied ridges

Using a ridge: furrow ratio of 30:30 cm, with crops planted in the furrow, untied ridges farming practice was evaluated.

3.4.3. Scenario (iii): planting basins

The planting basins, also known as "chololo" pits in Tanzania, of about 25 cm depth and 30 cm diameter, were assessed (Mati, 2005). The planting basin rows are set up roughly on the contour in the field.

3.4.4. Scenario (iv): combined rainfed management practice

An average of combined rainfed practices (Scenarios *i–iii*) was assessed.

3.4.5. Scenario (v): supplemental irrigation

Under supplemental irrigation practice, the streamflow diversions for irrigation were deducted from total streamflow and their impact on downstream water availability was assessed. The proportion of the catchment streamflow yield diverted for smallholder supplemental irrigation was 25% (Liebrand, 2006) based on the overall water allocation of 60% in both commercial and smallholder agriculture sectors in the catchment (DWAF, 2004).

3.4.6. Scenario (vi): combined rainfed management practice (Scenario iv) and maize price variation

Using the NAMC (2008) grain price projections, applicable to the study area, the impact of the maize grain price variation on family balance for rainfed farming was investigated by applying the price variation on the proportion of farm yield sold to the market. The maize price varied from 43 to 130% of the basis grain price of US$ 205 ton^{-1} (NAMC, 2008) of South Africa.

Table 1
Correlations between observed variables for farm types B and D, $n = 60$.

Test	Variable	Farm type B			Farm type D		
		Family balance	In-season rainfall	Yield	Family balance	In-season rainfall	Yield
Pearson correlation	Family balance	1.00	0.12	0.53	1.00	0.18	0.63
	In-season rainfall	0.12	1.00	0.21	0.18	1.00	0.27
	Yield	0.53	0.21	1.00	0.63	0.27	1.00
Significance (1-tailed)	Family balance		0.18	0.00		0.08	0.00
	In-season rainfall	0.18		0.06	0.08		0.02
	Yield	0.00	0.06		0.00	0.02	
Kurtosis			0.04	1.17		0.04	1.07
Skewness			0.83	0.12		0.83	1.25

4. Results and discussion

4.1. Statistical distribution check and correlation analysis

The statistical distribution checks for the input and output variables of observed data ($n = 60$) for both kurtosis and skewness coefficients are shown in Table 1. The coefficients of kurtosis and skewness greater than 0.05 for crop yield indicate that the variable has a normal distribution, while in-season rainfall, with a kurtosis coefficient of 0.042, indicates a distribution significantly different from a normal one (Sheskin, 2011). Evapotranspiration (not presented in Table 1, but in Figs. 8 and 9), which is challenging to measure at a particular field site showed higher correlation with family balance than in-season rainfall, which is easier to obtain in arid and semi-arid areas. Hence, in-season rainfall, which is easier to obtain in arid and semi-arid areas, was used. The correlation coefficient magnitudes between each generated input parameter and the output (family balance) showed that errors in crop yield (higher correlation) were more important than those of in-season rainfall (Table 1).

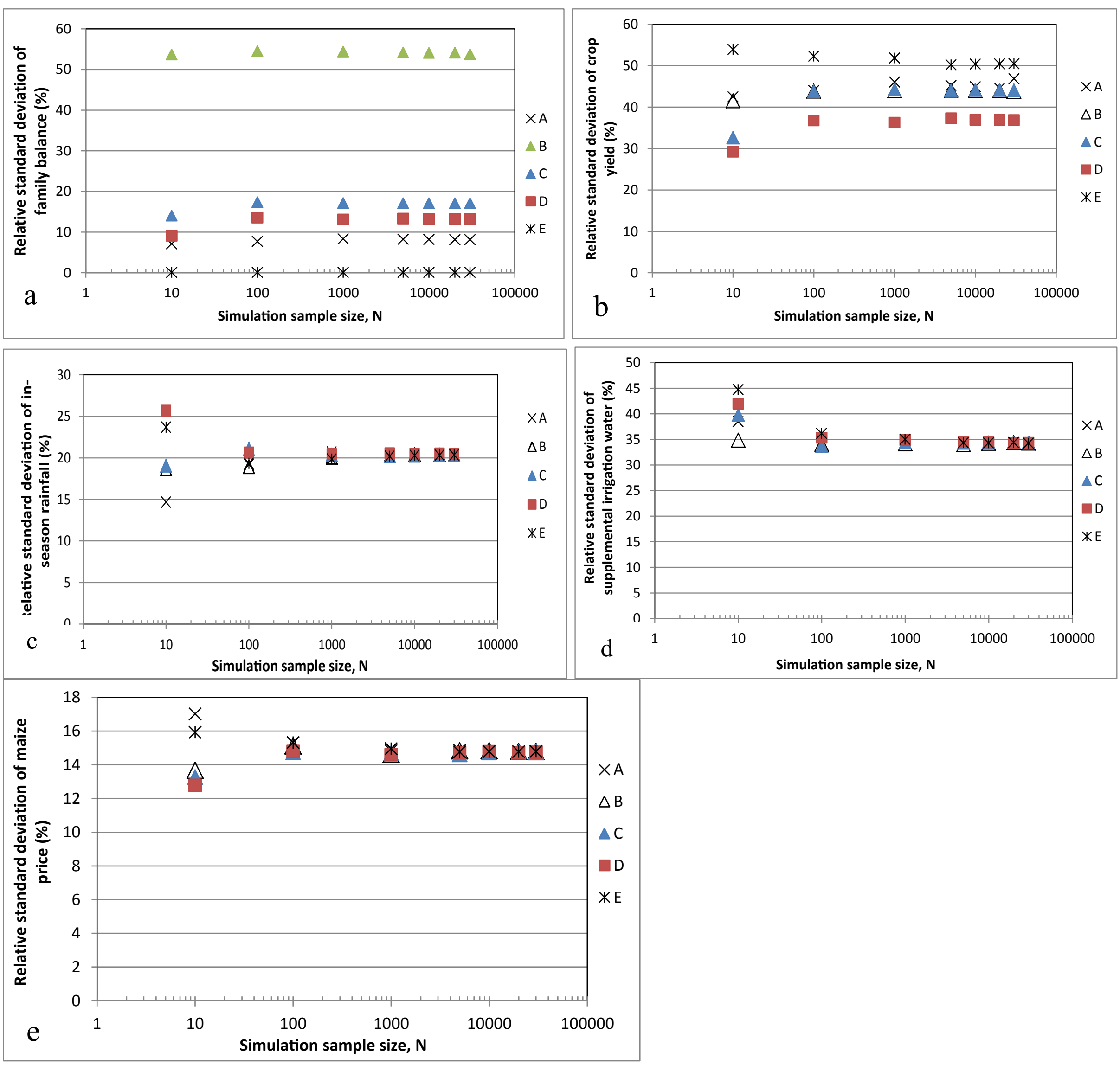

Fig. 2. Relative standard deviation (RSD) for (a) family balance, (b) crop yield, (c) in-season rainfall, (d) supplemental irrigation water flow, and (e) maize price variation as a function of the simulation sample size, N for farm types A–E.

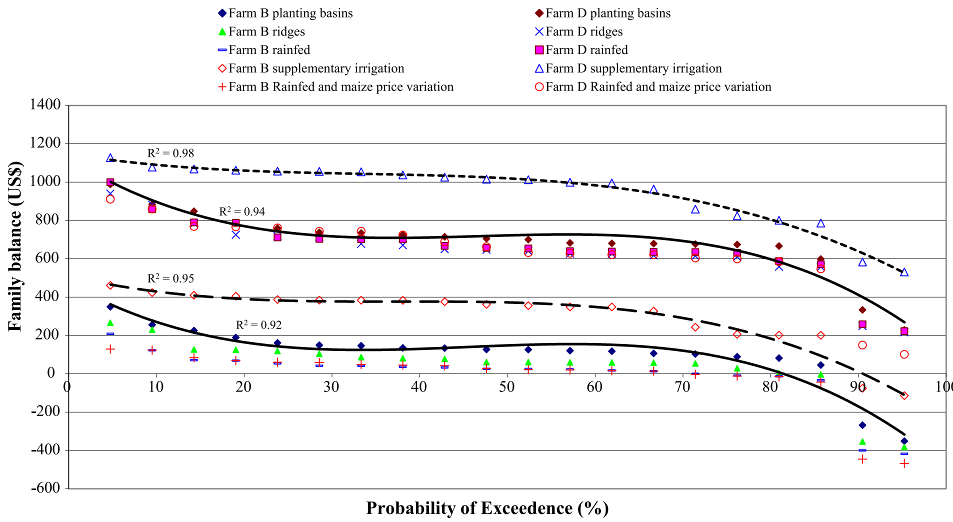

Fig. 3. Exceedance probability (%) of family balance per selected farm types B and D under different crop management scenarios using 20 years of simulated data under current climate variability.

The equations for farm types B and D relating family balance with in-season rainfall and maize grain yield for all six scenarios were estimated using product regression equation (Storlie and Helton, 2008), even though some of the independent variables are not normally distributed as evident by Kurtosis coefficient for in-season rainfall of 0.04 (Table 1). However, according to the Central Theorem (USEPA, 2000), whatever distributions are assigned to input variables, the output always tends to be normally distributed. For the three combined rainfed practices, a fit found from the data is represented by the regression models of the form $y = f(x_i, x_j)$ in Equations (4) and (5) for farm types B and D, respectively (Storlie and Helton, 2008). The regression models for the other farm types (A, C and E) are shown in Table 4. The correlation coefficients (R^2) of 0.86 (Equation (4)) and 0.90 (Equation (5)) from the multiple regression analysis of family balance, yield and in-season rainfall for farm types B and D, respectively, suggest a very strong relationship between family balance and the predictors of in-season rainfall and crop yield, often observed even in data scarce areas.

$$F_B = 154.07X - 190.24 \quad (4)$$

$$F_D = 221.52X - 252.24 \quad (5)$$

Table 2
Average gross margin and family balance per farm type for the two simulation periods.

Practice	Year	Farm A (US$)	Farm B (US$)	Farm C (US$)	Farm D (US$)	Farm E (US$)
Rainfed	2005–2014	639	−69	387	585	8137
(Scenario i)	2015–2024	735	59	471	720	8217
Ridges	2005–2014	657	−13	461	586	8144
(Scenario ii)	2015–2024	738	99	524	686	8223
Planting basins	2005–2014	733	42	529	631	8147
(Scenario iii)	2015–2024	817	162	602	749	8226
Supplemental	2005–2014	830	228	754	869	8158
irrigation	2015–2024	936	373	862	1024	8239
(Scenario v)						
Rainfed and maize	2005–2014	629	−82	350	553	8137
price variation	2015–2024	735	59	471	718	8217
(NAMC, 2008)						
(Scenario vi)						

where F_B and F_D is family balance for farm types B and D, respectively; $X = R^{0.1} \times Y^{0.5}$; R is the in-season rainfall (mm) and Y is the yield (ton ha^{-1}).

The non-linear dependence of family balance on in-season rainfall and crop yield shows that the family financial resource-base of smallholder farmers is multidimensional and highly related to physical climate, crop management practices and environment. Depletion of environmental resources, such as soil and water, puts some categories of rural people at risk of being trapped in the poverty cycle even when there is economic growth at country level. The regression relationships presented in this section feed into the error propagation analysis.

4.2. Generating random numbers and sample size

An optimal sample size, N for the Monte Carlo uncertainty analysis was attained by generating samples of random numbers, calculating the relative standard deviation (RSD) and checking the RSD till it was practically constant and independent from the simulation size (Espinosa-Paredes et al., 2012a, b). Sample sizes greater than 1000 of the independent parameters and family balance for farm types A to E were generally adequate as shown in Fig. 2a–e. Hence, the sample of 30,000 used, was more than adequate.

4.3. Family balance

The exceedance probabilities of family balance for different crop management practices for farm types B and D, calculated using Weibull formula, are shown in Fig. 3. The family balance plots reflect comparative performance of both farm types, with farm type B having lower family balance than farm type D. For 80% of the time the seasonal family balance was at least US$ 25 and US$ 600 for farm types B and D, respectively (Fig. 3). These results indicate that farm type B barely provides family food, and is unable to accumulate substantial savings as buffer for bad years, whereas farm type D with higher family balance is more food secure.

A summary of the family balance under different crop management practices for all five farm types is presented in Table 2. Farm type B has lower family balance compared to farm type D (Table 2) due to lower maize crop yields. If farm financial returns are consistently negative, then the farming system is unsustainable,

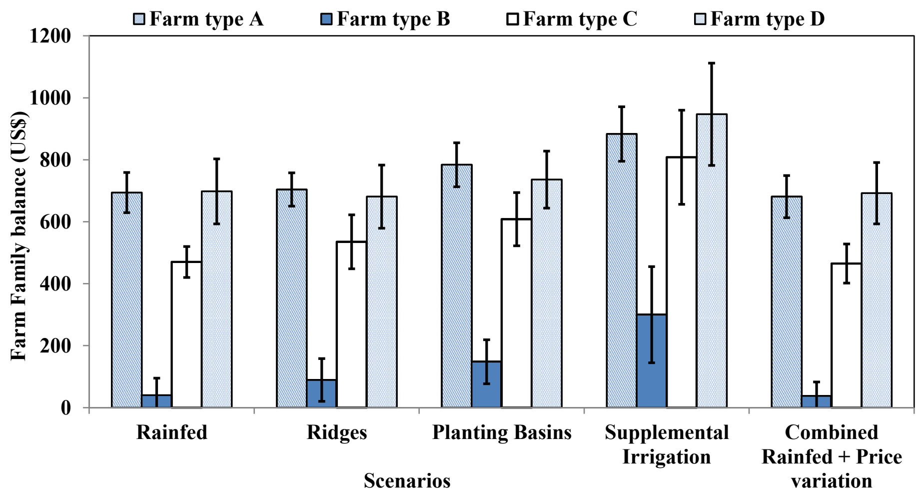

Fig. 4. Average family balance variation with scenarios investigated over the 20 years. The rectangular bars indicate the mean and the line bars indicate the standard deviation.

unless supplemented by off-farm income sources. All the farm types performed better for the second 10 years of simulation (2015–2024) compared to the first 10 years (2005–2014). This improvement in family balance is attributed to increased crop yields due to better in-season rainfall distribution in the second 10 years of simulation (not shown) and not necessarily due to higher total rainfall amount (Yenesew and Tilahun, 2009).

4.4. Family balance and scenarios simulated

The variation of the family balance with scenarios for the 20 years is presented in Table 2 and Fig. 4. Farm type B performed lowest amongst the farmers (Fig. 4). Farm type E, due to its high mean family balance ($8217/season), is left from being presented in Fig. 4 to ensure the clarity of the figure. Among the scenarios tested, supplemental irrigation performed best with mean family balance ranging from $300 (farm type B) to $947 (farm type D). This high performance is attributed to high crop yields realised under supplemental irrigation.

The marginal increase in family balance under improved crop-water management technologies compared to conventional farming practice is shown in Fig. 5. This marginal increase in family balance indicates the farm type that benefits the most or least by implementing the improved agricultural water management practices.

There is a general increase in family balance or benefits when farm types A, B, C and D implement improved crop-water management practices of ridges, planting basins and supplemental irrigation (Fig. 5). Farm type D benefits the most, while farm type E does not benefit from implementing the improved crop-water management practices. This can be explained by the intensive farming activities and fertiliser use under farm type D, while farm type E is not much involved in crop production (Magombeyi et al., 2012).

4.5. Sensitivity

The scatter graphs of dependent variable, family balance, and independent variable of crop yield from field observations for farm types B (Fig. 6) and D (Fig. 7) show a similar trend, indicating that their life strategies are similarly affected by crop yields. For farm types B and D that are heavily reliant on crop production, a

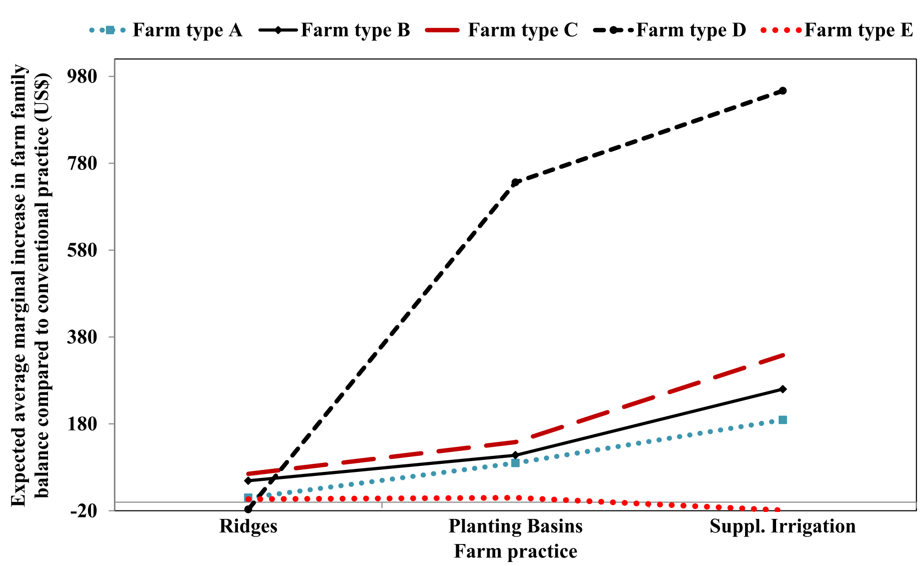

Fig. 5. Marginal increase in family balance under different crop management practices. Ridges (Scenario ii), planting basins (Scenario iii) and supplemental irrigation (Scenario v).

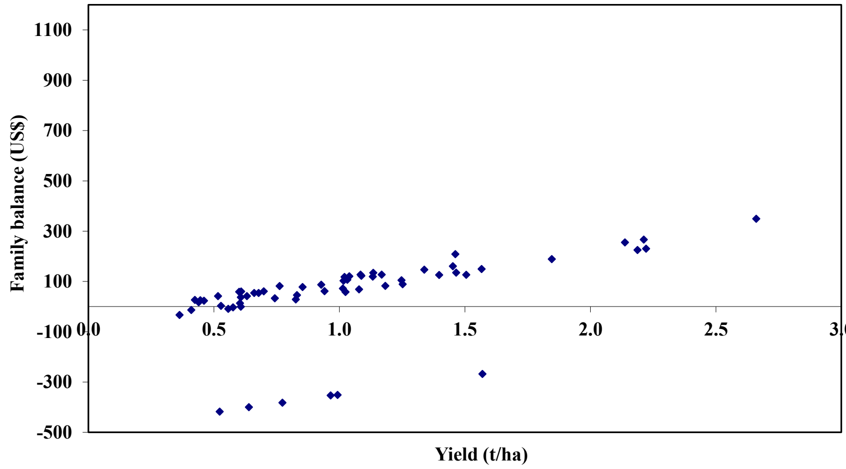

Fig. 6. Correlation of family balance and maize grain yield from field site observations for farm type B.

weakly non-linear relationship between family balance and crop yield was observed (Figs. 6 and 7), implying the dominance of the crop production term in Equation (2) over that of the non-farm income.

Although the relationship between in-seasonal rainfall and the family balance is weakly non-linear for both farm types B and D, the relationship of the latter with crop evapotranspiration (Figs. 8 and 9) is more strongly non-linear relationship and reflected in a higher

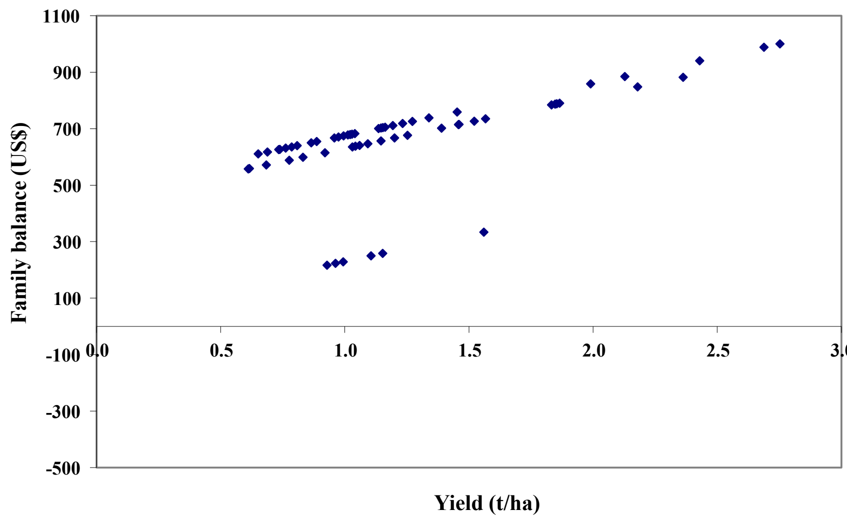

Fig. 7. Correlation of family balance and maize grain yield from field site observations for farm type D.

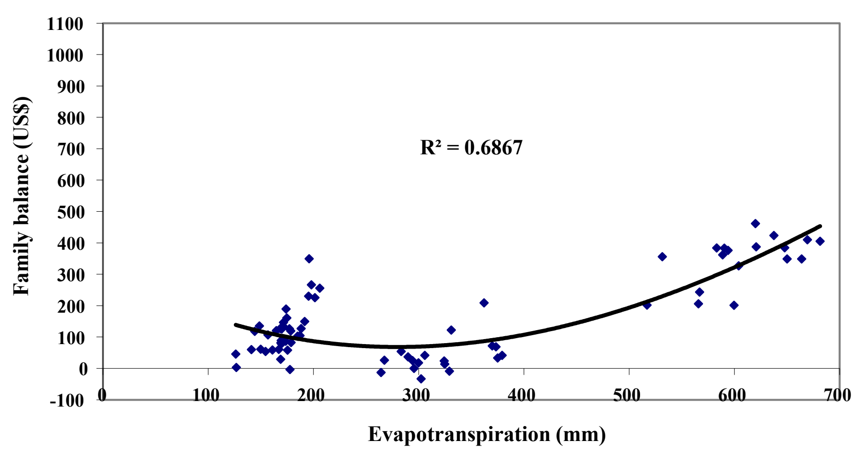

Fig. 8. Correlation of family balance and evapotranspiration from field site observations for farm type B.

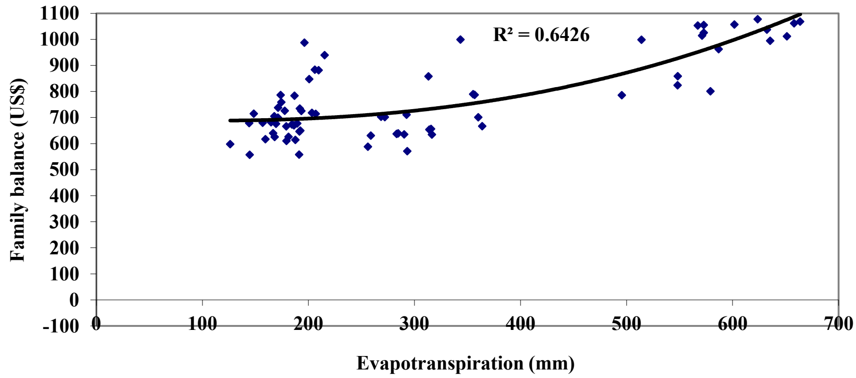

Fig. 9. Correlation of family balance and evapotranspiration from field site observations for farm type D.

correlation coefficient. Furthermore, the relationship of crop yield and in-season rainfall was non-linear (not shown) due to many factors that affect crop yield that could not be isolated from on-farm experimental observations. Similar non-linear relationships between crop yield and in-season rainfall are reported in Yenesew and Tilahun (2009). Crop yields vary from one farming season to another not only on the basis of total in-season rainfall, but on climatic factors, such as in-season rainfall distribution, intensity and amount, length of dry spells, sunshine hours, temperature and humidity, soil fertility, topography, crop management practice, and initial soil moisture at planting (Brouwer and Heibloem, 1986). With a fixed date for planting selected in the experimental fields, it was not possible to control all the explanatory variables due to weather variability.

The sensitivity of family balance resulting from 5% change in predictors of rainfall, supplemental irrigation water, maize price variation and crop yield are presented for all five farm types in Fig. 10a–c. The yield and crop market price play a greater role in food security, especially for farm type B, though their contributions vary with farm type. These contributions are insignificant for farm type E because its activities are mainly of an entrepreneurial nature in the farming sector. Similar family balance sensitivity trend was also observed for 10% change in predictors.

4.6. Impact of farming practices on streamflows

Streamflows diverted for supplemental irrigation ranged from 1220 to 4780 $m^3\ ha^{-1}\ yr^{-1}$, depending on the available flows and rainfall distribution. Over the 20-year simulation period, average annual streamflows at catchment level were 31 Mm^3 with conventional rainfed practice, 27 Mm^3 with untied ridges, 25 Mm^3 with planting basins and 14 Mm^3 with supplemental irrigation. These annual streamflow values indicate approximately 14%, 20% and 54% flow reductions for untied ridges, planting basins and supplemental irrigation practices, respectively. Although supplemental irrigation resulted in the highest reduction in streamflows that may negatively affect the environment, it also resulted in the highest crop yields that could improve livelihoods through increased family income and savings. Hence, there should be a trade-off between increasing crop yield and satisfying downstream water requirements, including the protection of the aquatic life.

4.7. Uncertainty propagation

The probability distribution graphs (Figs. 11 and 12) were derived from 30,000 Monte Carlo runs by varying in-season rainfall and maize crop yield simultaneously using the model Equations (4) and (5). The cumulative distribution functions (CDFs) and statistical measures for the 5th, 50th (median) and 95th percentile confidence levels for the different crop-water management practices for farm types B and D are shown in Figs. 11 and 12, respectively. The area between 5th and 95th gives the 90% confidence interval for the family balance.

Uncertainties associated with each parameter are quantified and ranked according to their importance to the overall family balance results, providing an aide in prioritising data collection and research efforts on highly ranked parameters. In this study, crop yield, which has a direct impact on family balance, is more important than in-season rainfall, because for the same in-season rainfall amount, different yields can be obtained depending on crop-water management practices employed.

The family balance was US$ 4–US$ 270 at 90% confidence interval under combined rainfed practices (Scenario iv) for farm type B. This family balance is reduced to between US$ 4 and US$ 132 at 90% confidence interval under maize price variation (Scenario vi) presented in Fig. 11. These results indicate family balance reduction by almost half due to maize price variation of 43–130% of the basis price of US$ 205 ton^{-1} in 2009. However, under supplemental irrigation practice (Scenario v), family balance increased to between US$ 233 and US$ 429 at 90% confidence interval, indicating increased and more reliable family savings under supplemental irrigation compared to combined rainfed practices (Scenario iv). However, these gains are reliant on the availability of water for supplemental irrigation.

Under combined rainfed practices (Scenario iv), the 90% family balance confidence interval for farm type D was between US$ 600 and US$ 900 and increased slightly to between US$ 600 and US$ 960 under maize price variation (Fig. 12). In contrast to farm type B which experienced a decline in family balance with maize price variation, the increase in family balance for farm type D is due to its intense and diversified farming practices which produced mean yield of 1.73 ton ha^{-1}, about twice that of B of 0.91 ton ha^{-1} (Table 3). The 90% family balance confidence intervals for farm types A (US$ 660–US$ 780) (Fig. 13) and C (US$ 420–US$ 590) (Fig. 14) for combined rainfed practices were reduced under maize price variation scenario, while that of farm type E (Fig. 15) was not affected by price variation due to its high reliance on employment income (Magombeyi et al., 2012). However, under supplemental irrigation practice, family balance increased to between US$ 900 and US$ 1140 for farm type D and between US$ 230 and US$ 430 for type B at 90% confidence interval, as water was no longer the main limiting factor for crop yield, but nutrient (CAWMA, 2007).

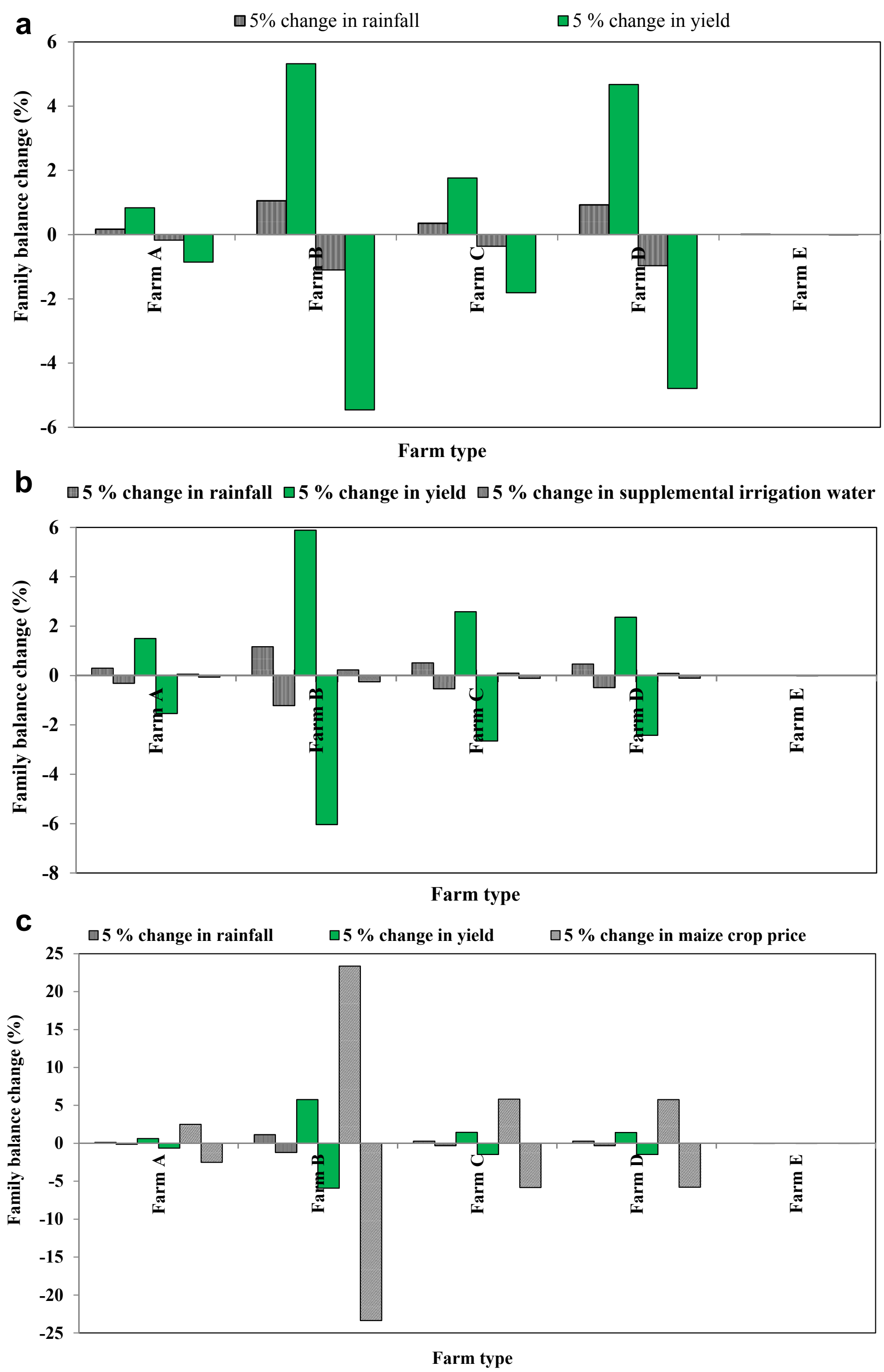

Fig. 10. Sensitivity in family balance with 5% change in rainfall and crop yield under combined rainfed practices (Scenario iv); (b) Sensitivity in family balance with 5% change in rainfall, supplemental irrigation water and crop yield under supplemental irrigation practice (Scenario v); (c) Sensitivity in family balance with 5% change in rainfall, maize crop price and crop yield under combined rainfed practices and maize price variation (Scenario vi).

In sum, resource-constrained farm type B with lower maize yields is more vulnerable to maize price variations than the more intensive and diversified farm type D. Based on the performance measure of maximizing family balance for enhanced food security and poverty reduction, supplemental irrigation offers the best strategy for smallholder farming, concurring with other results in Africa (CAWMA, 2007; Yenesew and Tilahun, 2009). However, the feasibility of wide adoption of this strategy is limited by available and proximity to water resources and infrastructure cost.

The combined uncertainty calculated for different scenarios per farm type based on an uncertainty propagation equation after Refsgaard et al. (2007) is presented in Table 3, while Table 4 presents the non-linear relationships of family balance with different input variables under different scenarios. Using the values of the relative standard deviation (ratio of standard deviation to mean) presented in Table 3, the crop yield contributes about twice (0.45) to family balance uncertainty compared to that of in-season rainfall (0.20) under combined rainfed practice (Scenario iv). This result

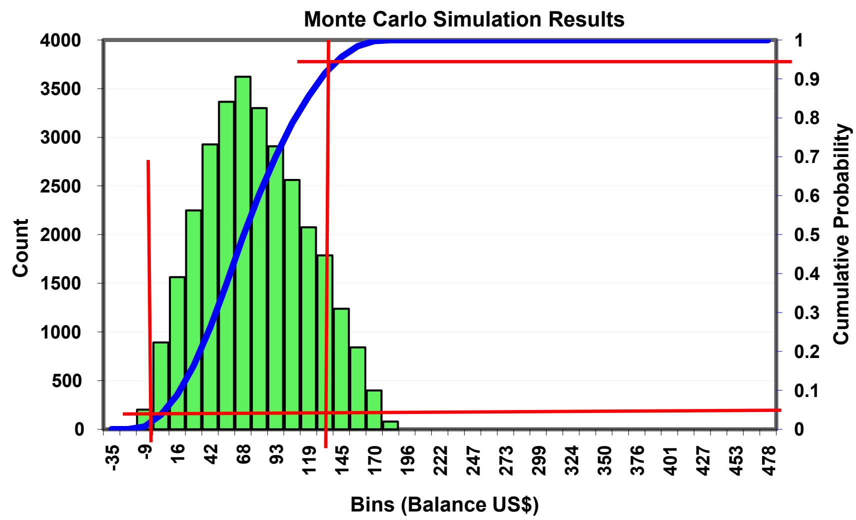

Fig. 11. Cumulative probability distribution of family balance for farm type B under combined rainfed practices with maize price variation (Scenario vi).

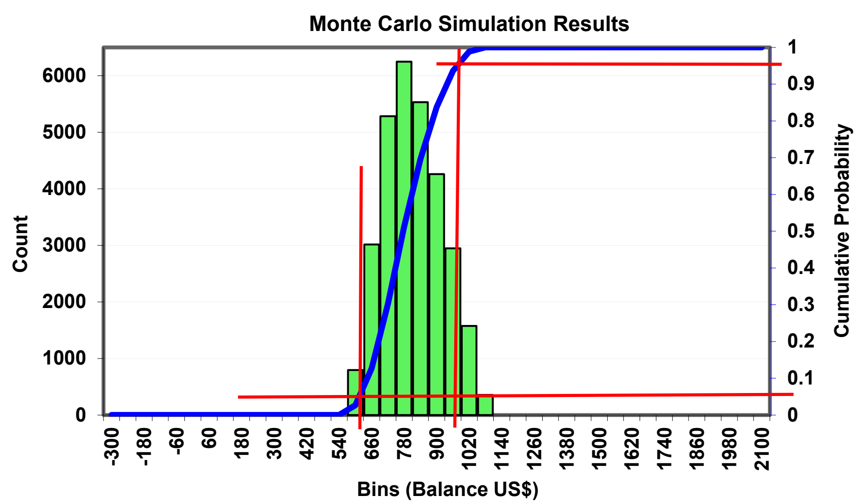

Fig. 12. Cumulative probability distribution of family balance for farm type D under combined rainfed practices with maize price variation (Scenario vi).

Table 3
Uncertainties under different scenarios per farm type.

Scenario/practice	Equation form	Farm type	Mean					Standard deviation					ϕ
			R (mm)	Y (t/ha)	P (US$/t)	Q (m^3/ha)	F (US$)	σ_R (mm)	σ_Y (t/ha)	σ_P (US$/t)	$\sigma_Q \times 10^6$ (m^3/ha)	σ_F (US$)	F %
Combined rainfed practices	$F = f(R, Y)$	A	521	1.44			780	106.0	0.65			384	49
		B	515	1.51			150	105.2	0.67			73	49
		C	519	1.53			616	105.4	0.67			298	48
		D	518	1.68			776	105.1	0.62			326	42
		E	521	1.45			8226	105.5	0.72			4444	54
Supplemental irrigation	$F = f(R, Y, Q)$	A	520	2.24		3013	894	105.7	0.54		1.791	413	46
		B	519	2.70		3006	336	106.7	0.44		1.812	145	43
		C	519	2.65		2982	832	104.5	0.47		1.820	365	44
		D	517	2.64		3001	1044	105.2	0.46		1.794	452	43
		E	521	2.64		2995	8238	105.9	0.46		1.811	3590	44
Maize price variation under rainfed	$F = f(R, Y, P)$	A	520	0.99	213		733	106.0	0.38	31		339	46
		B	519	0.91	212		72	105.5	0.32	32		31	43
		C	518	0.87	211		524	105.6	0.30	31		222	42
		D	520	1.73	213		786	105.5	0.60	32		335	43
		E	516	0.47	212		8218	104.9	0.16	31		3437	42

Notes: σ = standard deviation, ϕ = standard error, R = in-season rainfall, Y = yield and Q = sum of streamflow ha^{-1} $season^{-1}$ for supplemental irrigation, P = maize price in US$ per ton, F = family balance.

Table 4
Relationship between farm type family balance and input variables.

Scenario	Farm type	Relationship of family balance US$ (F)	Correlation coefficient (R^2)
Combined rainfed practices (iv)	A	$F = 119\ (Y^{0.5} \times R^{0.1}) + 521$	0.60
	B	$F = 154\ (Y^{0.5} \times R^{0.1}) - 190$	0.86
	C	$F = 196\ (Y^{0.5} \times R^{0.1}) + 181$	0.97
	D	$F = 222\ (Y^{0.5} \times R^{0.1}) - 252$	0.90
	E	$F = 9\ (Y^{0.5} \times R^{0.1}) + 8208$	0.97
Supplemental irrigation (v)	A	$F = 168\ (Y^{0.5} \times R^{0.1} \times Q^{0.01}) + 353$	0.66
	B	$F = 205\ (Y^{0.5} \times R^{0.1} \times Q^{0.01}) - 396$	0.78
	C	$F = 247\ (Y^{0.5} \times R^{0.1} \times Q^{0.01}) - 39$	0.95
	D	$F = 264\ (Y^{0.5} \times R^{0.1} \times Q^{0.01}) + 42$	0.85
	E	$F = 12\ (Y^{0.5} \times R^{0.1} \times Q^{0.01}) + 8195$	0.95
Maize price variation under combined rainfed practices (vi)	A	$F = 0.47\ (Y^{0.5} \times R^{0.1} \times P) + 552$	0.40
	B	$F = 0.46\ (Y^{0.5} \times R^{0.1} \times P) - 100$	0.61
	C	$F = 0.83\ (Y^{0.5} \times R^{0.1} \times P) + 220$	0.97
	D	$F = 0.88\ (Y^{0.5} \times R^{0.1} \times P) + 333$	0.84
	E	$F = 0.02\ (Y^{0.5} \times R^{0.1} \times P) + 8213$	0.90

Notes: R = in-season rainfall, Y = yield and Q = sum of streamflow ha^{-1} season^{-1} for supplemental irrigation, P = maize price in US$ per ton, F = family balance.

confirms earlier sensitivity analysis results (Table 1) that indicated a stronger correlation of crop yield (0.63) with family balance than with in-season rainfall (0.12).

Similarly, under maize price variation scenario, the contributions to family balance uncertainty, starting with the highest contributor are crop yield, in-season rainfall and maize price variation (Table 3). However, under supplemental irrigation practice, supplemental irrigation water availability contributes most to uncertainty followed by in-season rainfall and then crop yield (Table 3). This result is in agreement with on-farm results and indicates that supplemental irrigation practice in semi-arid areas is very important as it bridges crop-water stress periods that cause low crop yields under both conventional and improved rainfed practices (CAWMA, 2007; Yenesew and Tilahun, 2009).

Generally, a reduction (from a mean of 48–43%) in the family balance uncertainty is noted in Table 3, as input variables are increased from two to three. This reduction in uncertainty magnitude with increased number of input variables concurs with findings by Loucks and Van Beek (2005). However, Özkaynak et al. (2009) cautioned that while, adding new variables or model details reduce uncertainty, it may increase model complexity if the additional variable is difficult to measure.

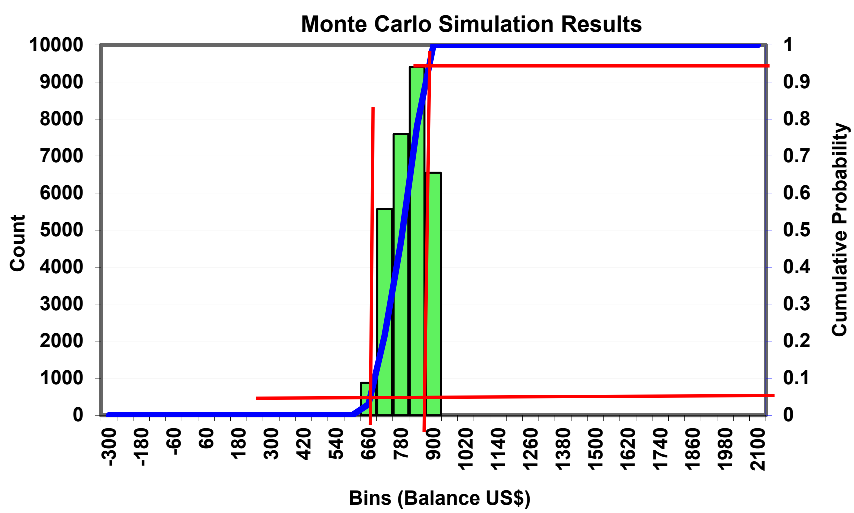

Fig. 13. Cumulative probability distribution of family balance for farm type A under combined rainfed practices (Scenario iv).

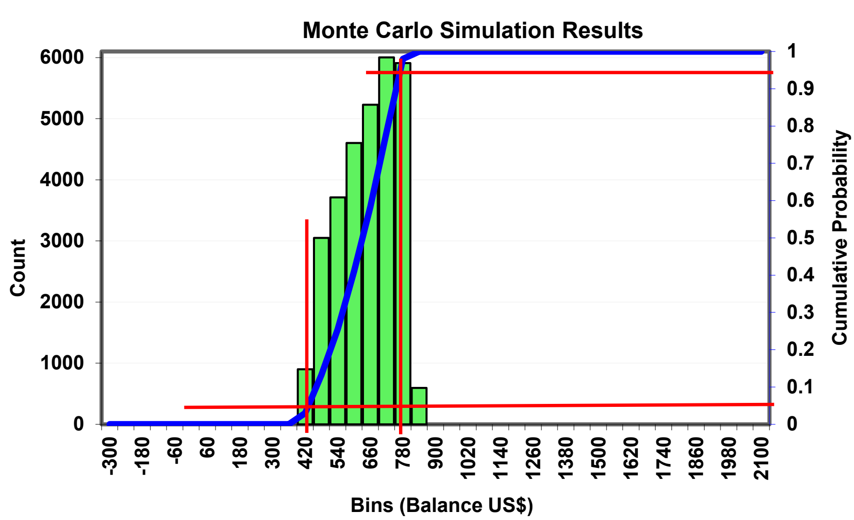

Fig. 14. Cumulative probability distribution of family balance for farm type C under combined rainfed practices (Scenario iv).

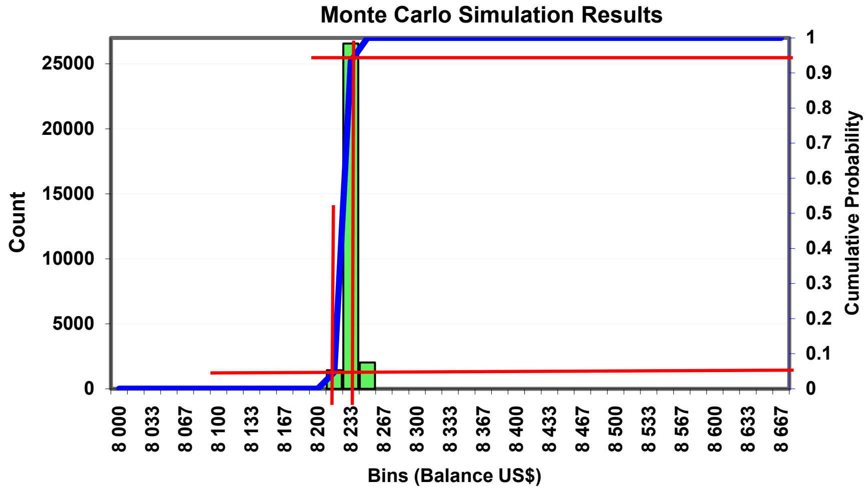

Fig. 15. Cumulative probability distribution of family balance for farm type E under combined rainfed practices (Scenario iv).

5. Conclusion

This paper illustrated the application of sensitivity and uncertainty analyses of family balance, an output from an integrated model that couples hydrology, agronomy and socio-economic aspects in rural smallholder farming systems in B72A catchment in northern South Africa. The possible impacts of different rainfall conditions, supplemental irrigation, crop-water management strategies, and maize grain price variations on smallholder food security, income and family balance/savings were evaluated. A high family balance indicates savings which can be accumulated to wealth at household level. The sensitivity analysis showed that crop yield, timely crop water availability and market price significantly affect smallholder food security and more effort is required in monitoring this information from the field to improve food security assessments in smallholder farming systems. The total in-season rainfall showed little influence on the total crop yield as the crop-water use efficiency depended on the crop-water management practice implemented to optimally use the available seasonal water. However, in-season rainfall distribution during the crop growing cycle is important. The magnitudes of family balance uncertainty were (42–54%) for combined rainfed, (42–46%) for maize price variation under combined rainfed, and (43–47%) for supplemental irrigation scenarios at 90% confidence level (Table 3). The family balance under resource-constrained farmers (farm type B) is most sensitive to commodity price variation, crop-water management practice as most of family food and income comes from agriculture, whereas farm type D, intensive and diversified irrigation farmers, is more food secure and benefits more from improved crop-water management practices. Hence, using improved agricultural productivity as one of the several ways to rural development, policies aimed at achieving food security and enhanced livelihoods among smallholder farmers should be targeted to most vulnerable farming groups in short to medium term. These policies should aim at supporting improved crop-water management technologies, marketing of produce and increased social protection and safety nets, such as public works programmes and cash/food transfers to ameliorate local and global market price fluctuations and food insecurity. If productivity is increased in these resource-constrained farmers labour can be re-allocated to other more productive sectors.

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