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Modelling the structure of dependence of stock markets in BRICS & KENYA: Copula GARCH approach

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DECLARATION

I declare that this work has not been previously submitted for the award of a degree by this or any other university. To the best of my knowledge and belief, the dissertation contains no material previously published or written by another person except where due reference is made in the thesis itself.

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Abstract

Background: Dependence structure is used widely to describe relationships between risks and provides estimation of risks for risk management purposes. Modeling dependence structure of stock returns is a difficult task when returns are having non elliptical distributions.
Objective: To examine the dependence pattern between the Kenya stock market return and BRICS stock market returns.
Methods: In this dissertation, we estimated the dependence using copula GARCH, an approach that combines copula functions and GARCH models. We applied this method to a stock market returns consisting of stock indices of Brazil, Russia, India, China and South Africa (BRICS) and Kenya stock market. We first used GARCH(1,1) to model the marginal distributions of each stock returns using different GARCH(1,1) specifications. Copula was then used to analyze the dependence between the BRICS stock market returns and Kenya stock market returns using the standardized marginal distributions derived from GARCH(1,1) residuals. The best fitting copula parameter was determined using the loglikelihood or AIC.
Results: Empirical results showed that GJR-GARCH model provided the best fit for Brazil, Russia, China and Kenya while E-GARCH model provided the best fit for India and South Africa. As for modeling the dependence structure, student t copula parameter provided the best fit for the marginal distributions of the returns.
Conclusion: Marginal models showed presence of volatility clustering which vanishes after crisis. To capture the dependence structure for bi variate data sets, Student t copula was considered to be the appropriate copula function.
Recommendation: Further research should be extended to examine the multivariate structure, a joint distribution of BRICS in terms of Multivariate GARCH. Also research should focus on specific time periods in order to ensure effectiveness in measurement and management of risks.

Key words : Non elliptical distributions, copula, GARCH model and BRICS.

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Dedication

This Project is dedicated to God for His amazing grace and my parents Mr and Mrs Otieno for their love, prayers and encouragement to further my studies.

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Chapter 1

1 INTRODUCTION

1.1 Background of the study

Any investor would wish to be among the first to identify a long term investment trend. This can be achieved by taking advantage of foreign investment activities. A foreign investment activity refers to investment made by foreigners in financial assets of another country for purpose of diversification and to get higher returns . Specifically, foreign investment activity in the stock market is encouraged through liberalization with aim of improving market activity (Kariguh, 2014). Investors can diversify risks, hedge against risks and earn returns in emerging and frontier markets by taking advantage of low correlation between this markets (Allen et al., 2011). There has been an increased levels of foreign direct investment (FDI) between BRICS nations and Africa with China, Brazil and India having greater number of projects than the rest.

BRICS nations which are Brazil, Russia, India, China and South Africa account for 25% of world GDP and 15% of world trade (Singh and Dube, 2014). Given the increased economic activities between these nations and other countries, there is need to understand the linkage between stock markets of this countries (Mensi et al., 2014; Chege, 2006). These nations are a major recipent of direct foreign investment and main global consumer of commodities (Adu et al., 2015; Mensi et al., 2014; Khan et al., 2001). Also many African countries are beneficiaries of FDI from BRICS and there is also demand for their exports from BRICS nations which shows that both parties are benefiting from the relationship. It is also important to understand the linkage between stock markets because fluctuations in world economies and financial conditions can be transmitted to BRICS stock market and affect performance of the other countries stock market.

Kenya which is the biggest foreign direct investor in other African countries according to EY's Africa Attractiveness Program 2016 is a major investment partner to BRICS. Nairobi stock exchange (NSE), one of the major stock markets in Africa attracting investors from all over the world (Obere, 2009) has experienced increased foreign investment activity resulting increased trading volumes and volume of capital raised. This study shall examine the relationship between BRICS stock market and Kenya stock market using an approach that combines different copula functions and GARCH models. Our interest is on dependence structure between this stock returns from a diversification and risk management perspective which is vital for portfolio managers and investors.

1.1.1 Kenya Stock market

Stock markets deals with exchange of shares, bonds and other instrument of money. Dealings involving shares and stocks started in 1920 under colonialism and first stock exchange was floated in 1922 based on gentleman's agreement since there were no existng rules and regulations.
Nairobi Stock exchange was formed in 1954 as a voluntary association of stock brokers

registered under the societies act. It has experienced tremendous growth since its inception. According to International Finance Corporation (IFC) in 1994 NSE was the best performing stock market which was also repeated in 2007. Investors in stock markets has increased over the years with Kenya having being the largest intra regional investor (e Young, 2016). Foreign investors participation began before independence but there was a decline after independence which resulted in formation of Foreign investment protection act in 1964 (Nyangoro, 2013). This has resulted to growth of NSE in terms of trading volumes and capital invested.

1.1.2 BRICS stock market

The acronym BRIC was formulated in 2001 by Jim O'Neil, of Goldman Sachs in a report showing the growth potential of the economies of Brazil, Russia, India and China. In 2011 it became BRICS with the inclusion of South Africa. Current and potential growth of BRICS countries has implications on their stock markets as well as other stock markets financial dependence with other stock markets. In terms of trade and investment, the economies of BRICS countries is increasing at a significant rate. According to world development indicators for stock market variables as at 2012, BRICS accounted for combined nominal GDP of US 16.039 million. Russia and South Africa are the smallest markets having 874 billion and 674 billion. The size of the stock market is 159% that of GDP. This provides a higher volume of shares traded as proportion of GDP with South Africa leading the pack at 81%. BRICS member states have established themselves as influential players in Africa. Collectively they are the largest trading partners as compared to the United States, with China being the second most trading partner in Africa behind United States. Thus understanding dependence structure of stock market in BRICS is crucial for portfolio managers, policymakers and researchers in Kenya.

1.1.3 Dependence structure

Dependence structure describes the relationship between risks and provides an estimation of risks (Longin and Solnik, 1995). Investors and policy makers are mostly interested in relationship across stock markets, their effect, relevance and implications for risk management (Van den Goorbergh et al., 2004). Risk measurement and management is vital for portfolio allocation and asset pricing (Pfaff, 2013). In risk modelling, a better understanding of dependence structure is crucial for diversification of risks and controlling financial contagion. Furthermore international diversification of portfolios is the source of an entirely new kind of world welfare gains from international economic relations (Grubel, 1968). Previous studies have shown that diversification of risks helps in management of portfolio.

Correlation measures the strength and direction of a linear relationship between two variables. Historically, correlation was used to measure and model dependence between variables, specifically Pearson correlation which looks at linear dependence. This method based on assumptions of normality was mostly used to measure linear dependence because of its ease in calculation (Embrechts et al., 2002). The linear correlation coefficient between X and Y is

$$\rho_{XY} = \frac{Cov(X,Y)}{\sigma_X \sigma_Y}, \tag{1}$$

where $Cov(X,Y) = E[(X - E(X))((Y - E(Y))]$ denotes co-variance between the random variable and σ_X and σ_Y are the standard deviations respectively.

$$r_{XY} = \frac{\sum_{i=1}^{n}(X_i - \bar{X})(Y_i - \bar{Y})}{\sqrt{\sum_{i=1}^{n}(X_i - \bar{X})^2}\sqrt{\sum_{i=1}^{n}(Y_i - \bar{Y})^2}}, \tag{2}$$

where r_{XY} is the sample co-variance correlation coefficient.

In cases where random variables are not normal or not elliptical this method will be partial and misleading thus there is need to understand dependence structure beyond linear correlation.

Embrechts et al. (1999) discusses the pitfall of using correlation in heavy tailed distributions. He argued that linear correlation under non linear strictly increasing transformation are not invariant that is the correlation of X and Y is not similar to correlation of $\log(X)$ and $\log(Y)$. He also advocates use of Kendall's tau and Spearman's rho instead of Pearson correlation. This is because the two methods lay out the process of concordance that is larger values of one variable are associated with large values of the other variable. The variables (X_1, Y_1) and (X_2, Y_2) are said to be concordant if

$$(X_1 > X_2) \text{ and } (Y_1 > Y_2), \tag{3}$$

hence it has the ability to capture the non linear dependence between the two variables.

Spearman rank correlation is an alternative to Pearson correlation which is based on ranks of the two data sets . A non parametric test used to identify and test the strength of a monotonic relationship between two sets of data. It is calculated as ;

$$\rho_{ranks} = 1 - \frac{6\sum d_i^2}{n\left(n^2 - 1\right)} \text{ where } d_i = x_i - y_i. \tag{4}$$

Deficiencies experienced in Pearson correlation are not seen in Spearman rank correlation. It is invariant under strictly increasing transformation and risks do not need to be of finite variance.

An extension to correlation which takes into account time variations and other conditions is the conditional correlation. Conditional correlation has been used under different conditions to take into account presence of outliers, trend, noise and non-linearity.

1.1.4 Copula

Nelsen (1999), defined copulas as functions that join multivariate distributions functions to their one-dimensional marginal distribution functions.The joint distribution function contains the marginal distributions and dependence of jointly distributed random variables.

Sklars theorem
let $x_1, ..., x_n$ be a random variable with a joint cumulative distribution function

$$F(x_1, ..., x_d) = P[X_1 < x_1 ....., X_d < x_d]. \tag{5}$$

Then $(x_1, ..., x_d)^T$ has a unique copula C such that

$$F(x_1, ..., x_d) = C[F_1(x_1), F_2(x_2) ....., F_d(x_d)]. \tag{6}$$

Copula is used to model dependence between stock returns since it allows modelling of dependence structure without studying marginal distributions (Bouyé et al., 2000). It address the issues multi variate GARCH models and correlations couldn't handle due to the asssumptions of multivariate normality (Patton, 2006). Since coefficient of correlation depends on the marginal distributions, its necessary to separate the dependence structure from the marginal distribution.

Patton et al. (2012) copula-models are flexible in modelling multivariate distributions by making it possible to fit models for the marginal distributions separately from the dependence structure. Embrechts et al. (2002) showed that copula functions are invariate to non- linearity increasing transformation of the data. Copula will enable modelling dependence of stock returns in extreme market conditions.

The aim of this study is to examine the dependence structure between BRICS and Kenya stock market. Copula will be used to capture the dependence structure between risks by building flexible models of the joint distributions of stock index returns and GARCH model will be used to capture the volatility of the risks. Traditional goodness of fit test such as AIC and log likelihood tests will be used to determine suitability of the copula model. This is because this test are based on direct comparison of observed data and a given copula and can be used in any dimensional copula (Kole et al., 2007).

1.2 Problem statement

Dependency enables investors and policy makers to explain fluctuations in one market from other markets and vice versa hence investors can apply it for diversification purposes. Correlation has been used to determine dependence between variables because of assumption of normal distribution (Embrechts et al., 1999). In this paper Embrechts et al. (1999) showed correlation as a source of confusion and emphasized on shortcomings of linear correlation. In situations where returns have non-ellipitical distribution especially in financial variables, copula functions are deployed as effective tool for determining dependency between variables (Embrechts et al., 2002).

Copula overcomes limitations of correlation, conditional correlation and GARCH models by determing the type of dependence structure and its degree(Jondeau and Rockinger, 2006). This study applies Copula GARCH. According to Jondeau and Rockinger (2006)- Copula GARCH models are models where some copula parameters are time varying in auto-regressive manner conditioned on past information. The dynamic parameters which are time varying will be suitable for financial time series data.

Although copula has been applied in most empirical research on dependence structure, the focus has be on developed and emerging nations (Dhesi, 2009; Yang et al., 2015; Van den Goorbergh et al., 2004; Mensah et al., 2016).

Most of the research focuses on developed and emerging markets, very little research has been carried out in reference to developed markets and frontiers markets or emerging and frontier markets. In this research, our interest is the emerging market and frontier markets. Although the risk involved in these markets is high, the returns are also high which presents a great opportunity for foreign investors . A different methodology using copula was applied in this research. The copula was used to investigate dependence structure between emerging markets represented by BRICS and frontier market represented by Kenya. The GARCH model was used to capture the volatility of the returns. Based on this method, time varying conditional correlation will ensure correlation coefficient will be changing over a period of time rather than being constant.This is necessary since it will enable investors or policymakers to establish the confidence bands of market fluctuations thus easy to make decisions on risks diversification.

1.3 Research Objectives

1.3.1 Main Objective

The main objective is to examine dependence pattern between the Kenya stock market and BRICS stock market. We shall also analyze and quantify the extent to which financial regulators and financial practitioners can take advantage of co movements between BRICS stock markets and Kenyan stock market.

1.3.2 Specific Objective

The specific objectives of this study are;

1. To develop a copula GARCH model for BRICS and Kenya stock market returns.
2. To conduct an empirical comparison performance of Copula-GARCH models under different GARCH specifications.
3. To examine the dependence structure between BRICS stock markets and NSE index and to ascertain the degree and direction.

1.4 Significance of the study

Globalization has a significant impact on a countries economic growth. Each country of BRICS has its unique comparative advantage that enables it to enjoy foreign direct investment. Understanding the dependence structure between this countries will help kenyan investors to take advantage of investment policies of BRICS (Khan et al., 2001). Also understanding dependence beyond simple correlation will help Investors in risk management and portfolio diversification who are interested in BRICS markets. Inclusion of non-linear distributions invalidates assumptions of using correlation since most of the stock returns are heavy tailed and skewed (Embrechts et al., 2002). There are several advantages of using the copula function. First, copula functions will allow modelling of the dependence structure of stock markets which are non linear and asymmetric. Secondly, copula approach in risk management has been used to produce multivariate time series models constructed by combining univariate time series models to form joint distributions (Patton et al., 2012).

Dependence structure will enables policy makers to explain fluctuations in one market in response to the other markets and vice versa.

Chapter 2

2 LITERATURE REVIEW

2.1 Introduction

This section provides a review of statistical and econometric approaches and models that have been applied in our assessment of the impact of the BRICS stock market. We will also review different approaches used by different authors in dependence structure, univariate GARCH modelling and use of copulas in different stock market.

2.2 Theoretical review

Foreign investors invest in other markets in order to diversify risks and increase returns. In order to improve market activity and access to foreign capital, foreign investment is encouraged through liberalization. Market liberalization reduces risk premium in the market and allows diversification of returns for risk mitigation among foreign and domestic investors. According to Henry (2000) stock market liberalization is whereby foreign investors are allowed to purchase shares in the country's stock market. From his paper, standard international asset pricing models (IAPMs) predicts that through sharing of risks between domestic and foreign investors, stock market liberalization may reduce country's cost of equity capital.

2.3 Empirical review

2.3.1 Dependence structure

In order to understand relationships between risks, there needs to be a linkage between different financial instruments. Dependence structure is used to explain this linkages. There is abundant empirical research on dependence structure but focusing on international markets Dhesi (2009), Yang et al. (2015) and (Wang and Cai, 2011). Various techniques have been applied in analyzing the dependence structure such as linear correlation, conditional correlation, and copula models Embrechts et al. (2002); Knif (2005); Silvennoinen and Teräsvirta (2012); Longin and Solnik (1995).

2.3.2 Correlation

Correlation is one of the most widely used dependence measure and it has been applied in many areas of finance such as Capital Asset Pricing Model (CAPM) and Arbitrage Pricing Theory (APT). Pearson correlation is one of the most widely used measure of linear dependence due to assumptions of normality and it ease in calculation (Embrechts et al., 2002). In situations where random variables are not normal or not elliptical this method will be partial and misleading thus there is need to understand dependence structure beyond linear correlation. Forbes and Rigobon (2002) showed that correlation suffers from conceptual deficiences when the co-movements relationship is non linear and cases where markets are so volatile. Correlation is mostly used for elliptical distributions which are polynomial functions has many drawbacks.

Embrechts et al. (1999) discusses the pitfall of using correlation in heavy tailed distributions. According to them, linear correlation as a measure of dependence correlation cannot tell everything about dependence structure since it is a scalar measure of dependency, possible values of coefficient of correlations depends on marginal distributions of risks which are not necessarily attainable. Perfectly positive dependent risks does not imply a correlation coefficient of 1 and perfectly negative dependent risks does not imply a coefficient of -1. They showed that zero correlation coefficient does not imply independence of risks and under transformations of risks, correlation is not invariant implying that correlation between X and Y cannot be similar to correlation of $log(x)$ and $log(y)$. They concluded that correlation was inappropriate for heavy tailed risks with infinite variances.

2.3.3 Conditional correlation

Modelling comovements of stock returns has become challenging due to the departure from Gaussian distributions and existence of tail dependence. Recently, research suggests that during market turmoils correlation changes over time which implies that risk analysis may be affected negatively or positively depending on diversification strategy, portfolio optimization or institution policies. In order to capture correlation changing over time, time varying conditional correlation is used. Time varying conditional correlation was applied to address this issues by using information from previous period to determine the conditional correlation. Engle (2002) proposed a new class of multivariate model Dynamic conditional correlation (DCC) to estimate time varying correlations. He found that this model performs well in variety of situations and had the flexibility of univariate GARCH models. He also observed volatility forecasts for multivariate and univariate were consistent with each other.

Another model time varying conditional correlation was proposed by (Knif, 2005). He revealed the contributions of world market volatility and national market volatility which are internal. He showed that time varying correlation between stock markets is dependent on world market volatility and national market volatility. Further research by Boyer, Gibson and Loretan (1999) showed that conditional correlation on which it is conditioned is highly non-linear hence it will be difficult to conclude that correct correlation is varying over time by comparison of estimated correlation conditioning on different values of returns.

In his paper "Time varying conditional dependence in Nairobi stock market ", Olukuru (2005) used Cook Johnson copula to examine Nairobi stock index returns. Using GARCH(1,1) processes with asymmetric student-t errors he examined that the returns are fat tailed and exhibit a significant unconditional skewness. Time varying copula was also used to examine the impact of introduction of the Euro on the dependence between European stock markets. GJR-GARCH-MA-t model together with Gaussian copula was used for marginal distribution and joint distributions respectively. The study showed that market dependence increased after introduction of currency.

Using extreme value theory to model the multivariate distribution of large returns, Longin and Solnik (1999) used the theory to specify the distribution of conditional correlation on large negative or positive returns with constant correlation. They found out that correlation of large positive returns is not inconsistent with multivariate normality while correlation of large negative return is much greater than expected. They also used US and UK markets whose distributions were multivariate normal with mean zero and a unit standard deviation to show that the conditional correlation of multi variate normal returns is usually lower than the correct correlation.

Financial returns according to Embrechts et al. (2002) have heavy tails than normal distributions and dependence between stock returns are usually non linear and asymmetric. Various methods have been used before to study the comovements of stock returns without considering the assumptions of normality. Multivariate generalized autoregressive conditional heteroskedasticity (MGARCH) models under assumptions of normality have been used to estimate dependency under different conditions. This methods such as dynamic conditional correlation (DCC) model by Engle (2002),the Baba, Engle, Kraft and Kroner (BEKK) model by Engle and Kroner (1995) and the varying correlation (VC) model by Tse and Tsui (2002) have been rejected by empirical findings Fama and French (1993); Longin and Solnik (2001); Richardson and Smith (1993); Mashal and Zeevi (2002), due to assumptions of normality. Another important issue relates to non-normal returns. How are we going to meausure dependency between assets when returns are not normal?. As a results of this restrictions, copula methods have been applied frequently in finance to address this drawbacks

2.3.4 Copula models

A copula is a function that links univariate marginals to multivariate marginals. It is considered a very powerful tool as it does not take into account any assumptions relating to distribution selected. It also allows decomposition of the joint distribution to marginal distributions and a copula. Different types of copula models have been applied by different scholars to study the dependence structure (Patton et al., 2012; Lee and Long, 2009; Mensah et al., 2016; Yang et al., 2015). Silvennoinen and Teräsvirta (2012) uses Smooth Transition Conditional Correlation (STCC) GARCH model to allow conditional correlation to change smoothly from one state to another as a function of the transitional variable. It was observed that the conditional correlation was higher during periods of high volatility than low. A two week lagged average of the daily absolute return of the S & P index was used as a transition variable.

Archimedian copulas has been used by Yang et al. (2015) to investigate dependence structure among international stock markets. The study revealed negative news had a larger impact on the degree of dependence structure than positive news about the market. It also showed evidence of existence of contagion during global and EU debt crisis.

Mensah et al. (2016) used static and time-varying copula to study dependence between the US and African stock markets. He showed that dependence structure between African and international markets varies overtime. He also observed that spillover effects on Africa due to bearish movement of stocks in US and UK was not significant. Rong and Trück (2014) applied copula model to investigate dependence structure between the returns from real estate investment trusts and equity markets in Australia. A significant positive correlation was observed and student t copula was found to be the best model for tail dependence. Also the study found out conditional copula approach outperforms DCC model and static variance-co-variance approach for risks of extreme negative returns.

Bozovic et al. (2009) used copula approach to study cross diversification. He applied copula to show that extreme events in markets have little effect in stocks traded in European emerging markets despite high tail dependence in upper and lower tail. In his paper, Mensah et al. (2016) examined dependence between two developed and four emerging African stock markets in copula framework. They observed that dependence structure varies overtime but weak and in extreme market conditions, dependence is generally weak.

Jondeau and Rockinger (2006) model dependency between stock market returns using copula garch approach. They observed that dependency is widely affected when returns move in same direction than when the move in opposite direction. Van den Goorbergh et al. (2004) applied uni-variate AR(p) - GARCH(1,1) model. Dependence across stock markets was allowed to vary overtime through a GARCH like auto-regressive conditional copula model. Strong evidence of conditional dependence between pairs was observed varying over time. Just like Rong and Trück (2014), Van den Goorbergh et al. (2004) found out that the copula approach was a superior model than DCC model . Using ARMA-GARCH model to study dependence pattern between stock market and foreign exchange market Kamal and Haque (2016) observed existence of asymmetric dependence from copula models with upper tail dependence in all pairs. Dhesi (2009) investigated volatility spillover effect between $FTSE\,100$ and $S\,\&\,P\,500$ stock indices based on multivariate GARCH-BEKK model, T-BEKK was employed to examine volatility spillover effect between different markets. They observed that there was a significant volatiltiy spillover effect between the UK and US stock markets. Under copula GARCH methodology, Dhesi (2009) concluded that time varying copula was better than other copula models. They also observed that symmetric Joe-Clayton (SJC) time varying copula improves the log-likelihood of estimation by accommodating difference in upper and lower tail dependence.

Bob (2013) combined copula functions, extreme value theory (EVT) and GARCH models to estimate portfolio value at risk (VAR) of stock indices from Germany, Spain, Italy and France. Asymmetric GARCH model and an EVT method were used to model the marginal distributions of log returns and copula function was used to link this marginal distributions into a multivariate distribution. Monte Carlo simulation approach was used to find estimates of portfolio VAR and backtesting methods were applied to determine goodness of fit. They observed that GARCH-EVT- student's t copula outperforms all other GARCH-EVT-copula models.

To calculate measure of risks and examine assumption of asymmetric dependence, Messaoud and Aloui (2015) used copula GARCH model. He used Glosten-Jagannathan-Runkle generalised autoregressive conditional heteroskedasticity (GJR-GARCH) model to deduce filtered residuals. To determine the empirical semi-parametric marginal cumulative distribution function, Messaoud and Aloui (2015) used an estimation of a generalized pareto distribution for the upper and lower tails. Copula functions were deployed to fit marginal distributions of filtered residuals and semi-parametric cumulative density function. Finally he applied the structure to compute value at risk and conditional value at risk.

Previous research have also proposed use of student t copula has the best fitting model for market data. (Jondeau and Rockinger, 2006) used the non parametric copulas to model conditional dependencies in international stock market. He used student t copula to capture tail dependence between stock market returns. (Church, 2012) used student t copula to investigate asymmetric dependence of asset return. He demonstrated that student t copula with individual degrees of freedom (SID) provides the best fit for market data as compared to other copula approaches. He went further and compared SID t copula and skewed t copula and concluded that SID t copula has a better fit with a single degree of freedom. Using Norwegian and Nordic geometric returns (Aas, 2004) used different copulas to model structural dependence of financial assets. He fitted a clayton, student t and Gaussian copula and observed that student t copula was the best fit relative to the others.

2.3.5 Application of Copula GARCH in BRICS

Different methodologies have also been applied to examine the comovements in BRICS stock market. Past studies have reported the existence of dependence between BRICS markets and other developed nations (Aloui et al., 2011). The paper did an empirical investigation of the extreme financial interdependences of some related emerging markets with the US. They observed a strong evidence of time-varying dependence between each of the BRICS markets and the US markets. They also observed high levels of dependencies in both bullish and bearish markets.

Using the concept of copulas, Vaz de Melo Mendes (2005) investigated the extent of integration between stock markets in emerging nations. Evidence of asymmetry was captured by both upper and lower tail dependencies. Caillault and Guegan (2005) used student and archimedian copulas to estimate tail dependence for the three tigers Thailand, Malaysia and Indonesia. They found that dependence structure is symmetric for Thailand and Malaysia and asymmetric for Thailand and Indonesia markets.

Time varying conditional copula approach (TVCC) was used by Hu (2010) to model dependence structure of Chinese and US stock markets. Findings showed that time varying does not always perform better than constant dependence model and in some short periods, upper tail dependence is much higher than lower tail dependence.

In his paper stock distribution in BRICSAdu et al. (2015) found out that Stock return distribution in BRICS is different from developed countries. The distribution was invariant to unit of measurement and returns also exhibit peakedness with fatter and longer tails.

2.3.6 Goodness of fit test

After estimating the parameters, goodness of fit test will be used to compare the copula models. There exists several goodness of fit tests for copula Genest et al. (2006); Fermanian (2005); Genest and Rémillard (2008). Goodness of fit tests have been used previously to evaluate fit of proposed copula model. This involves evaluating both the fitness of the margins and the overall fit of the copula model (Cherubini et al., 2004). Wang (2010) proposed two tests for parametric models belonging to the archimedian copula family. The test were based on fisher transformation of the correlation coefficient of a bivariate U, V. Based on simulations, they observed that both procedures perform well for large samples. Scaillet (2007) proposed a test statistic which is based on intergrated square difference between kernel estimator of copula density and kernel smoothed estimator of the parametric copula density. Genest et al. (2006) proposed a test focusing on blanket tests. The recommended use of a double parameter bootstrap procedure. Bob (2013); Li (2012) both used back testing for value at risk (VAR) to compare goodness of fit of the model.

Traditional methods of goodness of fit tests are Kolmogorov Smirnov tests and the Anderson-Darling tests as discussed in Kole et al. (2007). According to Kole et al. (2007), the tests compares dependence observed in the data and dependence implied in the data. Advantages of this is two tests is that they are applicable to any copula, They can be used to any dimensional copula and they show observed dependency can be captured accurately. Also instead of considering only a part of dependence pattern, the take into account complete dependence.

Use of information criterion has been widely used to determine the best fit copula parameter Ruppert (2011); **?**); Shams et al. (2013); Kamal and Haque (2016). Kamal and Haque (2016) used Akaike information criterion (AIC) to determine which copula is the best for estimating dependence between stock market and foreign exchange market. In this paper, three south Asian countries were used to examine degree of dependence of bivariate distribution of foreign exchange market return and stock market. Using AIC, they observed that gumbel copula reveals asymmetric dependence (upper tail), clayton reveals asymmetric tail dependence (lower tail) and student t copula exhibits symmetric tail dependence. **?** used AIC to determine which parametric copula has the best fit to capture asymmetry and non linearity in dependence structure. Focusing on clayton, gumbel and frank, they found out that gumbel copula has the best fit for dependence structure between two indices.

3 RESEARCH METHODOLOGY

This chapter discusses the dataset used, ARCH, GARCH and Copula GARCH methods that will be used to model the data,provide statistical inference and parameter estimations. Goodness of fit test was conducted to determine the best model for this data. The first three sections tackled the first objective. The aim was to come up with a copula GARCH model for BRICS and Kenya stock markets. The fourth section addressed different copula models that exists and they were compared empirically with the copula GARCH model developed using the stock market data. The last section determined the best fitting model using information criterion as proposed by (Church, 2012) .

3.1 Data

The data set used in this study comprises of daily closing prices of BRICS and Kenya stock market downloaded from Yahoo finance. The data is from the period January 2006 to February 2017. The stock indices used for the purpose of the study are as shown in Table 1 below.

Table 1: Stock Indices used

Country	Stock exchange (Abbreviation)	Index used (Abbreviation)
Brazil	Bolsa de Valores, Mercadorias & Futuros de São Paulo (BM&F BOVESPA)	IBOVESPA (BVSP)
Russia	Moscow Exchange	Russia Trading System Index (RTSI)
India	S&P Bombay Stock Exchange Sensitive Index	S & P BSE SENSEX
China	Hong Kong stock market (HKSE)	HANG SENG INDEX
South Africa	Johannesburg Stock Exchange (JSE)	FTSE/JSE Top 40 Stock Index
Kenya	NAIROBI STOCK EXCHANGE	NSE INDEX

3.2 Review of ARCH and GARCH

3.2.1 Autoregressive Conditionally Heteroscedastic (ARCH) Models

One way of testing the accuracy of models is by looking at the variations of the disturbance term. The ARCH model introduced by Engle (1982) addresses this issues. The model does not assume constant variance, it assumes that the conditional variance changes over time. Although the model parameters will still be unbiased, the standard error will give false impression of precision. ARCH model treats the error term as variance varying with time to be modelled (Engle, 2001).

let y_t be the stock market index. The return at time t is given by

$$r_t = \log\left(\frac{y_t}{y_{t-1}}\right) \tag{7}$$

where y_t is the price at time t and y_{t-1} is the price at time $t-1$.

The mean of returns is

$$\bar{r}_t = \frac{1}{n}\sum_{i=1}^{n} r_t \tag{8}$$

To capture volatility in the stock return, consider a linear regression model

$$r_t = \beta_0 + \beta_1 X_{t1} + \beta_2 X_{t2} + \beta_3 X_{t3} + \mu_t, \; \mu_t \sim N(0, \sigma_t^2) \tag{9}$$

To capture AR(q) with q lags we regressed

$$\sigma_t^2 = \beta_0 + \beta_1 \mu_{t-1}^2 + \beta_2 \mu_{t-2}^2 + \beta_3 \mu_{t-3}^2 \tag{10}$$

which is known an ARCH(q) model.
For an ARCH(1), we have

$$\mu_t = v_t \sigma_t, \; v_t \sim N(0,1) \tag{11}$$

$$\sigma_t^2 = \beta_0 + \beta_1 \mu_{t-1}^2 \tag{12}$$

where

$$\beta_0 > 0 \; and \; \beta_1 \geq 0.$$

Testing for ARCH effects
ARCH LM test was used to test for the ARCH effects that is the presence of autocorrelaton. LM test statistic is given by

$$LM = N.R^2$$

where N is the number of observations and R^2 is obtained from auxillary regression as squared multiple correlation coefficient. If one of the variables is highly statistically significant, value of R^2 will also be relatively high. The test statistics has a $\chi^2_{(m)}$ distribution, that is;

$$N \times R^2 \sim \chi^2_{(m)} \tag{13}$$

Hypothesis:

$$H_0 : \beta_0 = \beta_1 = 0$$
$$H_1 : \beta_i \neq 0$$

For this test if the $\chi^2_{(m)}$-test statistic in 13 is greater than tabulated value then we reject the null hypothesis. Incase where the error variance are correlated, there is an ARCH effect.

3.2.2 Generalized Autoregressive Conditional Heteroscedasticity (GARCH)

An extension of ARCH model proposed by Bollerslev (1986) and Taylor (1986). The model states that the conditional variance of u at time t depends on the squared error term and its conditional variance in the previous periods. That is GARCH(1,1) model is given by:

$$\sigma_t^2 = \beta_0 + \beta_1 \mu_{t-1}^2 + \alpha_1 \sigma_{t-1}^2 \tag{14}$$

where $\beta_0 \geq 0$, $\beta_1 > 0$ for $i = 1, \ldots, q$, and $\alpha_i \geq 0$ for $i = 1, \ldots,$

β_0 measures the reaction of conditional volatility to markets shocks, β_1 measures the persistence of conditional volatility and β_1 is large the volatility will take a long time to die out. $\alpha_1 + \beta_1$ measures the rate of convergence of conditional volatility to the long term unconditional volatility. The unconditional variance of u_t is given by

$$Var(u_t) = \alpha_0 \left[1 - (\alpha_0 + \beta_0)\right]^{-1} \tag{15}$$

where $\alpha_1 + \beta_1 < 1$ implies stationarity in variance. If $\alpha_1 + \beta_1 > 1$ which implies non stationarity in variance, the conditional variance forecast will not converge on unconditional value as horizons increases.

The model can be extended to GARCH(q,p) where q lagged terms are square error terms and p are terms of conditional variances;

$$\sigma_t^2 = \beta_0 + \sum_{i=1}^{q} \beta_i \mu_{t-i}^2 + \sum_{j=1}^{p} \alpha_j \sigma_{t-j}^2 \tag{16}$$

GARCH model is used to address the limitation of the ARCH model. It is more parsimonious and avoids overfitting. It also takes less consideration on breaching non–negativity constraints (Cuthbertson, 2004).

Many extensions and modifications of the GARCH modell have been formulated; in this research our focus will be on standard GARCH model,exponential GARCH model, GJR-GARCH model.

(i) Standard GARCH model
The standard GARCH(q,p) model denoted by sGARCH is given by;

$$\sigma_t^2 = \omega + \sum_{i=1}^{m} \varsigma_i u_{it} + \sum_{j=1}^{q} \beta_i u_{t-1}^2 + \sum_{j=1}^{p} \alpha_j \sigma_{t-j}^2 \tag{17}$$

where ω is the constant term and U_{it} denotes exogenous variables.

(ii) GARCH-M model
The volatility effect on the mean is incorporated under GARCH-M model. The GARCH-in-mean is given by

$$y_t = \mu + \delta\sigma_{t-1} + u_t, u_t \sim N(0, \sigma_t^2) \tag{18}$$

$$\sigma_t^2 = \beta_0 + \beta_1 \mu_{t-1}^2 + \alpha \sigma_{t-1}^2 \tag{19}$$

where δ can be interpreted as a risk premium since a positive and statistically significant δ can lead to increase in mean return due to increase in conditional variance leading to increase in risk.

(iii) Exponential GARCH model
Propose by Nelson,(1991). Its given by

$$\log(\sigma_t^2) = \beta_0 + \alpha \log(\sigma_{t-j}^2) + \gamma \frac{u_{t-1}}{\sqrt{\sigma_{t-j}}} + \beta_1 \left[\frac{| u_{t-1} |}{\sigma_{t-j}} - \sqrt{\frac{2}{\pi}} \right] \quad (20)$$

The model has advantages over the pure GARCH specifications. σ_{t-j} will be positive even if the parameters are negative since the $\log(\sigma_t^2)$ is modelled. Under eGARCH formulation, asymmetries are allowed since γ will be negative if the relationship between volatility and return is negative. eGARCH models employ conditionally normal errors because its easy in computational and intuitive interpretation rather than using generalised error distribution (GED) structure for the errors.

(iv) GJR-GARCH model It incorporated leverage effect by adding a parameter in the volatility equation for modelling asymmetric volatility clustering. The GJR-GARCH model that is Threshold GARCH model is written as

$$\sigma_t^2 = \beta_0 + \sum_{j=1}^{q} \beta_i u_{t-1}^2 + \sum_{j=1}^{p} \alpha_j \sigma_{t-j}^2 + \gamma u_{t-1}^2 \quad (21)$$

Where $I_{t-1} = 1$ if and only if $\varepsilon_{t-i} < 0$, denoting the indicator variable for negative ε_{t-i} where

$$\varepsilon_{t-i} = \begin{cases} 1 & \text{if } \varepsilon_{t-i} < 0 \\ 0 & \text{otherwise} \end{cases}$$

3.2.3 Estimation of GARCH models

Maximum Likelihood Estimation (MLE)
Since GARCH models are of non linear forms, ordinary least square (OLS) method cannot be applied hence maximum likelihood estimate (MLE) will be considered. Maximum likelihood estimate works by determining the most likely values of the parameters given the actual data.

Let $(x_1, x_2, \ldots, x_n)$ be a random sample of size n from a population where probability density function is $f(x, \theta)$. Then the likelihood function denoted by $L(x, \theta)$ is the joint density function of $(x_1, x_2, \ldots, x_n)$ that is;

$$\begin{aligned} L(x, \theta) =& f(x_1, \theta), f(x_2, \theta), \ldots f(x_n, \theta) \\ L(x, \theta) =& \prod_{i=1}^{n} f(x, \theta) \end{aligned}$$

The MLE $\hat{\theta}$ maximises the likelihood function $L(x, \theta)$ that is given the observed data, what is the value of the parameters containing useful information about the data. For a GARCH(q,p) model, assuming the shocks η has a conditional distribution, we construct joint density of $(x_1, x_2, \ldots, x_n)$ conditioned on x_0 and σ_0.

$$f_{x_1, x_2, \ldots, x_n | x_0, \sigma_0} (x_1, x_2, \ldots, x_n \mid x_0, \sigma_0) = \prod_{i=1}^{n} f_{x_t | x_{t-1}, \ldots, x_0, \sigma_0} (x_t \mid x_{t-1}, \ldots, x_0, \sigma_0)$$

The conditional likelihood will be

$$L(\alpha_0, \alpha_1, \beta_1, x) = \prod_{i=1}^{n} \frac{1}{\sigma_t} f_z\left(\frac{X_t}{\sigma_t}\right), \sigma_t = \sqrt{\alpha_0 + \alpha_1 X_{t-1}^2 + \beta_1 \sigma_{t-1}^2}$$

The log-likelihood function is

$$L(\alpha_0, \alpha_1, \beta_1, x) = \sum_{i=1}^{n} L(\theta); \quad \theta = (\alpha_0, \alpha_1, \beta_1)$$

The MLE maximises the log-likelihood by differentiating the log likelihood with respect to parameters θ

$$\frac{\partial}{\partial(\alpha_0, \alpha_1, \beta_1, x)} L(\alpha_0, \alpha_1, \beta_1, x) = \sum_{i=1}^{n} \frac{\partial L(\theta)}{\partial \theta}; \quad \theta = (\alpha_0, \alpha_1, \beta_1)$$

Model selection

Various GARCH specifications models will be estimated and information criterion such as AIC, BIC and will be used to pick the best fit model. A model with minimum AIC is preferred from a set of models. AIC can be expressed as;

$$AIC = -2\log(L) + 2k \tag{22}$$

where likelihood is probability of the data given a model, k represents free parameters in the model. BIC score gives the performance of model based on new dataset. It can be expressed as

$$BIC = -2\log(L) + d\log(W) \tag{23}$$

,

where W is the sample size of the intial dataset, d is the total number of parameters. A lower BIC in a set of models shows a better model. The term $d\log(w)$ is used to reduce the risk of over fitting which increases with increase in number of parameters.

3.2.4 Model diagnostics

It is necessary to explain how well the model can explain variations in dependent variables . Goodness of fit test is used to determine whether the model is a good measure and the significance of the parameters. For instance goodness of fit will be used to determine whether the GARCH model is a good model for measuring volatility. In this research, will consider different statistical tests to determine how good a fitted GARCH(1,1) model is.

Normality test

There are several test for normality which includes normal probability plots, histogram of residuals and jacque bera tests. In this research will focus on jarque bera test because it takes into consideration skewness of the data and peakedness of the data.

A Jacque bera test is used to test for normality of fitted residuals. It test whether coefficient of skewness and the coefficient of excess kurtosis are jointly zero. Skewness measures of the asymmetry of a probability distribution while kurtosis is the measure of the degree of peakedness of a distribution relative to the tails. A normal distribution is said to be symmetric and have a coefficient of kurtosis of 3.
Hypothesis

$$
\begin{aligned}
H_0 &: \text{The innovations } (\varepsilon_t) \text{ are normally distributed} \\
H_1 &: \text{The innovations are not normally ditributed.}
\end{aligned}
$$

The test statistic is computed as

$$
JB = N\left[\frac{s^2}{6} + \frac{(k-3)^2}{24}\right] \tag{24}
$$

Where N is the sample size, S is coefficient of skewness and k is coefficient of kurtosis. The statistic has a $\chi^2_{(2)}$ distribution hence we reject the null hypothesis if the p value is less than 0.05.

3.3 Copula functions

3.3.1 Copula

The word copula means "a tank", tie or bond. It was first employed by Abel,(1959) in the theorem describing the functions that join together one dimensional distributions to form one multivariate distribution functions.
Definition

A function $C : \mathbb{I}^d \longrightarrow \mathbb{I}$ is a copula if and only if the following properties hold:

- For every $j \in \{1, 2, \ldots, d\}$, $C(u) = u_j$ when all the components of u are equal to 1 with exception of j^{th} one that is equal to $u_j \in \mathbb{I}$;
- C is isotonic that is $C(u) \leq C(v)$ for all $u, v \in \mathbb{I}^d$ such that $u \leq v$;
- C is d-increasing.

Special case $d = 2$
A bivariate copula is a function $C : [0, 1]^2 \rightarrow [0, 1]$ such that;

- For all $u \in [0, 1]\,, C(u, 0) = C(0, u) = 0$
- $u \in [0, 1]\,, C(u, 1) = C(1, u) = u$
- For all $u, u', v, v' \in \prod$ with $u \leq u'$ and $v \leq v'$; $C(u', v') - C(u', v) - C(u, v') + C(u, v) \geq 0$

Sklars theorem
Consider two random variables X and Y with distribution functions

$$U = F(x) = P[X \leq x]$$

$$and$$

$$V = G(y) = P[Y \leq y]$$

. Since F and G are strictly increasing and continuous, we have

$$H(x, y) = P[X \leq x, Y \leq y]$$

The joint density H can be decomposed into products of the marginal densities f and g and the copula density, C. Consider $X = F^{-1}(U)$ and $Y = G^{-1}(V)$, since F and G are non decreasing and continous;

$$\frac{dx}{du} = \left[\frac{du}{dx}\right]^{-1} = \left(\frac{dF(x)}{dx}\right)^{-1} = [f(x)]^{-1}$$

and

$$\frac{dy}{dv} = \left[\frac{dv}{dy}\right]^{-1} = \left(\frac{dG(y)}{dy}\right)^{-1} = [g(y)]^{-1}$$

note that

$$\frac{dx}{dv} = \frac{dy}{du}$$

. Then

$$c(u, v) = \frac{h(F^{-1}(u), G^{-1}(v))}{f[F^{-1}(u)], g[G^{-1}(v)]}$$

.
From the above equation h will be expressed as a function of x and y using copula function C;

$$H(x, y) = f(x)g(y)c(F(x), G(y))$$

3.4 Copula families

Copulae can be classified into two broad categories namely archimedian copulae and elliptical copulae (Pfaff, 2013).

3.4.1 Elliptical copulas

Elliptical copulas use existing parametric distribution to capture dependence between variables. They are simple and simulations can be easily be carried out for copula model. According to Kurowicka et al. (2000) elliptical copulae are;

(i) Are continous and can have correlation values of $\rho \in (-1, 1)$

(ii) This copulae has linear regression and depicts equal correlation for partial and constant conditional correlations.

(iii) Ellipitical copulae combined with vines models can be used to represents high dimensional distributions in a better way.

.

Elliptical copulas has two forms; Gaussian copula and student t distribution.

Gaussian copula
Gaussian copula is based on normal distribution (Nelsen, 2003). It is defined as

$$C_\rho^{Ga}(v,z) = \Phi_{\sum}(\Phi^{-1}(v), \Phi^{-1}(z))$$

where $\sum$ is the bivariate covariance matrix, Φ is the cumulative density function of a standard normal distribution and $\Phi_{\sum}$ is the cumulative density function of a bivariate $\sim N(0,\sum)$.

Hence

$$C_\rho^{Ga}(v,z) = \int_{-\infty}^{\Phi^{-1}(v)} \int_{-\infty}^{\Phi^{-1}(z)} \frac{1}{2\pi\sqrt{(1-R_{12}^2)}} \exp\{\frac{R_{12}st - t^2 - s^2}{2(1-R_{12}^2)}\}dsdt$$

where R_{12} is the usual correlation coefficient of the corresponding bivariate normal distribution. Gaussian copula lacks both upper and lower dependence hence its application to risk modelling is limited.

Student t copula
Student t-copula has two dependence parameters, correlation coefficient R_{12} and degrees of freedom (v) where the later is responsible for controlling heaviness of tails. It is defines as

$$C_{v\Sigma}^t(u_1,u_2) = t_{v\Sigma}(t_v^{-1}(u_1), t_v^{-1}(u_2))$$

$$C_{v\Sigma}^t(u_1,u_2) = \int_{-\infty}^{(t_v^{-1}(u_1)} \int_{-\infty}^{t_v^{-1}(u_2)} \frac{1}{2\pi\sqrt{(1-R_{12}^2)}} \{1 + \frac{s^2 - R_{12}st + t^2}{v(1-R_{12}^2)}\}^{-\frac{v+2}{2}}$$

where Σ is correlation matrix,t_v is the cumulative distribution function and $t_{v\Sigma}$ is the cumulative distribution function of the multivariate distribution.
Student t-copula assumes asymmetric dependence for both the lower and upper tails of the joint distribution (Mensah et al., 2016). Thus there will be under estimation of dependence when there is asymmetric dependence since in most finance there is a stronger dependence between huge losses than huge gains.

3.4.2 Archimedian copulas

Archimedian copulas constitutes a class of both parametric and non parametric copulas used to model dependence between risks. Archimedian copulas have one dependency parameter and may take different forms.

Definition (Nelsen, 2007)
let φ be a continous strictly decreasing function from $[0,1]$ to $[0,\infty]$ such that $\varphi(1) = 0$ and let $\varphi^{[-1]}$ be the pseudo inverse of φ. Let C be the function from $[0,1]^2$ to $[0,1]$ given by $C(u,v) = \varphi^{[-1]}[\varphi(u) + \varphi(v)]$. Then C is a copula if and only if φ is convex.
The pseudo inverse of φ must be defined as

$$\varphi^{[-1]}(u) = \begin{cases} \varphi^{-1}(u) & 0 \le u \le \varphi(0) \\ 0 & \varphi(0) \le u \le +\infty \end{cases}$$

The different forms of archimedian copulas can be easily constructed and they have nice properties. They consists of Gumbel, Frank and Clayton.

Gumbel copula

Proposed in 1960 and is used to model asymmetry dependence. It is mostly used because of its ability to capture upper tail dependence and weak lower tail dependence.

The generating function is

$$\varphi(u) = [-\ln u]^{\theta}$$

Gumbel copula is

$$C_{\theta}^{Gu} = \exp\left[-([-\log u_1]^{\theta} + [-\log u_1]^{\theta})^{\frac{1}{\theta}}\right]$$

where θ is copula parameter restricted on the interval $[1, \infty)$. if $\theta \to \infty$ gumbel copula approaches the Frechet - Hoeffding upper bound and if $\theta \to 1$ then independence exists .If outcomes are strongly correlated at high values but less correlated at low values then gumbel copula is an appropriate choice.

Clayton copula

Clayton copula was introduced in 1978. It is mostly used because of the ability to capture lower tail dependence.

Copula generating function for Clayton is

$$\varphi(u) = (u^{-\theta} - 1)\theta^{-1}$$

Clayton copula is given by

$$C_{\theta}^{CL} = (u_1^{-\theta} + u_2^{-\theta} - 1)^{\frac{1}{\theta}}$$

where θ is the copula parameter.

If $\theta = 0$, then marginal distributions become independent. If $\theta \to \infty$ then it approximates the Frechet - Hoeffding upper bound.

Frank copula

The Frank copula (1979) exhibits no tail dependence because of its symmetry and allows maximum range of the dependence. It allows both positive and negative in the data. The copula generating function is given by

$$\varphi_{\theta}^{Fr}(u) = -\log\left[\frac{e^{\theta u} - 1}{e^{-\theta} - 1}\right]$$

Frank copula is given by

$$C_{\theta}^{Fr}(u_1, u_2) = -\theta^{-1}\log\left\{1 + \frac{(e^{-\theta u_1} - 1)(e^{-\theta u_2} - 1)}{[e^{-\theta} - 1]}\right\}$$

If $\theta \to +\infty$, then Frechet - Hoeffding upper bound will be attained and If $\theta \to -\infty$, then Frechet - Hoeffding lower bound will be attained. Frank copula attains independence case for $\theta = 0$. This copula is most suitable for modeling data characterized by weak tail dependence.

3.5 Copula GARCH models

Copula GARCH models are class of models where some of copula parameters are potentially time varying in auto-regressive manner conditioned on past information (Jondeau and Rockinger, 2006). Variance of individual returns can be modeled by fitting an AR-GARCH model. This is because financial returns are usually non-elliptical distributions hence applying conditional approach to filter out heteroscedastic behavior is important. Time varying copula allows copula parameters to vary over time which provides a better understanding of dependence structure.

In this paper suitable copula functions will be used to join uni-variate garch models. After constructing garch models and confirming that estimated parameters are statistically significant in the model, we obtain residuals to form a matrix. Copula functions will be used to model the residuals. Different copulas will be estimated for BRICS and Kenya returns and compare to determine which copula explains relation of any stock return of BRICS and Kenya.

3.6 Parameter estimation

To fit copula to our data, we have two estimation methods;

(a) Inference functions for margins (IFM)
The method proposed by Joe and Xu (2016) involves estimating parameter of marginal distributions and then estimating the parameter of the copula conditional on estimated parameters for the marginal distributions. Parameter of marginal distribution is estimated as follows;

(i) We get log likelihood function .
Given the probability density function (pdf) of univariate marginal distribution $f_i(x_i\theta_1)$ where θ_1 is the parameter of univariate marginal distribution

$$logl(\theta) = \sum_{i=1}^{n}\sum_{j=1}^{p} logf_i(x_{ij}, \theta_1). \tag{25}$$

(ii) We maximise the log likelihood function in order to get the parameter $\hat{\theta_1}$.

$$\hat{\theta_1} = argmax \sum_{i=1}^{n}\sum_{j=1}^{p} logf_i(x_{ij}, \theta_1) \tag{26}$$

(iii) We perform estimation of copula parameter θ_2

$$\hat{\theta_2} = argmax \sum_{t=1}^{n} logc[F_1(x_{i1}), F_2(x_{i2}), ..., F_n(x_{in}), \hat{\theta_1}] \tag{27}$$

The IFM estimator is defined as the vector $\hat{\theta_{IFM}} = (\hat{\theta_1}, \hat{\theta_2})$.

(b) Canonical maximum likelihood (CML)
In this method, the estimation of the parameter of copula is made without assuming any parametric form for marginal distributions. The sample data $(X_{1t}, ..., X_{nt})_{t=1}^{T}$ is transformed into uniform variates $(u_{1t}, ..., u_{nt})_{t=1}^{T}$ and then estimating the copula parameter. The procedure is as follows;

(i) Marginals are estimated using the empirical distribution

(ii) Copula parameters are estimated using the maximum likelihood estimation (MLE) approach.

$$\hat{\theta_2} = argmax_{\theta_2} \sum_{t=1}^{T} log\, c[\hat{F_1}(x_{i1}), \hat{F_2}(x_{i2}), ..., \hat{F_n}(x_{in}), \hat{\theta_1}] \tag{28}$$

3.6.1 Copula selection

Akaike information criterion (AIC), Schwarz Information Criterion (SIC) or Bayesian information criterion (BIC) will be used to compare the different copula models both parametric and non parametric copulas. AIC is calculates as shown in **??** above .

4 RESULTS AND DISCUSSIONS

4.1 Introduction

This section explores the application of the methods discussed in chapter 3. They include use of exploratory data analysis techniques on the data in order to achieve the objectives of the study. Statistical packages used for the analysis are R and excel. The first section of this chapter looks at the exploratory data analysis on BRICS and Kenya covering trading period between February 2010 to December 2016. The second section gives a summary of ARCH tests and univariate garch models, the third sections gives a summary of copula garch models and the last section compare the summary of the estimates of parametric and non parametric copula models.

4.2 Exploratory analysis

This section provides a summary of the data, correlation analysis as well as cross sectional analysis.

4.2.1 Data summary statistics

We empirically investigated the dependence between BRIC stock market and Kenya stock market return indices. Specifically data consists of emerging markets, Brazil, Russia, India, China and South Africa together with frontier economy represented by Kenya. The data is from February 2006 to August 2016 on a daily basis in different currencies for each country. The returns are calculated by taking log difference of the stock prices on two consecutive trading days.
First we look at summary statistics for the returns series by assessing the distribution of the return data.

From Table 2 China has the highest mean value of $(0.0112 \times 10^2 \pm 0.0451 \times 10^2)$ while India has the lowest mean value $(0.0538 \times 10^2 \pm 0.0302 \times 10^2)$. China has the highest variability $0.0531^2)$ while Kenya has the lowest variability SE 0.0077×10^2.

Table 2: Descriptive statistics (Percent)

	Brazil	Russia	India	China	South Africa	Kenya
mean	0.0332	0.0273	0.0538	0.0112	0.0466	0.0019
SE.mean	0.0352	0.0317	0.0302	0.0451	0.0251	0.0171
CI.mean.0.95	0.0691	0.0622	0.0593	0.0884	0.0492	0.0336
var	0.0324	0.0263	0.0239	0.0531	0.0165	0.0077
std.dev	1.8001	1.6204	1.5459	2.3053	1.2836	0.8760
coef.var	5,426.42	5,929.43	2,871.36	20,522.97	2,754.88	45,778.22

The chart in Figure 1 represents time series plots for BRICS countries. From the plot, all the markets showed an upward trend for the period between 2006 and 2008. From 2008 there was a bearish trend in all markets due to financial contagion experienced in US. as Russia was bearish between 2010 and 2012 but from 2013, the market was so volatile with big spikes hitting the lowest and highest closing prices for the year. India and South Africa have experienced an upward trend between 2009 and 2016. Russia and Brazil had bullish trends between 2009 and 2011 but lost momentum in 2011 and it fell with high volatility being observed in Brazil. China bounced back after the recession in 2008 and the bullish trend has been fairly well. Kenya took long to bounce back after the recession. After bouncing back in 2010 it fell again in 2011 but from 2012 it has maintained the bullish trend hitting the highest price since 2008.

Table 3: Jarque-Bera statistics

stock	**jacqbera.test**	**df**	**p.value**	**skewness**	**kurtosis**
Brazil	3903.79	2	0.00	0.24	5.97
Russia	12018.47	2	0.00	0.08	10.51
India	10066.73	2	0.00	0.34	9.59
China	8932.77	2	0.00	0.29	9.04
South Africa	1320.19	2	0.00	-0.09	3.48
Kenya	11813.13	2	0.00	0.83	10.28

The report from Table 3 also shows that none of the indices is normally distributed since Jarque-Bera statistics are highly significant and the data series are positively skewed and exhibit excess kurtosis. The summary statistics suggest that the probability distribution for BRICS are positively skewed indicating that right tails are fatter than the left tails.

Figure 2 in the appendix illustrates the variations in stocks in BRICS and Kenya stock market returns. It can be observed that the market were very unstable between 2006 and 2007 and between 2009 and 2010. After 2010 the market stabilized.

After checking the variations in the average return of each stock market, next is to check for presence of heteroskedasticity using second order moments (variance). The variance series is calculated by squaring the corresponding return series for each stock market. Figure 3 indicates presence of conditional heteroscedasticity of the squared series. It is highly experienced between 2008 and 2010 for all the six countries.

4.2.2 Correlation analysis

A scatter plot was used to investigate existence of any kind of relationship between the BRICS stock market and Kenya stock market returns as shown in Figure 4. There seems to be a relationship between Brazil and Russia, India and China, South Africa and China and India and South Africa. Each plot represent relationship between two stock markets. A positive correlation is represented by a positive slope while a negative correlation is represented by a negative slope. Presence of weak or zero correlation is represented by a random pattern.

Summary of correlation values of BRICS and Kenya stock market indices is provided in Table 4. JSE and BSE Sensex have the highest correlation coefficient of 0.9053. Hangseng and BSE Sensex have also a strong correlation of 0.6602. The focus of our study is between Kenya stock market returns and BRICS stock market returns. From Table 4 NSE appears to have a strong correlation with Hangseng and a weak correlation with Ibovespa. Only Kenya and Brazil stock market had a negative relationship but very weak.

Table 4: Correlation summary of stock indices

	Brazil	**China**	**India**	**Russia**	**South Africa**	**Kenya**
Brazil	1.00					
China	0.45	1.00				
India	0.17	0.66	1.00			
Russia	0.49	0.22	-0.33	1.00		
South Africa	0.10	0.58	0.91	-0.32	1.00	
Kenya	-0.11	0.56	0.25	0.33	0.26	1.00

4.2.3 Cross correlation analysis

Cross correlation analysis is helpful in identifying lags of the independent variables that might be useful in predicting the dependent variable. In our case we are interested in BRICS stock market data of past lags that might be useful in predicting NSE index returns. From Figure 5, a stable relationship from the past values of the NSE upto lag 0 is observed but the lag fades away as it decreases. This implies that Hangseng has a stronger influence on the past NSE values than on the future values. A similar relationship is observed between Micex and NSE.
An inverse proportional relationship is portrayed between NSE and Ibovespa. The correlation seems to be decreasing with increasing lags, which means that there was a stronger relationship in the past than in the present but fades off with positive lags. Ibovespa and NSE depicts positive correlation in past values than present values. This implies that India has a stronger influence on NSE in the past than in the present and a decline in relationship is observed with increasing negative lags implying a weaker relationship in future values of NSE.

4.2.4 Autocorrelation function (ACF) analysis

The autocorrelation function is used to check for linear dependence of a variable at different lags. From Figure 6, the sample ACF shows significant correlation at different lags for all the stock market returns. In Kenya stock market, NSE returns displays significant correlation with increasing lags. Ljung- Box test (Ljung and Box, 1978) is used to check whether auto correlation with different lags are zero. Ljung Box test statistics is defined by

$$Q_{LB} = n(n+2)\sum_{h=1}^{H}\frac{{\hat{\rho}_e}^2(h)}{n-h} \tag{29}$$

where ${\hat{\rho}_e}^2(h)$ is auto correlation function at lag h and H is the number of auto correlation that have been tested. For a significant level α, we reject the H_0 if ;

$$Q_{LB}(k) > \chi^2_{1-\alpha}(k)$$

From the results of ljung box test in Table 5, it was observed that there was a strong statistical evidence of rejecting the null hypothesis implying presence of auto correlation at 5% confidence level in all stock markets returns.

Table 5: P-value of ljung box test for return series at 5% confidence level

Stock	Box.stats	p.value
Brazil	30.76	0.00
Russia	25.94	0.00
India	12.74	0.00
China	2.38	0.00
South Africa	92.68	0.00
Kenya	390.71	0.00

Partial auto-correlation function (PACF) is also used to check for linear dependence at different lags but it removes the effect of intervening correlations. From Figure 7 below, all the stock returns appear to be serially correlated. NSE and JSE have a decreasing serial correlation with increasing lag as observed in Figure 7 below.

For conditional heteroscedasticity of return series, we plot the ACF of the squared returns which represents variance series. From 8 below, it is seen that the variance processes are largely correlated for each square return series which is significant for all the stock markets. All stock markets appears to have presence of ARCH effects during the first few lags but with increase in lags, the serial correlation decreases for all stock market returns with NSE decreasing at a faster rate than the rest hence becoming insignificant. From Figure 8, a GARCH model with lagged variances and lagged squared returns are better for modeling return series. ARCH LM test will thus be used to check for serial correlation in BRICS and Kenya stock market returns.

4.3 Univariate GARCH modelling

One of the objective of the study is to develop a copula GARCH model for the BRICS and Kenya stock market. This can be achieved by fitting a univariate GARCH model estimating the copula model. We will first test for the presence of ARCH effect in the mean of the daily returns for each stock market using lagrange multiplier tests (ARCH-LM tests). The test will cover lags 1, 5, 10 and 20.
Hypothesis;

$$H_o \text{ :No ARCH effect}$$
$$H_1 \text{ :Presence of ARCH effect}$$

;

Table 6: Lagrange multiplier test for ARCH effects daily stock returns.

Stock	Brazil	Russia	India	China	South Africa	Kenya
Arch1	71.07	144.11	37.75	311.39	120.90	518.27
df	1.00	1.00	1.00	1.00	1.00	1.00
p.value	0.00	0.00	0.00	0.00	0.00	0.00
Arch5	526.82	342.10	199.03	608.57	510.45	569.00
df	5.00	5.00	5.00	5.00	5.00	5.00
p.value	0.00	0.00	0.00	0.00	0.00	0.00
Arch10	734.06	417.32	276.27	678.90	641.06	579.20
df	10.00	10.00	10.00	10.00	10.00	10.00
p.value	0.00	0.00	0.00	0.00	0.00	0.00
Arch20	892.26	687.55	291.54	714.13	690.47	592.05
df	20.00	20.00	20.00	20.00	20.00	20.00
p.value	0.00	0.00	0.00	0.00	0.00	0.00

The results from table 6 shows that we reject Ho for presence of no ARCH effects at 5% level of significance for all the six countries. This provides evidence of presence of conditional heteroscedasticity in the mean of BRICS returns hence GARCH model that accounts for volatility needs to be employed in modeling BRICS and Kenya stock returns.

Table 7: AIC values of the GARCH(1,1) model under various specifications for each of the stock returns

	AIC					
Returns	**sGARCH**	**sGARCH-M**	**eGARCH**	**gjrGARCH**		**MIN**
Brazil	3.75	3.75	3.74	3.73		3.73
Russia	4.12	4.12	4.10	4.10		4.10
India	3.32	3.32	3.31	3.31		3.31
China	3.39	3.39	3.37	3.37		3.37
South Africa	3.01	3.01	2.98	2.99		2.98
Kenya	-7.06	-7.06	-7.05	-7.06		-7.06

A GARCH(1,1) model was fitted for each of BRICS stock returns and Kenya stock returns to capture volatility. AIC and BIC was used to compare the different specifications of GARCH models for each country's stock market return. A model with lower AIC or BIC was selected. Table 7 shows a summary of AIC for the six stock market returns . The table shows that eGARCH volatility model is selected for India returns and South Africa returns. GJrGARCH model is selected for Brazil, Russia, China and Kenya. Each stock return was then fitted with their respective GARCH model specifications.

Table 8: Estimates of the GARCH(1,1) parameters under various specifications for each of the stock returns

Returns	Parameters	Estimate	Std. Error	t value	Pr(¿—t—)
BRAZIL	μ	0.0023	0.0285	0.0799	0.9363
	α	0.0504	0.0133	3.7759	0.0002
gjrGARCH	α_1	0.0105	0.0079	1.3397	0.1803
	β_1	0.9185	0.0127	72.4004	0.0000
	γ_1	0.1102	0.0170	6.4983	0.0000
RUSSIA	μ	0.0185	0.0332	0.5572	0.5774
	α	0.0829	0.0167	4.9711	0.0000
gjrGARCH	α_1	0.0232	0.0087	2.6526	0.0080
	β_1	0.9067	0.0121	74.8841	0.0000
	γ_1	0.0993	0.0140	7.0705	0.0000
INDIA	μ	0.0448	0.0172	2.6034	0.0092
	α	0.0168	0.0030	5.6267	0.0000
eGARCH	α_1	-0.0796	0.0111	-7.1969	0.0000
	β_1	0.9803	0.0016	604.0744	0.0000
	γ_1	0.1949	0.0223	8.7226	0.0000
CHINA	μ	0.0245	0.0229	1.0703	0.2845
	α	0.0339	0.0074	4.5779	0.0000
gjrGARCH	α_1	0.0258	0.0074	3.4709	0.0005
	β_1	0.9087	0.0113	80.6763	0.0000
	γ_1	0.0959	0.0157	6.1176	0.0000
SOUTH AFRICA	μ	0.0223	0.0189	1.1814	0.2374
	α	0.0033	0.0026	1.2806	0.2004
eGARCH	α_1	-0.1163	0.0099	-11.7637	0.0000
	β_1	0.9857	0.0014	724.1415	0.0000
	γ_1	0.1030	0.0174	5.9127	0.0000
KENYA	μ	0.0222	0.0625	0.3554	0.7223
	α	1.7316	0.2405	7.2010	0.0000
gjrGARCH	α_1	0.2565	0.0265	9.6884	0.0000
	β_1	0.6788	0.0250	27.1582	0.0000
	γ_1	0.1274	0.0339	3.7598	0.0002

From Table 8, the parameter β_1 measure the persistence in conditional volatility irrespective of anything happening in the market. The β_1s are all positive and relatively large above 0.9 except stock market return for NSE which means that volatility takes long to decay for kenya stock market returns.
It can also be seen that the leverage effect γ are positive and statistically significant at 95% confidence level which implies that positive innovations are more destabilizing than negative innovations.
The sum of $\alpha_1 + \beta_1$ is less than one for all stock markets implying that there is no violation of stability requirement for GARCH(1,1). Volatility is persistent if $\alpha_1 + \beta_1 = 1$ and

less persistent if $\alpha_1 + \beta_1 < 1$ and explosive if $\alpha_1 + \beta_1 > 1$. It is observed that NSE has the highest volatility $\alpha_1 + \beta_1 = 0.9353$ whereas JSE has the lowest volatility persistent $\alpha_1 + \beta_1 = 0.8694$
. The GARCH(1,1) models for each stock return can be written as follows;

Brazil gjrGARCH(1,1) model

$$\begin{aligned} r_t(IBOVESPA) =& 0.00227(\pm 0.02845) + \varepsilon_t \\ \sigma_t^2 =& 0.0504(\pm 0.02845) + 0.01054(\pm 0.00786)\varepsilon_{t-1}^2 + 0.91851(\pm 0.01269)\sigma_{t-1}^2 \\ &+ 0.11021(\pm 0.01696)\varepsilon_{t-1}^2 I_{t-1} \end{aligned}$$

The asymmetry sign is positive and statistically significant at 95% confidence level.

Russia gjrGARCH(1,1) model

$$\begin{aligned} r_t(RTSI) =& 0.01854(\pm 0.0.0332) + \varepsilon_t \\ \sigma_t^2 =& 0.08294(\pm 0.01668) + 0.02318(\pm 0.08874)\varepsilon_{t-1}^2 + 0.90673(\pm 0.01211)\sigma_{t-1}^2 \\ &+ 0.09927(\pm 0.01404)\varepsilon_{t-1}^2 I_{t-1} \end{aligned}$$

The asymmetry sign is positive and statistically significant at 95% confidence level.

India eGARCH(1,1) model

$$\begin{aligned} r_t(BSE) =& 0.04482(\pm 0.01722) + \varepsilon_t \\ \log[\sigma_t^2] =& 0.01682(\pm 0.0299) - 0.0796(\pm 0.01106)\left[\frac{\varepsilon_{t-1}}{\sigma_{t-1}^2} - \sqrt{\frac{2}{\pi}}\right] + 0.98026(\pm 0.0.00162)\log(\sigma_{t-1}^2) \\ &+ 0.1949(\pm 0.02234)\frac{\varepsilon_{t-1}}{\sqrt{\sigma_{t-1}^2}} \end{aligned}$$

The asymmetry sign is positive and statistically significant at 95% confidence level.

China gjrGARCH(1,1) model

$$\begin{aligned} r_t(HANGSENG) =& 0.02449(\pm 0.02288) + \varepsilon_t \\ \sigma_t^2 =& 0.03394(\pm 0.00742) + 0.02577(\pm 0.00742)\varepsilon_{t-1}^2 + 0.90874(\pm 0.01126)\sigma_{t-1}^2 \\ &+ 0.09592(\pm 0.01568)\varepsilon_{t-1}^2 I_{t-1} \end{aligned}$$

The asymmetry sign is positive and statistically significant at 95% confidence level.

South Africa eGARCH(1,1) model

$$\begin{aligned} r_t(JSE) =& 0.02231(\pm 0.018882) + \varepsilon_t \\ \log[\sigma_t^2] =& 0.00332(\pm 0.0259) - 0.1163(\pm 0.00989)\left[\frac{\varepsilon_{t-1}}{\sigma_{t-1}^2} - \sqrt{\frac{2}{\pi}}\right] + 0.98573(\pm 0.00136)\log(\sigma_{t-1}^2) \\ &+ 0.10302(\pm 0.01742)\frac{\varepsilon_{t-1}}{\sqrt{\sigma_{t-1}^2}} \end{aligned}$$

The asymmetry sign is positive and statistically significant at 95% confidence level.

Kenya gjrGARCH(1,1) model

$$\begin{aligned} r_t(NSE) =& 0.02222(\pm 0.06253) + \varepsilon_t \\ \sigma_t^2 =& 1.73157(\pm 0.24046) + 0.25648(\pm 0.02647)\varepsilon_{t-1}^2 + 0.67882(\pm 0.025)\sigma_{t-1}^2 \\ &+ 0.12739(\pm 0.03388)\varepsilon_{t-1}^2 I_{t-1} \end{aligned}$$

The asymmetry sign is positive and statistically significant at 95% confidence level.

4.3.1 Diagnostic tests

Diagnostic tests are used to determine whether our fitted GARCH(1,1) model with different specifications for different stock markets are adequate. Three test were carried our, jarque bera (JB) test for testing normality of residuals, ljung box test for checking for serial correlation of fitted residuals. Under this test both residuals of returns and squared residuals of returns of each stock market of BRICS and Kenya. The final test is ARCH LM test which was used to check for presence of errors in the fitted residuals.

(i) Jacque bera test of normality
From Table 3 the Jacque bera test of normality rejects presence of normality on the fitted residuals for all stock market returns. A normal q-q plot was also used to support the presence of deviations from normality of each of the fitted residuals.

From Figure 9, all stock market returns shows signs of normality for large part of the residuals but departures from normality is observed at the tails for each stock market returns.

(ii) Ljung Box test
The residuals of different GARCH(1,1) specifications for the six countries were analyzed to check for auto correlation of the fitted residuals. From ljung box test, the null hypothesis is that the fitted residuals are identically independently distributed.

Table 9: Summary of Weighted Ljung-Box Test on Standardized and Residuals Squared Residuals

stock	ljung-Box (R)	p.value	ljung-Box (R^2)	p.value
Brazil	0.01	0.93	5.69	0.02
Russia	19.20	0.00	0.87	0.35
India	17.30	0.00	1.08	0.30
China	1.06	0.30	5.61	0.02
South Africa	0.35	0.55	1.80	0.18
Kenya	189.20	0.00	7.29	0.01

From the Table 9 test statistics for ljung box (R)and ljung box (R^2) are insignificant for Ibovespa, Hangseng and JSE implying that we fail to reject the null hypothesis for absence of serial correlation. For Micex, BSE SENSEX and NSE,we reject the null hypothesis that fitted residuals are independently distributed since the p value is less than 0.05 and conclude that that their is significant auto correlation at different lags for all the stock market returns residuals. For the squared return residuals, there is no siginificant autocorrelation for the three stock markets at 95% confidence level. The ACF of the standardized residuals as show in Figure 10 obtained for the different GARCH specifications for each country depicts a better model than that of the ACF of the raw stock market returns for all countries.

The ACF of squared standardized residuals also shows there are no significant spikes of the fitted residuals and there is no significant auto correlation in all stock markets residuals for the variances as shown in Figure 11.

4.4 Estimation of Copula parameter

After obtaining standardized residuals from the different specifications of GARCH models, next step was to model the marginals distribution using copula. The marginal standardized residuals were first transformed to uniform distribution. The copula families used in modeling the marginal standardized distributions are parametric copulas; clayton, frank, joe and gumbel copula and non parametric includes student t and Gaussian copula. Since our interest is on dependence structure between BRICS stock market and Kenya stock market, we consider the following market pairs ; Brazil-Kenya, Russia-Kenya, India-Kenya, China-Kenya and South Africa-Kenya and we estimate copula model parameters. The best fit copula parameter was determined using AIC and the log likelihood function. The results are as follows;

4.4.1 Copula Parameters Brazil-Kenya stock market

Table 10 presents a summary of different copula parameters. The student t copula provides the best fit since it has the lowest AIC and the highest log likelihood. The bivariate student t copula has two parameters, the degrees of freedom and correlation coefficient. From the table the correlation coefficient is -0.02164 and the degrees of freedom is 11.69825.

Table 10: Copula parameter estimation Brazil-Kenya

Copula family	Estimates	Log likelihood	AIC
t1	11.70	9.61	-15.22
t2	-0.02	9.61	-15.22
Gaussian	-0.03	1.13	-0.25
Frank	-0.09	0.31	1.38
Clayton	-0.01	0.05	1.90

4.4.2 Copula Parameters Russia-Kenya stock market

From Table 11 clayton copula provides the best fit since it has a lower AIC.

Table 11: Copula parameter estimation Russia-Kenya

Copula family	Estimates	Log likelihood	AIC
Clayton	0.05	3.36	-4.72
t1	0.02	4.18	-4.36
t2	17.28	4.18	-4.36
Normal	0.02	0.47	1.06
Frank	0.08	0.25	1.51
Gumbel	1.01	0.22	1.57
Joe	1.00	0.05	1.90

4.4.3 Copula Parameters India-Kenya stock market

From Table 12 Student t copula provides the best fitting copula since it has a lower AIC as compared to other copula parameters.

Table 12: Copula parameter estimation India-Kenya

Copula family	Estimates	Log likelihood	AIC
t1	0.02	29.12	-54.24
t2	6.17	29.12	-54.24
Clayton	0.05	2.66	-3.31
Gumbel	1.02	1.85	-1.70
Joe	1.02	1.55	-1.11
Frank	0.11	0.44	1.12
Normal	0.01	0.07	1.85

4.4.4 Copula Parameters China-Kenya stock market

From Table 13 Student t copula provides the best fitting copula since it has a lower AIC as compared to other copula parameters.

Table 13: Copula parameter estimation China-Kenya

Copula family	Estimates	Log likelihood	AIC
t1	0.000378	11.58	-19.16
t2	9.51	11.58	-19.16
Clayton	0.04	2.00	-2.00
Frank	-0.04	0.05	1.90
Normal	0.000335	0.00	2.00

4.4.5 Copula Parameters South Africa-Kenya stock market

From Table 14 Student t copula provides the best fitting copula since it has a lower AIC as compared to other copula parameters.

Table 14: Copula parameter estimation South Africa-Kenya

Copula family	Estimates	Log likelihood	AIC
t1	-0.02	29.60	-55.19
t2	6.10	29.60	-55.19
Normal	-0.03	1.34	-0.67
Frank	-0.09	0.29	1.41
Clayton	0.00	0.01	1.98

Chapter 5

5 CONCLUSION AND RECOMMENDATIONS

5.1 Discussions

Financial time series are characterized with fat tails, dependence and volatility clustering which presents a serious challenge when modeling. Modeling financial returns is necessary in order to understand volatility shocks between stock markets and degree of co-movements of different stock markets. This would help in diversification of risks and portfolio allocation.

In this thesis, copula GARCH is used to model dependence structure between BRICS stock market returns and Kenya stock market returns. The paper first discusses the short comings of Pearson correlation in measuring dependence which includes being invariant under strictly increasing transformations. Copula was used to examine the co movements between BRICS stock market return and Kenya stock market return between $2006-2016$ period. This method addresses the short comings of Pearson correlation by separating marginal distributions from the joint distributions. The paper focused on parametric copulas; Frank, Gumbel, Clayton and Joe copula and non parametric which has gaussian copula and student t copula.

Preliminary analysis is first carried out on BRICS and Kenya stock market returns to verify typical properties of returns such as normality. We found that the returns were heavily tailed and positvely skewed with presence of ARCH effects which was confirmed by Ljung box test. We fitted GARCH(1,1) with different specifications and AIC was used to select the GARCH(1,1) for each country stock market returns. GjrGARCH(1,1) was found to be the best fit for IBOVESPA, RTSI, Hangseng and NSE while eGARCH(1,1) was the best fitted model for BSE SENSEX and JSE. It was also observed that NSE had the highest volatility persistent of 0.9353 whereas JSE had the lowest volatility persistent of 0.8694. This results are similar to (Aloui et al., 2011; Kasman, 2009) where JSE was found to have a lower volatility persistent and China had the highest volatility persistent where only BRIC countries were considered in the research. The two models, gjrGARCH(1,1) and eGARCH(1,1) was used as filter for the data to provide residuals. The standardized residuals were used to estimate the marginal distribution for each series by transforming them to uniform distributions. Copula was used to fit the uniform distributions and the best fitting copula parameter was selected using AIC. The results show that the student t copula provides the best fit for all combinations of BRICS and Kenya stock market return. Similar results were achieved by (Jondeau and Rockinger, 2006; Church, 2012; Aas, 2004)

5.2 Conclusion

5.2.1 Volatility analysis

This section discusses volatility patterns in BRICS stock market returns and Kenya stock market return.

Brazil
Volatility is quite persistent since the coefficient is close to 1. $\alpha+\beta$ coefficients are positive in line with our constraints. γ coefficient representing asymmetry is also positive and statistically significant implying that bad news have less impact on returns. A different results was obtained in Aloui et al. (2011); Morales and Gassie (2011), the coefficient of asymmetry was negative implying that bad news have a strong effect on returns than positive news. The model was found not to be appropriate since α_1 was not statistically significant as it was also observed in (Morales and Gassie, 2011).

Russia
The coefficients $\alpha, \alpha_1, \beta_1, \gamma$ are positive and statistically significant in the model. This imply volatility persistent is higher since $\alpha+\beta$ is near 1. The asymmetry coefficient is also positive and statistically significant implying positive innovations are more destabilizing than negative innovations. A contrary feedback was observed in Aloui et al. (2011). The volatility persistent might be due to natural resources such as oil and gas which supports its economy much.

India
The coefficients α, β_1 are positive and statistically significant in the model. The ARCH effect is less than zero which contradicts our non negativity assumptions and findings in Aloui et al. (2011); Morales and Gassie (2011). The γ coefficient is positive and statistically significant which implies Indian market return is affected by asymmetric information hence its very necessary to differentiate positive and negative news since it may have an impact on returns.

China
The coefficients measuring volatility persistent are statistically significant at 95% confidence level which implies the return is affected by volatility persistent. Coefficient of asymmetry is positive and statistically significant implying bad news have less impact on returns.

South Africa
The coefficients $\alpha + \beta$ is less than one and statistically significant in the model. The volatility persistent is the weakest as compared to other BRICS nations. ARCH effect is negative and small implying that volatility will take a short time to die out. The coefficient of asymmetry is positive and statistically significant in the model hence impact of bad news on returns will be low.

Kenya
The coefficients $\alpha, \alpha_1, \beta_1, \gamma$ are positive and statistically significant in the model. The stock market return had the highest volatility persistent of 0.9358 which is near 1 implying that rate of convergence of conditional volatility to long term unconditional volatility is high. The asymmetry effect is positive and significant implying negative news has less impacts on returns.

5.2.2 Copula parameter analysis

The research considered two parametric copulas; Gaussian copula which has no tail dependence and student t copula which exhibits symmetric tail dependence and non parametric copula; Frank copula which has no tail dependence, Gumbel copula exhibiting upper tail dependence and Clayton copula showing lower tail dependence. Student t copula provided the best dependence for each market pair. This copula can be generalized to more than two assets which is an advantage for multivariate analysis. Student t copula also provides tail dependence better tail dependence than normal copula (Jondeau and Rockinger, 2006). The dependence is measured by copulas parameter hence from the Table 10-14, the parameters imply presence of tail dependence between BRICS and Kenya stock market. This implies that BRICS and Kenya stock markets may tend to rise and fall together during periods of economic boom and recession.

5.3 Suggestions for further research

Dependence structure of the stock market return in this paper was bi variate since we were looking at Kenya and the other countries individually that is; Brazil-Kenya, Russia-Kenya, India-Kenya, China-Kenya and South Africa Kenya. Combinations of the five countries to BRICS introduces a new dimension of multivariate hence to model this combined stock market return, further research should be conducted to incorporate multivariate analysis using joint dsitributions of BRICS. Use of multivariate GARCH models is also recommended for further research other than univariate GARCH models.

Use of different currency is a big set back since fluctuations of exchange rates is one of major factors in determining stock markets movements. For future research, the study can be conducted using a common currency for example use of Morgan Stanley Share Index (MSCI) since it will provide a base for comparisons and study of cointergration.

Use of parametric and non parametric methods indicates presence of heavy tails in the stock returns. We propose a semi parametric copula which should be able to take into consideration presence of upper and lower tail dependence. Also combining both Clayton and Gumbel copula should further be incorporated on this data set since it has the ability to capture both tail dependence.

Lastly, presence of co-movements between BRICS stock market and Kenya stock market should be of concerned to financial practitioners and regulators in making financial decisions and controlling financial contagion respectively. Hence further research should focus on specific time periods in order to ensure effectiveness in measurement and management of risk.

References

Aas, K. (2004). Modelling the dependence structure of financial assets: A survey of four copulas.

Adu, G., Alagidede, P., and Karimu, A. (2015). Stock return distribution in the brics. *Review of Development Finance*, 5(2):98–109.

Allen, F., Otchere, I., and Senbet, L. W. (2011). African financial systems: A review. *Review of Development Finance*, 1(2):79–113.

Aloui, R., Aïssa, M. S. B., and Nguyen, D. K. (2011). Global financial crisis, extreme interdependences, and contagion effects: The role of economic structure? *Journal of Banking & Finance*, 35(1):130–141.

Bob, N. K. (2013). Value-at risk estimation. a garch-evt-copula approach. *Mathematiska institutionen.*

Bouyé, E., Durrleman, V., Nikeghbali, A., Riboulet, G., and Roncalli, T. (2000). Copulas for finance-a reading guide and some applications. *Available at SSRN 1032533.*

Bozovic, M., Karapandza, R., Urosevic, B., and Advantage, R.-R. C. (2009). A copula approach to cross-market diversification. *London School of Economics & Political Science.*

Caillault, C. and Guegan, D. (2005). Empirical estimation of tail dependence using copulas: application to asian markets. *Quantitative Finance*, 5(5):489–501.

Chege, M. (2006). 2 economic relations between kenya and china, 1963–2007. *The Guardian (London).*

Cherubini, U., Luciano, E., and Vecchiato, W. (2004). *Copula methods in finance.* John Wiley & Sons.

Church, C. (2012). The asymmetric t-copula with individual degrees of freedom. *g.*

Cuthbertson, K. (2004). Introductory econometrics for finance, chris brooks, cambridge university press, cambridge, 2002.

Dhesi, L. X. G. (2009). Dependence structure analysis using copula-garch model: An application to uk and us stock markets. .

e Young, E. (2016). Staying the course. *Africa attractiveness program 2016.*

Embrechts, P., McNeil, A., and Straumann, D. (2002). Correlation and dependence in risk management: properties and pitfalls. *Risk management: value at risk and beyond*, pages 176–223.

Embrechts, P., Mcneil, E., and Straumann, D. (1999). Correlation: pitfalls and alternatives. In *Risk Magazine.* Citeseer.

Engle, R. (2001). Garch 101: The use of arch/garch models in applied econometrics. *The Journal of Economic Perspectives*, 15(4):157–168.

Engle, R. (2002). Dynamic conditional correlation: A simple class of multivariate generalized autoregressive conditional heteroskedasticity models. *Journal of Business & Economic Statistics*, 20(3):339–350.

Engle, R. F. (1982). Autoregressive conditional heteroscedasticity with estimates of the variance of united kingdom inflation. *Econometrica: Journal of the Econometric Society*, pages 987–1007.

Engle, R. F. and Kroner, K. F. (1995). Multivariate simultaneous generalized arch. *Econometric theory*, 11(01):122–150.

Fama, E. F. and French, K. R. (1993). Common risk factors in the returns on stocks and bonds. *Journal of financial economics*, 33(1):3–56.

Fermanian, J.-D. (2005). Goodness-of-fit tests for copulas. *Journal of multivariate analysis*, 95(1):119–152.

Forbes, K. J. and Rigobon, R. (2002). No contagion, only interdependence: measuring stock market comovements. *The journal of Finance*, 57(5):2223–2261.

Genest, C., QUESSY, J.-F., and Rémillard, B. (2006). Goodness-of-fit procedures for copula models based on the probability integral transformation. *Scandinavian Journal of Statistics*, pages 337–366.

Genest, C. and Rémillard, B. (2008). Validity of the parametric bootstrap for goodness-of-fit testing in semiparametric models. In *Annales de l'IHP Probabilités et statistiques*, volume 44, pages 1096–1127.

Grubel, H. G. (1968). Internationally diversified portfolios: welfare gains and capital flows. *The American Economic Review*, 58(5):1299–1314.

Henry, P. B. (2000). Stock market liberalization, economic reform, and emerging market equity prices. *The Journal of Finance*, 55(2):529–564.

Hu, J. (2010). Dependence structures in chinese and us financial markets: a time-varying conditional copula approach. *Applied Financial Economics*, 20(7):561–583.

Joe, H. and Xu, J. J. (2016). The estimation method of inference functions for margins for multivariate models. *•*.

Jondeau, E. and Rockinger, M. (2006). The copula-garch model of conditional dependencies: An international stock market application. *Journal of international money and finance*, 25(5):827–853.

Kamal, J. B. and Haque, A. E. (2016). Dependence between stock market and foreign exchange market in south asia: A copula-garch approach. *The Journal of Developing Areas*, 50(1):175–195.

Kariguh, B. W. (2014). The relationship between foreign investment activity and market return at the nairobi securities exchange. *h*.

Kasman, A. (2009). The impact of sudden changes on the persistence of volatility: Evidence from the bric countries. *Applied Economics Letters*, 16(7):759–764.

Khan, T. I., Barua, U., and Bhuiya, M. I. I. (2001). Brics economy: An appealing investment opportunity on the international stage. .

Knif, J. (2005). *What drives correlation between stock market returns? International evidence*. PhD thesis, Mays Business School.

Kole, E., Koedijk, K., and Verbeek, M. (2007). Selecting copulas for risk management. *Journal of Banking & Finance*, 31(8):2405–2423.

Kurowicka, D., Misiewicz, J., and Cooke, R. (2000). Elliptical copulae. In *Proc of the International Conference on Monte Carlo Simulation-Monte Carlo*, pages 209–214.

Lee, T.-H. and Long, X. (2009). Copula-based multivariate garch model with uncorrelated dependent errors. *Journal of Econometrics*, 150(2):207–218.

Li, Y. (2012). Garch-copula approach to estimation of value at risk for portfolios. .

Ljung, G. M. and Box, G. E. (1978). On a measure of lack of fit in time series models. *Biometrika*, pages 297–303.

Longin, F. and Solnik, B. (1995). Is the correlation in international equity returns constant: 1960–1990? *Journal of international money and finance*, 14(1):3–26.

Longin, F. and Solnik, B. (1999). Correlation structure of international equity markets during extremely volatile periods1. Technical report, Working Paper, Department of Finance, ESSEC Graduate Business School, France.

Longin, F. and Solnik, B. (2001). Extreme correlation of international equity markets. *The journal of finance*, 56(2):649–676.

Mashal, R. and Zeevi, A. (2002). Beyond correlation: Extreme co-movements between financial assets. *Unpublished, Columbia University.*

Mensah, J. O., Alagidede, P., et al. (2016). How are africa's emerging stock markets related to advanced markets? evidence from copulas. Technical report, Economic Research Southern Africa.

Mensi, W., Hammoudeh, S., Reboredo, J. C., and Nguyen, D. K. (2014). Do global factors impact brics stock markets? a quantile regression approach. *Emerging Markets Review*, 19:1–17.

Messaoud, S. B. and Aloui, C. (2015). Measuring risk of portfolio: Garch-copula model. *Journal of Economic Integration*, pages 172–205.

Morales, L. and Gassie, E. (2011). Structural breaks and financial volatility: Lessons from bric countries.

Nelsen, R. B. (1999). An introduction to copulas, volume 139 of lecture notes in statistics.

Nelsen, R. B. (2003). Properties and applications of copulas: A brief survey. In *Proceedings of the First Brazilian Conference on Statistical Modeling in Insurance and Finance,(Dhaene, J., Kolev, N., Morettin, PA (Eds.)), University Press USP: Sao Paulo*, pages 10–28. Citeseer.

Nelsen, R. B. (2007). *An introduction to copulas.* Springer Science & Business Media.

Nyangoro, O. (2013). Foreign portfolio flows and stock market performance in kenya: Case of nairobi securities exchange. In *A presentation made at the CSAE conference on Economic Development in Africa.*

Obere, J. A. (2009). *Essays on an emerging stock market: the case of Nairobi stock exchange (Statistical distribution of returns, market seasonality and reactions to dividend announcements.* PhD thesis, University of Nairobi.

Olukuru, L. J. (2005). *Time-varying conditional dependence in Nairobi Stock Market.* PhD thesis, University of Glasgow.

Patton, A. J. (2006). Modelling asymmetric exchange rate dependence. *International economic review*, 47(2):527–556.

Patton, A. J. et al. (2012). Copula methods for forecasting multivariate time series. *Handbook of economic forecasting*, 2:899–960.

Pfaff, B. (2013). *Financial Risk Modelling and Portfolio Optimization with R*. John Wiley and Sons.

Richardson, M. and Smith, T. (1993). A test for multivariate normality in stock returns. *Journal of Business*, pages 295–321.

Rong, N. and Trück, S. (2014). Modelling the dependence structure between australian equity and real estate markets-a copula approach. *Australasian Accounting Business & Finance Journal*, 8(5):93.

Ruppert, D. (2011). *Statistics and data analysis for financial engineering*. Springer.

Scaillet, O. (2007). Kernel-based goodness-of-fit tests for copulas with fixed smoothing parameters. *Journal of Multivariate Analysis*, 98(3):533–543.

Shams, S., Haghighi, F. K., et al. (2013). A copula-garch model of conditional dependencies: Estimating tehran market stock exchange value-at-risk. *Journal of Statistical and Econometric Methods*, 2(2):39–50.

Silvennoinen, A. and Teräsvirta, T. (2012). Modelling conditional correlations of asset returns: A smooth transition approach. .

Singh, S. P. and Dube, M. (2014). Brics and the world order: A beginner's guide. *Available at SSRN 2443652*.

Tse, Y. K. and Tsui, A. K. C. (2002). A multivariate generalized autoregressive conditional heteroscedasticity model with time-varying correlations. *Journal of Business & Economic Statistics*, 20(3):351–362.

Van den Goorbergh, R. et al. (2004). A copula-based autoregressive conditional dependence model of international stock markets. Technical report, Netherlands Central Bank, Research Department.

Vaz de Melo Mendes, B. (2005). Asymmetric extreme interdependence in emerging equity markets. *Applied Stochastic Models in Business and Industry*, 21(6):483–498.

Wang, A. (2010). Goodness-of-fit tests for archimedean copula models. *Statistica Sinica*, pages 441–453.

Wang, H. and Cai, X. (2011). A copula based garch dependence model of shanghai and shenzhen stock markets. *Unpublished D-level Essay in Statistics, Dalarna University, Sweden (June 2011), http://www. statistics. du. se/essays/D11_HuilingWang_XinhuaCai. pdf (accessed February 10, 2013).*

Yang, L., Cai, X. J., Li, M., and Hamori, S. (2015). Modeling dependence structures among international stock markets: Evidence from hierarchical archimedean copulas. *Economic Modelling*, 51:308–314.

A Appendix

A.1 List of figures

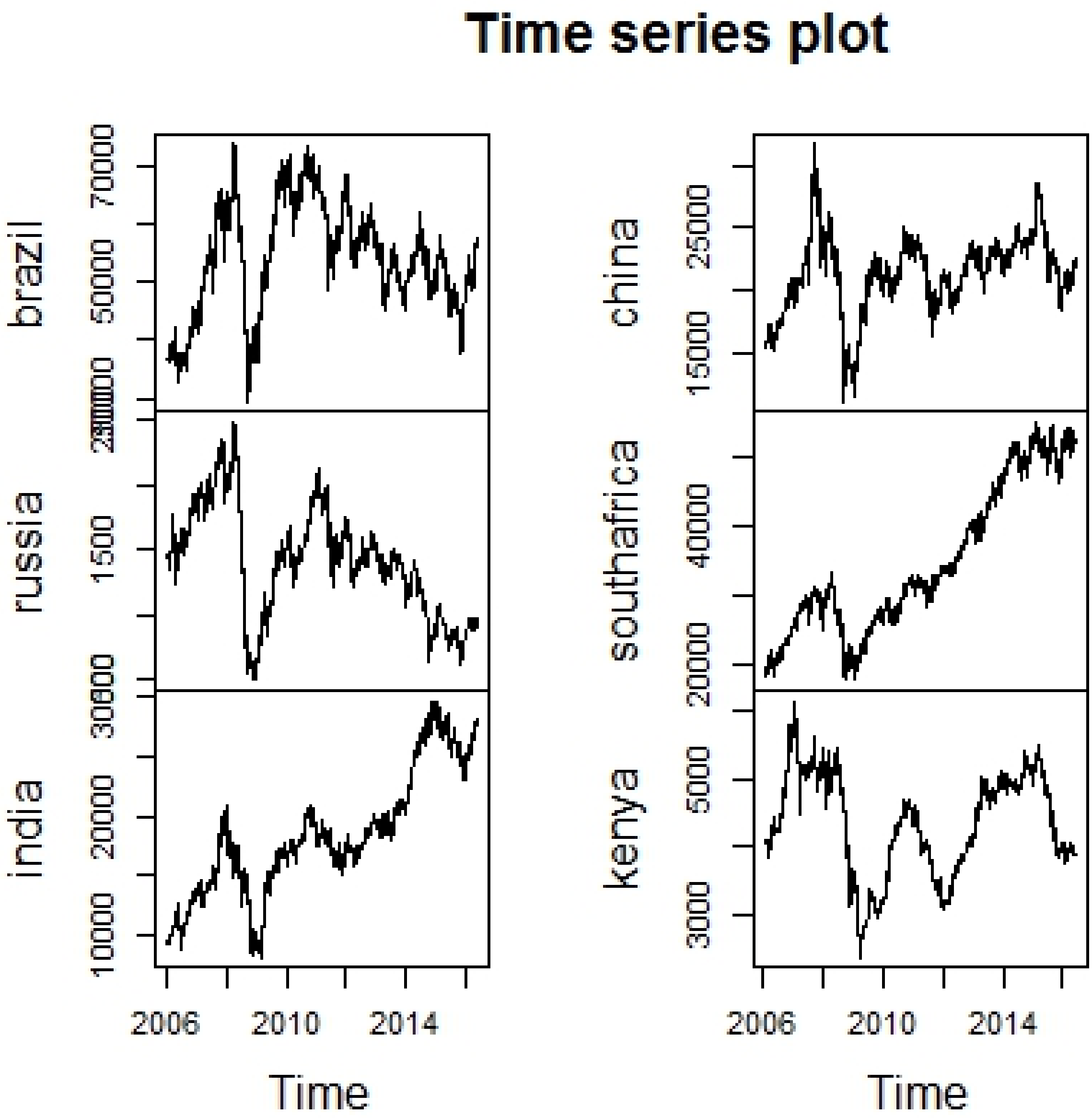


Figure 1: Time series plots

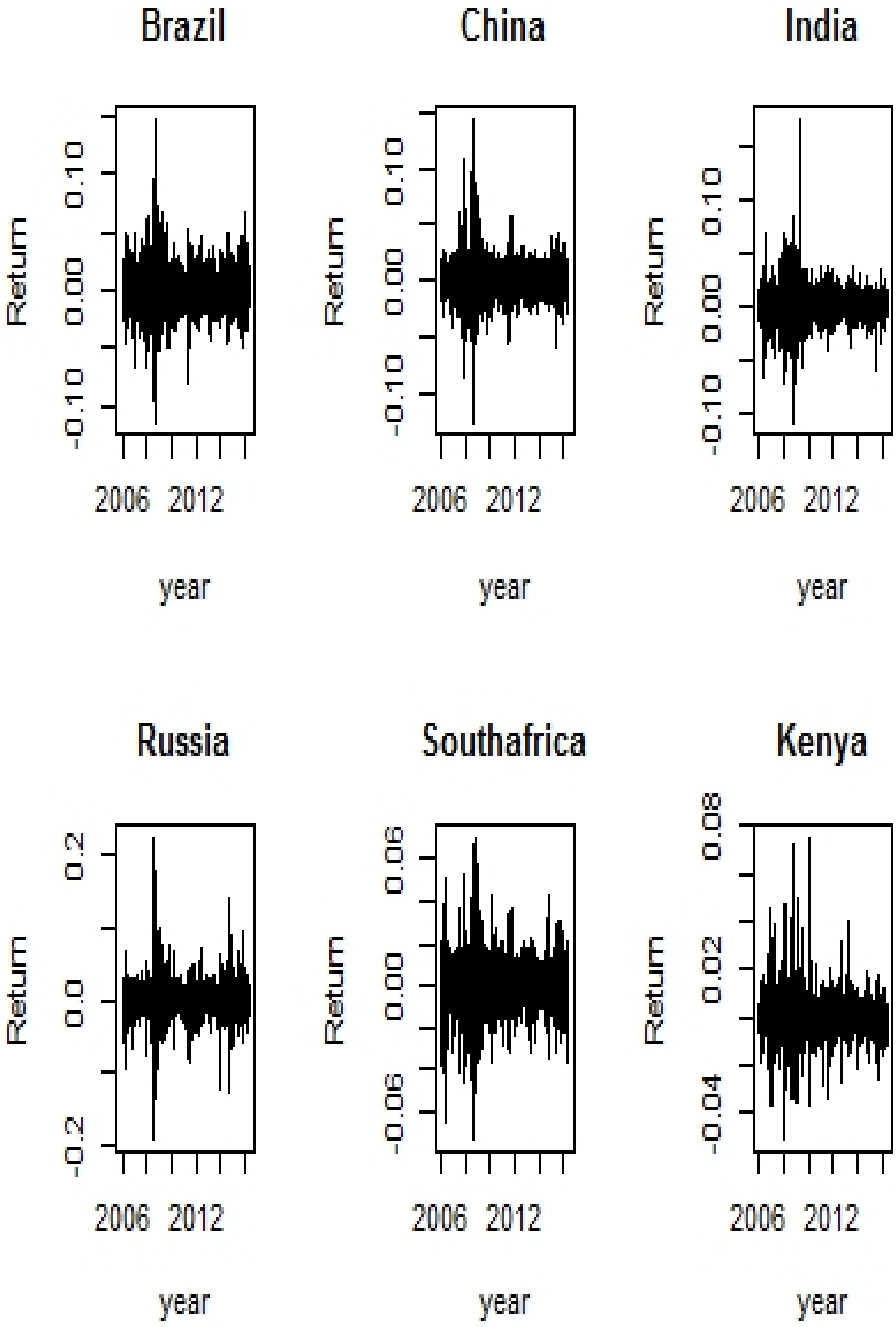


Figure 2: Time path of daily returns for Brazil, Russia, India, China, South Africa and Kenya

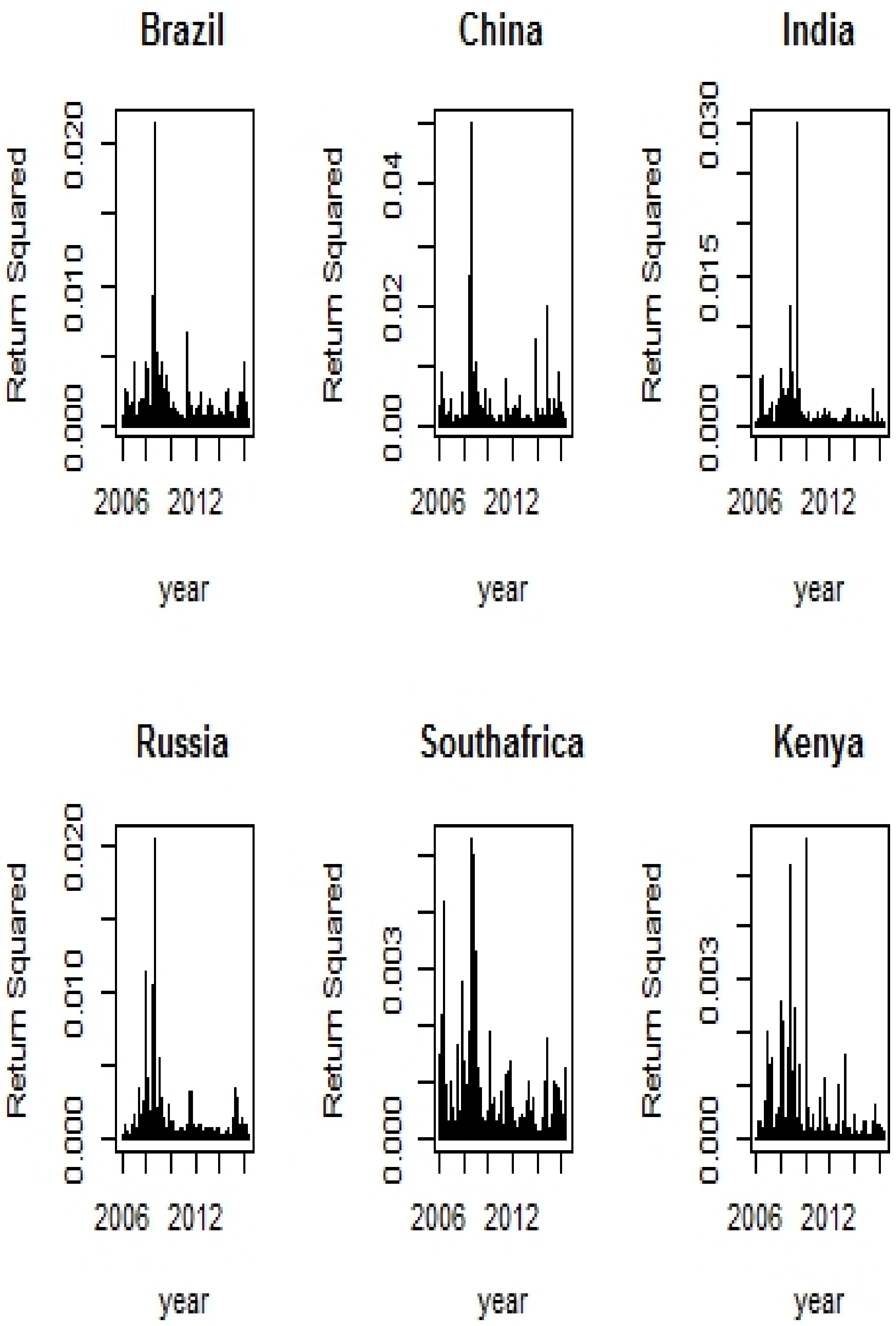


Figure 3: Time path of daily squared returns for Brazil, Russia, India, China, South Africa and Kenya

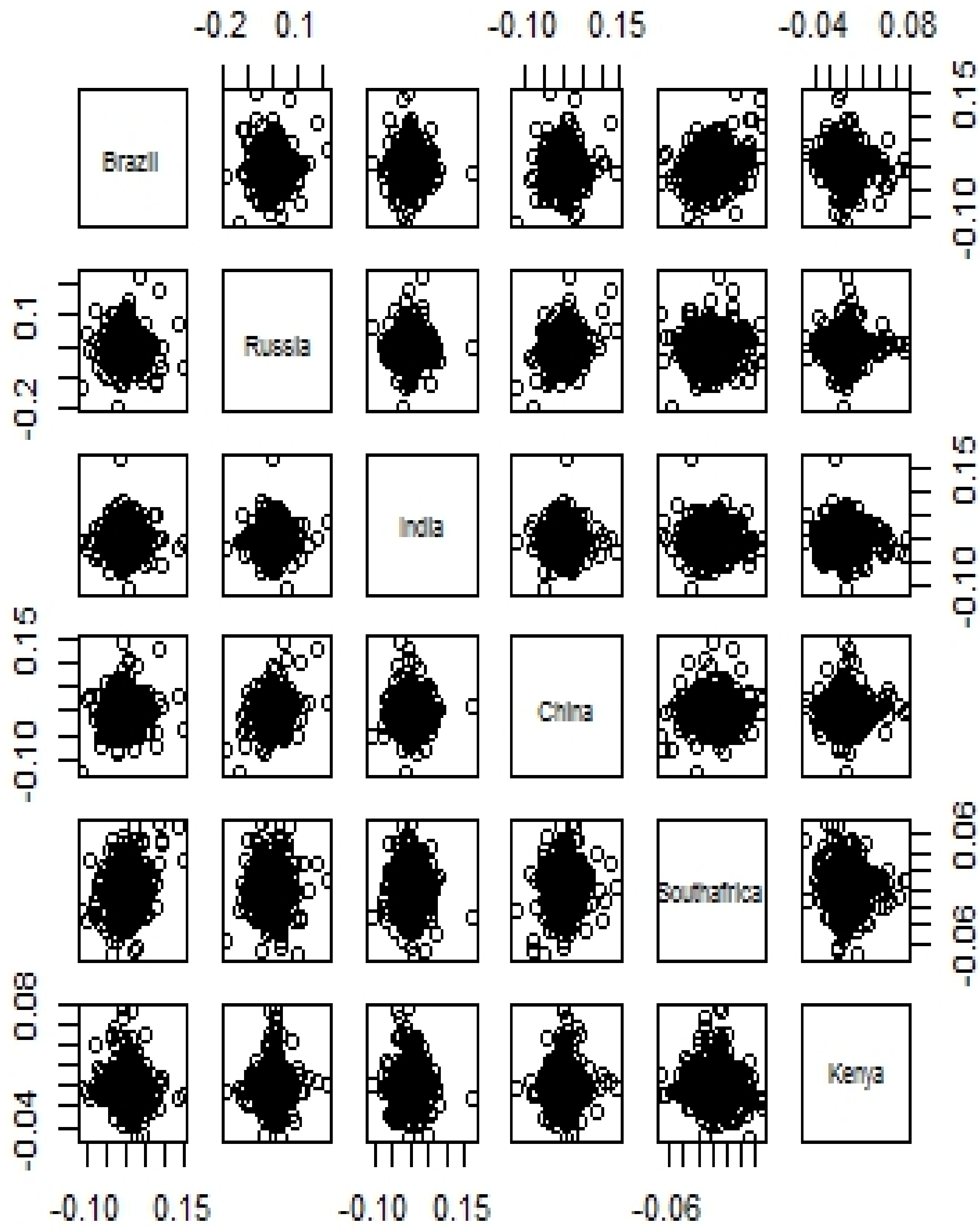


Figure 4: Scatter plot

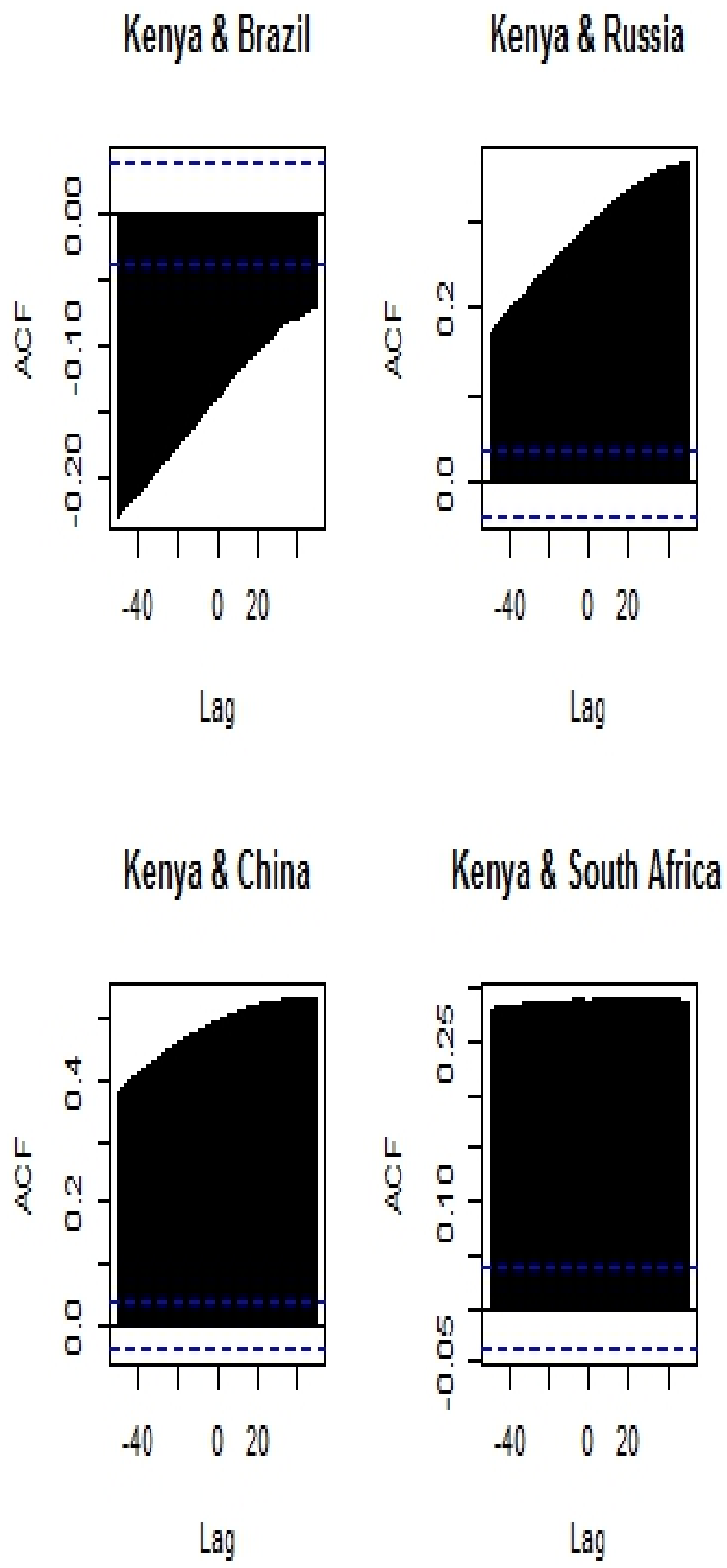


Figure 5: Cross-correlation plots

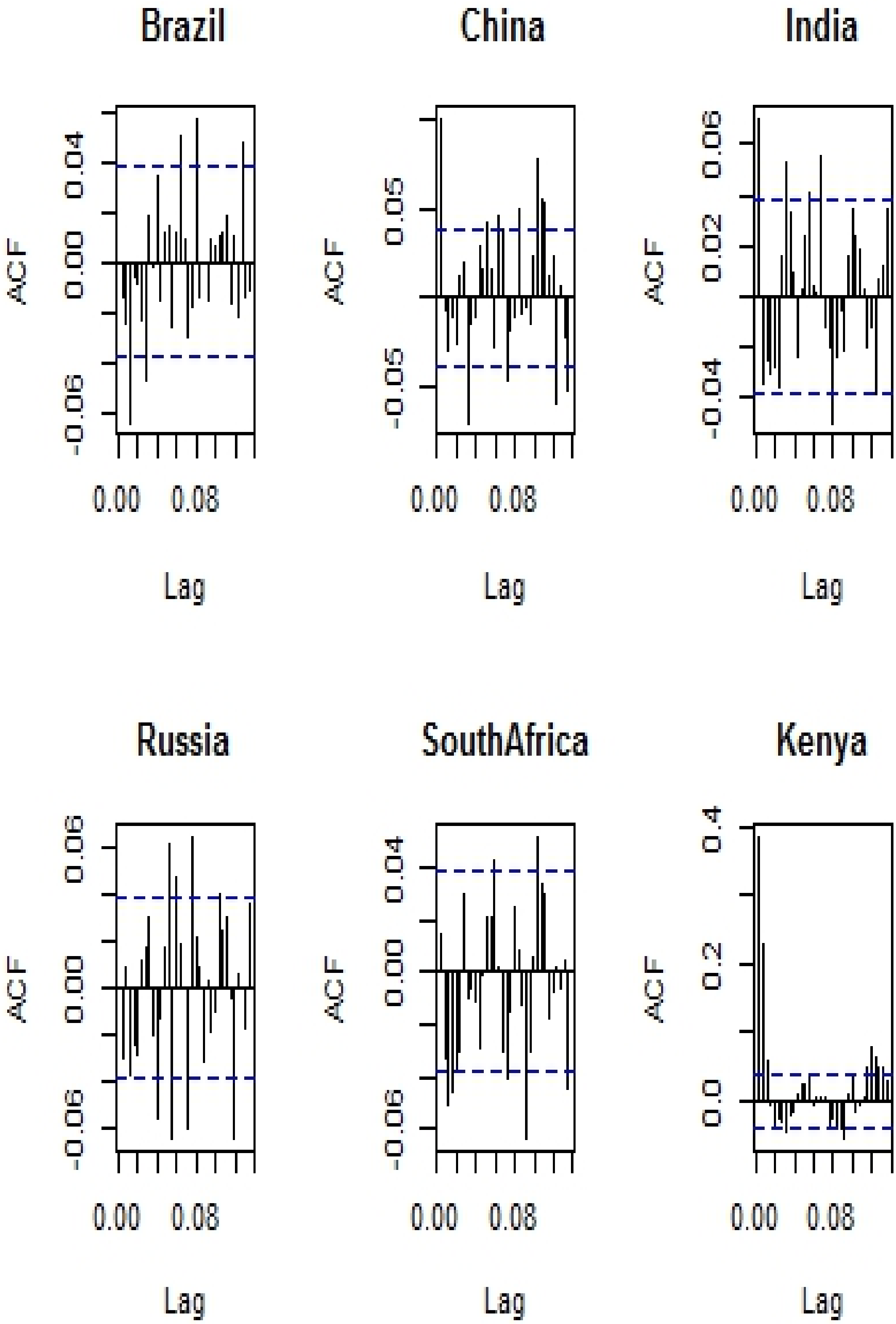


Figure 6: Auto Correlation Function (ACF)of returns series

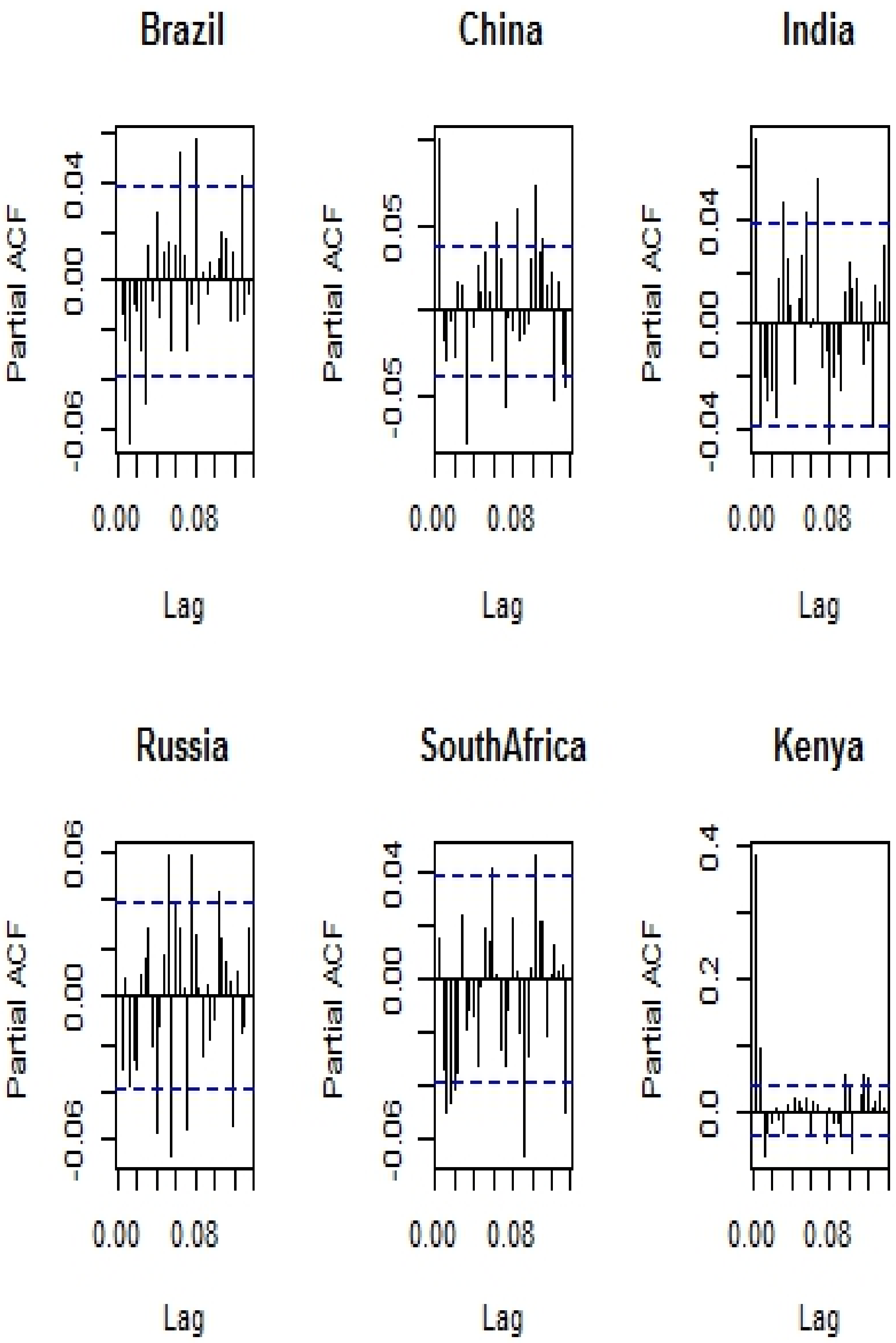


Figure 7: Partial Auto Correlation Function (PACF) of returns series

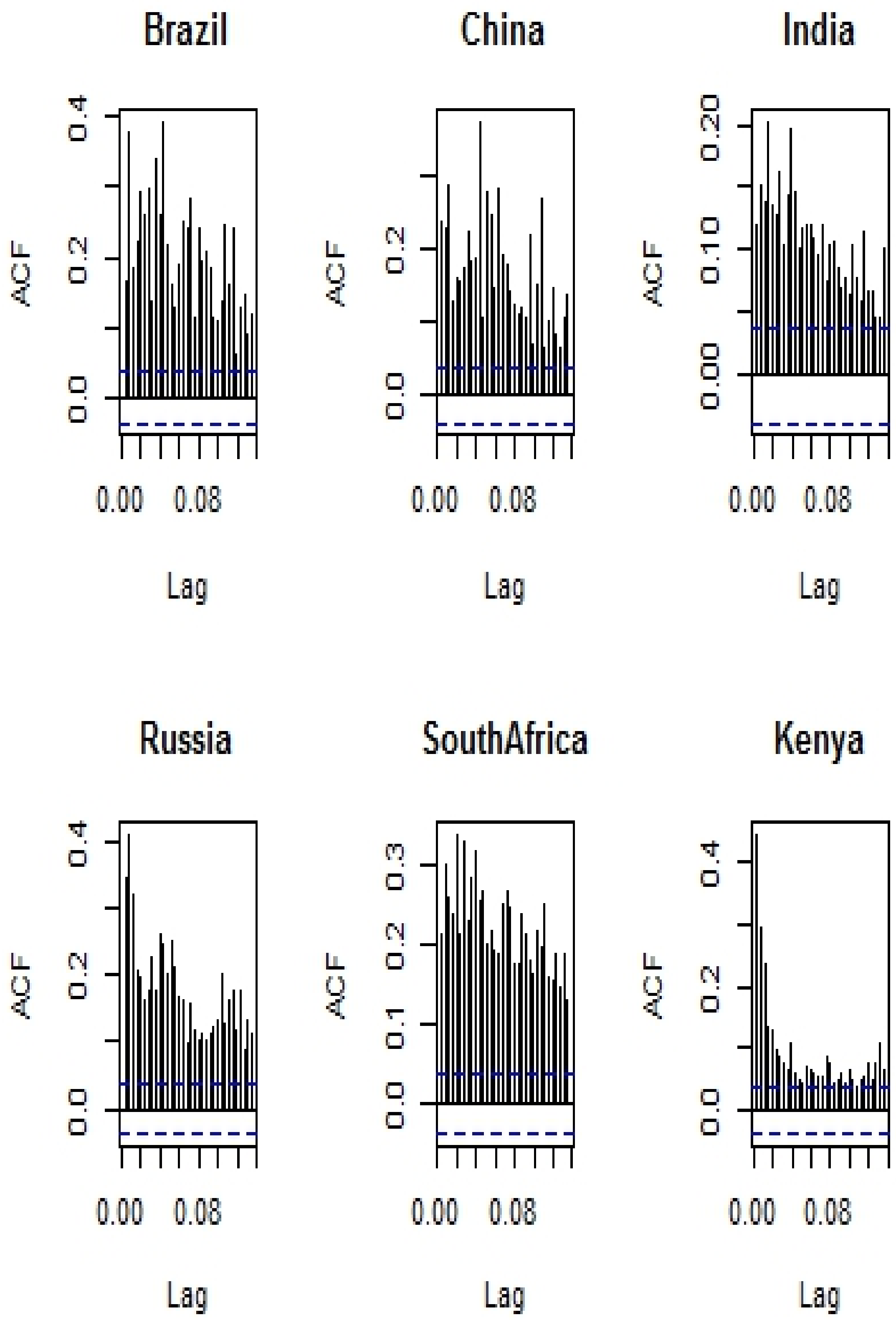
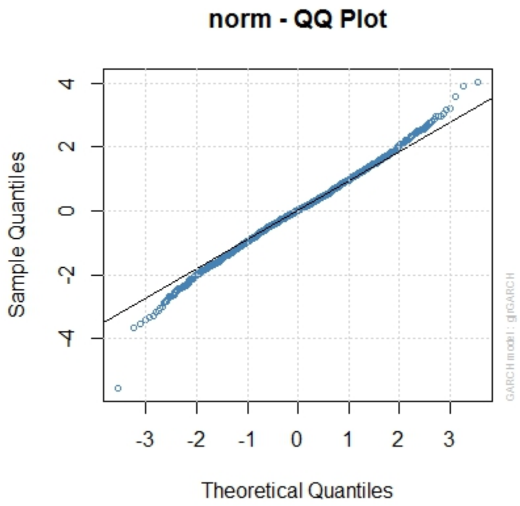
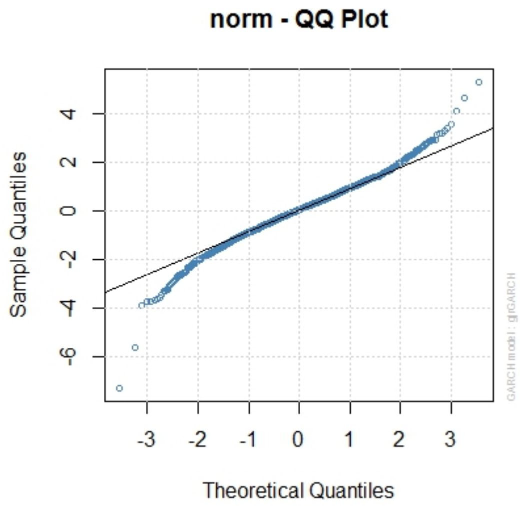


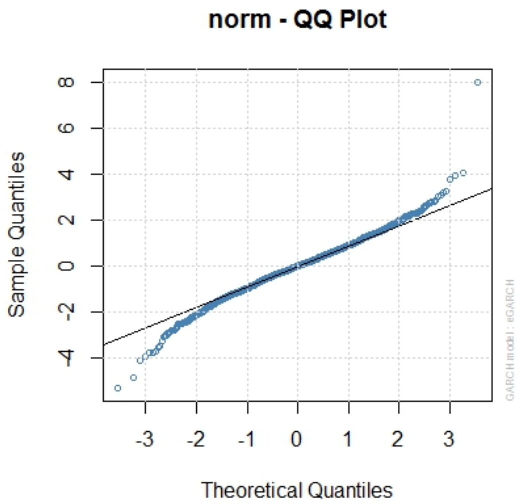
Figure 8: Auto Correlation Function (ACF) of squared returns series



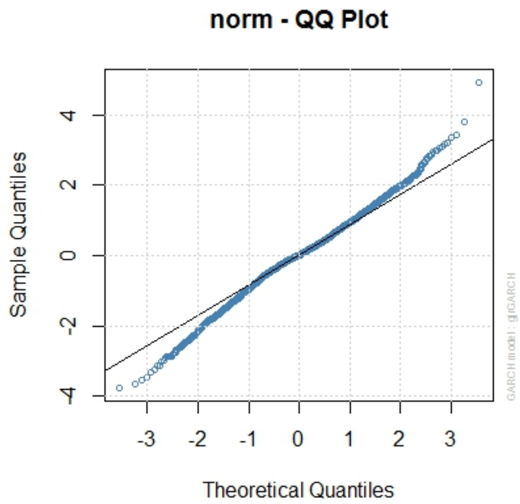
(a) Ibovespa



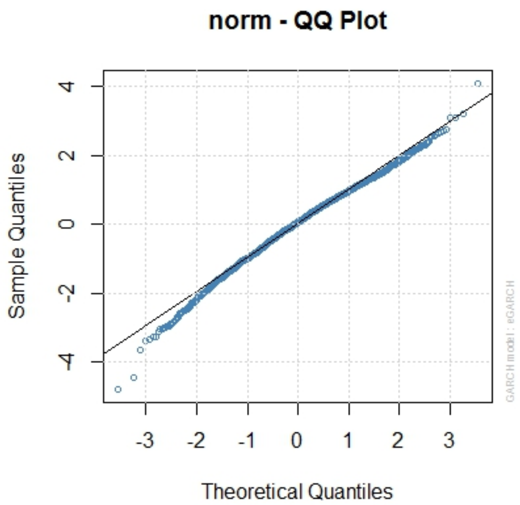
(b) RTSI



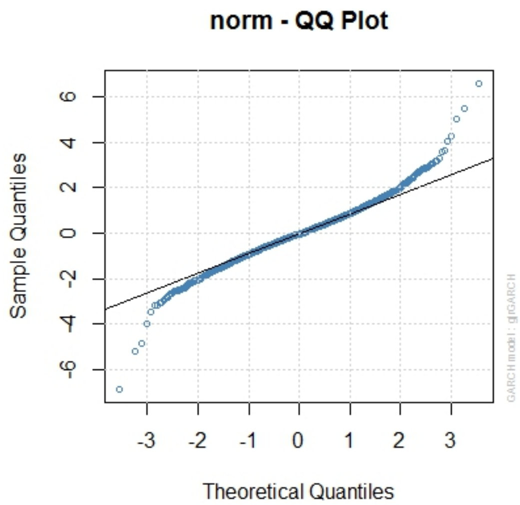
(c) BSE SENSEX



(d) HANGSENG

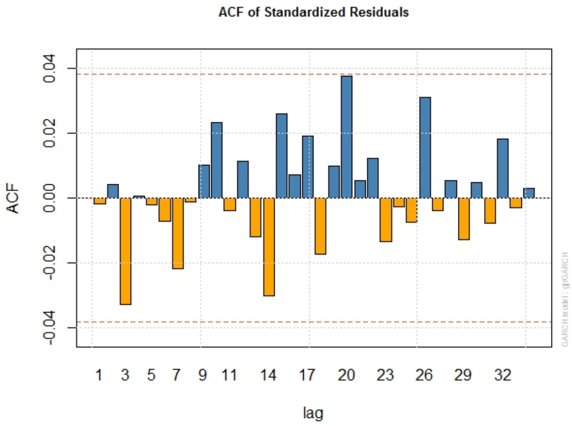


(e) JSE

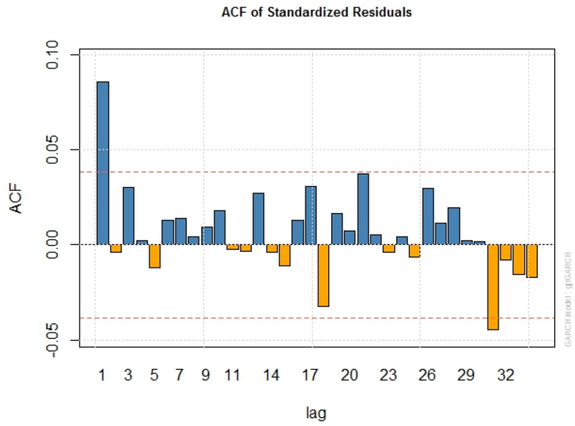


(f) NSE

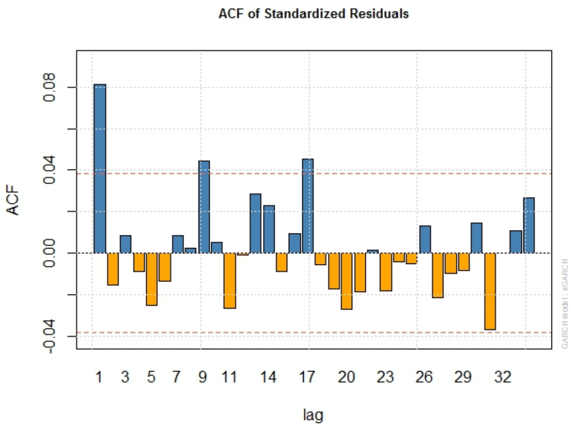
Figure 9: Normality Q-Q plots of the GARCH(1,1) residuals; BRICS and Kenya stock returns



(a) Ibovespa



(b) RTSI



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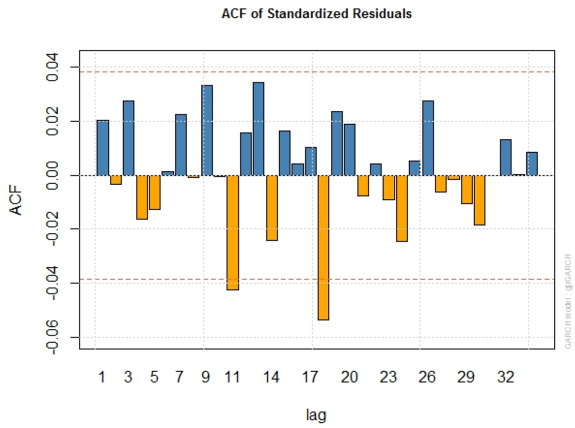

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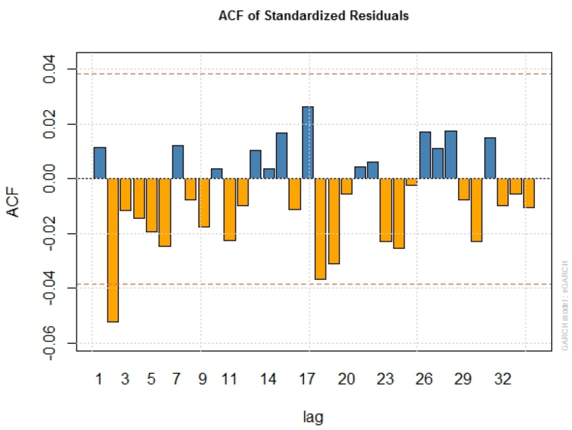
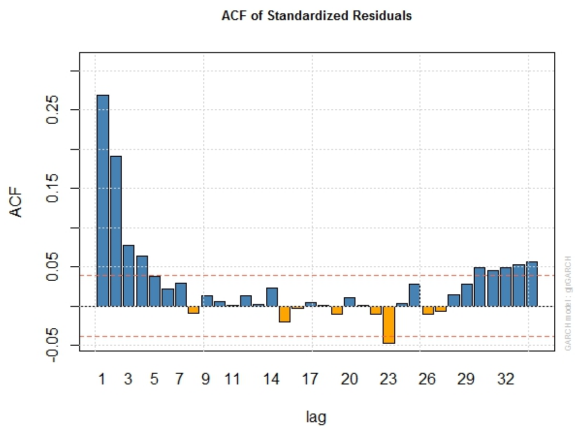
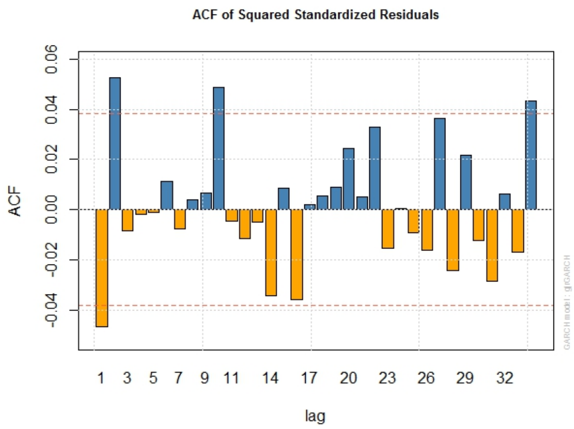


(e) JSE

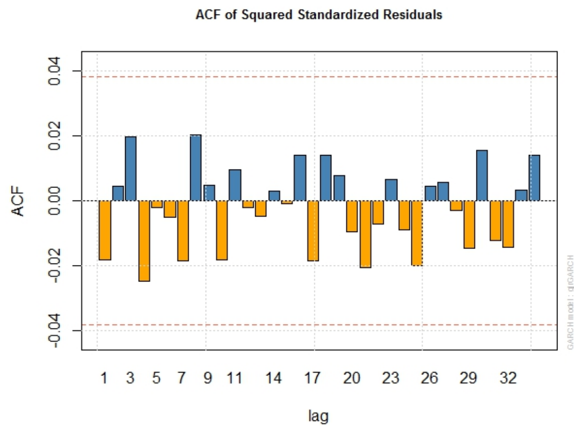


(f) NSE

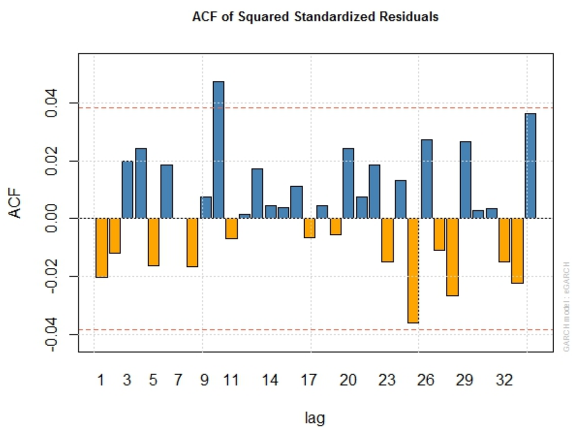
Figure 10: Auto Correlation Function (ACF) of standardized residuals



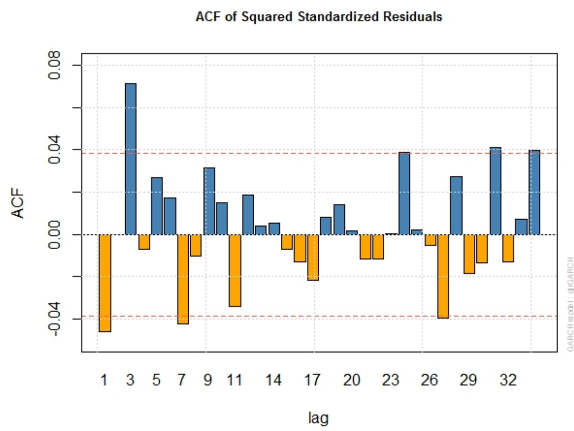
(a) Ibovespa



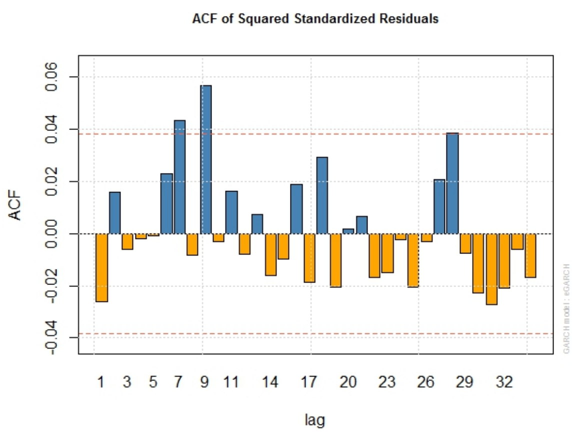
(b) RTSI



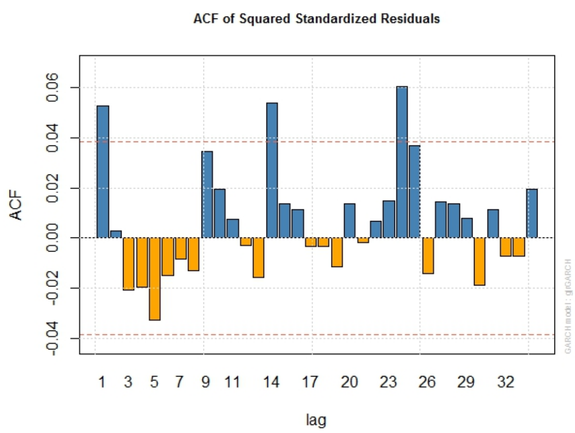
(c) BSE SENSEX



(d) HANGSENG



(e) JSE



(f) NSE

Figure 11: Auto Correlation Function (ACF) of squared standardized residuals

A.2 data

```
setwd("C:/Users/User/Google Drive/r")
data=read.csv("data1.csv")
%b=as.numeric(data$brazil)
%c=as.numeric(data$china)
%i=as.numeric(data$india)
%r=as.numeric(data$russia)
%s=as.numeric(data$southafrica)
%k=as.numeric(data$kenya)
%pairs(data[,-1])
```

A.3 Time series plots

```
rb=ts(b,frequency=252,start=c(2006,1))
rc=ts(c,frequency=252,start=c(2006,1))
ri=ts(i,frequency=252,start=c(2006,1))
rr=ts(r,frequency=252,start=c(2006,1))
rs=ts(s,frequency=252,start=c(2006,1))
rk=ts(k,frequency=252,start=c(2006,1))
data2=data.frame(rb,rc,ri,rr,rs,rk)
```

A.4 Getting returns

```
rtb=diff(b)/b[-length(b)]
rtc=diff(c)/c[-length(c)]
rti=diff(i)/i[-length(i)]
rtr=diff(r)/r[-length(r)]
rts=diff(s)/s[-length(s)]
rtk=diff(k)/k[-length(k)]
data2.ts=ts(data2,start=c(2006,12),frequency=252)
%plot(data2.ts)
%pairs(data2.ts[,-1])
```

```
#data summary statistics
round(stat.desc(data2.ts,basic=F),4)
%round(stat.desc(data5,basic=F),4)
%round(stat.desc(data4,basic=F),4)
%round(stat.desc(data4,basic = T),2)
```

A.5 ljung box test statistics

```
boxb=Box.test(rtnb,type="Ljung",lag=1,fitdf=1)
boxr=Box.test(rtnr,type="Ljung",lag=1,fitdf=1)
boxi=Box.test(rtni,type="Ljung",lag=1,fitdf=1)
boxc=Box.test(rtnc,type="Ljung",lag=1,fitdf=1)
boxs=Box.test(rtns,type="Ljung",lag=1,fitdf=1)
```

```
boxk=Box.test(rtnk,type="Ljung",lag=1,fitdf=1)
boxb=Box.test(rtnb,type="Ljung",lag=1,fitdf=1)
box=c(boxb,boxr,boxi,boxc,boxs,boxk)

stock=c("ibosvespa","Micex","Bse sensex","Hangseng","Jse","Nse")
box.stats=c(boxb$statistic,boxr$statistic,boxi$statistic,boxc$statistic,boxs$statis
df=c(boxb$parameter,boxr$parameter,boxi$parameter,boxc$parameter,boxs$parameter,box
p.value=c(boxb$p.value,boxr$p.value,boxi$p.value,boxc$p.value,boxs$p.value,boxk$p.v
box.test=data.frame(stock,box.stats,df,p.value)
```

A.6 ARCH LM test lag 1

```
archb1=ArchTest(rtnb,1)
archr1=ArchTest(rtnr,1)
archi1=ArchTest(rtni,1)
archc1=ArchTest(rtnc,1)
archs1=ArchTest(rtns,1)
archk1=ArchTest(rtnk,1)
arch1=c(archb1,archr1,archi1,archc1,archs1,archk1)
stock=c("ibosvespa","Micex","Bse sensex","Hangseng","Jse","Nse")
Arch1=c(archb1$statistic,archr1$statistic,archi1$statistic,archc1$statistic,archs1$
df1=c(archb1$parameter,archr1$parameter,archi1$parameter,archc1$parameter,archs1$pa
p.value1=c(archb1$p.value,archr1$p.value,archi1$p.value,archc1$p.value,archs1$p.val
arch.test1=data.frame(stock,Arch1,df1,p.value1)
```

A.7 GARCH models

```
Univariate GARCH Brazil
specb_sGARCH=ugarchspec(variance.model=list(model="sGARCH"),mean.model=list(armaOrd
specb_GARCHM=ugarchspec(variance.model=list(model="sGARCH"),mean.model=list(armaOrd
specb_eGARCH=ugarchspec(variance.model=list(model="eGARCH"),mean.model=list(armaOrd
specb_gjrGARCH=ugarchspec(variance.model=list(model="gjrGARCH"),mean.model=list(arm
fitGARCHb1=ugarchfit(data=rtnb,spec=specb_sGARCH) ##Standard GARCH
fitGARCHb2=ugarchfit(data=rtnb,spec=specb_GARCHM) ## GARCH in mean
fitGARCHb3=ugarchfit(data=rtnb,spec=specb_eGARCH) ## Exponential GARCH
fitGARCHb4=ugarchfit(data=rtnb,spec=specb_gjrGARCH) ## Threshhold GARCH
```

A.8 Estimting copula parameter

```
##Estimting copula parameter brazil
fit.Nb<- fitCopula(normalCopula(),  data = Ub,method="ml")
fit.tb<- fitCopula(tCopula( dispstr = "un"),data = Ub,method="itau.mpl")
fit.Cb<- fitCopula(claytonCopula(), data = Ub,method="ml")
##fit.Gb<- fitCopula(gumbelCopula(), data = Ub,method="ml") tau<0
fit.Fb<- fitCopula(frankCopula(), data = Ub,method="ml")
##fit.Jb= fitCopula(copula=joeCopula(),data=Ub,method="ml") contains NA values
```

```
#loglik
blN=fit.Nb@loglik
blt=fit.tb@loglik
blC=fit.Cb@loglik
blF=fit.Fb@loglik
loglikb=c(blN,blt,blC,blF)

#estimate
beN=fit.Nb@estimate
bet=fit.tb@estimate
beC=fit.Cb@estimate
beF=fit.Fb@estimate
estimateb=c(beN,bet,beC,beF)

#AIC
aicNb=-2*fit.Nb@loglik + 2*length(fit.Nb@estimate)
aictb=-2*fit.tb@loglik + 2*length(fit.tb@estimate)
aicCb=-2*fit.Cb@loglik + 2*length(fit.Cb@estimate)
aicFb=-2*fit.Fb@loglik + 2*length(fit.Fb@estimate)
aicb=c(aicNb,aictb,aicCb,aicFb)
b.cop=data.frame(loglikb,aicb)
```