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The optimisation of the angle of inclination of a solar collector to maximise the incident solar radiation

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Abstract

Irradiation data, recorded on vertical surfaces facing north, south, east and west and on a horizontal surface every ten minutes during daylight hours from January–December 1992 in Valencia, Spain, have been compared with estimated solar irradiation from inclined-surface models. Results show that Hay's model most accurately reproduces the variation in irradiation on all vertical surfaces.

Hay's model has been used to find the hourly variation in the optimum tilt angle for a South-facing solar collector in Valencia, Spain, and also to calculate the yearly average of this angle. This method has been compared with the results provided by another model that uses average monthly values of daily irradiation derived from the same experimental data, to calculate average monthly values of the optimum tilt angle. The results show that the method involving monthly averages is more accurate and easier to work with. © 1999 Elsevier Science Ltd. All rights reserved.

1. Introduction

Accurate modelling of solar irradiation on tilted surfaces at several orientations is needed for most of the practical applications of solar energy. Horizontal solar global radiation is the most commonly measured parameter; many of the previous studies [1–3] have concentrated on estimating diffuse and beam components of solar global

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radiation on a horizontal surface. Modelling of solar irradiation on non-horizontal surfaces is more complex due to the effect of diffuse radiation anisotropy over the sky dome, and therefore needs additional information, provided in most cases, by beam irradiation at normal incidence.

A number of models have been proposed to estimate solar radiation on tilted surfaces [4–10]. In the present work several models are compared with experimental data obtained in Valencia, Spain. This location is situated near the coast on the east of the Iberian peninsula. The data measured correspond to global solar irradiation on vertical surfaces oriented north, south, east and west, global solar irradiation on a horizontal plane and beam solar irradiation at normal incidence. The aim of the comparison is to find the best model that can be used to calculate solar irradiance on tilted planes in Valencia.

Working with the same irradiation data as in the evaluation of the models, the most accurate model is used to find the hourly variation in the optimum tilt angle for a south-facing solar collector in Valencia, Spain, and also to calculate the yearly average of this angle. This method is compared with the results provided by another model that uses average monthly values of daily irradiation derived from the same experimental data in order to calculate average monthly values of the optimum tilt angle.

The location where the measurements were performed (Faculty of Physics, Burjassot, Valencia) is situated at 40 m above sea-level at a latitude of 39.5°N. Obstructions above the horizon are in general less than 4°, except a small zone in the west. A previous paper [11] describes site obstructions and the measuring set-up detail.

The experimental data were obtained every ten minutes for the period from 1st January–30 June 1992. Due to experimental errors there are uncertainties of up to 5% in the values of global radiation and 3% in the values of direct radiation.

2. Models for calculating the solar radiation on an inclined plane

All the models assume that the total radiation ($I_{\mathrm{T}\beta\mathrm{Ap}}$) at a given orientation, azimuth, A_{p}, and inclination, β, is the sum of the direct ($I_{\mathrm{b}\beta\mathrm{Ap}}$), diffuse ($I_{\mathrm{d}\beta\mathrm{Ap}}$) and reflected ($I_{\mathrm{r}\beta\mathrm{Ap}}$) radiation:

$$I_{\mathrm{T}\beta\mathrm{Ap}} = I_{\mathrm{b}\beta\mathrm{Ap}} - I_{\mathrm{d}\beta\mathrm{Ap}} + I_{\mathrm{r}\beta\mathrm{Ap}} \tag{1}$$

In this study the reflected radiation is not considered, because an artificial horizon is used on the measuring apparatus to block out any light reflected from the ground.

For an inclined plane the direct radiation is calculated as follows:

$$I_{\mathrm{b}\beta\mathrm{Ap}} = I_{\mathrm{n}} \cos\theta, \tag{2}$$

where I_{n} is the direct normal irradiance and $\cos\theta$ is given by:

$$\cos\theta = \cos\gamma \sin\beta \cos(A_{\mathrm{s}} - A_{\mathrm{p}}) + \sin\gamma \cos\beta, \tag{3}$$

where γ is the solar altitude and A_{s} is the solar azimuth.

Calculating the flux of diffuse sky radiation is a much more complicated problem.

The most accurate method is to solve the equation of radiative transfer for a turbid atmosphere and then integrate it over the dome of the sky. This, however, produces complicated functions that use up a lot of computer time.

Many attempts have been made to produce an approximate model for the distribution of diffuse sky radiation. In this paper we will analyse radiation models that use historical data of global radiation falling on a horizontal plane and direct radiation at normal incidence in order to predict the diffuse radiation on a tilted surface by using various geometrical relationships.

The simplest model assume an isotropic distribution of diffuse radiation, whereas the more complicated models divide the sky up into different zones and use factors that account for the uneven distribution of the diffuse radiation.

In the case of overcast skies the assumption of isotropy is valid because the clouds act like a perfectly diffusing, homogeneous and infinite layer, which gives an isotropic diffusion. However, this assumption is not valid for clear or partially clouded skies where there is an increase in diffuse radiation in the zones near to the sun and to the horizon.

2.1. Isotropic model

This model, attributed both to Liu and Jordan [12] and to Kondratyev and Manovla [13], was the first to apply form factors to the study of solar radiation. It assumes that diffuse radiation is isotropically distributed across the hemispherical sky and can be written in the form:

$$I_{\mathrm{d}\beta\mathrm{Ap}} = I_{\mathrm{d00}} \frac{1+\cos\beta}{2}, \tag{4}$$

where I_{d00} is the diffuse radiation on a horizontal plane.

In this model the radiation is independent of the orientation, A_{p}, of the plane. We would therefore expect it to produce good results for very overcast skies only, where the increase in intensity near to the sun is almost negligible.

2.2. Temps–Coulson model

Temps and Coulson [10] developed a model from readings taken in clear skies. They introduced geometrical terms into the isotropic model to take into account the brightening of the sky in the region of the sun and at the horizon:

$$I_{\mathrm{d}\beta\mathrm{Ap}} = I_{\mathrm{d00}} \left(\frac{1+\cos\beta}{2}\right)\left(1-\sin^3\frac{\beta}{2}\right)(1+\cos^2\theta\sin^3(90-\gamma)). \tag{5}$$

The first additional term is the correction for the area near the horizon and the second is that for the zone around the solar disc. It should be pointed out that, apart from only being valid for clear skies, a weakness of this model is that for a horizontal collector, the expression does not reduce to I_{d00}.

2.3. Klucher model

Klucher [6] introduced the factor:

$$F = 1 - \left(\frac{I_{d00}}{I_{T00}}\right)^2 \tag{6}$$

to modulate eqn (5) as the sky changed from clear to overcast, giving:

$$I_{d\beta Ap} = I_{d00}\left(\frac{1+\cos\beta}{2}\right)\left(1+F\sin^3\frac{\beta}{2}\right)(1+F\cos^2\theta\sin^3(90-\gamma)) \tag{7}$$

where I_{T00} is the total radiation on a horizontal plane.

Under overcast conditions, when the ratio of diffuse to total intensity, I_{d00}/I_{T00}, is unity, this model reduces to the isotropic model. Under the clear sky where I_{d00}/I_{T00} is small, it reduces to the Temps–Coulson model.

2.4. Hay model

In this model [5, 14] diffuse radiation is considered to be the sum of two parts: that which comes from the area around the solar sphere and the isotropic radiation that comes from the rest of the sky. The contribution made by each component depends on the transmissivity of the atmosphere; this is included in the model in the form of the coefficient k, which Hay calls the index of anisotropy:

$$k = \frac{I_n}{I_{sc}}, \tag{8}$$

where I_{sc} is the solar constant whose recommended value, found by experiment is 1367 ± 7 Wm^{-2}. When I_n is large there is a big amount of direct radiation meaning that the transmissivity of the atmosphere is high; when it is small, the transmissivity is low.

The diffuse radiation from around the solar disc is given by the term:

$$I_{dsc} = I_{d00}\frac{k\cos\theta}{\sin\gamma}, \tag{9}$$

using a form analogous to that used to calculate the direction radiation, seeing as that is the main contributing factor to this diffuse component.

The isotropic radiation from the rest of the sky is given by:

$$I_{dsk} = I_{d00}\frac{(1-k)(1+\cos\beta)}{2}, \tag{10}$$

where the $(1-k)$ factor shows the fact that the isotropic component is most important when there is little direct radiation.

It follows that the total diffuse radiation is given by:

$$I_{d\beta Ap} = I_{d00}\left(k\frac{\cos\theta}{\sin\gamma} + \frac{(1-k)(1+\cos\beta)}{2}\right). \quad (11)$$

Hay designed this model for south-facing surfaces. For different orientations, especially north, there is often no contribution from direct radiation. In this case this model reduces to the isotropic model.

There exist more complicated models [4, 8, 9, 11] and some of them, like the Perez model, provide better results for all the planes [15]. Nevertheless, if the study is focused on the south plane, accurate results can be obtained using some of the above mentioned models.

3. Evaluation of models

RMSD (root mean square difference) and MBD (mean bias difference) are statistical estimators commonly used to evaluate the accuracy of models. Since the two estimators differ and can give different results, all two were calculated for each model and orientation. RMSD gives positive values whilst MBD may be positive or negative, with positive values corresponding to overestimation by the model. Although solar irradiation papers usually use absolute values when evaluating errors, we have used relative values for the RMSD in order to compare results.

Table 1 shows the errors for the total radiation for each one of the four models and four orientations. Standard deviation and relative standard deviation are also included. The error table show that, in general, the models slightly over-estimate the radiation arriving planes oriented to the north and west, where the mean error is greater than zero, whereas they underestimate the radiation on south and east-oriented surfaces, where higher levels of radiation would be expected.

For south-facing planes, where most of the incident radiation is direct, Temps and Coulson, Klucher and Hay models have similar small RMSD, probably because their diffuse radiation predictions are almost negligible due to the dominance of the term predicting direct radiation. The isotropic model has a larger error because it is designed to predict radiation for overcast skies, where light from the solar disc is almost totally dispersed resulting in an isotropic distribution of the diffuse sky radiation. Therefore it is more accurate when the direct component is small.

For the north-facing plane, where there is almost no direct radiation, the Isotropic model has the smallest errors because the anisotropy introduced by the other models is not relevant for a plane where there is little or no direct radiation. Also, for a north-facing collector Temps and Coulson model is significantly less accurate than the other models because it does not approximate to the Isotropic model when there is only a small amount of direct radiation.

An overall evaluation of results shows that Hay model most accurately predicts the variation in global irradiation over all vertical faces, except for that facing north. For a north-facing plane the Isotropic model has smaller errors, although it underestimates

Table 1
Errors for the estimated total radiation in vertical planes

	Isotropic	T&C	Klucher	Hay
Vertical plane facing north				
MBD	9.7	40.4	34.4	14.3
RMSD	28.7	55.8	50.2	34.1
S.D.	27.0	38.5	36.5	31.0
$RMSD_R$ (%)	35.1	68.4	61.5	41.8
% S.D.	33.0	47.2	44.8	38.0
Vertical plane facing south				
MBD	−58.2	−13.3	−23.3	−8.5
RMSD	76.7	39.0	46.7	31.9
S.D.	49.8	36.7	40.5	30.7
$RMSD_R$ (%)	22.5	11.5	13.7	9.4
% S.D.	14.6	10.8	11.9	9.0
Vertical plane facing east				
MBD	−41.8	−1.7	−10.3	−15.1
RMSD	79.1	63.3	64.7	51.0
S.D.	67.2	63.3	63.9	48.8
$RMSD_R$ (%)	31.7	25.4	26.0	20.5
% S.D	26.9	25.4	25.6	19.5
Vertical plane facing west				
MBD	−17.2	28.1	16.0	23.0
RMSD	53.6	48.1	49.4	46.6
S.D.	50.8	39.1	46.8	40.5
$RMSD_R$ (%)	31.7	28.5	29.2	27.6
% S.D.	30.0	23.1	27.7	24.0

the amount of radiation more than the Hay model does; so the Hay model would still be recommended for use on all planes.

The results of the south-facing plane show that all the models gave a good agreement, but the Hay model was slightly better than the rest and is only marginally more complicated to use than the isotropic model which is the simplest.

In comparison with other studies, in particular Utrillas et al. [11] which was carried out at the same site, but using only data recorded during the winter months, the errors are of approximately the same order. The errors in this study are slightly smaller, but this can be attributed to the fact the data recorded during the summer months for this evaluation are likely to be more accurate due to the greater intensity of direct normal irradiation and the absence of cloud cover in Valencia in the summer.

4. Optimum tilt angle based on yearly average solar radiation

From the results in the previous section it can be seen that the south-facing plane gives the best results for all the models and also it is that plane that receives the most radiation during the course of a year. It follows that henceforth all calculations will be carried out on south-facing planes only.

In order to use a model to determine the best inclination for a south-facing solar collector, a mathematical model describing the variation in the global and direct irradiation over the course of a year needed to be developed. Ideally, to describe the variation over the course of a year we would need to use values of the total daily irradiation. However, this is not possible with the Hay model, as it can only use instantaneous values of radiation intensity, or values integrated over a short period of time that the solar altitude can be considered constant, at maximum one hour.

It was decided instead to use the experimental data of irradiation integrated over ten min at different times of the day and to see how this varied over the course of a year. This would also enable us to see how the optimum inclination angle varied over the course of a day. Different equations were tested to see which best described the variation in radiation over the course of a year then the most accurate was selected to be used in place of the experimental data used previously in the evaluation of the models to give an expression that would describe the total radiation as a function of the tilt angle, assuming a south-facing collector.

For every hour from 07.00–17.00 h the value of irradiation recorded on the hour was plotted against the day of the year. All the graphs had approximately an obvious sinusoidal appearance, so a cosine graph was the first choice. Following the Duffie and Beckman's approximation [16] and taking account that the cosine cycle should not begin on the first day of the year, but on the 21st December (the winter solstice when there is the minimum number of daylight hours) the first curve tried was:

$$I_{\mathrm{T00}} = a - b\cos\left(\frac{360}{365}(D+11)\right), \tag{12}$$

where a and b are constants and D is the day of the year.

Equation (12) was used to find the global radiation on a horizontal surface.

Table 2 lists the equations describing the variation in radiation for each time. All equations are of the form: $I_{\mathrm{T00}} = a - b\cos \mathrm{D}'$, where $\mathrm{D}' = (360/365)$(day of year). Although the curves gave a good fit to the data points for global radiation there are still small errors. The size of these errors depends on the value of cos((360/365)(D+11), so vary constantly, being larger in winter and smaller in the summer. The maximum and minimum errors therefore refer to the errors at midwinter and midsummer respectively.

For direct radiation of normal incidence the fit of these curves was significantly poorer than for the global radiation. This is because the direct radiation is affected more by the presence of clouds, so, many of the data points are lower than would be expected at that time of year. Again all equations are of the form $I_{\mathrm{n}} = a - b\cos \mathrm{D}'$, where $\mathrm{D}' = (360/365)$(day of year), and are shown in Table 3.

Table 2
Equations describing the daily variation in global horizontal irradiation recorded at different times

Time (h)		Max. error (%)	Min. error (%)	R
07.00	$I_{T00} = 80.54–193.24 \cos D'$	7.9	6.7	0.94
08.00	$I_{T00} = 229.62–229.06 \cos D'$	3.0	1.7	0.97
09.00	$I_{T00} = 393.96–254.93 \cos D'$	2.5	1.1	0.97
10.00	$I_{T00} = 548.47–260.35 \cos D'$	5.8	0.8	0.97
11.00	$I_{T00} = 654.76–268.27 \cos D'$	2.1	0.6	0.97
12.00	$I_{T00} = 702.02–267.93 \cos D'$	2.0	0.6	0.97
13.00	$I_{T00} = 691.95–264.25 \cos D'$	2.0	0.5	0.97
14.00	$I_{T00} = 621.09–261.93 \cos D'$	2.1	0.6	0.97
15.00	$I_{T00} = 503.58–253.85 \cos D'$	2.1	0.7	0.97
16.00	$I_{T00} = 346.15–234.35 \cos D'$	2.1	0.9	0.97
17.00	$I_{T00} = 188.09–184.68 \cos D'$	1.8	2.6	0.96

Table 3
Equations describing the daily variation in direct normal irradiation recorded at different times

Time (h)		Max. error (%)	Min. error (%)	R
07.00	$I_n = 161.89–224.64 \cos D'$	18.1	12.6	0.64
08.00	$I_n = 307.85–217.25 \cos D'$	8.4	3.8	0.77
09.00	$I_n = 465.12–136.48 \cos D'$	11.3	2.4	0.62
10.00	$I_n = 571.79–83.555 \cos D'$	16.7	1.8	0.46
11.00	$I_n = 624.37–75.978 \cos D'$	16.1	1.4	0.46
12.00	$I_n = 638.99–74.246 \cos D'$	15.2	1.3	0.47
13.00	$I_n = 627.23–84.732 \cos D'$	13.8	1.4	0.50
14.00	$I_n = 594.75–102.06 \cos D'$	11.5	1.4	0.57
15.00	$I_n = 525.47–132.99 \cos D'$	9.1	1.6	0.67
16.00	$I_n = 406.56–166.33 \cos D'$	7.5	2.2	0.75
17.00	$I_n = 262.51–188.91 \cos D'$	9.0	4.1	0.74

The horizontal diffuse radiation, is given by eqn

$$I_{d00} = I_{T00} - I_n \sin \gamma. \tag{13}$$

From experimental data, we have determined the curve that describes the variation in the diffuse horizontal irradiation at 12 noon (12.00 h), calculated using the above method. This curve, plotted in Fig. 1, corresponds to eqn:

$$I_{d00} = 702.02 - 267.93 \cos D' - (638.99 - 74.24 \cos D') \sin \gamma.$$

$$\text{Max. error} = 15.3\%; \quad \text{min. error} = 1.4\% \tag{14}$$

The errors shown with eqn (14) are calculated from the quadratic sum of the errors in direct and global irradiation, hence the maximum and minimum values.

Using the Hay model, the diffuse horizontal and direct normal radiation was

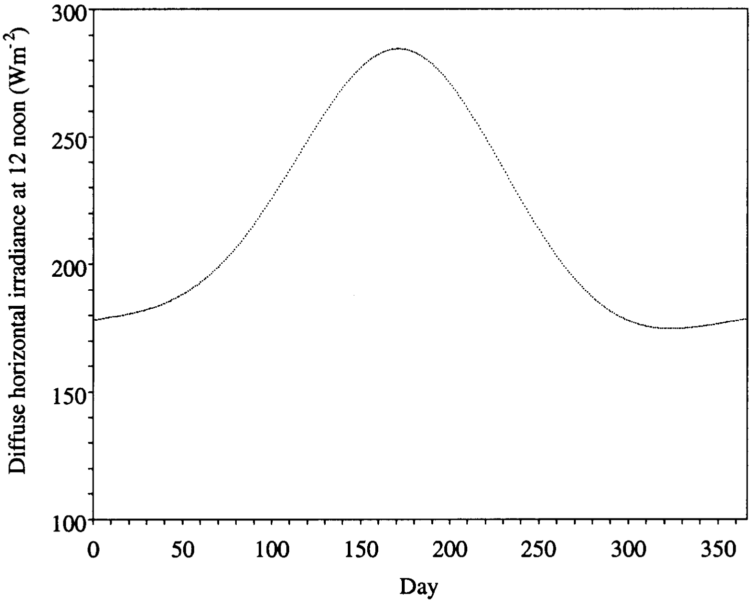

Fig. 1. Variation in I_{d00} at 12.00 h over the course of a year.

calculated for each hour. This enabled the global radiation on a south-facing surface at inclination, β, to be calculated using equations (1) and (11). Values of β were taken at 5° intervals from 0–90°. For each value of β the global irradiation was calculated for the same time each day. These values were summed to give an annual total which was then plotted as a function of β.

In order to compare results, the curves plotted at 08.00, 12.00 and 14.00 h are shown in Fig. 2. The curves appeared to have a quadratic form, so the first equations tried were quadratic polynomials. However, it was found that the accuracy could be improved by using a third order polynomial (cubic) graph. The equations of the graphs are written in the form $H_{T00} = a + b\beta + c\beta^2 + d\beta^3$. Table 4 shows the equations for all other times of day and the errors in the coefficients are also shown.

By differentiating the equations listed in Table 4 with respect to β, and then putting the differentials equal to zero, the value of β that gives the highest annual value of radiation was found for each time. This was repeated for every hour. Figures 3 and 4 show that the angle, β_m, that gives the greatest amount of annual radiation varies depending on the time of day. Figure 5 shows this variation plotted as a function of the time of day. Table 5 lists the optimum angle of inclination for each time and the corresponding error in β_m.

From Fig. 5 we can see that between 10.00–14.00 h the optimum angle of inclination varies only slightly with an average of 31.13°. A reasonable hypothesis is that seeing as we would expect the majority of the solar radiation to be collected in the hours around midday, a solar collector inclined at the average optimum angle for those hours would collect the most radiation. For testing this hypothesis in Fig. 6 the global

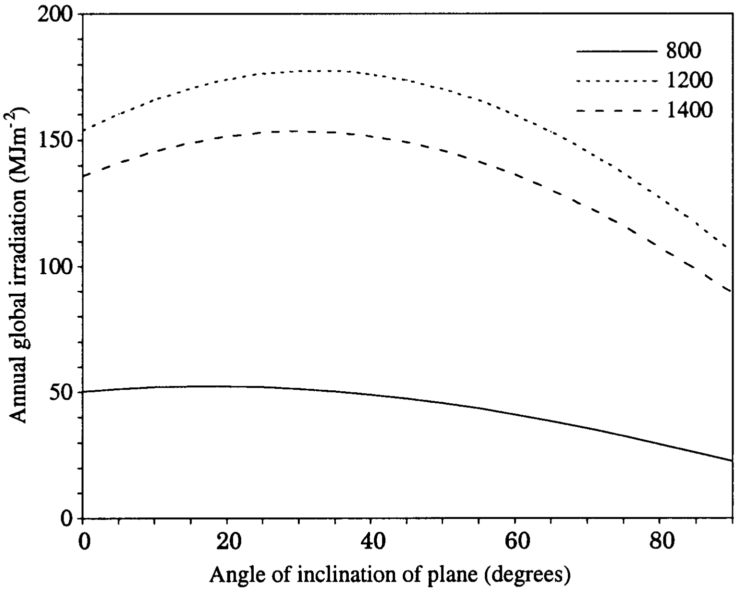

Fig. 2. Variation in the annual global radiation with the inclination of the plane at 08.00, 12.00 and 14.00 h.

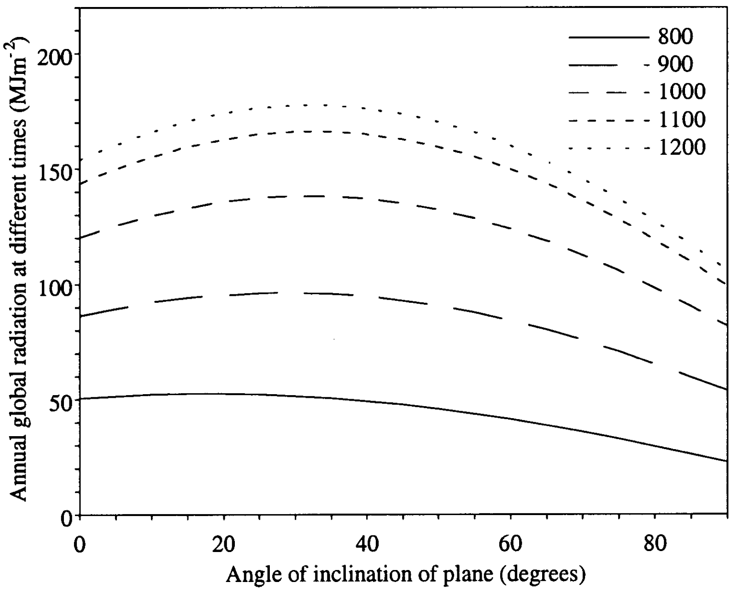

Fig. 3. Variation in annual global radiation with the inclination of the plane, from 07.00–12.00 h.

Table 4
Coefficients for the equation $H_{T00} = a + b\beta + c\beta^2 + d\beta^3$

Time (h)	a	b	c	d
08.00	83,860 (±80)	424 (±8)	−13.0 (±0.2)	0.0281 (±0.0014)
09.00	143,610 (±160)	1231 (±16)	−23.5 (±0.4)	0.033 (±0.003)
10.00	200,000 (±200)	1970 (±20)	−33.4 (±0.6)	0.039 (±0.004)
11.00	238,800 (±300)	2430 (±30)	−40.2 (±0.7)	0.045 (±0.005)
12.00	255,600 (±300)	2570 (±30)	−42.8 (±0.8)	0.048 (±0.006)
13.00	252,400 (±300)	2440 (±30)	−41.6 (±0.8)	0.049 (±0.006)
14.00	226,200 (±300)	2040 (±20)	−36.5 (±0.7)	0.047 (±0.005)
15.00	183,670 (±190)	1419 (±19)	−28.4 (±0.5)	0.043 (±0.004)
16.00	126,230 (±110)	687 (±11)	−18.2 (±0.3)	0.036 (±0.002)
17.00	68,570 (±40)	47 (±4)	−8.90 (±0.11)	0.0284 (±0.0008)

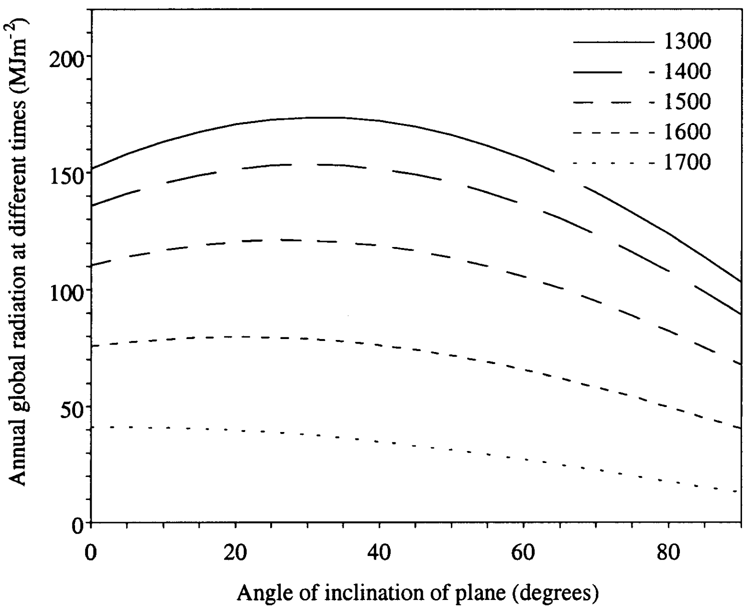

Fig. 4. Variation in annual global radiation with the inclination of the plane, from 12.00–17.00 h.

horizontal radiation collected in the hours around midday are taken. Due to the fact that data from the whole year are used, it is impossible to fit a curve to this graph. Using data from a selected number of days in summer and winter, the evolution of the radiation with the time follows the approximation:

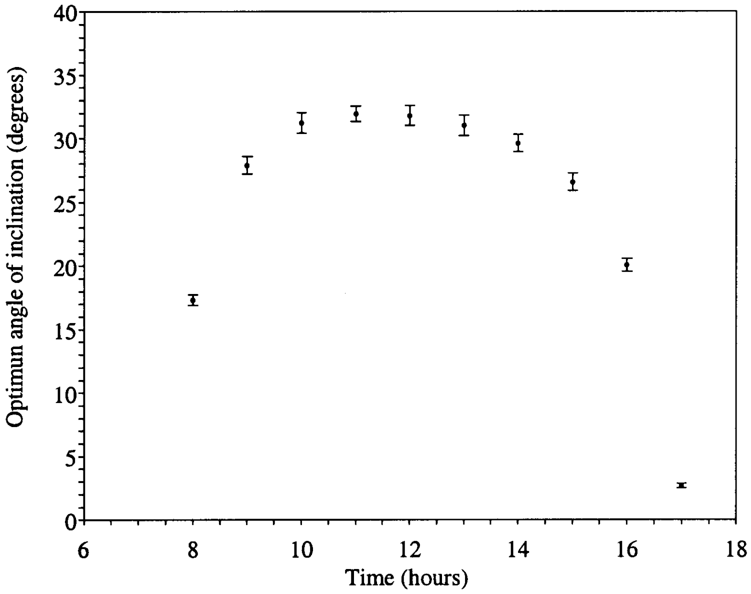

Fig. 5. Variation of the optimum angle of inclination over the period of a day.

Table 5
Optimum tilt angle for each h

Time (h)	β_m (degrees)	Error in β_m (degrees)
08.00	17.3	0.4
09.00	27.9	0.7
10.00	31.2	0.8
11.00	32.0	0.6
12.00	31.8	0.8
13.00	31.0	0.8
14.00	29.6	0.7
15.00	26.6	0.7
16.00	20.1	0.5
17.00	2.7	0.2

$$I_{T00} = 1960(\pm 190) - 1080(\pm 70)t + 190(\pm 10)t^2 - 12.1(\pm 0.5)t^3 + 0.244(\pm 0.011)t^4 \quad (15)$$

in summer and

$$I_{T00} = 8200(\pm 1400) - 3600(\pm 500)t + 480(\pm 60)t^2 - 28(\pm 4)t^3 + 0.58(\pm 0.07)t^4 \quad (16)$$

in winter.

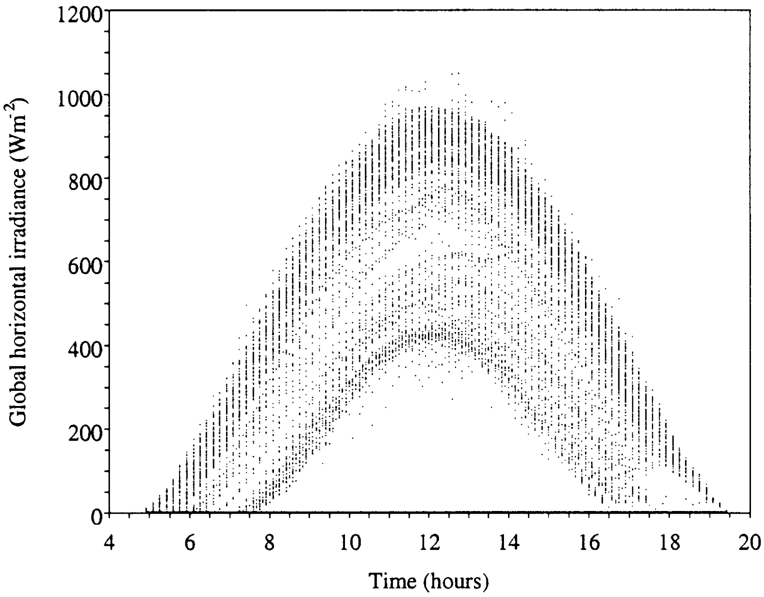

Fig. 6. Variation in global horizontal radiation with the time of day, for the whole year.

Correlations coefficients $R = 0.99$ and $R = 0.97$ were found respectively for eqns (15) and (16).

By integrating eqns (15) and (16) it was found that in summer the amount of radiation received between 10.00–14.00 h was 47% of the daily total, and in winter that figure rose to 67%. Therefore it is reasonable to suppose that an average yearly optimum tilt angle would be approximated 31°.

5. Optimum tilt angle based on monthly average solar radiation

In a recent study by Tiris and Tiris [17] the optimum inclination angle for a solar collector in Gebze, Turkey was found using monthly averages of total daily irradiation. This would give the optimum tilt angle for each month of the year so that a solar collector could be adjusted accordingly. Following the procedure used by Tiris and Tiris [17], we have used an isotropic model [18] for estimating the monthly average daily radiation on a tilted surface in terms of the tilt angle, β. In this sense the following expression can be used:

$$H_{\mathrm{T}\beta} = RH_{\mathrm{T}} \tag{17}$$

where

$$R = \left(1 - \frac{H_{\mathrm{d}}}{H_{\mathrm{T}}}\right) R_{\mathrm{b}} + H_{\mathrm{d}} \frac{1+\cos\beta}{2H_{\mathrm{T}}} + \rho \frac{1-\cos\beta}{2}, \tag{18}$$

and R_b is a function of the transmittance of the atmosphere, ρ is the ground reflectance, β is the inclination of the plane, and H_T and H_d are the monthly average daily total and diffuse irradiation on a horizontal plane, respectively. The final term of this equation refers to reflected radiation which is being ignored in this study, so eqn (17) becomes:

$$H_{T\beta} = (H_T - H_d)R_b - H_d \frac{1+\cos\beta}{2}. \tag{19}$$

R_b depends on the atmospheric cloudiness, water vapour and particulate concentration, but Liu and Jordan (1960) offer the following approximation:

$$R_b = \frac{\cos(\phi-\beta)\cos\delta\sin\omega_s + (\pi/180)\omega_s\sin(\phi-\beta)\sin\delta}{\cos\phi\cos\delta\sin\omega_0 + (\pi/180)\omega_0\sin\phi\sin\delta} \tag{20}$$

where ω_0 and ω_s are the sunset angles of, respectively, a horizontal and an inclined plane.

In order to obtain H_T and H_d values for each month, graphs were plotted of global horizontal and direct normal radiation against time, as, for example, in Fig. 7. The spread of results for each time of day refers to the variation in radiation levels over the course of each month. Hence, a curve fitted to these results describes the monthly average variation. By integrating the equation of this curve from the hour of sunrise to sunset the monthly average of daily radiation could be found. A fourth order polynomial curve was chosen to fit the curves shown in Fig. 7. Since the diffuse

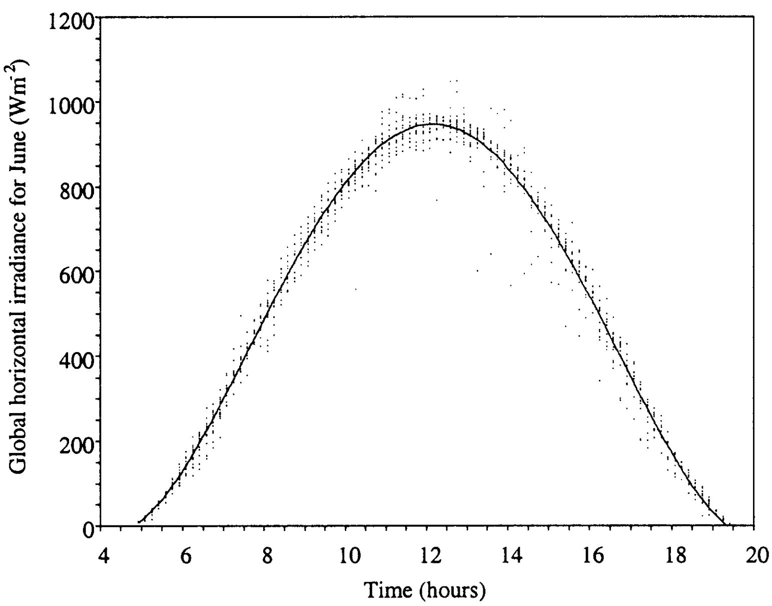

Fig. 7. Variation in global horizontal radiation with time for the month of June.

Table 6
Monthly averages of daily global and diffuse horizontal radiation

Month	H_T (MJm^{-2} day^{-1})	Deviation in H_T (%)	H_d (MJm^{-2} day^{-1})	Deviation in H_d (%)
January	9.4	9.5	3.9	23
February	13.0	5.9	5.3	26
March	18.8	5.7	7.1	32
April	23.8	1.6	8.3	26
May	25.8	6.7	9.9	23
June	28.2	15	10.3	14
July	26.3	3.1	8.8	28
August	23.8	3.6	8.7	32
September	21.3	37	6.8	24
October	15.1	6.4	4.9	43
November	11.4	12.4	2.7	66
December	8.2	2.2	3.2	26

horizontal radiation was also needed, it was also calculated, using a fourth order polynomial approach.

In order to integrate the equations analytically the average times of sunset and sunrise for each month had to be found. The obtained global horizontal, H_T, and diffuse horizontal, H_d, monthly average daily values are listed in the Table 6. Moreover Table 6 shows the deviation of the values of H_T and H_d from values calculated directly from experimental data.

Using eqn (19) it was possible to plot graphs of the variation in average monthly global radiation with the angle of inclination of the plane, shown in Fig. 8a and b. The optimum tilt angle for each month is shown in Table 7.

For each month the average daily global irradiation was found for the optimum tilted surface, the one at 31°, and a horizontal surface. These results are listed in Table 8. The errors shown for H_{opt} and H_{31} are calculated using the errors propagation method. It can be seen that the optimum tilt angle increases towards the beginning and end of each year. This is also the time when the greatest improvement is made on the amount of radiation collected by a horizontal collector. The total amount of irradiation incident on a horizontal collector at Valencia is calculated to be 6836 MJm^{-2} per year. The use of a collector at 31° instead of a horizontal collector represents an increase of about 7% in the total amount of irradiation received. Varying optimum tilt angle, the annual irradiation received by the collector is about 9000 MJm^{-2}, which represents a significant improvement of about 13%.

6. Discussion

The extent to which the Hay model is useful for predicting the optimum tilt angle for a solar collector is limited when compared to the model that uses monthly averages

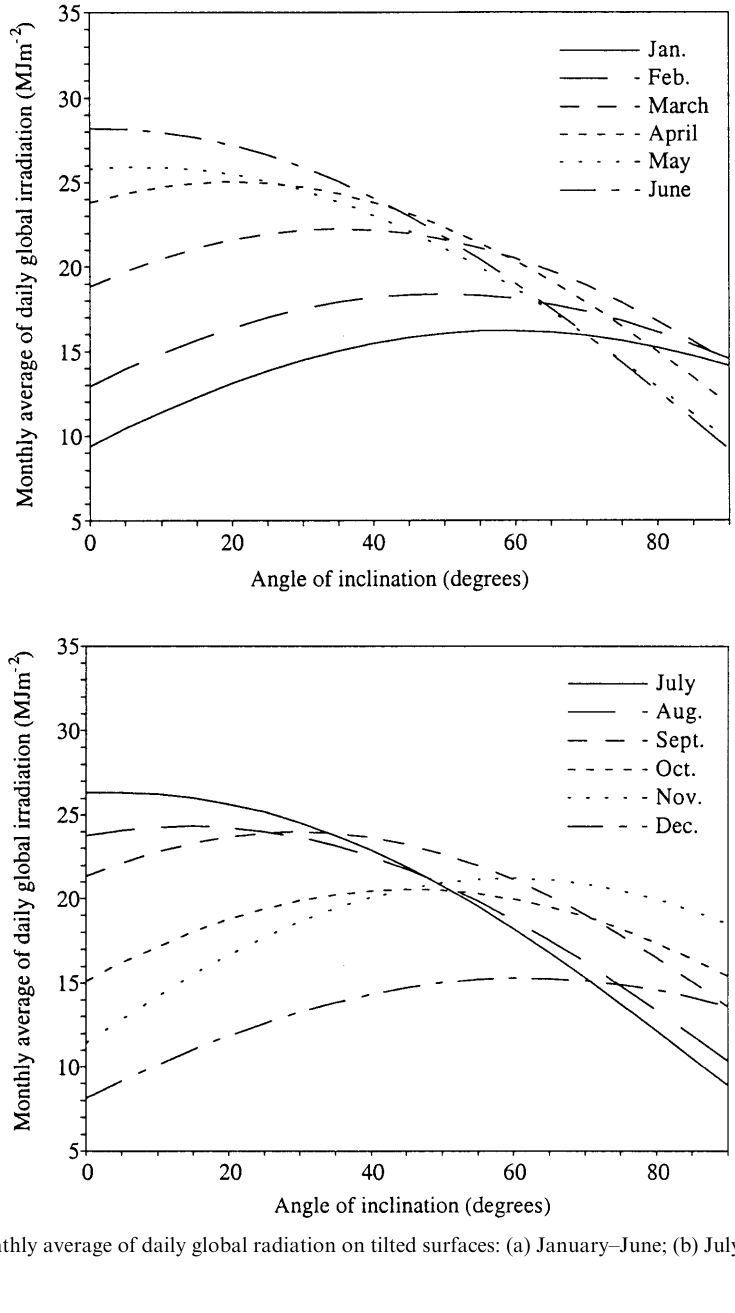

Fig. 8. Monthly average of daily global radiation on tilted surfaces: (a) January–June; (b) July–December.

of direct normal irradiation. The major problem with using the Hay model is that it restricts the data studied to values of irradiation integrated over a period of no more than one hour, which was why in this study it was only used with instantaneous values

Table 7
Optimum tilt angle for each month

Month	β_m (degrees)
January	58.5
February	49.0
March	35.0
April	20.0
May	4.50
June	0
July	3.0
August	14.5
September	29.0
October	45.5
November	60.0
December	61.0
Yearly average	31.0

Table 8
Monthly average of daily global radiation on optimum tilted surface and surface tilted at 31°

Month	H_{opt} (MJm^{-2} day^{-1})	Deviation in H_{opt} (%)	H_{31} (MJm^{-2} day^{-1})	Deviation in H_{31} (%)
January	16.2	17	14.6	14
February	18.4	12	17.6	10
March	22.3	9.4	22.2	9.1
April	25.0	4.0	24.6	2.5
May	25.9	6.3	23.9	6.3
June	28.2	13	25.4	13
July	26.3	3.0	24.1	3.0
August	24.3	3.6	23.4	3.9
September	24.0	29	24.0	29
October	20.6	14	20.0	11
November	21.2	40	18.8	36
December	15.3	14	13.4	10
Yearly total	8142 (MJm^{-2})	13	7664 (MJm^{-2})	12

of irradiation recorded every ten min. This means that it is impossible to obtain an overall picture of how much radiation can be collected over longer periods of time such as a day or a year. However, the amount of irradiation received using the yearly average of the optimum tilt angle is only 6% less than when changing the tilt every month and the former method may be preferred because it would involve cheaper equipment and less work to leave the tilt angle the same all year round.

However, by averaging the monthly values of the optimum tilt angle obtained using the second method, the same yearly average of 31° is attained. Also, there are several other reasons why this method is preferable to using the Hay model. Firstly, the curve fits to the plots of the variation in global horizontal and direct normal radiation for each month are more accurate than the curve fits to the plots that show daily variation in the same values at fixed times. On the other hand the method of using monthly averages of daily irradiation gives more useful results as it allows the calculation of daily and annual totals of incident irradiation. Finally, although using the Hay model predicts the hourly variation in the optimum tilt angle, this does not take account of the time of year, which must be considered if the incident irradiation is to be maximised. The monthly optimum tilt angle increases towards the beginning and end of each year, so it is probable that the hourly optimum tilt angles predicted also need to be altered slightly to reflect this monthly variation.

The results presented in this work concerning the optimum tilt angle are very similar to those obtained by Tiris and Tiris [17], as should be expected from the similar latitudes of Gebze and Valencia. Nevertheless, the present study predicts slightly higher values of incident irradiation, due probably to lower levels of clouds cover in Valencia.

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