

Pre-print of a review published in *Phenomenological Reviews*, 6 April 2017,
<http://reviews.ophen.org/2017/04/06/stefan-roski-bolzano-conception-of-grounding/>

Stefan Roski: Bolzano's Conception of Grounding

Title: Bolzano's Conception of Grounding

Series: Studies in Theoretical Philosophy vol. 5

Author: Stefan Roski

Publisher: Vittorio Klostermann

Release Date: January 2017

Format: Paperback €59

Pages: X+270

Reviewed by: Petter Sandstad (Universität Rostock)

Bernard Bolzano (1781-1848) was a Catholic priest and held, until his politically motivated dismissal by the Austrian Emperor Franz in 1819/20, a professorship on religious doctrine at the University of Prague. He was a polymath, writing on philosophy of religion, political philosophy, ethics, aesthetics, and social reform (and engaged in philanthropic work). Yet his most notable and original work, largely ignored during his life, is on mathematics, logic, and philosophy of science. This part of Bolzano's work was influential upon many students of Franz Brentano (Smith 1994:156), e.g., Carl Stumpf, Alexius Meinong, Kazimierz Twardowski, and not least Edmund Husserl: "Bernard Bolzano's *Wissenschaftslehre* [...]far surpasses everything the world-literature has to offer in the way of a systematic sketch of logic. [...]we must count him as one of the greatest logicians of all time" (*Prolegomena*, appendix to §61, Husserl 2001:142). Bolzano's notion of 'proposition in itself' (*Sätze an sich*) is a predecessor of Brentano's 'immanent content', Frege's 'thought' (*Gedanke*), and Husserl's 'ideal meaning'; and more generally, Bolzano's view that propositions are, in Husserl's terminology, ideal or abstract entities was highly influential (Smith 1994:185-190).

There has recently been a vast expansion in the literature on Bolzano's theoretical philosophy. In 2014 the first complete English translation of Bolzano's main work from 1837, the *Wissenschaftslehre* (Theory of Science), was published in four volumes (Bolzano 2014). Some noteworthy recent monographs are Morscher's (2008) *Bolzano's Life and Work* and Lapointe's (2011) *Bolzano's Theoretical Philosophy*. For a brief introduction see Morscher (2014) and Šebestik (2016). What is of interest for the purpose of this review is the fact that Bolzano was, as claimed by Roski, perhaps the first to have a *proper*, though tentative and fragmentary, theory of grounding (55; here and henceforth references to page numbers alone refer to the book here under review).

Grounding is a topic which has received immense attention these last 15 years or so (see Correia & Schnieder 2012; Bliss & Trogon 2016). It is often seen as a replacement for the controversial notions of truth-makers and of supervenience, where it is taken as a virtue of all these notions that they allow for non-eliminative theories, for instance of the mental or of social reality. Still, grounding is perhaps an even more controversial notion. For instance, the notion is criticized for being too coarse-grained (Wilson 2014). Some examples of grounding, especially conducive to Bolzano's conception, are (216):

*Moral facts obtain because non-moral ones do. E.g., Metternich's actions are bad because they yield to a decrease of the overall distribution of happiness.

*It is true that p because p . E.g., it is true that Socrates is brave because Socrates is brave.

*A conjunction is true because its conjuncts are true. E.g., Socrates is Greek and Socrates is a philosopher because Socrates is Greek, and because Socrates is a philosopher.

The use of 'because' indicates that a relation of grounding holds between two true propositions (the relata). 'Because' is here not used in an evidential and epistemological sense; rather, but rather to express an explanatory metaphysical priority-relation.

Roski's *Bolzano's Conception of Grounding* is the first book-length treatment solely devoted to Bolzano's theory of grounding. The author is concerned with Bolzano's mature theory of grounding as expressed in his main work, the *Wissenschaftslehre* (especially *WL* §198-221). Aside from a few notes, the book does not discuss Bolzano's transition from his immature to his mature theory (especially, both from 1810, the *Aetiologie* and the *Beyträge zu einer begründeteren Darstellung der Mathematik*; but cf. Centrone 2016). Nor does it discuss Bolzano's influence on his successors, e.g., Husserl. By all meanings of the term, this is a scholarly work—both in its varied use of texts from Bolzano's collected works (the *Bernard Bolzano Gesamtausgabe*), and in its discussion of relevant secondary literature. As I am no scholar of Bolzano, I will not engage with Roski's interpretation. Rather, I will focus on the value of this book for a general reader, who is either already familiar with the recent grounding-literature and interested in learning more on Bolzano's view, or interested in a historically guided introduction to grounding.

The book is a revision of Roski's 2013 doctoral dissertation, from which two chapters have been omitted (one on Bolzano's early work and one on his conception of science) and one chapter added (ch.5). It consists of an introduction (ch.1); a presentation of key notions from the *Wissenschaftslehre*, especially deducibility (ch.2); one chapter on Bolzano's pure logic of grounding (ch.3), and a very long chapter on his impure logic of grounding (ch.4). Roski takes the distinction between pure and impure from Fine (2012). The pure logic of

grounding “contains principles that hold for every case of grounding, irrespective of [...]the relata”, whereas impure logic of grounding “consists of more specific principles that mostly apply only to truths from deductive or *a priori* sciences” (16). The book concludes with a comparison between Bolzano’s theory and contemporary theories of grounding (ch.5).

Chapter 1. Introduction

The introduction not only contains the customary brief description and motivation of the book, methodological remarks, and outline of the chapters. It further gives a brief description of grounding. Bolzano’s terminology is that of a relation of grounding (*Abfolge*) which holds between two relata (both truths), the ground (*Grund*) and the consequence (*Folge*). Unfortunately, the translation of ‘*Abfolge*’ as ‘grounding’ is nowhere defended in the book; in Bolzano (2014) it is translated as ‘the relation of ground and consequence’, and ‘grounding’ might be a misleading translation. Roski characterizes grounding as an explanatory consequence relation—thus, the consequence not only follows deductively (Roski explains this notion in ch.2, see below) from the ground(s), but the ground is also explanatory of the consequence (2). Further, grounding is an ordering relation and is as such asymmetric: *a* grounds *b*, yet *b* does not ground *a*. Roski presents two of Bolzano’s examples of grounding (for both these examples, though presumably not in general, either proposition is deducible from the other): First, the truth that the atmospheric pressure has dropped partially grounds the truth that the barometer stands lower. Second, from Euclid’s *Elements* (Prop I.1), the truth that for every two points *a* and *b*, there is a point *c* such that lines $ab = ac = bc$ grounds the truth that the circumferences of any two circles with common radius *ab* and centres *a*, *b*, lying on the same plane, intersect at a point *c* equidistant from *a* and *b*.

Historically, Roski situates Bolzano within what he names, following Betti & de Jong (2010), the *Classical Model of Science*. This model goes back to Aristotle’s *Posterior Analytics*, and Roski says that similar views are defended *inter alia* by Arnauld & Nicole’s *Port Royal Logic*, Leibniz, and Wolff. Its central idea is that science should be structured axiomatically (e.g., Euclid’s axiomatization of geometry). Earlier discussions of this model simply call it the *Ancient Axiomatic Theory* (Scholz 1930/1975). This accords with the traditional interpretation of Aristotle’s *Posterior Analytics* (dating back at least to Philoponus’ 6th century commentary), although it is by no means the only possibility (for an interpretation more along the line of a Husserlian formal ontology, where each science is a regional ontology, see my (2016)). In the remainder of the book Euclid, not Aristotle, will represent this model (although Bolzano considered Euclid’s *Elements* to be highly flawed).

Roski notes strong similarities between Bolzano's two examples (especially the barometer) and Aristotle's example that "it is not because the planets [C] do not twinkle [B] that they are near [A] – rather, because they are near they do not twinkle." (*APo* I 13, 78^a37-38) For one, we have a similar use of 'because'. Further, Aristotle says that either truth deductively follows from the other: 'B belongs to C, and A belongs to B, therefore A belongs to C' is a valid syllogism, but it gives the wrong explanation (*aitia*). The correct explanation has A (being near) as the explanatory middle term: 'A belongs to C, B belongs to A, therefore B belongs to C.' This illustrates Aristotle's distinction between a merely valid and a demonstrative (explanatory) syllogism (a modern variant is Bromberger's flagpole-counterexample to the Hempel-Oppenheim-account). Similarly, Bolzano speaks of demonstrations (*Begründungen*) which present the objective ground for a truth (10-11). A further similarity, unremarked by Roski, is Bolzano's distinction between objective and subjective grounds, where the latter corresponds to the evidential/epistemological use of 'because'. Thus, while the fall in atmospheric pressure is the objective ground of the fall of the barometer, conversely the fall of the barometer is the subjective ground of the fall in atmospheric pressure. This is the same as Aristotle's distinction between what is prior to us and what is prior in nature/*simpliciter* (*APo* I 2, 72^b25-32). Third, Roski does not mention the strong similarity between Aristotle's view throughout the *Posterior Analytics* and Bolzano's view that the "generality of the premises is thus, as it were, measured in terms of the *terminus medius*. [...]the highest possible generality is given by the case in which *terminus medius* and predicate idea have the same extension." (179)

Finally, Roski presents six claims central to Bolzano's conception of grounding (12):

- (I) Grounding proceeds from more to less general truths.
- (II) Grounding proceeds from simple to more complex truths. (Roski argues in ch.4 that Bolzano came to reject this claim).
- (III) The grounds of a given consequence are uniquely determined.
- (IV) There are ungrounded truths.
- (V) Grounding is an asymmetric relation.
- (VI) Grounding gives rise to deductive economy.

Chapter 2. Objective truth, variation & truth-preservation

Roski presents some key notions from Bolzano's *Wissenschaftslehre*, which are prerequisites for understanding Bolzano's theory of grounding.

For Bolzano, the relata of grounding are true propositions (*Sätze an sich*). Propositions are abstract objects (Bolzano merely speaks of non-existing objects), bearers of truth and falsity, and composed of ideas in themselves (*Vorstellungen an sich*) (20-21). Ideas are in turn either complex (*zusammengesetzt*), e.g. [prime number] and also [prime], or simple (*einfach*), e.g. [not] (as is customary in Bolzano-scholarship, square brackets indicate propositions or ideas). Further, ideas can either refer to one or more objects, or to no object at all. Both propositions and ideas Bolzano sharply distinguishes from their corresponding mental items, viz. judgements and subjective ideas (in Husserlian terminology, propositions and ideas are ideal, not immanent, contents).

Bolzano understands the distinction between intuitions (*Anschaunungen*) and concepts (*Begriffe*) somewhat different from Kant. An intuition “is defined as a simple idea that has exactly one object” (24), e.g., ‘this pen’, and a concept is “(a)ny idea that is not an intuition and that does not contain an intuition as a part” (25). The third group, “ideas that aren’t intuitions, but contain at least one intuition as a part” (25), Bolzano calls mixed ideas (*gemischte Vorstellung*). Correspondingly, Bolzano distinguishes between intuitional propositions, i.e. propositions containing at least one intuition, and conceptual propositions: for Bolzano roughly equivalent to *a posteriori* and *a priori* truths (26).

Bolzano holds that all propositions have the form [A has *b*]; [has] is an objectless idea (*copula*), [A] is a subject-idea and [*b*] is a predicate-idea. This requires Bolzano to heavily paraphrase ordinary propositions, e.g. ‘there are cows’ into [[Cow] has objectuality] and ‘there are no real square roots of -1’ into [[Real square root of -1] has lack of objectuality]. A proposition, [A has *b*], is then defined by Bolzano as true “just in case A is objectual and *all* objects that fall under [A] have at least one of the properties that fall under [*b*].” (29)

Bolzano’s notion of deducibility (*Ableitbarkeit*) constitutes not only a significant improvement compared to his contemporaries’ alternative accounts (19), but is also, with its focus on truth-preservation under variation, close to the now standard Tarskian definition (43-44). However, it is highly unorthodox in that it is a ternary relation: deducibility holds “between a collection of premises, a collection of conclusions *and* a collection of ideas that are considered to be variable.” (45) Thus “Bolzano’s notion validates many arguments that are not logically valid on a modern understanding of the notion.” (47) Fortunately, Bolzano also has a binary relation of logical deducibility which is close to our modern understanding, defined such that the collection of premises Δ is logically deducible from Γ iff Δ is deducible from Γ when all non-logical ideas are considered to be variable. It is logical deducibility that is important for Bolzano’s theory of grounding (47). Further relevant to grounding, Roski

presents a notion of mutual deducibility/equivalence (*Gleichgültigkeit*), e.g. as in the example of the barometer; and a notion of exact deducibility (*genaue Ableitbarkeit*) i.e. an argument containing no redundant premises or ideas.

Chapter 3. Explanatory priority: Bolzano's pure logic of grounding

Roski reconstructs Bolzano's theory of the pure logic of grounding, i.e. the logical properties of grounding independent of its relata.

Bolzano distinguishes between complete and partial grounds, and between immediate and mediate grounds. A partial ground is a part of a complete ground, e.g., the truth of a conjunct is a partial ground of the truth of a conjunction. A mediate ground is a ground of an immediate ground, e.g., in Euclid's *Elements* Post I.1 is a mediate ground of Prop I. 19, while Prop I.5 and I.18 are its immediate (complete) ground (64). Bolzano only applies the partial/complete distinction to immediate grounds (65), and thus we have a tripartite distinction between partial immediate grounds, complete immediate grounds, and mediate grounds (and correspondingly for consequences). Bolzano explicitly takes the complete immediate ground to be the only genuine case of grounding (66). However, Bolzano-scholars disagree on which of the following two cases should be considered basic: Either the relation of a complete ground to its complete consequence, or the relation of a complete ground to one of its partial consequences. Roski favours the second alternative, thus naming this relation 'grounding'. The first he names 'complete grounding'.

Roski first roughly follows Bolzano's presentation of immediate grounding (*WL* §198-215) in his section 3.4, and then gives a more concise reconstruction in 3.5. In the latter, Roski shows that most properties of pure grounding can be derived from three basic principles: The asymmetry of partial grounding, the uniqueness of grounding (i.e., there is only one complete ground of a consequence), and the existence of fundamental truths (i.e., some truths are ungrounded). Both partial grounding and complete grounding are defined in terms of the primitive notion of grounding. From this Roski derives, first, the theorem that there is no overlap between any complete ground and its complete consequence; second, the asymmetry of complete grounding; third and fourth, the irreflexivity of complete grounding and partial grounding; fifth, that complete consequences do not overlap; sixth, the uniqueness of complete grounds; seventh and eighth, anti-monotonicity of grounding and of complete grounding; and, ninth, failure of transitivity and cut. In order to derive the last properties of pure ground, two further principles must be supplied, corresponding to what is sometimes called *Aristotle's insight* (cf. *Metaphysics* Θ 9, 1051^b6-9; and *Categories* 5, 4^b8-10 and 12,

14^b18-22): “Every truth ϕ and every collection of truths $\{\phi_1, \dots, \phi_n\}$ is the complete ground of a truth of the form $[\phi \text{ is true}]$ and $[\text{Each proposition in } \{\phi_1, \dots, \phi_n\} \text{ is true}]$ respectively” (71). From this Roski derives, first, the seriality of complete grounding, grounding, and partial grounding (i.e., every collection of truths has another collection of truths as its complete consequence, every collection of truths is a complete ground of another truth, and every truth is a partial ground of another truth); second, linkedness of partial grounds (i.e., there are truths ϕ, θ, ψ such that ϕ partially grounds θ and θ partially grounds ψ); third, internal dependence (i.e., “that truths which form the complete ground of a given truth may themselves stand in grounding relations” (97)); and fourth, anti-amalgamation (i.e., pairs of complete grounds and consequences cannot be fused).

The chapter also discusses mediate grounding, showing it to be irreflexive, transitive, and asymmetrical (103), and briefly discusses the possibility of infinite grounding trees.

Chapter 4. Simplicity and economy: Bolzano’s impure logic of grounding

The main source for Bolzano’s impure logic of grounding, where only conceptual truths are the relata of impure grounding, is *WL* §221. In what is by far the longest chapter of the book (109-213), Roski gives a very thorough discussion of §221 (most of which I am unable to discuss here for reasons of space), including a discussion of some tensions between Bolzano’s pure and impure logic.

The fundamental notion in this chapter is complexity. Grounds are required to be less complex than their consequences. Bolzano strictly defines complexity in terms of the number of ‘simple parts’ in a proposition, such that a proposition with more simple parts is more complex than a proposition with less simple parts. Roski here argues that the relevant parts are not the type but rather the token occurrences (e.g. of the simple idea [not]) (115). In addition, grounds should avoid redundancy and therefore not contain logically analytic truths nor consist of logically dependent propositions.

Here enters the tension with Bolzano’s pure logic, which says that every truth and every collection of truths grounds some other truth. Yet, not every truth satisfies the simplicity and economy principles. To resolve this tension, Roski argues that “Bolzano should let go of the seriality of grounding, and consequently of $(\text{Truth}_{\text{coll}})$ ” (159), where ‘ $(\text{Truth}_{\text{coll}})$ ’ is Aristotle’s insight applied to collections of truths.

Chapter 5. Bolzano’s logic of grounding and the logic of metaphysical grounding

Roski generally notes much convergence between Bolzano's and contemporary theories of grounding, most strongly for his pure logic of ground. However, Bolzano held pretty much the opposite view from the contemporary, regarding ground of quantificational truths (229-231): He took truths containing an existential quantifier, e.g., 'there is something', as basic, rather than as grounded in a truth without the existential quantifier; and he took a truth containing a universal quantifier as partially grounding one of its instances.

Further, Roski earlier (10) notes that grounding does not hold among two truths because a relation of ontological dependence holds between the objects mentioned in the truths. Dependence- or priority-relations has no further explicit role in Roski's discussion of Bolzano, yet they are central to many contemporary theories of grounding (especially to Benjamin Schnieder's work). I would have liked some discussion on this point.

Summary

The book is written in clear and concise English, and does not presuppose previous knowledge of Bolzano. There are some typos, none of which should present any obstacle to understanding the text. The logical notation used is introduced and explained, and a list of symbols, definitions, and principles is included. However, I must note some shortcomings of the index. Preferably, the index should have contained names of other scholars of Bolzano discussed in the book. Similarly, some of the entries are incomplete (e.g., a number of passages mentioning Aristotle have not been included in the index). Further, the Bolzano-scholar would, I think, appreciate an *index locorum* for passages from Bolzano.

While the book is mainly a scholarly work rather than a theoretical contribution, Roski is too modest when he asserts that "the book will not contribute anything new to the debate on metaphysical grounding" (2-3). For he later says: "Bolzano did not merely anticipate many views that are part and parcel of the current debate. He also went beyond them in interesting ways." (232) The presentation of these ideas can be said to be an addition to the debate, even though this still leaves the possible application and defence of these ideas as a task for future research. In conclusion, Roski's book should be of strong interest to anyone interested in Bolzano or grounding.

Acknowledgements:

Ansten Klev, Ludger Jansen, and Georg Fuellen provided helpful feedback to a draft of this review.

Bibliography:

- Betti, A. & de Jong, W.R.(2010) “The Classical Model of Science: A Millenia-old Model of Scientific Rationality”, *Synthese* 174:185-203.
- Bliss, R. & Trogdon, K.(Winter 2016) “Metaphysical Grounding”, *Stanford Encyclopedia of Philosophy*, E.N. Zalta (ed.), <https://plato.stanford.edu/archives/win2016/entries/grounding/>
- Bolzano, B.(2014) *Theory of Science*, 4 vols., P. Rusnock & R. George (translators), Oxford: OUP.
- Centrone, S.(2016) “Early Bolzano on Ground-Consequence Proofs”, *Bulletin of Symbolic Logic* 22:215-237.
- Correia, F. & Schnieder, B.(eds.)(2012) *Metaphysical Grounding: Understanding the Structure of Reality*, Cambridge: CUP.
- Fine, K.(2012) “The Pure Logic of Ground”, *Review of Symbolic Logic* 5:1-25.
- Husserl, E.(2001) *Logical Investigations* Vol. 1, J.N. Findlay (translator), London/New York: Routledge.
- Lapointe, S.(2011) *Bolzano's Theoretical Philosophy*, Basingstoke: Palgrave Macmillan.
- Morscher, E.(2008) *Bolzano's Life and Work*, Sankt Augustin: Academia.
- Morscher, E.(Fall 2014) “Bernard Bolzano”, *Stanford Encyclopedia of Philosophy*, E.N. Zalta (ed.), <https://plato.stanford.edu/archives/fall2014/entries/bolzano/>
- Sandstad, P.(2016) “The Formal Cause in the *Posterior Analytics*”, *Filozofski vestnik* 37:7-26.
- Scholz, H.(1930/1975) “The Ancient Axiomatic Theory”, J. Barnes (translator), *Articles on Aristotle: I. Science*, Barnes/Schofield/Sorabji(eds.), London: Duckworth:50-64.
- Šebestik, J.(Spring 2016) “Bolzano's Logic”, *Stanford Encyclopedia of Philosophy*, E.N. Zalta (ed.), <https://plato.stanford.edu/archives/spr2016/entries/bolzano-logic/>
- Smith, B.(1994) *Austrian Philosophy: The Legacy of Franz Brentano*, Chicago/La Salle, Illinois: Open Court.
- Wilson, J.M.(2014) “No work for a theory of Grounding”, *Inquiry* 57:535-579.