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1 Component and resultant forces

An apple is falling from the tree. According to Newton's second law of motion, its acceleration is determined by the sum of all the forces exerted on it. Let us assume that in that case only two forces act on the apple : the gravitation exerted by the earth on the apple (call it $\overrightarrow{G}$) and the resistance or friction exerted by the air on it (call it $\overrightarrow{F}$). According to Newton second law :

$$\overrightarrow{F} + \overrightarrow{G} = \overrightarrow{R} = m\overrightarrow{a}$$

$\overrightarrow{F}$and $\overrightarrow{G}$ are called the component forces and $\overrightarrow{R}$ is called the resultant force. How are the entity and operations mentionned in this equation to be interpreted metaphysically ? I shall here assume realism about newtonian forces : there are *sui generis* forces, be they component or resultant, that are irreducible to accelerating masses[1]. (This entails that one at least of the two « = » above does not refer to an identity : Newton's second law is not a definition of forces.)

The question I want to adress here is the following : among those tree vectorial representations $\overrightarrow{F}$, $\overrightarrow{G}$ and $\overrightarrow{R}$, which refer to real forces ? There are four possible answers :

1. None of them : there are no forces, neither $\overrightarrow{F}$, $\overrightarrow{G}$ nor $\overrightarrow{R}$ refer to *sui generis* forces. I have just rejected this option, *ex hypothesis*.
2. $\overrightarrow{F}$and $\overrightarrow{G}$ but not $\overrightarrow{R}$. There are component forces, but no resultant forces. Resultant forces are just mathematical fictions (Creary, 1981, Johansson, 2004, pp. 167-8, Molnar, 2003, 194-198).

[1]See Wilson (2007) and Massin (2009) for defense of realism about newtonian forces.

3. $\overrightarrow{R}$ but not $\overrightarrow{F}$and $\overrightarrow{G}$. There are resultant forces, but no component forces : component forces are just mathematical fictions. (Cartwright, 1983, 54-73, Wilson, 2009).
4. The three of them : $\overrightarrow{F}$, $\overrightarrow{G}$ and $\overrightarrow{R}$. There are both component forces *and* resultant forces.

I favour the fourth version. Let us call it generous realism about forces :

generous realism about forces : both component and resultant forces are real.

A quick, but inconclusive, way to defend generous realism is to rely on the reciprocal conceptual dependency between component and resultant forces. Conceptually, there cannot be component without compounds, nor compounds, or resultants, without components. If there are only component forces, then they are not really *component* ; and if there are only resultant forces then there are not really *resultant*.

This remark however is only of limited scope. Those who say that there are only component (or resultant) forces will sensibly grant that the component/resultant distinction occurs at the representational level (the level of vector calculus) and has not counterpart in reality. But they will insist that it still makes sense to ask which of the representational force-vectors do refer to real forces : $\overrightarrow{F}$ and $\overrightarrow{G}$, or $\overrightarrow{R}$? The claim that only component forces are real, for instance, will be read as the claim that only forces represented by component-vectors are real.

Despite the faillure of this easy way towards generous realism, it is important to notice that this solution is still, *prima facie*, the most attractive one, because we apparently need both type of forces :

1. *The initial case for component forces.*
   (a) That they are distinct component forces is first suggested first by the fact that different component forces relate different bodies to the body under consideration. Here, the air friction related the air to the apple while the gravitation relate the earth to the apple.
   (b) Second, each of these force has not only different relata, but also receive different explanation or sources. Gravitation is described by Newton's law of gravitation, while air resistance or drag is a complex force, which in fluid dynamics describes as grounded in skin friction and form drag.
   (c) Third, those force have not only different sources, but also different properties. Force instance gravitation is a volumic force that acts

at each inner point of the apple, while air resistance only act on its skin. Each force falls under the scope of a distinct law, and if we want to maintain that laws of nature state some facts, what Cartwright (1983, 54) calls a facticity view of laws, whe should welcome component forces in our ontology.

(d) Finally, that $\overrightarrow{F}$ and $\overrightarrow{G}$ are real distinct forces is attested by the fact that one or the other might suddenly stop to act. Suppose the air is suddenly annhilated : the apple will continue to fall but under the influence of the $\overrightarrow{G}$ alone.

2. *The initial case for resultant forces*. That resultant forces are real is suggested by the fact that they cause accelerations. These are the resultant forces, not the component forces that figures in Newton's second law (Wilson, 2009). Most often, component forces do no cause any acceleration but remain latent : here for instance neither $\overrightarrow{F}$ nor $\overrightarrow{G}$ cause an actual acceleration corresponding to its intensity : there are no such acceleration of the apple. The only acceleration that happens is caused by $\overrightarrow{R}$. If the kinematic behaviour of bodies is to be explained by appeal to forces, as any realist about forces should agree, then resultant forces are needed because component ones cannot do the job.

Having both component and resultant forces in our ontology would therefore be nice. Some killjoys however have argued that we cannot have the cake and eat it. They endorse a stingy realism about forces :

stingy realism about forces : either component forces or resultant forces are real.

2 The problem

Creary (1981) and Wilson (2009) are both stingy realists about forces and though they disagree on which type forces are real, they agree on the reason why both type of forces cannot be real together. Developping an initial suggestion of Creary, Wilson has recently proposed the following *reductio ab absurdum* of generous realism :

P1 Resultant and component forces are real .

P2 Resultant and component forces are wholly distinct.

P3 The resultant force acting on a body is a sufficient cause of its acceleration. The same is true for the component forces acting on it.

C Resultant and components forces causally overdetermined their effect.

In newtonian worlds where $\overrightarrow{F}$ and $\overrightarrow{G}$ are missing but $\overrightarrow{R}$ is still there, or in newtonian worlds where $\overrightarrow{R}$ is missing but $\overrightarrow{F}$ and $\overrightarrow{G}$ are still there, the apple would still accelerate as it does. Generous realism about forces leads to regular causal overdetermination of the effects of forces.

One weakness of this argument however is that not everybody will agree that it is a *reductio* : causal overdetermination might not be a bad thing. Sider (2003) argues that is unproblematic as long as one of the complete cause at least depend on the other. If $\overrightarrow{R}$ depends on $\overrightarrow{F}$ and $\overrightarrow{G}$, or the reverse, then the overdetermination of component and resultant forces might be innocuous.

There is I think a more important objection to generous realism about forces which even friends of dependent overdetermination should grant. Instead of pointing towards the overdetermination of the effects by the forces, it points towards the destruction of those effects. Newton's second law asks us to sum all the distinct forces that acts on a body in order to determine its kinematic behaviour. If both component forces and resultant forces were real and distinct, we should the add them together. The acceleration of the apple would then be caused by :

$$\overrightarrow{F} + \overrightarrow{G} + \overrightarrow{R}$$

But this is silly : not only would the predicted acceleration be of twice the intensity of the real acceleration. But we would be engaged in a dramatic regress since $\overrightarrow{S}$, the resultant of the sum of $\overrightarrow{F}$, $\overrightarrow{G}$ and $\overrightarrow{R}$ is also a sui generis forces exerted on the apple. $\overrightarrow{S}$ should therefore be added in turn, etc. So we get a second reductio of generous realism about forces :

P1 Resultant and component forces are real .

P2 Resultant and component forces are wholly distinct.

P3 The acceleration of a body is determined, according to Newton's second law by the sum of all the wholly distinct forces acting on it.

C The acceleration of a body is determined by the sum of the component forces and of their resultant.

While some generous realists about forces might be happy with regular causal overdetetermination, there is no reasonnable way for them to bite the bullet in that second case.

3 Wholly distinct ?

These two objections to generous realism however, focus on a specific kind of generous realism, which I shall call prodigal realism :

prodigal realism about forces : component and resultant forces are real and wholly distinct.

I agree that prodigal realism is false, because of at least one of the two objections mentionned. However, the generous realist about force might reject P2 in those objections. I endorse moderately generous force realism (see fig. 1) :

moderately generous realism about forces : component and resultant are real but not wholly distinct.

What the moderately generous realist claims is that component and resultant forces are partially identical. $\overrightarrow{R}$ is not wholly distinct from the sum of $\overrightarrow{F}$ and $\overrightarrow{G}$.

Before trying to see where exactly component and resultant forces overlap, let us mention one *prima facie* reason in favour of that view. Suppose the apple is falling in the void. In that case only one force is exerted on it : the gravitation of the earth. Is this force component or resultant ? The natural answer is « both ». In the case where a single force is acting on a body, there is an identity between the component and the resultant force.

4 Component forces are not parts of resultant ones

How are component and resultant forces overlapping ? The most intuitive answer is to claim that component forces are parts of resultant forces. The vector addition should be understood as a mereological sum. This view however is to be rejected for at least two reasons.

First, in case of equilibrium, the sums of the component forces is null. This is a problem because if component forces are parts of resultant forces, whe have to say that in those cases there is a resultant force. But we have also to say that this resultant force has no intensity and no direction. Being realist about resultant forces is one thing, but being realist about resultant forces having no intentisty and no direction is another. It would be more

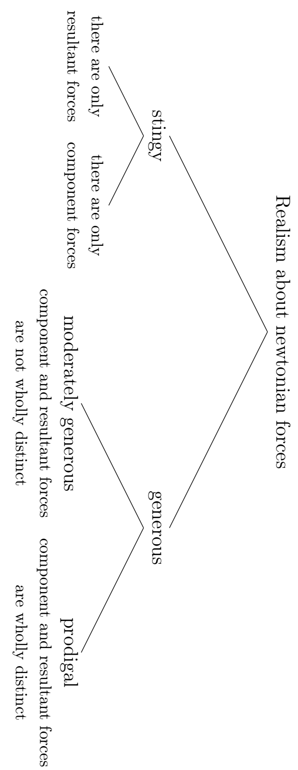

Fig. 1 – Varieties of forces realism

natural to say that in such circonstances there is no resultant force. But this cannot be the case if component forces are parts of resultant ones.

A second objection to such a view appeals to an analogy between force-vectors on the one hand, and kinematic-vectors such as displacement, velocity or accelerations vectors on the other. Take any kinematic vectors such as the actual acceleration of the apple. It can be analysed in many components acceleration-vectors, such as a first downward acceleration toward the left, and a second downard acceleration towards the right. But such component accelerations did not occur : they cannot be parts of the actual resultant acceleration of the apple. Russell (also quoted by Wilson, 2009) writes :

> Let there be three particles A, B, C. We say that B and C both cause accelerations in A, and we compound these two accelerations by the parallelogram law. But this composition is not truly addition, for the components are not *parts* of the resultant. The resultant is a new term, as simple as its components, and not by any means their sums.

I think the last sentence is true and remains true when the vectors under considerations are force-vectors rather than acceleration ones. Resultants forces are not made up of the component forces from which they result.

5 Concurrent forces and prevention

The proposal I want to defend is the reverse of the previous one : component forces are not parts of resultant forces, rather, *resultants forces are parts of component forces*. In order to flesh out this proposal, I need first to shed light on the nature of the relation between different concurring forces.

Different forces can act on a body. They can have different directions, different intensities and be of different types (gravitational, magnetic, etc.). All the (wholly) distinct forces acting on a body are called concurrent. Concurrent forces is a more neutral name for component forces : stingy realist who thinks that component forces are there only real ones think that there are only concurrent forces in reality.

concurrent forces : wholly distinct forces that act on a same body.

The crucial question is to understand the exact nature of this relation of concurrence between force. It cannot be only a matter of a juxtaposition of forces at a time in a body. Spatio-temporal coincidence is a too weak relation here for it cannot explain the way the different current forces combine in order to cause the motion of the body under consideration. If the relation between

the forces is not spatio-temporal, what could it be ? It is not itself a force. Forces do not exert forces on each other. In order to understand the nature of this concurrent, let us consider a special case of concurrent forces, namely antagonist forces :

antagonist forces : concurrent forces that have the same intensity but opposite directions.

Consider the apple when it is still hanging on the tree. The earth exert on it its gravitional force, but the branch of the tree exert on it an opposite force which exactly compensate for the gravitation. As a result, the apple does not move. The force exerted by the branch on the apple prevents the force exerted by the earth on the apple to cause the acceleration it would have caused, had it been alone.

This suggests the that the relation between concurrent forces is a causal relation of *prevention*. Concurring forces prevent each other to cause the acceleration they would have cause, had they been alone.

6 Resultant forces as residual component forces

We are now in a position to understand how resultant forces can be component forces. In the example of the apple hanging on the tree, the two concurrent forces were preventing each other to cause any acceleration of the apple. As a result, the resultant force was zero, which we should understand as meaning there was no resultant force.

Now consider again the case of the apple falling toward the earth, its fall being slowed down that the air resistance. The gravity and the air resistance are of opposite directions but have different intensity : because the air resistance has a lower intensity that the gravitation in that case, the apple accelerates towards the earth. I shall call those kind of forces opposite but not antagonist :

oppositive forces : concurrent forces that have opposite direction (they might have or not the same intensity).

The air resistance prevents the gravitational force to exert its full effect. In the vaccum, the apple would have accelerated more quickly toward the earth. My proposal is that we should interpret this as meaning that *the air resistance prevents parts of the gravitational force to exert its effect.* The rest of the gravitational force, which is not counteracted by the air resistance causes the acceleration. This unimpeded part of $\overrightarrow{G}$, is nothing but $\overrightarrow{R}$, the resultant force.

The proposal is then that a resultant force is one of the component force, namely the residual component force which does not meet any antagonist force. Resultant force is the unimpeded part of one of one of the component force : it is a residual force. Vectorial sums of forces should understood as substraction rather that as addition. An equivalent but less misleading mathematical representation of the resultant force acting on the apple is then :

$$\overrightarrow{R} = \overrightarrow{G} - \overleftarrow{F}$$

Each of these forces refers to a force concurring in the apple and the sign « – » refer to the relation of opposition between forces. Before looking at the costs of such a view, it is worth underlining its advantages :

1. It gives an explanation of the relationship between the concurrent forces, namely in terms of mutual causal prevention. Concurrent forces causally prevent each other from causing the accelerations they would have caused, had they been alone.
2. It allows for generous realism about forces which, as we have seen, is *prima facie* plausible. Both component forces and resultant forces are real. There are also distinct : resultant forces are just those component forces which are not counteracted by other antagonist component forces.
3. It avoids prodigal realism about forces. Though distinct, component forces are resultant forces are not wholly distinct : resultant forces are identical with the unimpeded component forces. Any risk of causal overdetermination of effects, or destruction of effects, is avoided.
4. It allows us to maintain that in case of equilibrium, there are no resultant forces rather that a null resultant force.

If true, *pace* the stingy realist, we can have the cake and eat it.

7 The costs (1) : gunky forces ?

« A stodgy cake », will he finickily reply. The first assumption of the proposed solution is that forces have parts that are themselve forces. According to the picture proposed, the gravitation exerted on the apple can be splitted in two parts : on part that enters in a mutual prevention relation with the air resistance force ; another part that causes the accelerations of the apple. Those part of forces are not spatio-temporal, so on first worry is that we are

using mereological tools where we are not allowed to do it (Wilson, 2009). But let us assume that there is a more liberal use of the part/whole relation that do not restrict it to spatio-temporal entities.

Even so the forces present picture entail that forces have infinitely many parts. Suppose the apple is no falling in the air, but in the water, or in any other gaz or liquid whose resistance force is inferior to the gravitation. For each of these stuff, the gravitational force exerted by the earth on the apple will be cutted in two distinct parts, depending of the intensity of the resistance force exerted by the stuff on the apple. This suggest that forces are infinitely divisible according to their intensity. There is however an intuitive appeal to the claim that two forces

8 The costs(2) : the double causal power of forces

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