On the Combined Effect of Periodic Signals and Coloured Noise on Velocity Uncertainties

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ABSTRACT

The velocity estimates and their uncertainties derived from position time series of Global Navigation Satellite System (GNSS) stations are affected by seasonal signals and their harmonics, and the statistical properties, i.e. the stochastic noise, contained in the series. If the deterministic model (linear trend and periodic terms) to describe the time series is not accurate enough, then this will alter the stochastic model and the resulting effect on the velocity uncertainties can be perceived as a consequence of a misfit of the deterministic model applied to the time series. In other words, the effect of insufficiently modeled seasonal signals will propagate into the stochastic model and falsify the results of the noise analysis besides the velocity estimates and their uncertainties. In this study we provide the General Dilution of Precision (*GDP*) of velocity uncertainties as being a ratio between uncertainties of velocities determined when two different deterministic models are applied while accounting for stochastic noise at the same time. In this newly defined GDP the first deterministic model includes a linear trend, while the second one includes a linear trend and seasonal signals. These two are tested with the assumption of power-law noise in the data. The more seasonal terms were added to the series, the more biased the velocity uncertainties become, especially for short time scales. With the increasing time span of observations, the assumption of seasonal signals becomes less important and the power-law character of the residuals starts to play a crucial role in the determined velocity uncertainties. With reference frame and sea level applications in mind we argue that even smaller change in *GDP* than 5% could be considered as significant. This means, that 7 and 9 years of continuous observations is the threshold for white and flicker noise, while

17 years are required for random-walk to make *GDP* to decrease below 5% and to omit periodic oscillations in the GNSS-derived time series taking only the noise model into consideration.

Key words: Time-series analysis; Numerical modeling; Satellite geodesy; Reference systems.

1 INTRODUCTION

Today, Global Navigation Satellite System (GNSS) measurements, in particular those from the Global Positioning System (GPS), are fundamental to many geodetic and geophysical investigations and are frequently used during the construction of kinematic reference frames, such as the International Terrestrial Reference Frame 2014 (ITRF2014) (Altamimi *et al.* 2016). From the processing or re-processing of the observables of permanently installed GNSS stations, daily or weekly geocentric coordinate solutions are obtained, from which position time series are formed. The primary product from the analysis of these time series is often the linear rate of change, or velocity, and the associated uncertainty (Zhang *et al.* 1997).

The velocities are assumed to represent the linear movement of the Earth's crust or land due to tectonic plate motions (Larson *et al.* 1997, Altamimi *et al.* 2012) or the viscoelastic relaxation associated with glacial isostatic adjustment (GIA) (Johansson *et al.* 2002, Bradley *et al.* 2009). Besides this, almost all sites within the global network of GPS stations also show some non-linear, periodic motions, which have been associated primarily with the seasonal changes on Earth. However, some stations may also exhibit motions of non-linear and non-periodic character (Shih *et al.* 2008, Bogusz 2015). This may be due to the elastic response of the Earth's crust from the rapid ice mass loss at the polar ice sheets or mountain glaciers (Wahr *et al.* 2013), being located in the deforming zones near plate boundaries (Wdowinski *et al.* 2004) or areas of oil and gas or groundwater extraction (Munekane *et al.* 2004). In this study we will not deal with cases showing such non-linear and non-periodic behavior.

Blewitt & Lavallée (2002) were one of the first to investigate the effect of periodic signals on GPS time series and today it is widely acknowledged that such seasonal terms affect GPS and other geodetic time series (e.g. Bos *et al.* 2010, Davis *et al.* 2012). The causes are almost completely recognized and can be grouped into categories suggested by Dong *et al.* (2002): real geophysical effects of atmospheric (e.g.

Tregoning & van Dam 2005), hydrological (e.g. van Dam *et al.* 2001) or ocean loadings (e.g. van Dam *et al.* 2012) with thermal expansion (Romagnoli *et al.* 2003) and numerical artifacts of navigation satellite systems. Penna & Stewart (2003) described aliased periodic signals in the coordinate time series due to under-sampling of residual diurnal and semi-diurnal tidal signatures. Griffith & Ray (2013) extended this analysis for the largest waves in the International Earth Rotation and Reference Frame Service (IERS) Conventions 2010 diurnal and semi-diurnal tidal polar motion model assuming 24-h sampling. The second technique-related error is associated with satellite orbits (draconitics). Agnew & Larson (2007) found that for daily sampling rates in GPS-derived coordinates this period will alias to a frequency of 1.04333 cpy. Ray *et al.* (2008) compared harmonics obtained using different techniques (GPS, VLBI and SLR) when they discovered an anomalous peak in the GPS derived time series, which was not present in other series. They explained it to be related to the interval required for the constellation to repeat its inertial orientation with respect to the sun (GPS year). Amiri-Simkooei (2013) upon the analysis of the JPL (Jet Propulsion Laboratory) data processed at the GPS Analysis Center obtained a period of 351.6±0.2 days. Finally, multipath (King *et al.* 2012), insufficient modeling of antennas (Sidorov 2016), errors in network, the adjustment that transfer from fiducial stations or the inclusion of the scale (Tregoning & van Dam 2005) contribute to constellation-specific periodic signals in the time series.

From this it is clear that the deterministic model of a GNSS position time series needs to include the parameters for both the linear and periodic motions at a given station (Bevis & Brown, 2014). Furthermore, in a true geodetic approach the model must provide a means to obtain the most realistic uncertainties associated with the parameter estimates in order to provide confidence limits at a given significance level for these. In this respect, we model the time series, which have been pre-processed for outliers, offsets and gaps, with Least Squares Estimation as:

$$x(t) = x_0 + v \cdot t + \sum_{i=1}^{n} \left[AC^i \cdot \cos\left(\omega_i^T \cdot t\right) + AS^i \cdot \sin\left(\omega_i^T \cdot t\right) \right] + \varepsilon(t) \tag{1}$$

where x_0, v, AC^i, AS^i, n are the intercept, velocity, cosine and sine terms of i^{th} harmonic and the number of harmonics of angular velocity ω of period T, respectively. The term $\varepsilon(t)$ contains the residuals, all variations not explicitly modeled and, therefore, disregarded by this station motion model. It is now commonly known that the residuals do not follow a strict random (white noise with spectral index $\kappa = 0$) behavior but follow that of a combination of white and coloured noise, with the latter often being modeled as a power-law process (Agnew 1992). This temporally-correlated noise is due to mismodelling in GNSS satellites orbits, Earth Orientation Parameters, large-scale atmospheric or hydrospheric effects (flicker noise with spectral index $\kappa = -1$; Williams *et al.* 2004, Beavan 2005, Amiri-Simkooei *et al.* 2007, Bos *et al.* 2008, Teferle *et al.* 2008, Kenyeres & Bruyninx 2009, Santamaria-Gomez *et al.* 2011, Klos *et al.* 2015a), or the local environment and monumentation (random walk process with spectral index $\kappa = -2$; Johnson & Agnew 1999, Beavan 2005). Even when a perfect monument is considered with the environment being transparent to the signal, the noise would be flicker and arise from low-frequency fluctuations of the satellite clocks (Dutta & Horn 1981). Beyond the above, there are also some seasonal features that add more correlated noise to GNSS data. As stated by Johnson & Agnew (1999), if time series were too short, it is more difficult to detect any change due to correlation than it would be for long data. As is now widely acknowledged the stochastic properties, i.e. the stochastic model of the residuals significantly influence the magnitude of the uncertainties associated with the parameters (Zhang *et al.* 1997, Langbein & Johnson 1997, Mao *et al.* 1999, Williams 2003) and the whiter the noise, the faster the velocity uncertainty decays with increasing time series length. Therefore, the noise character has the greatest impact on velocity uncertainty and errors of the deterministic model estimated at the same time.

In this research we provide a General Dilution of Precision (*GDP*) of the velocity uncertainties being the ratio of uncertainties of velocities arising from two different assumptions of the deterministic model. The first of them assumes a linear velocity only, while the second is a combination of linear velocity and periodic components. Each of the assumptions is presented with certain noise models starting from white, moving on to flicker and end up with random walk, the most extreme case for GNSS position time series. The determined errors of the velocities are being discussed along the two previous papers of Blewitt & Lavallée (2002) and Bos *et al.* (2010) which provided the first discussion of this topic. We show using simulated data that our approach gives additional advantages to GNSS time series analysis in comparison to the ones mentioned before. Finally we apply the newly developed formulae to real GNSS time series of selected stations in Europe.

2 DILUTION OF PRECISION

Blewitt & Lavallée (2002) developed a model to calculate the bias level while one may not account for periodic signals of annual frequency. The velocity bias expressed in mm/yr introduced by them was a zero-crossing oscillatory function of data span, tending to zero for infinitely long time series. According to this function they discovered the "zero-bias theorem" for unbiased velocity near integer-plus-half years and introduced the ratio of uncertainties of velocities v_1 and v_2. They called it the Dilution of Precision (*DP*) and estimated its value as (Blewitt & Lavallée 2002):

$$DP(\tau) = \left[1 - \frac{6}{(\pi \cdot f \cdot \tau)^2} \cdot \frac{\left(\cos(\pi \cdot f \cdot \tau) - \frac{\sin(\pi \cdot f \cdot \tau)}{\pi \cdot f \cdot \tau} \right)^2}{1 - \frac{\cos(\pi \cdot f \cdot \tau) \cdot \sin(\pi \cdot f \cdot \tau)}{\pi \cdot f \cdot \tau}} \right]^{-\frac{1}{2}} \tag{2}$$

where τ is the time span, v_1 and v_2 denote the velocity determined without and with accounting for periodic terms of frequency f, respectively. Figure 1 shows the *DP* value which increases towards infinity when the time span is shorter than 1 year. A number of maxima of oscillations in *DP* can be noticed for integer years. These come from periodic terms in (1). For certain epochs starting from 1.5 years with a step of 1 year $DP = 1$. It results from the fact that the velocity uncertainties for a model with linear velocity and model with seasonal terms added are equal to each other. Besides, for a time span longer than 3.5 years the $DP < 1.05$, which means that the difference between both variances is below 5%. However, Blewitt & Lavallée (2002) considered only white noise, in consequence assumed that the character of residuals ε has no or little impact on the estimated uncertainties.

However, it is expected that when power-law noise is added, the time when the difference between both variances is below 5% will increase, as the power-law noise process adds temporal correlation to the time series. That is why Bos *et al.* (2010) discussed the results of Blewitt & Lavallée (2002) by empirically analyzing the effect of periodic signals using six stations, but assuming a combination of white and coloured noise. They noticed, that the choice of character of the stochastic part may be much more important than that of the seasonal part in the deterministic model. This is due to the fact that velocity uncertainty depends mostly on the spectral index and amplitude of the power-law process. They concluded that the knowledge of the noise characteristics of GNSS time series is crucial when velocities and their uncertainties are determined. Furthermore, they observed a shift in the minimum of the DP from the integer-plus-half years towards integer-plus-a-quarter years position. In this paper we confirm the results found by Bos *et al.* (2010), but deliver the mathematic formulas for that.

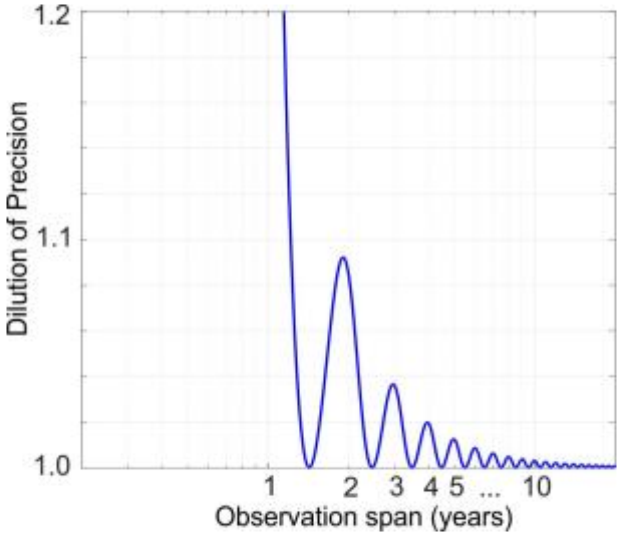

Figure 1. Dilution of precision for models: with linear velocity and linear velocity plus periodic signals (recomputated from Blewitt & Lavallée 2002).

Let us assume that the vector of residuals $\boldsymbol{\varepsilon}$ arising from deterministic model matrix $\hat{\mathbf{x}}$ fitted into (1) are computed from:

$$\boldsymbol{\varepsilon} = \hat{\mathbf{x}} - \mathbf{x} \tag{3}$$

Then we can write down the observations in (1) as:

$$\mathbf{x} = \mathbf{A}\boldsymbol{\theta} + \boldsymbol{\varepsilon} \tag{4}$$

where $\mathbf{A}$ is the model or design matrix, $\boldsymbol{\theta}$ is a vector with parameters of the model and $\boldsymbol{\varepsilon}$ is the vector of residuals. When the parameters of the model are determined by means of Maximum Likelihood Estimation (MLE), $\boldsymbol{\theta}$ is given by:

$$\hat{\boldsymbol{\theta}} = \left(\mathbf{A}^T \cdot \mathbf{C}_{\varepsilon\varepsilon}^{-1} \cdot \mathbf{A}\right)^{-1} \cdot \mathbf{A}^T \cdot \mathbf{C}_{\varepsilon\varepsilon}^{-1} \cdot \mathbf{x} \tag{5}$$

with $\mathbf{C}_{\varepsilon\varepsilon}$ being the covariance matrix for the vector of residuals $\boldsymbol{\varepsilon}$. The covariance matrix of the parameters of the model is:

$$\mathbf{C}_{\hat{\theta}\hat{\theta}} = \mathbf{A}^{-1} \cdot \mathbf{C}_{\varepsilon\varepsilon} \cdot \mathbf{A}^T \tag{6}$$

If the vector of residuals $\boldsymbol{\varepsilon}$ follows a power-law process such as:

$$\varepsilon_i = \sum_{j=0}^{N-1} h_j \cdot w_{i-j} + v_i \tag{7}$$

which is a convolution of white noise of $w_i \in \mathrm{N}(0, \sigma_{pl})$ and another white noise of $v_i \in \mathrm{N}(0, \sigma_{wn})$ where σ_{pl} and σ_{wn} are the standard deviations of noise, respectively. N is the number of data, while h is defined by the recursive formula (Bos *et al.* 2008):

$$\begin{aligned} h_0 &= 1 \\ h_i &= \left(\frac{-\kappa}{2} + i - 1\right)\frac{h_{i-1}}{i}, \forall i > 0 \end{aligned} \tag{8}$$

with κ being the spectral index, which when appropriately assigned may characterize the stochastic part of the time series (Mandelbrot & Van Ness 1968). Then the covariance matrix of the residuals can be written as:

$$\mathbf{C}_{\varepsilon\varepsilon il} = \sigma_{pl}^2 \sum_{j=0}^{N} h_j \cdot h_{j+|i-N|} + \sigma_{wn}^2 \cdot \delta_{iN} \tag{9}$$

or as:

$$\mathbf{C}_{\varepsilon\varepsilon} = \sigma_{pl}^2 \cdot \mathbf{L} \cdot \mathbf{L}^T + \sigma_{wn}^2 \cdot \mathbf{I} \tag{10}$$

L is a lower triangular Toeplitz matrix with coefficients of:

$$L_{ij} = \begin{cases} h_{i-j} & \forall (i-j) \geq 0 \\ 0 & \forall (i-j) < 0 \end{cases} \tag{11}$$

If we assume a deterministic model with a power-law character of the residuals as in (7), we get the covariance matrix of the determined parameters (6) as:

$$\mathbf{C}_{\hat{\theta}\hat{\theta}} = \sigma_{pl}^2 \cdot \mathbf{E}(\kappa) + \sigma_{wn}^2 \left[\mathbf{A}^T \cdot \mathbf{A}\right]^{-1} \tag{12}$$

where the covariance matrix **E** results from the power-law process:

$$\mathbf{E}(\kappa) = \left[\left(\mathbf{L}^{-1} \cdot \mathbf{H}\right)^T \left(\mathbf{L}^{-1} \cdot \mathbf{H}\right)\right]^{-1} \tag{13}$$

Now, we can inverse (11) into:

$$L_{ij}^{-1} = \begin{cases} g_{i-j} & \forall (i-j) \geq 0 \\ 0 & \forall (i-j) < 0 \end{cases} \tag{14}$$

with:

$$g_0 = 1$$
$$g_i = \left(\frac{\kappa}{2} + i - 1\right) \cdot \frac{g_{i-1}}{i}, \ \forall i > 0 \tag{15}$$

These coefficients whiten any fractionally differenced Gaussian noise process as:

$$w_m = \sum_{i=0}^{N-1} g_i \cdot \varepsilon_{m-i} \tag{16}$$

with ε defined in (7).

3 MODELS

If we assume that the deterministic model of GNSS time series follows a linear trend with a vector of parameters built as:

$$\boldsymbol{\theta} = [x_0, v]^T \tag{17}$$

where v is the slope or velocity and x_0 is the intercept and the model matrix $\mathbf{A}$:

$$\mathbf{A} = \begin{bmatrix} 1 & t_0 \\ \vdots & \vdots \\ 1 & t_{N-1} \end{bmatrix} \tag{18}$$

then the variance of velocity that is estimated with the assumption of the general power-law process is determined with:

$$C_{vv} = \frac{\sigma_{pl}^2}{\sum_{i=0}^{N}\left(\sum_{j=0}^{i} g_j \cdot t_{i-j}\right)^2 - \frac{\left\{\sum_{i=0}^{N-1}\left[\left(\sum_{j=0}^{i} g_j \cdot t_{i-j}\right)\left(\sum_{j=0}^{i} g_j\right)\right]\right\}^2}{\sum_{i=0}^{N-1}\left(\sum_{j=0}^{i} g_j\right)^2}} \tag{19}$$

The proper modelling of the seasonal terms may directly influence on reliability of determined parameters, such as the velocity. Blewitt & Lavallée (2002) showed, that seasonal term influences the uncertainty of velocity. However, they did not consider the power-law character of residuals, but assumed them to follow a white noise process. Bos *et al.* (2010) noticed that not only do the seasonal terms affect the linear velocity when being improperly removed, but the noise properties are much more important for a reliable estimation of the velocity error. These directly affect the uncertainty of velocity due to different shapes of covariance matrix, as in (9-10).

Let us assume now a deterministic model with linear velocity and annual term:

$$\mathbf{A} = \begin{bmatrix} 1 & t_0 & \cos(2\cdot\pi\cdot f\cdot t_0) & \sin(2\cdot\pi\cdot f\cdot t_0) \\ \vdots & \vdots & \vdots & \vdots \\ 1 & t_{N-1} & \cos(2\cdot\pi\cdot f\cdot t_{N-1}) & \sin(2\cdot\pi\cdot f\cdot t_{N-1}) \end{bmatrix} \tag{20}$$

The vector of parameters is built here as:

$$\boldsymbol{\theta} = \left[x_0, v, AC, AS\right]^T \tag{21}$$

where *AC* and *AS* are the annual cosine and sine term, respectively.

Now, we can compute the inverse of covariance matrix for the general power-law process $\mathbf{E}(\kappa)^{-1}$ (13) as:

$$
\begin{aligned}
E^{-1}_{x_0,x_0} &= \sum_{i=0}^{N-1}\left(\sum_{j=0}^{i} g_j\right)^2 \\
E^{-1}_{x_0,v} &= \sum_{i=0}^{N-1}\left[\left(\sum_{j=0}^{i} g_j \cdot t_{i-j}\right)\left(\sum_{j=0}^{i} g_j\right)\right] \\
E^{-1}_{x_0,AC} &= \sum_{i=0}^{N-1}\left[\left(\sum_{j=0}^{i} g_j\right)\left(\sum_{j=0}^{i} g_j \cdot \cos\left(2\cdot\pi\cdot f\cdot t_{i-j}\right)\right)\right] \\
E^{-1}_{x_0,AS} &= \sum_{i=0}^{N-1}\left[\left(\sum_{j=0}^{i} g_j\right)\left(\sum_{j=0}^{i} g_j \cdot \sin\left(2\cdot\pi\cdot f\cdot t_{i-j}\right)\right)\right] \\
E^{-1}_{v,v} &= \sum_{i=0}^{N-1}\left(\sum_{j=0}^{i} g_j \cdot t_{i-j}\right)^2 \\
E^{-1}_{v,AC} &= \sum_{i=0}^{N-1}\left[\left(\sum_{j=0}^{i} g_j \cdot t_{i-j}\right)\left(\sum_{j=0}^{i} g_j \cdot \cos\left(2\cdot\pi\cdot f\cdot t_{i-j}\right)\right)\right] \\
E^{-1}_{v,AS} &= \sum_{i=0}^{N-1}\left[\left(\sum_{j=0}^{i} g_j \cdot t_{i-j}\right)\left(\sum_{j=0}^{i} g_j \cdot \sin\left(2\cdot\pi\cdot f\cdot t_{i-j}\right)\right)\right] \\
E^{-1}_{AC,AC} &= \sum_{i=0}^{N-1}\left(\sum_{j=0}^{i} g_j \cdot \cos\left(2\cdot\pi\cdot f\cdot t_{i-j}\right)\right)^2 \\
E^{-1}_{AC,AS} &= \sum_{i=0}^{N-1}\left[\left(\sum_{j=0}^{i} g_j \cdot \cos\left(2\cdot\pi\cdot f\cdot t_{i-j}\right)\right)\left(\sum_{j=0}^{i} g_j \cdot \sin\left(2\cdot\pi\cdot f\cdot t_{i-j}\right)\right)\right] \\
E^{-1}_{AS,AS} &= \sum_{i=0}^{N-1}\left(\sum_{j=0}^{i} g_j \cdot \sin\left(2\cdot\pi\cdot f\cdot t_{i-j}\right)\right)^2
\end{aligned}
\tag{22}
$$

with g being defined using (15). Now, this inverse covariance matrix can easily transferred to (12) to estimate the variances of determined parameters.

4 SIMULATED SERIES

In order to test the General Dilution of Precision formulas derived above we carried out a number of evaluations. For these we simulated time series of up to a maximum length of 25 years, which is at the time of writing the longest term of available GNSS time series. Synthetic data were created based on two approaches for the deterministic part: the first included annual and semi-annual terms, the second included all significant periods i.e. all tropical and draconitic terms up to 9th harmonic plus fortnightly and the Chandlerian period as it may be present at some stations at island and coastal sites (Richard Gross, private communication, 2015), see Bogusz & Klos (2015) for more details. Then, we added temporal correlation in form of white plus power-law noise, built an inverse covariance matrix as in (22) and transferred it to estimate the variances of determined parameters as in (12). We assumed four different spectral indices equal to 0, -1, -1.5 and -2 that indicate pure white, pure flicker, power-law noise between flicker and random-walk process and pure random-walk noise, respectively, with $\sigma_{pl}^2 = 1$ and $\sigma_{wn}^2 = 1$. White noise was added to flicker, power-law and random-walk, when assumed.

Figure 2 shows the error of the velocity for white, flicker and random-walk noise. For time series shorter than 70 days, the error of velocity is much higher for white and flicker noise assumptions than for random-walk noise. This changes when data becomes to be longer than 70 days. The error of velocity is the smallest for white noise assumption. Random-walk noise delivers the greatest errors of parameters. The longer the data is, the greater is the difference between errors delivered for random-walk noise and flicker or white noise assumptions. Having assumed white, flicker and random-walk noises, for 20 years of data, one will obtain the error of velocity equal to 10^{-3}, 10^{-2} and 10^{-1} mm/yr, respectively.

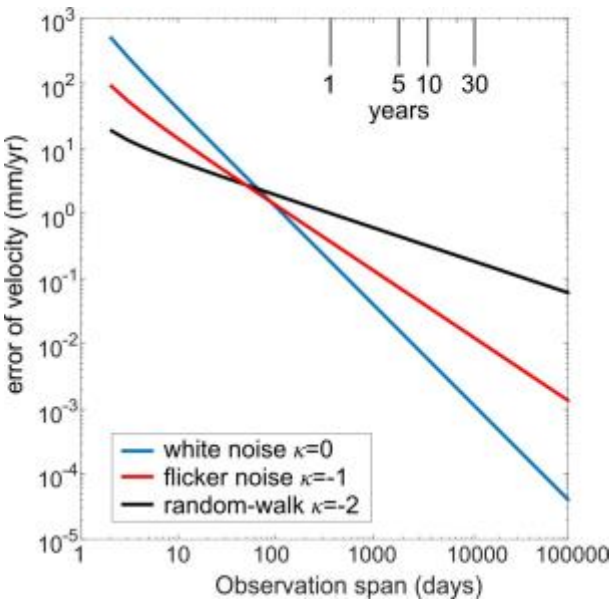

Figure 2. Error of velocity σ_v (in mm/yr) for different lengths of time series in double logarithmic scale. The integer spectral indices are examined. We assumed that $\sigma_{pl}^2 = 1$ and $\sigma_{wn}^2 = 1$.

We estimated the relative differences in velocity variances for two deterministic models: with linear velocity denoted as σ_{v1}^2 and with linear velocity plus seasonal terms denoted as σ_{v2}^2 as:

$$\Delta\sigma_v^2 = \frac{\sigma_{v1}^2 - \sigma_{v2}^2}{\sigma_{v1}^2} \tag{23}$$

Adding seasonal terms results in oscillations in the estimated velocity error when the length of time series changes. These of course are not so obvious any more, when a spectral index of power-law dependencies increases. The same has been already noticed by Bos *et al.* (2010). The largest differences between two models are being observed for the random-walk noise assumption. As the spectral index was shifted from 0 towards -3, the differences between velocity uncertainty will of course enlarge much more. Due to the above, the largest differences between two models are being expected for badly monumented stations (Langbein & Johnson 1997, Beavan 2005, Klos *et al.* 2015b).

A difference between the two uncertainties estimated with and without periodic terms can be better understood by computing a ratio between them. We called it the General Dilution of Precision (*GDP*) in accordance with Blewitt & Lavallée (2002) and as in (2), but taking into consideration a power-law noise in the stochastic part, as was underlined in (12). We have adopted two approaches: widely used annual and semi-annual terms to be subtracted and the approach consistent with Bogusz & Klos (2015): tropical and draconitics up to their 9th harmonics plus Chandlerian and fortnightly. Figures 3 & 4 show a *GDP* for white, flicker, random-walk and power-law noise of spectral indices equal to 0, -1, -2 and -1.5, respectively.

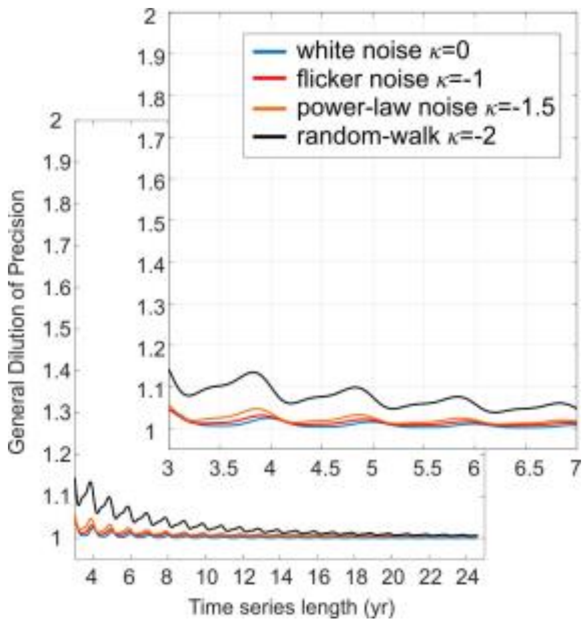

Figure 3. General Dilution of Precision (*GDP*) for white noise (blue), flicker noise (red), power-law noise of spectral index equal to -1.5 (orange) and random-walk (green) plotted for deterministic model containing linear velocity plus annual and semi-annual oscillations.

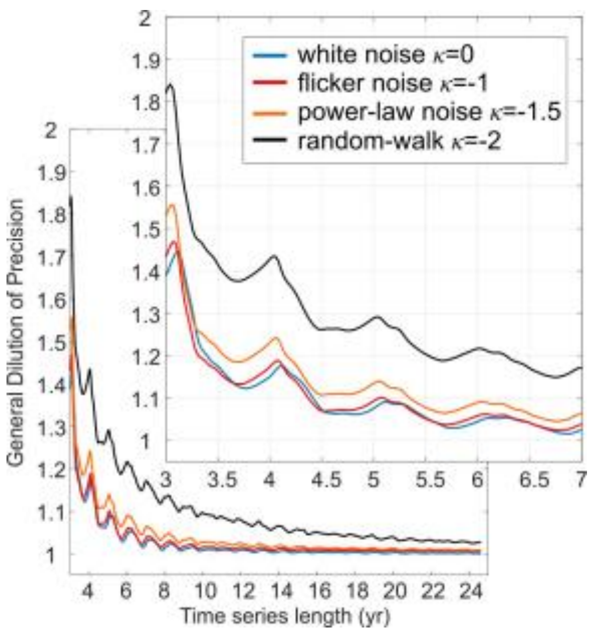

Figure 4. General Dilution of Precision (*GDP*) for white noise (blue), flicker noise (red), power-law noise of spectral index equal to -1.5 (orange) and random-walk (black) plotted for deterministic model containing linear velocity plus extended model of periodicities of all tropical and draconitic terms up to 9th harmonic plus fortnightly and the Chandlerian period, according to Bogusz & Klos (2015).

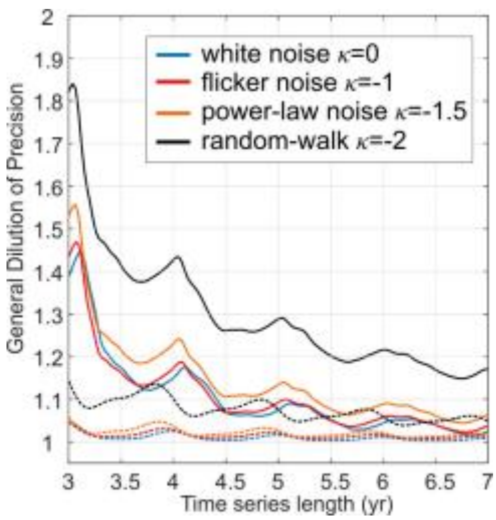

Figure 5. Comparison between GDPs estimated for white noise (blue), flicker noise (red), power-law noise of spectral index equal to -1.5 (orange) and random-walk (black) plotted for deterministic model containing 1) linear velocity plus

annual and semi-annual oscillations (dashed lines) and 2) linear velocity plus extended model of periodicities of all tropical and draconitic terms up to 9th harmonic plus fortnightly and the Chandlerian period, according to Bogusz & Klos (2015) (solid lines).

We computed the *GDP* values for harmonics of one year. They increased local maxima in the *GDP* plot. It can be noticed that in comparison to Blewitt & Lavallée (2002) who considered the annual term, the semi-annual signal also increases the local minimum of *GDP* with a white noise assumption. It means, that adding the power-law noise to the annual curve or adding a semi-annual term to white noise causes an increase in the velocity uncertainty even at those points where the estimated velocity should not be biased. The more seasonal terms are added to the series, the more biased is the velocity uncertainty, especially for short time scales. Having compared Figures 3 and 4 (Figure 5) it is clear, that the type of deterministic model affects the velocity uncertainty and makes *GDP* to reach the value of 5% (we adopted this number following Blewitt & Lavallée 2002) after 9 years, rather than 4, as was expected for annual plus semi-annual terms (a case of white and flicker noise). The value of 5% means an increase in velocity uncertainty of 0.025 mm/yr, when a typical error of velocity of 0.5 mm/yr is considered. However, with the increasing demand on velocities from reference frame and sea level applications, we argue that even lower changes than 5% in *GDP* could be considered as significant. All the above shows, that periodic terms affect the velocity uncertainty much more at short time scales than they do for long-term data leaving the values of velocity unbiased. With the increasing time span of observations, the assumption of seasonal signals becoming less important is validated. Here, the power-law character of the residuals plays a crucial role in determining the velocity uncertainty. In this way, 7, 9 and 17 years is enough, respectively, for white, flicker and random-walk to make *GDP* to decrease below 5% and to omit periodic oscillations in the GNSS-derived time series taking only noise model into consideration. So, providing a time series long enough, the assumed periodicities will not affect the velocity uncertainty as much as the noise would. This is why we should focus on obtaining the best estimate for the spectral index of each geodetic time series such as position, sea level or zenith total delay.

5 REAL GNSS TIME SERIES

In order to confirm our theoretical approach above with real data we used position time series of continuous GNSS stations produced at the Jet Propulsion Laboratory (JPL) using Precise Point Positioning (PPP) (Zumberge *et al.* 1997) with integer ambiguities resolved (Bertiger *et al.* 2010). Further details on the data processing strategy are published at: https://gipsy-oasis.jpl.nasa.gov. In this study we picked 115 stations with time series of different lengths from 5 to 23 years and showing no specific or unusual behavior. This is to ensure that our results are not compromised by stations, which, for example, did not follow the assumption of a linear evolution in the coordinates. The station distribution is indicated in Figure 6. The daily time series of the North, East and Up components were pre-processed for outliers, offsets and gaps if necessary and then analyzed with the reformulated Maximum Likelihood Estimation (MLE) as implemented in the Hector software (Bos *et al.* 2013). The stochastic model was assumed to be a combination of a white plus power-law process along with two different deterministic models: (1) velocity and (2) velocity with all tropical and draconitic terms up to 9th harmonic plus fortnightly and the Chandlerian period. Table 1 presents the results of median seasonal amplitudes of annual (tropical), semi-annual, three- and four-monthly periods together with their errors. The median amplitudes of the annual term are at the level of 1.65, 1.78 and 4.22±0.20 mm for the North, East and Up component, respectively. Almost all median amplitudes of the 2nd, 3rd and 4th harmonics of the annual term fall below 1 mm with a mean error equal to 0.10 and 0.25 mm for horizontal and vertical changes.

The length of data of a minimum of 15 years is long enough for a noise process in GNSS time series to become a dominant in *GDP* over seasonal signals. The ratio of velocity biases for the Up component is rapidly varying in time due to the generally small values of the velocity itself. Hence, we decided not to focus on it further.

Figure 6 presents *GDP* values for globally distributed set of stations of series of different lengths from 5 to 23 years. From these figures we can easily notice, that the *GDP*s for the North and East components remain close to one for a majority of stations with minimum value of 0.94, maximum values of 1.04 and medians close to 1.00. This is in good agreement with the theoretical formulae derived above. The greatest deviations of values of *GDP* were obtained for stations affected by large seasonal signals, as e.g. BRAZ (Brazil) or

KOUR (French Guiana). *GDP* for European stations generally tend to fall below 1% for all series examined here.

6 CONCLUSIONS & DISCUSSION

Not accounting for the seasonal signals results in an increase of the autocorrelation or temporally-correlated noise within the time series. This, in turn, influences the stochastic model when the periodic signals are not properly assumed. In this way, if we were certain about the presence of seasonal signals in the GNSS time series and did not model them, the residuals would resemble more a flicker noise. This would lead to increased uncertainties for all parameter estimates. Again, when the seasonal signals are properly modelled, another issue that may cause an artificial increase in the velocity uncertainty is an improperly assumed noise model itself. When flicker and random-walk are being compared to a power-law process there may be an underestimation of the error bounds by a factor of two.

Some points can be easily noticed and raised for deeper discussion from the presented results. Along with the increasing spectral index, the amplitudes of oscillations also increase. This arises from the fact that any power-law process with $\kappa<0$ brings a correlation between amplitudes of seasonal terms and velocity. In this way, the *GDP* value is much higher for any time series length considered. The strong peaks of oscillations as seen in the *GDP* are indicated for short time scales, especially for the random-walk case. The applied oscillations play a significant role, even much more important than the a priori assumed noise character. The noise character starts to become important for time series longer than 9 years. The local minima and maxima of *GDP* are also being enlarged together with a change of spectral index from 0 to -2. This shows, that the *GDP* may differ from integer-plus-half years by Blewitt & Lavallée (2002), who considered only white noise. This is clearly noticed for the case of random-walk and has already been empirically confirmed by Bos *et al.* (2010). In this research, we provided mathematic formulas for the findings of Bos *et al.* (2010) and confirm their correctness with synthetic and real data.

Table 1. Median values of amplitudes of tropical periodicities (365.25 days) in the GNSS time series with a range of standard deviations (STD) determined using power-law dependencies in the time series. A range of absolute values of standard deviations are presented in the table.

Component	Amplitude [mm]				Range of absolute values of STD [mm]			
	1 cpy	2 cpy	3 cpy	4 cpy	1 cpy	2 cpy	3 cpy	4 cpy
North	1.65	0.53	0.29	0.18	0.07-0.21	0.05-0.15	0.04-0.12	0.04-0.11
East	1.78	0.59	0.23	0.17	0.05-0.36	0.04-0.24	0.04-0.20	0.04-0.17
Up	4.22	1.96	0.34	0.34	0.18-0.57	0.16-0.44	0.15-0.39	0.14-0.36

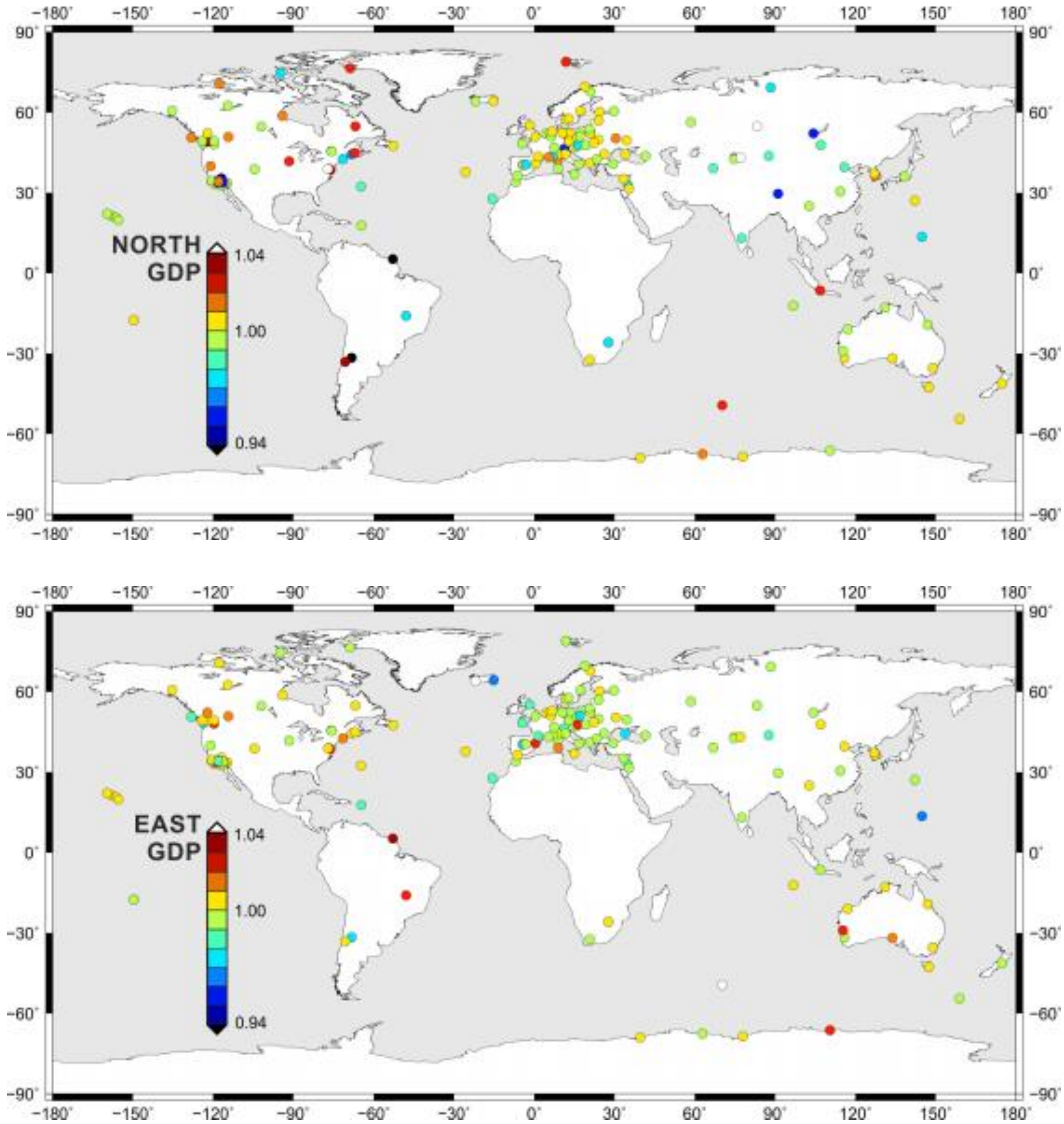

Figure 6. General Dilution of Precision (*GDP*) for North and East components.

In this study we focused on white plus power-law noise, as generally the best estimate for the stochastic model of GNSS time series. We showed that periodic signals are more important for short time scales, whereas the stochastic noise plays a significant role when the length of the time series increases. Also, with increasing spectral indices, the *GDP* decreases more slowly. We discussed a previously published approach, which indicated that 3.5 years of data are enough for the *GDP* to fall below 5%. When more seasonal signals and their harmonics were added to the deterministic model: periodicities of all tropical and draconitic terms up to 9th harmonic plus fortnightly and the Chandlerian period, the *GDP* requires 9 years to fall below 5% for white and flicker noise model. We have also discovered, that the noise character starts to become more important than the periodic signals for time series longer than 9 years. And finally, Blewitt & Lavallée (2002) used the value of 5% to calculate the minimum velocity bias. However, this value is disputable. With the increasing demand on velocities, we argue that even smaller change in *GDP* could be considered as significant. This means, that 7 and 9 years of continuous observations is the threshold for white and flicker noise, while 17 years is enough for random-walk to make the *GDP* to decrease to below 5% and to omit periodic signals in the GNSS-derived time series, taking only the noise model into consideration.

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JPL repro2011b time series accessed from ftp://sideshow.jpl.nasa.gov/pub/JPL_GPS_Timeseries/repro2011b/raw/ on 2015-08-13.

Maps in Fig. 6 were prepared with GMT software (Wessel et al., 2013).

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