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AMPLIFICATION OF VIBRATIONS DUE TO THE REPETITION OF THERMAL SHOCKS IN A PULSED FAST REACTOR

by

J. RANDLES

1968

Joint Nuclear Research Center
Ispra Establishment - Italy

Reactor Physics Department
Reactor Theory and Analysis

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The problem is essentially that of studying the interference between an existing wave (the cumulative product of all previous pulses) and the wave suddenly introduced at a given moment by the latest pulse. A recurrence formula relating the amplitude of vibration at any two consecutive pulses is derived.

This equation is solved in two ways. First, the asymptotic solution (representing steady oscillatory equilibrium with the imposed pulse train) is obtained analytically and found to possess the expected resonances for pulsation periods equal to integral multiples of the (fundamental mode) period of oscillation. Second, the detailed solution describing the pulse-by-pulse approach of the vibrations to their asymptotic level is obtained numerically. Simple approximate formulae giving the height and half-width of the resonances and the timescale of all transients are derived and should prove useful in design studies.

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SUMMARY

The objective of this paper is to provide a method of evaluating the amplification of fuel slug vibrations caused by the fast repetition of thermal shocks during the normal operation of a pulsed fast reactor. Such a method is required primarily for the assessment of fuel element safety.

From a study of the response of a fuel slug to a discontinuous temperature rise, it is concluded that only the fundamental mode axial vibrations generated during a pulse can survive long enough to interact with the vibrations generated by the next pulse. The theory of amplification is therefore developed on the basis of the fundamental mode alone.

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KEYWORDS

FAST REACTORS
PULSES
THERMAL SHOCK
VIBRATIONS
NUMERICALS

CONTENTS

INTRODUCTION

The temperature rise during the normal excursions of a pulsed fast reactor is so rapid that longitudinal elastic vibrations are excited in the fuel slugs[1,2,3]. These vibrations are gradually dissipated between pulses by several mechanisms, the most important being: (1) transfer of vibrational energy from the fuel slug through areas of contact with the cladding and other components; (2) internal friction and other non-adiabatic effects within the fuel.

Provided the first of these mechanisms is operative, the damping will almost certainly be strong enough to remove the oscillations generated by each pulse before the arrival of the next. However, because of the long life of each fuel charge, the probability of minor deformations in the fuel elements is not negligible and it may happen that the areas of contact through which acoustic energy propagates out of the fuel slug may be temporarily lost. If this occurs, the damping of the oscillations created by the pulses may be considerably reduced, since it is now known experimentally[4] that the internal friction of some metallic fuels is very small.

Thus, there arises a problem in which the vibrations stimulated in one of the fuel slugs by a certain pulse are still present when the next pulse arrives. The new burst of oscillations introduced by this pulse then interfere with the residual vibrations and, if the phase is appropriate, amplification occurs. When repeated over many hundreds of pulses, the amplification may become so large as to constitute a danger to the fuel element.

The following example can serve as an illustration of the above general discussion. In the SORA reactor[5], the fuel slug is seated in the base of the cladding which thereby carries its whole weight. In addition to this weight, a spring mechanism in the upper part of the cladding provides additional downward pressure to ensure that the slug always remains in its seating after the recoil following every pulse. The constraints thus existing at both ends of the slug provide a very efficient mechanism

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for the removal of vibrational energy and the decay time of all oscillations is expected to be adequately small. However, with the occurrence of small distortions in the fuel element, there is no guarantee that the spring mechanism will not fail and if this happens, repeated pulsing may shake the fuel slug free of the constraints at both ends, its weight being supported by a lateral "pinching" from the cladding. In such a case, the damping of the vibrations would depend almost entirely on the non adiabatic effects in the fuel which, as stated previously, may not provide a conveniently low decay time. The oscillations would then be free to build up to levels where the stresses become very large and the main danger will be the possibility that the slug will fall back into its seat and transfer these stresses destructively to the cladding.

In order to assess the likelihood of such dangers in advance, it is essential to develop a method of calculating the vibration amplitudes to be expected from regular pulsing when the damping is small. The purpose of the present paper is to provide such a method.

1. VIBRATIONS INDUCED BY A SINGLE STEP RISE IN TEMPERATURE

As has been shown previously(3), a uniform step rise in temperature produces a thermoelastic response in a long, straight, continuous fuel slug very close to that produced by the real temperature rise. Assuming that the ends of the slug are free (in conformity with the above described accident condition) and that the damping is zero, the oscillory part of the axial displacement field, $\xi(x,t)$, generated by a sudden temperature rise T (at t = 0) imposed on a <u>stationary</u> slug is given by(1)

$$\xi(x,t) = -\alpha L T \frac{8}{\pi^2} \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)^2} \sin\left[(2n+1)\frac{\pi x}{2L}\right] \cos\left[(2n+1)\frac{\pi c t}{2L}\right] \qquad (1)$$

where α is the coefficient of linear thermal expansion of the fuel, L its half length and c the speed of longitudinal waves propagating along the slug. If E is the Youngs modulus of the fuel and ρ its density, then $c = \sqrt{E/\rho}$. The position x is measured relative to the mid-point of the slug.

Equation (1) informs us that the disturbance generated by a sudden temperature rise consists of a superposition of an infinite number of standing waves. We see, however, that the fundamental mode (n = 0) is very dominant, accounting for a fraction $8/\pi^2$ (81%) of the total.

If we now incorporate the energy dissipation due to internal friction and heat conduction into the theory by means of first order perturbation theory, we obtain, according to the appendix, the results

$$\xi(x,t) = -\alpha L T \frac{8}{\pi^2} \sum_{n=0}^{\infty} \frac{(-1)^n e^{-t/\tau_n}}{(2n+1)^2} \sin\left[(2n+1)\frac{\pi x}{2L}\right] \cos\left[(2n+1)\frac{\pi c t}{2L}\right] \qquad (2)$$

where

$$\tau_n = \frac{2E/\omega_n^2}{F + \frac{\alpha^2 T_0 E k}{\rho C_p^2}} \qquad (3)$$

and

$$\omega_n = (2n+1)\frac{\pi c}{2L} , \qquad n = 0,1,2.... \qquad (4)$$

are the basic resonance frequencies of the fuel slug. In these formulae, F is a fundamental internal friction parameter (see appendix) for the fuel, k its thermal conductivity and C_p it specific heat at constant pressure.

Although not of great importance in the further development of the theory, it should be noted here that equations (1) and (2) neglect the dispersion occuring for wavelengths comparable with the diameter of the slug. In addition, the internal friction F, while a constant for very small vibrations, may increase dramatically if the amplitude of oscillation exceeds even modest limits. The effect of heat conduction, described by the second term in the denominator of equation (3), is generally negligible and can be ignored.

From equation (2) it is easy to see that every wave mode present in the disturbance is damped independently of the others, while equations (3) and (4) reveal that this damping increases very rapidly with the mode number, n. The first harmonic (n = 1), for example, is damped nine times more rapidly than the fundamental mode (n = 0); the second harmonic (n = 2) twenty five times more rapidly. Hence, the overall profile of the disturbance changes quickly with time and, for $t > \tau_1$, only the fundamental mode remains. Particle displacements are then given by

$$\xi(x,t) = -\frac{8}{\pi^2}\alpha L T\, e^{-t/\tau} \sin kx \cos \omega t \quad , \quad t > \frac{\tau}{9} \qquad (5)$$

where the symbols $\tau = \tau_0$, $\omega = \omega_0 = \pi c/2L$ and $k = \omega_0/c$ have been used for brevity.

2. AMPLIFICATION OF VIBRATIONS DUE TO REPEATED PULSING

Let us suppose that the temperature rise T, generated by a normal power pulse, is repeated at regular intervals p. The effect of the first pulse has already been described above. Because of the small contribution and heavier damping of all the higher modes, only the fundamental mode will be considered. Thus, the oscillating component of the particle displacement due to the first pulse will be given by

$$\xi_1 = a e^{-t/\tau} \sin kx \cos \omega t \tag{6}$$

where

$$\left.\begin{aligned} a &= -\frac{8}{\pi^2}\alpha L T \\ \omega &= \frac{\pi c}{2L} \\ k &= \frac{\pi}{2L} \end{aligned}\right\} \tag{7}$$

The decay time τ will be known from experimental data.[4]

After a time p, the second pulse will occur and a further burst of oscillations will be superimposed on (6). If taken alone, the displacement field generated by this second burst would be

$$\xi_2' = \begin{cases} 0 & \text{for } t < p \\ a\, e^{-(t-p)/\tau} \sin kx \cos \omega(t-p) & \text{for } t > p \end{cases} \tag{8}$$

but the <u>actual</u> displacement field ξ_2 during the interval $p < t < 2p$ is given by the <u>sum</u> of (6) and (8):

$$\xi_2 = a e^{-(t-p)/\tau} \sin kx \left[e^{-p/\tau} \cos \omega t + \cos \omega(t-p) \right] \tag{9}$$

This linear superposition procedure is the well known technique for describing interference. Its validity rests on the linearity of the wave equation.

If we now write

$$\beta = e^{-p/\tau} \tag{10}$$

$$\mathcal{A}_2 = \sqrt{(\beta + \cos\omega p)^2 + \sin^2\omega p}$$

and
$$\tan\phi_2 = \frac{\sin\omega p}{\beta + \cos\omega p}$$

equation (9) becomes

$$\xi_2 = \mathcal{A}_2\, a\, e^{-(t-p)/\tau} \sin kx \cos(\omega t - \phi_2) \tag{11}$$

from which we observe that interference causes an amplification $\mathcal{A}_2$ and phase shift ϕ_2 .

With the arrival of the third pulse at t = 2p a further burst of oscillations is generated. In isolation, this would give a displacement

$$\xi_3' = \begin{cases} 0 & \text{for } t < 2p \\ a\, e^{-(t-2p)/\tau} \sin kx \cos\omega(t-2p) & \text{for } t > 2p \end{cases} \tag{12}$$

but the actual displacement ξ_3 for $2p < t < 3p$ results from the linear combination of (11) and (12):

$$\xi_3 = a\, e^{-(t-2p)/\tau} \sin kx \left[\mathcal{A}_2\, e^{-p/\tau} \cos(\omega t - \phi_2) + \cos\omega(t-2p) \right]$$

i.e.

$$\xi_3 = \mathcal{A}_3 a\, e^{-(t-2p)/\tau} \sin kx \cos(\omega t - \phi_3) \tag{13}$$

where

$$\mathcal{A}_3 = \sqrt{(\beta \mathcal{A}_2 \cos\phi_2 + \cos 2p\omega)^2 + (\beta \mathcal{A}_2 \sin\phi_2 + \sin 2p\omega)^2} \tag{14}$$

and

$$\phi_3 = \tan^{-1}\left[\frac{\beta \mathcal{A}_2 \cos\phi_2 + \cos 2p\omega}{\beta \mathcal{A}_2 \sin\phi_2 + \sin 2p\omega}\right] \tag{15}$$

are the new amplification and phase shift respectively.

It is very easy to see that repeated application of the above argument for all pulses leads to the following formula for the displacement field after the arrival of the (m+1)'th pulse ($mp < t < (m+1)p$):

$$\xi_{m+1} = \mathcal{A}_{m+1} a\, e^{-(t-mp)/\tau} \sin kx \cos(\omega t - \phi_{m+1}) \tag{16}$$

where the amplification is governed by the recurrence relation

$$\mathcal{A}_{m+1} = \left[(\beta \mathcal{A}_m \cos\phi_m + \cos m\omega p)^2 + (\beta \mathcal{A}_m \sin\phi_m + \sin m\omega p)^2\right]^{\frac{1}{2}} \tag{17}$$

and the phase angle by

$$\phi_{m+1} = \tan^{-1}\left[\frac{\beta \mathcal{A}_m \sin\phi_m + \sin m\omega p}{\beta \mathcal{A}_m \cos\phi_m + \cos m\omega p}\right] \tag{18}$$

the initial conditions being

$$\left.\begin{aligned} \mathcal{A}_1 &= 1 \\ \phi_1 &= 0 \end{aligned}\right\} \tag{19}$$

Equation (17), (18) and (19) describe in detail the development of the amplitude of fuel slug oscillations due to the repeated pulsing of the reactor. By solving these equations numerically it is possible to obtain the amplification occuring for any number of pulses. Such solutions are presented and discussed in a later section. Of more interest for the operation of a pulsed fast reactor is the final steady value $\mathcal{A}$ attained by the amplification after a <u>large</u> number of pulses. This value is given by the asymptotic solution of equations (17)-(19).

3. ASYMPTOTIC SOLUTION: CUMULATIVE AMPLIFICATION

From equations (17) and (18) it is easy to see that, when the amplification reaches its asymptotic value $\mathcal{A}_m = \mathcal{A}$, the phase angle acquires the simple relation:

$$\phi_m = \phi + m\omega p \tag{20}$$

where ϕ is a constant (depending on ωp and β) to be determined. Substitution of (20) into (17) leads to the result

$$(1-\beta^2)\mathcal{A}^2 - 2\beta\cos\phi \cdot \mathcal{A} - 1 = 0 \tag{21}$$

while substitution into (18) gives

$$\phi+(m+1)\omega p=\tan^{-1}\left[\frac{(\beta\mathcal{A}\cos\phi+1)\sin m\omega p+(\beta\mathcal{A}\sin\phi)\cos m\omega p}{(\beta\mathcal{A}\cos\phi+1)\cos m\omega p-(\beta\mathcal{A}\sin\phi)\sin m\omega p}\right] \tag{22}$$

Writing

$$\Psi = \tan^{-1}\left[\frac{\beta\mathcal{A}\sin\phi}{\beta\mathcal{A}\cos\phi+1}\right]$$

we get from (22)

$$\phi = \Psi-\omega p$$

and therefore

$$\tan(\phi+\omega p) = \frac{\beta\mathcal{A}\sin\phi}{\beta\mathcal{A}\cos\phi+1}$$

On expanding the tangent and solving for $\mathcal{A}$ we obtain

$$\beta\mathcal{A} = -(\sin\phi\cot\omega p+\cos\phi) \tag{23}$$

Equations (21) and (23) are a pair of simultaneous equations for $\mathcal{A}$ and ϕ . The amplification $\mathcal{A}$ can be eliminated by substituting (23) into (21), the result (after some manipulation) being

$$\sin\phi\cot\omega p+\cos\phi = \pm\beta\frac{\sin\phi}{\sin\omega p} \tag{24}$$

On combining this result with equation (23) we get

$$\mathcal{A} = \mp\frac{\sin\phi}{\sin\omega p} \tag{25}$$

where the upper negative sign (or lower positive sign) in (25) corresponds to the upper positive sign (or lower negative sign) in (24).

Although both sets of signs in these equations represent a formally valid solution of equations (21) and (23), only the upper signs are meaningful physically. This can be seen by rewriting (24) in the form

$$\cot\phi = -\frac{\cos\omega p \mp \beta}{\sin\omega p} \tag{26}$$

squaring (25) and using (26) to eliminate $\sin^2\phi$. We obtain

$$\mathcal{A}^2 = \frac{1}{\sin^2\omega p + (\cos\omega p \mp \beta)^2}$$

where again the order of the signs corresponds to that in (24) and (25). Examination of this result shows that the amplification has a sharp resonance at $\omega p = 2N\pi$ for the upper sign (N = an integer) and at $\omega p = (2N+1)\pi$ for the lower sign. Since it is quite clear that resonance will occur only if the pulsation period p is an exact multiple of the oscillation period $2\pi/\omega$, we conclude that the former case is the correct one and that the lower signs in the above equations are spurious. The correct asymptotic behaviour of the system due to the cumulative amplification of oscillations is therefore given by the equation

$$\mathcal{A} = \left[\frac{1}{\sin^2\omega p + (\cos\omega p - \beta)^2}\right]^{1/2} \tag{27}$$

Denoting the frequency of the fundamental longitudinal mode of the fuel slugs by f and recalling equation (10) for β , $\mathcal{A}$ can be written as

$$\mathcal{A} = \left[\frac{1}{\sin^2 2\pi fp + (\cos 2\pi fp - e^{-p/\tau})^2}\right]^{1/2} \tag{28}$$

In figure 1, $\mathcal{A}$ is plotted as a function of the number of cycles fp in a pulsation period for several values of the damping p/τ. These values are specifically chosen to cover range expected in the SORA reactor, i.e. $0.02 < p/\tau < 0.2$ [4]. The assumed values of fp are completely unimportant since $\mathcal{A}$ is a periodic function of this parameter and it is only necessary to cover a complete cycle, i.e. $N - \frac{1}{2} < fp < N + \frac{1}{2}$, where N is any integer. For the particular case of the SORA reactor, however, $fp \approx 54$ and one can imagine that N = 54 if one requires definiteness.

Figure 1 reveals the extreme sharpness of the resonance at $fp = N$. The maximum is given by

$$\mathcal{A}_{max} = \frac{1}{1 - e^{-p/\tau}} \tag{29}$$

which, for the values of p/τ considered, is reasonably well given by the approximation

$$\mathcal{A}_{max} \approx \frac{\tau}{p} \tag{30}$$

In the same approximation, the half-width W is given by

$$W \approx \frac{\sqrt{3}}{\pi} \frac{p}{\tau} \approx 0{\cdot}55 \frac{p}{\tau} \tag{31}$$

It is interesting to note that the amplification is <u>less</u> than unity for more than 60% of the fp range. The references to later figures on figure 1 indicate the points where the detailed transient behaviour of the amplification has been evaluated and the figure numbers tell where a plot of these transients will be found.

In figure 2, the variables have been interchanged and $\mathcal{A}$ is plotted as a function of p/τ for various values of fp. Here, we see that, except very near to the resonance, the amplification is very insensitive to the damping. This proves that the phase of the oscillations is usually the determining factor, while the damping serves to limit the amplification only in the resonance region.

4. DETAILED SOLUTION: TRANSIENTS

In order to solve equations (17)-(19) for the detailed growth of the amplification $\mathcal{A}_m$ with the number of pulses m, a Fortran computer programme AMPLOS has been written. This programme operates on the IBM 360/65 machine and utilizes the CALCOMP routine for obtaining automatic plots of $\mathcal{A}_m$ against m. It also evaluates the asymptotic amplification from equation (28).

Figures 3-10 present a selection of calculated transients which cover the entire range of interest. Several values of fp (N, $N \pm 0.1$, $N \pm 0.3$ and $N \pm 0.5$) were assumed but, for the damping p/τ, it was sufficient to treat only the two extreme cases $p/\tau = 0.02$ and 0.2.

Since the theory requires only the values of fp and p/τ and never p and τ separately, figures 3-10 (as well as 1 and 2) are rather general. However, it is useful at this point to state that, for SORA, these parameters have the values:

$$p = 0.02 \text{ sec}, \qquad f \approx 2700 \text{ cps}, \qquad 0.1 < \tau < 1 \text{ sec}^{(4)}$$

The abscissa measuring the pulse number m in each of figures 3-10 has been calibrated with time intervals corresponding to the 0.02 sec pulsation period of the SORA reactor.

The detailed solutions for $fp = N$ with $p/\tau = 0.02$ and 0.2 are shown in figures 3 and 4 respectively. Comparison reveals that, while the amplification $\mathcal{A}_m$ takes about 200 pulses (4 sec for SORA) to reach its asymptote when $p/\tau = 0.02$ ($\tau = 1$ sec for SORA), the case with $p/\tau = 0.2$ ($\tau = 0.1$ forSORA) requires only about 20 pulses (0.4 sec for SORA). Thus, it is clear that the time τ_A required to accomplish steady oscillation with $\mathcal{A}_m = \mathcal{A}$ is directly proportional to τ, the approximate relation being

$$\tau_A = 4\tau \tag{32}$$

Detailed solutions for $fp = N \pm 0.1$ with $p/\tau = 0.02$ and 0.2 are given in figures 5 and 6 respectively. The time τ_A required to reach equilibrium is again given by (32) although the approach to steady oscillation is now different. Instead of rising monotonically to the asymptotic level, $\mathcal{A}$, the oscillations undergo beats. For the case $p/\tau = 0.02$, the amplitude of oscillation increases with the first four pulses but diminishes again with the 5th - 9th pulses. The 10th - 14th pulses bring increases in $\mathcal{A}_m$, but the 15th - 19th bring decreases. Continuing in this way, $\mathcal{A}_m$ approaches its asymptotic value of 1.63 via a series of fluctuations. The case for $p/\tau = 0.2$ is very similar.

Figures 7 and 8 give the results for $fp = N \pm 0.3$ with $p/\tau = 0.02$ and 0.2 respectively. Again, equation (32) for the time to equilibrium is satisfied and, again, beats in the oscillation amplitude occur.

The detailed solutions for $fp = N \pm 0.5$ with $p/\tau = 0.02$ and 0.2, shown in figures 9 and 10 respectively, reveal a very striking beats pattern. For example, in figure 9 the first pulse brings $\mathcal{A}_m$ almost to zero, the second returns it almost to unity, the third again renders it very small, the fourth again to near 1. Gradually, the fluctuations converge and after a time τ_A (given by (32)) almost vanish.

The transients depicted here are very severe in the sense that pulsing is assumed in the theory to begin abruptly at t = 0. In the actual operation of a pulsed fast reactor, such sudden changes are most unlikely. During start-up, the pulses will begin very small and build up gradually until the desired energy yield (fuel temperature rise) is achieved. Thus, the level of oscillation will be in asymptotic equilibrium with the imposed pulses and the largest oscillation amplitude present will occur in the fuel slug with the largest value of the asymptotic amplification $\mathcal{A}$.

If unforseen circumstances do materialize, however, and the temperature rise T per pulse begins to rise faster than planned, the asymptotic amplification $\mathcal{A}$ will still apply provided that T does not change appreciably during the time τ_A, i.e. provided that

$$\frac{1}{T}\frac{dT}{dt} < \frac{1}{4\tau} \qquad (33)$$

APPENDIX

Dissipation of Vibrations due to Internal Friction and Heat Conduction

The instantaneous rate of energy dissipation per unit volume at any point within a solid disturbed by a wave is given by[6]

$$\dot{\mathcal{E}}' = -\frac{\kappa}{T_0}(\nabla \tilde{T})^2 - 2\eta\left(\dot{u}_{ik} - \frac{1}{3}\dot{u}_{jj}\,\delta_{ik}\right)^2 - \zeta\,\dot{u}_{jj}^{\,2} \qquad \text{(A1)}$$

where u_{nm} is the strain tensor and $\tilde{T}$ the temperature perturbation accompanying the wave, κ is the thermal conductivity, T_o the undisturbed mean temperature and η and ζ the two viscosity constants of the solid (assumed isotropic). In this formula and all what follows, we use the well known convention of tensor analysis that repeated indices imply summation, e.g.

$$u_{ik}^{\,2} = u_{ik}u_{ik} = \sum_{n=1}^{3}\sum_{m=1}^{3} u_{nm}^{\,2}$$

$$u_{jj} = \sum_{n=1}^{3} u_{nn}$$

The first term of equation (A1) gives the dissipation due to thermal conduction, the second that due to shearing friction and the third that due to the friction accompanying density changes. Since the disturbance caused by a wave is nearly adiabatic, it is a very good approximation[6] to take

$$\tilde{T} = -\frac{\alpha T_0 E}{C_p\,\rho(1-2\nu)}\,u_{jj}$$

where α is the coefficient of linear thermal expansion, E the adiabatic Young's modulus, C_p the specific heat at constant pressure, ρ the density and ν the adiabatic Poisson ratio of the solid. Inserting this expression into (A1) and averaging $\dot{\mathcal{E}}'$ over a cycle, we get for the <u>average</u> rate of dissipation the result

$$\dot{\mathcal{E}} = -\frac{\alpha^2 T_0 E^2 \kappa}{\rho^2 C_p^2 (1-2\nu)^2} \left\langle (\nabla u_{jj})^2 \right\rangle - 2\eta \left\langle (\dot{u}_{ik} - \tfrac{1}{3}\dot{u}_{jj}\,\delta_{ik})^2 \right\rangle - \zeta \left\langle \dot{u}_{jj}^{\,2} \right\rangle \tag{A2}$$

where the square brackets denote time averages taken over a single cycle.

In order to obtain the characteristic time for the damping of the wave we require not only the mean dissipation rate $\dot{\mathcal{E}}$ but also $\mathcal{E}$, the mean energy per unit volume itself. This is given by

$$\mathcal{E} = \rho \left\langle \dot{u}_i^{\,2} \right\rangle \tag{A3}$$

where $\left\langle \dot{u}_i^{\,2} \right\rangle$ denotes the average of the square of the particle velocity $\dot{u}_i$ taken over a single cycle. The negligible change in the amplitude of the wave during a cycle has been ignored in (A3).

It is well known that the fall in time of the amplitude A of a wave is governed by an equation of the form

$$A = A_0\, e^{-t/\tau_d} \tag{A4}$$

or, since $\mathcal{E} \propto A^2$

$$\mathcal{E} = \mathcal{E}_0\, e^{-2t/\tau_d} \tag{A5}$$

where τ_d is the amplitude damping time. On combining (A2), (A3) and (A5) we get immediately the following formula for τ_d :

$$\tau_d = \frac{2\rho \left\langle \dot{u}_i^{\,2} \right\rangle}{2\eta \left\langle (\dot{u}_{ik} - \tfrac{1}{3}\dot{u}_{jj}\,\delta_{ik})^2 \right\rangle + \zeta \left\langle \dot{u}_{jj}^{\,2} \right\rangle + \dfrac{\alpha^2 T_0 E^2 \kappa}{\rho^2 C_p^2 (1-2\nu)^2} \left\langle (\nabla u_{jj})^2 \right\rangle} \tag{A6}$$

Let us now consider the axial displacement field $\xi(x,t)$ provoked in a fuel slug by a sudden temperature rise. For very small times (before significant damping occurs) $\xi(x,t)$ is given by equation (1). This equation reveals that ξ consists of an infinite number of independent waves

$$\xi(x,t) = \sum_{n=0}^{\infty} \left\{ \xi_n(x,t) + \pi_n(x,t) \right\} \tag{A7}$$

where

$$\left.\begin{aligned} \xi_n(x,t) &= \frac{1}{2} a_n \sin \omega_n\left(\frac{x}{c} + t\right) \\ \pi_n(x,t) &= \frac{1}{2} a_n \sin \omega_n\left(\frac{x}{c} - t\right) \end{aligned}\right\} \tag{A8}$$

and

$$\omega_n = (2n+1)\frac{\pi c}{2L} \tag{A9}$$

Because of the dissipative processes, however, a_n cannot be a constant but must gradually die away according to the equation

$$a_n = a_n^o \, e^{-t/\tau_n} \tag{A10}$$

where, because of the need to reproduce equation (1) at t = 0, a_n^o is given by:

$$a_n^o = -\frac{8\alpha LT}{\pi^2} \frac{(-1)^n}{(2n+1)^2}$$

The damping time τ_n of the n'th mode can be evaluated by using (A6). The strain tensor u_{ik} for the wave $\xi_n(x,t)$ is given by[6]

$$\left.\begin{aligned} u_{xx} &= \frac{\partial \xi_n}{\partial x} \\ u_{yy} &= -\nu u_{xx} \\ u_{zz} &= -\nu u_{xx} \\ u_{xy} = u_{xz} &= u_{yz} = 0 \end{aligned}\right\} \tag{A11}$$

Although (A11) is not true for the high modes with wavelengths comparable to or less than the fuel slug diameter, the fact that such modes are unimportant makes the error allowable. Applying the previously mentioned rules for tensor manipulation we now obtain

$$u_{jj} = (1-2\nu)\frac{\partial \xi_n}{\partial x} = \frac{a_n \omega_n}{2c}(1-2\nu)\cos\omega_n\left(\frac{x}{c}+t\right)$$

Hence

$$\left\langle \dot{u}_{jj}^{2} \right\rangle = \frac{1}{2}\left[\frac{a_n \omega_n^2}{2c}(1-2\nu)\right]^2 \tag{A12}$$

and

$$\left\langle (\nabla u_{jj})^2 \right\rangle = \frac{1}{2}\left[\frac{a_n \omega_n^2}{2c^2}(1-2\nu)\right]^2 \tag{A13}$$

Similarly

$$\begin{aligned}
(\dot{u}_{ik}-\tfrac{1}{3}\dot{u}_{jj}\delta_{ik})^2 &= \dot{u}_{ik}^2 - \frac{2}{3}\dot{u}_{jj}\dot{u}_{ik}\delta_{ik} + \frac{1}{9}\dot{u}_{jj}^2\delta_{ik}^2 \\
&= \dot{u}_{ik}^2 - \frac{1}{3}\dot{u}_{jj}^2 \\
&= (1+2\nu^2)\left[\frac{\partial^2 \xi_n}{\partial x \partial t}\right]^2 - \frac{1}{3}(1-2\nu^2)\left[\frac{\partial^2 \xi_n}{\partial x \partial t}\right]^2 \\
&= \frac{2}{3}(1+\nu)^2\left[\frac{\partial^2 \xi_n}{\partial x \partial t}\right]^2 \\
&= \frac{2}{3}(1+\nu)^2\left[\frac{a_n \omega_n^2}{2c}\right]^2 \sin^2\omega_n\left(\frac{x}{c}+t\right)
\end{aligned}$$

Hence

$$\left\langle (\dot{u}_{ik}-\tfrac{1}{3}\dot{u}_{jj}\delta_{ik})^2 \right\rangle = \frac{1}{3}\left[\frac{a_n \omega_n^2}{2c}(1+\nu)\right]^2 \tag{A14}$$

For the particle velocity $\dot{u}_i$ associated with ξ_n we have

$$\dot{u}_x = \dot{\xi}_n = \frac{a_n \omega_n}{2} \cos \omega_n \left(\frac{x}{c} + t \right) , \qquad \dot{u}_y \ \& \ \dot{u}_z \sim u_x^2$$

and therefore

$$\langle \dot{u}_i^2 \rangle = \frac{1}{2} \left[\frac{a_n \omega_n}{2} \right]^2 \qquad \text{(A15)}$$

Substituting (A12), (A13), (A14) and (A15) into equation (A6), rearranging and recalling that $c^2 = E/\rho$, we see that the damping time τ_n of the n'th mode is given by

$$\tau_n = \frac{2E/\omega_n^2}{F + \frac{\alpha^2 T_0 E k}{\rho C_p}} \qquad \text{(A16)}$$

where the friction term F is given in terms of the viscosities by

$$F = \frac{4}{3}(1+\nu)^2 \eta + (1-2\nu)^2 \zeta \qquad \text{(A17)}$$

Naturally, execution of the above analysis in terms of the forward moving wave π_n gives the same result.

Collecting together the above results, we arrive at equations (2), (3) and (4) of section 1.

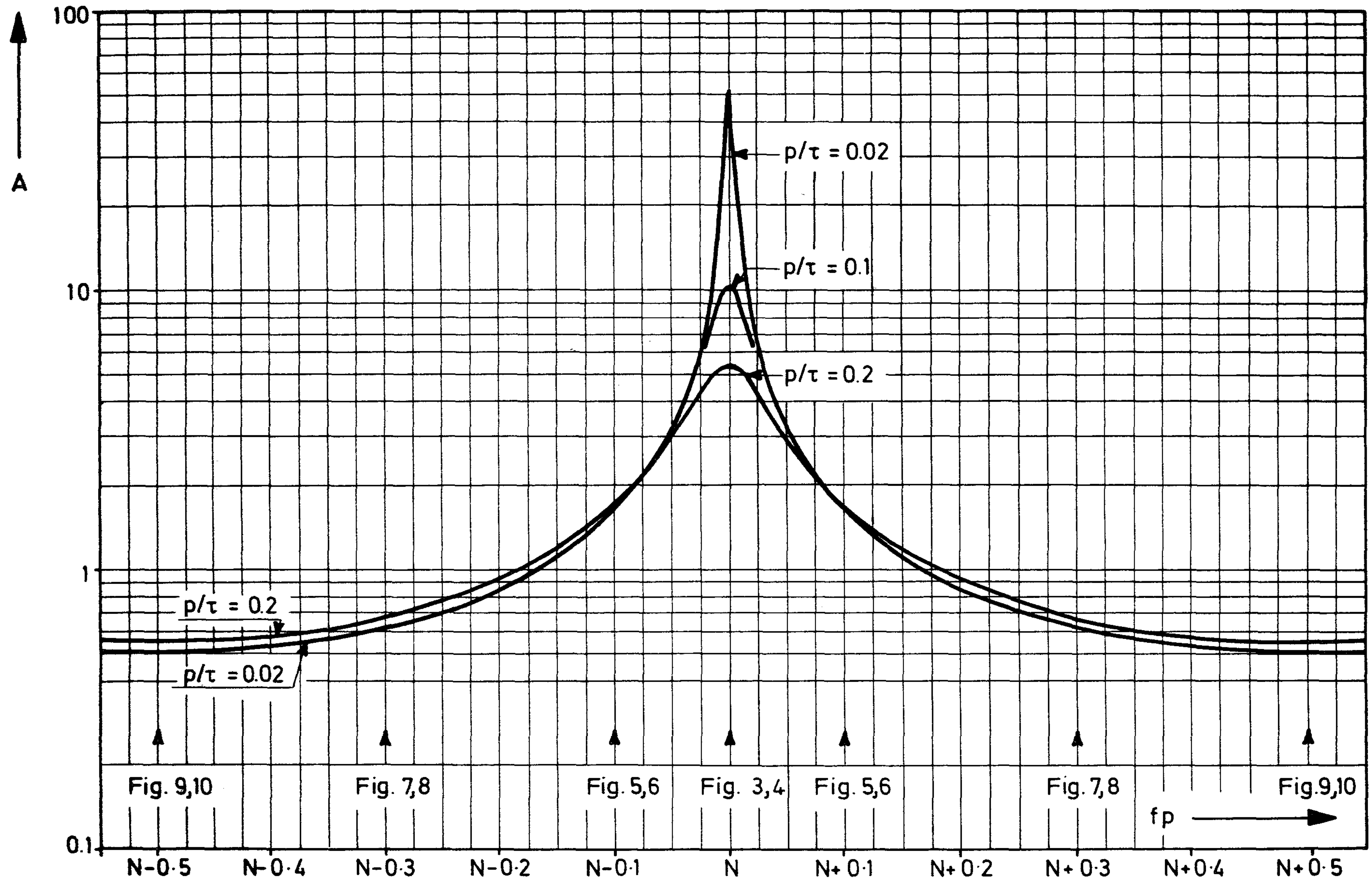

FIGURE 1 ASYMPTOTIC AMPLIFICATION A AS A FUNCTION OF THE NUMBER OF CYCLES PER PULSATION PERIOD, fp, FOR VARIOUS VALUES OF THE DAMPING, p/τ.

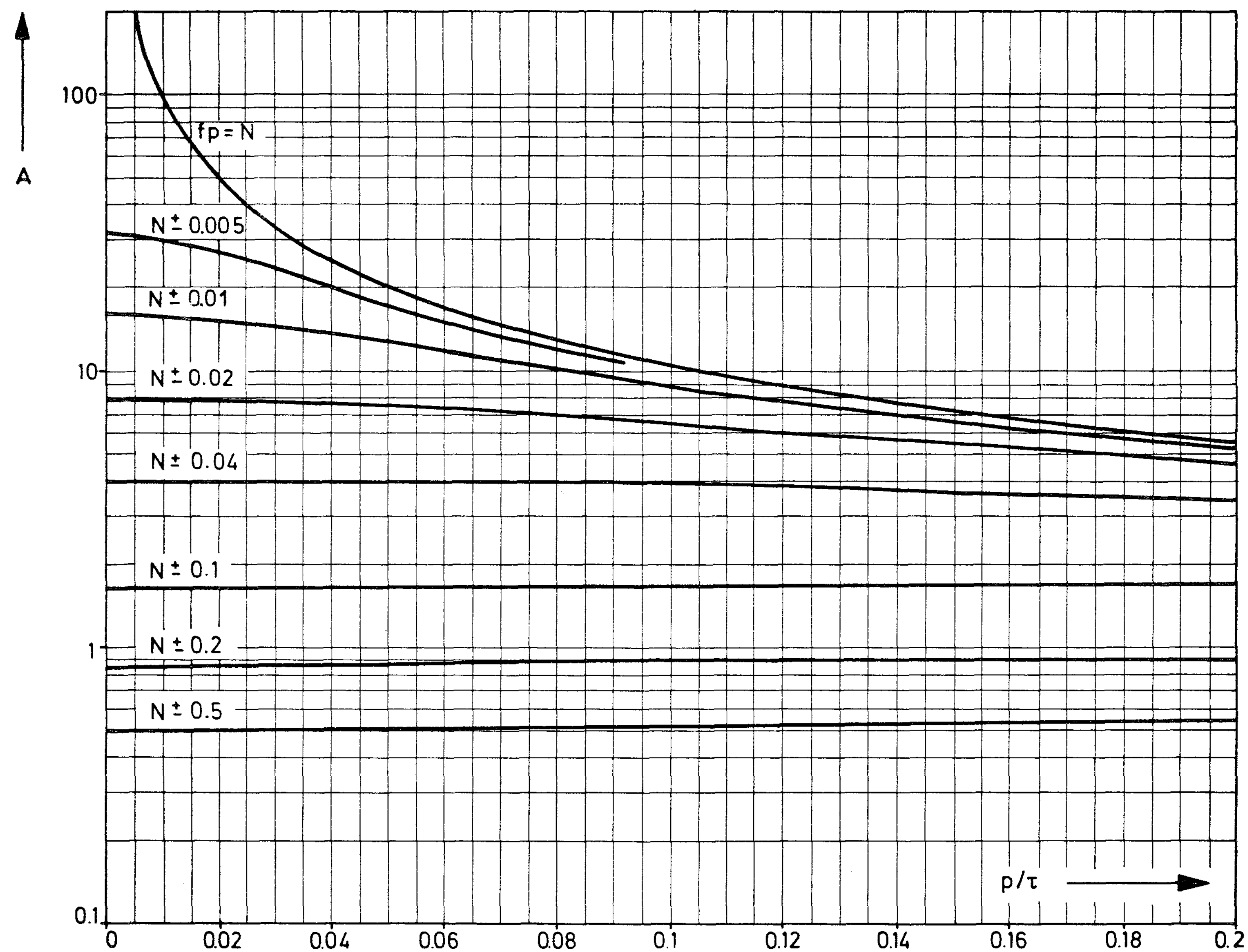

FIGURE 2 ASYMPTOTIC AMPLIFICATION A AS A FUNCTION OF THE DAMPING, p/τ, FOR VARIOUS VALUES OF fp (CYCLES / PULSATION PERIOD).

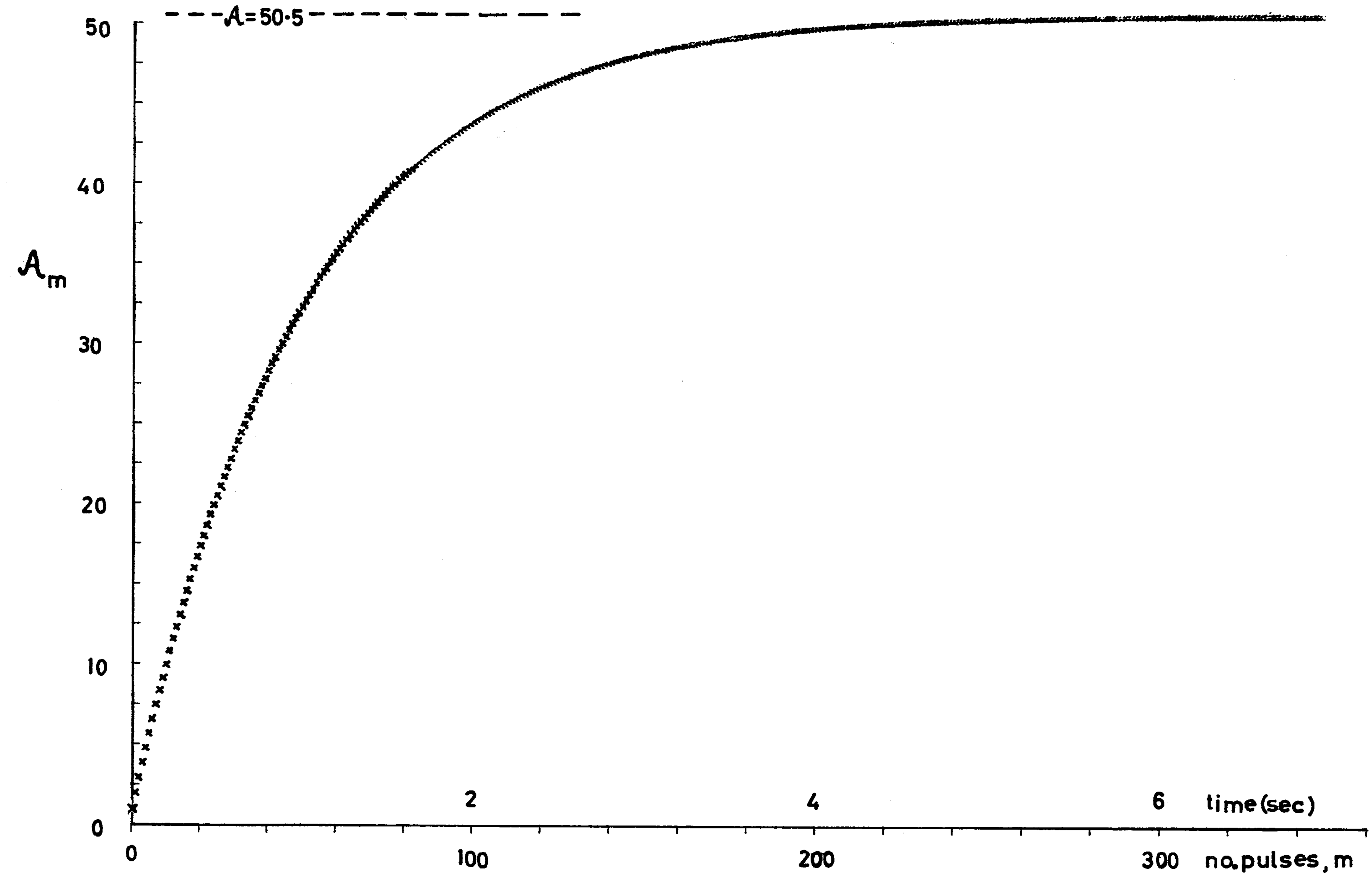

FIGURE 3 DETAILED SOLUTION FOR $fp = N, \frac{P}{\tau} = 0{\cdot}02$

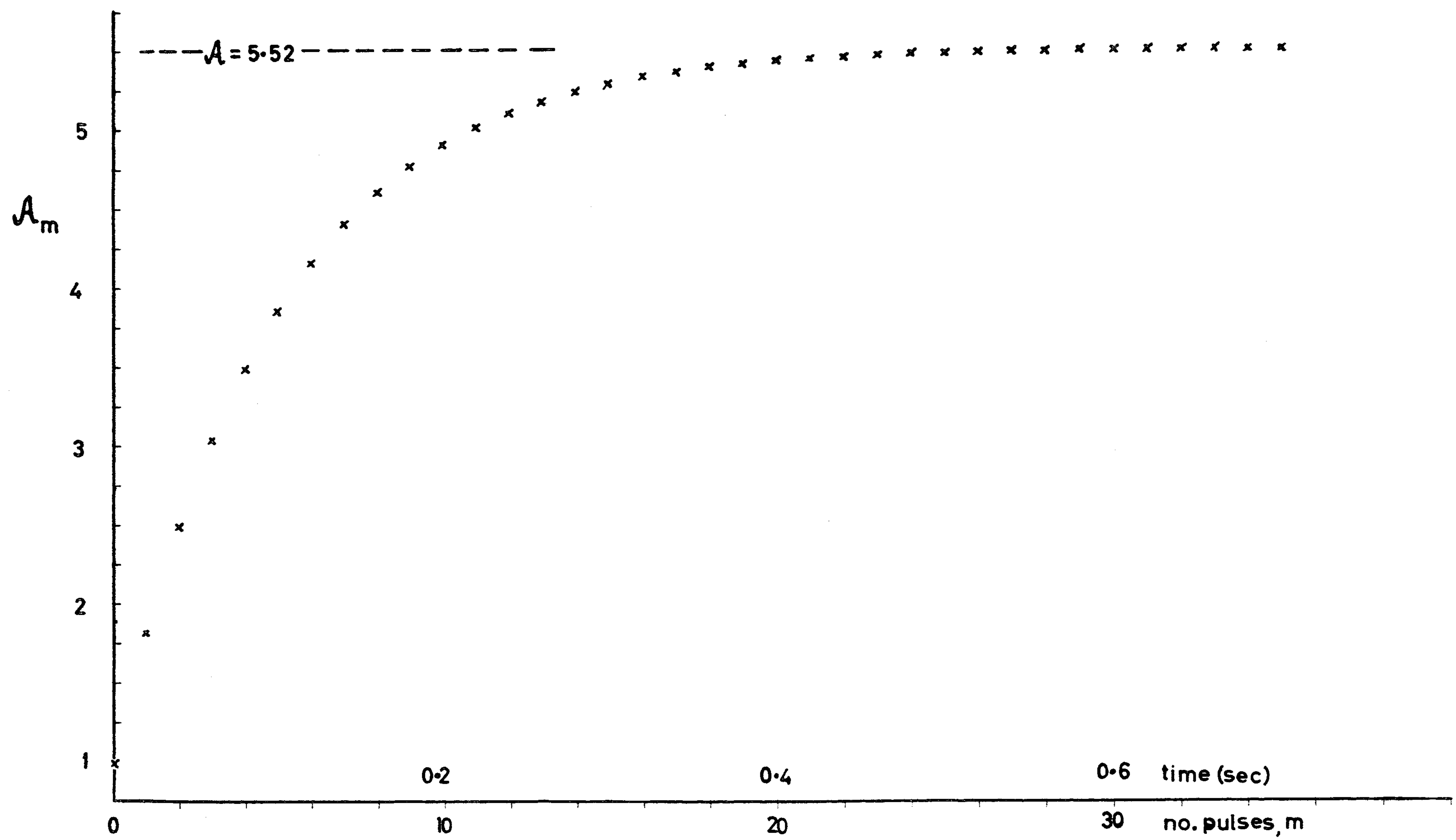

FIGURE 4 DETAILED SOLUTION FOR $fp = N$, $\frac{p}{\tau} = 0{\cdot}2$

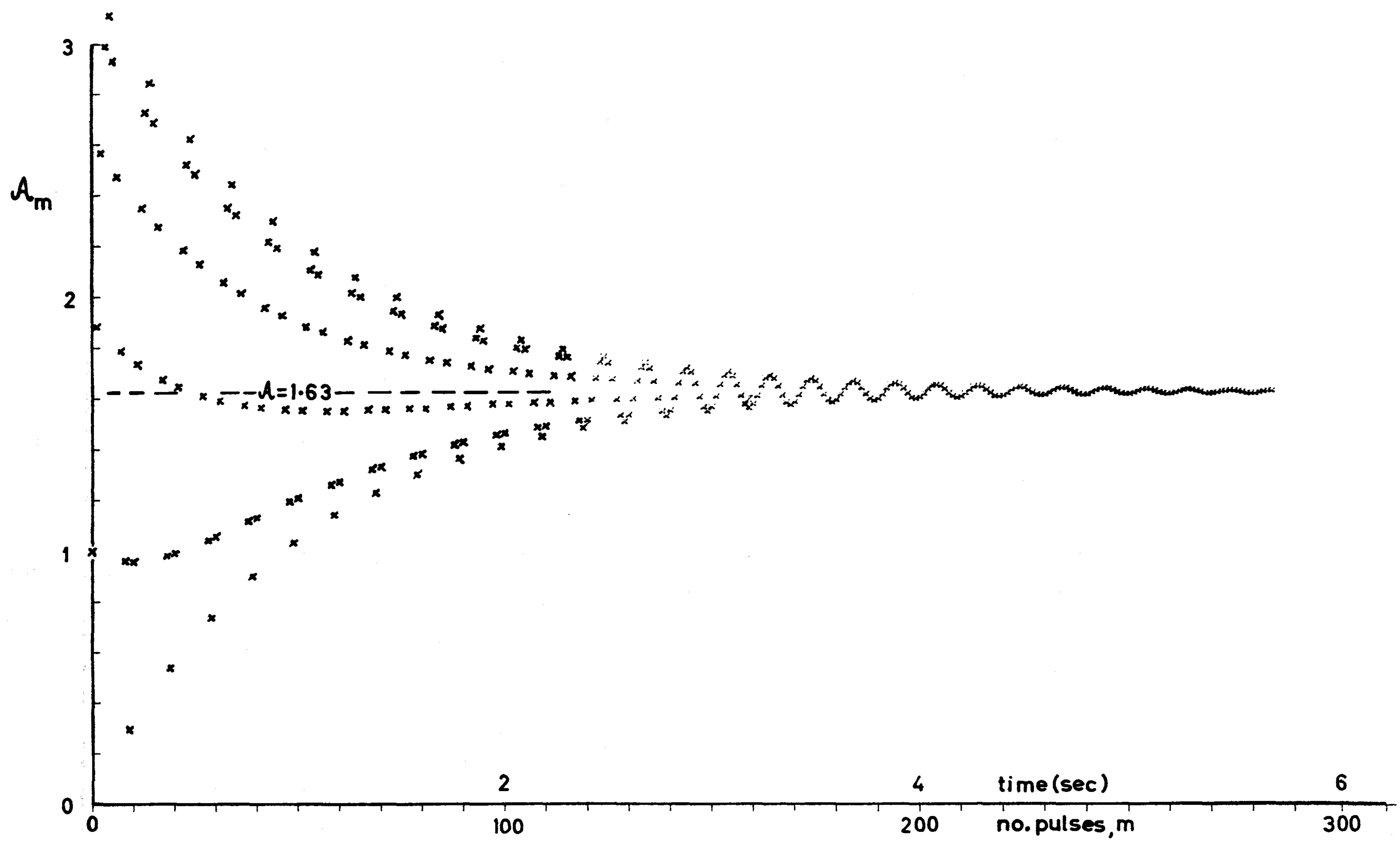

FIGURE 5 DETAILED SOLUTION FOR $fp = N \pm 0{\cdot}1$, $\frac{p}{\tau} = 0{\cdot}02$

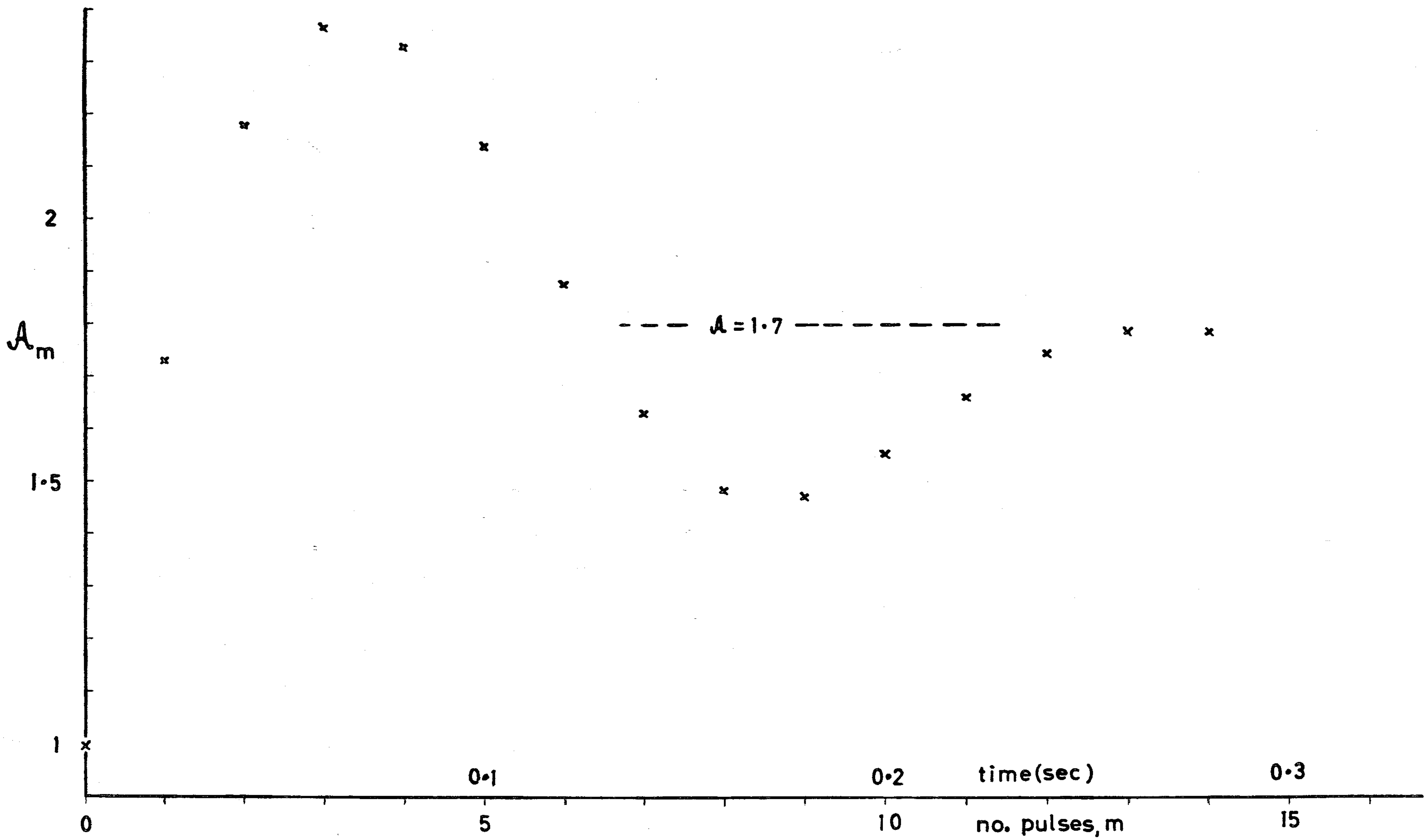

FIGURE 6 DETAILED SOLUTION FOR $fp = N \pm 0·1$, $\frac{p}{\tau} = 0·2$

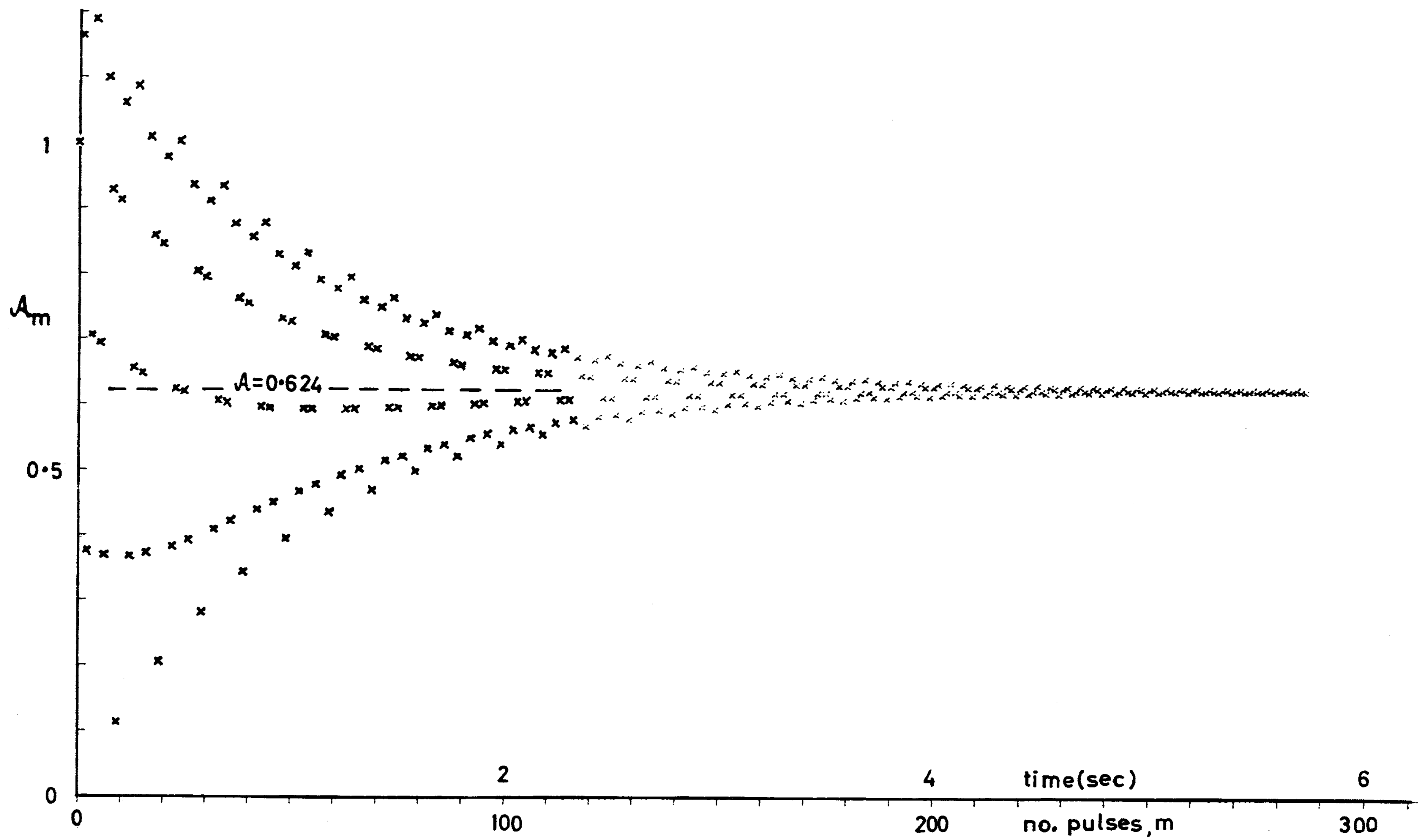

FIGURE 7 DETAILED SOLUTION FOR $fp = N \pm 0{\cdot}3$, $\frac{p}{\tau} = 0{\cdot}02$

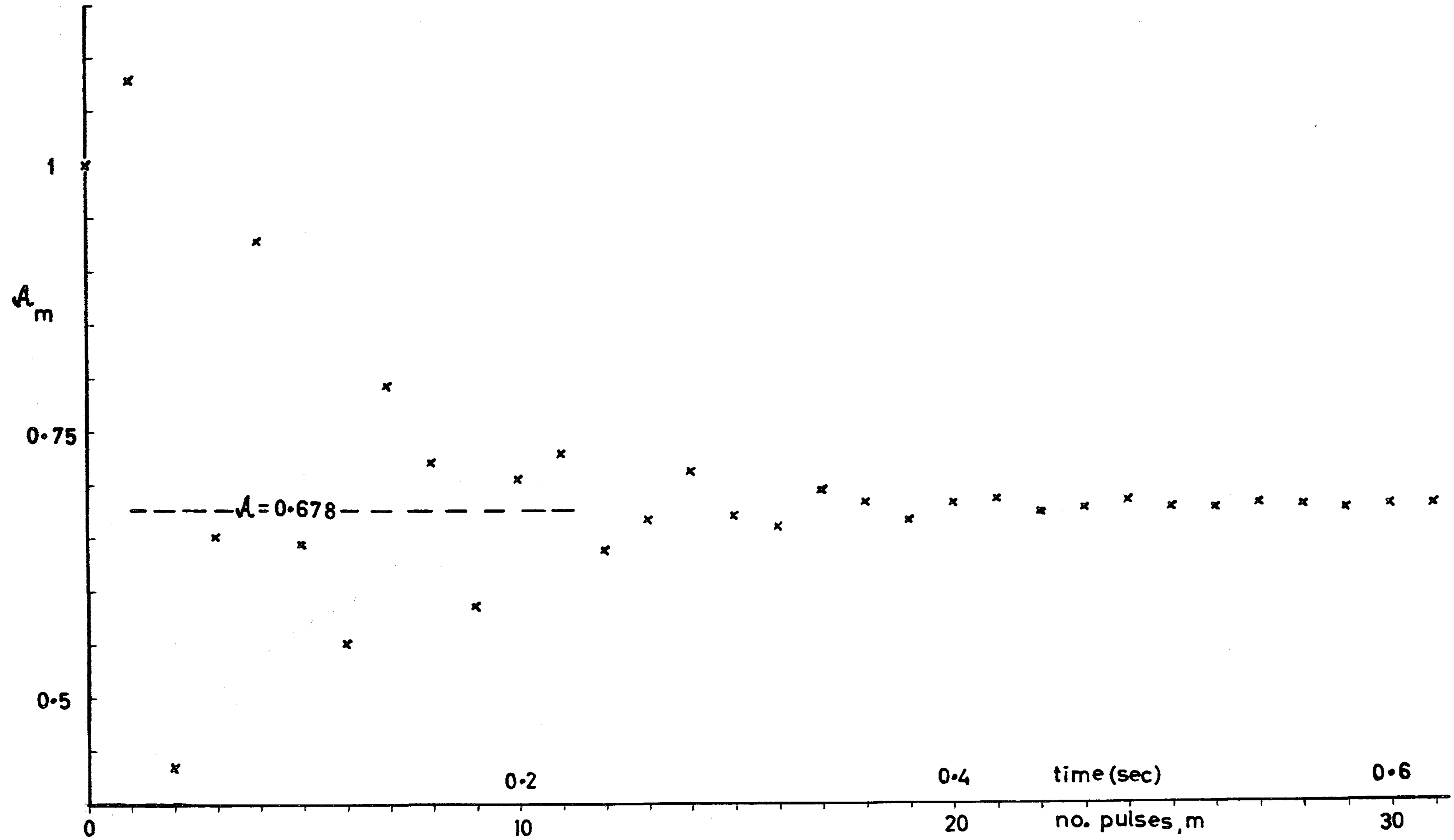

FIGURE 8 DETAILED SOLUTION FOR $fp = N \pm 0{\cdot}3$, $\frac{p}{\tau} = 0{\cdot}2$

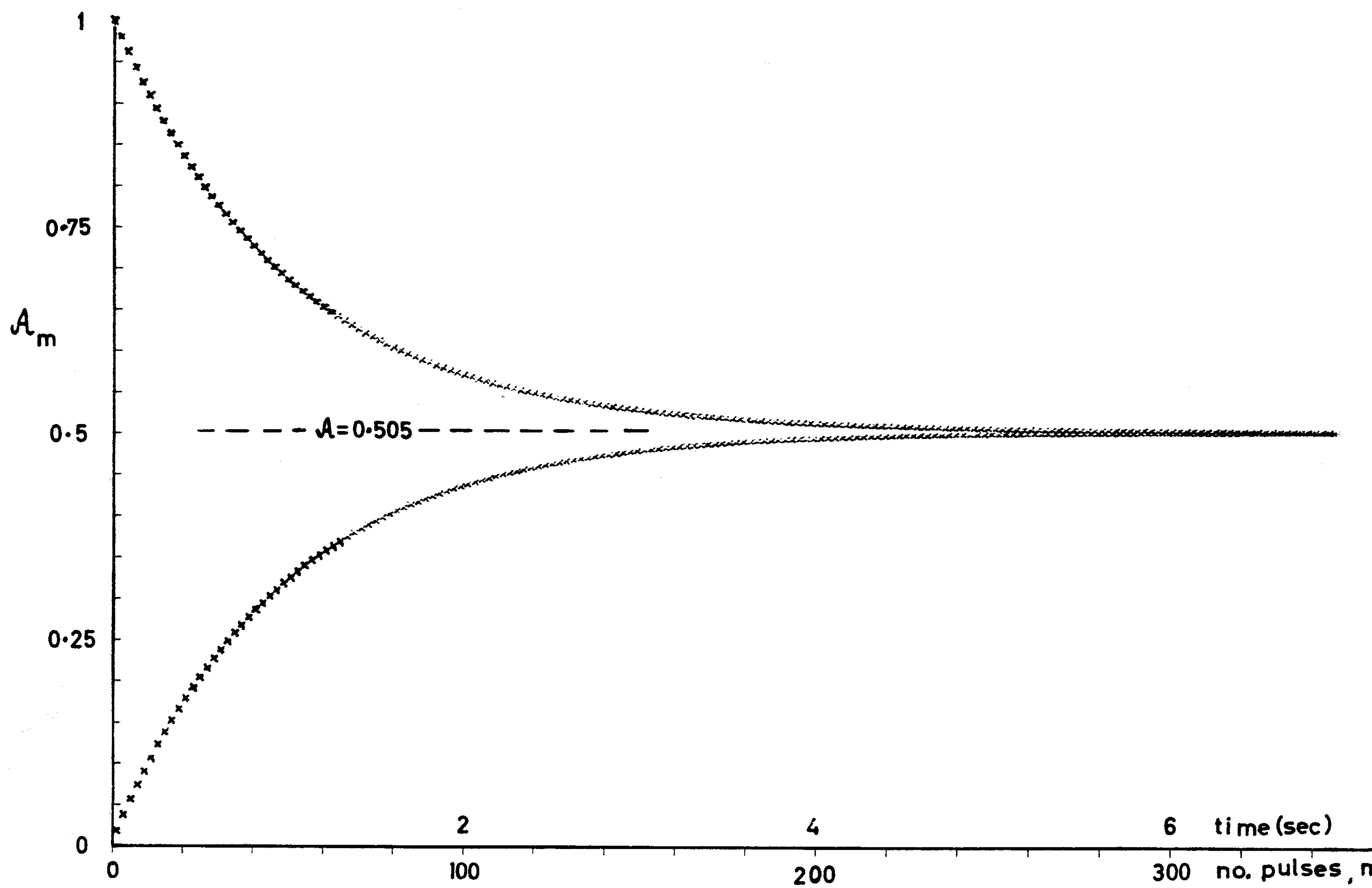

FIGURE 9 DETAILED SOLUTION FOR $fp = N \pm 0{\cdot}5$, $\frac{p}{\tau} = 0{\cdot}02$

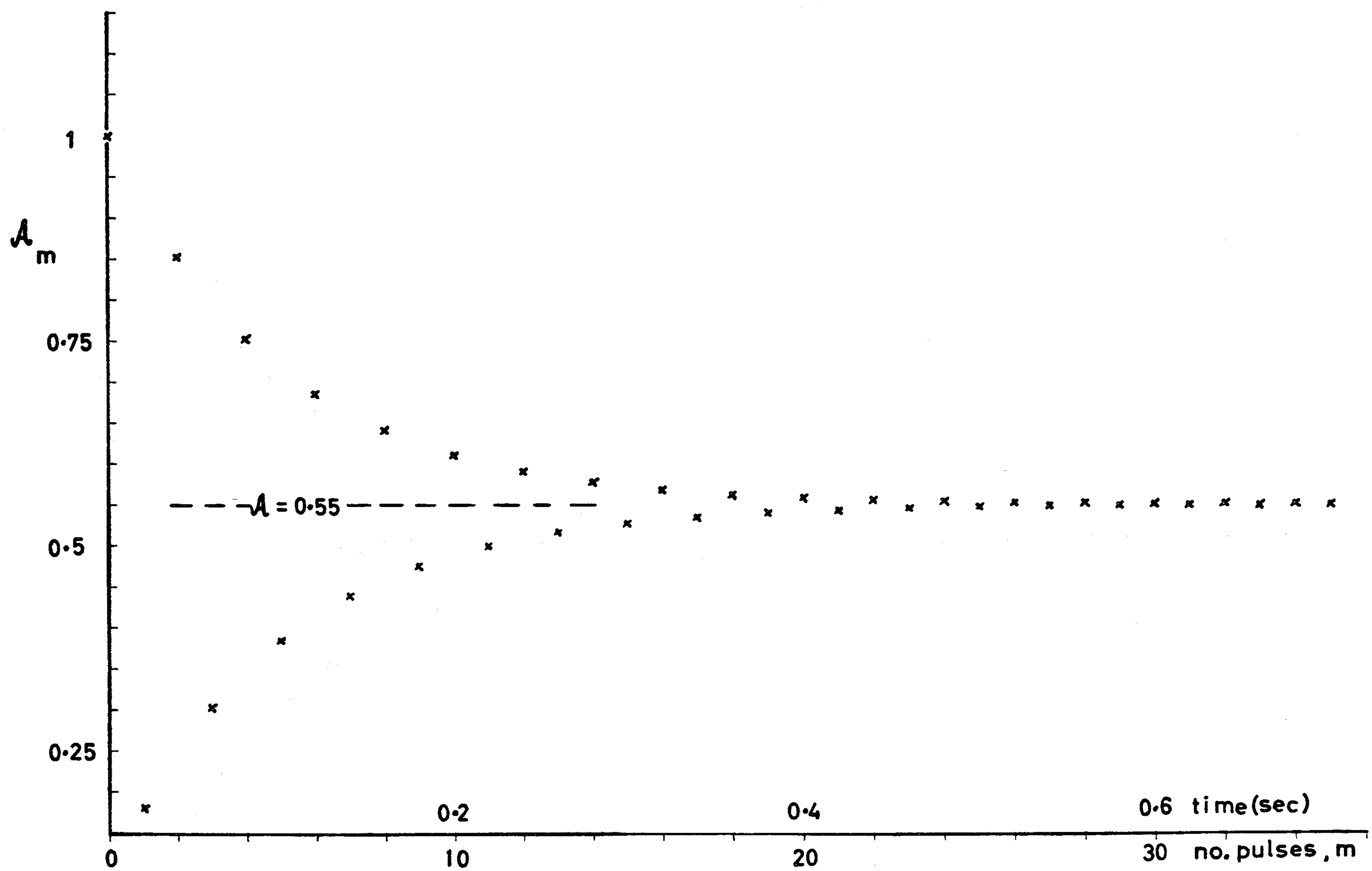

FIGURE 10 DETAILED SOLUTION FOR $fp = N \pm 0{\cdot}5$, $\frac{p}{\tau} = 0{\cdot}2$

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Alfred Nobel

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