EUR 2474.e

ASSOCIATION

EUROPEAN ATOMIC ENERGY COMMUNITY – EURATOM
GESELLSCHAFT FÜR KERNFORSCHUNG, KARLSRUHE – GfK

SIMULATION OF Pu-239 FUELED CORES BY MEANS OF Pu-239 - U-235 FUELED CORES IN FAST CRITICAL ASSEMBLIES

by

A. ANCARANI

1965

Work performed at the Institut für Angewandte Reaktorphysik
Karlsruhe, Germany

Association No. 009 - 63 - 1 RAAD

LEGAL NOTICE

This document was prepared under the sponsorship of the Commission of the European Atomic Community (EURATOM).

Neither the EURATOM Commission, its contractors nor any person acting on their behalf:

1° — Make any warranty or representation, express or implied, with respect to the accuracy, completeness, or usefulness of the information contained in this document, or that the use of any information, apparatus, method, or process disclosed in this document may not infringe privately owned rights; or

2° — Assume any liability with respect to the use of, or for damages resulting from the use of any information, apparatus, method of process disclosed in this document.

This report can be obtained at the price of Belgian Francs 125, from: PRESSES ACADEMIQUES EUROPEEN. NES - 98, chaussée de Charleroi, Brussels 6.

Please remit payments to:
— BANQUE DE LA SOCIETE GENERALE (Agence Ma Campagne) - Brussels - account No. 964.558,
— BELGIAN AMERICAN BANK AND TRUST COMPANY - New York - account No. 22.186,
— LLOYDS BANK (Europe) Ltd. - 10 Moorgate - London E.C. 2,

giving the reference: "EUR 2474.e - SIMULATION OF Pu-239 FUELED CORES BY MEANS OF Pu-239 - U-235 FUELED CORES IN FAST CRITICAL ASSEMBLIES."

Printed by Snoeck-Ducaju & Fils, Ghent.
Brussels, november 1965.

EUR 2474.e

ASSOCIATION

EUROPEAN ATOMIC ENERGY COMMUNITY – EURATOM
GESELLSCHAFT FÜR KERNFORSCHUNG, KARLSRUHE – GfK

SIMULATION OF Pu-239 FUELED CORES BY MEANS OF Pu-239 - U-235 FUELED CORES IN FAST CRITICAL ASSEMBLIES

by

A. ANCARANI

1965

Work performed at the Institut für Angewandte Reaktorphysik
Karlsruhe, Germany

Association No. 009 - 63 - 1 RAAD

CONTENTS

Simulation of Pu-239 Fueled Cores by Means of PU-239 - U235 Fueled Cores in Fast Critical Assemblies (*)

1. INTRODUCTION

The use of Zero Power Assemblies to study Power Reactors and simulate their behaviour under various operating conditions is of common practice for the development and the understanding of tthe physics of new systems. Particularly useful are the Zero Power Assemblies in the case of fast reactors, mainly because of safety reasons. In a general way the results obtained from integral experiments on such machines are to be compared with the results of calculations based on cross section data obtained from differential experiments: in principle the comparison of integral experiments and calculation results might be used to refine the input cross sections data. As a matter of fact, it is found that this is possible when comparing systems of the same material concentration and composition, while the results are not satisfactory when comparing dilute systems and concentrated ones.

Future large Fast Power Reactors (1 000 MWth and up) will most probably be of the first kind because of technological difficulties of obtaining very high power densities. As a consequence, such reactors will be fueled with dilute fissionable materials and the same will be true when simulating them on a Zero Power Assembly. That means that such an assembly, if it has to have the same fuel concentration as the full scale reactor in order to be representative of its behaviour, will easily ask for cores sizes up to several thousand liters and Pu loadings of the order of 1000 kg. Unfortunately, the amounts of Pu now available, aside from safety considerations, will not permit to build up so big fast cores in Zero Power Assemblies. In order to avoid such difficulties, two methods have been so far proposed [1, 2]: first, the use of dilution materials, with small capture cross sections, which are to be added to Pu in a homogeneous way so as to have minimal critical masses (while the spectrum will be softened); second, the choice of a zoned reactor, the central part of which has the exact Pu composition of the reactor under study, while the surrounding zone (driver) is made up with fuel of different composition, these fissile isotopes being, for distance, Pu-239 or, more likely, U-235.

In the present study, the second method is considered in some detail in order to solve two main problems:

1 — maximum size of the reactors which is possible to simulate with given quantities of Pu;

(*) Manuscript received on December 8, 1964.

2 — reliability on flux spectrum, for physical measurements, of the zoned reactors in comparison with that of the full scale ones.

As just mentioned, it is clear that limited amounts of Pu in zoned reactors will ask the use of at least another fissile isotope to make up for the cores critical masses; U-235 only is here constantly considered, which will bring forth a problem relative to the maximum amounts of this isotope one will need. For the moment being, we do not concern ourselves with such details and, as far as the isotopic composition of Pu is concerned, we will consider it, for simplicity, as 100 per cent pure Pu-239, though several percent of higher isotopes of Pu might be present in the real cases.

2. SCOPE OF THE PRESENT STUDY

From what precedes, it follows that the general schemes of the reactors to be examined are like the ones in Fig. 1, a, b, c, (spherical or cylindrical geometry).

In a general way, the study of the properties of the full scale reactors (Fig. 1, a) by means of zoned reactors (fig. 1, b, c) can be related to the coupled reactor theory [3]. Such a reference will be useful in further developments of this study, when dynamic properties will be considered.

There are several ways to design a zoned reactor. One, Fig. 1, b, can be to have in the inner zone and in the driver the same fissile isotope, but with different concentration $y=N_{\text{fertile}}/N_{\text{fissile}}$; another solution can be to have different fissile isotopes in the inner zone and the driver and having or not the same y's; still another, Fig. 1, c, can be to have the same or different fissile isotopes in the inner zone and the driver, having or not the same y's and with a decoupler zone between the two.

It is not possible to predict beforehand on a theoretical basis wich of the just mentioned solutions will meet the required neutron conditions with limited amounts of fissile materials available. That is why extensive calculations have to be performed and their results compared. In the present study, the inner zone contains constantly Pu-239 (metal or ovide) and the driver contains U-235 (metal or oxide); the y's of inner zone and driver can differ; the influence of a decoupler is considered in several cases.

For a given composition (Fissile isotopes: Fertile isotopes: Coolant: Structure), the reactors core containing only Pu-239 will be the smallest in comparison with similar zoned reactors. It is then to be expected that, both Pu-239 and U-235 being available in definite amounts, the range of possible reactors will be somewhat restricted, especially when dealing with very diluted systems. Apart from these considerations, the results of a series of calculations will be given here in a general way in order to discuss for each reactor

a) critical mass
b) neutron spectrum.

The last information is sought in order to determine the minimum diameter of the inner Pu containing zone (of a zoned test system) such that, inside it, the neutron spectrum be equivalent to that of the reference system; in order to be meaningful, such an equivalence is deemed necessary at least over a few neutron mean free paths. Once this problem is solved and the equivalence regions defined, the inner Pu containing zone of the zoned test system can be treated as the corresponding portion of the reference reactor; in this way, and at least in a certain volume of the inner zone, physical measurements can

be performed with a confidence of obtaining dependable results relative to the reference reactor.

3. APPLICATION MEANS

Two points of practical character are to be decided upon and are now briefly examined.

The first refers to the calculation means: constant use has been made of an already available unidimensional code in the diffusion approximation [4], and that because of the survey calculations type here needed. A more sophisticated treatment in cylindrical geometry, while closer to the reality of a Fast Power Reactor and of the related Zero Power Assembly, would certainly give better results as far as critical masses are concerned. As for the spectra flux informations we are seeking, it is believed that the qualitative results will remain valid, at least in the chosen frame of approximation. Of course, should neutron transport codes have been available, the results would have been more convenient for our purposes.

The second point refers to the cross section set to be introduced into the calculations. Constant use has been made of the 16-group YOM set [5]. In so doing, we have deliberately not been concerned with the influence of the cross-section data on the results, since we were looking for the outputs to be consistent. Of course a finer decoupling of energy spectrum, i.e. a number of groups greater than 16, would give us, conceptually at least, the possibility of a better physical interpretation of the phenomena under study.

4. CORES AND BLANKETS COMPOSITION

In order to define the volumetric core composition of the various reactors to calculate, let be [6]:

α = coolant %
β = structure material and canning %
γ = fissile isotopes %
δ = fertile isotopes %

so that

$$\alpha + \beta + \gamma + \delta = 1 \tag{1}$$

Besides, let be

$$\beta = \beta_o + x\,(\gamma+\delta) \tag{2}$$

where

β_o = refers to the structural material only
x = proportionality term between the canning and the fuel

and, if

$$\delta = \gamma y \tag{3}$$

where

$$y = \frac{N_{\text{fertile}}}{N_{\text{fissile}}},$$

we get from (2)

$$\beta = \beta_o + x\gamma\,(1+y) \tag{2'}$$

Once α, β_o, x and y have been chosen, one obtains

$$\gamma = \frac{1 - (\alpha+\beta_o)}{x + xy + y + 1} \tag{4}$$

and β and δ follow from (2') and (3) respectively.

For the calculations, the following constants have been chosen throughout all the present study:

CORE (reference systems or zoned test systems)

$\alpha = 0.40$
$\beta_o = 0.15$
$x = 0.30$
$y_1 = 4, 5, 6$ (reference systems and inner zone in zoned test systems)
$y_2 = 1, 1.5, 2, 3, 4, 5, 6$ (driver zone in zoned test systems)

BLANKET (reference systems or zoned test systems)

$\alpha = 0.10$
$\beta = 0.20$
$\delta = 0.70$

In the case of metallic reactors, the cores are supposed to contain

Na-23 (coolant)
Fe-56 (structure and cladding)
Pu-239 or U-235 (fissile isotope)
U-238 (fertile isotope)

In the case of oxide reactors, the cores are supposed to contain

Na-23 (coolant)
Fe-56 (structure and cladding)
PU-239 or U-235 (fissile isotope)
O-16 (chemically bounded either to Pu-239 or U-235 and to U-238)
U-238 (fertile isotope)

Blanket composition has been assumed the same both in the case of metal and of oxide reactors:

Na-23 (coolant)
Fe-56 (structure and cladding)
U-238 (fertile isotope)

The densities of the different materials are assumed to be:

$\rho_U = \rho_{Pu} = 19$ g/cm^3
$\rho_{UO_2} = \rho_{PuO_2} = 9$ g/cm^3
$\rho_{Na} = 0.84$ g/cm^3
$\rho_{Fe} = 7.86$ g/cm^3

and, both in core and blanket of every reactor here considered, no void is supposed to be present: that and the fact that the structure an cladding material is considered as 100 % Fe (instead of SS with its composite structure) will slightly influence the results.

Table I gives the composition in atoms/cm^3 for metallic cores when y=4, 5, 6. Table II gives similar informations for oxide cores when y=1, 1.5, 2, 3, 4, 5, 6.

Table 1

COMPOSITION OF METALLIC CORES

Component	Isotope	$y = 4$		$y = 5$	
		V.F.	atoms/cm^3 $\times 10^{24}$	V.F.	atoms/cm^3 $\times 10^{24}$
Coolant	Na-23	$\alpha = 0.40$	0.0088	$\alpha = 0.40$	0.0088
Structural and canning material	Fe-56	$\beta = 0.2539$	0.02150533	$\beta = 0.25385$	0.021501095
Fuel	Pu-239 or U-235	$\gamma = 0.0692$	0.0033216	$\gamma = 0.05769$	0.00276912
Breeding material	U-238	$\delta = 0.2769$	0.0132912	$\delta = 0.28846$	0.01384608

Component	Isotope	$y = 6$		Blanket	
		V.F.	atoms/cm^3 $\times 10^{24}$	V.F.	atoms/cm^3 $\times 10^{24}$
Coolant	Na-23	$\alpha = 0.40$	0.0088	$\alpha = 0.10$	0.0022
Structural and canning material	Fe-56	$\beta = 0.25385$	0.021501095	$\beta = 0.20$	0.01694
Fuel	Pu-239 or U-235	$\gamma = 0.04945$	0.0023736	——	——
Breeding material	U-238	$\delta = 0.2967$	0.0142416	$\delta = 0.70$	0.0336

5. CRITICAL MASSES CALCULATIONS

Critical mass calculations have been performed for a number of reflected spherical reactors using the already mentioned code [4]. The results are listed in Tables IV to XXV and, in a graphical form, are shown in Figures 2 to 23.

Every table includes data (critical radii, critical masses, etc.) relative to ten reactors (except in the few cases where calculation results were not obtained). The last reactor in every table is to be considered the ideal full scale Pu-239 fueled reactor (Fig. 1,a) to analyze, should this isotope be available in an unrestricted basis: such reactor is characterized by a value of $y=4$, 5, 6 and the same or a different y characterizes the first reactor listed in each table (except that this is of the type of Fig. 1a, but U-235 fueled) and which is included for comparison reasons. (The ideal Pu-239 fueled reactors, for which $y=4$, 5, 6, are, respectively, number 16, 26 and 36 and in what follows we shall frequently refer to them and to the related reactors through the use of terms like "group 16", "group 26", "group 36"). The other eight reactors are zoned ones of the type of Fig. 1, b, with a y_1 of the inner zone equal to the y of the full scale Pu-239 fueled reactor (16, 26 or 36) and a y_2 of the driver zone equal to the y of the full scale U-235 fueled reactor (first one of each table). Besides, these eight reactors, are characterised by a driver zone, the thickness of which is decreasing from 40 cm to 5 cm.

Table II

COMPOSITION OF OXIDE CORES

Component	Isotope	$y = 1$			$y = 1.5$		
			V.F.	atoms/cm^3 $\times 10^{24}$		V.F.	atoms/cm^3 $\times 10^{24}$
Coolant	Na-23	$\alpha = 0.40$	0.40000	0.0088	$\alpha = 0.40$	0.40000	0.0088
Structural and canning material	Fe-56	$\beta = 0.25384$	0.25385	0.021501095	$\beta = 0.25385$	0.25385	0.021501095
Fuel	Pu-239 or U-235	$\gamma = 0.17308$ (*)	0.07237	0.0034737156	$\gamma = 0.13846$ (*)	0.05789	0.0027788922
Breeding material	U-238	$\delta = 0.17308$ (*)	0.07237	0.0034737156	$\delta = 0.20769$ (*)	0.08684	0.0041683383
Oxygen	O-16		0.14474	0.0138948624		0.14473	0.013894461

Component	Isotope	$y = 2$			$y = 3$		
			V.F.	atoms/cm^3 $\times 10^{24}$		V.F.	atoms/cm^3 $\times 10^{24}$
Coolant	Na-23	$\alpha = 0.40$	0.40000	0.0088	$\alpha = 0.40$	0.4000	0.0088
Structural and canning material	Fe-56	$\beta = 0.25385$	0.25385	0.021501095	$\beta = 0.25385$	0.25385	0.021501095
Fuel	Pu-239 or U-235	$\gamma = 0.11538$ (*)	0.04824	0.0023156766	$\gamma = 0.08654$ (*)	0.03618	0.0017368578
Breeding material	U-238	$\delta = 0.23077$ (*)	0.09649	0.0046315539	$\delta = 0.25961$ (*)	0.10855	0.0052103727
Oxygen	O-16		0.14473	0.013894461		0.14473	0.013894461

(*) Including Oxygen

Table II (Continued)

COMPOSITION OF OXIDE CORES

Component	Isotope	$y = 4$	V.F.	atoms/cm^3 $\times 10^{24}$	$y = 5$	V.F.	atoms/cm^3 $\times 10^{24}$
Coolant	Na-23	$\alpha = 0.40$	0.40000	0.0088	$\alpha = 0.40$	0.40000	0.0088
Structural and canning material	Fe-56	$\beta = 0.2539$	0.2539	0.02150533	$\beta = 0.25385$	0.25385	0.021501095
Fuel	Pu-239 or U-235	$\gamma = 0.0692$ (*)	0.02893	0.001388844	$\gamma = 0.05769$ (*)	0.02412	0.0011578383
Breeding material	U-238	$\delta = 0.2769$ (*)	0.11578	0.005557383	$\delta = 0.28846$ (*)	0.12061	0.0057893922
Oxygen	O-16		0.14471	0.013892454		0.14473	0.013894461

Component	Isotope	$y = 6$	V.F.	atoms/cm^3 $\times 10^{24}$	Blanket	V.F.	atoms/cm^3 $\times 10^{24}$
Coolant	Na-23	$\alpha = 0.40$	0.40000	0.0088	$\alpha = 0.10$	0.10	0.0022
Structural and canning material	Fe-56	$\beta = 0.25385$	0.25385	0.021501095	$\beta = 0.20$	0.20	0.01694
Fuel	Pu-239 or U-235	$\gamma = 0.04945$ (*)	0.02068	0.0009924615	——	——	——
Breeding material	U-238	$\delta = 0.29670$ (*)	0.12061	0.0057893922	$\delta = 0.70$	0.70	0.0336
Oxygen	O-16		0.14129	0.0135637074	——	——	——

(*) Including Oxygen

Table III

CRITICAL DIMENSIONS AND CRITICAL MASSES
OF REFLECTED METAL-FUELED FAST REACTORS

N	Core					Blanket			y
	Radius cm	Volume l	Fissile Isotope		U-238	Thick--ness cm	Volume l	U-238 kg	
				kg					
1	39.26	253.49	U-235	328.38	1331	50	2725.49	36173	4
2	46.58	423.38	U-235	457.23	2316	50	3350.35	44466	5
3	54.98	696.04	U-235	644.33	3916	50	4149.84	55077	6
4	25.60	70.26	Pu-239	92.57	369	50	1739.50	23087	4
5	29.54	107.97	Pu-239	118.60	591	50	1999.88	26542	5
6	33.66	159.80	Pu-239	150.45	899	50	2293.21	30435	6

The same criteria hold for the reactors of the type of Fig. 1, c; here a decoupler is introduced between the inner zone and the driver. Two different materials have been considered for the decoupler, namely U-238 and HS-208; this second isotope is a fictition heavy pure elastic scatterer, to which have been attributed the same density of lead and a constant $\sigma_{el} = 12$ b.

In the graphs the calculations results are offered as functions of the ratio Pu-239 mass/(Pu-239 + U-235) masses, so that the abscissae from 0 to 1 take account of the ten reactors of each table, from the full scale U-235 fueled one to the full scale Pu-239 fueled one. Masses of fissile isotopes and radii of inner zone and total core are reported.

All the reactors here considered are reflected by a blanket 50 cm thick and containing U-238 as fertile material. This is true both for the oxide fueled and the metal fueled reactors, the last ones being included only in order to have some extreme elements of comparison.

In Fig. 24, 25 and 26 a comparison is possible among reactors of group 16, group 26 and group 36 respectively. The influence of varying y_2 for a constant y_1 and the influence of a decoupler are here presented as far as the radii of the different reactors are concerned. One must note that the thickness of the decoupler, when existing, has been kept constant and equal to 5 cm in order to make comparison possible; for the same reasons the thickness of the blanket has been kept constant too, though the influence of the last one cannot have a strong bearing (a few per cent of the total radius especially for diluted oxide reactors) on the overall dimensions and hence on the masses of fissile isotopes.

The figures from 2 to 23 will enable to solve the problem of the reactors which will be possible to build with a given amount of Pu-239. Besides, they allow to determine the amount of U-235 necessary, in every case, to obtain criticality. They show, as was known, that the masses of fissile isotopes increase with y, and that the introduction of a decoupler is of little or no use, as far as critical masses are concerned, depending on the chosen isotope.

More informations can be derived from Fig. 24 (group 16), Fig. 25 (group 26) and Fig. 26 (group 36). Here critical radii (of inner zone and of total core) are reported

Table IV

CRITICAL DIMENSIONS AND CRITICAL MASSES OF REFLECTED OXIDE-FUELED FAST REACTORS
GROUP 4-4

N.	Inner zone				Driver zone				Blanket			Total core radius cm
	Radius cm	Volume l	Pu-239 kg	U-238 kg	Thickness cm	Volume l	U-235 kg	U-238 kg	Thickness cm	Volume l	U-238 kg	
* 7	76.36	1865.02	1010.18	4094	—	—	—	—	50	6586	87411	76.36
8	28.12	93.19	51.34	205	40	1231.17	666.86	2703	50	5580	74056	68.12
9	30.77	122.07	67.25	268	35	1069.84	579.48	2348	50	5308	70450	65.77
10	33.32	155.01	85.39	340	30	908.62	492.16	1995	50	5032	66791	63.32
11	35.85	192.93	106.28	424	25	750.64	406.59	1648	50	4761	63192	60.85
12	38.41	237.32	130.74	521	20	597.31	323.53	1311	50	4502	59750	58.41
13	41.02	289.07	159.25	635	15	447.24	242.25	982	50	4255	56473	56.02
14	43.90	354.43	195.26	778	10	301.56	163.34	662	50	4042	53652	53.90
15	46.96	433.76	238.96	952	5	153.83	83.32	338	50	3852	51127	51.96
16	50.32	533.65	293.99	1171	—	—	—	—	50	3695	49043	50.32

* Except for this reactor which contains U-235 only

Table V

CRITICAL DIMENSIONS AND CRITICAL MASSES OF REFLECTED OXIDE-FUELED FAST REACTORS
GROUP 5-5

N.	Inner zone				Driver zone				Blanket			Total core radius cm
	Radius cm	Volume l	Pu-239 kg	U-238 kg	Thickness cm	Volume l	U-235 kg	U-238 kg	Thickness cm	Volume l	U-238 kg	
* 17	96.13	3721.45	1680.44	8510	—	—	—	—	50	9350	124099	96.13
18	38.73	243.27	111.73	556	40	1800.54	813.04	4117	50	6891	91457	78.73
19	41.04	289.53	132.97	662	35	1552.12	700.87	3548	50	6545	86870	76.04
20	43.25	338.80	155.60	775	30	1307.29	590.31	2990	50	6196	82229	73.25
21	45.67	399.01	183.25	912	25	1079.40	487.41	2468	50	5882	78062	70.67
22	48.21	469.48	215.62	1074	20	860.11	388.39	1967	50	5590	74195	68.21
23	51.03	556.54	255.61	1273	15	649.22	293.16	1485	50	5337	70834	66.03
24	53.83	653.31	300.05	1494	10	435.94	196.85	997	50	5089	67536	63.83
25	56.95	773.68	355.33	1769	5	222.20	100.33	508	50	4881	64783	61.95
26	60.56	930.19	427.21	2127	—	—	—	—	50	4730	62778	60.56

* Except for this reactor which contains U-235 only

Table VI

CRITICAL DIMENSIONS AND CRITICAL MASSES OF REFLECTED OXIDE-FUELED FAST REACTORS
GROUP 6-6

N.	Inner zone				Driver zone				Blanket			Total core radius cm
	Radius cm	Volume l	Pu-239 kg	U-238 kg	Thickness cm	Volume l	U-235 kg	U-238 kg	Thickness cm	Volume l	U-238 kg	
* 27	123.20	7832.15	3031.51	17911	—	—	—	—	50	13930	184880	123.20
28	50.23	530.74	208.94	1214	40	2545.97	985.44	5822	50	8473	112455	90.23
29					35				50			
30	54.61	682.02	268.49	1560	30	1854.77	717.91	4242	50	7679	101917	84.61
31	57.07	778.54	306.49	1780	25	1536.83	594.84	3514	50	7334	97333	82.07
32	60.21	914.12	359.86	2090	20	1247.13	482.71	2852	50	7085	94036	80.21
33	62.43	1019.05	401.17	2330	15	925.23	358.12	2116	50	6723	89224	77.43
34	65.43	1173.52	461.99	2684	10	624.45	241.70	1428	50	6469	85853	75.43
35	68.69	1357.44	534.39	3104	5	318.54	123.29	728	50	6250	82953	73.69
36	72.43	1591.43	626.51	3639	—	—	—	—	50	6095	80891	72.43

* Except for this reactor which contains U-235 only

Table VII

CRITICAL DIMENSIONS AND CRITICAL MASSES OF REFLECTED OXIDE-FUELED FAST REACTORS
GROUP 5-4

N.	Inner zone				Driver zone				Blanket			Total core radius cm
	Radius cm	Volume l	Pu-239 kg	U-238 kg	Thickness cm	Volume l	U-235 kg	U-238 kg	Thickness cm	Volume l	U-238 kg	
* 37=7	76.36	1865.02	1010.18	4094	—	—	—	—	50	6586	87411	76.36
38	31.33	128.88	59.19	295	40	1391.66	753.79	3055	50	5962	79127	71.33
39	34.93	178.55	82.00	408	35	1254.02	679.24	2753	50	5793	76889	69.93
40	38.22	233.89	107.42	535	30	1096.12	593.71	2406	50	5591	74206	68.22
41	41.67	303.16	139.23	693	25	938.36	508.26	2060	50	5411	71819	66.67
42	45.11	384.56	176.62	879	20	771.75	418.02	1694	50	5233	69452	65.11
43	48.66	482.49	221.60	1103	15	597.95	323.88	1313	50	5069	67281	63.66
44	52.36	601.18	276.11	1375	10	414.45	224.49	910	50	4926	65374	62.36
45	56.42	752.26	345.49	1720	5	218.25	118.21	479	50	4823	64016	61.42
46=26	60.56	930.19	427.21	2127	—	—	—	—	50	4730	62778	60.56

* Except for this reactor which contains U-235 only

Table VIII

CRITICAL DIMENSIONS AND CRITICAL MASSES OF REFLECTED OXIDE-FUELED FAST REACTORS
GROUP 6-4

N.	Inner zone				Driver zone				Blanket			Total core radius cm
	Radius cm	Volume l	Pu-239 kg	U-238 kg	Thickness cm	Volume l	U-235 kg	U-238 kg	Thickness cm	Volume l	U-238 kg	
* 47=7	76.36	1865.02	1010.18	4094	—	—	—	—	50	6586	87411	76.36
48	34.76	175.93	69.26	402	40	1574.34	852.74	3456	50	6384	84729	74.76
49	39.12	250.71	98.70	573	35	1454.73	787.96	3193	50	6304	83661	74.12
50	43.65	348.44	137.17	797	30	1325.18	717.78	2909	50	6246	82896	73.65
51	48.23	469.88	184.98	1075	25	1174.95	636.41	2579	50	6193	82199	73.23
52	52.88	619.46	243.87	1417	20	1002.17	542.82	2200	50	6151	81633	72.88
53	57.52	797.25	313.85	1823	15	800.47	433.57	1757	50	6107	81046	72.52
54	62.35	1015.12	399.63	2321	10	570.99	309.28	1253	50	6085	80760	72.35
55	67.38	1281.48	504.48	2930	5	306.97	166.27	674	50	6089	80810	72.38
56=36	72.43	1591.43	626.51	3639	—	—		—	50	6095	80891	72.43

* Except for this reactor which contains U-235 only

Table IX

CRITICAL DIMENSIONS AND CRITICAL MASSES OF REFLECTED OXIDE-FUELED FAST REACTORS
GROUP 6-5

N.	Inner zone				Driver zone				Blanket			Total core radius cm
	Radius cm	Volume l	Pu-239 kg	U-238 kg	Thickness cm	Volume l	U-235 kg	U-238 kg	Thickness cm	Volume l	U-238 kg	
* 57=17	96.13	3721.45	1680.44	8510	—	—	—	—	50	9350	124099	96.13
58	44.35	365.29	143.80	835	40	2148.19	970.03	4913	50	7643	101443	84.35
59	47.42	446.69	175.85	1022	35	1898.65	857.35	4342	50	7381	97964	82.42
60	50.49	539.20	212.27	1233	30	1645.25	742.92	3762	50	7123	94538	80.49
61	53.50	641.43	252.51	1467	25	1384.84	625.33	3167	50	6862	91067	78.50
62	57.00	775.73	305.39	1774	20	1136.59	513.23	2599	50	6668	88497	77.00
63	60.50	927.59	365.17	2121	15	875.14	395.17	2001	50	6477	85964	75.50
64	64.16	1106.35	435.54	2530	10	602.12	271.89	1377	50	6309	83734	74.16
65	68.00	1317.09	518.50	3012	5	312.42	141.08	714	50	6165	81825	73.00
66=36	72.43	1591.43	626.51	3639	—	—	—	—	50	6095	80891	72.43

* Except for this reactor which contains U-235 only

Table X

CRITICAL DIMENSIONS AND CRITICAL MASSES OF REFLECTED OXIDE-FUELED FAST REACTORS
GROUP 4-4-D1

N.	Inner zone				Decoupler			Driver zone				Blanket			Total core radius cm
	Radius cm	Volume l	Pu-239 kg	U-238 kg	Thickness cm	Volume l	U-238 kg	Thickness cm	Volume l	U-235 kg	U-238 kg	Thickness cm	Volume l	U-238 kg	
*67=7	76.36	1865.02	1010.18	4094	—	—	—	—	—	—	—	50	6585	87411	76.36
68	38.38	236.79	130.45	520	5	105.13	1993	40	2086.13	1129.95	4579	50	7511	99688	83.38
69	40.51	278.53	153.44	611	5	116.38	2207	35	1791.28	970.24	3932	50	7126	94576	80.51
70	42.20	314.84	173.44	691	5	125.69	2383	30	1486.88	805.37	3264	50	6694	88841	77.20
71	43.58	346.71	191.01	761	5	133.55	2532	25	1188.45	643.72	2609	50	6237	82777	73.58
72	44.94	380.27	209.49	835	5	141.56	2684	20	911.45	493.69	2001	50	5795	76908	69.94
73	46.23	413.84	227.99	908	5	149.33	2831	15	653.67	354.06	1435	50	5360	71141	66.23
74	47.52	449.37	247.56	986	5	157.31	2983	10	416.75	225.73	915	50	4943	65606	62.52
75	48.88	489.14	269.47	1074	5	165.99	3147	5	199.84	108.24	439	50	4551	60407	58.88
76=16	50.32	533.65	293.99	1171	—	—	—	—	—	—	—	50	3695	49043	50.32

* Except for this reactor which contains U-235 only

Table XI

CRITICAL DIMENSIONS AND CRITICAL MASSES OF REFLECTED OXIDE-FUELED FAST REACTORS
GROUP 5-5-D1

N.	Inner zone				Decoupler			Driver zone				Blanket			Total core radius cm
	Radius cm	Volume l	Pu-239 kg	U-238 kg	Thick-ness cm	Volume l	U-238 kg	Thick-ness cm	Volume l	U-235 kg	U-238 kg	Thick-ness cm	Volume l	U-238 kg	
*77=17	96.13	3721.45	1680.44	8510	—	—	—	—		—	—	50	9350	124099	96.13
78	51.52	572.72	263.04	1310	5	183.46	3478	40	3010.00	1359.19	6883	50	9409	124875	96.52
79	52.70	613.19	281.62	1402	5	191.60	3633	35	2532.32	1143.49	5791	50	8836	117267	92.70
80	53.76	650.65	298.83	1488	5	198.97	3772	30	2079.04	938.80	4754	50	8261	109646	88.76
81	54.75	687.59	315.79	1572	5	206.09	3908	25	1556.47	747.99	3788	50	7700	102189	84.75
82	55.84	729.42	335.00	1668	5	214.00	4057	20	1269.69	573.34	2904	50	7170	95156	80.84
83	56.97	774.60	355.76	1771	5	222.36	4216	15	913.29	412.40	2089	50	6664	88450	76.97
84	58.07	820.13	376.66	1875	5	230.62	4373	10	583.27	263.38	1334	50	6174	81935	73.07
85	59.35	875.75	402.21	2003	5	240.50	4560	5	280.93	126.86	642	50	5724	75973	69.35
86=26	60.56	930.19	427.21	2127	—	—	—	—	—	—	—	50	4730	62778	60.56

* Except for this reactor which contains U-235 only

Table XII

CRITICAL DIMENSIONS AND CRITICAL MASSES OF REFLECTED OXIDE-FUELED FAST REACTORS
GROUP 4-4-D2

N.	Inner zone				Decoupler			Driver zone				Blanket			Total core radius cm
	Radius cm	Volume l	Pu-239 kg	U-238 kg	Thickness cm	Volume l	kg	Thickness cm	Volume l	U-235 kg	U-238 kg	Thickness cm	Volume l	U-238 kg	
* 87=7	76.36	1865.02	1010.18	4094.02	—	—	—	—	—	—	—	50	6586	87411	76.36
88	26.59	78.74	43.38	172.84	5	53.30	607	40	1404.79	760.90	3084	50	5993	79536	71.59
89	29.07	102.85	56.66	225.77	5	62.73	715	35	1214.37	657.76	2666	50	5690	75523	69.07
90	31.28	128.15	70.60	281.31	5	71.81	818	30	1019.47	552.19	2238	50	5366	71212	66.28
91	33.30	154.73	85.24	339.66	5	80.68	919	25	827.22	448.06	1816	50	5030	66762	63.30
92	35.21	182.83	100.72	401.35	5	89.48	1019	20	641.96	347.72	1409	50	4693	62284	60.21
93	37.13	214.35	118.09	470.53	5	98.79	1126	15	467.75	253.36	1027	50	4369	57981	57.13
94															
95	41.09	290.52	160.05	637.74	5	119.50	1361	5	148.45	80.41	326	50	3768	50013	51.09
96=16	50.32	533.65	293.99	1171.45	—	—	—	—	—	—	—	50	3695	49043	50.32

* Except for this reactor which contains U-235 only

Table XIII

CRITICAL DIMENSIONS AND CRITICAL MASSES OF REFLECTED OXIDE-FUELED FAST REACTORS
GROUP 4-3

N.	Inner zone				Driver zone				Blanket			Total core radius cm
	Radius cm	Volume l	Pu-239 kg	U-238 kg	Thickness cm	Volume l	U-235 kg	U-238 kg	Thickness cm	Volume l	U-238 kg	
* 127=7	76.36	1865.02	1010.18	4094.02	—	—	—	—	50	6586	87411	76.36
128	19.54	31.25	17.22	69	40	852.84	577.69	1755	50	4621	61335	59.54
129	23.53	54.54	30.04	120	35	785.15	531.84	1616	50	4514	59914	58.53
130	27.36	85.76	47.25	188	30	704.64	477.31	1450	50	4393	58299	57.36
131	31.04	125.30	69.03	275	25	611.99	414.54	1260	50	4258	56507	56.04
132	34.67	174.61	96.19	383	20	509.95	345.43	1050	50	4119	54672	54.67
133	38.30	235.32	129.64	517	15	398.92	270.22	821	50	3983	52862	53.30
134	42.12	313.07	172.47	687	10	280.09	189.73	576	50	3868	51338	52.12
135	46.09	410.01	225.88	900	5	148.45	100.56	306	50	3768	50013	51.09
136=16	50.32	533.65	293.99	1171	—	—	—	—	50	3695	49043	50.32

* Except for this reactor which contains U-235 only

Table XIV

CRITICAL DIMENSIONS AND CRITICAL MASSES OF REFLECTED OXIDE-FUELED FAST REACTORS
GROUP 4-2

N.	Inner zone				Driver zone				Blanket			Total core radius cm
	Radius cm	Volume l	Pu-239 kg	U-238 kg	Thickness cm	Volume l	U-235 kg	U-238 kg	Thickness cm	Volume l	U-238 kg	
* 137=7	76.36	1865.02	1010.18	4094	—	—	—	—	50	6586	87411	76.36
138	7.80	1.99	1.10	4	40	455.59	411.45	833	50	3461	45937	47.80
139	12.82	8.83	4.87	19	35	449.25	405.73	822	50	3463	45958	47.82
140	17.87	23.92	13.18	52	30	435.65	393.44	797	50	3468	46021	47.87
141	22.98	50.83	28.00	112	25	411.84	371.94	753	50	3477	46152	47.98
142	28.15	93.44	51.47	205	20	374.16	337.91	685	50	3493	46359	48.15
143	33.52	157.79	86.93	346	15	320.73	289.66	587	50	3527	46814	48.52
144	38.99	248.23	136.75	545	10	244.19	220.53	447	50	3570	47386	48.99
145	44.56	370.71	204.23	814	5	139.30	125.81	255	50	3624	48100	49.56
146=16	50.32	533.65	293.99	1171	—	—	—	—	50	3695	49043	50.32

* Except for this reactor which contains U-235 only

Table XV

CRITICAL DIMENSIONS AND CRITICAL MASSES OF REFLECTED OXIDE-FUELED FAST REACTORS
GROUP 4-1

N.	Inner zone				Driver zone				Blanket			Total core radius cm
	Radius cm	Volume l	Pu-239 kg	U-238 kg	Thickness cm	Volume l	U-235 kg	U-238 kg	Thickness cm	Volume l	U-238 kg	
* 147=7	76.36	1865.02	1010.18	4094	—	—	—	—	50	6586	87411	76.36
148												
149												
150	5.60	0.73	0.41	2	30	188.32	255.12	258	50	2439	32365	35.60
151	10.98	5.55	3.06	12	25	189.56	256.80	260	50	2467	32746	35.98
152												
153	24.09	58.52	32.24	128	15	191.58	259.54	263	50	2711	35985	39.09
154	32.33	141.60	78.01	311	10	176.20	238.71	242	50	2980	39546	42.33
155	41.32	295.45	162.76	649	5	120.76	163.61	166	50	3327	44151	46.32
156=16	50.32	533.65	293.99	1171	—	—	—	—	50	3695	49043	50.32

* Except for this reactor which contains U-235 only

Table XVI

CRITICAL DIMENSIONS AND CRITICAL MASSES OF REFLECTED OXIDE-FUELED FAST REACTORS
GROUP 5-3

N.	Inner zone				Driver zone				Blanket			Total core radius cm
	Radius cm	Volume l	Pu-239 kg	U-238 kg	Thickness cm	Volume l	U-235 kg	U-238 kg	Thickness cm	Volume l	U-238 kg	
*157=17	96.13	3721.45	1680.44	8510	—	—	—	—	50	9350	124099	96.13
158	20.59	36.57	16.79	84	40	895.19	606.38	1842	50	4734	62827	60.59
159	25.52	69.62	31.97	159	35	858.88	581.78	1768	50	4726	62726	60.52
160	30.35	117.05	53.76	268	30	803.45	544.23	1654	50	4707	62478	60.35
161	35.16	182.11	83.64	416	25	730.05	494.52	1503	50	4688	62218	60.16
162	40.07	269.51	123.78	616	20	638.47	432.49	1314	50	4678	62087	60.07
163	45.00	381.70	175.31	873	15	523.08	354.32	1077	50	4671	61987	60.00
164	50.12	527.33	242.19	1206	10	382.82	259.31	788	50	4683	62155	60.12
165	55.32	709.31	325.77	1622	5	210.22	142.40	433	50	4705	62448	60.32
166=26	60.56	930.19	427.21	2127	—	—	—	—	50	4730	62778	60.56

* Except for this reactor which contains U-235 only

Table XVII

CRITICAL DIMENSIONS AND CRITICAL MASSES OF REFLECTED OXIDE-FUELED FAST REACTORS
GROUP 5-2

N.	Driver zone				Inner zone				Blanket			Total core radius cm
	Radius cm	Volume l	Pu-239 kg	U-238 kg	Thickness cm	Volume l	U-235 kg	U-238 kg	Thickness cm	Volume l	U-238	
*167=17	96.13	3721.45	1680.44	8510	—	—	—	—	50	9350	124099	96.13
168												
169												
170	18.74	27.56	12.66	63	30	457.40	413.09	837	50	3547	47080	48.74
171	24.78	63.70	29.26	146	25	452.88	409.00	829	50	3644	48364	49.78
172	31.40	129.68	59.56	297	20	439.14	396.59	803	50	3798	50412	51.40
173	38.52	239.46	109.98	548	15	402.78	363.76	737	50	4005	53154	53.52
174	45.93	405.98	186.46	928	10	327.06	295.37	598	50	4247	56361	55.93
175	53.36	636.30	292.24	1455	5	196.17	177.16	359	50	4497	59680	58.36
176=26	60.56	930.19	427.21	2127	—	—	—	—	50	4730	52778	60.56

* Except for this reactor which contains U-235 only

Table XVIII

CRITICAL DIMENSIONS AND CRITICAL MASSES OF REFLECTED OXIDE-FUELED FAST REACTORS
GROUP 5-1

N.	Inner zone				Driver zone				Blanket			Total core radius cm
	Radius cm	Volume l	Pu-239 kg	U-238 kg	Thickness cm	Volume l	U-235 kg	U-238 kg	Thickness cm	Volume l	U-238 kg	
*177=17	96.13	3721.45	1680.44	8510	—	—	—	—	50	9350	124099	96.13
178												
179												
180												
181												
182												
183	26.14	74.79	34.35	171	15	216.80	293.71	297	50	2879	38213	41.14
184	36.93	211.03	96.92	483	10	222.02	300.78	305	50	3382	44887	46.93
185	49.18	498.22	228.82	1139	5	167.94	227.51	230	50	4070	54017	54.18
186=26	60.56	930.19	427.21	2127	—	—	—	—	50	4730	62778	60.56

* Except for this reactor which contains U-235 only

Table XIX

CRITICAL DIMENSIONS AND CRITICAL MASSES OF REFLECTED OXIDE-FUELED FAST REACTORS
GROUP 6-3

N.	Inner zone				Driver zone				Blanket			Total core radius cm
	Radius cm	Volume l	Pu-239 kg	U-238 kg	Thickness cm	Volume l	U-235 kg	U-238 kg	Thickness cm	Volume l	U-238 kg	
*187=27	123.20	7832.15	3031.51	17911	—	—	—	—	50	13930	184880	123.20
188	21.62	42.32	16.66	97	40	937.65	635.14	1930	50	4845	64303	61.62
189	27.26	84.82	33.39	194	35	925.94	627.21	1906	50	4915	65229	62.26
190	33.26	154.18	60.69	353	30	906.46	614.01	1866	50	5026	66703	63.26
191	39.50	258.15	101.63	590	25	865.85	586.50	1782	50	5164	68535	64.50
192	46.00	407.72	160.51	932	20	796.54	539.56	1639	50	5334	70793	66.00
193	52.77	615.52	242.31	1408	15	688.23	466.19	1416	50	5538	73505	67.77
194	59.46	880.36	346.58	2013	10	523.12	354.35	1077	50	5737	76137	69.46
195	66.04	1206.48	474.96	2759	5	295.30	200.03	608	50	5926	78655	71.04
196=36	72.43	1591.43	626.51	3639	—	—	—	—	50	6095	80891	72.43

* Except for this reactor which contains U-235 only

Table XX

CRITICAL DIMENSIONS AND CRITICAL MASSES OF REFLECTED OXIDE-FUELED FAST REACTORS
GROUP 5-5-D2

N.	Inner zone				Decoupler			Driver zone				Blanket			Total core radius cm
	Radius cm	Volume l	Pu-239 kg	U-238 kg	Thickness cm	Volume l	kg	Thickness cm	Volume l	U-235 kg	U-238 kg	Thickness cm	Volume l	U-238 kg	
*217=17	96.13	3721.45	1680.44	8510	—	—	—	—	—	—	—	50	9350	124099	96.13
218	36.51	203.93	93.66	466	5	95.77	1091	40	1969.08	889.15	4503	50	7259	96346	81.51
219	38.49	238.81	109.68	546	5	105.69	1204	35	1680.81	758.98	3844	50	6860	91046	78.49
220	40.35	275.18	126.38	629	5	115.50	1316	30	1401.32	632.78	3205	50	6458	85712	75.35
221	42.17	314.09	144.25	718	5	125.50	1430	25	1134.88	512.46	2595	50	6063	80472	72.17
222	43.97	356.12	163.56	814	5	135.82	1547	20	882.40	398.45	2018	50	5679	75376	68.97
223	45.89	404.80	185.92	926	5	147.26	1678	15	646.19	291.79	1478	50	5321	70626	65.89
224	47.96	462.07	212.22	1057	5	160.11	1824	10	423.19	191.09	968	50	4992	66255	62.96
225	50.19	529.71	243.28	1211	5	174.59	1989	5	209.27	94.50	479	50	4691	62262	60.19
226=26	60.56	930.19	427.21	2127	—	—	—	—	—	—	—	50	4730	62778	60.56

* Except for this reactor which contains U-235 only

Table XXI

CRITICAL DIMENSIONS AND CRITICAL MASSES OF REFLECTED OXIDE-FUELED FAST REACTORS
GROUP 6-6-D2

N.	Inner zone				Decoupler			Driver zone				Blanket			Total core radius cm
	Radius cm	Volume l	Pu-239 kg	U-238 kg	Thickness cm	Volume l	kg	Thickness cm	Volume l	U-235 kg	U-238 kg	Thickness cm	Volume l	U-238 kg	
*227=27	123.20	7832.15	3031.51	17911	—	—	—	—	—	—	—	50	13930	184880	123.20
228	47.16	439.27	172.93	1005	5	155.06	1767	40	2684.16	1038.93	6138	50	8755	116197	92.16
229	48.68	483.36	190.28	1105	5	164.74	1877	35	2273.60	880.02	5199	50	8251	109513	88.68
230	50.50	539.46	212.37	1234	5	176.63	2012	30	1902.01	736.19	4350	50	7803	103559	85.50
231	52.36	601.19	236.67	1375	5	189.21	2156	25	1549.46	599.73	3543	50	7373	97849	82.36
232	54.27	669.49	263.56	1531	5	202.62	2308	20	1214.29	470.00	2777	50	6962	92399	79.27
233	56.23	744.87	293.23	1703	5	216.88	2471	15	894.05	346.05	2045	50	6570	87198	76.23
234	58.45	836.31	329.23	1912	5	233.52	2660	10	589.77	228.28	1349	50	6220	82557	73.45
235	60.98	949.74	373.89	2172	5	253.31	2886	5	294.76	114.09	674	50	5919	78554	70.98
236=36	72.43	1591.43	626.51	3639	—	—	—	—	—	—	—	50	6095	80891	72.43

* Except for this reactor which contains U-235 only

Table XXII

CRITICAL DIMENSIONS AND CRITICAL MASSES OF REFLECTED OXIDE-FUELED FAST REACTORS
GROUP 4-3-D2

	Inner zone				Decoupler			Driver zone				Blanket			Total core radius cm
	Radius cm	Volume l	Pu-239 kg	U-238 kg	Thickness cm	Volume l	kg	Thickness cm	Volume l	U-235 kg	U-238 kg	Thickness cm	Volume l	U-238 kg	
*237=7	76.36	1865.02	1010.18	4094	—	—	—	—	—	—	—	50	6586	87411	76.36
238	17.18	21.24	11.70	47	5	24.46	279	40	961.28	651.14	1978	50	4906	65115	62.18
239	21.83	43.56	23.99	96	5	37.31	425	35	909.08	615.79	1871	50	4868	64604	61.83
240	26.00	73.62	40.56	162	5	51.17	583	30	825.99	559.50	1700	50	4778	63413	61.00
241	29.42	106.68	58.77	234	5	64.16	731	25	708.03	479.60	1457	50	4609	61170	59.42
242	32.54	144.28	79.48	317	5	77.26	880	20	576.31	390.38	1186	50	4411	58545	57.54
243	35.26	183.63	101.16	403	5	89.72	1022	15	433.50	293.64	892	50	4178	55455	55.26
244															
245	40.52	278.68	153.53	612	5	116.42	1326	5	145.02	98.23	298	50	3714	49297	50.52
246=16	50.32	533.65	293.99	1171	—	—	—	—	—	—	—	50	3695	49043	50.32

* Except for this reactor which contains U-235 only

Table XXIII

CRITICAL DIMENSIONS AND CRITICAL MASSES OF REFLECTED OXIDE-FUELED FAST REACTORS
GROUP 5-4-D2

N.	Inner zone				Decoupler			Driver zone				Blanket			Total core radius cm
	Radius cm	Volume l	Pu-239 kg	U-238 kg	Thickness cm	Volume l	kg	Thickness cm	Volume l	U-235 kg	U-238 kg	Thickness cm	Volume l	U-238 kg	
*247=7	76.36	1865.02	1010.18	4094	—	—	—	—	—	—	—	50	6586	87411	76.36
248	30.00	113.10	51.94	259	5	66.50	758	40	1587.55	859.90	3485	50	6414	85128	75.00
249	33.26	154.12	70.78	352	5	80.48	917	35	1412.41	765.03	3100	50	6197	82252	73.26
250	36.37	201.50	92.54	461	5	95.06	1083	30	1226.13	664.13	2692	50	5966	79181	71.37
251	39.05	249.48	114.58	571	5	108.62	1237	25	1021.11	553.08	2242	50	5689	75504	69.05
252	41.76	304.96	140.06	697	5	123.19	1404	20	817.97	443.05	1796	50	5421	71945	66.76
253	44.37	365.84	168.02	837	5	138.15	1574	15	613.12	332.09	1346	50	5149	68338	64.37
254	46.98	434.22	199.42	993	5	153.93	1754	10	408.98	221.52	898	50	4884	64820	61.98
255	49.79	516.95	237.42	1182	5	171.91	1959	5	206.34	111.76	453	50	4648	61686	59.79
256=26	60.56	930.19	427.21	2127	—	—	—	—	—	—	—	50	4730	62778	60.56

* Except for this reactor which contains U-235 only

Table XXIV

CRITICAL DIMENSIONS AND CRITICAL MASSES OF REFLECTED OXIDE-FUELED FAST REACTORS
GROUP 6-5-D2

N.	Inner zone				Decoupler			Driver zone				Blanket			Total core radius cm
	Radius cm	Volume l	Pu-239 kg	U-238 kg	Thickness cm	Volume l	kg	Thickness cm	Volume l	U-235 kg	U-238 kg	Thickness cm	Volume l	U-238 kg	
*257=17	96.13	3721.45	1680.44	8510	—	—	—	—	—	—	—	50	9350	124099	123.20
258	42.17	314.12	123.65	718	5	125.50	1430	40	2334.86	1054.32	5339	50	8036	106659	87.17
259	44.79	376.32	148.14	861	5	140.63	1602	35	2036.23	919.47	4656	50	7704	102250	84.79
260	47.22	440.91	173.57	1008	5	155.43	1771	30	1731.50	781.87	3960	50	7354	97596	82.22
261	49.85	518.79	204.23	1186	5	172.30	1963	25	1441.25	650.80	3296	50	7038	93407	79.85
262	52.28	598.47	235.60	1369	5	188.66	2149	20	1145.96	517.47	2621	50	6704	88970	77.28
263	54.86	691.69	272.30	1582	5	206.88	2357	15	858.87	387.83	1964	50	6397	84898	74.86
264	57.64	802.23	315.82	1835	5	227.39	2591	10	576.01	260.10	1317	50	6121	81241	72.64
265	60.50	927.59	365.17	2121	5	249.51	2843	5	290.67	131.25	665	50	5851	77791	70.50
266=36	72.43	1591.43	626.51	3639	—	—	—	—	—	—	—	50	6095	80891	72.43

* Except for this reactor which contains U-235 only

Table XXV

CRITICAL DIMENSIONS AND CRITICAL MASSES OF REFLECTED OXIDE-FUELED FAST REACTORS
GROUP 6-6-D1

N.	Inner zone				Decoupler			Driver zone				Blanket			Total core radius cm
	Radius cm	Volume l	Pu-239 kg	U-238 kg	Thickness cm	Volume l	kg	Thickness cm	Volume l	U-235 kg	U-238 kg	Thickness cm	Volume l	U-238 kg	
*267=27	123.20	7832.15	3031.51	17911	—	—	—	—	—	—	—	50	13930	184880	123.20
268	65.71	1188.49	467.88	2718	5	292.47	5545	40	4203.07	1626.84	9612	50	11703	155321	110.71
269	66.10	1209.60	476.19	2766	5	295.79	5608	35	3497.29	1353.66	7998	50	10930	145057	106.10
270	66.80	1248.42	491.47	2855	5	301.86	5723	30	2868.43	1110.25	6560	50	10233	135808	101.80
271	67.51	1288.98	507.44	2948	5	308.12	5842	25	2286.84	885.15	5230	50	9562	126901	97.51
272	68.43	1342.24	528.41	3069	5	316.24	5996	20	1757.76	680.36	4020	50	8944	118698	93.43
273	69.22	1389.59	547.05	3178	5	323.37	6131	15	1262.51	488.67	2887	50	8329	110540	89.22
274	70.18	1448.11	570.08	3312	5	332.07	6296	10	809.00	313.13	1850	50	7759	102977	85.18
275	71.34	1520.82	598.71	3478	5	342.71	6498	5	390.67	151.21	893	50	7236	96036	81.34
276=36	72.43	1591.43	626.51	3639	—	—	—	—	—	—	—	50	6095	80891	72.43

* Except for this reactor which contains U-235 only

so that a direct comparison can be made among reactors of the same group. In each of them the radius of the full-scale Pu-239 fueled reactor is the basic reference, 50.32 cm for reactor No. 16, 60.56 cm for reactor No. 25 and 72.43 cm for reactor No. 36. One can easily see that using a value for $y_2 < y$, the inner core radius and the total radius diminish; for y_1 sufficiently bigger than y_2 we can reach criticality with a reactor having a radius smaller than the reference one. It is interesting to note that for a particular choice of y_2 we can attain criticality with a reactor having the same radius as the reference one and the right composition in the central part. Considerations on flux spectra (see next section) will permit to decide if in so doing we attain ideal conditions for experimental measurements too.

Turning to the influence of the decoupler on critical radii, we see that U-238 has the effect of increasing them and is, then, of no use. HS-208 is responsible for a slight increase in the inner core radius for values of the ratio Pu-239 mass/(Pu-239 + U-235) masses between 0 and about 0.2; from 0.2 on, the effect of HS-208 is to slightly decrease the inner core radius; the total core radius can be higher (with reference to the same couple y_1-y_2 without decoupler) at the beginning, but usually smaller for the above ratio close to unity. The use of this decoupler is then such to reduce the amount of Pu-239 necessary and the more for the more diluted cases.

It would have been desirable to study the influence of the decoupler thickness on critical radii and/or the influence of other isotopes. Of course, as HS-208 is a fictitions heavy pure elastic scattering isotope with arbitrary physical properties, our results have only a limited interest: it seems advisable, anyway, to see which ones, among real heavy isotopes, have physical properties convenient for use as decoupler mediums.

Again, critical masses (or radii) considerations are not enough if taken alone: we will see in the next section if the use of a decoupler is convenient from the flux spectra point of view.

6. COMPARISON METHODS

Once the problem of the possible reactors to make up with the available amounts of Pu-239 (and of U-235) has been solved, one has to test the zoned reactors, against the full scale ones as far as the reliability on flux spectrum. The ideal situation would be to have the flux spectrum of a zoned and that of the full scale reference reactor exactly the same both in energy groups and in space. These two conditions are too stringent if one wants them to hold for all the volume of the examined reactors, but can be met in a reasonable way if we limit our attention to the central part of the cores.

Intuitively, it is clear that if we have a full scale Pu-239 reactor A with a certain y_1 and a zoned reactor B characterized by the same value y_1 in the inner zone (the driver zone being U-235 fueled, with y_2 equal or not to y_1), the central part of the two reactors should behave approximatively in the same way, especially for high values of y_1, which means for reactors of big dimensions. The meaning to attribute to the central part of a reactor, i.e. the length of the radius to consider, stems from the experiments to perform and the volume they will require. One of the most important experiments foreseen in a zero power fast critical facility will deal with the measurement of the Doppler Effect: to this purpose a loop of square cross section is likely to be installed in the central part of the reactor. Due to the practice of using square tubes to make up the cores, the edge of the loop cross section will be an integer multiple of the edge of the elementary tube.

With reference to square cross section tubes of 10 cm side, it is possible to design a Doppler loop to be located in the place of an array of 3×3 (or 4×4) normal tubes. That seems enough, if the inner zone has a diameter bigger than, say, 40 (or 50) cm, while smaller Doppler loops can be designed for the cases when the inner zone has a more limited diameter. Actually, because of the nature of the Doppler experiment and the consequent temperature requirements (possibly, liquid sodium could be present), only the central part in the 3×3 (or 4×4) tubes array would be used for the measurements. It is normally assumed that, inside an experimental volume, outer media influence can be disregarded if one considers volumes about three mean free paths away from boundaries: it is easy to see that this condition is fulfilled in our cases and that the zone inside a 3×3 (or 4×4) tubes array has a diameter of about 1.5 and 2 times respectively the lenght of 3λ. An estimation of the value of λ for fast reactors characterised by $y = 4$, 5, 6 is reported in Appendix A; there, the adopted scheme of calculation is rather rough but the results reliable enough to our purposes. They show a λ of about 7 cm in the three cases, which confirms the possibility of this kind of experiments in the chosen volume.

We shall examine in the next section the reactors which are possible to use for a Doppler measurement if a Doppler loop of great dimensions is to be installed in them. This experiment, anyway, is only one of a series to be foreseen, the other ones requiring the same or smaller volumes. In any case the validity of the measurements is based on the possibility of having, as already mentioned, convenient flux spectra.

In a general way, a standard criterium of comparison between the flux spectra of two reactors does not exist, which suggest to compare the results of different methods. Difficult as it is to define the equivalence of a test core and of a full scale core from the neutronic point of view, several criteria can be proposed and used. The degree of confidence of each of them is to be considered as related to the particular experimental measure one wants to perform. In what follows we try then to list several possible criteria, each one allowing a certain kind of comparison.

Talking of comparison, we have to consider reactors inside the same group, i.e. reactors characterized by the same y_1 to be compared with the full-scale Pu-239 fueled one. The influence of different driver thickness and that of a decoupler can then be evaluated. In so doing we will be able to attribute a certain numerical value to the similarity (or to the difference) between two reactors: the reverse problem, to design a reactor having a core composition such that its similarity to another reactor has a certain numerical value in a certain comparison frame, cannot be explicitely stated and its solution can be achieved only tentatively and by interpolation.

6.1. Direct comparison of flux spectra

The direct comparison of flux spectra of two reactors as given by the results of calculations is of no use. As a matter of fact, the values of the fluxes so obtained are only proportional to the real ones, the constant of proportionality being different for each reactor.

Several ways can be followed to establish a comparison between two reactors on the flux spectra, e.g., to impose the rate of power (like, for instance, one fission per second) or to normalize the fluxes to a given value. If this second scheme is followed, let be, for a reactor R

N = number of energy groups considered;
M = point along the radius where the flux is calculated ($M = 0$ at the origin, taken at the center of the reactor; everywhere, M equals the distance from the origin);
φ_{MN} = neutron flux density per unit lethargy at a point M in the Nth group.

With reference to a point M along the radius, we consider the sum

$$\sum_{N=1}^{N} \varphi_{MN} \cdot \Delta u_N = A_M \tag{1}$$

were Δu_N is the lethargy interval relative to group N. We can write (1) in the following form

$$\sum_{N=1}^{N} \frac{100\, \varphi_{MN} \Delta u_N}{A_M} = 100 \tag{1'}$$

or

$$\sum_{N=1}^{N} \bar{\varphi}_{MN} \Delta u_N = 100 \tag{1"}$$

so defining

$$\bar{\varphi}_{MN} = \frac{100\, \varphi_{MN}}{A_M} = \frac{100\, \varphi_{MN}}{\sum_{N=1}^{N} \varphi_{MN} \Delta u_N} \tag{2}$$

which is the normalized flux we are seeking, the normalization being defined according to (1"). If we normalize in the same way the flux of a reactor R' and for the same values of N and M, we obtain

$$\bar{\varphi'}_{MN} = \frac{100\, \varphi'_{MN}}{A'_M} = \frac{100\, \varphi'_{MN}}{\sum_{N=1}^{N} \varphi'_{MN} \Delta u_N} \tag{2'}$$

and similarly for any other reactor we want to compare with R.

Examples of comparison among reactors of the same group are given in Figs. 27, 28 and 29 for groups 16, 26 and 36 respectively. In the three instances, the histograms refer to the center of the reactors. Similar diagrams are needed for all the points along the radius where comparison is deemed necessary.

The method gives an overall idea about the flux spectra of several reactors to be compared and points out the differences and similarity from the reference reactor for every energy interval. That can be useful when a particular experiment is strongly energy dependent, but this method has two main disadvantages: first, it does not show if and how there is a compensation from some energy groups fluxes to others; second, it requires several graphs (one for each point along the radius to be examined) from which it is not easy to deduce how fast the flux differences between two reactors vary radius-wise.

6.2. Mismatch factor (Pinco Pallino Parameter - PPP)

An easy comparison between the flux spectra of any two reactors of a certain group can be performed through a mismatch factor PPP (Pinco Pallino Parameter) defined as

$$PPP = \left[\frac{\sum_{N=1}^{N} (\bar{\varphi}_{MN} - \overline{\varphi'}_{MN})^2}{N} \right]^{1/2} \tag{3}$$

where the simbols have the meaning already explained in 6.1.

Clearly, PPP is a function of the point M; results of several calculations, with reference to the reactors examined in section 5, are represented in Figures 30 to 44. Here, the values taken along the x axis (corresponding to the radius of the examined reactors) are limited between 0 and 20 cm, according to the reasons already given on the volume to use in a reactor for performing experimental measurements.

The main advantages of introducing the PPP are: first, the comparison of any reactor with the reference one is made in a global way, irrespective of which energy group contributes positively or negatively, since we make use of the square of differences; second, the graphs give a clear idea (even if qualitative, due to the arbitrary definition of PPP itself) of how much a reactor differs from another and how this difference varies radius-wise; third, a visual comparison of several reactors with the one of reference is very easy, the graphical representation allowing to decide which reactor is more convenient.

As it is apparent, the mismatch factor PPP is based, more than on physics considerations, on a simple mathematical formulation (root mean square) well known from the statistics and the theory of errors. One interesting feature of PPP, as compared for instance with other methods, is that it furnishes absolute values after choosing the base reference in the more convenient way.

6.3. Average cross sections

The average cross section for a certain reaction of a given isotope along the radius r of a reactor can be defined

$$\bar{\sigma}(r) = \frac{\int \sigma(u)\varphi(u,r)du}{\int \varphi(u,r)du} \tag{4}$$

or, in a summation form,

$$\bar{\sigma}_M = \frac{\sum_{N=1}^{N} \sigma_N \varphi_{MN} \Delta u_N}{\sum_{N=1}^{N} \varphi_{MN} \Delta u_N} \tag{4'}$$

which can easily be rewritten in an equivalent form making use of normalized fluxes

$$\bar{\sigma}_M = \frac{1}{100} \sum_{N=1}^{N} \sigma_N \bar{\varphi}_{MN} \Delta u_N \tag{4''}$$

The cross section to be considered can be anyone of those listed in the library used (fission, capture, absorption, etc.) and the results will show how $\bar{\sigma}_M$ of the reference reactor and $\bar{\sigma}_M$ of the reactor to compare varies with radius. This methods, like the one described in 6.1. does not give an absolute value of the differences between two flux spectra.

6.4. Spectral index

The spectral index (SI) being the value of the ratio of two measures made by means of two detectors in the same neutron flux, we can write

$$\mathrm{SI} = \frac{\int \sigma_1(u)\varphi(u,r)\,du}{\int \sigma_2(u)\varphi(u,r)\,du} \tag{5}$$

or, in the usual summation form,

$$\mathrm{SI} = \frac{\sum_{N=1}^{N} \sigma_{1N}\varphi_{MN}\Delta u_N}{\sum_{N=1}^{N} \sigma_{2N}\varphi_{MN}\Delta u_N} = \frac{\sum_{N=1}^{N} \sigma_{1N}\bar{\varphi}_{MN}\Delta u_N}{\sum_{N=1}^{N} \sigma_{2N}\bar{\varphi}_{MN}\Delta u_N} \tag{5'}$$

where use is made of normalized fluxes. The chosen σ_1 (and σ_2) can be any convenient one: the adoption of fission cross section is suggested by the fission counters which are of common use in such kind of experimental measurements.

As in 6.3., we can here note that the spectral index method does not allow to obtain an absolute value of the differences between two flux spectra. It can, however, give us informations about the riches of a spectrum in neutrons of one or another energy. The results of the calculations presented in Figs. 45, 46, and 47 are obtained using U-235 fission cross section as a reference.

6.5. Statistical weight

In an analysis of simulation possibilities made with purposes similar to ours, A.V. CAMPISE [7] states the following conditions in order to ensure transposition from test to full-scale-core systems:

1. Statistical weight of the test section (taken inside the test core) nearly identical to that of the full-scale system;
2. Neutron flux spectrum identical in the two cases;
3. Material composition nearly identical in the two cases.

Conditions 2 and 3 having already been included in our present analysis, condition 1 can be derived as follows.

Consideration of the Boltzmann equation for the diffusion of neutrons, together with its adjoint form, allows to determine the perturbation caused by a pure absorbing material introduced into a reactor at position r_o. The reactivity change in the total reactor is

$$\rho^*(r_o) = -\frac{\iint \sigma_a^*(u)\psi(u,\Omega,r_o)\varphi^*(u,\Omega,r_o)\,du d\Omega}{\iiint \frac{\chi(u)}{4\pi}\psi(u,\Omega,r)\nu\sigma_f(u,r)\varphi^*(u,\Omega,r)du d\Omega dr} \tag{6}$$

where

$\sigma_a^*(u)$ = functional variation in the absorption cross section of the pure absorbing material;
$\psi(u,\Omega,r)$ = neutron importance;
$\varphi^*(u,\Omega,r)$ = perturbed flux;
$\chi(u)$ = relative fission spectrum;
ν = average number of neutrons per fission;
$\sigma_f(u,r)$ = fission cross section.

If we put

$$\delta\sigma_a = -\iint \sigma_a^*(u)\psi(u,\Omega,r_o)\varphi^*(u,\Omega,r_o)\,du d\Omega \tag{7}$$

and

$$I = \iiint \frac{\chi(u)}{4\pi}\psi(u,\Omega,r)\nu\sigma_f(u,r)\varphi^*(u,\Omega,r)du d\Omega dr \tag{8}$$

then (6) becomes

$$\rho^*(r_o) = \frac{\delta\sigma_a}{I} \tag{9}$$

In (7), $\sigma_a^*(u)$ is a property of the inserted material only and the product

$$\psi(u,\Omega,r_o)\varphi^*(u,\Omega,r_o) = \text{S.W.}$$

is the statistical weight of the reactor system at the point r_o and lethargy u; S.W. is a dependent variable of the reactor system. The expression for I is the total rate of production of neutron importance. The reactivity change due to a pure absorber in the test core is, from (9),

$$[\rho^*(r_o)]_{TC} = \frac{[\delta\sigma_a]_{TC}}{I_{TC}} \tag{9'}$$

and that in the full-scale core (same absorber at the same location r_o):

$$[\rho^*(r_o)]_{FC} = \frac{[\delta\sigma_a]_{FC}}{I_{FC}} \tag{9"}$$

If the S.W.-s are the same in the two cores, then, from (7)

$$[\delta\sigma_a]_{TC} = [\delta\sigma_a]_{FC} \tag{10}$$

and from (9'), (9")

$$[\rho^*(r_o)]_{FC} = [\rho^*(r_o)]\frac{I_{TC}}{I_{FC}} \tag{11}$$

While (11) allows to obtain $[\rho^*(r_o)]_{FC}$ once $[\rho^*(r_o)]_{TC}$ has been measured and the two systems are known (I_{TC} and I_{FC} core properties of the reactors only and not of the experiments to be performed on them, so that they can be calculated once for all), equality (10) corresponds to stating

$$[\varphi^*(u,\Omega,r_o)\psi(u,\Omega,r_o)]_{TC} = [\varphi^*(u,\Omega,r_o)\psi(u,\Omega,r_o)]_{FC} \qquad (10')$$

i.e. to having in the two reactors identical statistical weights in correspondence of the same values of r_o, which in practice will take on the values of the region of interest as explained at the beginning of the present section.

From what precedes, one sees that the calculations to carry out will be of two kinds:

a) determination of the S.W.-s, through the neutron importance (radius-wise) and the knowledge of the flux perturbed by the introduction of a chosen absorber (radius-wise);

b) determination of the ρ^*-s, through the $\delta\sigma_a$-s and I-s.

The calculations under a) ask for the solution, respectively, of the adjoint Boltzmann's equation (ψ) and the Boltzmann's equation for the perturbed flux (φ^*). On the other hand, the calculations under b) will enable us, through (11), to foresee the outcome of a particular (and important) class of experiments.

In the case of an ideal absorber for which $a = \frac{1}{v}$, (7) can be written

$$\delta\sigma_a = -\frac{a}{E_o^{1/2}} \int\int \psi(u,\Omega,r_o)\varphi^*(u,\Omega,r_o)e^{u/2}du d\Omega \qquad (7')$$

where:

a = constant;

E_o = upper limit of energy (10 MeV in the YOM scheme).

Another way will be to consider the absorber made up of, say, two isotopes, namely U-238 and Pu-239 in a determined ratio (for instance 5:1 or, in general, the y ratio of the reactor under examination); upon collapsing the 16-groups in one group by standard methods, a constant σ_a for the chosen sample can be assumed.

Apart from practical considerations (perturbation theory calculations asking for codes more complex than the ones which can take care of the methods examined in 6.1, 6.2, 6.3 and 6.4), the use of statistical weight as a comparison standard looks very attractive. This method is equivalent to stating that two systems can be considered equivalent if they react in the same way to the same external stimulus. Again the validity of this criterion is to be weighted against particular experimental measurements and used together with one or the other of the already mentioned methods.

7. RESULTS OF CALCULATIONS BASED ON COMPARISON METHODS

The philosophy of comparison criteria has been briefly outlined in the introductory part of section 6, followed by a description of some possible methods. With the help of the code described in Appendix B, all these methods have been applied in order to test groups 16, 26 and 36 of the already discussed reactors.

In Figures 27, 28 and 29 normalized fluxes of some reactors inside each group are showed as a function of lethargy. The spectra here represented refer to the center of the examined reactors ($R = 0$), but the values of the fluxes vary so little along the radius (for at least 20-30 cm) that these representations can be assumed valid over the volume of interest.

In Figures 30 to 44, mismatch parameters (PPP) are given. The results of the other methods are not shown, either because the informations are similar to the ones obtained from PPP, or because of the reasons already given in section 6. Besides the variations of σ_f, σ_c, σ_a as a function of radius, are too small to be as much representative as the variations of PPP.

An exception is made for the spectral indexes (Figures 45, 46, 47), due to the fact that they indicate where a spectrum is softer or harder than a reference one.

Let us examine Fig. 30 relative to reactors of group 16 for which $y = 4$ (similar results can be derived from Fig. 35 and Fig. 40, respectively for $y = 5$ and $y = 6$). It is apparent that the thinner the driver, the smaller is the corresponding value of PPP for the same R. The use of a y_2 smaller than y_1 does not seem, in general, to improve the situation as given by y_2 equal to y_1: anyway, this trend does not look very regular. The introduction of a decoupler usually acts in the sense to produce a bigger PPP in comparison with the same y_1-y_2 case without decoupler. Here again the trend does not seem very regular.

All in all it is possible to say that the more dilute the reactors, the more they are easy to compare. For a given thickness of the driver and for every y_1, PPP increases when the ratio y_2/y_1 diminishes, but always taking on limited values for small driver thicknesses (5-10 cm). For big driver thicknesses, PPP diminishes a little when diminishing the ratio y_2/y_1.

Another way to compare reactors has been attempted through the substitution of absorbing materials at different places of the reactors. The results of these calculations are shown in Figures 48 and 49 for reactors of group $y_1 = 5$. Fig. 48 gives the reactivity variations when $a\frac{1}{v}$ absorber is used, and Fig. 49 the reactivity variation when the absorber is natural Boron. In a general way, one can say that the information so obtained show the same trends as the PPP.

8. CONCLUSIONS AND FURTHER WORK

While not absolutely conclusive, the results of the PPP calculations allow to say:

1) the simulation of a reactor of the type of Fig. 1 a with one of the type of Fig. 1 b (or 1 c) is possible from the flux spectra point of view in a central part about 15-20 cm radius;

2) the difference between a zoned reactor and a full scale one stays almost constant over about 10 cm radius, which allows to consider the 10 cm radius volume as a volume of "constant difference".

3) the influence of a decoupler is not very clear and should require further investigations.

Due to the extensive kind of investigations needed in order to examine the possible criteria for comparing flux spectra of similar reactors, it would have been advisable to examine a number of cases more numerous than the ones here considered. Only in such a way it will be possible to see the real influence of driver thickness, decoupler thickness and composition, y_2/y_1 ratio and so on. It is hoped that, in so doing, it would be possible to understand why, like in some case of group 36, the value of PPP decreases, instead of increasing, as a function of radius.

ACKNOWLEDGMENTS

This work has been accomplished during the time the author was assigned at I.A.R. (Institut für Angewandte Reaktorphysik), Kernforschungszentrum, Karlsruhe. He wishes to express his appreciation to Prof. W. HAEFELE for all the stimulating discussions about the subject.

Particular thanks are given to Miss M.P. FERRANTI, who wrote the PPP code and operated it.

REFERENCES

[1] Edwards A.G., *The assessment of plutonium cross section data by fast integral experiments*, TRG Report 439 (D) (1963).

[2] Meneghetti D. and Ishikawa H., *Feasibility of Pu-239 - U-235 fueled cores to predict Pu-239 fueled core dimensions*, ANL-6559 (June 1962).

[3] Avery R., *Theory of coupled reactors*, A/conf/15/P/1858, Geneva (1858).

[4] Münzner W., *MGP - Multigruppenprogram*, Programmbeschreibung N. 11, Kernforschungszentrum Karlsruhe (1963).

[5] Yiftah S., Okrent D. and Moldauer P.A., *Fast reactor cross sections*, Pergamon Press (1960).

[6] Häfele W., *Rechenprogramm für den gasgekühlten schnellen Brüter*, INR-TA/R-PSB-Notiz, Nr. 27/60, Karlsruhe (1960).

[7] Campise A.V., *Advanced epithermal thorium reactor (A.E.T.R.) physics*, Seminar on the physics of fast and intermediate reactors, I.A.E.A., Vienna (1961). *Proceedings*, Vol. III, pages 335-354.

APPENDIX A

EVALUATION OF THE MEAN FREE PATH FOR TYPICAL HOMOGENEOUS FAST REACTORS

Using a one-group theory, the reciprocal of the mean free path is given by

$$\alpha = \sum_i n_i (\sigma_{tr})_i \text{ cm}^{-1} \qquad \text{(a)}$$

where n_i is the number of nuclei of the i-th kind present in the core of the considered homogeneous fast reactor, and $(\sigma_{tr})_i$ is the corresponding value of the transport cross section. The n_i values are given in Table 2 and the $(\sigma_{tr})_i$ values are as follows (ANL-5800, pag. 421):

	σ_{tr} (barns)
Pu-239	6.8
U-235	6.8
U-238	6.9
Fe-56	2.7
Na-23	3.3
O-16	3.5

Upon performing the simple calculation, the reciprocal values of α for oxide fueled reactors give, for the mean free paths, numerical results of $\lambda = \sim 7.05$ cm, almost constant for values of y going from 1 to 6. For comparison sake, one can see that the same calculations for the metal fueled reactors of Table 1 give for the mean free path numerical results of $\lambda = \sim 4.96$ cm almost constant for values of y going from 4 to 6.

APPENDIX B

P P P - PINCO PALLINO PROGRAM

1. Introduction

The Pinco Pallino Program (PPP) has been written in FORTRAN coding system for the IBM-7070 data processing system and is intended to supply several informations about nuclear reactors, as specified in what follows. The main purposes of PPP are to obtain complementary data beside the ones usually provided by reactors calculations programs and to enable the comparison among reactors of similar or different composition. While it has been established with reference to fast reactors as calculated in the frame of the 16 Group YOM set of cross sections, PPP can be used, upon performing slight modifications, for any kind of reactors and in connection with a different cross sections set.

The required input data consist of the group Cross Section Library and, radius wise, both neutron and adjoint fluxes as provided by multigroup calculations of the reactors of interest.

The output data are referred to particular points along the radius of the chosen reactors (considered in spherical geometry) and consist of the following:

a. normalized neutron flux
b. mismatch factor (Pinco Pallino Parameter)
c. average fission cross section
d. average absorption cross section
e. average capture cross section
f. spectral index
g. normalized adjoint flux
h. statistical weight

2. Input data

The calculation of the reactors of interest is based on a chosen group cross section set; the same set is introduced to supply the necessary numerical values of cross sections

(see 1-c, d, e, f, i). Only a limited number of isotopes are of use in the PPP, namely:

U-238
U-235
Pu-239
Pu-240
Pu-241

(to which U-233 and Th-232 could be added for completeness). When the results of calculations imply several isotopes, the above mentioned sequence is constantly adopted.

Neutron and adjoint fluxes can be furnished either through cards punched as a part of nuclear reactors calculations [see, for instance, MGP-Multigruppenprogram by W. Münzner (Karlsruhe Programmbeschreibung N. 11 [4])] or through card hand-punched from previous results. Due to the point-wise (radius-wise) results foreseen, neutron and adjoint fluxes are to be furnished point-wise (radius-wise), too. Reference to spherical geometry will simplify somewhat the calculations but will not limit the validity of the investigations. Anyway, the program holds when referring to other geometries if input data are consistent (for instance if taken rad us-wise or axis-wise in cylindrical geometry and x- or y- or z-dimension-wise in parallelepiped geometry), and the results are taken at the same points and along the same lines. In what follows, constant reference is made to spherical geometry.

3. Output data

The output data list has been presented sub 1). The printed results include the values of neutron and adjoint fluxes too (see paragraph 5 for presentation of listing).

Here follows a description of the quantities mentioned sub 1).

a. *Normalized neutron flux (PHINOR)*

Let be for a reactor R

N = number of energy groups considered ($N = 16$ in the YOM set);
M = point along the radius where the flux is calculated ($M = 0$ at the origin, taken at the center of the reactor; everywhere, M equals the distance from the origin to a considered point);
φ_{MN} = neutron flux as furnished from calculations at point M in the Nth group.

With reference to a fixed point M along the radius, we consider the sum

$$\sum_{N=1}^{N} \varphi_{MN} \cdot \Delta u_N = A_M \tag{1}$$

which can be written:

$$\sum_{N=1}^{N} \frac{100\, \varphi_{MN} \Delta u_N}{A_M} = 100 \tag{1'}$$

or

$$\sum_{N=1}^{N} \overline{\varphi}_{MN} \Delta u_N = 100 \tag{1''}$$

so defining

$$\bar{\varphi}_{MN} = \frac{100\,\varphi_{MN}}{A_M} = \frac{100\,\varphi_{MN}}{\sum_{N=1}^{N} \varphi_{MN}\Delta u_N} \tag{2}$$

$\bar{\varphi}_{MN}$ gives the normalized flux we are seeking.

b. *Mismatch Factor (Pinco Pallino Parameter — PPP)*

If we normalize the flux of a reactor R' for the same values of N and M used in 3.a, we obtain:

$$\bar{\varphi'}_{MN} = \frac{100\,\varphi'_{MN}}{A'_M} = \frac{100\,\varphi'_{MN}}{\sum_{N=1}^{N} \varphi'_{MN}\Delta u_N} \tag{2'}$$

From (2) and (2') we define a mismatch factor (PPP — Pinco Pallino Parameter) as:

$$\text{PPP} = \left[\frac{\sum_{N=1}^{N} (\bar{\varphi}_{MN} - \bar{\varphi'}_{MN})^2}{N} \right]^{1/2} \tag{3}$$

PPP is a function of the point M.

c. *Average fission cross section — (SIGMAF)*

This part of the program allows to ponderate, over the flux, the fission cross section of the isotopes listed sub 2) according to the expression

$$\bar{\sigma}_f(r) = \frac{\int \sigma_f(u)\varphi(u,r)du}{\int \varphi(u,r)du} \tag{4}$$

which is reduced to a ratio of summations:

$$\bar{\sigma}_{fM} = \frac{\sum_{N=1}^{N} \sigma_{fN}\varphi_{MN}\Delta u_N}{\sum_{N=1}^{N} \varphi_{MN}\Delta u_N} \tag{4'}$$

and is evaluated radius-wise. It can easily be seen that (4') is equivalent to

$$\bar{\sigma}_{fM} = \frac{1}{100} \sum_{N=1}^{N} \sigma_{fN}\bar{\varphi}_{MN}\Delta u_N \tag{4''}$$

d. *Average capture cross section — (SIGMAC)*

Similary to $\bar{\sigma}_f\ (r)$ we define:

$$\bar{\sigma}_c(r) = \frac{\int \sigma_c(u)\varphi(u,r)du}{\int \varphi(u,r)du} \tag{5}$$

which is reduced to a ratio of summations:

$$\begin{aligned} \bar{\sigma}_{cM} &= \frac{\sum_{N=1}^{N} \sigma_{cN}\varphi_{MN}\Delta u_N}{\sum_{N=1}^{N} \varphi_{MN}\Delta u_N} \\ &= \frac{1}{100}\sum_{N=1}^{N} \sigma_{cN}\bar{\varphi}_{MN}\Delta u_N \end{aligned} \tag{5'}$$

and is evaluated radius-wise.

e. *Average absorption cross section — (SIGMAA)*

From the definition:

$$\sigma_a = \sigma_f + \sigma_c \tag{6}$$

assumed valid for the cases under examinaton, we get:

$$\bar{\sigma}_{aM} = \bar{\sigma}_{fM} + \bar{\sigma}_{cM} \tag{6'}$$

f. *Spectral index — (SPECTRAL INDEX)*

The spectral index is the value of the ratio of two measures made by means of two detectors in the same neutron flux. We have then:

$$\mathrm{SI} = \frac{\int \sigma_1(u)\varphi(u,r)du}{\int \sigma_2(u)\varphi(u,r)du} \tag{7}$$

The chosen σ is here the fission cross section; the reference isotope can be any convenient one. In the usual summation form, we have:

$$\begin{aligned} \mathrm{SI} &= \frac{\sum_{N=1}^{N} \sigma_{1N}\varphi_{MN}\Delta u_N}{\sum_{N=1}^{N} \sigma_{2N}\varphi_{MN}\Delta u_N} \\ &= \frac{\sum_{N=1}^{N} \sigma_{1N}\bar{\varphi}_{MN}\Delta u_N}{\sum_{N=1}^{N} \sigma_{2N}\bar{\varphi}_{MN}\Delta u_N} \end{aligned} \tag{7'}$$

g. *Normalized adjoint flux — (PHIADNOR)*

Let N and M have the same meaning as in 3.A) and

ψ_{MN} = adjoint flux (neutron importance) as furnished from calculations at a point M in the Nth group.

With reference to a fixed point M along the radius, we consider the sum

$$\sum_{N=1}^{N} \psi_{MN} \cdot \Delta u_N = B_M \tag{8}$$

which can be written

$$\sum_{N=1}^{N} \frac{\psi_{MN} \cdot \Delta u_N}{B_M} = 1 \tag{8'}$$

or

$$\sum_{N=1}^{N} \bar{\psi}_{MN} \cdot \Delta u_N = 1 \tag{8''}$$

so defining

$$\bar{\psi}_{MN} = \frac{\psi_{MN}}{B_M} = \frac{\psi_{MN}}{\sum_{N=1}^{N} \psi_{MN} \cdot \Delta u_N} ; \tag{9}$$

$\bar{\psi}_{MN}$ gives the normalized adjoint flux (total neutron importance, constantly equal to one along the radius).

h. *Statistical weight (STAT. WEIGHT)*

The statistical weight function at a point r_o of a reactor is defined:

$$\mathrm{SW}(u) = \varphi^*(u, r_o)\psi(u, r_o) \tag{10}$$

or, with our usual notations,

$$\mathrm{SW}_M = \varphi^*_{MN} \cdot \psi_{MN} \tag{10'}$$

where

φ^*_{MN} = perturbed flux, as furnished by calculations, at a point M in the Nth group;

ψ_{MN} = adjoint flux (neutron importance), as furnished by calculations, at a point M in the Nth group.

Using for φ^*_{MN} and ψ_{MN} the corresponding values normalized according to 3.a) and 3.g), we have:

$$\mathrm{SW}_M = \bar{\varphi}^*_{MN} \cdot \bar{\psi}_{MN} \tag{10''}$$

As a first approximation and/or in the absence of φ^*_{MN}, one can use φ_{NM} as to get:

$$\mathrm{SW}_M = \sim \bar{\varphi}_{MN} \cdot \bar{\psi}_{MN} \tag{10'''}$$

4. Source statements

Here follows a description of the input statements supplied through punched cards.

1st statement (data presented in the form of fixed point constants)

M = number of points along the radius (M max = 50, but always a multiple of 5);
N = number of energy groups (N max = 40);
NC = number of reactors examined (NC = arbitrary; the code is such that of a given group of reactors, anyone of them can be used as a reference in the sense of 3.b), where the mismatch factor is defined);
MN = number of isotopes (MN max = 8; see point 2);
$N2$ = number of energy groups time two if the adjoint fluxes are used;
$L1$ = number of group constants one wants to take from GROUCO ($L1$ max = 4);
MP = 1 when neutron fluxes (and adjoint fluxes) are integrated over the lethargy and is then necessary to get them from

$$\varphi_N = \frac{\varphi_n}{\Delta u}$$

for the neutron fluxes and similarly

$$\psi_N = \frac{\psi_n}{\Delta u}$$

for the adjoint fluxes;
= 2 when the given neutron and adjoint fluxes are the real ones.

2nd statement (data presented in the form of floating point constants)
DELTA = Δu = group lethargy interval; Δu takes on as many values as the number of energy groups (N).

3rd statement (fixed point constants)
IG = group identification for neutron and adjoint fluxes ($N2$ values).

4th statement (fixed point constants; 5 alphabetic or numerical characters for each word)
MI = isotopes names according to the ones used in GROUCO (for instance Ub-235) and in the sequence to be constantly used here (see 2 above).

5th statement (fixed point constants; 5 alphabetic or numerical characters for each word)
LD = name of group constants according to the ones used in GROUCO, first σ_f then σ_c;
LGS = name of the Neutron Cross Section Set introduced on the multigroup program used to calculate the reactors here examined (3 words of 5 characters each: of lately, the identification for LGS was 16-GRYOMSETOO5.

6th statement (floating point constants)
R = distance value (in cm), from the center, of each of the M points chosen along the radius.

7th statement (fixed point constants)
NR = I reactor name (3 words of 5 alphanumerical characters);
II reference reactor name (1 word of 5 alphanumerical characters);
III = 1: fluxes cards hand punched;
= 2: fluxes cards automatically punched in the course of the MGP calculations.

8th statement (floating and fixed point constants)

This statement refers to the cards supplying fluxes data. These cards, either hand punched or funished through the MGP [4], must conform to the FORMAT of MGP itself, and contain:

1 — value of the radius at which the flux is to be taken (floating point constant)
2 — value of the flux (floating point constant)
3 — value of the current (this data being of no interest for PPP, any number will do, provided it be of the floating constant type)
4 — group number followed by:
0 if one is dealing with neutron fluxes
1 if one is dealing with adjoint fluxes
5 — identification of the reactor.

NOTE — Statements No. 7 and 8 must be repeated for every reactor which has to be considered.

5. Results listing

The results listing for one reactor takes four pages. The first line of each page gives the name of the considered reactor (for instance, REACTOR SKIN 52) and the one of the reference reactor (for instance, REFERENCE 36). The second line of each page gives the values in cm of the distance from the center to the point examined along the radius: these distances are used as heading for all the following results.

The first page contains:

a. Table of neutron fluxes as supplied to the program,
b. Table of normalized neutron fluxes,
c. Pinco Pallino Parameters (mismatch factors).

The second page contains:

a. Table of $\overline{\sigma_f}$, average fission cross section; each line of the table refers to a different isotope in the sequence of number 2) above.
b. Table of $\overline{\sigma_c}$, average capture cross section, a line for each isotope.
c. Table of $\overline{\sigma_a}$, average absorption cross section, a line for each isotope.
d. Table of spectral indexes, again a line for each isotope; this table is headed by the name of the reference isotope.

The second page bis contains:

Tables of spectral indexes when they are calculated with reference to several isotopes and the second page is not enough. The reference isotope is indicated at the bottom of each table.

The third page contains:

a. Table of adjoint fluxes as supplied to the program.
b. Table of normalized adjoint fluxes.

The fourth page contains:

Table of statistical weights.

6. Search of errors in the inputs of each reactor

The calculations performed with the aid of PPP ask for quite a large number of cards. In order to avoid, or diminish, the possibility of errors in the input data, a special error search testing has been introduced. When the program stops because a card is wrong, this error search testing allows to find where the mistake is.

1 — The cards, where the needed fluxes are listed, are counted after having been read. Once the reading for a particular reactor is over, a check is made about their number. If the number is wrong, the machine prints ERROR IN INPUT OF ... (name of the reactor).

2 — The 5th figure in every card is the reactor identification. If such a figure does not stay the same for all the cards relative to a single reactor, the machine prints ERROR IN CARD ... (number of the irregular card). When this happens either the 5th figure is missing, or wrong, or a card from another reactor has been inadvertently inserted.

7. Number of storage memories and calculation time

A. The total number of storage memories needed for the entire program amounts to 5126. Of these, 1286 are used for the various statements and 3840 for input and output data. These latter are so distributed:

PHI	400
PHINOR	400
PHIAD	400
PHIADN	400
SIGMA	1280
SIGMAF	40
SIGMAC	40
SIGMAA	40
SPECTRAL INDEX	320
STAT WEIGHT	200
PPP	5
Various	315
Total	3840

B. The reading of the values of σ_f and σ_a (see section 3 c, d and e) takes 20 seconds. The entire calculation relative to one reactor (see section 3) takes 20 seconds when supplying the computer with hand punched card. In the case of having the computer itself picking up the necessary cards among the ones obtained from preceding MGP results, the time will be longer (but saving time on the hand-punching business): anyway, little can be said on this respect as experience is lacking.

C. When M (see 3,a and 4, 1st statement) is greater than 5, it is possible to repeat the comparison for any other group of five points, if so it is desired.

FIG. 1. Geometry of the reflected metal and oxide-fueled fast reactors studied

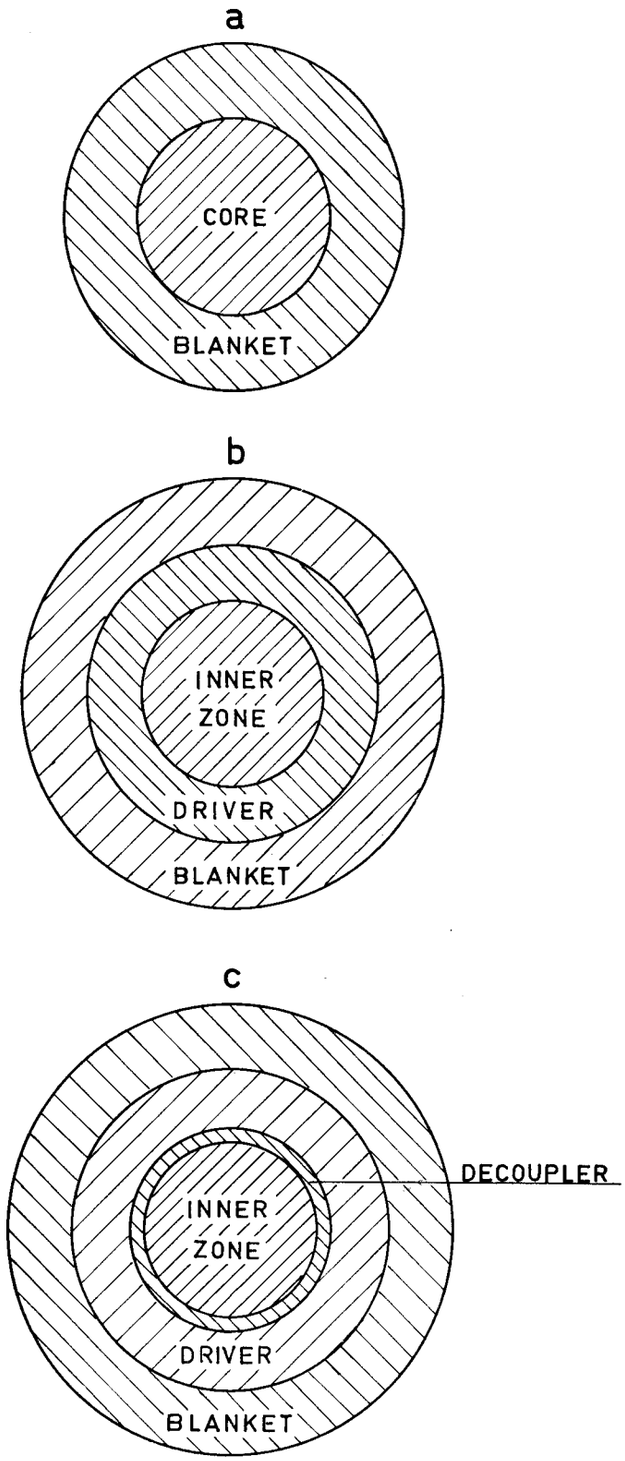


FIG. 2_Critical dimensions and critical masses of reflected spherical two-region core oxide-fueled fast reactors

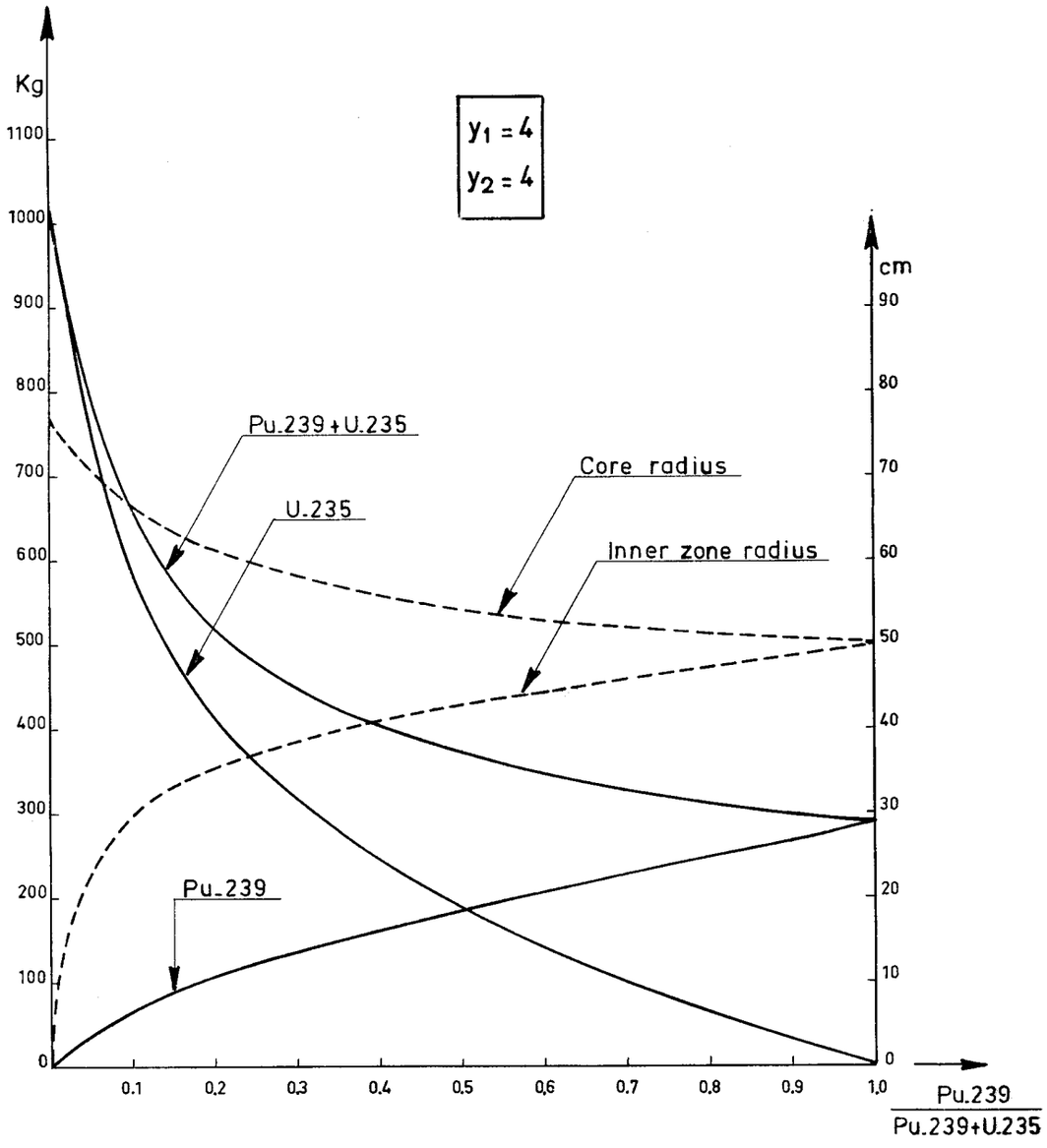


FIG 3_ Critical dimensions and critical masses of reflected spherical two-region core oxide-fueled fast reactors

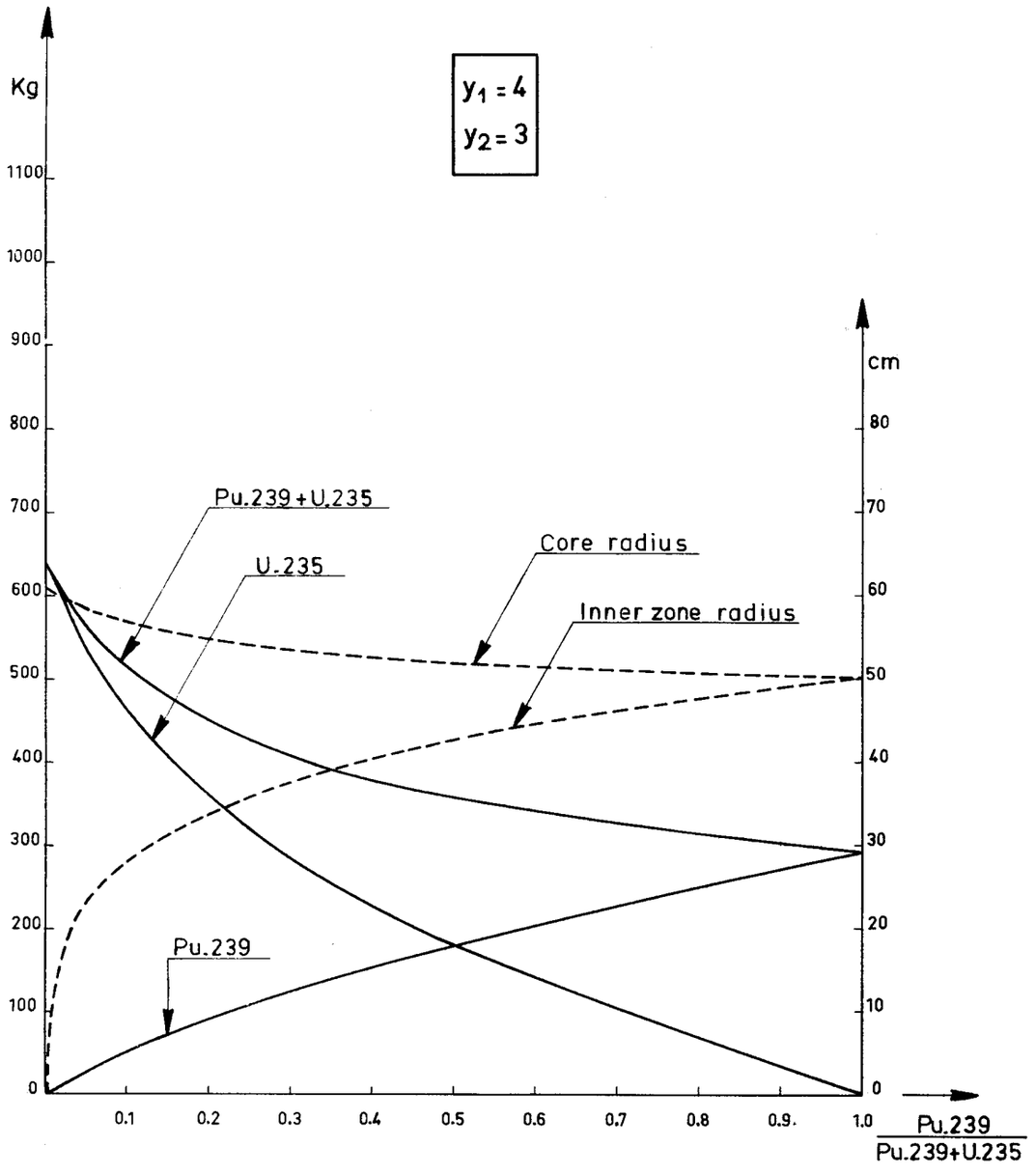


FIG. 4_Critical dimensions and critical masses of reflected spherical two-region core oxide-fueled fast reactors

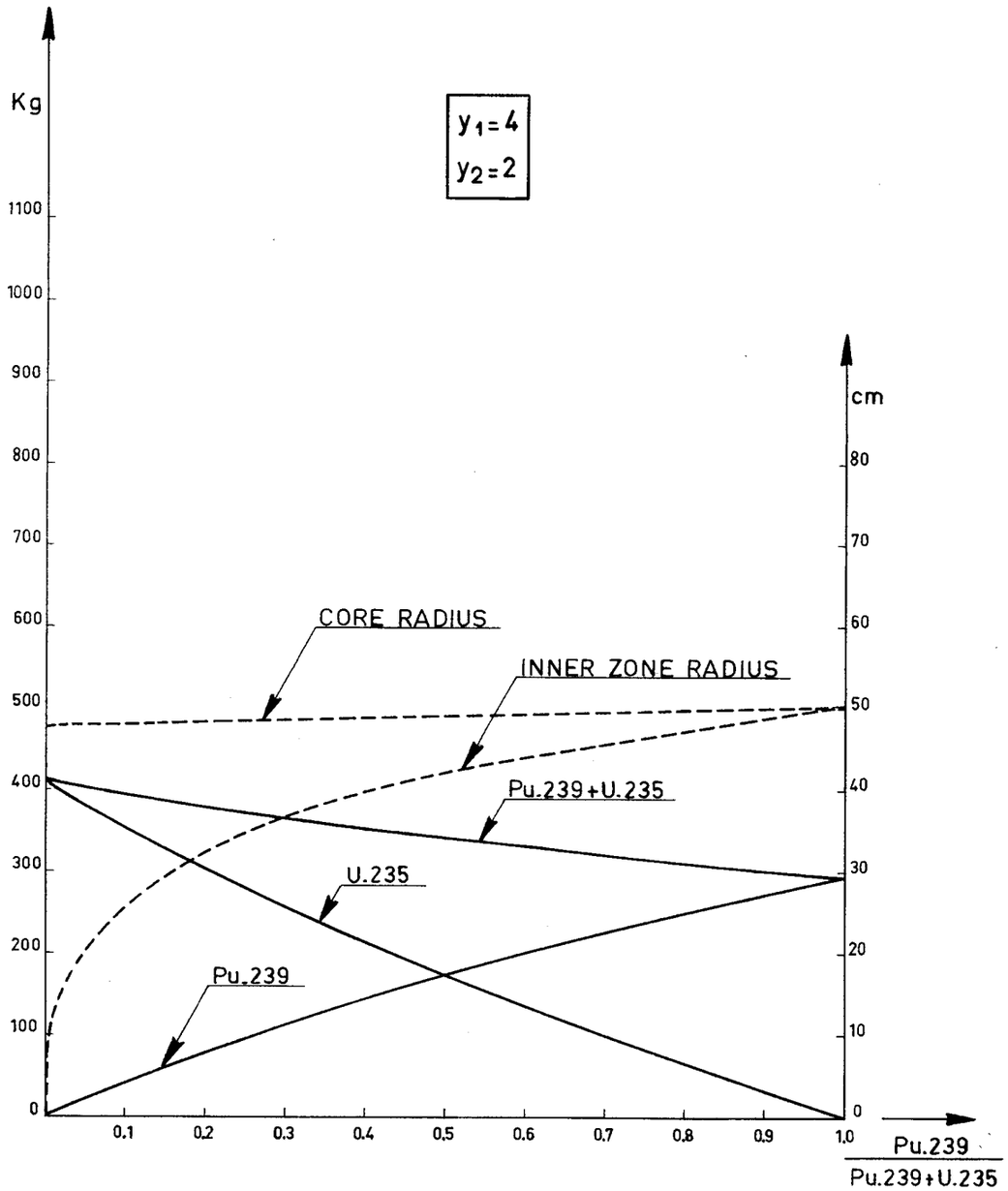


FIG. 5 _ Critical dimensions and critical masses of reflected spherical two-region core oxide-fueled fast reactors

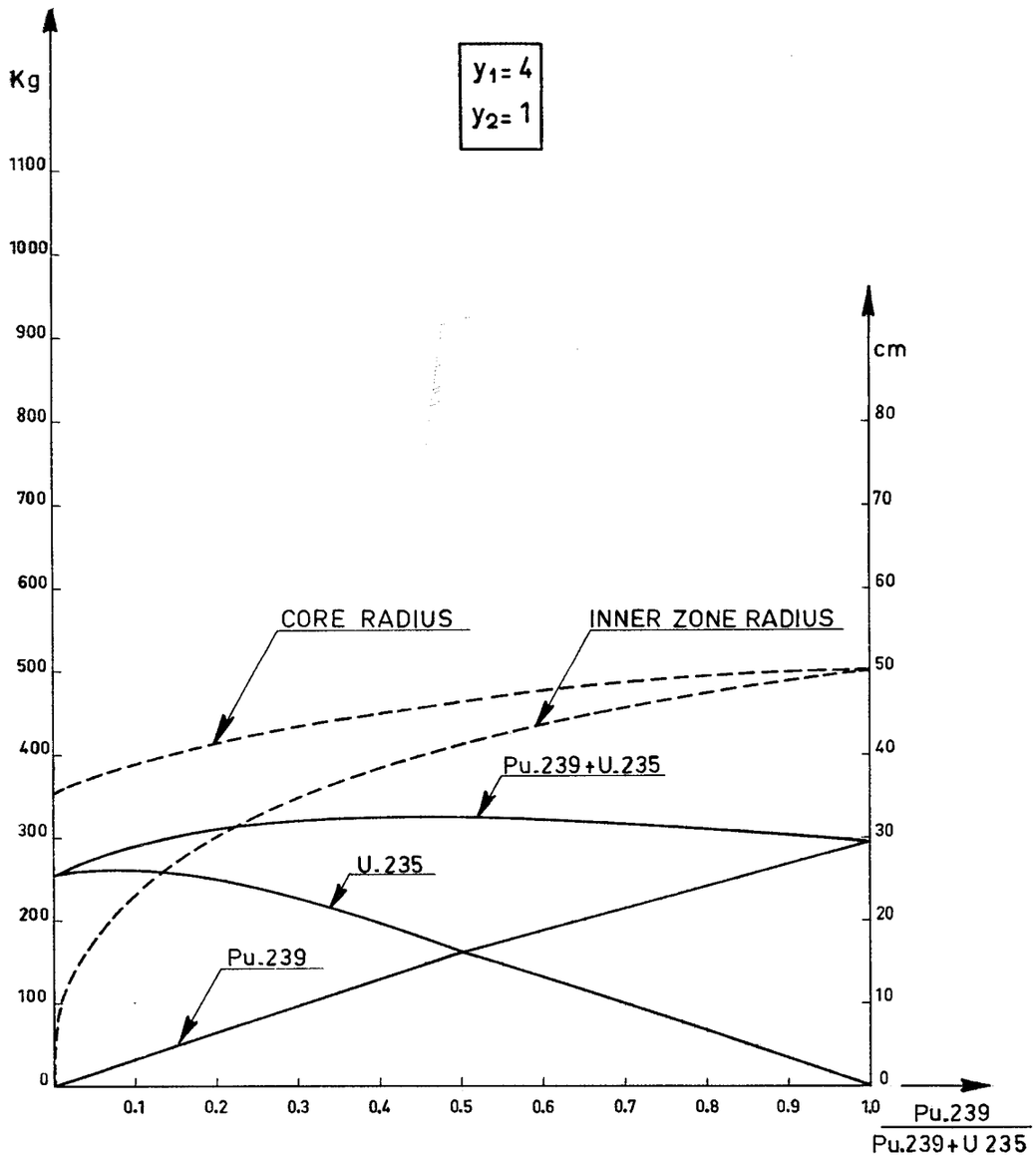


FIG. 6 - Critical dimensions and critical masses of reflected spherical two-region core oxide-fueled fast reactors

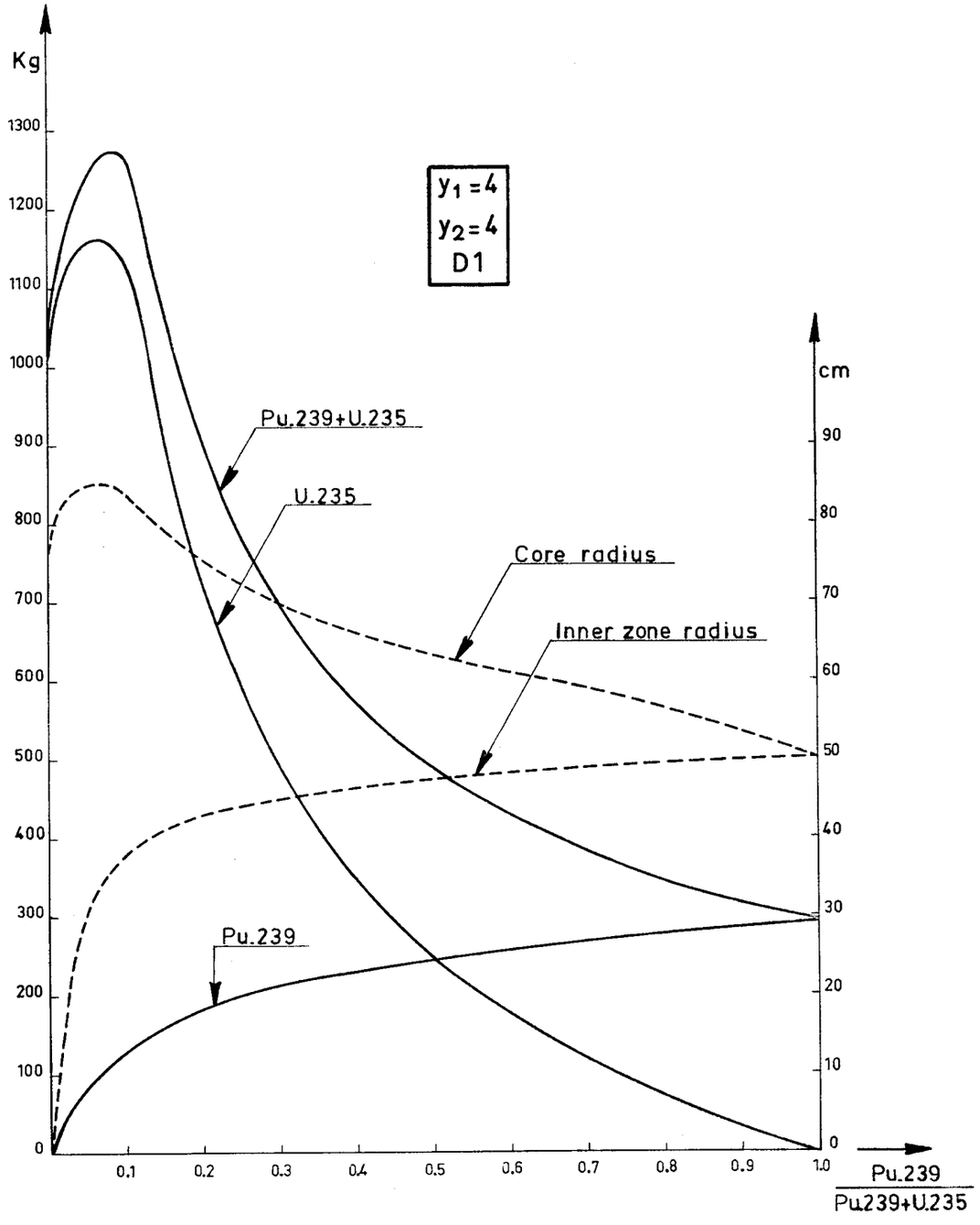


FIG. 7 - Critical dimensions and critical masses of reflected spherical two-region core oxide-fueled fast reactors

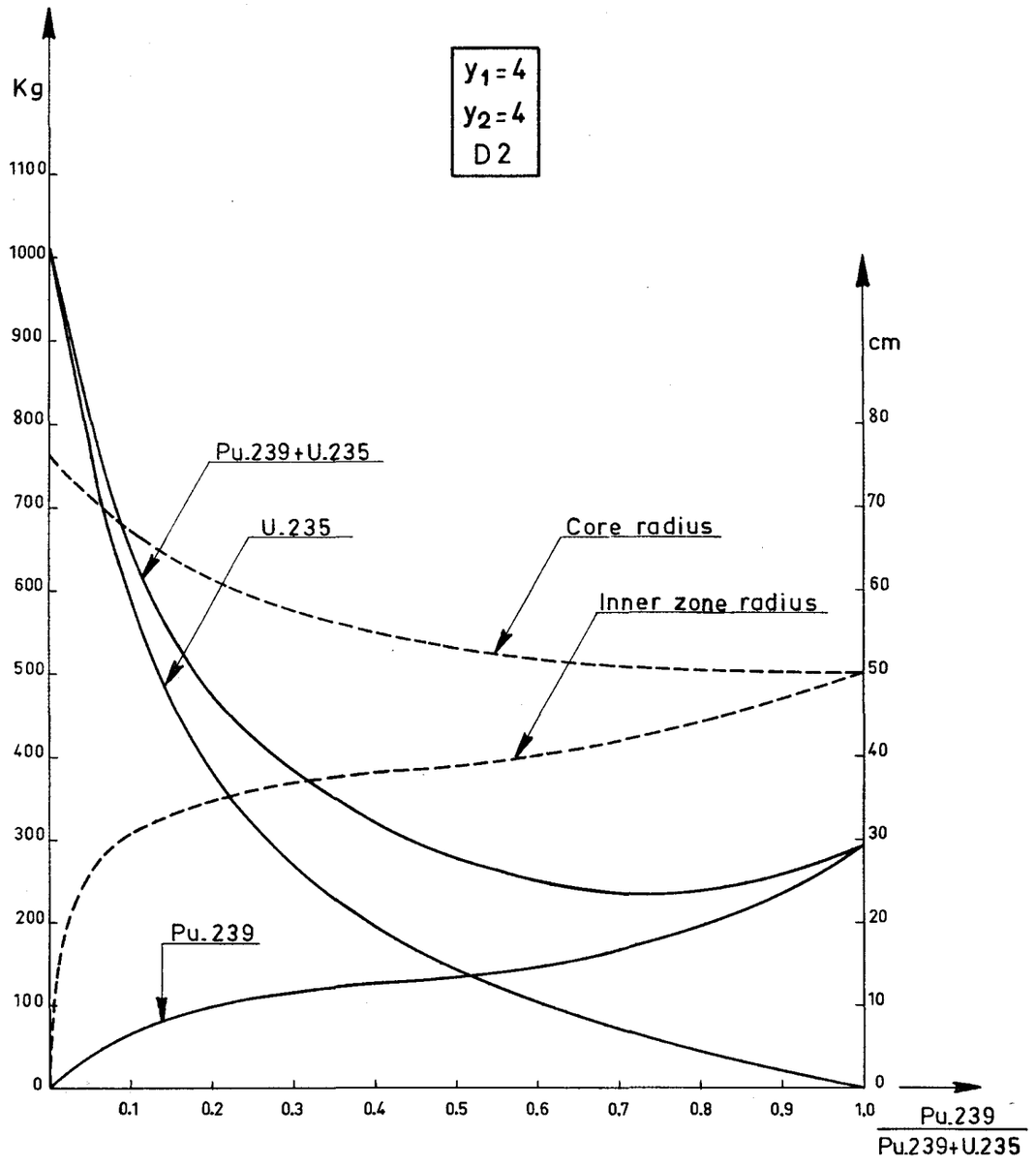
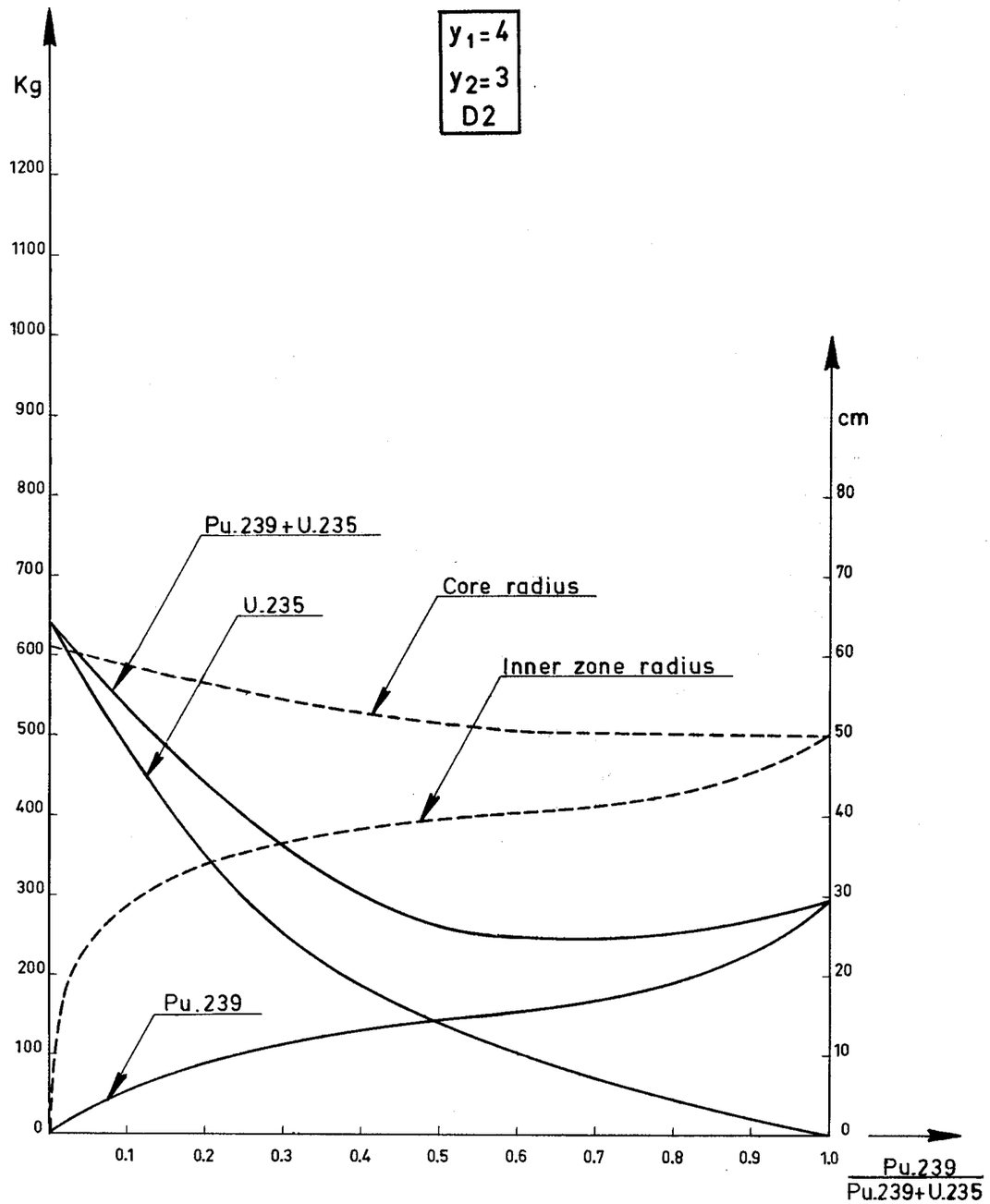


FIG. 8 _ Critical dimensions and critical masses of reflected spherical two-region core oxide-fueled fast reactors



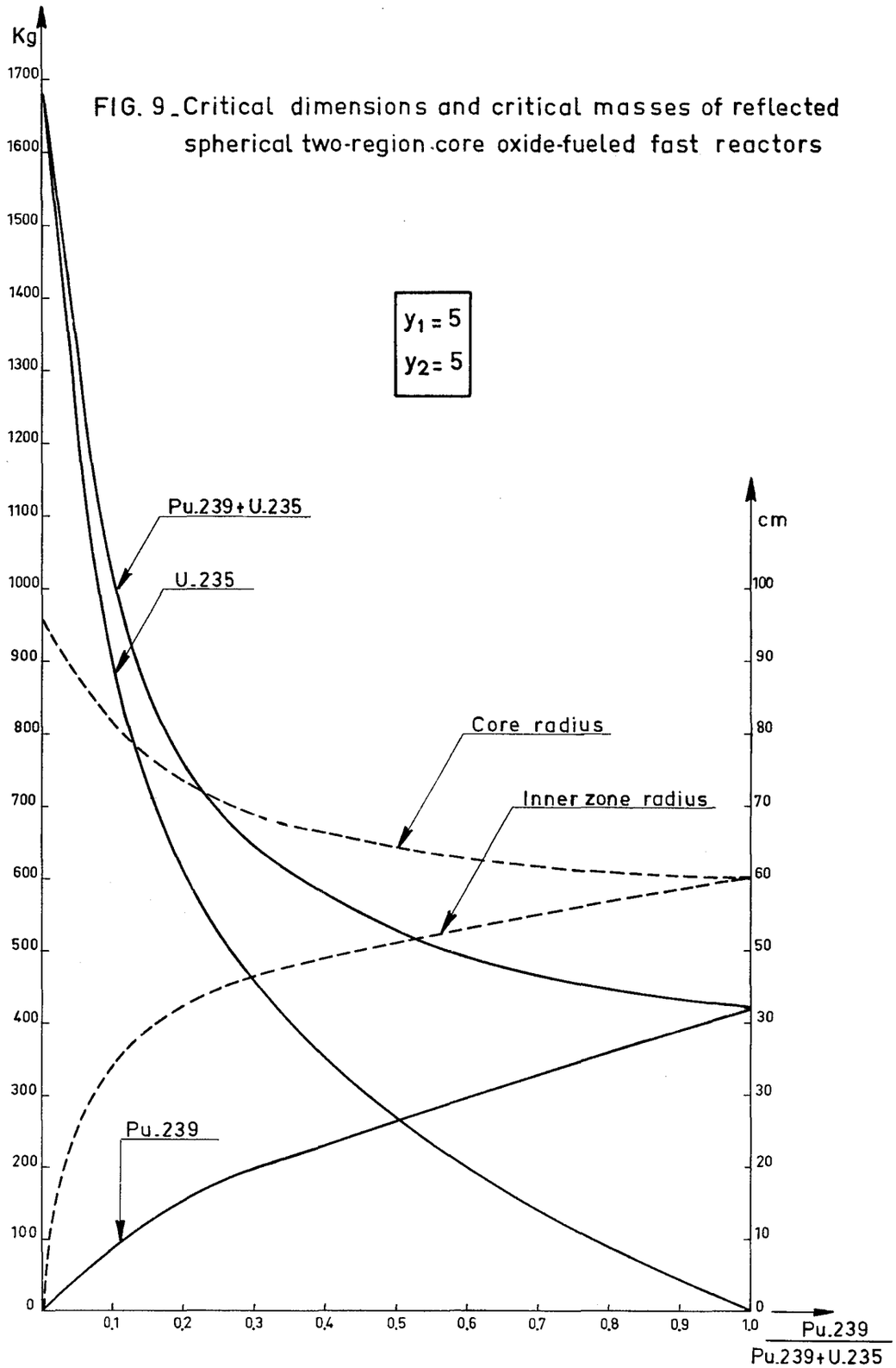


FIG. 9 _Critical dimensions and critical masses of reflected spherical two-region core oxide-fueled fast reactors

FIG. 10_ Critical dimensions and critical masses of reflected spherical two-region core oxide-fueled fast reactors

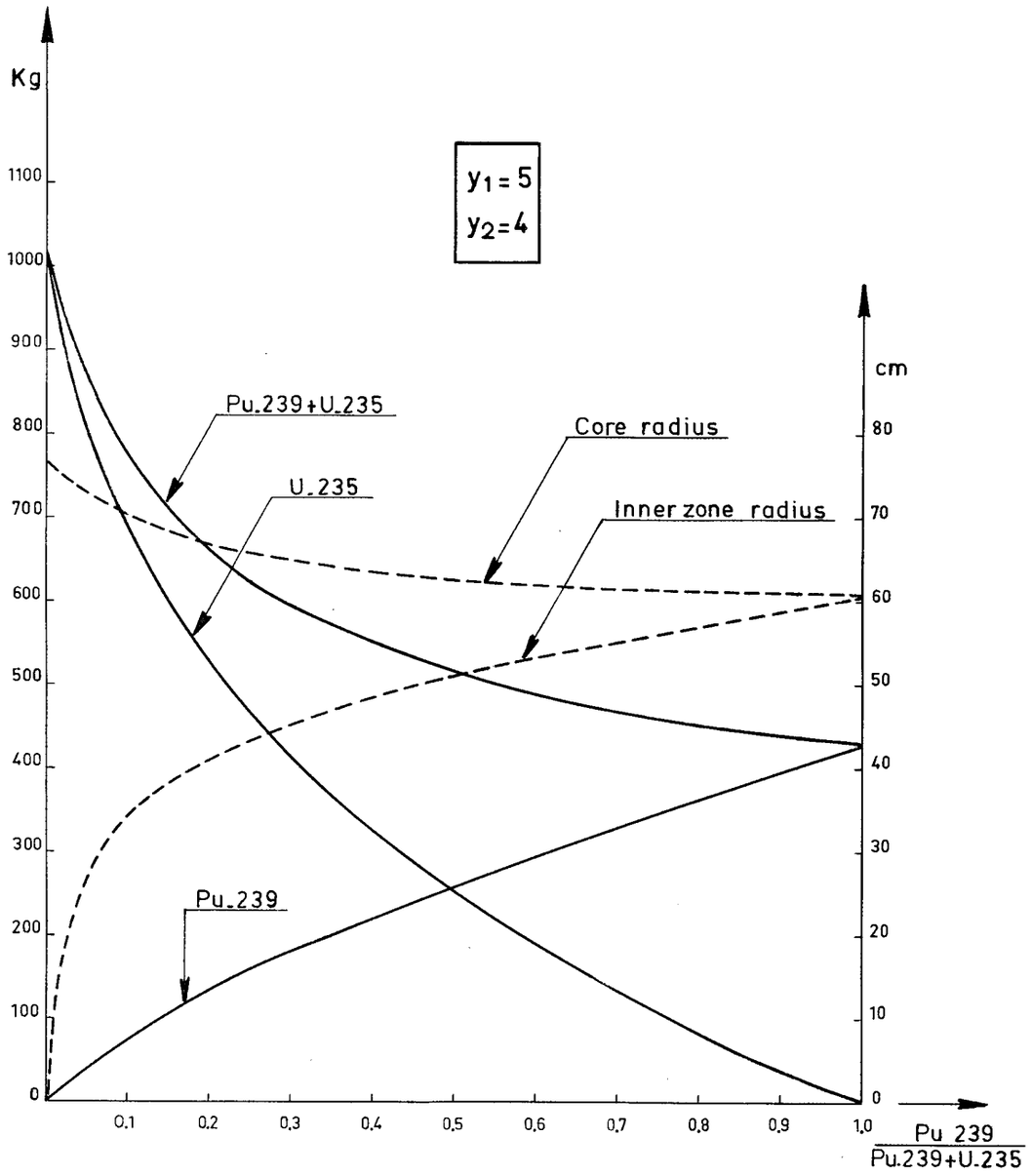


FIG. 11_Critical dimensions and critical masses of reflected spherical two-region core oxide-fueled fast reactors

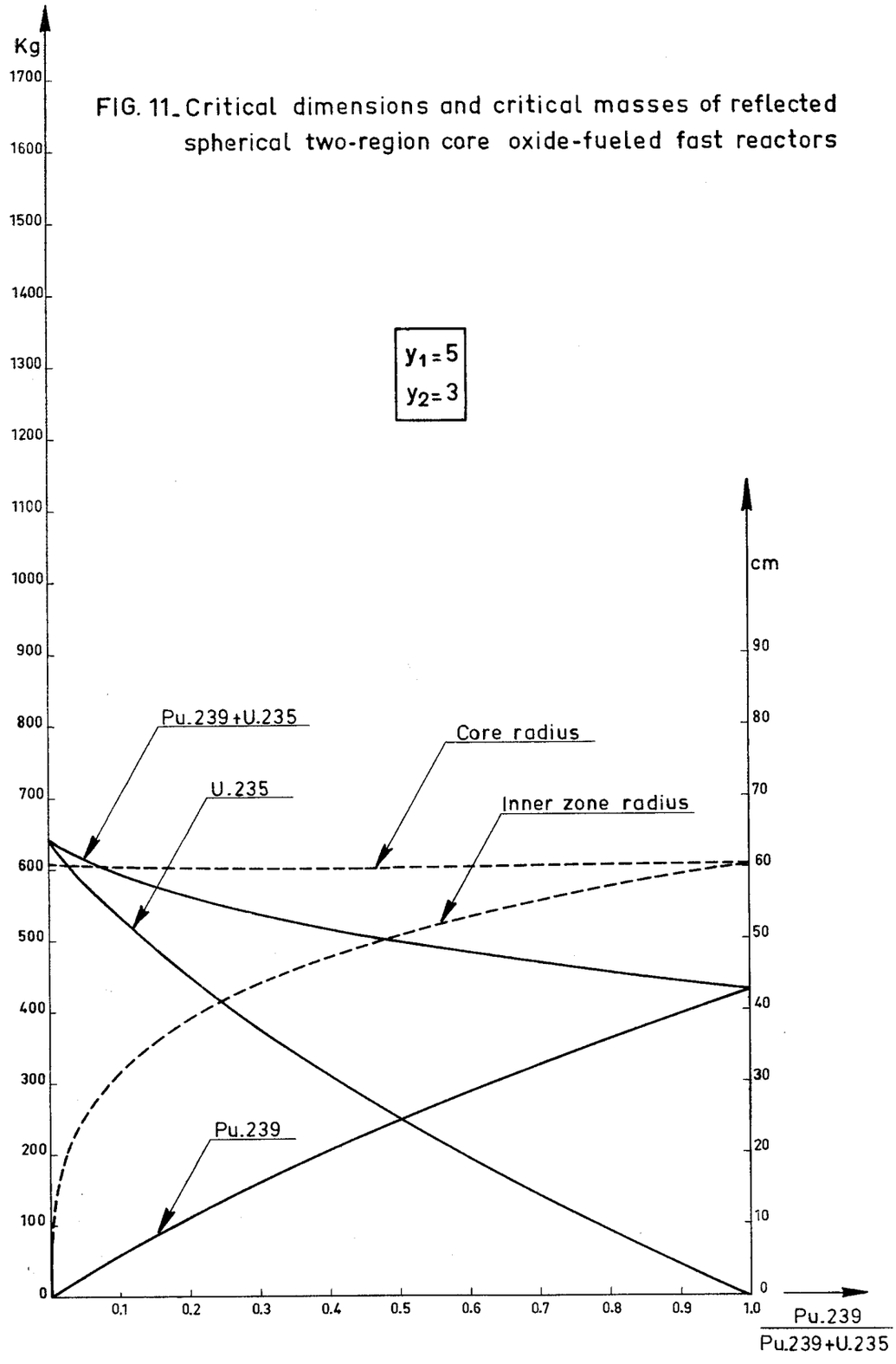


FIG. 12_Critical dimensions and critical masses of reflected spherical two-region core oxide-fueled fast reactors

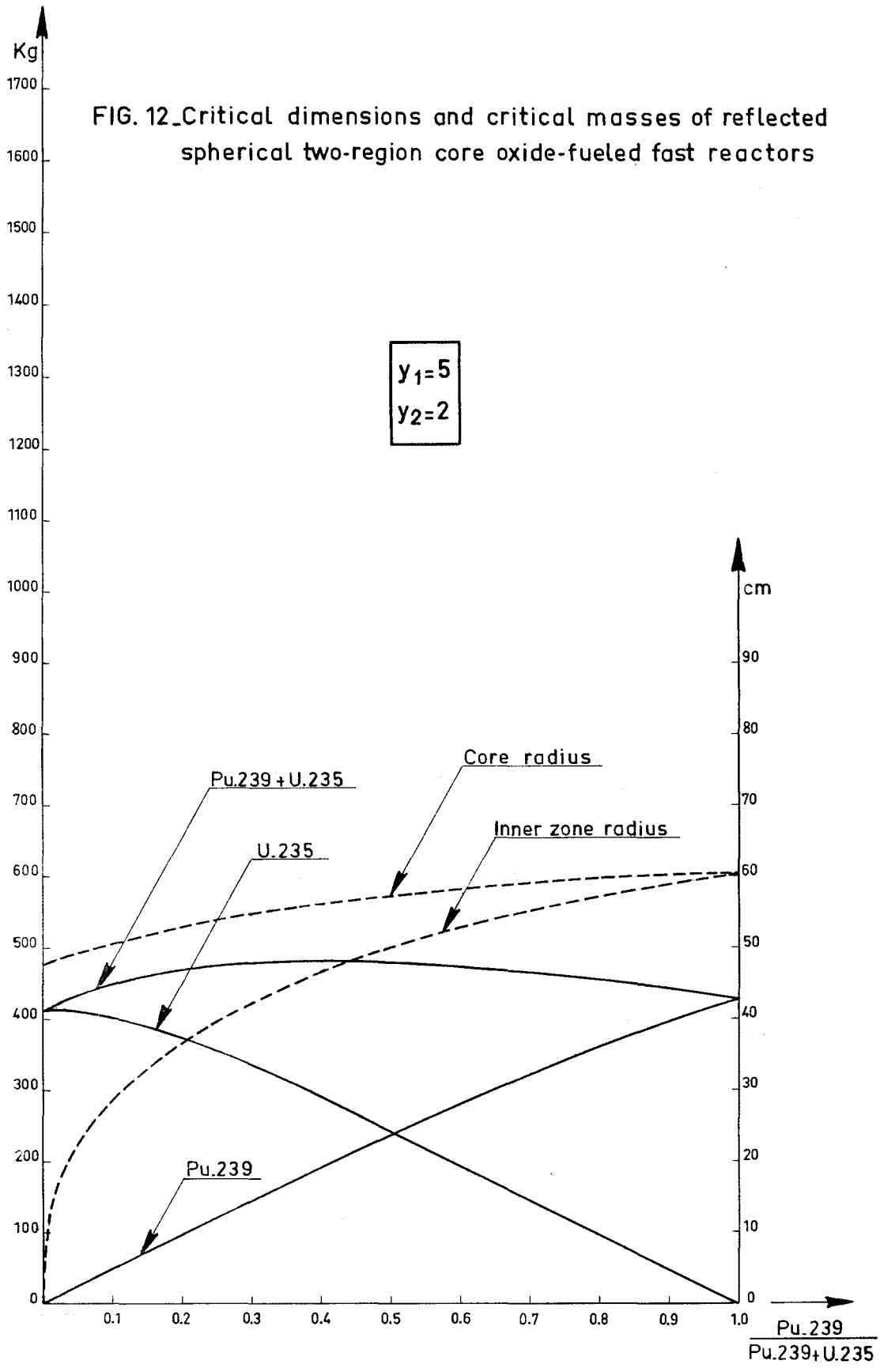
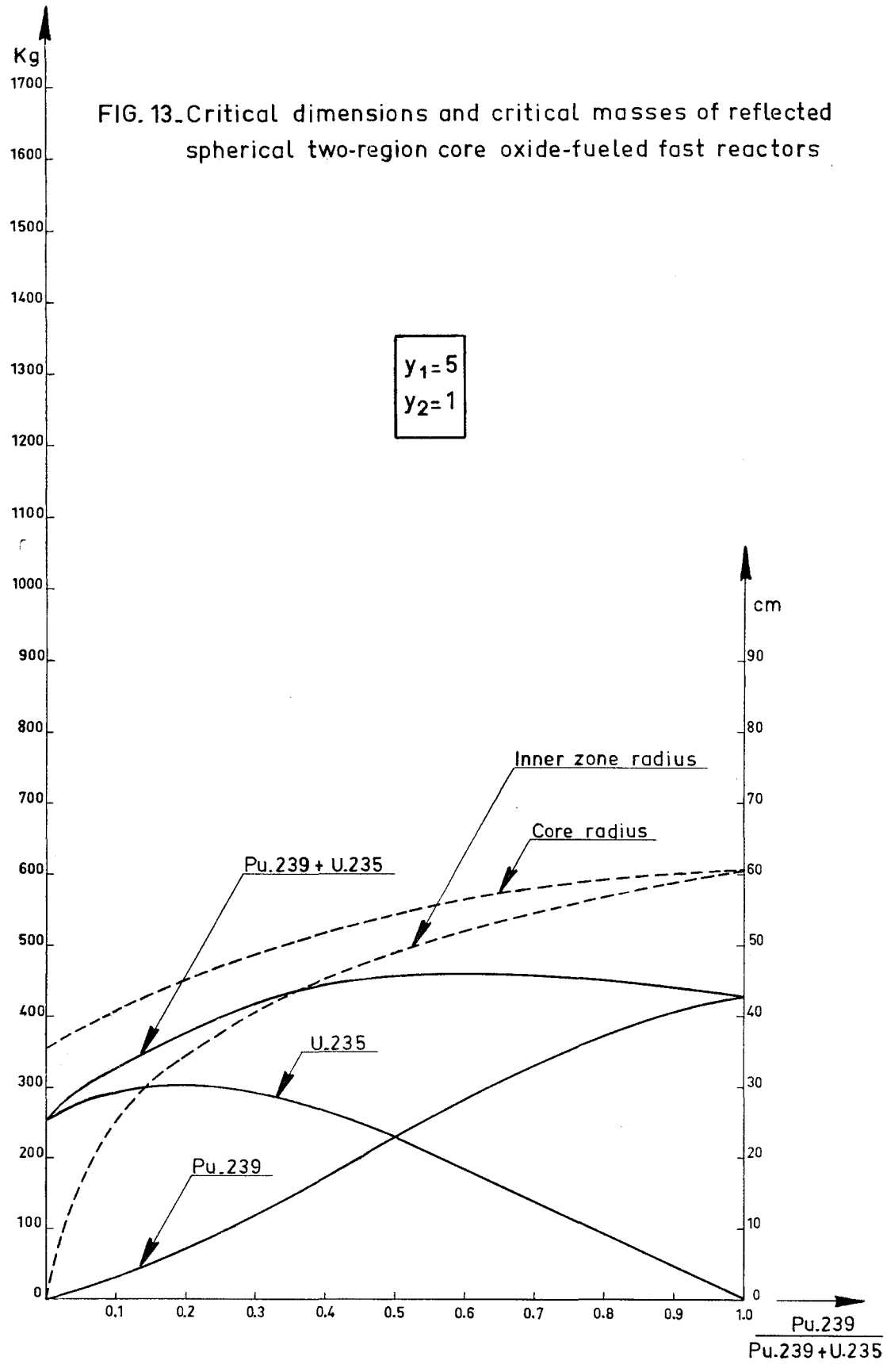


FIG. 13_Critical dimensions and critical masses of reflected spherical two-region core oxide-fueled fast reactors



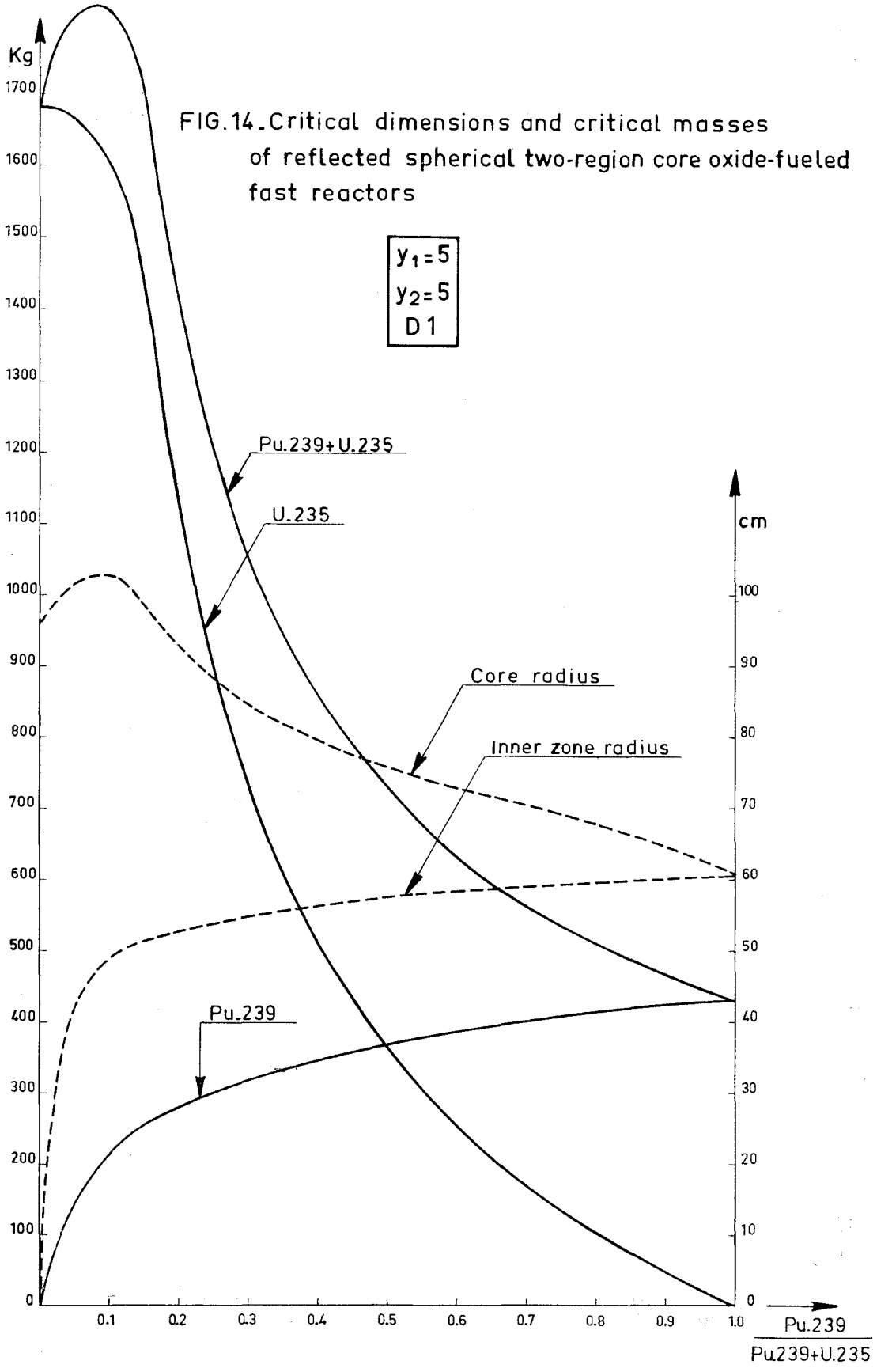


FIG.14.Critical dimensions and critical masses of reflected spherical two-region core oxide-fueled fast reactors

FIG. 15_Critical dimensions and critical masses of reflected spherical two-region core oxide-fueled fast reactors

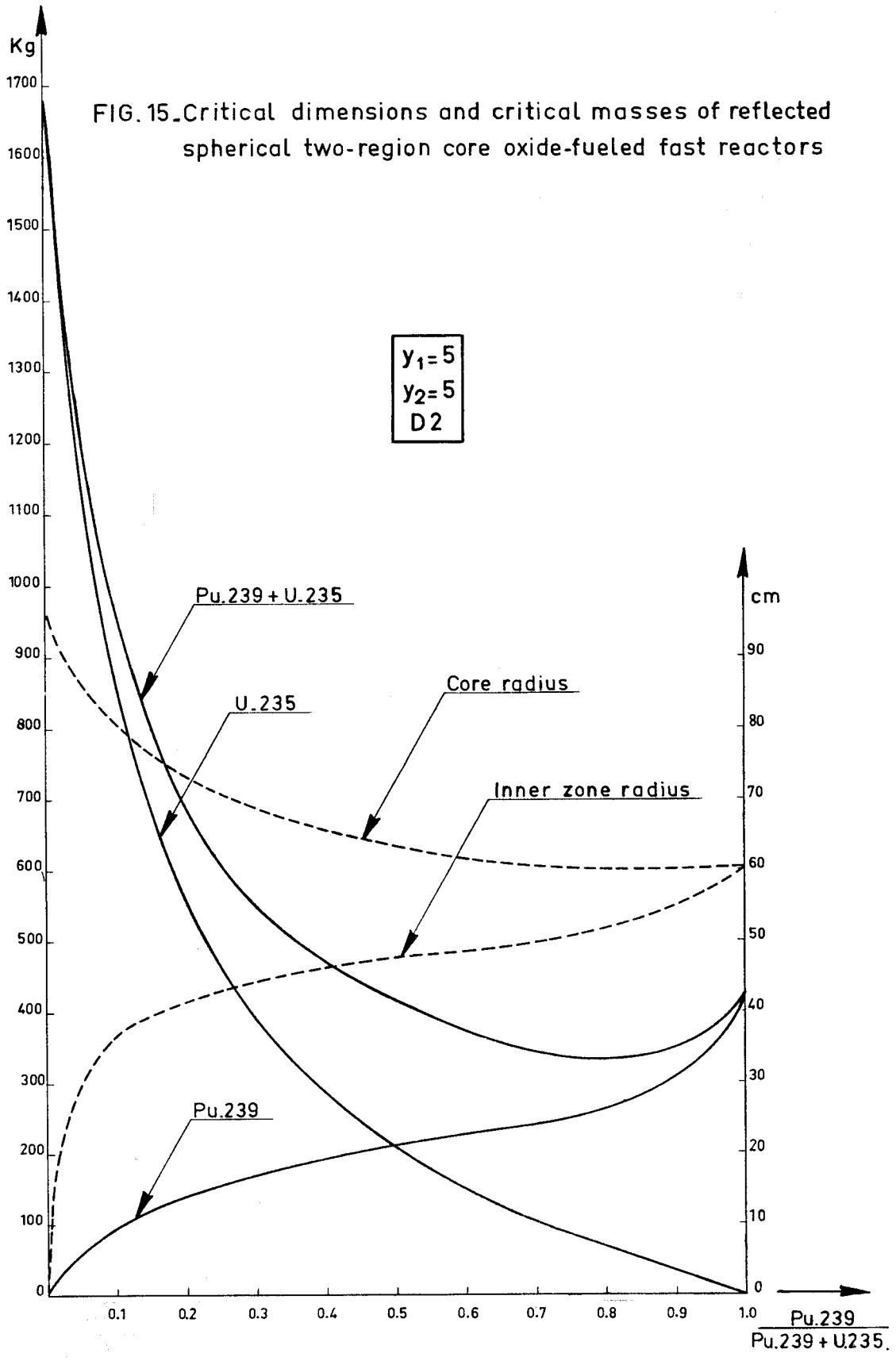


FIG. 16_Critical dimensions and critical masses of reflected spherical two-region core oxide-fueled fast reactors

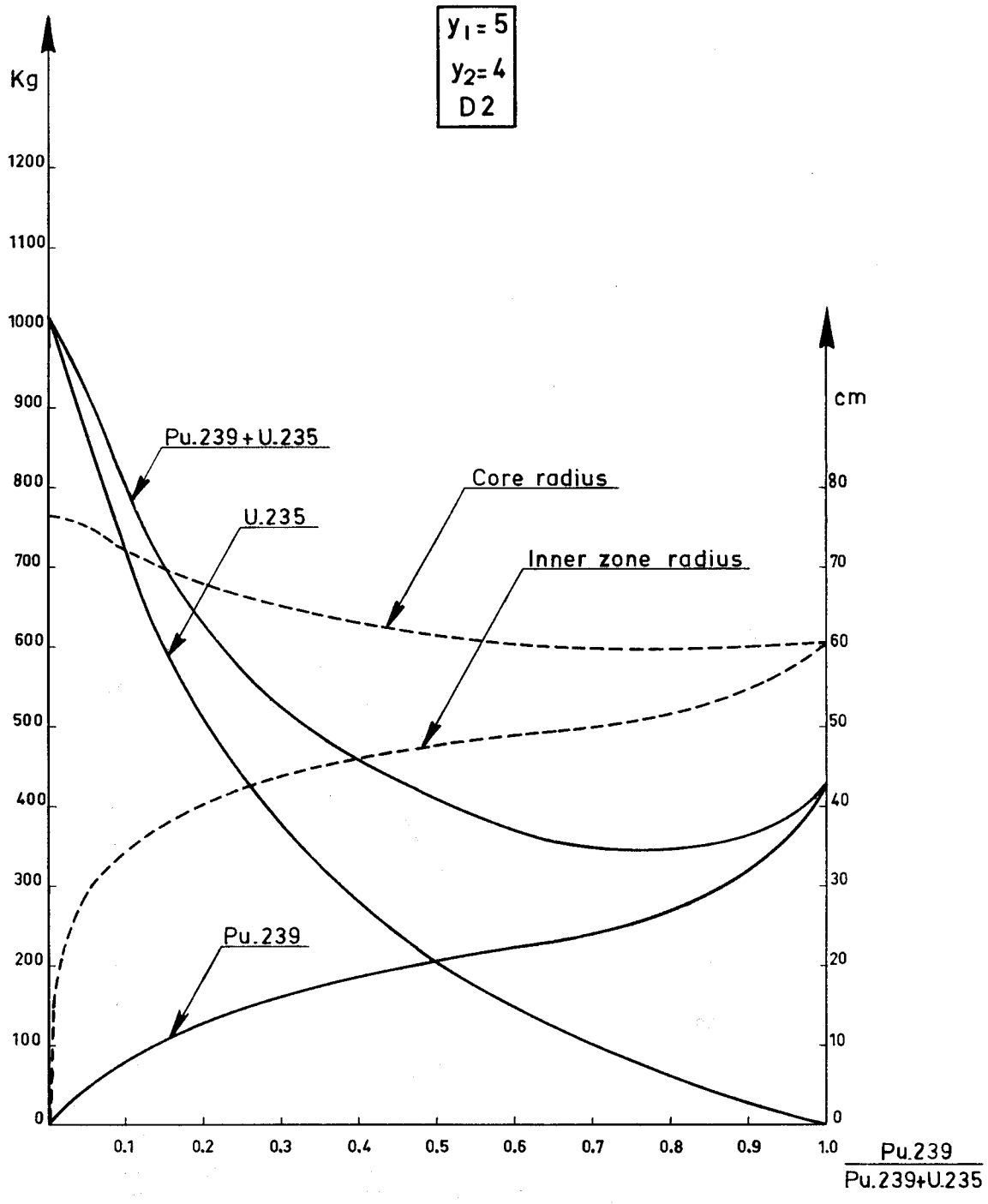
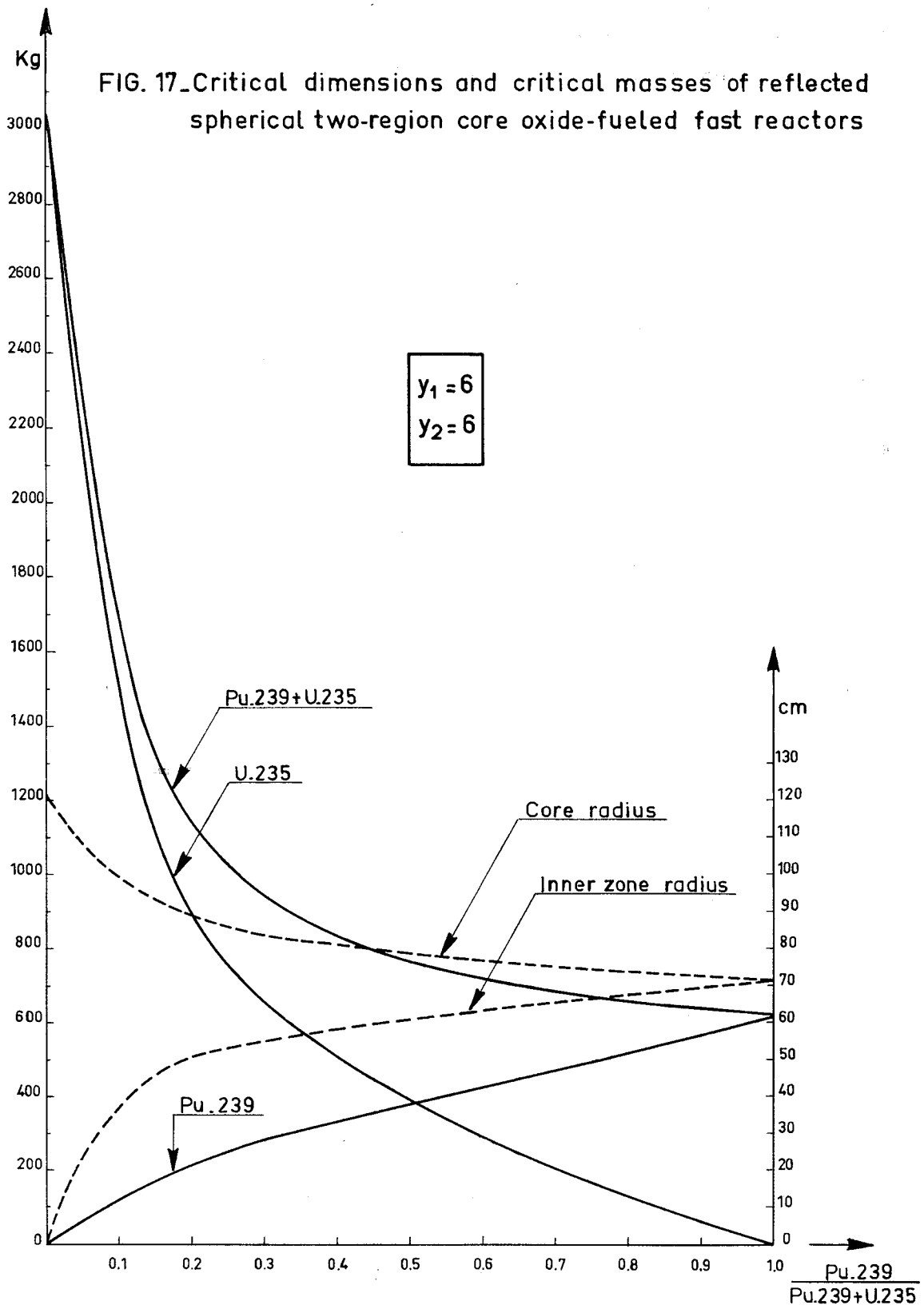


FIG. 17_Critical dimensions and critical masses of reflected spherical two-region core oxide-fueled fast reactors



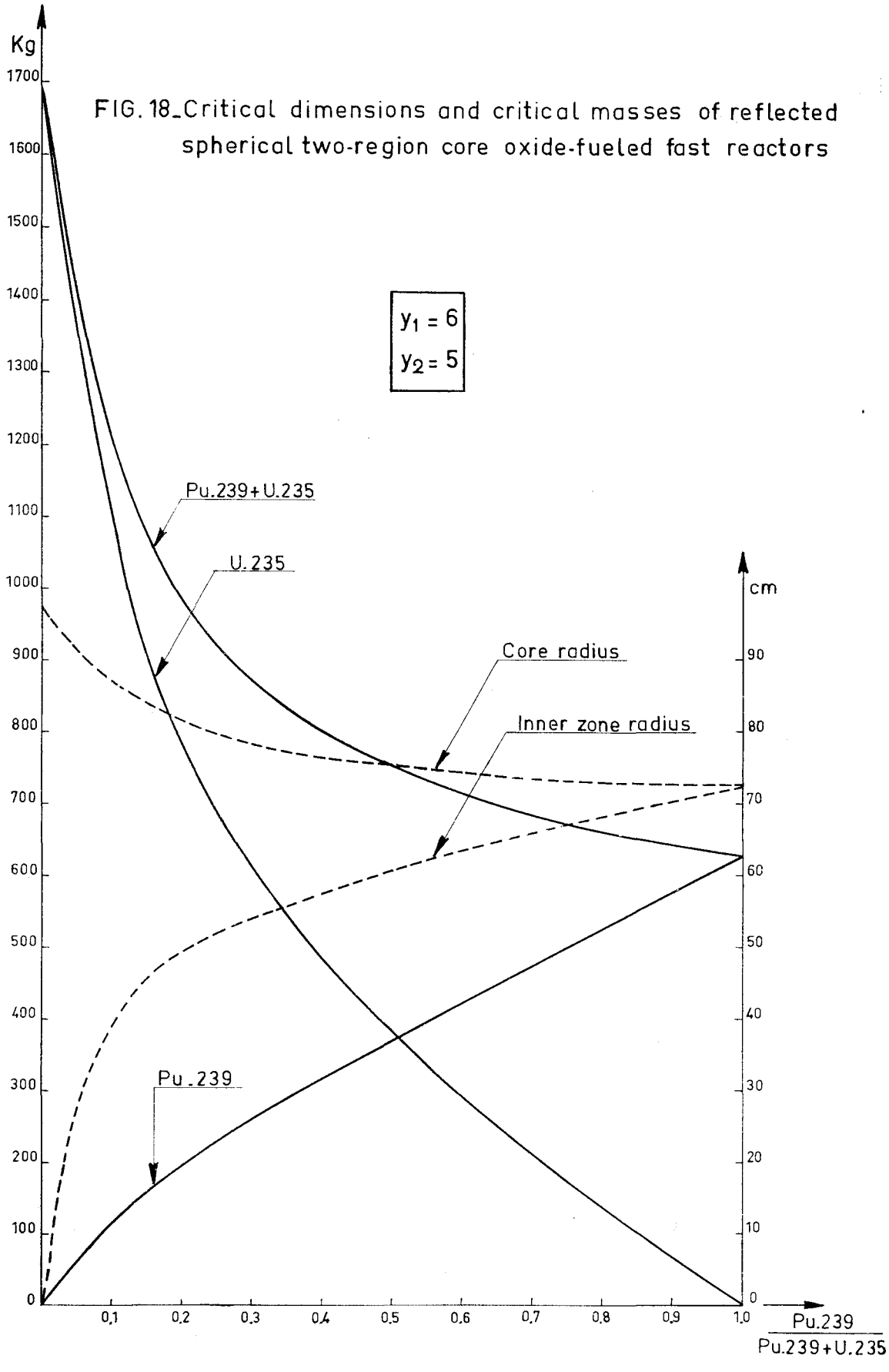


FIG. 18_Critical dimensions and critical masses of reflected spherical two-region core oxide-fueled fast reactors

FIG. 19 _ Critical dimensions and critical masses of reflected spherical two-region core oxide-fueled fast reactors

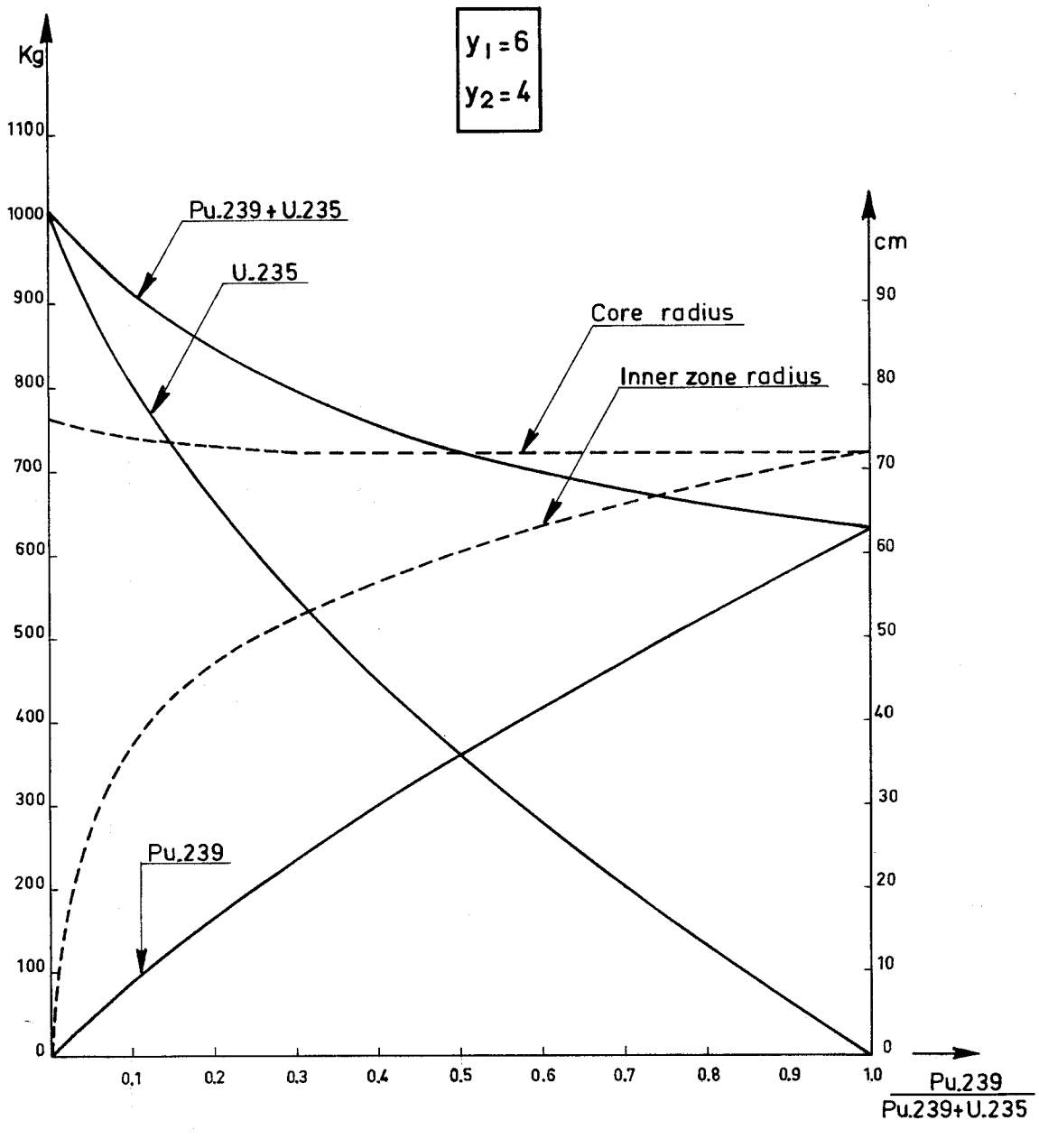


FIG. 20_Critical dimensions and critical masses of reflected spherical two-region core oxide-fueled fast reactors

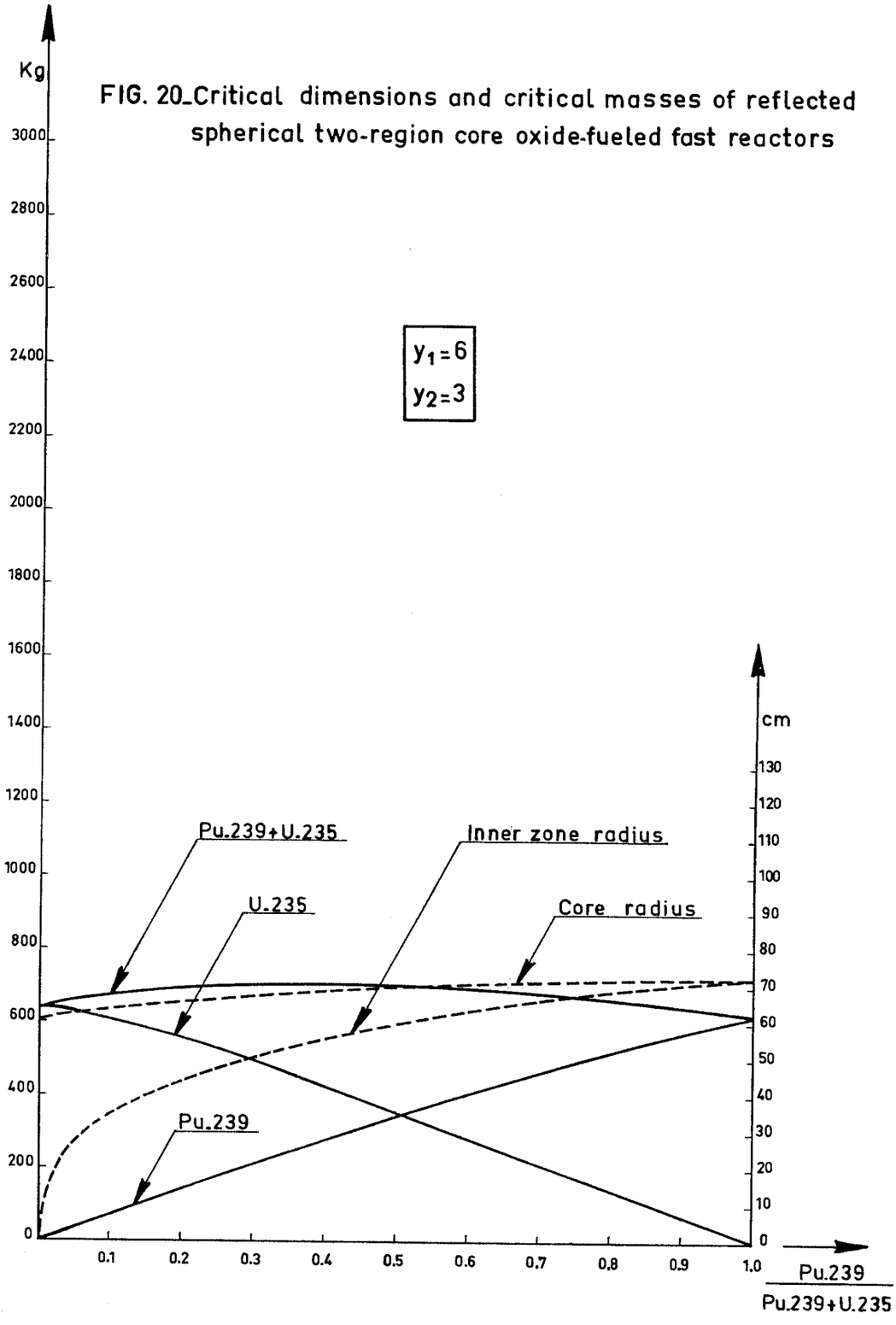


FIG. 21_Critical dimensions and critical masses of reflected spherical two-region core oxide-fueled fast reactors

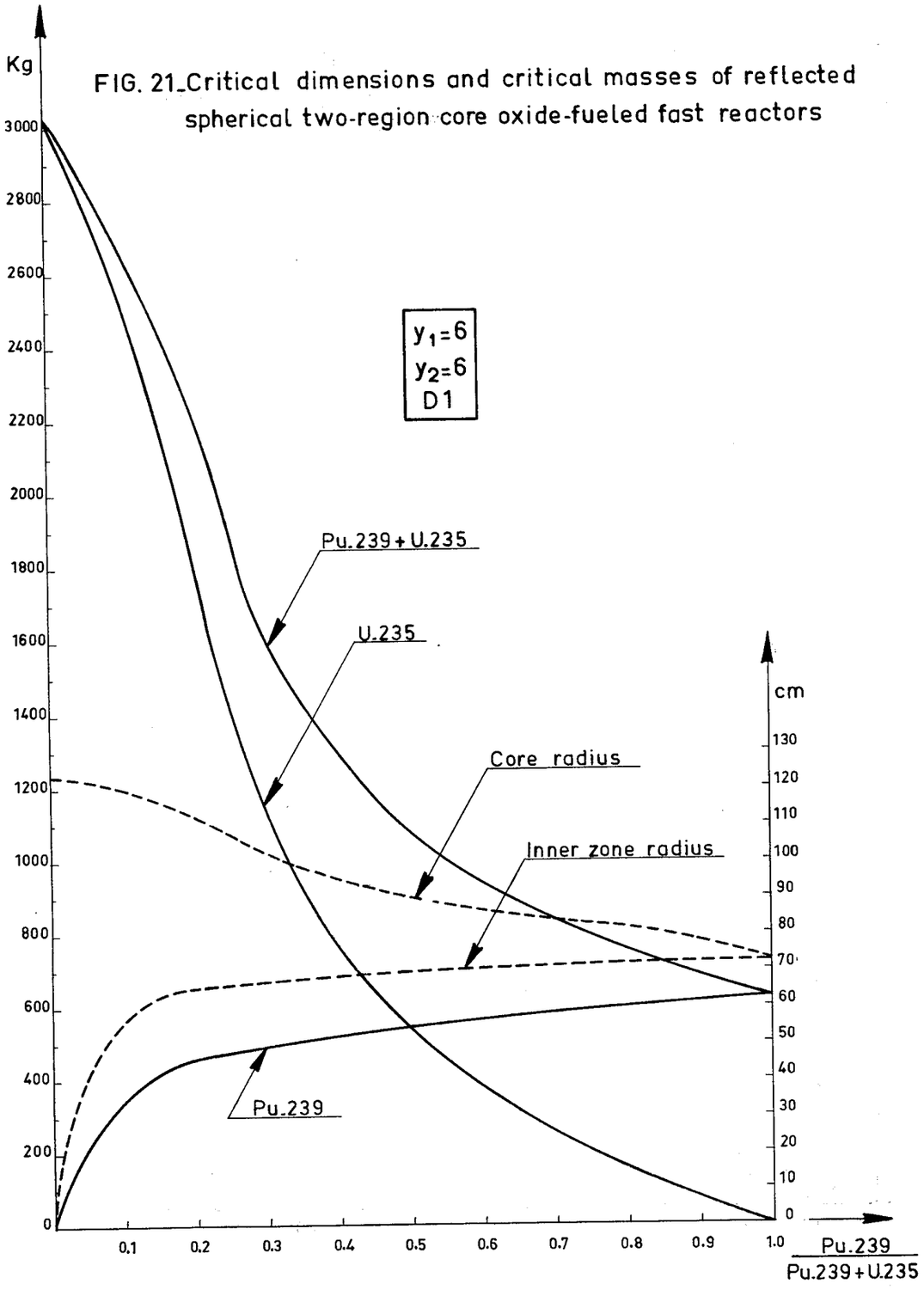


FIG. 22_Critical dimensions and critical masses of reflected spherical two-region core oxide-fueled fast reactors

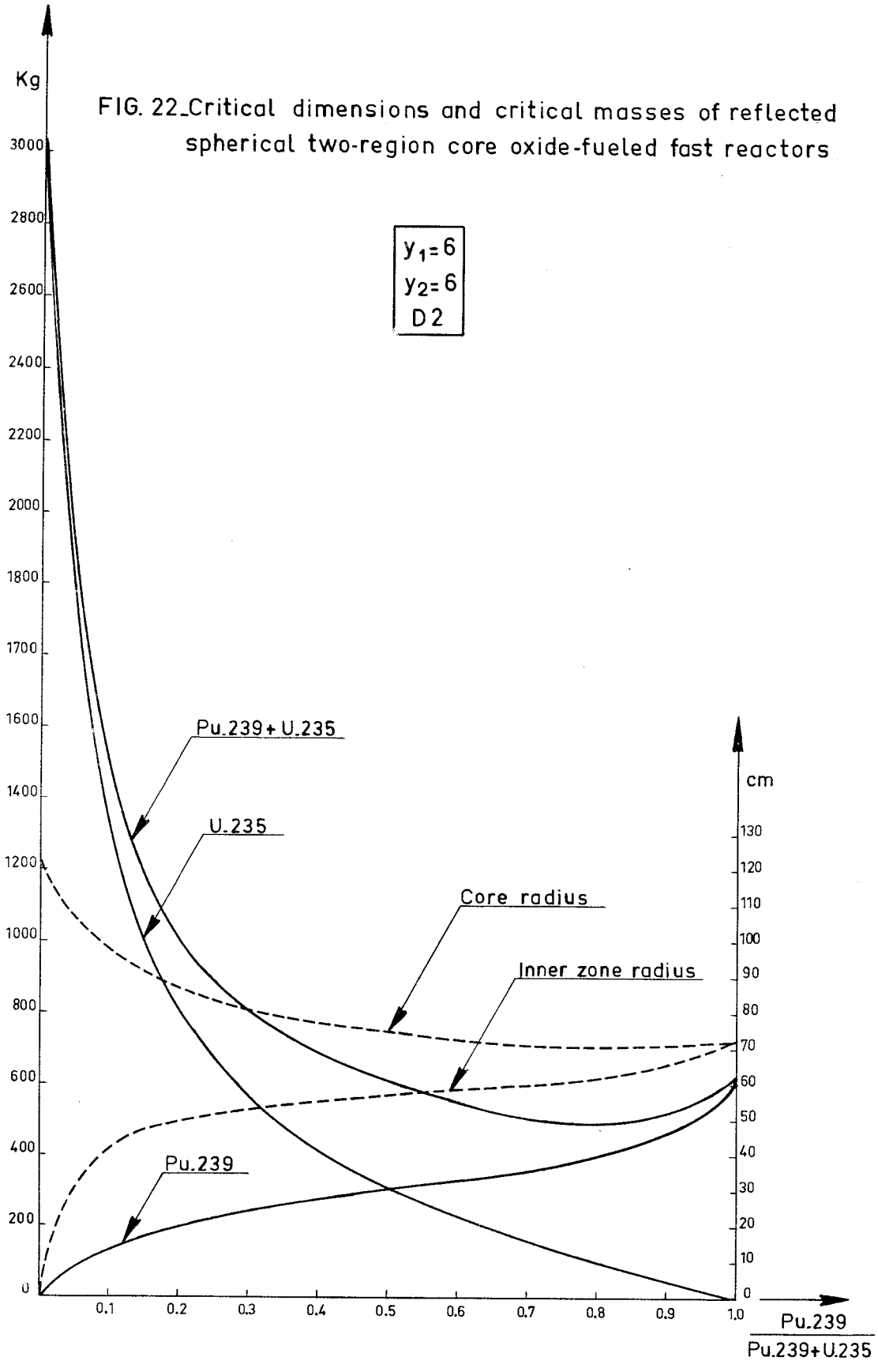
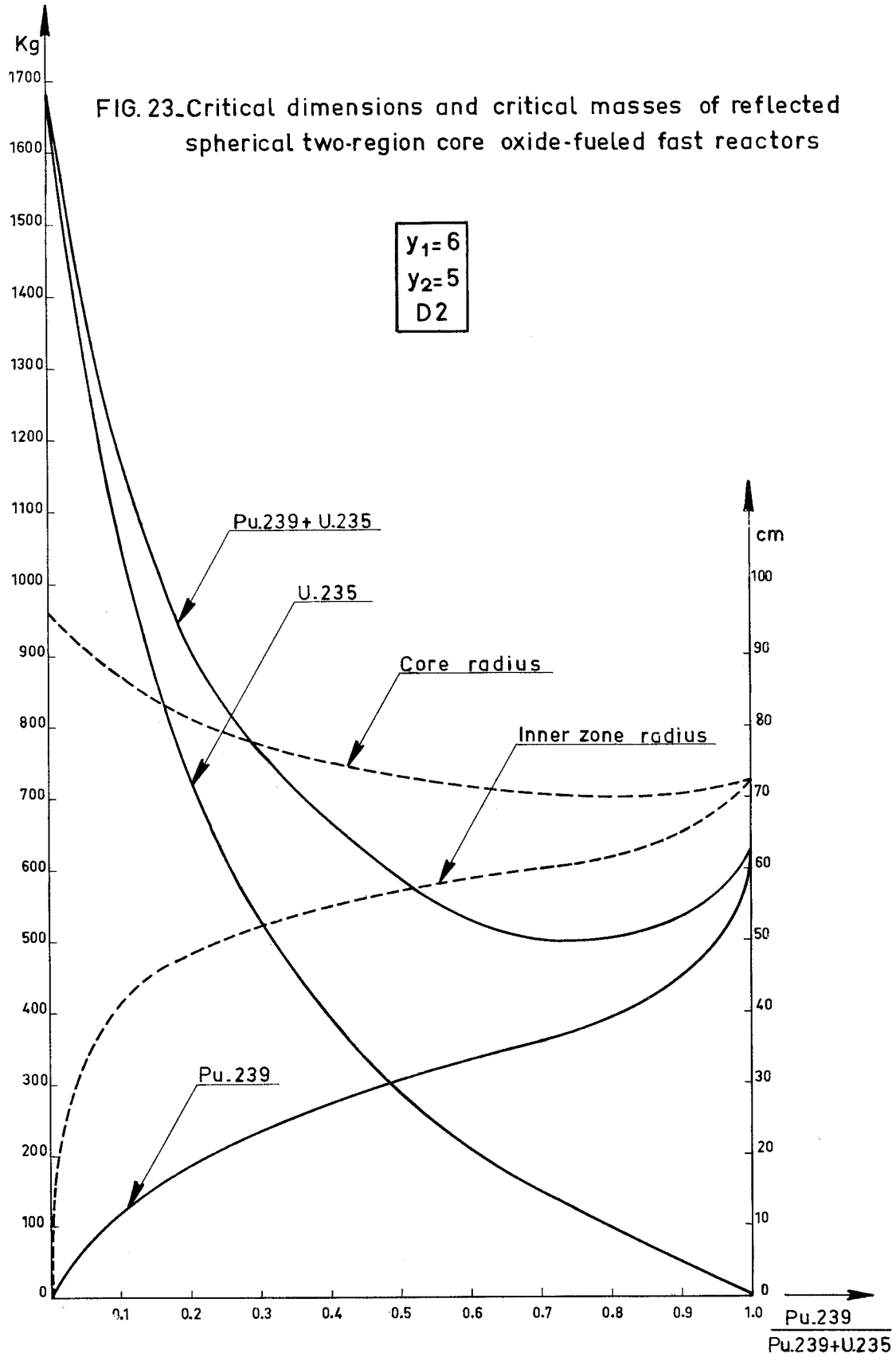


FIG. 23_Critical dimensions and critical masses of reflected spherical two-region core oxide-fueled fast reactors



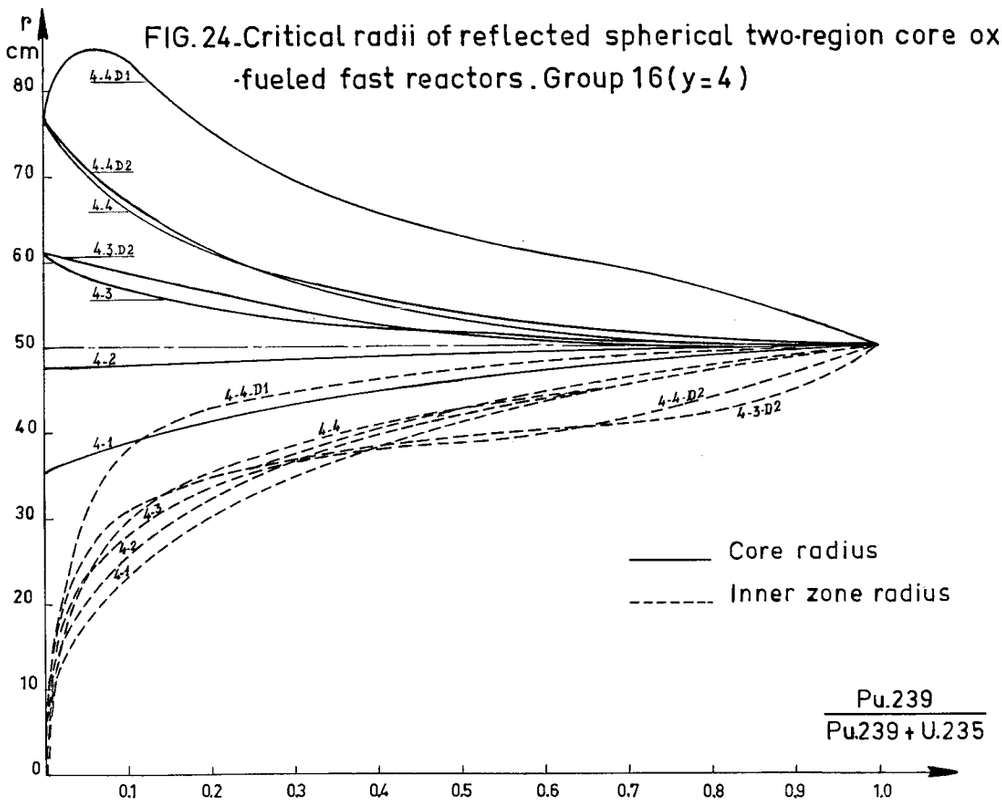


FIG. 24. Critical radii of reflected spherical two-region core oxide-fueled fast reactors. Group 16 (y = 4)

FIG. 25.-Critical radii of reflected spherical two-region core oxide-fueled fast reactors. Group 26 (y = 5)

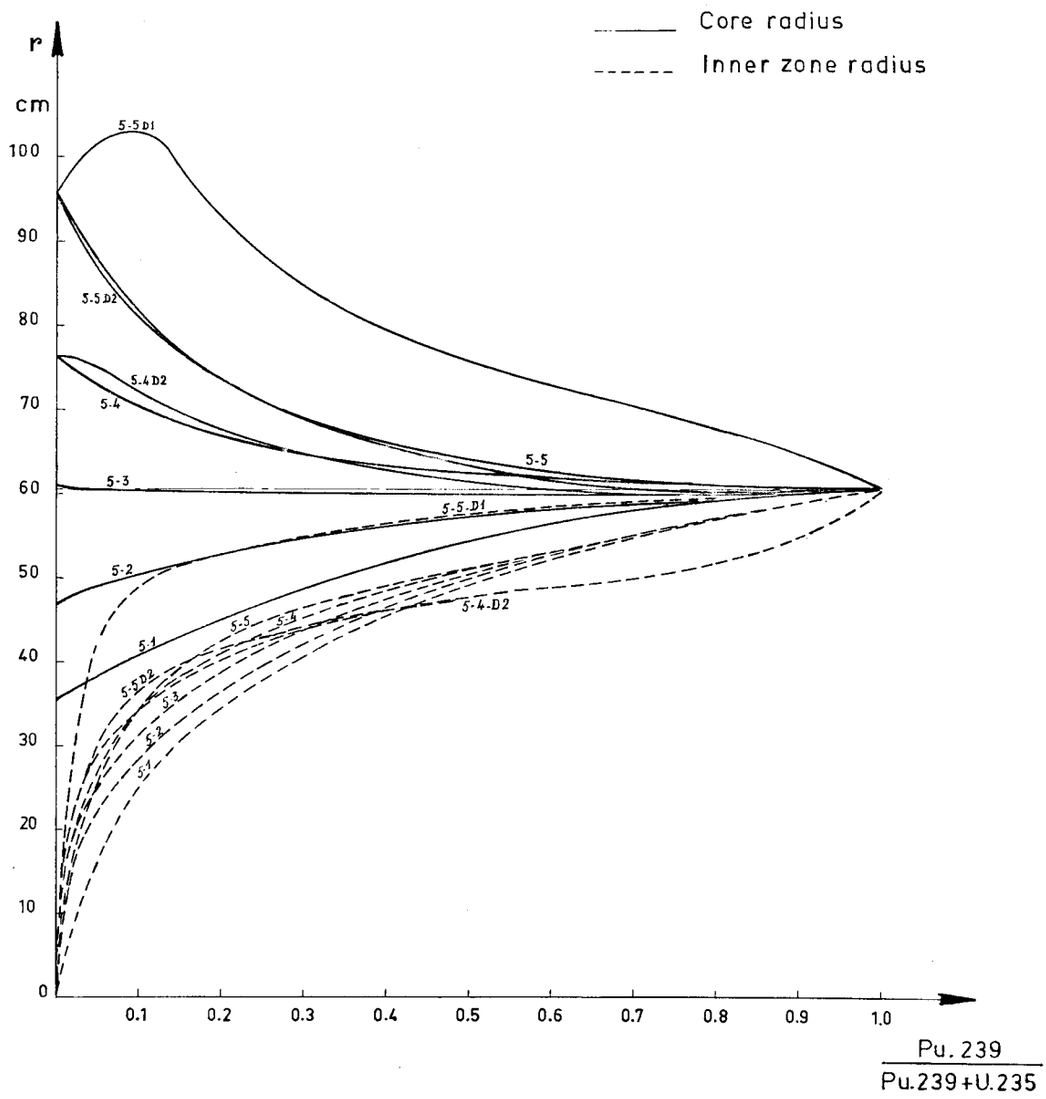


FIG. 26. Critical radii of reflected spherical two-region core oxide-fueled fast reactors. Group 36 (y = 6)

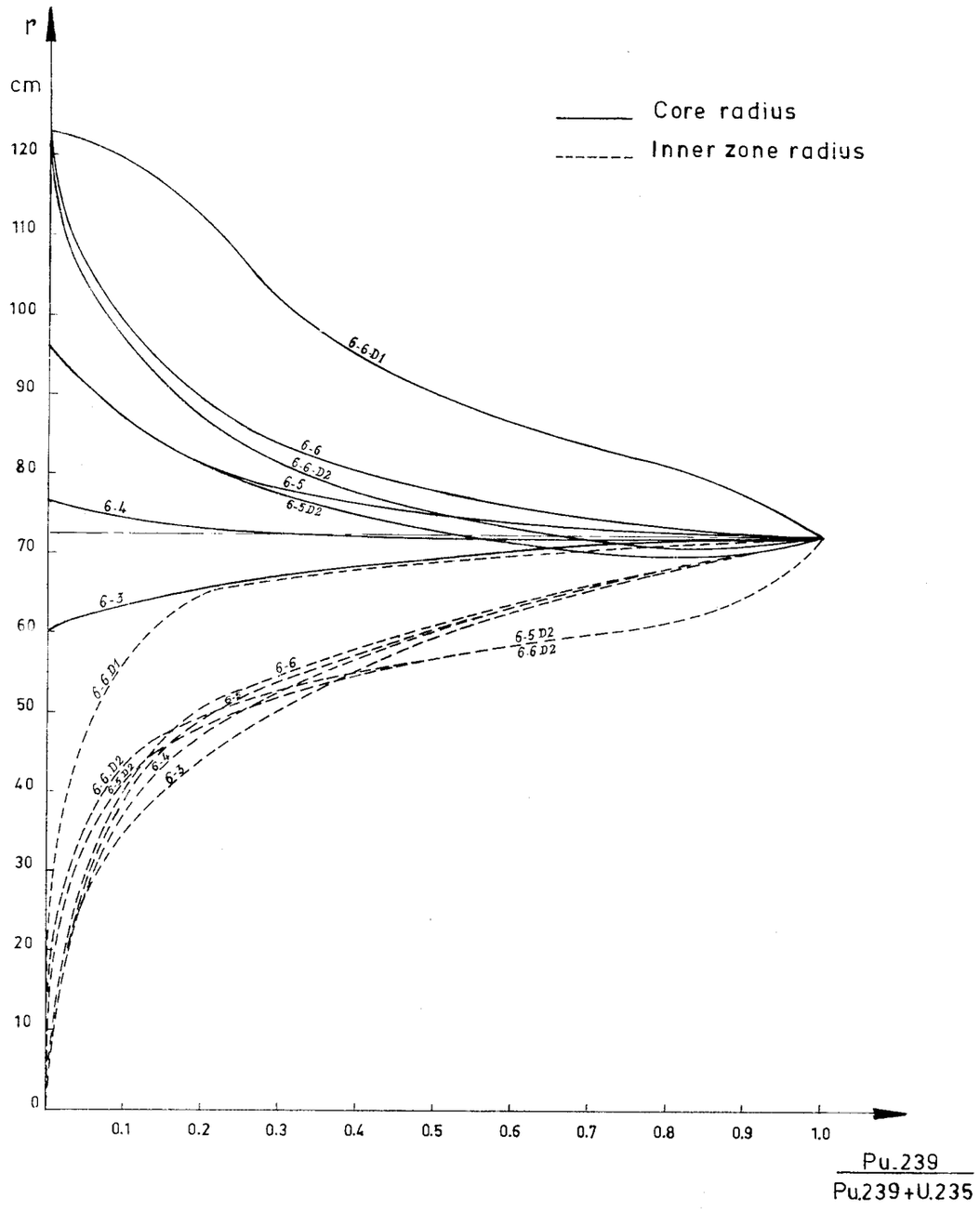


FIG. 27_Normalized neutron spectra at the center of reflected spherical two-region core oxide-fueled fast reactors. Group 16 (y=4)

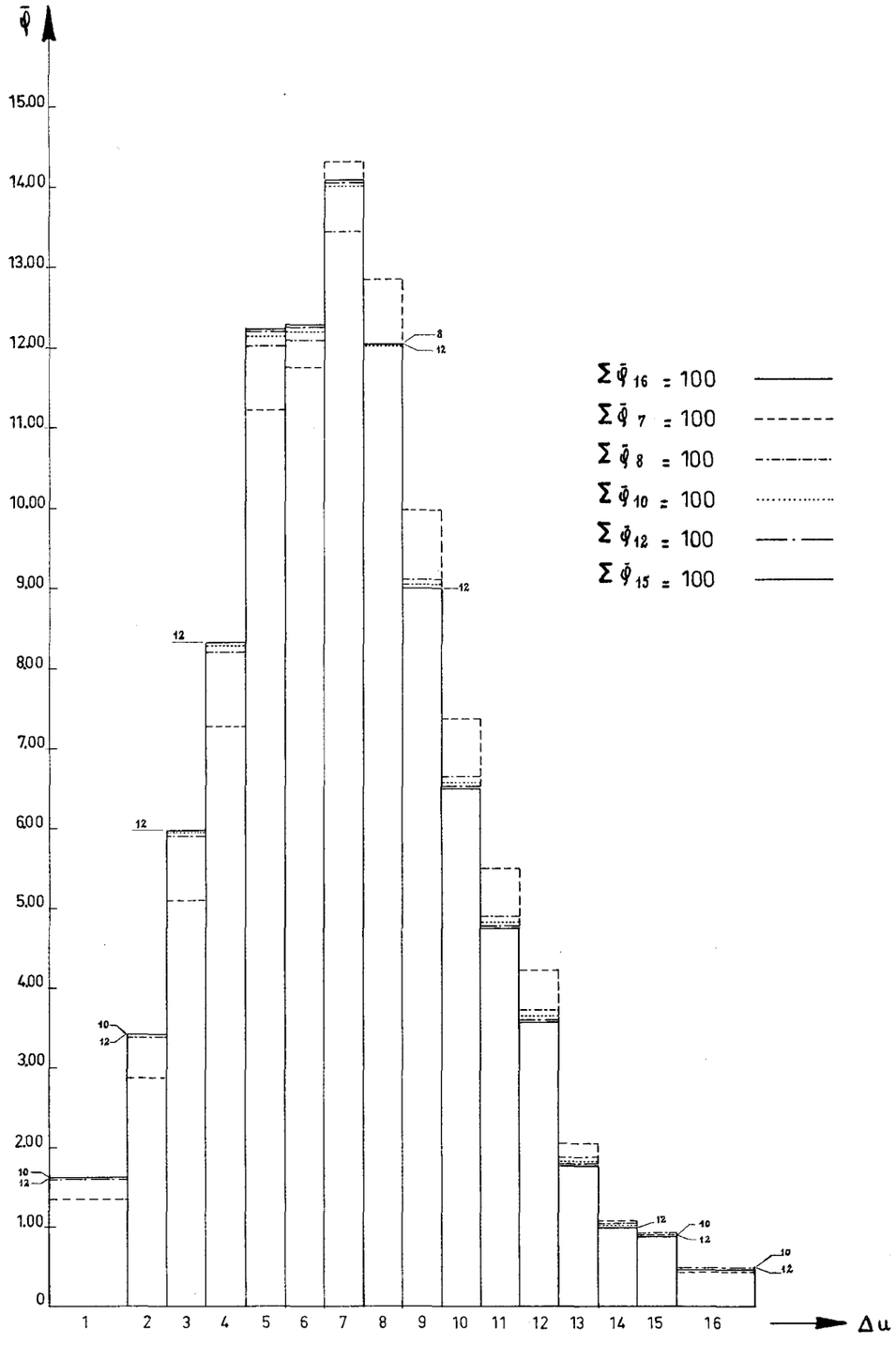


FIG. 28. Normalized neutron spectra at the center of reflected spherical two-region core oxide-fueled fast reactors. Group 26 (y = 5)

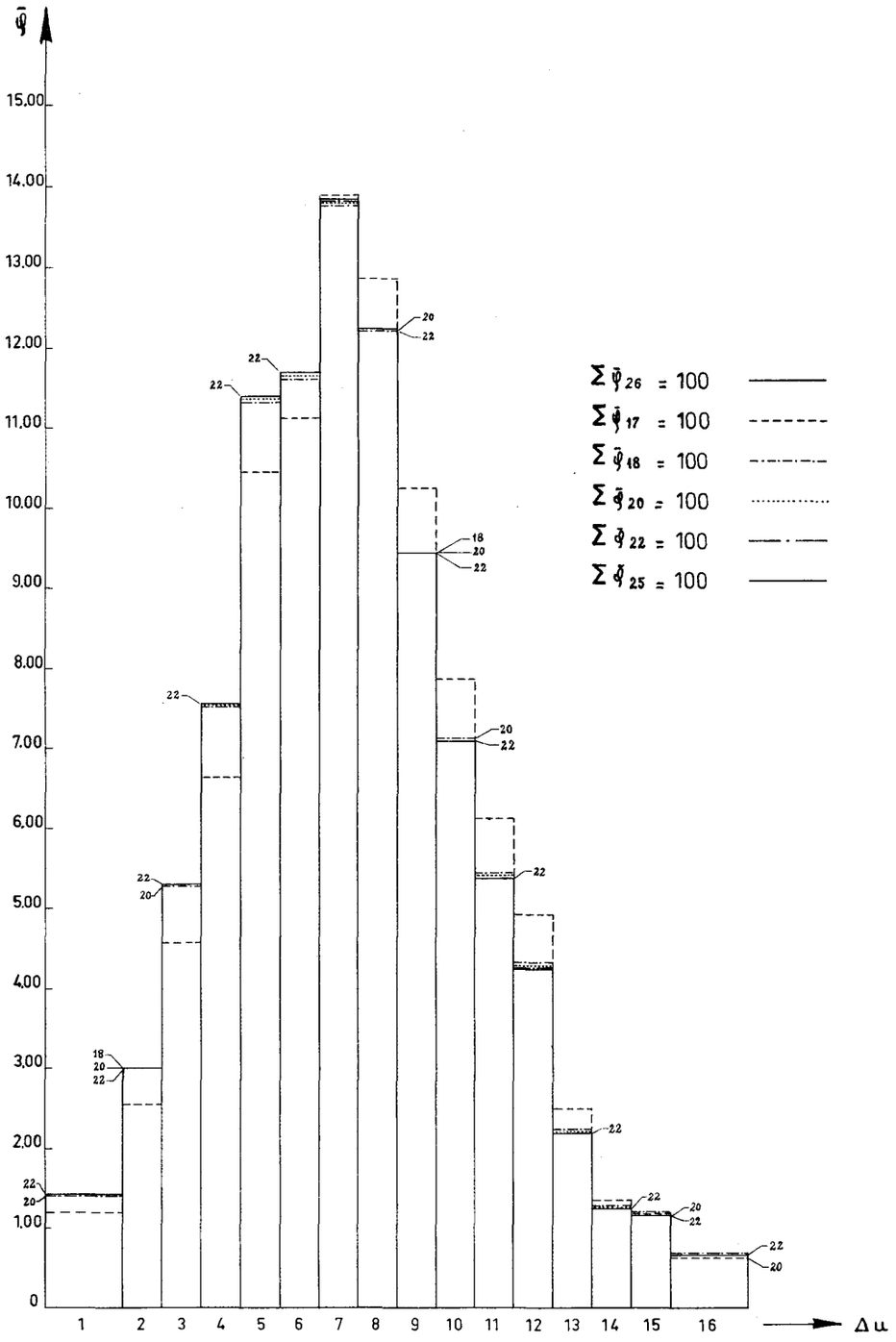


FIG. 29. Normalized neutron spectra at the center of reflected spherical two-region core oxide-fueled fast reactors. Group 36 (y = 6)

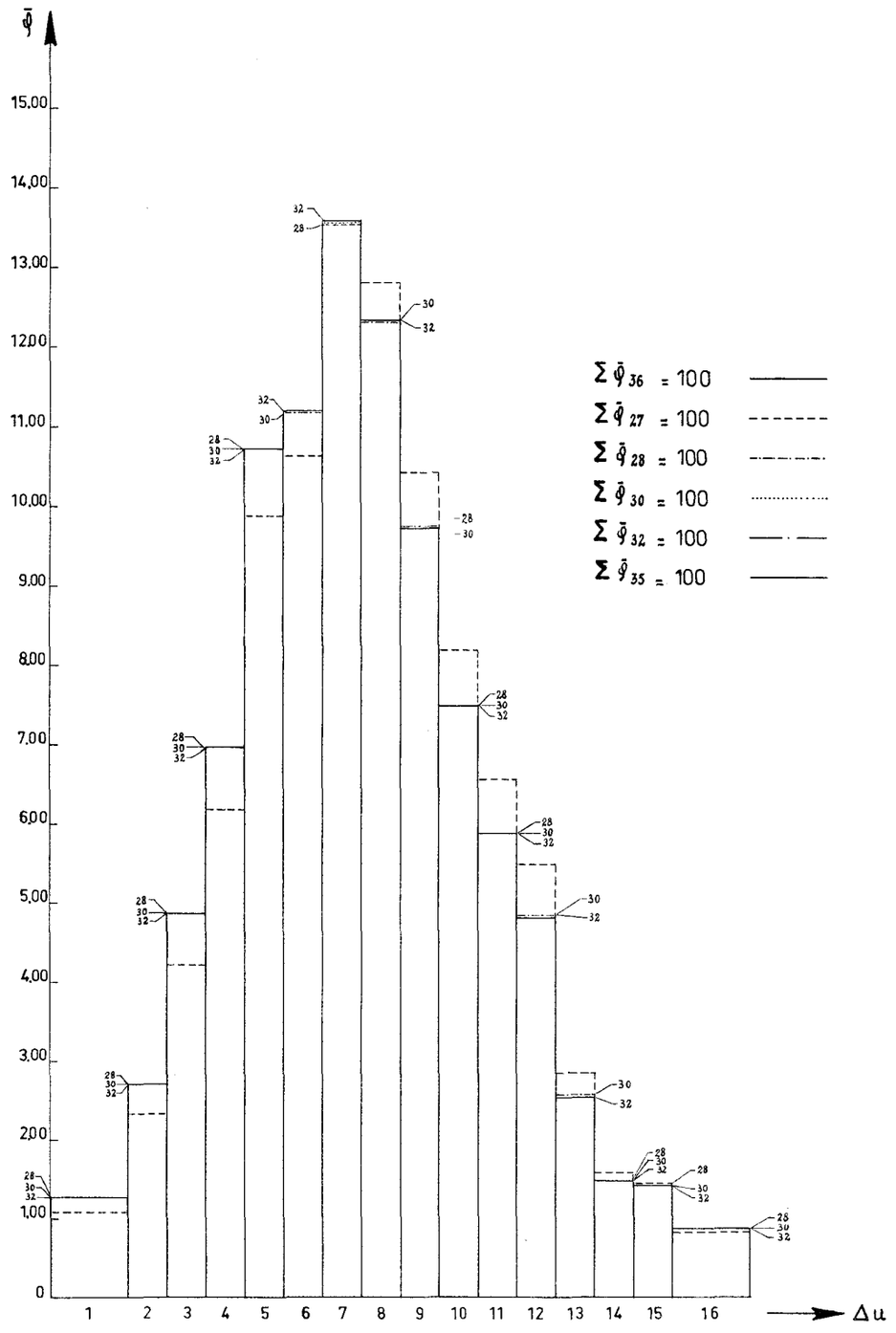


FIG. 30. Mismatch parameter(PPP) of the reactors of the group16 (y = 4)

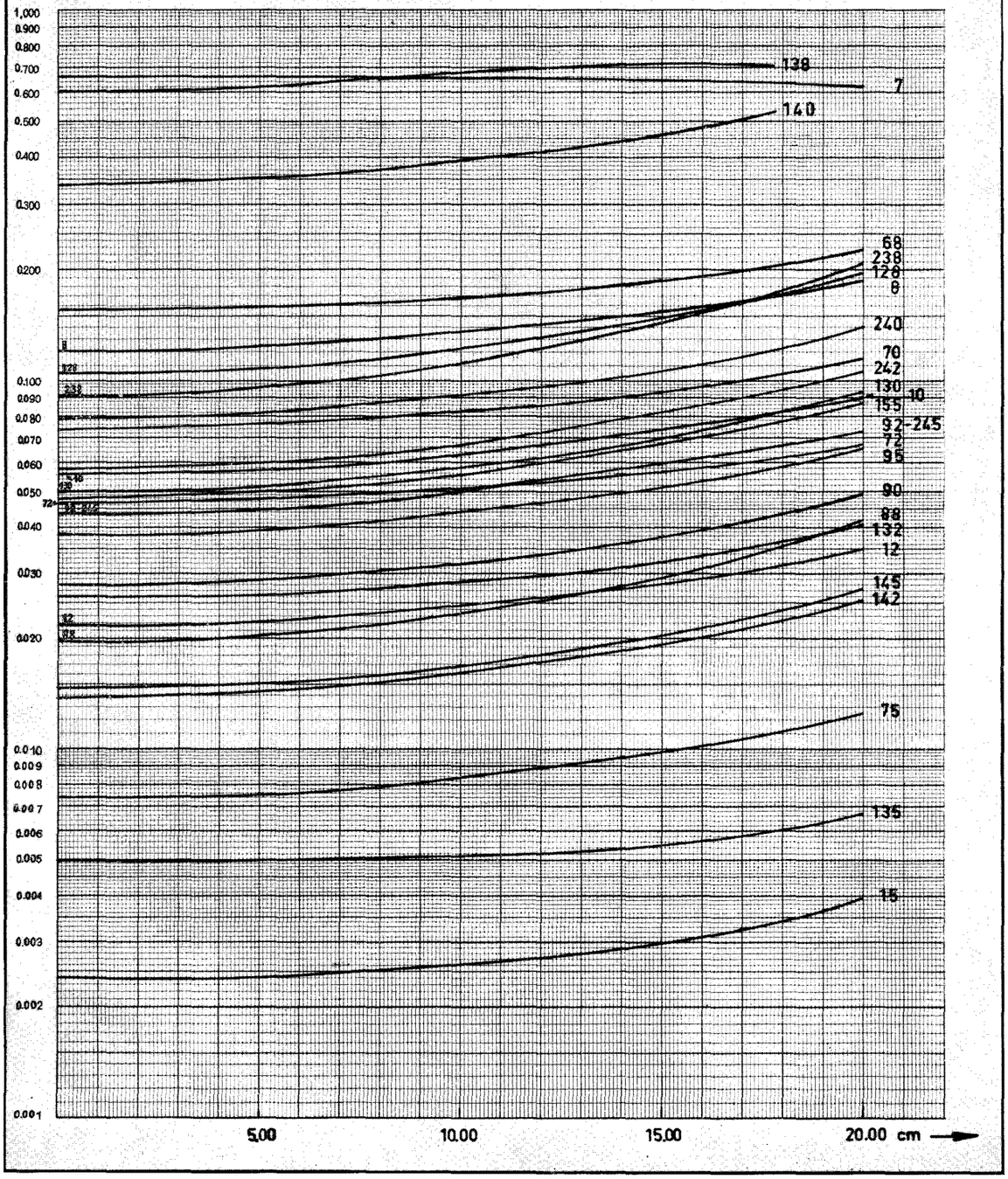


FIG. 31. Mismatch parameter(PPP)of the reactors of the group 16 (y = 4)

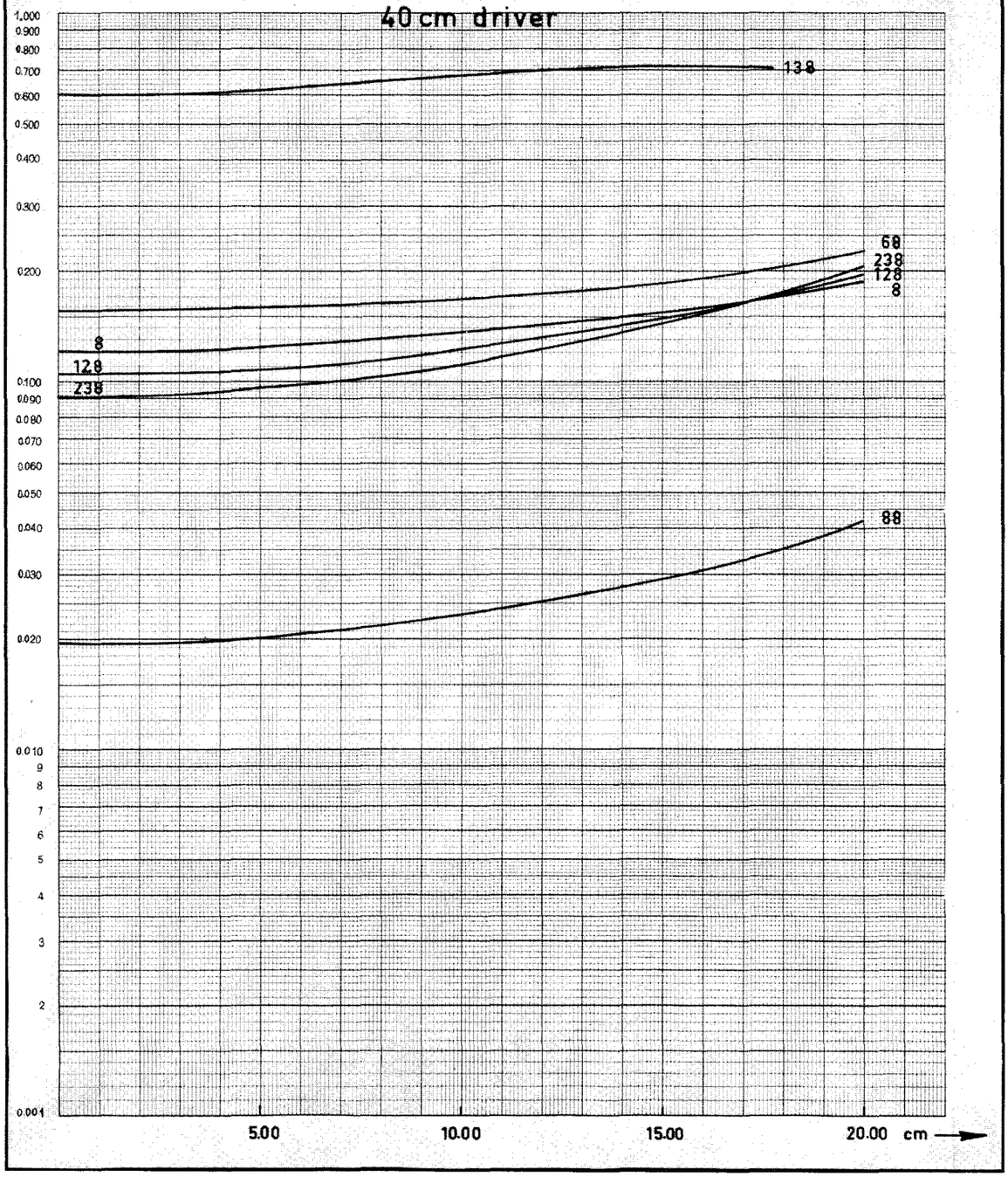


FIG.32.Mismatch parameter(PPP)of the reactors of the group16 (y = 4)

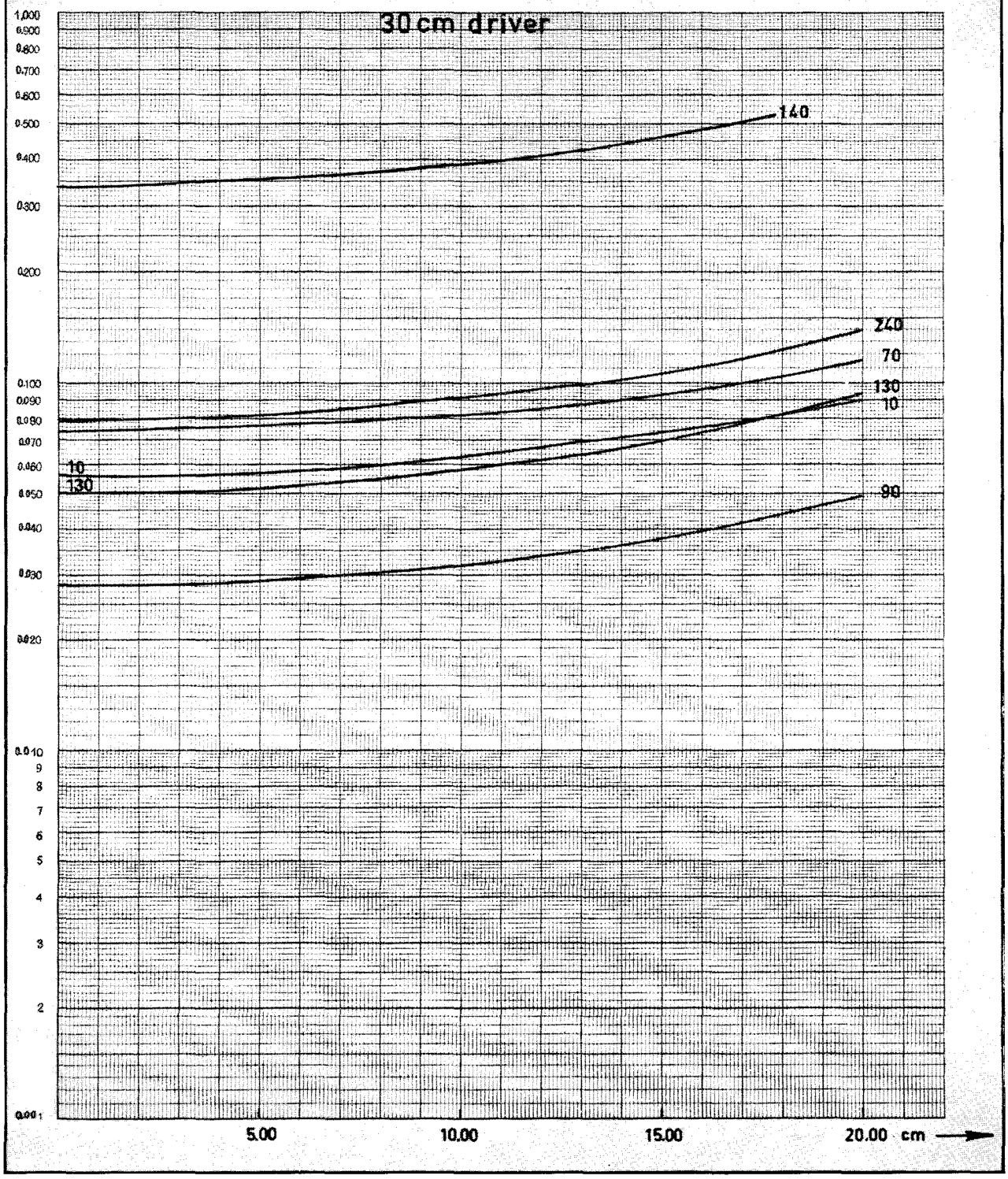


FIG.33. Mismatch parameter(PPP) of the reactors of the group16 (y = 4)

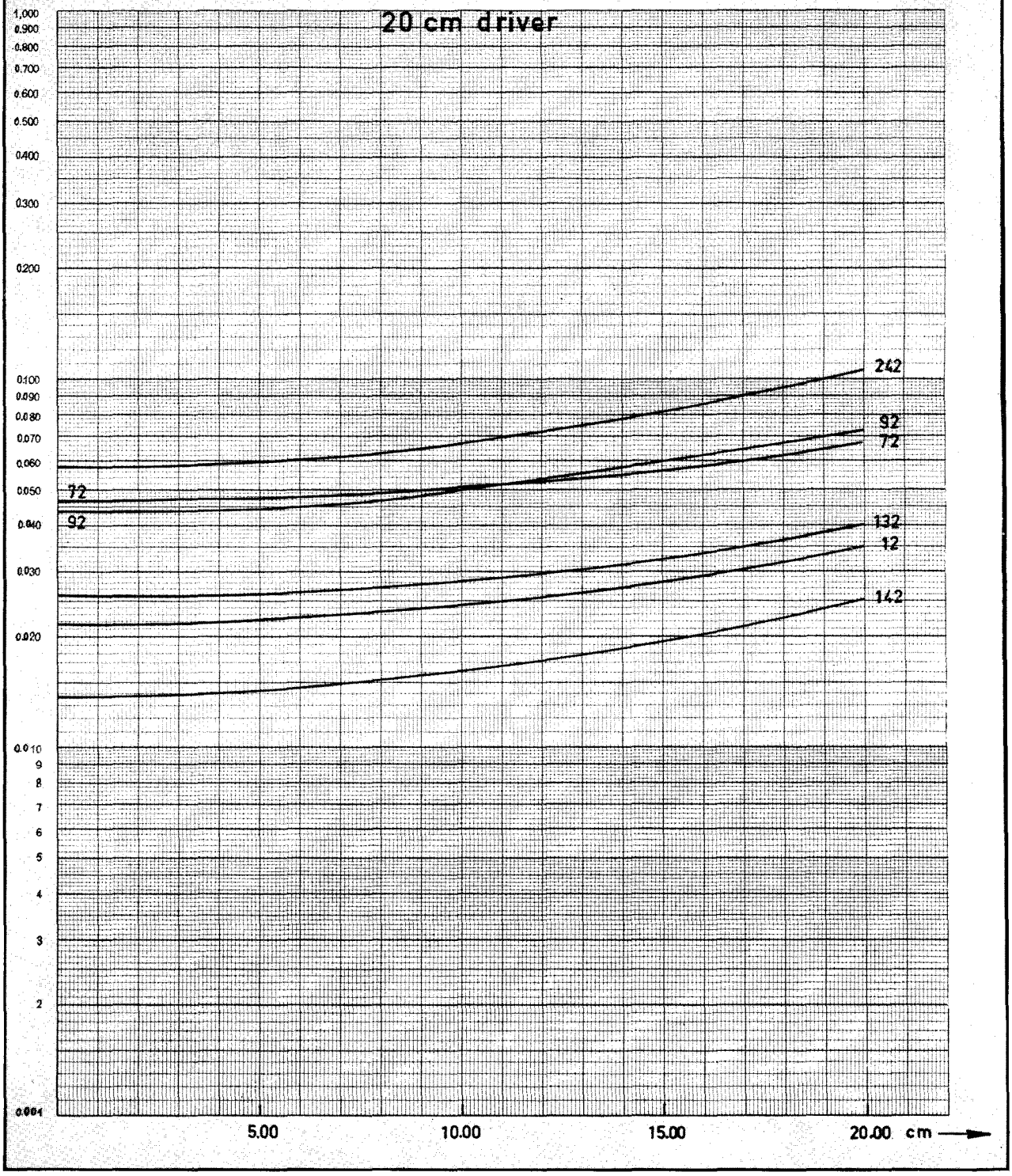


FIG.34.-Mismatch parameter (PPP) of the reactors of the group 16 (y = 4)

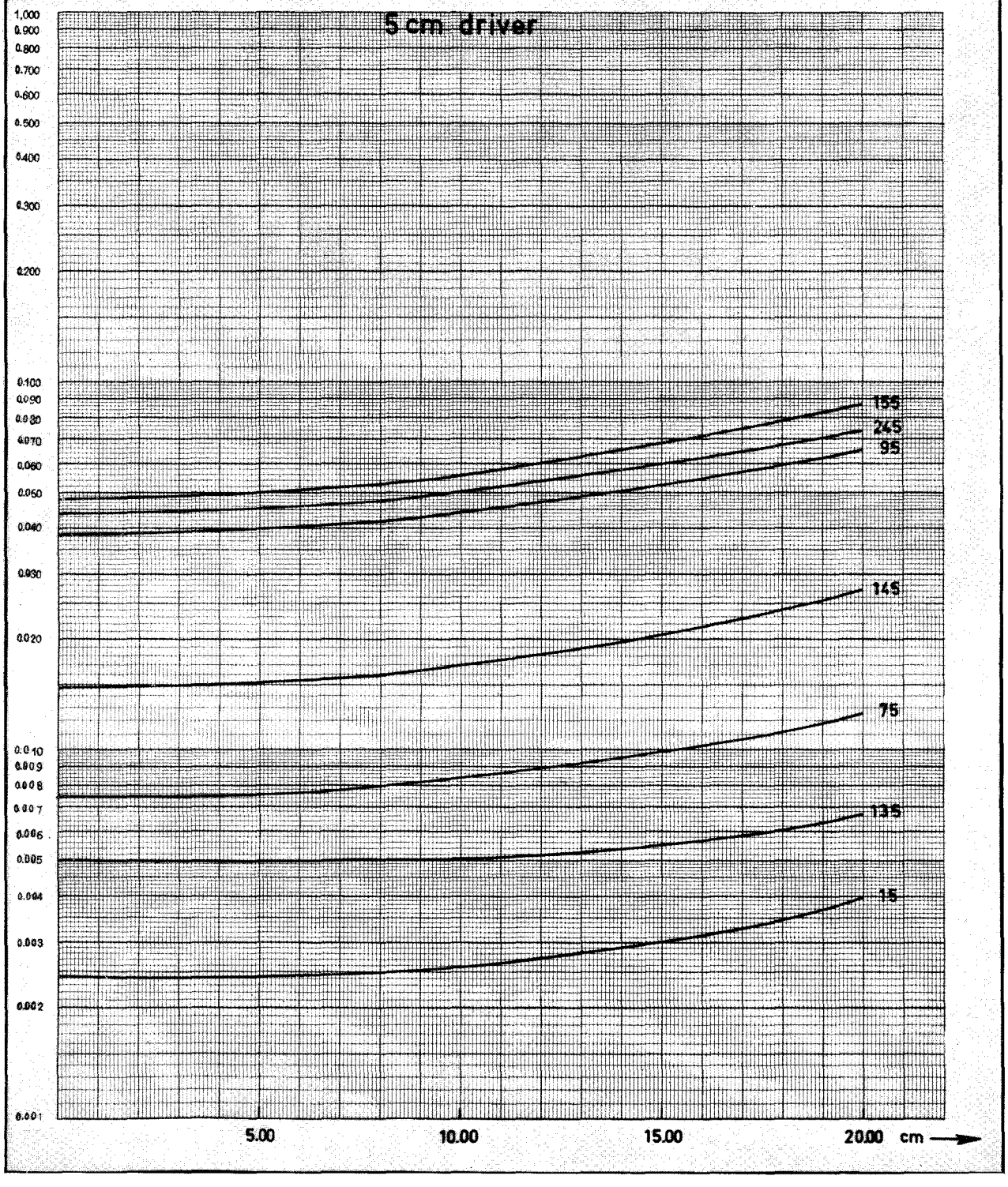


FIG.35. Mismatch parameter (PPP) of the reactors of the group 26 (y = 5)

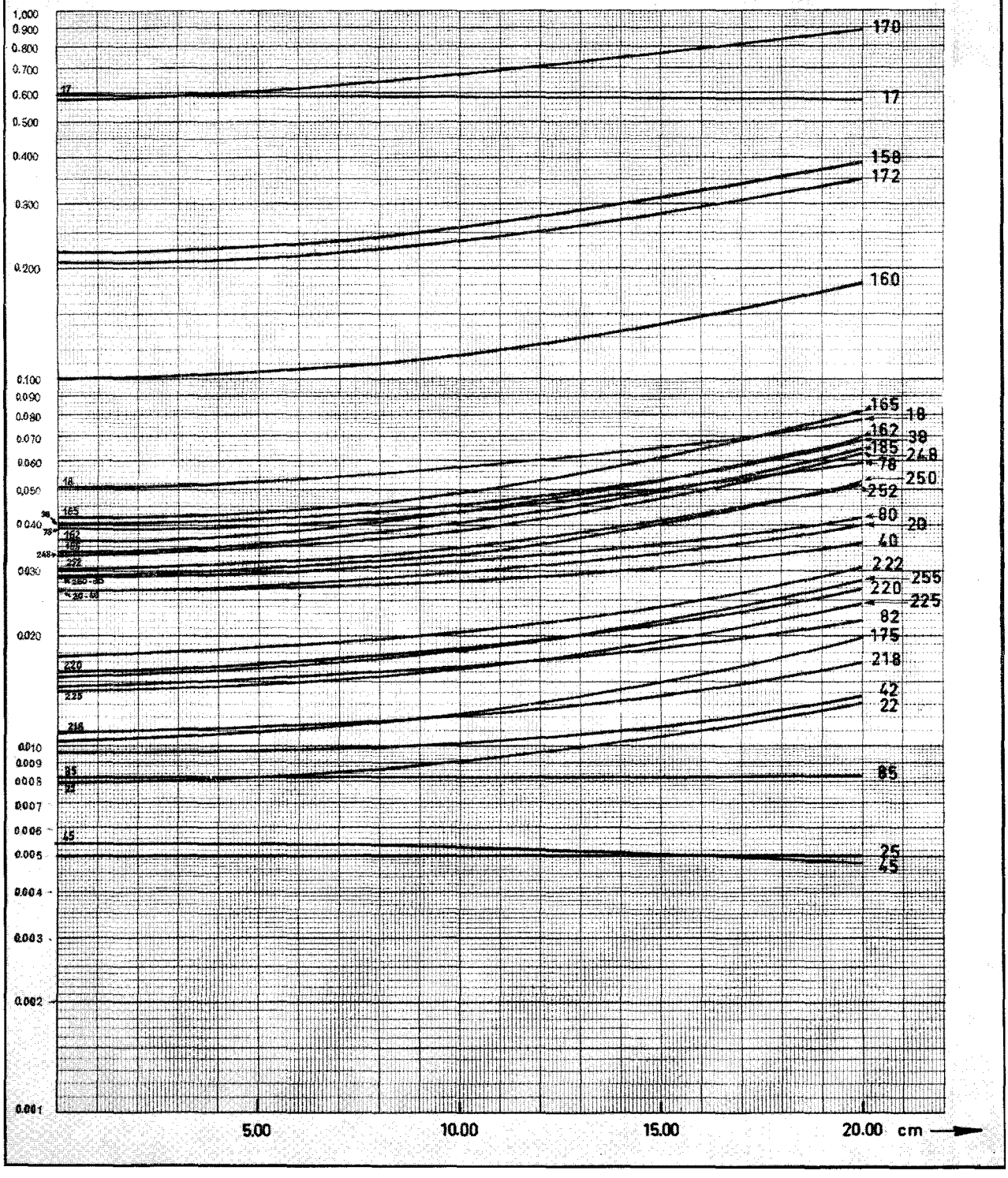


FIG.36. Mismatch parameter (PPP) of the reactors of the group 26 (y = 5)

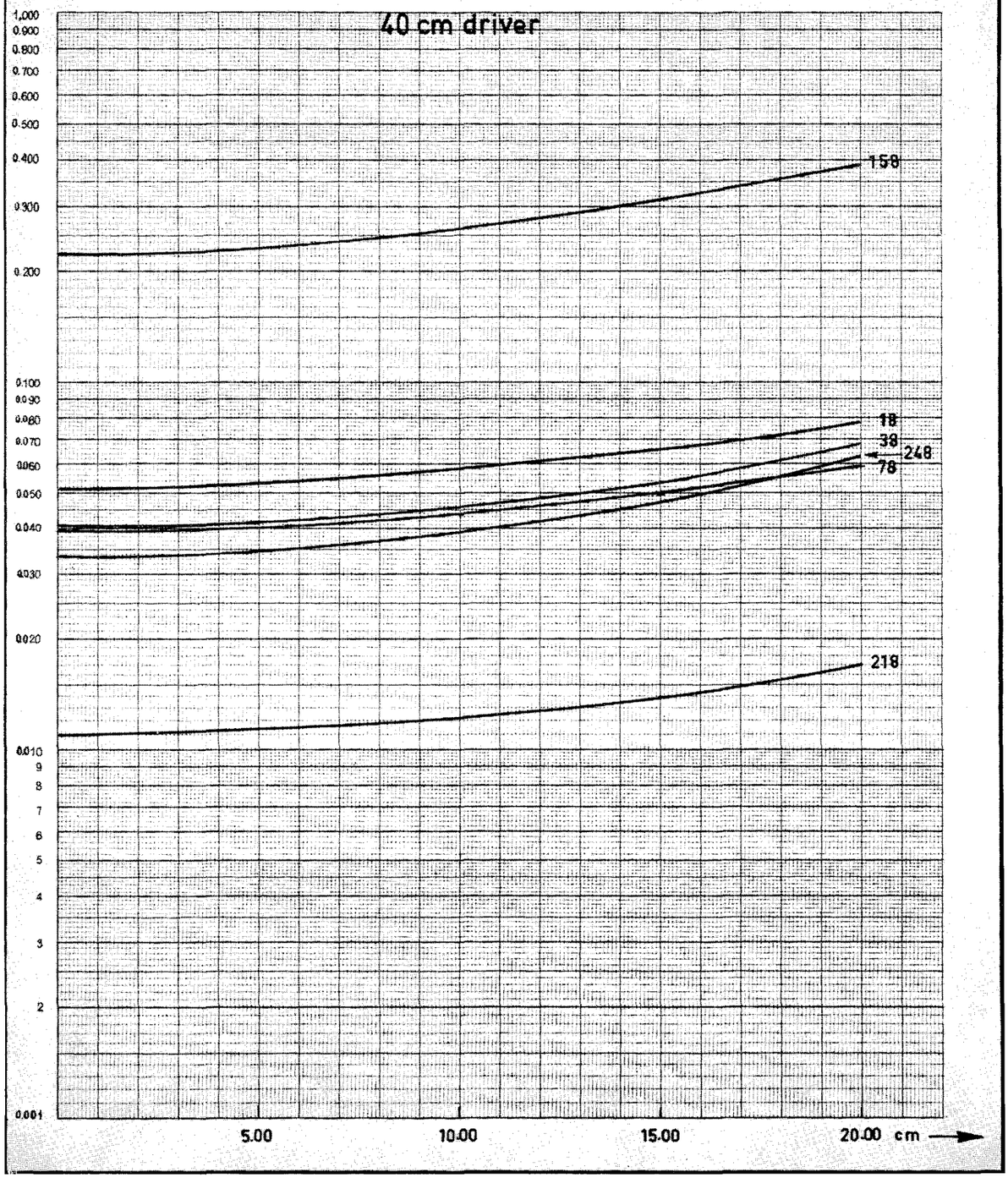


FIG.37. Mismatch parameter (PPP) of the reactors of the group 26 (y = 5)

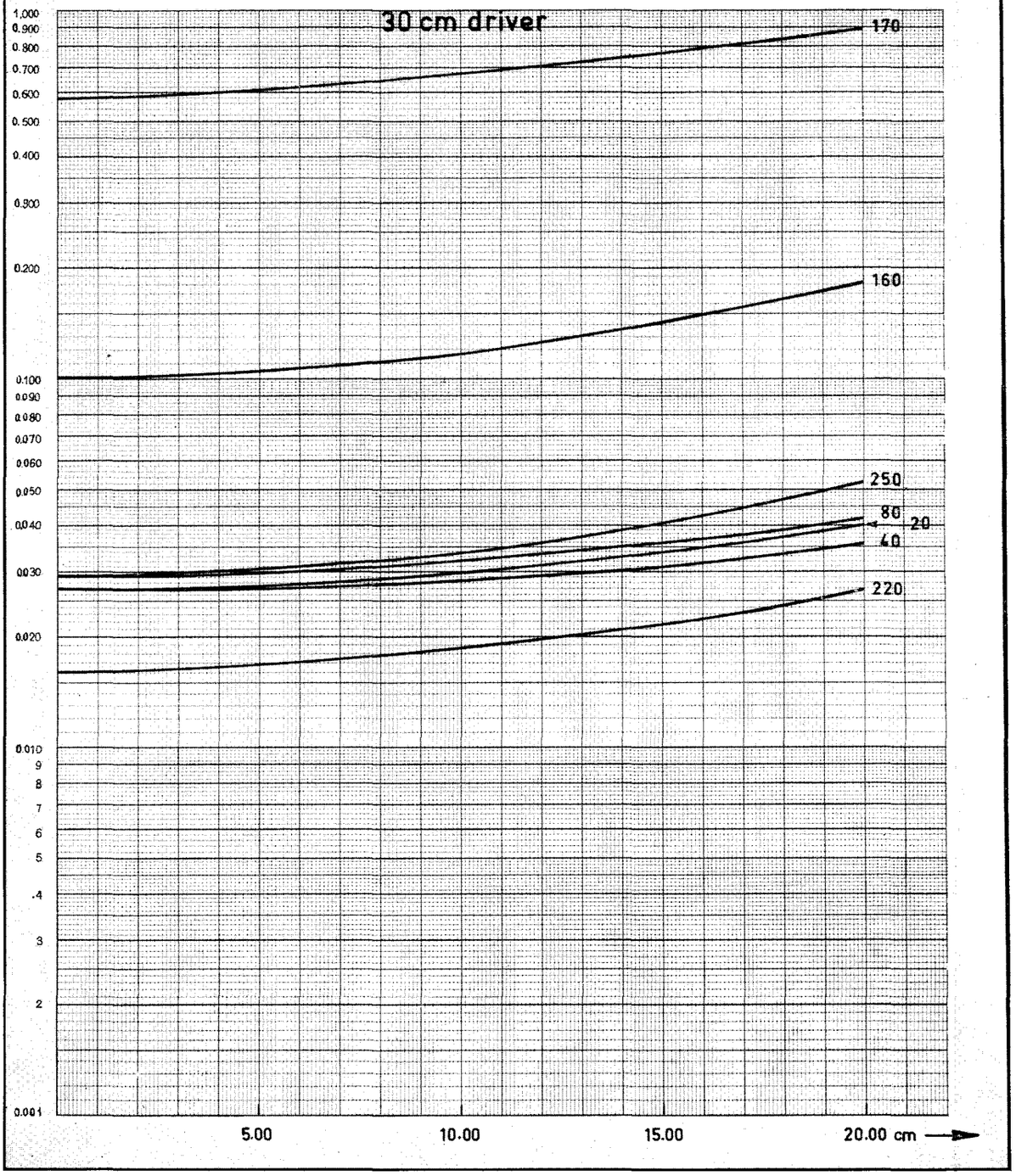
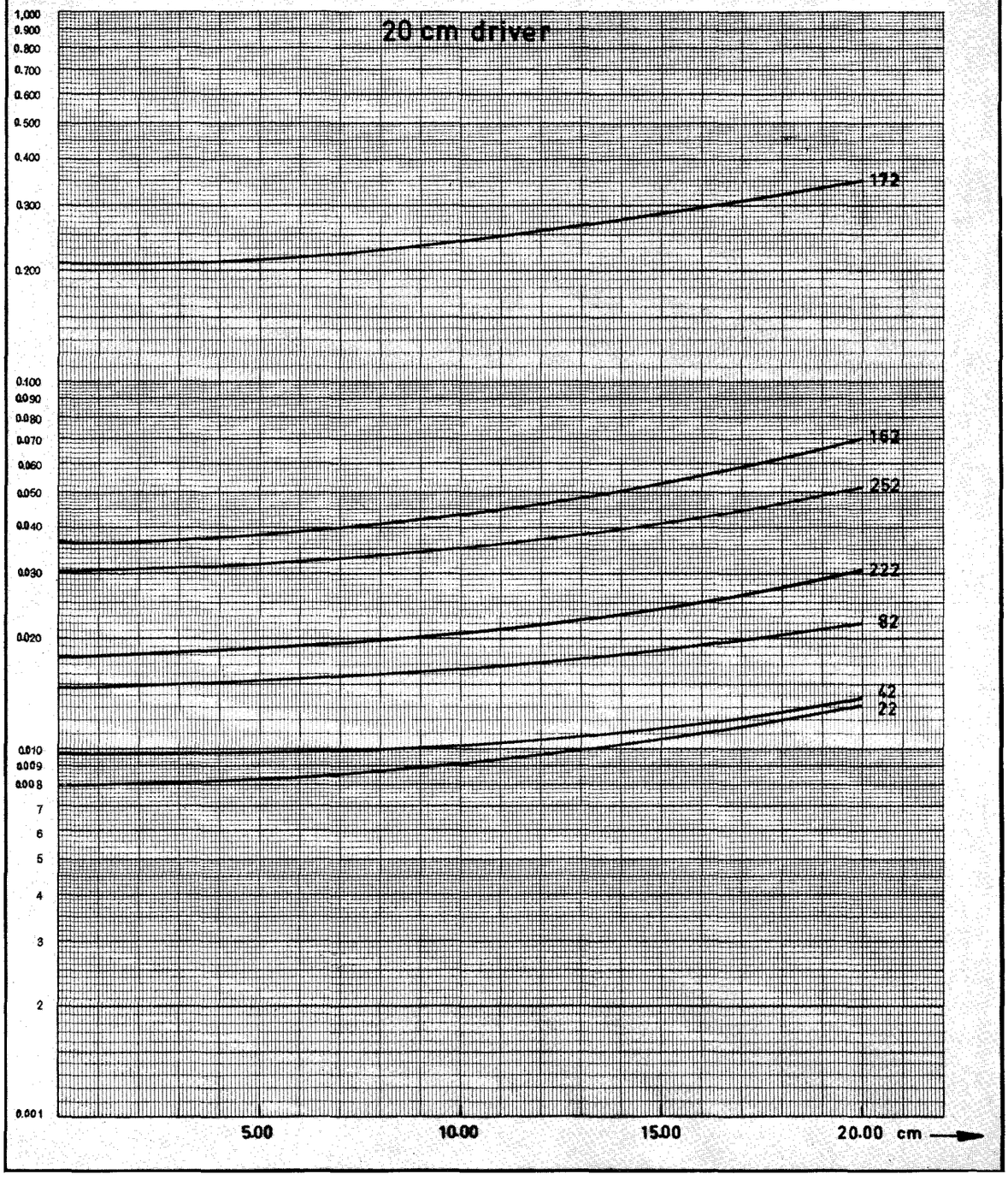


FIG.38. Mismatch parameter (PPP) of the reactors of the group 26 (y=5)



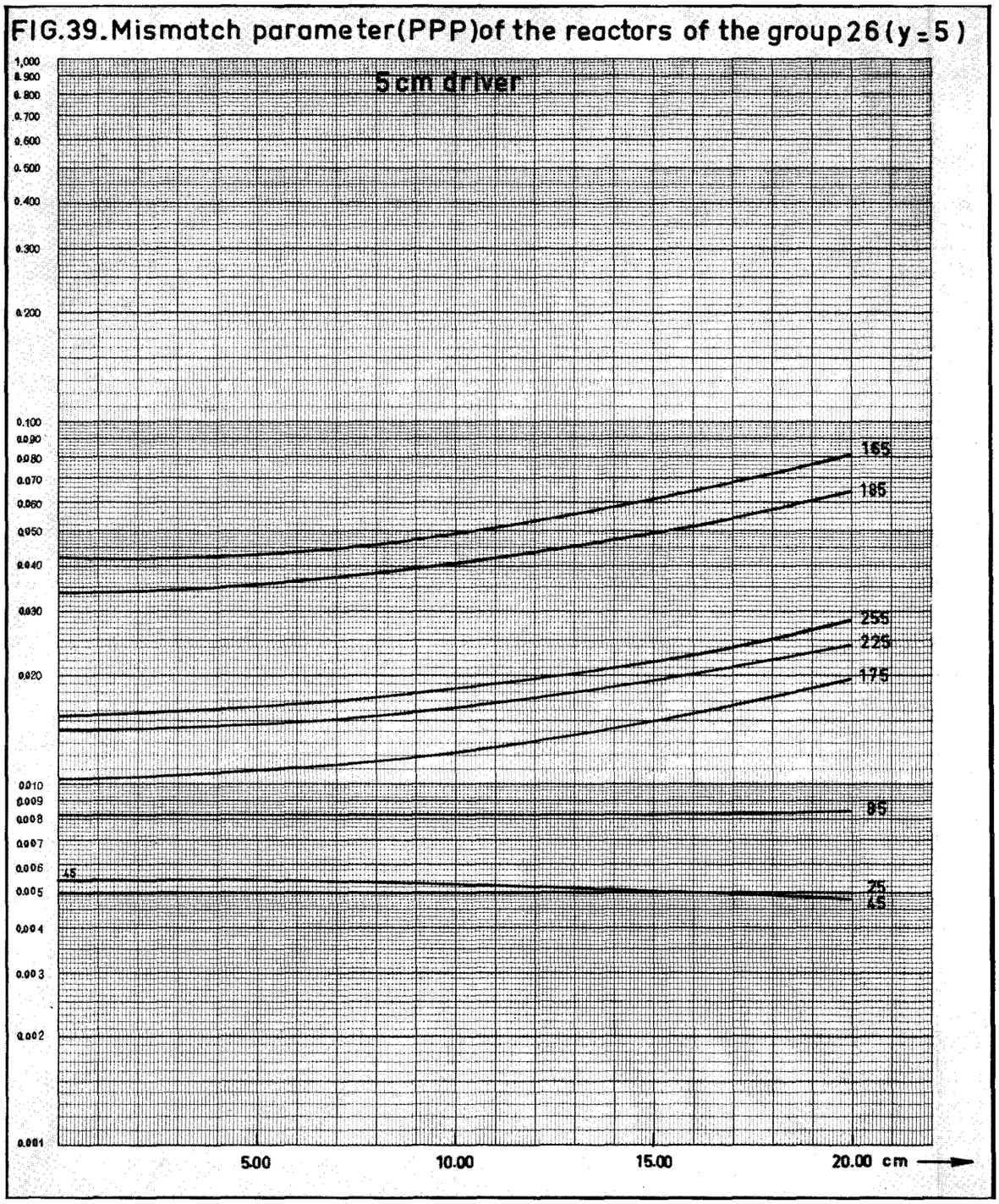


FIG.39. Mismatch parameter (PPP) of the reactors of the group 26 (y = 5)
5 cm driver
1,000
0.900
0.800
0.700
0.600
0.500
0.400
0.300
0.200
0.100
0.090
0.080
0.070
0.060
0.050
0.040
0.030
0.020
0.010
0.009
0.008
0.007
0.006
0.005
0,004
0.003
0.002
0.001
165
185
255
225
175
85
45
25
45
5.00
10.00
15.00
20.00 cm

FIG. 40. Mismatch parameter (PPP) of the reactors of the group 36 (y = 6)

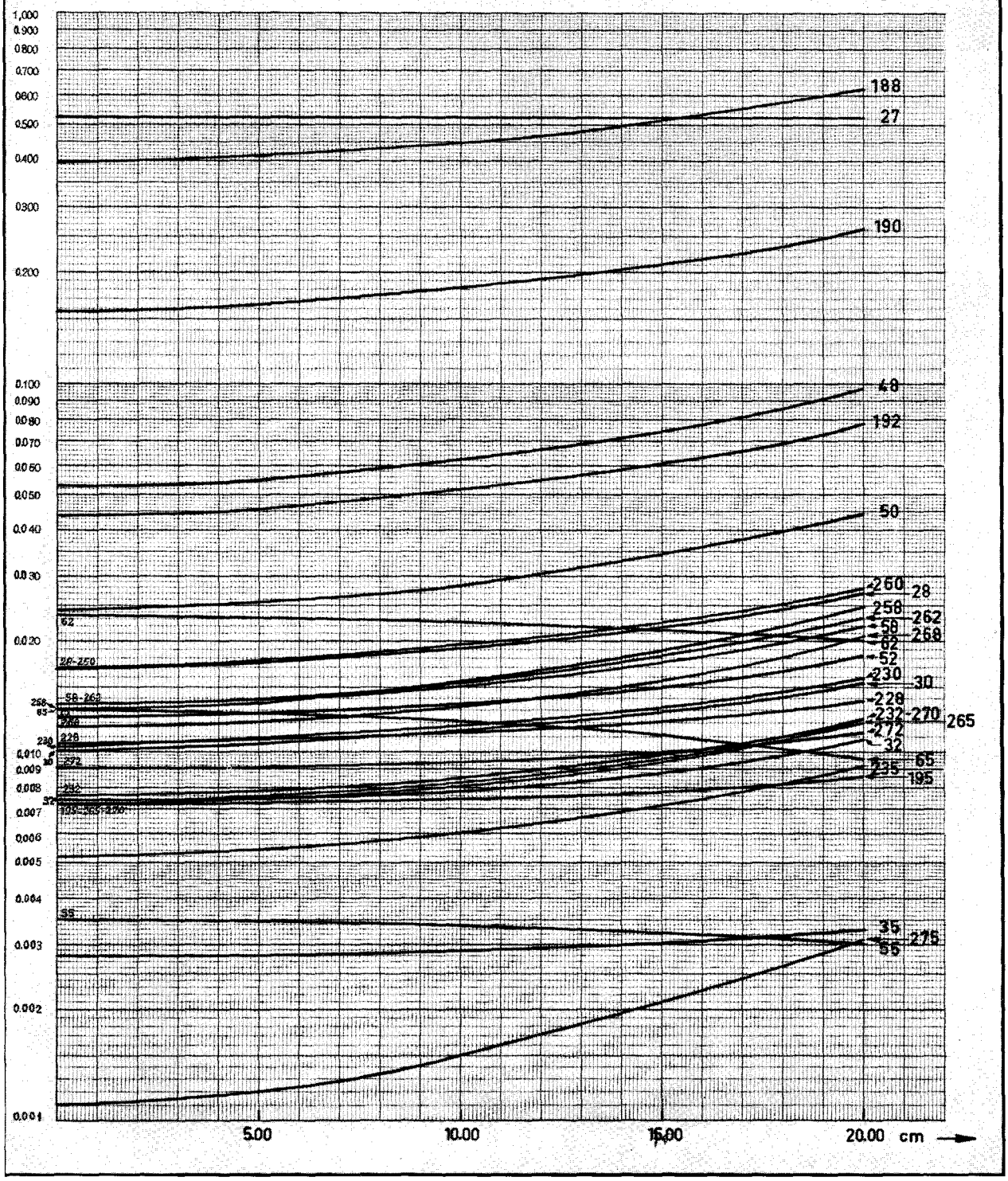


FIG.41_Mismatch parameter(PPP)of the reactors of the group 36 (y = 6)

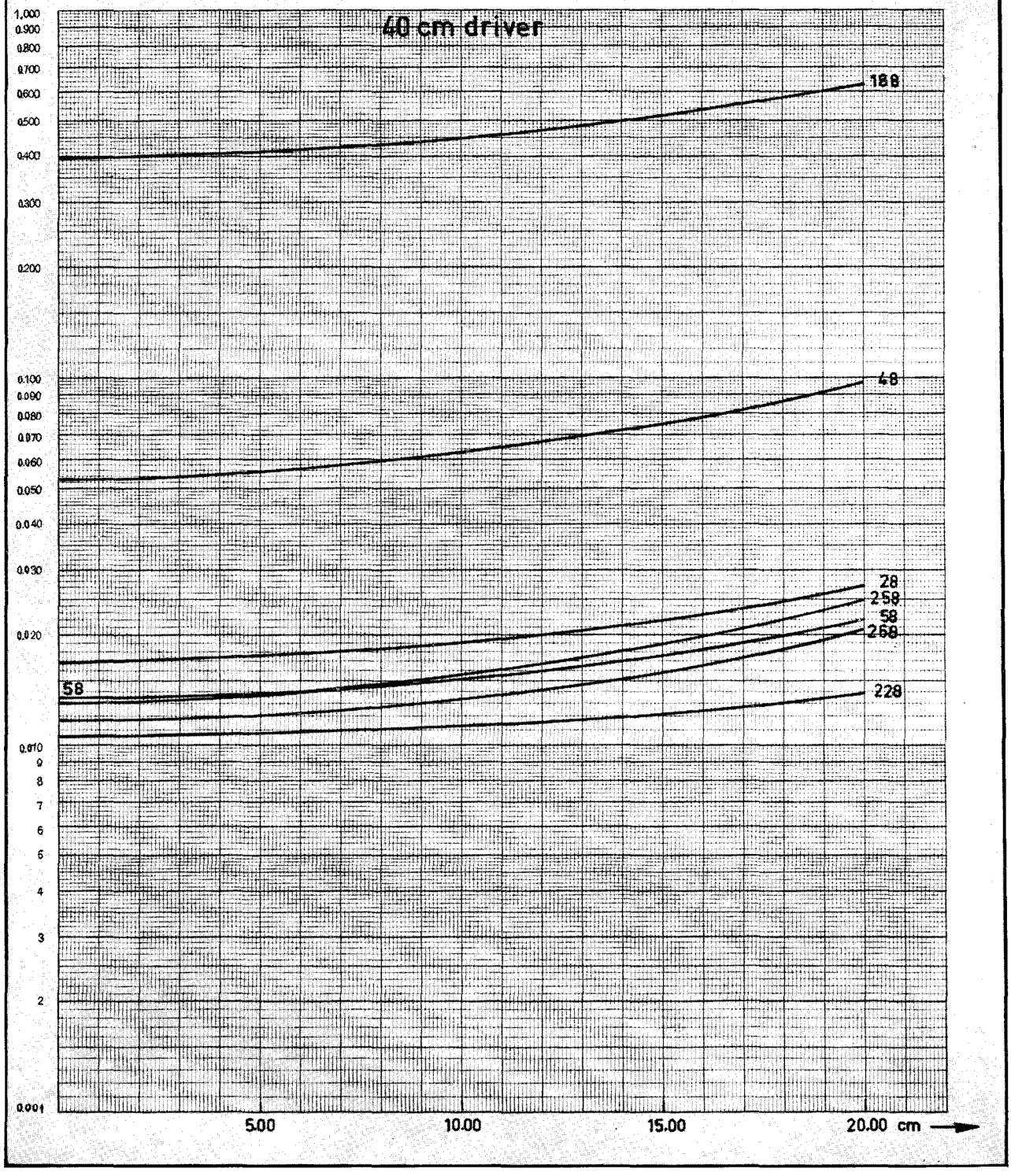


FIG.42.Mismatch parameter(PPP)of the reactors of the group 36 (y= 6)

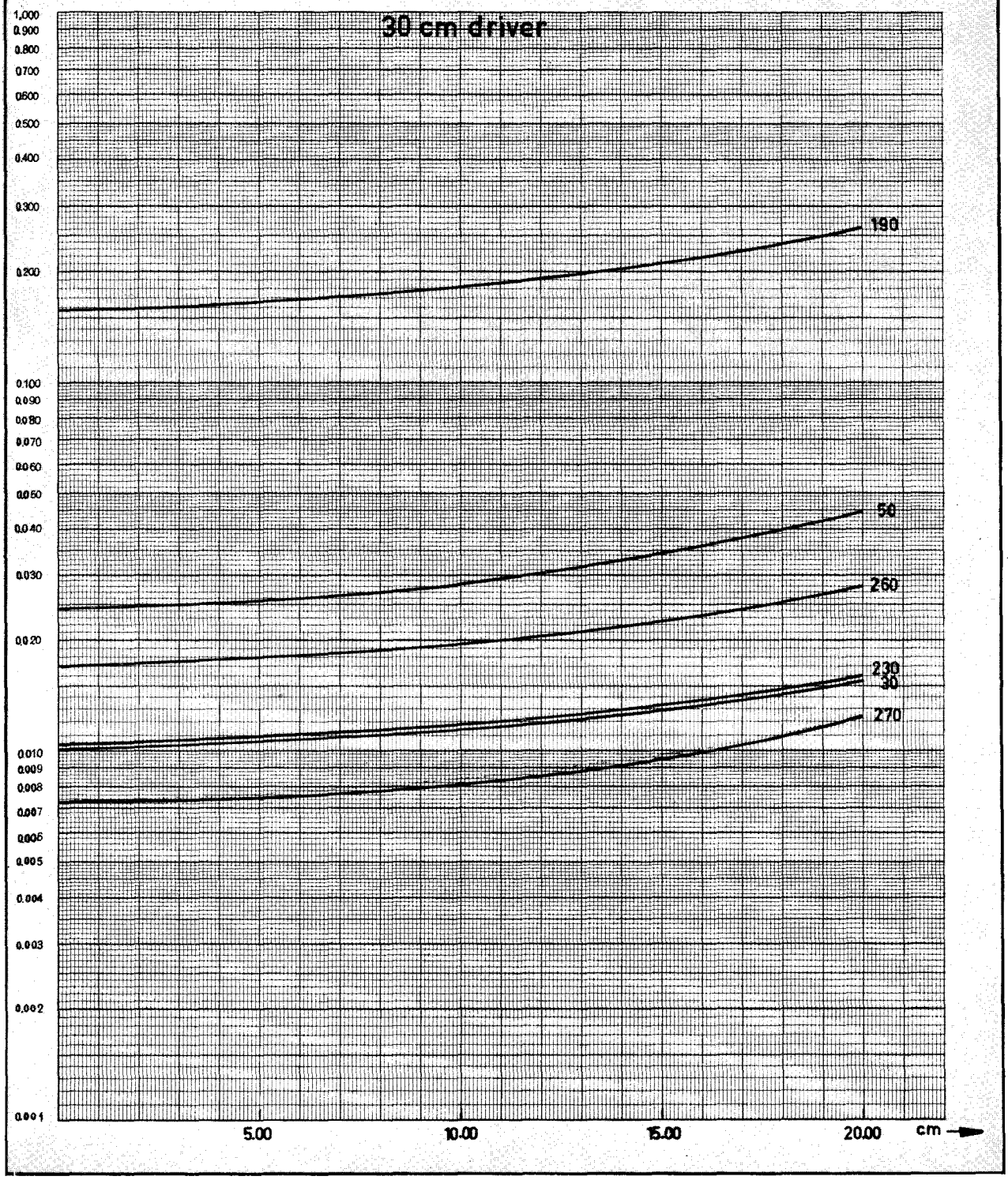


FIG. 43. Mismatch parameter (PPP) of the reactors of the group 36 (y = 6)

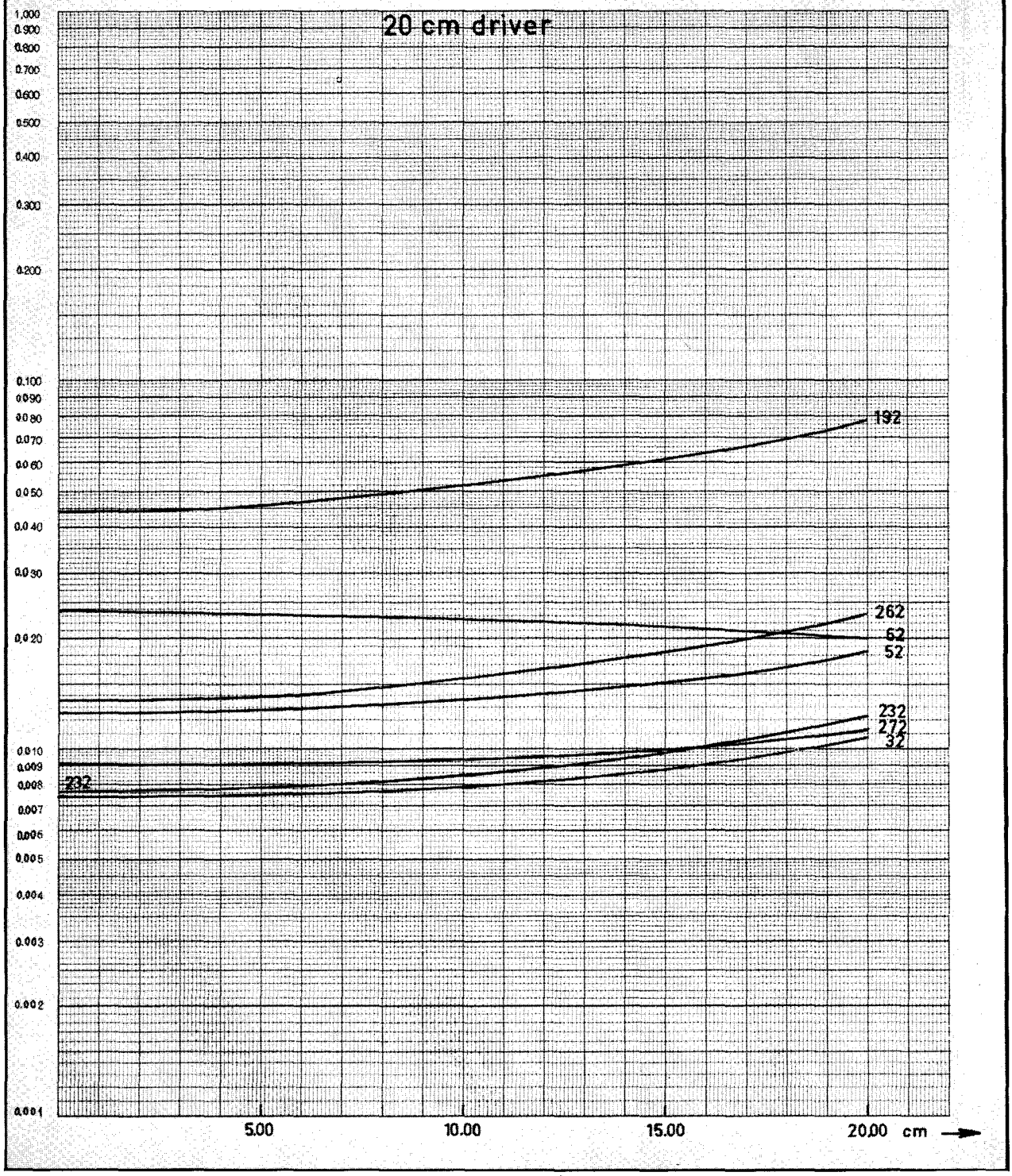
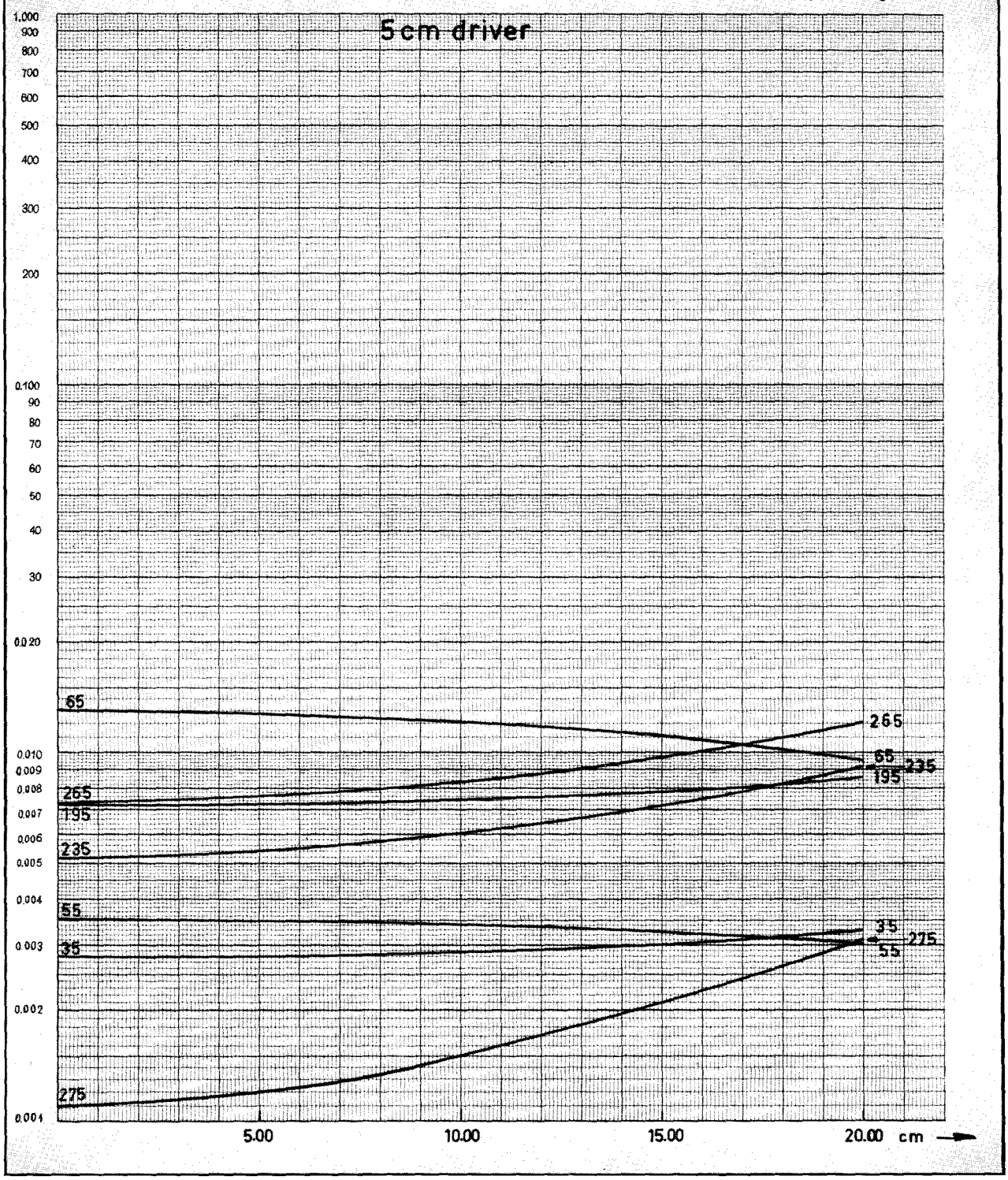


FIG. 44. Mismatch parameter (PPP) of the reactors of the group 36 (y = 6)



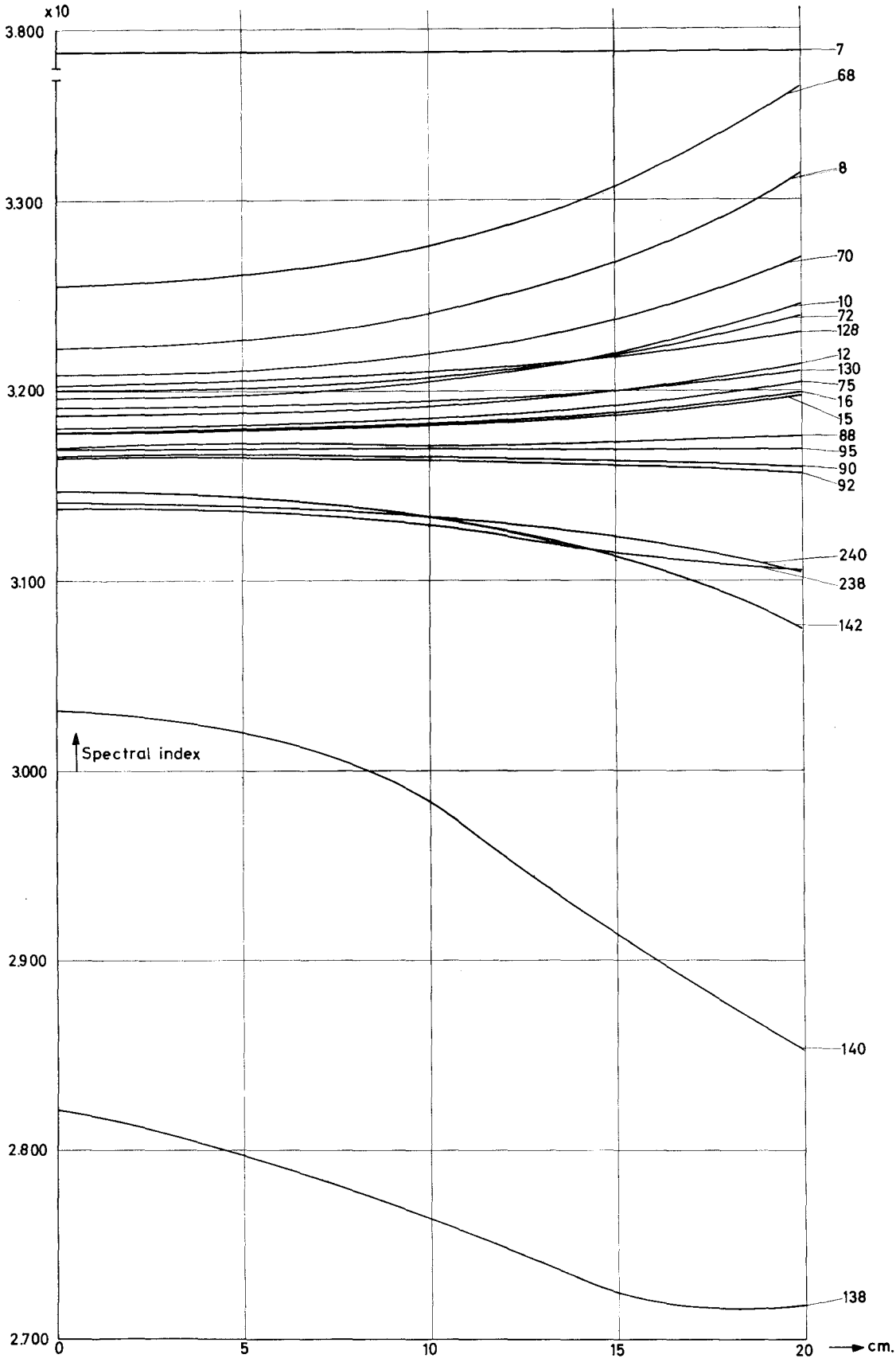


Fig. 45
Spectral index for Pu-239 (reference isotope: U-238) for reactors having $y_1 = 4$.

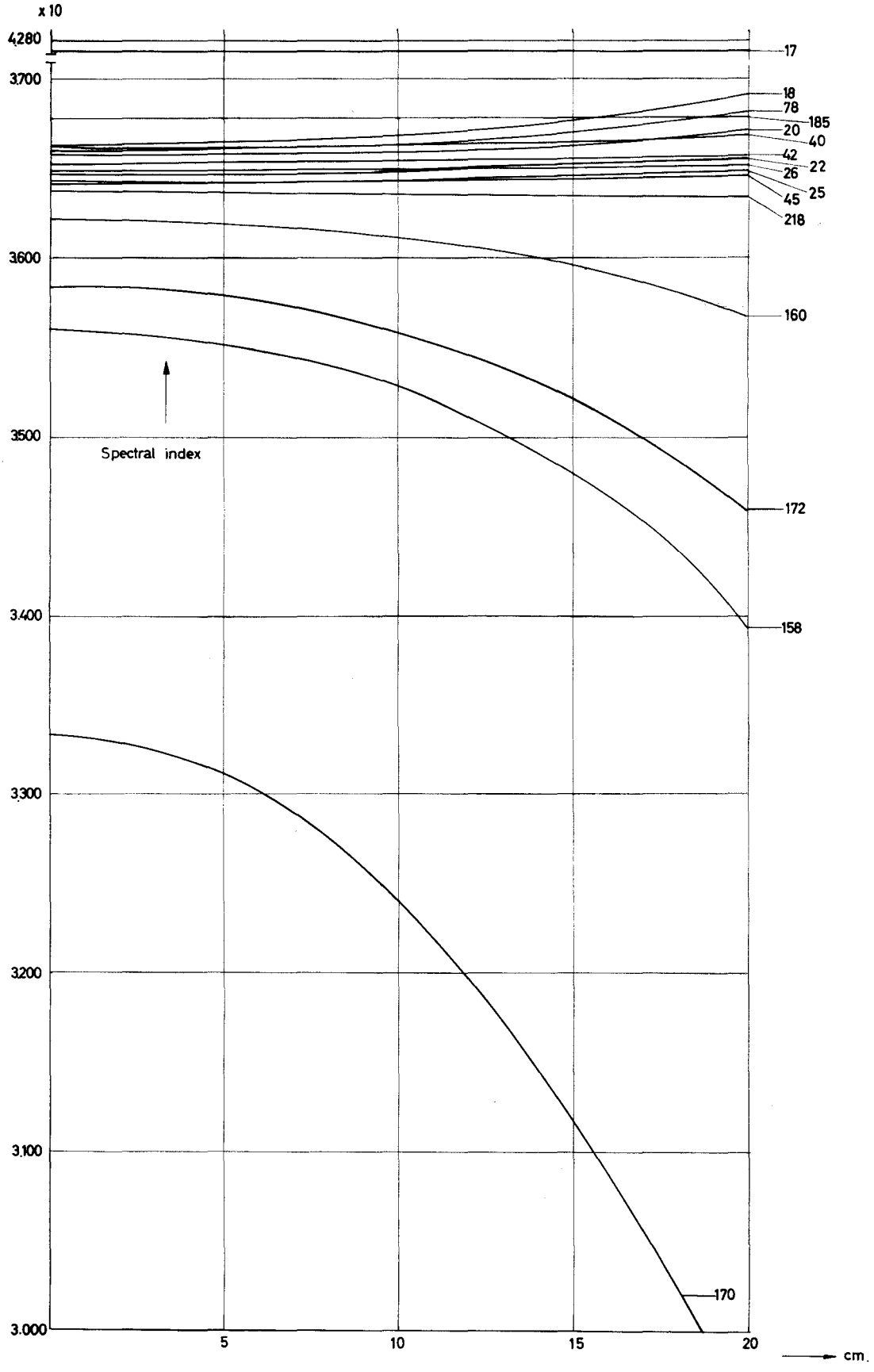


Fig. 46
Spectral index for Pu-239 (reference isotope: U-238) for reactors having $y_1 = 5$.

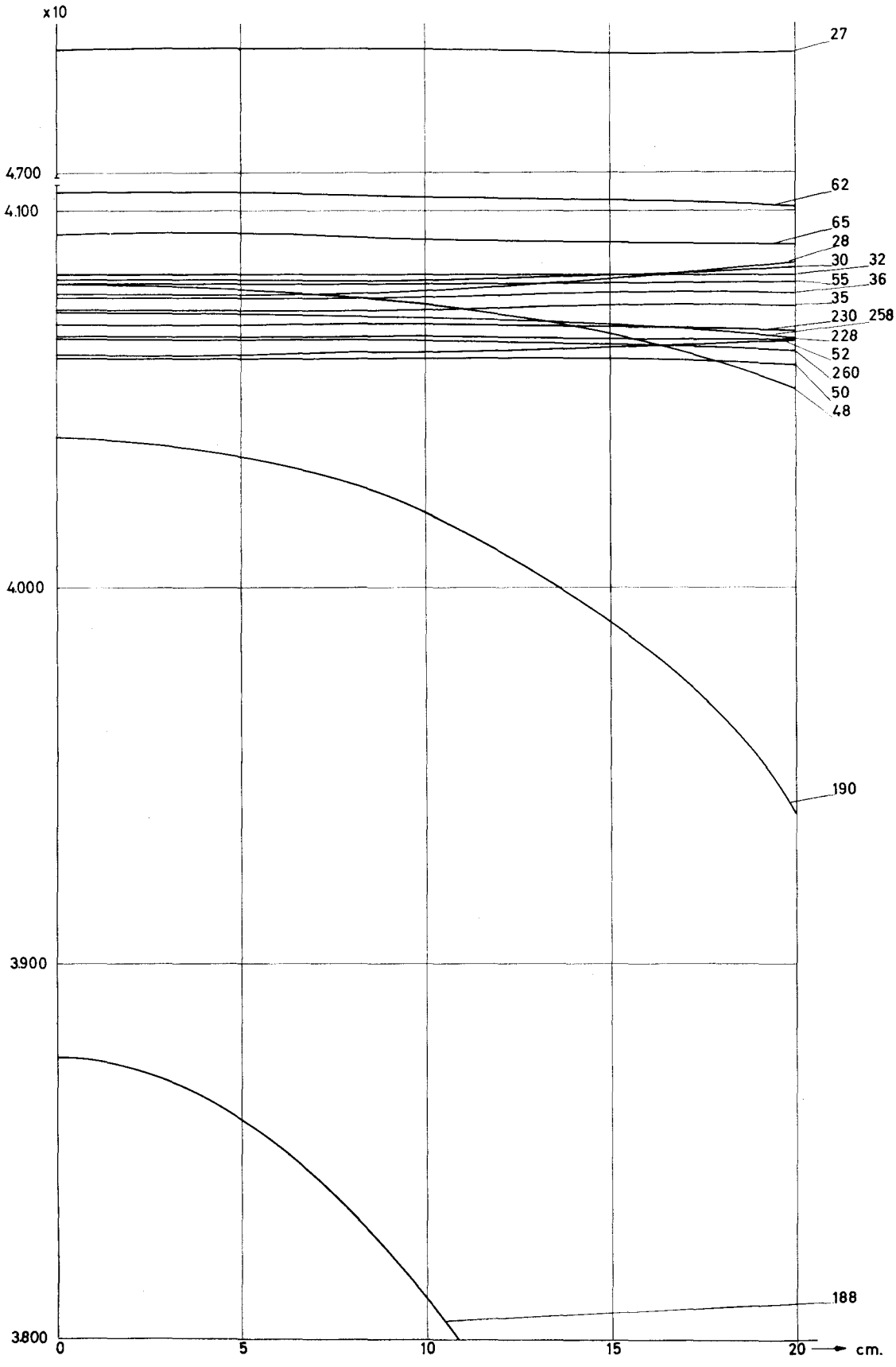


Fig. 47
Spectral index for Pu-239 (reference isotope: U-238) for reactors having $y_1 = 6$.

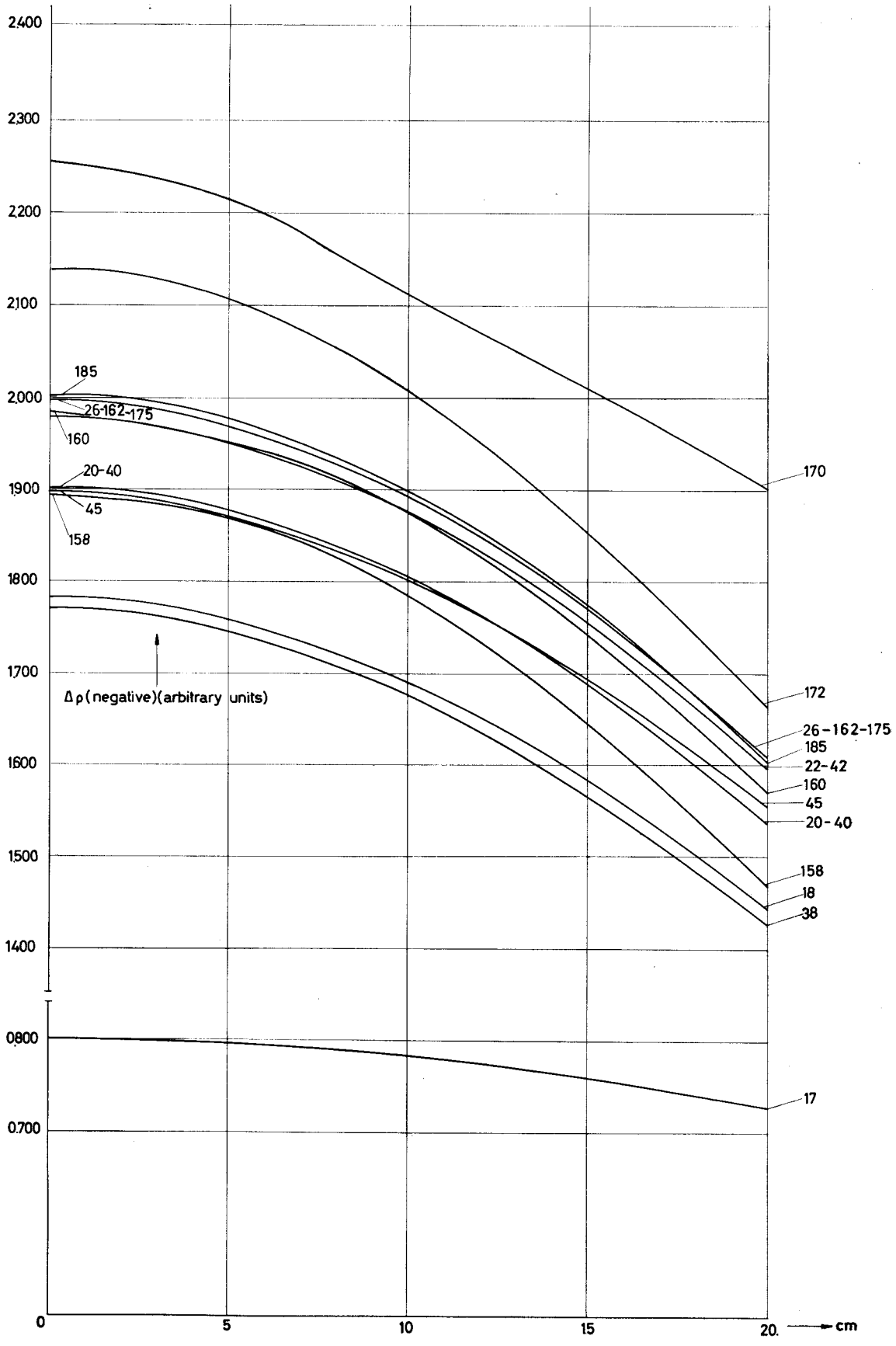


Fig. 48
Reactivity variation due to a B-*nat sample introduction at different points in reactors having* $y_1 = 5$.

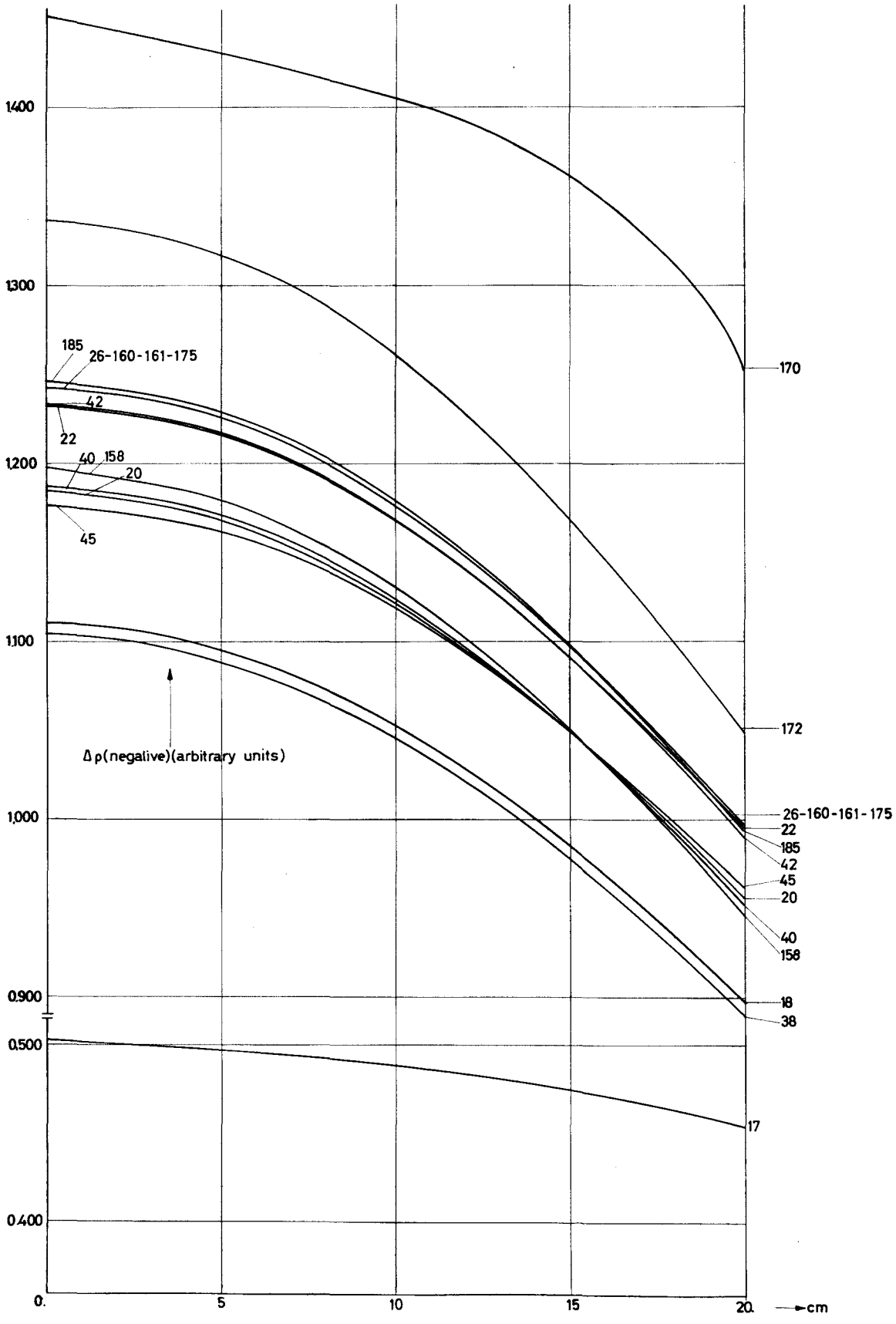


Fig. 49
Reactivity variation due to a $^1/_v$ absorber sample introduction at different points in reactors having $y_1 = 5$.