

**EUR 5039 e**

COMMISSION OF THE EUROPEAN COMMUNITIES

**PRESSURE HISTORY DURING FLASHING CAUSED  
BY A SUDDEN EXPANSION**

by

G. FRIZ and W. RIEBOLD

1974



**Joint Nuclear Research Centre  
Ispra Establishment - Italy  
Technology Division**



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Luxembourg, May 1974 - 32 Pages - 7 Figures - B.Fr. 50,—

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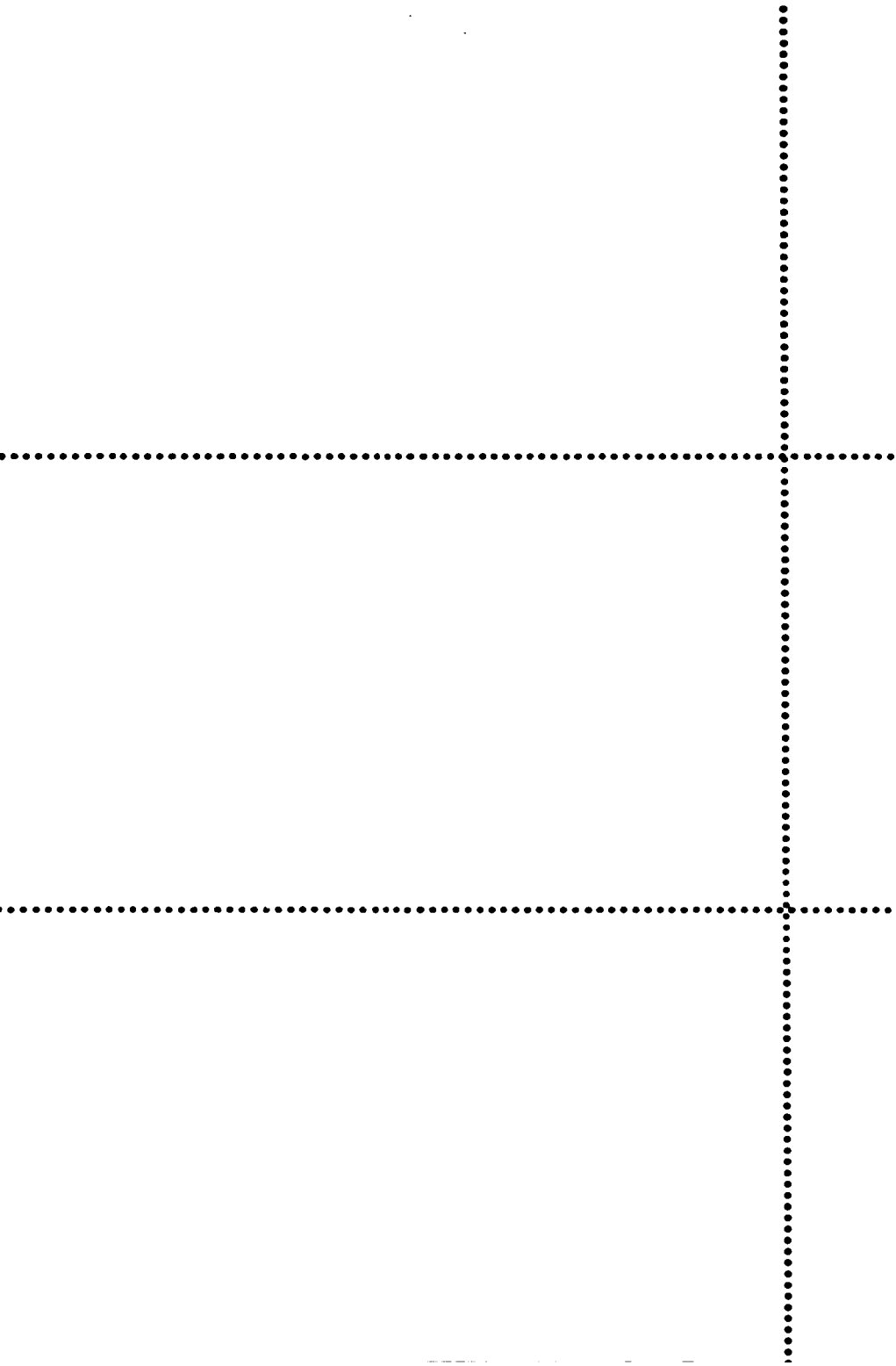
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## **ABSTRACT**

The pressure history in a flashing liquid which follows a volume step is calculated on the basis of the « thin thermal layer model » considering also the rise of the bubbles formed during the initial volume step. The number  $N$  of bubbles per  $\text{cm}^3$  is introduced as a parameter. The numerical calculations were done for water in the temperature range from  $280^\circ$  to  $340^\circ$  C.  $N$  varied from 1.0 to  $10^4$  ( $1/\text{cm}^3$ ). Pressure curves and the « half value time  $t_h$  » of return to the equilibrium pressure are presented as functions of initial water temperature, bubble number and initial pressure step.

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## 1. Introduction

The knowledge of the flashing process, i. e. a sudden formation and growth of a high number of vapour bubbles caused by a quick expansion of a liquid, is of importance for reactor safety considerations. This phenomenon occurs in the primary cooling circuit of a pressurized water reactor, when a rupture in the system leads to a rapid pressure fall. During the flashing the mixture of water and vapour bubbles deviates strongly from the thermodynamic equilibrium. This means that the pressure-volume relation of the mixture, which is well defined in thermodynamic equilibrium, does not hold during the flashing period.

In the last years several computer codes have been developed to calculate the thermohydraulic history in a primary circuit after a rupture in the system, as for example: BRUCH-D from LRA-GARCHING, the RELAP-Series from GENERAL ELECTRIC, the KAPCOR-code from GAAA in France and others. All these codes contain the assumption of thermodynamic equilibrium. As a consequence of this fact these codes cannot exactly describe the first period after the rupture, during which the flashing water deviates from the equilibrium.

Recently, first steps have been done to consider non-equilibrium in blow down codes with simplified models. In BLAST 2, for example, the evaporator rate is assumed to be proportional to the difference between water and vapour temperature. The unknown factor of proportionality was chosen on the basis of a comparison between calculations and experiments. Interesting results have been published on the recent Meeting of Reactor Heat Transfer at Karlsruhe (Oct., 1973)<sup>(1, 2)</sup>.

Nevertheless, there remains the task to clarify the nature and the numerical values of the empirical factors in function of initial and blow down conditions. Furthermore, it has to be studied whether these models are suffi-

ciently realistic to describe well the pressure-volume relation during flashing or if another, more sophisticated, theoretical description must replace the simple formulas.

This lack of information induced the German Reactor Licensing authorities to sponsor a research contract with the C. C. R. -Ispra for an experimental study of non-equilibrium phenomena, especially the flashing process.

To describe quantitatively the deviation from thermodynamic equilibrium of the flashing water, both the volume and the pressure must be known simultaneously. The planned experimental apparatus allows a controlled variation of the volume and a dynamic measurement of the pressure, i. e. in the experiments the volume represents the independent variable and the pressure characterizes the deviation from thermodynamic equilibrium. The measuring program will start with the most simple volume-time function, that is a stepwise increase of the volume.

This report deals with the theoretical description of the pressure history which follows such a step-function of the volume.

## 2. Theory

### 2.1 Assumptions and Neglections

We assume that the water is completely free from gas- or vapour bubbles and has a uniform temperature before the flashing starts. The volume step causes, under these conditions, a pressure step which follows the adiabatic line of the liquid phase. At this point arises the first problem: flashing occurs in the superheated region, i. e. when the pressure  $p$  has fallen below the saturation value  $p_0$ . In this region, however, table values of the specific volume of liquid water do not exist. For this reason the value of

$$\left(\frac{\partial p}{\partial v_1}\right)_{s=\text{const}} \quad \text{at } T = T_o = T_{\text{sat}}$$

was maintained within the superheated region and applied to the liquid phase throughout the whole flashing process.

It is assumed that a certain number  $N$  of bubbles per  $\text{cm}^3$  is created simultaneously with the stepwise expansion, i. e. the creation time delay has been neglected. This number  $N$  represents the most uncertain factor in the theory. In reality it will depend on several factors, as for example: initial temperature, impurities, gas content etc. The bubble density  $N$  will be, for this reason, introduced as a parameter and varied in a wide range.

The growth velocity of the created bubbles is mainly determined by the heat conduction in the liquid. The influence of inertia and surface tension forces, vanishes very quickly. This was shown by FOSTER and ZUBER<sup>(3)</sup>, who developed an equation for the bubble growth near the critical size. Applying their formula to water at  $320^\circ\text{C}$ , the calculated time to reach the size, where these forces are negligible was very small compared with the time scale which is expected in our experiments (from  $10^{-2}$  to 0.5 sec for the "half value time" of the pressure). For this reason inertia and surface tension have been disregarded in the theory. The experiments only, however, can decide whether this neglect was admissible or not.

During the bubble growth the liquid around the bubble moves in radial direction. This movement deforms the shape of the radial temperature distribution. The effect increases markedly the heat flux to the bubble and has to be taken into consideration. This was done by introducing the "thin thermal layer hypothesis" in the model. This hypothesis allows a considerable simplification of the mathematical formalism giving, ne-



vertheless, very satisfactory results.

The interaction of the temperature fields of neighboured bubbles has also been disregarded, which leads to a small error of the asymptotic value of the pressure curves (at 320°C about 0.7% of the initial pressure step).

The rise of the bubbles by buoyancy forces, however, has to be considered. The increased heat transfer by bubble rise leads to a faster return to thermodynamic equilibrium. By using the NUSSELT-number of small spheres in a stationary flow, this additional heat transfer has been - in an approximative way - taken into account.

During the whole flashing process, the bubbles are assumed to have a spherical shape.

With these major assumptions the problem has been formulated in terms of a differential-integral equation. The derivation of this equation will be outlined in the next chapters.

## 2.2 The Single Bubble Growth Law

The spherical bubble grows by evaporation from the bubble surface. A certain mass flow  $\dot{M}$  increases the bubble volume  $V_B$  following the equation:

$$\dot{M} = \frac{d}{dt} (\rho_v \cdot V_B) \quad (1)$$

With: 
$$V_B = \frac{4}{3} \pi R^3 \quad (2)$$

(R = bubble radius)

and the mass flow per cm<sup>2</sup> bubble surface:

$$\dot{m} = \dot{M}/4\pi R^2 \quad (3)$$

we obtain:

$$\dot{m} = \rho_v \frac{dR}{dt} + \frac{1}{3} \cdot R \cdot \frac{d\rho_v}{dt} \quad (4)$$

Assuming a constant vapour temperature  $T_o$  (this assumption leads to an error of about 1%), we obtain:

$$\rho_v / \rho_{vo} = p / p_o \quad (5)$$

and equation (4) becomes:

$$\frac{dR}{dt} + \frac{1}{3} \cdot \frac{R}{p} \cdot \frac{dp}{dt} = \frac{\dot{m}}{\rho_v} \quad (4a)$$

The evaporation requires the heat flux  $q$  (per  $\text{cm}^2$ ) to the surface:

$$q = -L \cdot \dot{m} \quad (6)$$

(The positive heat flux is, as usual, defined in positive  $r$ -direction. A positive evaporation  $\dot{m}$  requires thus a negative heat flux).

With (6) equation (4a) becomes:

$$\frac{dR}{dt} + \frac{1}{3} \cdot \frac{R}{p} \cdot \frac{dp}{dt} = \frac{-q}{L \cdot \rho_v} \quad (4b)$$

### 2.3 The Pressure- Bubble Volume Relation

Within each  $\text{cm}^3$  of the initial water volume  $V_{10}$  a number  $N$  of bubbles is created, forming a bubbly two-phase mixture. The bubbles are assumed to have all the same diameter.

The water phase of this mixture follows the adiabatic line. We characterize this line by the adiabatic elasticity coefficient  $\alpha$  which is defined by:

$$\alpha = - \frac{v_{10}}{p_o} \cdot \left( \frac{\partial p}{\partial v_1} \right)_s \quad (\text{at } T = T_o) \quad (7)$$

Remaining (with the liquid phase) on the adiabatic line throughout the whole flashing process  $v_1$  depends only on  $p$  and we can write:

$$\frac{dp}{p_o} = - \frac{\alpha}{v_{10}} \cdot dv_1 \quad (7a)$$

Introducing  $v_1 = V_1 / M_1$  and finite differences instead of differentials, one obtains:

$$\frac{p-p_o}{p_o} = \alpha \left( \frac{V_1 - V_{10}}{V_{10}} - \frac{M_1 - M_{10}}{M_{10}} \right) \quad (7b)$$

The actual water volume  $V_1$  is equal to the initial volume  $V_{10}$  (before flashing) plus the volume step  $\Delta V$  (which causes the flashing) minus the sum of the bubble volumes, hence:

$$V_1 = V_{10} + \Delta V - V_{10} \cdot N \cdot V_B \quad (8)$$

The initial water mass  $M_{10}$  is diminished by evaporation of the mass  $M_v$ , i. e.:

$$M_1 = M_{10} - M_v \quad (9)$$

or:  $-(M_1 - M_{10}) / M_{10} = M_v / M_{10} \quad (9a)$

With the relation:

$$M_v / M_{10} = \frac{\rho_v \cdot V_B \cdot N}{\rho_l} \quad (10)$$

and introducing:  $\rho_{vo} / \rho_{10}$  instead of  $\rho_v / \rho_l$  the pressure relation becomes:

$$\frac{p_o - p}{p_o} = \alpha \left[ \frac{\Delta V}{V_{10}} - N \cdot V_B \cdot (1 - \rho_{vo} / \rho_{10}) \right] \quad (11)$$



We now define a certain bubble radius  $R_o$  which has the following meaning: when the bubbles have reached the radius  $R_o$ , the pressure has returned to the equilibrium pressure  $p_o$ . For  $R_o$ , we obtain the expression:

$$R_o^3 = \frac{\Delta V}{V_{10}} \cdot \frac{3}{4\pi \cdot N \cdot (1 - \rho_{vo}/\rho_{10})} \quad (12)$$

and (11) can be written as:

$$\frac{p}{p_o} = 1 - \alpha \cdot \frac{\Delta V}{V_{10}} \cdot \left[ 1 - (R/R_o)^3 \right] \quad (11a)$$

Immediately after the initial volume step  $\Delta V$ ,  $R$  is still zero and the pressure jumps down to a value called  $p_m$ . From equation (11a) follows:

$$\frac{p_m}{p_o} = 1 - \alpha \cdot \frac{\Delta V}{V_{10}} \quad (13)$$

Introducing (13) in (11a) we obtain:

$$\frac{p}{p_o} = \frac{p_m}{p_o} + \left( 1 - \frac{p_m}{p_o} \right) \cdot \left( \frac{R}{R_o} \right)^3 \quad (14)$$

#### 2.4 The Heat Flux to the Bubble

Each single bubble, expanding by evaporation, requires a certain heat flux, which is transported by heat conduction from the liquid phase to the bubble surface. According to PLESSET and ZWICK<sup>(4)</sup> and other authors, we assume now that the temperature fall from liquid bulk temperature  $T_o$  to the surface temperature  $T$  occurs in a small "thermal boundary layer  $b$ " which is defined by:

$$\frac{T_o - T}{b} = \left( \frac{\partial \vartheta}{\partial r} \right)_{r=R} \quad (15)$$

(with  $\vartheta = \vartheta(r)$  being the temperature profile in the liquid).

If  $b$  is small compared to the bubble radius and the medium around the bubble is at rest, the temperature gradient is well known as function of time:

$$\left(\frac{\partial \theta}{\partial r}\right)_{r=R} = \frac{T_o - T}{\sqrt{\pi K_o t}} \quad (16)$$

( $K$  = thermal diffusivity)

and we have the relation:

$$b = \sqrt{\pi K_o t} \quad (17)$$

By differentiation one obtains:

$$\frac{db}{dt} = \frac{\sqrt{\pi K}}{2\sqrt{t}} = \frac{\pi K}{2} \cdot \frac{1}{b} \quad (18)$$

We now must consider, however, that the liquid around the bubble is moving in radial direction and the shape of the temperature curve is continuously deformed.

To obtain the total chagement of  $b$  we introduce the bubble radius  $R$ , which characterizes the movement, as second variable, i. e.:

$$b = b(t, R) \quad (19)$$

From this follows:

$$\frac{db}{dt} = \left(\frac{\partial b}{\partial t}\right)_{R=\text{const}} + \left(\frac{\partial b}{\partial R}\right)_{t=\text{const}} \cdot \frac{dR}{dt} \quad (19a)$$

We now identify the first term on the right hand of (19a) with equation (18), because (18) was derived considering a conducting medium at rest ( $R=\text{const}$ ), hence:

$$\left(\frac{\partial b}{\partial t}\right)_{R=\text{const}} = \frac{\pi K}{2} \cdot \frac{1}{b} \quad (18a)$$

The second term in (19a) follows from the continuity equation: at  $t = \text{const}$  we consider the deformation of  $b$  only by the movement of the

conducting liquid. As  $b$  at  $dt = 0$  does not spread out relative to the liquid, the volume occupied by the boundary layer  $b$  must remain constant, i. e.

$$b \cdot R^2 = \text{const} \quad (\text{for } t = \text{const})$$

or:

$$\left( \frac{\partial b}{\partial R} \right)_{t=\text{const}} = -2 \frac{b}{R} \quad (20)$$

Equation (19a) now becomes:

$$\frac{db}{dt} = \frac{\pi K}{2} \cdot \frac{1}{b} - 2 \frac{b}{R} \frac{dR}{dt} \quad (21)$$

which can also be written in the form:

$$\frac{d(b^2)}{dt} + b^2 \frac{4}{R} \cdot \frac{dR}{dt} = \pi \cdot K \quad (22)$$

This is a first order linear differential equation for  $b^2$  where  $R$  is a pure time function (in our case even a monotonous time function).

The general solution of this equation is:

$$b^2 = \frac{\pi K}{R^4} \cdot \int_{t'}^t R^4 d\varphi + \frac{C}{R^4} \quad (23)$$

with an arbitrary lower limit  $t'$ . The solution, which obeys the initial condition  $b = 0$  at  $t = t'$ , is the interesting one in our problem. This leads to  $C = 0$  and thus we write:

$$b^2 = \frac{\pi K}{R^4} \cdot \int_{t'}^t R^4 d\varphi \quad (23a)$$

From (23a) we obtain now the differential of the heat flux caused by a temperature step  $dT$  occurring at the time  $t'$  at the bubble surface:

$$dq_{\text{inst}}(t, t') = \lambda \cdot \frac{dT}{b} = \frac{\lambda \cdot dT}{\sqrt{\pi K}} \cdot \frac{R^2(t)}{\left[ \int_{t'}^t R^4 d\varphi \right]^{1/2}} \quad (24)$$



The heat flux at the time  $t$  is obtained by integrating (24) over  $t'$  from 0 to  $t$ :

$$q_{inst} = \frac{\lambda}{\sqrt{\pi K}} \cdot R^2 \int_0^t \frac{\left(\frac{dT}{dt}\right)_{t=t'} dt'}{\left[\int_{t'}^t R^4 d\phi\right]^{\frac{1}{2}}} \quad (25)$$

The index "inst" was used to distinguish between this (insteady) part of the total heat flux and the heat flux by bubble rise, which has now to be considered. The rise of the bubbles, caused by the buoyancy forces, results in an additional heat flux which, relative to  $q_{inst}$ , increases with increasing bubble diameter. A thorough study of DERGARABEDIAN's measurements<sup>(5)</sup> gives an indication of the existence of this effect. With higher bubble radius the measuring points tend to lie above the theoretical curve calculated with a heat flux following equation (25).

It is almost hopeless to pretend an exact theoretical description of this problem. It would require the simultaneous solution of the NAVIER-STOKES equation and the heat conduction equation in a moving liquid, a highly complex mathematical problem. For this reason, the additional heat flux by bubble rise has been taken into account in an approximative way. To this end the well known formula for the heat flux to spheres in a stationary flow<sup>(6)</sup> has been simply added to the flux  $q_{inst}$  in equation (25). Neglecting the effect of PRANDTL'-number (in the interesting range is  $Pr \approx 1$ , and in the formula appears only  $Pr \approx 1/3$ ) we have:

$$\tilde{q}_{rise} = \frac{\lambda}{2R} (2 + 0.37 \cdot Re^{0.6})(T - T_0) \quad (26)$$

Test calculations, however, with  $\tilde{q}_{rise}$  added to  $q_{inst}$  resulted in a too high heat flux in the early stage of bubble growth. This can be explained by the fact, that the displacement of the bubbles by rise during the first

milliseconds is small compared to the bubble diameter and the temperature field, required for the heat flux  $\tilde{q}_{\text{rise}}$  of (26), is not yet developed. Therefore, the velocity dependent part of (26) was corrected by multiplying it with a time dependent function f:

$$f = 1 - e^{-\beta \mu^2} \quad (27)$$

with  $\mu = W_{\text{rise}} \sqrt{\frac{dR}{dt}}$ .

The factor  $\beta$  was then chosen to obtain the best fit with DERGARABEDIAN's measurements.

With this correction the total heat flux to the bubble becomes:

$$q = \frac{\lambda}{\sqrt{\pi K}} \cdot R^2 \int_0^t \frac{\left(\frac{dT}{dt}\right)_{t=t'} dt'}{\left[ \int_{t'}^t R^4 d\phi \right]^{\frac{1}{2}}} + \frac{\lambda}{2R} (2 + 0.37 \text{Re}^{0.6} \cdot f) (T - T_0) \quad (28)$$

The bubble rise velocity  $W_{\text{rise}}$  (necessary for f and Re) was obtained by equalizing buoyancy forces with friction forces. The calculation method is described in the appendix.

## 2.5 The Equations of Pressure History

We now dispose of all elements to set up the equations for the time curve of pressure p as a consequence of a volume step  $\Delta V$ .

At first we must correlate the temperature step  $\Delta T = T - T_0$  and the differential dT with a  $\Delta p$  and a dp respectively. To this end we identify the temperature T of the bubble surface with the saturation temperature belonging to the actual pressure p. This assumption was also made by PLESSET and ZWICK and led, as already mentioned, to a good agreement with the experiments. Furthermore, we linearize the saturation

line in the neighbourhood of  $T_o$  and obtain:

$$T - T_o = \left( \frac{dT}{dp} \right)_{\text{sat}} \cdot (p - p_o) \equiv \gamma \cdot (p - p_o) \quad (30)$$

and:  $dT = \gamma \cdot dp$

$\gamma$  is taken at the initial water temperature  $T_o$ .

Introducing the dimensionless variables:

$$\begin{aligned} R/R_o &\equiv \psi & \text{and } p/p_o &\equiv \psi \\ p_m/p_o &\equiv \psi_m \end{aligned} \quad (31)$$

we obtain from the equations (5), (4b) and (14) the growth law:

$$\frac{d\psi}{dt} = \frac{1}{2\psi - \psi_m} \cdot \frac{-q}{L \cdot \rho_{vo} \cdot R_o} \quad (32)$$

The integral in the heat flux equation (28) becomes with an initial pressure step from  $p_o$  to  $p_m$  at the time  $t = 0$ :

$$J = \int_0^t \frac{\left( \frac{dT}{dt} \right)_{t=t'} dt'}{\left[ \int_{t'}^t R^4 d\varphi \right]^{\frac{1}{2}}} = \gamma \cdot \frac{p_m - p_o}{\left[ \int_0^t R^4 d\varphi \right]^{\frac{1}{2}}} + \gamma \cdot \int_0^t \frac{\left( \frac{dp}{dt} \right)_{t=t'} dt'}{\left[ \int_{t'}^t R^4 d\varphi \right]^{\frac{1}{2}}} \quad (33)$$

The first term on the right hand is the result of the integration with the pressure step function only, the second term contains the pressure history after the step excluding the step function itself.

We introduce now the dimensionless time

$$\tau \equiv t \cdot \frac{K}{R_o^2} \quad ; \quad \tau' \equiv t' \cdot \frac{K}{R_o^2} \quad (34)$$

and a new variable defined by:

$$\sigma \equiv \int_0^{\tau} v^4 d\xi \quad \text{with } \xi \equiv \varphi \cdot \frac{K}{R_o^2} \quad (35)$$

From (35) follows:

$$d\sigma = v^4 d\tau \quad (35a)$$

and

$$\int_{\tau'}^{\tau} v^4 d\xi = \sigma(\tau) - \sigma(\tau') \equiv \sigma - \sigma' \quad (35b)$$

If  $\sigma$  is a monotonous function of  $\tau$  the integral J in (33) can be written:

$$J = \frac{\gamma p_o \sqrt{K}}{R_o^3} \left[ \frac{\psi_m^{-1}}{\sqrt{\sigma}} + \int_0^{\sigma} \frac{\left(\frac{d\psi}{d\sigma}\right)_{\sigma=\sigma'} d\sigma'}{\sqrt{\sigma - \sigma'}} \right] \quad (36)$$

The equations (14), (27), (32), (33) and (36) together give the growth law in the following dimensionless form:

$$\frac{d v^3}{d\sigma} = \frac{3BD}{1+2D v^3} \left[ \frac{1}{\sqrt{\sigma}} - \int_0^{\sigma} \frac{\left(\frac{d v^3}{d\sigma}\right)_{\sigma=\sigma'} d\sigma'}{\sqrt{\sigma - \sigma'}} + \frac{1-v^3}{v^3} (\sqrt{\pi} + 0.328 \cdot Re^{0.6} \cdot f) \right] \quad (37)$$

with the definitions:

$$B \equiv \frac{\gamma \cdot p_o \cdot c_1 \cdot \rho_{l0}}{\sqrt{\pi} \cdot L \cdot \rho_{v0}} \quad (c_1 = \text{specific heat of the liquid})$$

$$D \equiv (1 - \psi_m) / \psi_m$$

$$f \equiv 1 - e^{-\beta \mu^2}$$

$$\mu \equiv W_{\text{rise}} / \frac{dR}{dt} \quad (37a)$$

Equation (37) relates the dimensionless volume differential  $d(v^3)$  to  $d\sigma$ , which is linked with the time differential  $d\tau$  by (35a). Equation (14) combines  $v^3$  with the dimensionless pressure  $p/p_o$ , hence the desired pres-

sure-time function as a consequence of a volume step can be calculated. The REYNOLDS-number  $Re$  and the function  $f$  must be calculated with  $W_{rise}$ , which is determined by the actual bubble diameter.

## 2.6 The Method of Numerical Calculation

The integral in (37) requires for each calculation step two memory places. The computer program must thus be written in such a form that the number of necessary steps to reach a certain accuracy results as small as possible. Test calculations with different possible independent parameters ( $d\tau$ ,  $d\sigma$ ,  $d(v^3)$ ,  $dv$ ) revealed  $dv$  to be the most convenient one.

With the definitions:

$$y \equiv \sqrt{\sigma}$$

and

$$I \equiv \int_0^{\sigma} \frac{\left(\frac{dv^3}{d\sigma}\right)_{\sigma=\sigma'} d\sigma'}{\sqrt{\sigma - \sigma'}} \quad (38)$$

equation (37) can be written:

$$\delta y = \frac{(1+2Dv^3) \cdot v^2 \cdot \delta v}{2DB \left[ 1 - I \cdot y + y \cdot \frac{1-v^3}{v^3} (\sqrt{\pi} + 0.328 \cdot Re^{0.6} \cdot f) \right]} \quad (39)$$

The integral  $I$  to be used in (39) for step number  $n+1$  was calculated in the form:

$$I = 2 \cdot \sum_{i=1}^n \left(\frac{\delta v^3}{\delta \sigma}\right)_i \left[ \sqrt{\sigma_{n+1} - \sigma_i} - \sqrt{\sigma_{n+1} - \sigma_{i+1}} \right] \quad (40)$$

where  $\left(\frac{\delta v^3}{\delta \sigma}\right)_i$  is a medium value taken over the step  $i$ .

As the variables  $y$  and  $v$  start at zero, the first step had to be cal-



culated separately. This was done by putting I and Re in (39) to zero and resolving the remaining equation in a small interval.

For each step the values of R and  $dR/dt$  of the previous step are taken for the calculation of  $W_{rise}$  and  $\mu$ .

In the appendix the listing of the FORTRAN IV-program is given and a confrontation of the code-symbols with the symbols used in these chapters.

### 3. Results and Conclusions

Fig. 1 shows the adiabatic elasticity coefficient  $\alpha$  of liquid water at the saturation line and Fig. 2 the factor B as function of temperature. The values have been obtained with the help of the steam tables<sup>(7)</sup>.

In Fig. 3 a comparison is made between calculated curves and experiments of DERGARABEDIAN. The curves were calculated with equation (37) considering, however, the experimental boundary condition  $p = \text{const}$ . This condition is fulfilled by maintaining zero the integral in (37). The figure shows the deviation of the measuring points from the theoretical curve without bubble rise (calculated with  $Re = 0$ ) and the good agreement with the curve which includes the bubble rise. The factor  $\beta$  resulting in the best fit was  $\beta = 0.0784$ . This factor was then maintained throughout the other calculations.

The pressure history for water at  $300^{\circ}\text{C}$  with different bubble densities N are given in Fig. 4. The initial pressure step is 10% of the equilibrium pressure  $p_0$ , i. e.  $\psi_m = 0.9$ . The pressure step is the consequence of a relative volume step  $\Delta V/V_0 = 1.47 \times 10^{-3}$ .

The time for the pressure to reach the value  $\psi_m + (1 - \psi_m) \cdot 0.5$  is

called the "half value time  $t_h$ ". It represents a characteristic measure of the time scale for the return to thermodynamic equilibrium;  $t_h$  depends on temperature  $T_o$ , initial pressure step (or volume step) and on bubble density  $N$ . Fig. 5 shows this half value time  $t_h$  as function of initial temperature  $T_o$  at a given bubble density of  $N = 100/\text{cm}^3$  and with an initial pressure step of 10% from equilibrium value ( $\psi_m = 0.9$ ). Fig. 6 shows  $t_h$  as function of  $\psi_m$  at a given temperature. Fig. 7 finally shows the influence of the bubble density  $N$  on the half value time with an initial temperature of  $T_o = 300^\circ\text{C}$ . The results indicate a reasonable range of time for return to equilibrium after a stepwise disturbance from 0.01 to 0.5 seconds.

The experiments now must decide which bubble density  $N$  describes in the best way the measured pressure history after a volume step.

As regards the application of the results in blow down codes, further work has to be done in the following two directions:

- the theory has to be extended to arbitrary volume-time functions;
- the resulting pressure-time functions must be compared to the results of the simplified models, as for example with that used in the BLAST-2 code, with the aim to improve these models or to replace them by more sophisticated formulations.

#### 4. Appendix

##### 4.1 Bubble Rise Velocity and Re-Number

The rise velocity  $W_{\text{rise}}$  of the bubbles is calculated making a simple counterbalance of buoyancy forces and flow resistance by friction. The effects of bubble acceleration and radial expansion and also the tangential movement of the bubble surface have been neglected, (i. e. we take the resistance law of solid spheres). These neglects are justified by the fact that also in the heat flux formula the effect of bubble rise is considered only in an approximative way and the resulting curves are fitted to the experimental results of DERGARABEDIAN by the choice of the empirical factor  $\beta$ .

The flow resistance is:

$$W = c_w \cdot \pi \cdot R^2 \cdot \frac{\rho_1}{2} \cdot W_{\text{rise}}^2 \quad (40)$$

The buoyancy force (with  $\rho_v \ll \rho_1$ ):

$$A = g \cdot \rho_1 \cdot \frac{4}{3} \pi R^3 \quad (41)$$

Equalizing (40) and (41) we obtain:

$$X = c_w \cdot Re^2 = R^3 \frac{32}{3} \cdot \frac{g \rho_1^2}{2} = \nu^3 \frac{32}{3} \frac{g \rho_1^2 R_o^3}{\eta^2} \quad (42)$$

with:  $Re = \rho_1 \cdot W_{\text{rise}} \frac{2R}{\eta}$

The grouping  $c_w \cdot Re^2$  can be correlated to Re (in the case of solid spheres) by the following equation:

$$\log Re = -1.2990 + 0.95798 \cdot X - 0.035276 \cdot X^2 \quad (43)$$

The determination of Re and  $\mu$ , necessary in equation (39) for the calculation of a certain step, is done in the following way:

From equation (12), the water properties  $\rho_1$  and  $\eta$  and the  $\nu$ -value of the previous calculation step we obtain with (42) the value X. With equation (43) we calculate Re and from Re we obtain:

$$W_{\text{rise}} = \text{Re} \cdot \frac{\eta}{\rho_1 2R} \quad (44)$$

and

$$\mu = W_{\text{rise}} / \frac{dR}{dt} \quad (45)$$

taking the values of R and  $dR/dt$  from the previous calculation step.

In the code the equations are slightly transformed by introducing non-dimensional values. Equation (42) can be written:

$$c_w \cdot \text{Re}^2 = \nu^3 \frac{8}{\pi} \cdot \underbrace{\frac{g \cdot \rho_1^2}{\eta^2 \cdot \alpha (1 - \rho_{vo}/\rho_{lo})}}_{\text{AT}} \cdot \frac{1 - \psi_m}{N} \quad (42a)$$

AT is a function of the water properties. This value must be given as entrance parameter in the code calculations.

Expression (45) can be transformed in

$$\mu = 0.5 \cdot \text{Pr} \cdot \text{Re} \cdot \frac{1}{\nu \cdot \frac{dy}{d\tau}} \quad (45a)$$

(In the code the grouping  $\beta \cdot \mu^2$  is called B2). The best fit value for  $\beta$  was:

$$\beta = 0.0784 \ .$$

#### 4.2 Listing of the Computer Code

From the listing, given hereafter, we understand that five entrance parameters are necessary for the calculations. These parameters are:

<u>Code Symbol</u>	<u>Report Symbol</u>
PM	$\psi_m$ (see 31)
B	B (see 37a)
AT	AT (see 42a)
PR	Pr (PRANDTL-number of water)
ZN	N (bubble density)

The values of dimensionless pressure, time and bubble radius are printed. Each 20th step is printed, i. e. in total 100 values, as the total step number is limited to 2,000. Independent variable is  $\nu$  (in the code R). The step  $\delta\nu$  (in the code called HR) is 0.0005.



LEVEL 20

MAIN

DATE = 73149

18/30/23

FLASHING

```

IMPLICIT REAL*8(A-H,D-Z)
DIMENSION S(2001),DVS(2000)
9 READ (5,1) PM,B,AT,PR,ZN,KN
1 FORMAT (5D15.6,I10)
WRITE (6,2) PM,B,AT,PR,ZN
2 FORMAT ('1',5X,'PM=',D15.6,5X,'B=',D15.6,5X,'AT=',D15.6,5X,'PR=',D
115.6,5X,'ZN=',D15.6//)
HR=0.0005D0
HRH=0.500*HR
H=HR*HR*HR
D=(1.000-PM)/PM
F=2.000*D
GR=0.500/D/B*HR
APN=(1.000-PM)/ZN
AR=AT*APN
QA=1.77245D0
A=(DSQRT(2.000*QA/(3.000*B*D)+1.000)-1.000)/QA/2.000
DY=A*H
Y=0.000
S(1)=0.000
T=0.000
SI=0.000
N=0
DT=0.000
10 N=N+1
Y=Y+DY
R=HR*DFLOAT(N)
V=R*R*R
X=DLG10(AR*V)
RELG=-1.299D0+0.95798D0*X-0.035276D0*X*X
RE=14.000**RELG
BETA=0.14000*PR*RE*DT/HR/R
IF(BETA.GT.10.000) GO TO 12
B2=BETA*BETA
EXB2=1.000-DEXP(-B2)
GO TO 13
12 EXB2=1.000
13 RE05=EXB2*RE**0.600
QNU=Y/V*(1.77245D0+0.328D0*RE05)*(1.000-V)
QN=1.000-Y*SI+QNU
IF(QN.LT.1.000-9) GO TO 50
S(N+1)=Y*Y
DS=S(N+1)-S(N)
DT=DS/(R-HRH)**4.000
T=T+DT
DY=GR*(1.000+F*V)*(R+HRH)**2.000/QN
DVS(N)=6.000*(R-HRH)*(R-HRH)*HR/DS
14 SS=0.000
DO 20 K=1,N
SK=(DSQRT(S(N+1)-S(K)) DSQRT(S(N+1)-S(K+1)))*DVS(K)
20 SS=SS+SK
SI=SS
IF(MOD(N,20).EQ.0) GO TO 40
IF(N.EQ.2000) GO TO 60
GO TO 10
40 P=PM+(1.000-PM)*V
WRITE (6,3) P,T,R
3 FORMAT ('1',10X,D15.6,10X,D15.6,10X,D15.6)
IF(N.EQ.2000) GO TO 60
GO TO 10
50 WRITE (6,4)
4 FORMAT ('1', 'NENNER QN NAHE NULL'//)
P=PM+(1.000-PM)*V
WRITE (6,3) P,T
60 IF(KN.GT.0) GO TO 70
GO TO 3
70 STOP
END

```

4.3 List of Symbols

Symbol		Meaning (equation number)	Dimension
report	code		
A	-	buoyancy force	dyn
B	B	grouping (37a)	-
D	D	grouping (37a)	-
I	SI	integral (38)	-
J	-	integral (33)	$^{\circ}\text{C}/\text{cm}^2/\text{s}$
K	-	thermal diffusivity	$\text{cm}^2/\text{s}$
L	-	heat of evaporation	J/g
M	-	mass content	g
$\dot{M}$	-	evaporating mass per second	g/s
N	ZN	bubble density	$1/\text{cm}^3$
Pr	PR	PRANDTL-number	-
Re	RE	REYNOLDS-number	-
R	-	bubble radius	cm
$R_o$	-	final bubble radius	cm
T	-	temperature of bubble surface	$^{\circ}\text{C}$
V	-	volume	$\text{cm}^3$
$V_B$	-	bubble volume	$\text{cm}^3$
$\Delta V$	-	volume step	$\text{cm}^3$
W	-	flow resistance	dyn
$W_{\text{rise}}$	-	bubble rise velocity	cm/s
X	X	$c_w \cdot \text{Re}^2$ (42)	-
b	-	thermal layer thickness	cm
$c_l$	-	specific heat of liquid	$\text{J}/\text{g}/^{\circ}\text{C}$
$c_w$	-	coefficient of flow resistance	-
f	EXB2	function (27)	-
g	-	gravity acceleration	$\text{cm}/\text{s}^2$
i	-	current index	-

(contd.)

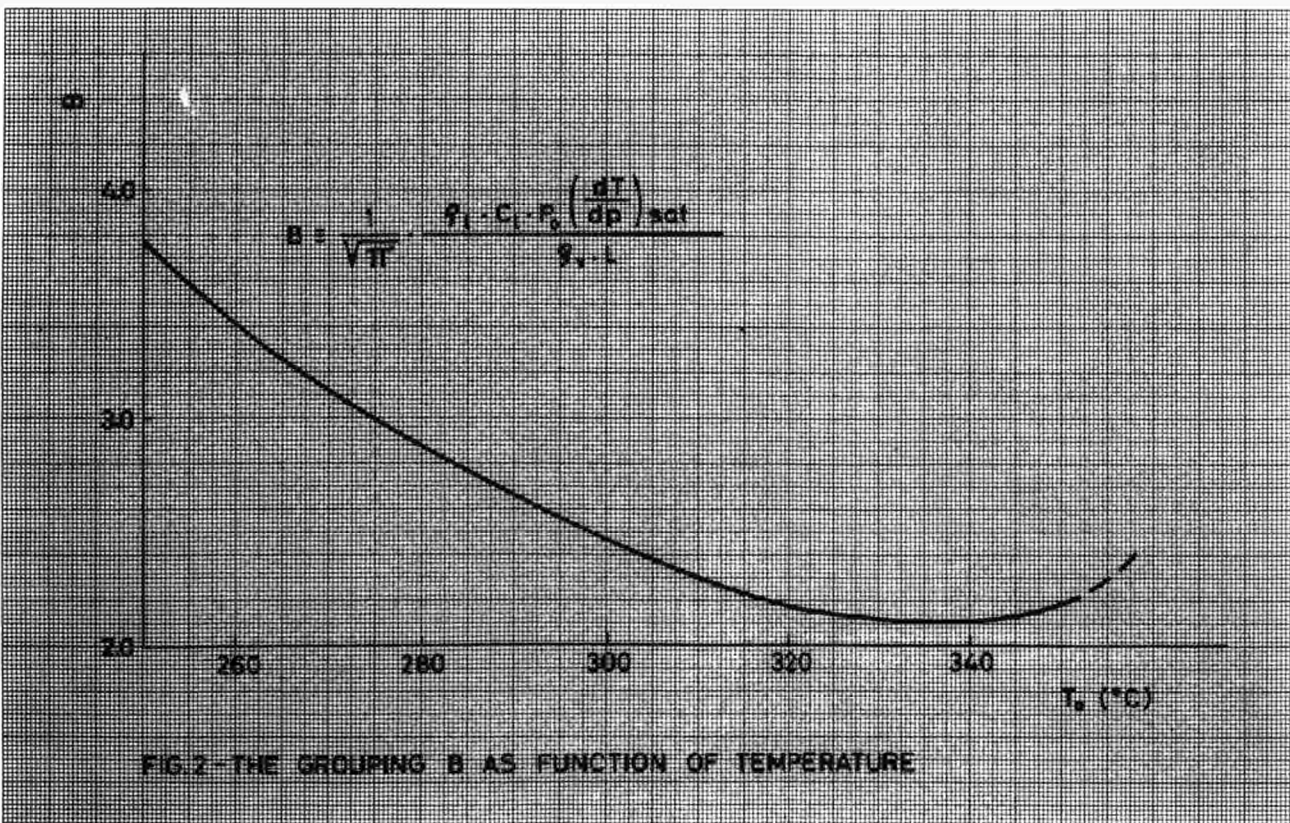
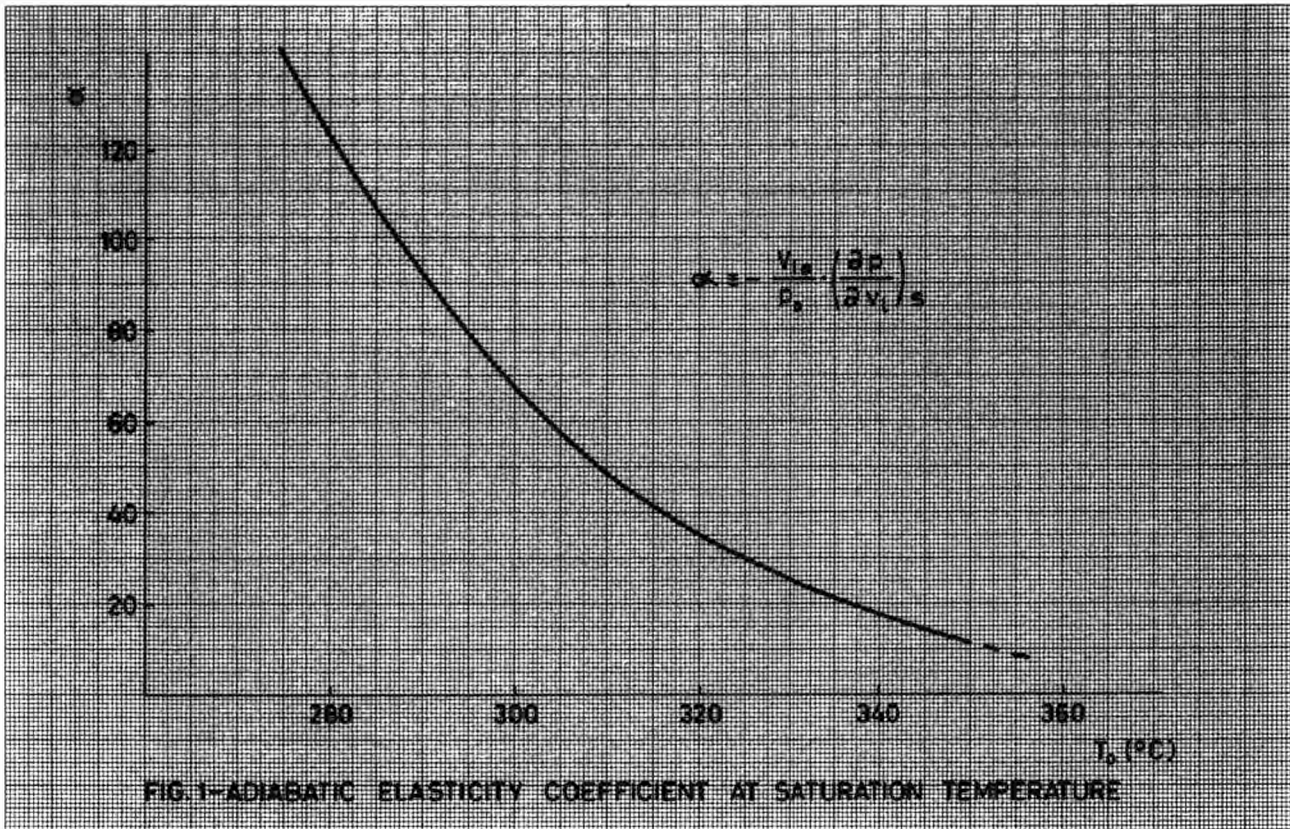
Symbol		Meaning (equation number)	Dimension
report	code		
m	-	evaporating mass per second and cm <sup>2</sup>	g/cm <sup>2</sup> /s
n	-	current index	-
p	-	pressure	dyn/cm <sup>2</sup>
q	-	heat flux	J/cm <sup>2</sup> /s
r	-	radius coordinate	cm
s	-	specific entropy	J/g/°C
t, t'	-	time	s
v	-	specific volume	cm <sup>3</sup> /g
y	Y	variable (38)	-
α	-	elasticity coefficient	-
β	-	empirical factor	-
γ	-	slope of saturation curve (30)	°C. cm <sup>2</sup> /dyn
δ	-	finite difference	-
η	-	water viscosity	g/cm <sup>2</sup> /s
θ	-	temperature (radial distribution)	°C
λ	-	thermal conductivity	J/s/cm/°C
μ	-	ratio (27)	-
ν	R	dimensionless bubble radius	-
ξ	-	integration variable (dimensionless time)	-
ρ	-	density	g/cm <sup>3</sup>
σ, σ'	S	variable (35)	-
τ, τ'	T	dimensionless time	-
φ	-	integration variable (time)	s
ψ	P	dimensionless pressure	-

Subscripts:

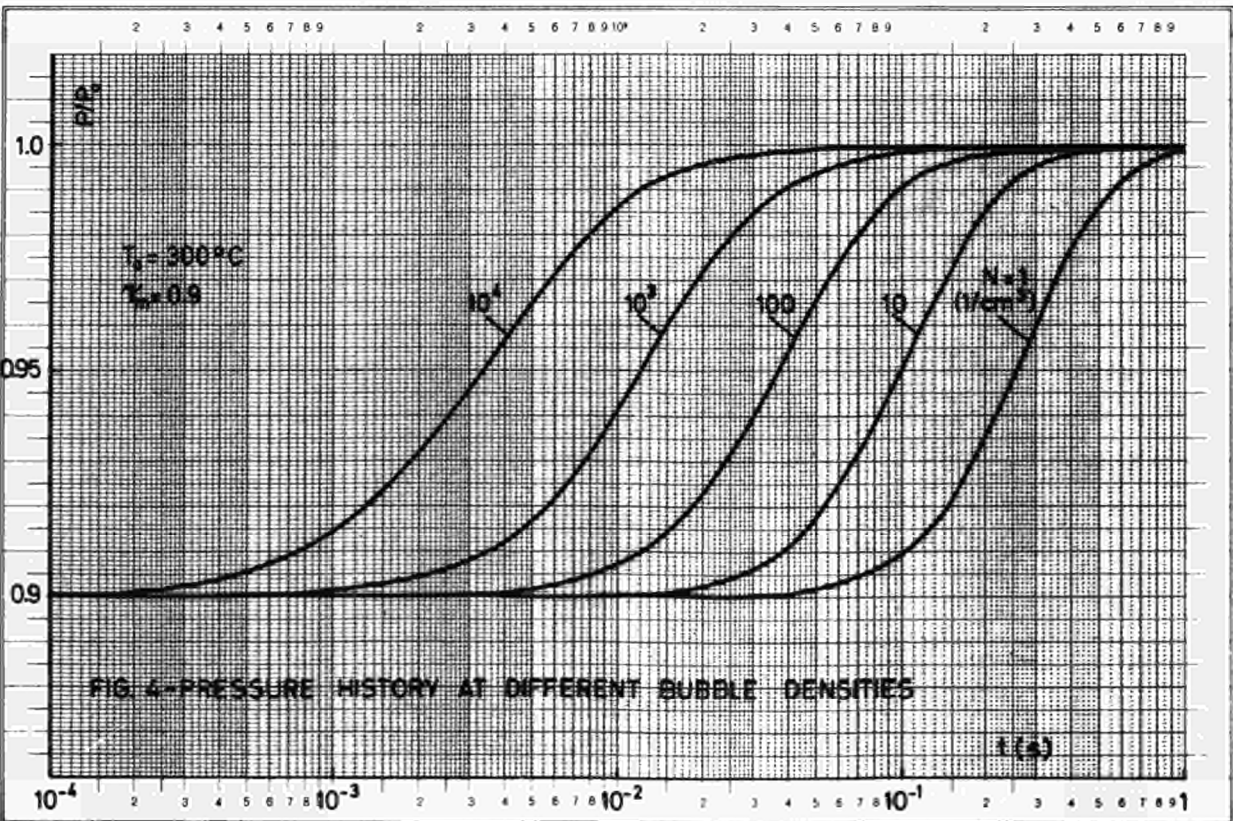
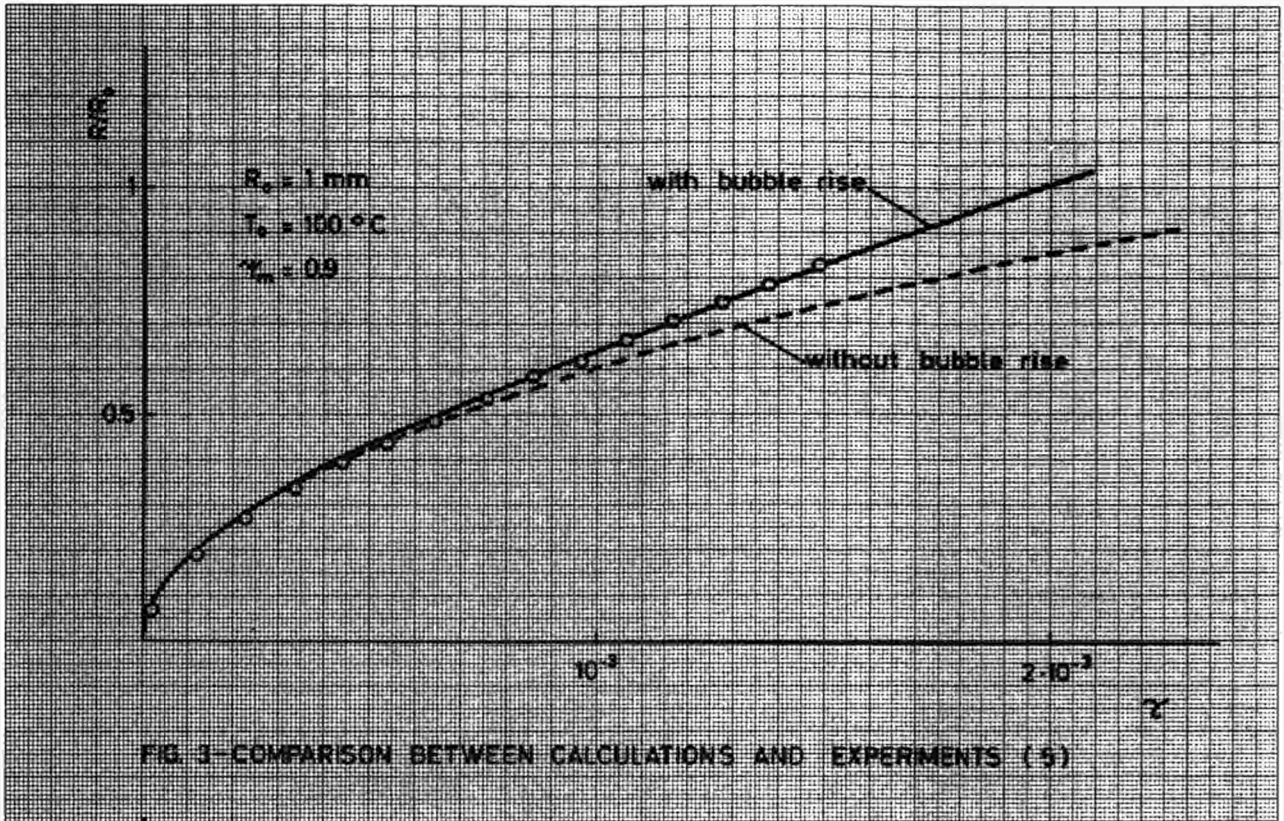
v	vapour
l	liquid
o	initial conditions (except $R_o$ )
sat	saturation
m	minimum value (after the step)
rise	bubble rise
inst	non-stationary

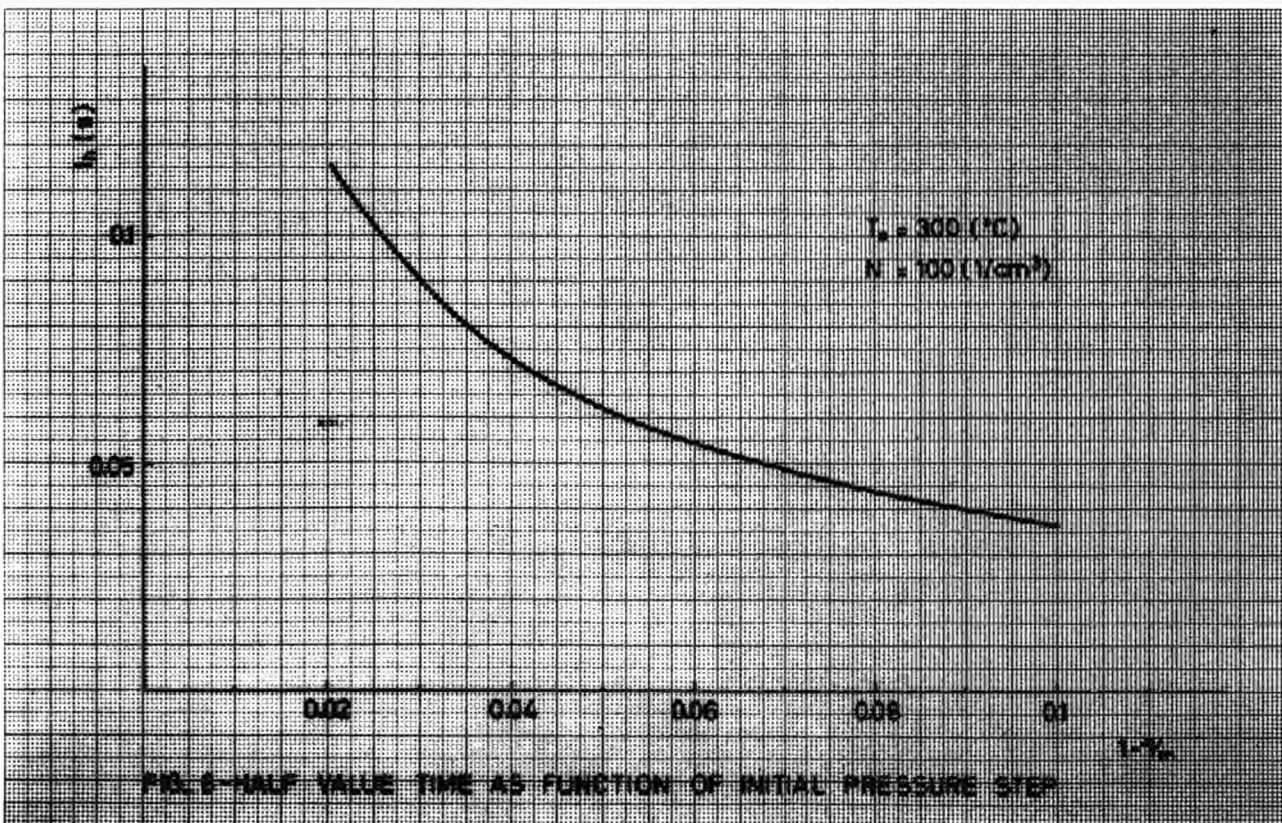
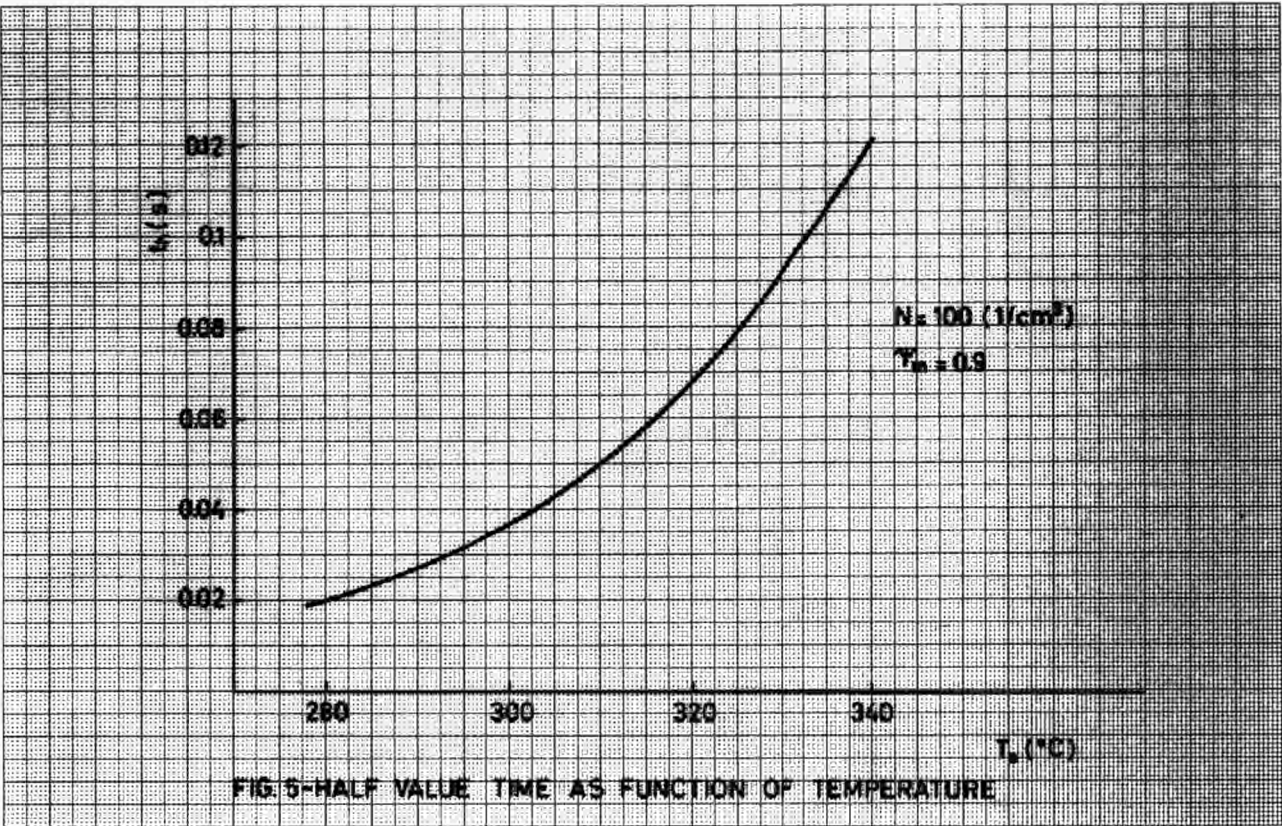
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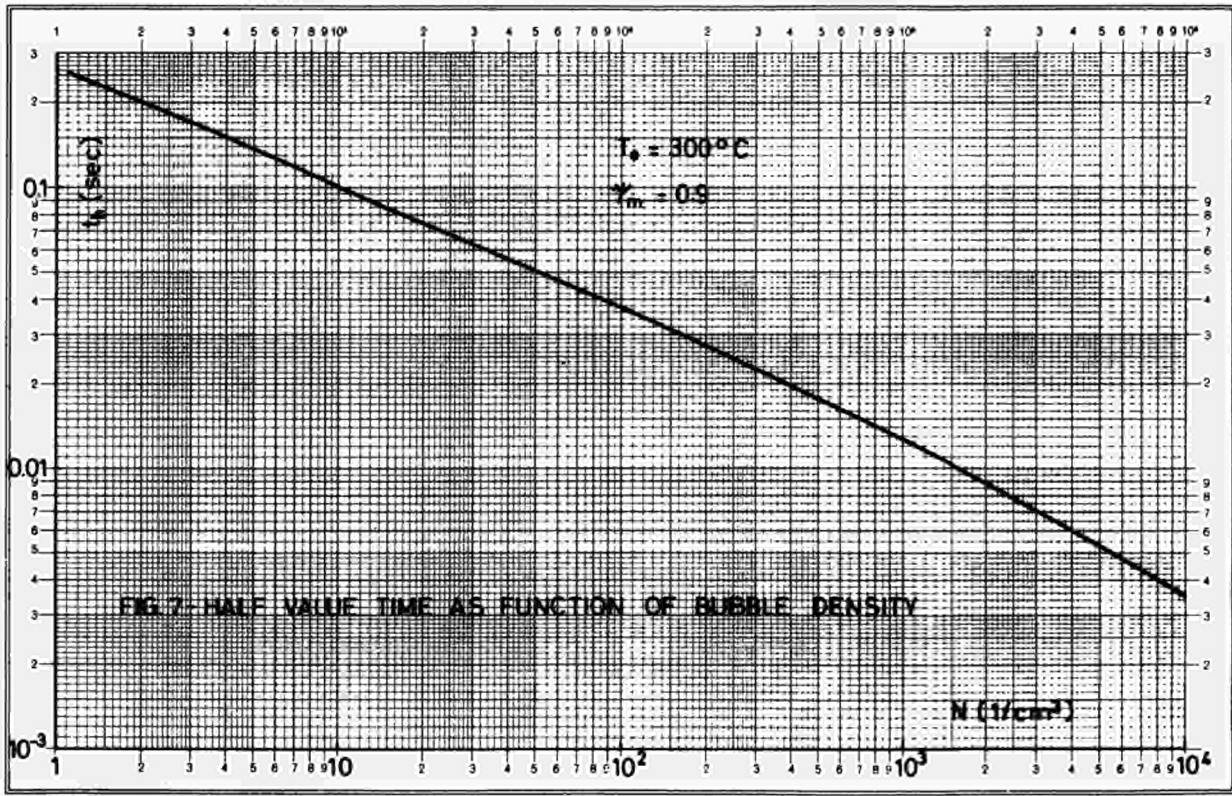












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