Validation of a tolerance analysis simulation procedure in assemblies

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Abstract

A simulation of tolerance analysis in assemblies using Sigmund CAT (Computer Aided Tolerancing) software is validated through the example of an automobile locking device. Simulation with CAT, applying criteria on both the statistical distribution and the rivet pin position in the hole used in the example, will allow us to predict the functional dimension tolerances in these assemblies with greater accuracy in the preliminary design phase. These tolerances will subsequently define the manufacturing specifications. The statistical distribution, in the example, that best fits the overall set of tolerances, is the triangular distribution followed by the normal distribution; the position of the rivet pin axis in its hole is off-centre by 53% with regard to its maximum value.

Keywords: (Computer Aided Tolerancing) CAT, hole pin float deviation, tolerance simulation and analysis in assemblies, statistical distributions in manufacturing.

1 Introduction

Ceglarek [1] affirmed that 65-70% of all design changes and failures in the aerospace and the automotive industries are related to dimensional and geometrical variation caused by a lack of technological expertise and knowledge needed for accurate predictions of process variations during the product design stage. Likewise, Gerth [2] revealed that the incorrect assignment of tolerances in component parts is one of the principal causes for the rejection of mechanisms, either because of incorrect functioning or because of poor assembly. Product Design Teams (PDTs) often spend a great deal of time on the critical dimensions of the product, but pay little attention to the tolerances of non-critical dimensions, the cumulative effect of which will subsequently cause problems in the manufacture of the mechanism and its assembly and functioning. The teams of PDTs, having analyzed the functioning and assembly of a mechanism, should identify the Functional Requirement (FR) with which they should comply, such as for example the values of maximum and minimum clearance. Each FR that is identified is not limited by the dimensions and tolerances that affect their chain of dimensions. In this way, we know the functional elements of the different parts that influence this FR, but we do not know the values of their dimensions or their Dimensional and Geometrical Tolerances (DGTs).

The initial values of the DGTs are normally estimates based on the accumulated experience of the PDTs. CAT (Computer Aided Tolerancing) software simulates the DGT estimates for each component part. It confirms whether and to what degree the FR of the mechanism complies with those estimates. The DGTs are modified in the search for optimal manufacturing costs that are compatible with the FR. The DGT modification process and subsequent simulation with the CAT software can be repeated any number of times, until it is considered that the DGT results fit the FR as closely as possible.

The majority of 3D CAT software packages simulate the different DGTs using the Monte Carlo method. A set of parameters has to be introduced in order to perform the CAT simulation:

- The types of DGTs that appear in the blueprints.
- The values of the DGTs.
- The types of statistical distributions of the DGTs.
- The position of each element in the different assemblages with clearance.

The reliability of the results from the CAT simulation will depend on the parameters that are inputted in the CAT and their fit with the reality of the manufacturing and assembly processes.

Our objective in this article is to validate the simulation procedure in the CAT for a standard case in the automobile industry, the riveting of a pin on a base plate on which a pulley, cog, or some type of latch will be placed.

The types and values of the DGTs appear in the blueprint, but the statical distributions of the DGTs and the position of the rivet pin axis in a hole are parameters that have to be estimated. This is the reason why, in this paper, we seek to identify the values that best fit those two aforementioned parameters, using the specific example of an automobile lock to do so and subsequently validating the simulation method with the CAT software.

2. Literature Review

Unfortunately, despite consulting different knowledge databases, we have found no clear information on the position of a rivet pin axis in a hole, nor on the statistical distribution that should be used in the simulation of each of the DGTs that appear in the technical drawing plans.

Despite all this, in the majority of cases in which the type of statistical distribution is unknown, it is simulated with the normal distribution, which has the inconvenience of an infinite range model that neither covers the asymmetrical tolerances, nor the confidence level of 100%, nor is it satisfactory when the number of tolerances in each chain of dimensions is low. The use of a truncated normal distribution would represent an improvement, as it rejects all the components that fall outside the tolerances. Kuo [3] confirmed that the use of the mean, standard deviation, coefficient of skewness, and coefficient of kurtosis values of the normal truncated distribution with a mean shift gives similar results to those of the Monte Carlo method, with minor variations of 0.5%, provided that the resulting distribution of the dimension chain is close to a normal distribution and its coefficient of kurtosis is close to three.

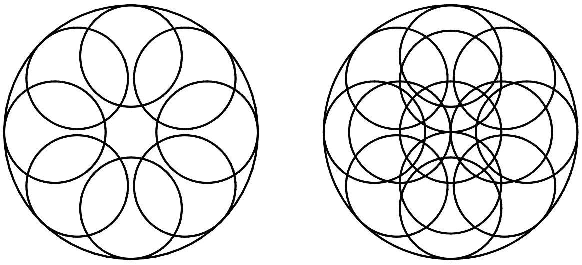
Bjørke [4] selected the beta statistical distribution from among various others for the dimensional tolerances, justifying it because the beta distribution: 1) covers the actual range of distributions from normal to rectangular; 2) is of a finite range; and, 3) includes asymmetrical cases.

The triangular distribution is also finite, permits asymmetry and is easy to use. The rectangular distribution has the least accurate fit of all the standard distributions, but guarantees larger frequencies than in reality at the extremes and is applied when the manufacturing process is unknown.

However, regardless of the types of statistical distributions that are employed in each of the DGTs that are applicable in each chain of dimensions, on the basis of the Central Limit Theorem (CLT), it may be assumed that the assembly tolerance of each FR follows a normal distribution [5]. This approach has greater certainty, the greater the number of DGTs found in the chain of dimensions.

The Monte Carlo method is the simplest and most popular means of simulating the statistical tolerances for the assemblies in the 3D CAT, [6]. The advantages of using Monte Carlo simulation are that it may be used in all types of distributions for the components i.e. it is not only restricted to normal distributions. Some of the most widely used statistical distributions in the CAT are: beta, gamma, normal, triangular, uniform and Weibull.

The previous literature has even less to say on the relation to the position of the rivet pin axis in the hole. Bjørke [4] completed a preliminary classification plan, with a distributed direction in the axis of the hole, and proposed two options to model the clearance between the axis and the hole, which are: 1) both the axis and the hole are in contact (Fig 1-a); and, 2) the position of the axis in the hole is distributed (Fig 1-b).



a) In contact b) Distributed

Fig. 1. Position of the axis in the hole

In three dimensions, we can apply the earlier criteria to the two extreme points of the rivet pin axis in the hole, as may be seen in figure 2. It may therefore be assumed that the points of centres *a* and *b* of the rivet pin axis are randomly distributed around the variables *r1, r2, α1* and *α2* with regard to the centre O_1 and O_2 of the hole (Fig. 3). So, we can say that the centre of a point on the axis is defined by two random variables: radial deviation (*r)* and angular deviation (*α*), both limited by the contact of the axis in the hole.

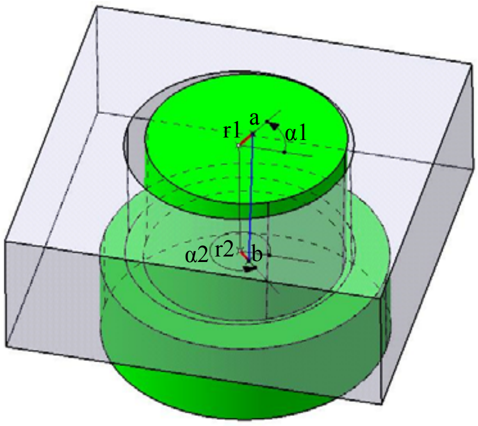
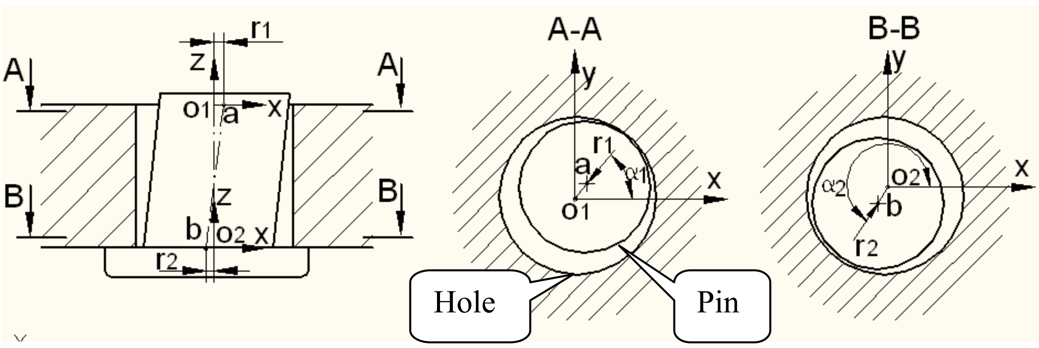


Fig. 2. 3D position of the axis in the hole



a) Cross-sectional side view b) Cross-sectional top view at a and b

Fig. 3. Radial and angular deviation.

3. Theoretical framework of tolerance analysis in assemblies with 3D CAT

Shen [7] pointed out that 1D and 2D tolerance analyses are a poor approximation of 3D analysis and that 1D and 2D tolerance analysis and assemblability analysis are insufficient in themselves.

We present a summary of the theoretical framework of a 3D CAT package, in order to understand how it analyses tolerance. Chase [8] classified the methods of tolerance analysis in: 1-Worst Case (WC), 2- Statistical (for example Root Sum Square (RSS) and method of moments), and 3- Monte Carlo. Comparing the different methods of tolerance analysis, the author came to the conclusion that the Monte Carlo method is the most satisfactory as it allows: 1) the use of normal and non-normal distributions; and 2) linear and non-linear assembly models. It is assumed that the sensitivities are constant in the linear models.

If the aim is to analyze the functional requirement *(y)* of an assembly, in accordance with the variables *(x_i)* that define its chain of dimensions, $y = f(x_1, x_2 .... x_i, ... x_n)$, where f is an unknown function in the majority of mechanisms, then:

1: The values of *(x_i)* are assigned at their maximum and minimum limit values to complete the worst-case analysis in 3D. The FR tolerance of the assembly *Ty*, with an approximation to Taylor's equation is:

$$Ty = \sum_{i=1}^{n} \left| \frac{\partial f}{\partial x_i} \right| Tx_i$$

where, Tx_i is the tolerance at dimension x_i and $\frac{\partial f}{\partial x_i}$ is the first partial derivative of the function, is known as the Sensitivity.

2: The RSS method is employed on the basis of 3 dimensions in the FR, calculating the tolerance by $Ty = \sqrt{\sum_{i=1}^{n} \left(\frac{\partial f}{\partial x_i} \right)^2 \cdot Tx_i^2}$, assuming that the variables are independent.

3.-. The Monte Carlo method estimates the dimensional variation in an assembly, due to the dimensional and geometric variations of the different components of the assembly. The input variables *(x_i)* are random variables that follow a statistical distribution. The 3D CAT then follows an iterative process that estimates: the average and the typical deviation, the asymmetry coefficients and the kurtosis of the output variable *(y)*, after having simulated the random variables x_i *n* times, as in figure 4.

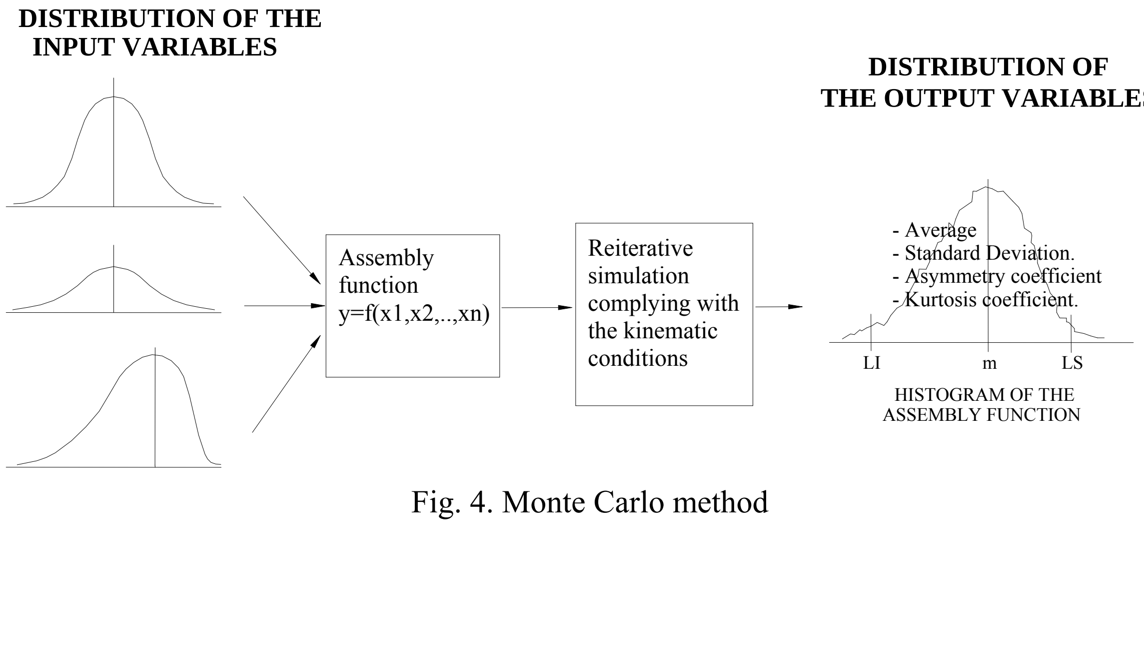


Fig. 4. Monte Carlo method

Arya [9] concluded that if the input distributions *xi* are normal in a 1D Monte Carlo tolerance analysis, then the output variable *y* will be normal. But, if the inputs *xi* are a mixture of normal and uniform, the resulting distribution of *y* will be a symmetrical distribution that is more pointed than normal, which, in this second case, increases the percentage possibility of rejection.

According to Franciosa [10] and Mäkeläinen [11], the main commercial CAT software packages are: VisVSA, eMTolMate, CETOL 6σ, 3DCS, Mechanical Advantag and Sigmund. Our choice was to work with Sigmund software [12] that applies the Monte Carlo method.

Other authors like Tlija [13] used CAD packages instead of CAT packages, simulating in the worst cases, the behaviour of the surfaces that intervene in the FR. The plane surfaces and the axes of the real holes are obtained by displacement and torsion of their respective theoretical elements that are shown in the blueprint.

4. Methodology and validation model

The following elements are needed to validate the tolerance analysis system:

1. A 3D CAT software package (in our case, Sigmund). The use of a CAT 3D software package other than Signmund should not vary the results, provided the Monte Carlo method is used. Sigmund applies the simulation method and completed the simulation in various steps: 1- Assemblies of the components, where it is necessary to define the reference systems in each component to indicate the deviation of the hole-pin float; 2-Definition of the deviations of the pieces, where the dimensions and tolerances of the hole-pin float are introduced, and radial and angular variations of the union; 3-Definition of the FR under study; 4-Simulation of the model, taking into account the different statistical distributions that are assumed; 5-Confirmation of the results.
2. A system of measurement that allows us to compare the real measured parameters with the dimensions that are simulated in the CAT. This is done with a 29 led Metris laser scanner, displayed on a Krypton optical camera, with a standard calibration deviation of 0.027 mm.
3. A mechanism, where the different dimensions and tolerances can be measured. The statistical distributions of the tolerances and the position of the rivet pin axis in the hole may be established with the different measurements.

The mechanism under study is a vehicle locking device (Fig. 5a). The FR to be verified is the distance between the axes of the two rivet pins that are included in the lock (Fig. 5b). The pieces that affect this FR are: the base plate (Fig. 5c) and the two rivet pins of equal size (Fig. 5d).

Fig. 5a. Lock

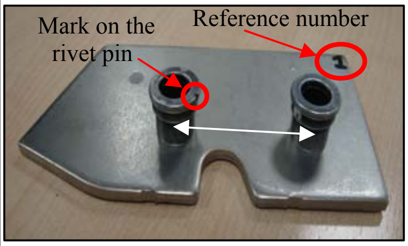


Fig. 5b. Inter-axial distance

Fig. 5c. Base-plate

Fig. 5d. Rivet pin

A total of 50 lock assemblies were prepared, for which purpose 50 base-plates, 50 right rivet pins and 50 left rivet pins were needed. In the first phase, the pieces were assigned a reference number (ver Fig 5b) and digitalized. In the second phase, the two pins were riveted onto the base plate with the rivet gun, where the pieces of each assembly had the same reference number and were placed in the position that had previously been marked. In the third phase, the assembly in figure 5b was once again digitalized. The rivet pin is marked so as to be able to position the individual pin by rotation in the CAD and make that mark coincide with the same mark of the pin that has already been riveted (Fig. 5b). By doing so, we ensure that the positions of both the rivet pin in the CAD model and the riveted pin in the assembly coincide.

The point clouds of the digitalizations of the pieces were inputted into CATIA (3D CAD software) and its reverse engineering module “*Quick Surface Reconstruction*” generated the features (cylinder, planes), as in the example of figure 6, with which we can represent the axes of the pieces and measure their features.

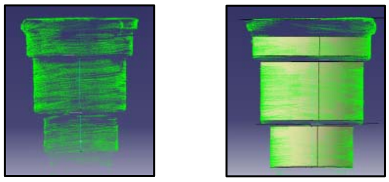


Fig 6. Generating features

The positioning of the rivet pin in CAD is done on the upper plane, at the intersection with axis B and the point of the mark (Fig. 7). The positioning of the base plate in CAD is done in the same way, but is now on three planes (Fig. 8).

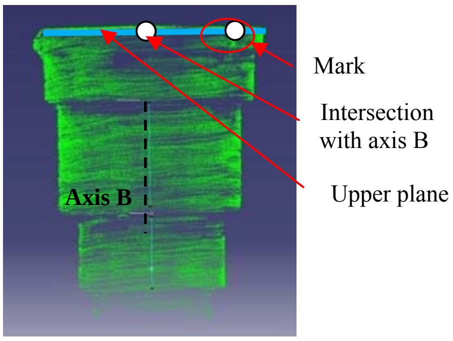


Fig. 7. Rivet Pin references.

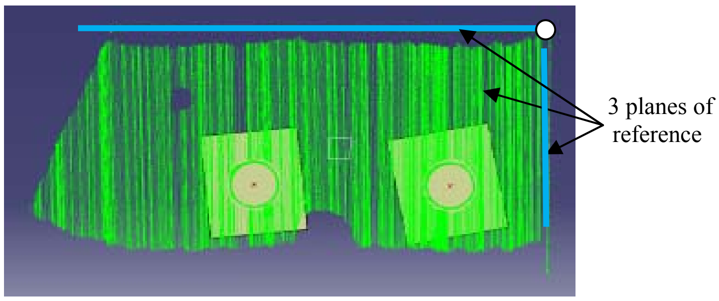


Fig. 8. . Base plate references.

The point cloud of each riveted assembly (rivet pins and base plate) were also introduced into CATIA, so as to identify the referenced positions of each part in the assembly (Fig. 9). Subsequently, in the CAD, these references of the pins and the individual base plate are made to coincide with the same references of the pins and the base plate of the assembly.These operations were repeated in each of the 50 assemblies (Fig. 10). This assembly had to be done in the CAD, in order to establish the radial and the angular deviations of the rivet pin axis in the hole (see figure 2 and 3), as the smallest cylinder of the pin is not visible after riveting, and can not therefore be directly measured.

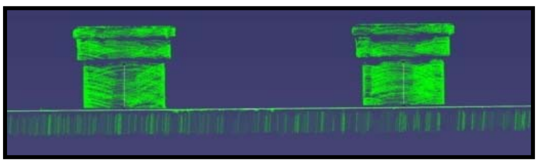


Fig. 9. Digitalization of the riveted assembly

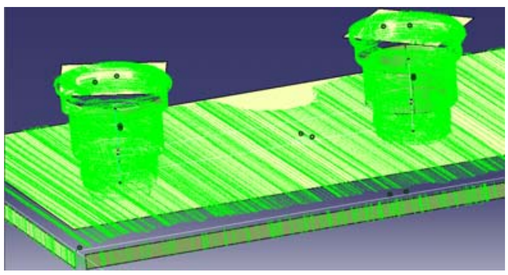


Fig. 10. Assembly in CATIA of the digitalized pieces and the riveted assembly

The blueprints of the rivet pin and the base plate were available in which the following distances were measured: 4 parameters in each rivet, as shown in figure 11. (1: Diameter at mid-height; 2: Inferior diameter; 3: Coaxiality; 4: Height of middle cylinder), and 7 parameters in the base plate, as shown in figure 12 (1: Left hole flatness; 2: Right hole flatness; 3: Left hole diameter; 4: Right hole diameter; 5: Perpendicularity on the left; 6: Perpendicularity on the right; 7: Inter-axial distance). Figure 13 shows the positions of the rivet axes in their respective holes that were measured in the assemblies (Distances A, B, C, and D of the respective axes 1 and 2) and the distance between the centres of figure 14 (Distance 3 between points E-F and G-H).

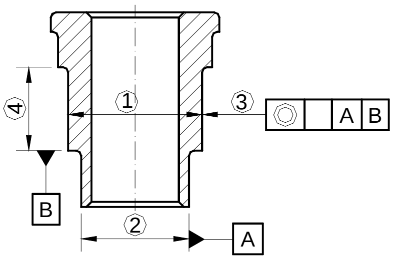


Fig. 11. Rivet pin

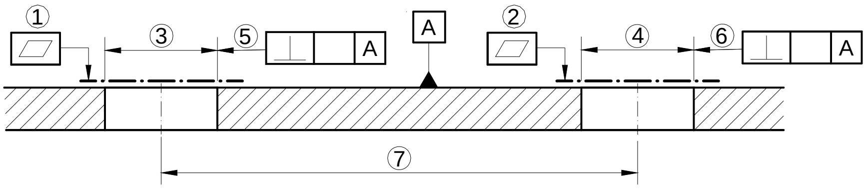


Fig. 12. Base-plate measurements

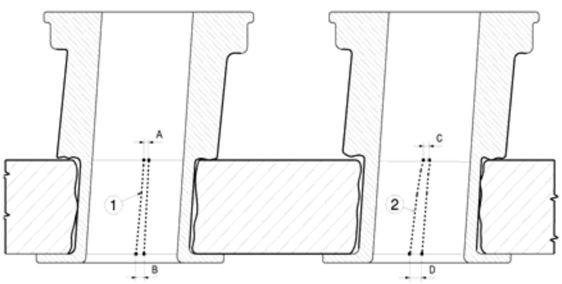


Fig. 13. Deviation of the rivet pin axes in the holes

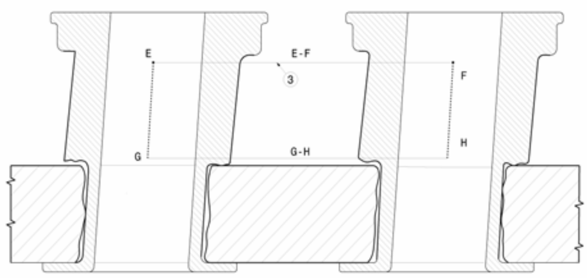


Fig. 14. Distance between the centres in the assembly

For example, to measure the diameter of coaxial tolerance of the rivet pin in figure 11:

1- Axis A is defined by a least squares fitting of the point cloud of cylinder 2 (Fig. 15).
2- Axis B is defined by a least squares fitting of the point cloud of cylinder 1.
3- Points 1 and 2 of the intersection of axis B are defined with the blueprint.
4- The maximum distance of points 1 and 2 to axis A are calculated. Twice this maximum distance is the measured diameter of the coaxiality of axis B with respect to axis A.

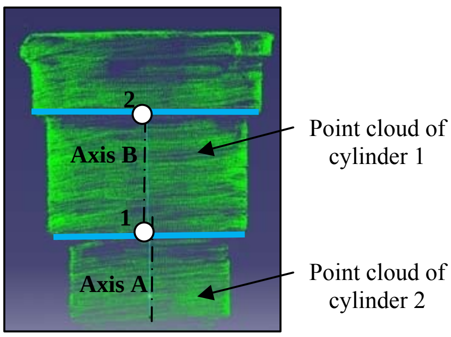


Fig. 15. Measurement of coaxiality.

In total, 21 parameters were each measured 50 times, of which 11 were dimensional tolerances and 10 were geometric tolerances: 15 were measured on the separate component parts and 6 were measured on the assembly.

As points A, B, C, and D in figure 13 are not visible, the position of the pin is defined by identifying points E, F, G and H (Fig. 14) in the assembly by means of a scanner, in order to determine the deviation of the pin in the hole. To do so, the following process is used: 1- the point cloud of the assembly are inputted into CATIA (Fig. 9), the cylindrical features and points E, F, G and H in the assembly are defined (Fig. 10); 2- the features of the cloud points of the pins and the base plate in CATIA (Fig. 6, 7 8 and 15) are determined, making points E, F, G, and H of the points coincide with those same points in the assembly; the marks of the rivet pins should also coincide with their position in the assembly, so as not to place the pins in a twisted position; 3- In CAD, the positions of points A, B, C and D may be identified on the pin, so as to measure the deviation of these points with regard to the respective centres of the holes in the base plate. A simulation of the distances E-F and G-H in CAT 3D was performed to validate this procedure as the correct one, and these distances were also defined by a 1D tolerance analysis using the WC method and the RSS method.

The 1D tolerance stack up of the assembly is shown in figure 16.

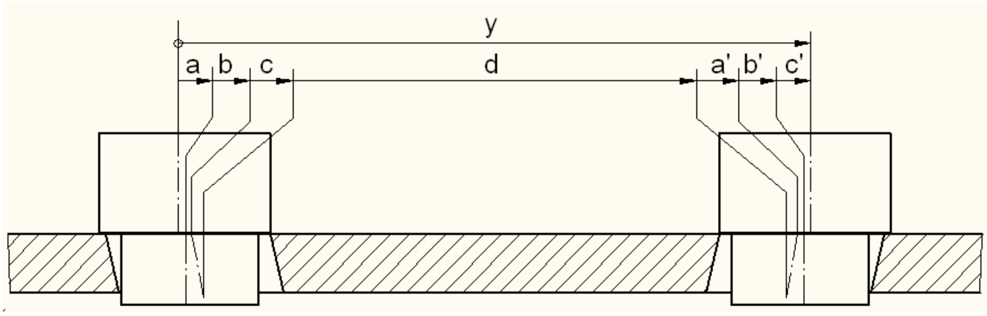


Fig. 16. 1D tolerance stack up

Where:

y is the condition to verify (distances E-F and G-H). $y = a + b + c + d + a' + b' + c'$

- a and a' are defined by the tolerance coaxialities of the rive pin with a value of 0.2 mm.
- b and b' express the float position of the pin in the hold. The diameter of the hole in the base plate is $10.6_{0}^{+0,1}$, and that of the riveted axis is $10.5_{-0,1}^{0}$, therefore the maximum displacement of the pin in the hole is $(r_{\max})_{hole} - (r_{\min})_{pin} = 5.35 - 5.2 = 0.15mm$
- c and c' are defined by the tolerances of perpendicularity of the holes in the base plate with a value of 0.1mm.
- d is the tolerance between the centres of the hole in the base plate, with a dimension of $44.9 \pm 0,1$

The tolerance of condition y is $Ty = \sum Tx_i = 2(Ta + Tb + Tc) + Td = \pm 0.6mm$ with the WC method, and the same condition is $Ty = \sqrt{\sum Tx_i^2} = \sqrt{2(Ta^2 + Tb^2 + Tc^2) + Td^2} = \pm 0.28mm$ with the RSS method. Therefore the tolerance values of y are 44.9 ± 0.6 with the WC method and 44.9 ± 0.28 with the RSS method.

So as to validate the method of determining the hole-pin float position in the hole, we should confirm that the values of the distances E-F and G-H are within the extreme values of y in the WC (44.9 ± 0.6) and their limit values are close to the values of the RSS method (44.9 ± 0.28).

EasyFit statistical analysis software [14] was used to confirm that the statistical distributions were adjusted to a significance level of below 0.01 in the 21 parameters, which permits direct comparison of the sampled data in a maximum of 60 types of statistical distributions, among which are the most widely used distributions in the CAT packages: beta, gamma, normal, triangular, uniform and Weibull. The fitness tests that

EasyFit use to test the goodness of fit of the data with the different statistical distributions are: Kolmogorov-Smirnov, Anderson-Darling and Chi-squared.

5. Results and analysis

5.1. Statistical distributions

The statistical software package, EasyFit, generated the data shown below as an example in table 1. In it, a value of 1 is assigned to the statistical distribution with the best fit, 6 for the distribution with the worse fit, and N/A if there is no fit.

Table 1. Fitness tests of the mid-height diamenter of the right rivet pin (dimension 1).

#	Distribution	Kolmogorov Smirnov		Anderson Darling		Chi-squared	
		Statistic	Order	Statistic	Range	Statistic	Order
1	Beta	0.09766	3	0.43909	1	3.6315	4
2	Gamma	0.1053	4	0.43917	2	3.4953	2
3	Normal	0.10543	5	0.44022	3	3.496	3
4	Triangular	0.09185	1	0.44032	4	2.6567	1
5	Uniform	0.09613	2	11.752	6	N/A	
6	Weibull	0.14589	6	1.5188	5	5.1735	5

The average value of the "order" column of the three statistical in table 1 was calculated, in order to perform the goodness-of-fit study for all the parameters and statistical distributions. In table 2, the boxes with a shaded background show the best distribution of each parameter. The parameters of the pieces and the assembly are identified in figures 11, 12, 13, and 14.

Table 2. Goodness of fit test of all the measurements

	Right Rivet Pin				Left Rivet Pin			
	1	2	3	4	1	2	3	4
BETA	2.66	5.33	N/A	N/A	3	**2**	4	N/A
GAMMA	2.66	**1.66**	2	4	**1**	2	4	4
NORMAL	3.66	2.66	3.66	3	2	2.33	2.66	3
TRIANGULAR	**2**	**1.66**	**1**	**1.66**	4	3.66	4.33	**1.66**
UNIFORM	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
WEIBULL	5.33	4.33	3.33	2	5	5	**1.66**	2

	Base Plate							Assembly					
	1	2	3	4	5	6	7	A	B	C	D	E-F	G-H
BETA	N/A	2.33	4.66	N/A	N/A	3	3.33	2.66	**2**	N/A	3	**1.33**	N/A
GAMMA	5	**2**	3.66	4.66	4.33	4.33	3	4.33	3.33	**1**	2.66	3	2.33
NORMAL	**1.66**	3	2.66	3.66	**1.33**	4	**2**	**1.66**	3.66	4.33	3.66	3.33	3.33
TRIANGULAR	3.66	3.66	3	2	2.33	3	4.33	2	4	3.33	**2**	2.66	**1**
UNIFORM	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
WEIBULL	4	4	**1.33**	**1**	2.33	**1**	2.33	4.66	2.33	2	5.33	5.33	4.66

When studying all the average scores together in table 2 of the statistical distributions, we can say that:

- the triangular distribution has the best fit, if we take the average of all the values of table 2, with an average value of 2.71 (see table 3).
- the triangular distribution has the best fit, if the number of tolerances that best fit each type of distribution are added up, amounting to a total of 7 shaded boxes in table 2, in the triangular distribution.
- if we take into account the two earlier criteria, the triangular distribution has the best fit, followed by the normal distribution.

Tabla 3. Prueba de bondad de ajuste de todas las tolerancias

	Average	**Number**
BETA	3.02 -N/A	3
GAMMA	3.09	4
NORMAL	**2.91**	**4**
TRIANGULAR	**2.71**	**7**
UNIFORM	N/A	N/A
WEIBULL	3.28	4

A second possible study with the data in table 2 is to establish the distribution that best fits both the dimensional tolerances and the geometric tolerances. Remember that 11 of the tolerances are dimensional tolerances and 10 are geometric tolerances. Table 4 is obtained on the same basis as table 3, where the triangular distribution is better adjusted to the dimensional tolerances and the normal distribution to the geometric tolerances. It is worth noting that when the geometric tolerances are averaged out, the beta distribution has the lowest value, however, it can not be taken into account, because various tolerances fail to fit that distribution at a level of significance of $\alpha<0.01$.

Table 4. Fitness test of the dimensional and geometric tolerances

	Dimensional Tolerances		**Geometric Tolerances**	
	Average	**Number**	**Average**	**Number**
BETA	3.19 – N/A	2	2.83- N/A	1
GAMMA	2.91	2	3.30	2
NORMAL	2.87	1	2.96	3
TRIANGULAR	2.51	5	2.93	2
UNIFORM	N/A	N/A	N/A	N/A
WEIBULL	3.48	2	3.06	2

A third study with the data in table 2 was done by dividing all the tolerances into two groups: 1) tolerances of the pieces; and, 2) tolerances of the assemblies. In table 5, the most satisfactory distribution is clearly the triangular distribution, both for the parts and for the assembly. The normal distribution is a second type of distribution that may be used in the parts.

Tabla 5. Fitness test of the parts in the assembly

	Parts		**Assembly**	
	Average	**Number**	**Average**	**Number**
BETA	3.36- N/A	1	2.25- N/A	2
GAMMA	3.22	3	2.77	1
NORMAL	2.75	3	3.32	1
TRIANGULAR	2.79	5	2.49	2
UNIFORM	N/A	N/A	N/A	N/A
WEIBULL	2.976	4	4.05	0

5.2. Position of the rivet pin axis in the hole

The second part of the research is to confirm the position of the rivet pin in the hole. The data measured with the scanner and the data obtained in the Sigmund simulation are those of table 6, which correspond to the dimensions in figures 11-14. Each measurement in Sigmund from table 6, was obtained from a graph like that of figure 17, which in this example corresponds to dimension 1 of the right rivet pin.

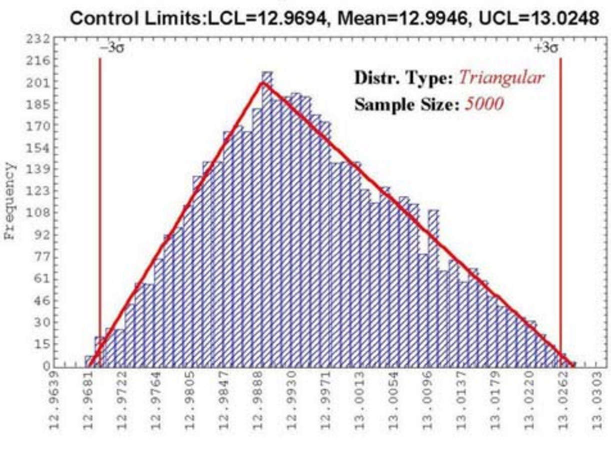


Fig. 17. Sigmund measurements of dimension 1 of the right rivet pin

Table 6. Fitness test of the pieces and in the assembly

		Scanner measurements			Sigmund measurements		
	Dimension	**Av.**	**Max.**	**Min.**	**Av.**	**Max.**	**Min.**
Right Rivet Pin	**1**	12.996	13.024	12.968	12.995	13.025	12.969
	2	10.407	10.438	10.376	10.406	10.438	10.374
	3	0.169	0.292	0.066	0.170	0.285	0.077
	4	8.120	8.158	8.062	8.119	8.160	8.065
Left Rivet Pin	**1**	12.993	13.026	12.968	12.993	13.027	12.959
	2	10.404	10.431	10.376	10.404	10.426	10.381
	3	0.168	0.268	0.06	0.169	0.277	0.058
	4	8.120	8.158	8.062	8.119	8.160	8.065
Base Plate	**1**	0.030	0.035	0.025	0.030	0.037	0.025
	2	0.030	0.035	0.025	0.030	0.034	0.026
	3	10.727	10.763	10.687	10.727	10.767	10.687
	4	10.729	10.770	10.686	10.729	10.762	10.690
	5	0.044	0.069	0.014	0.044	0.074	0.014
	6	0.039	0.071	0.01	0.039	0.071	0.010
	7	44.881	44.913	44.852	44.881	44.914	44.849
Assembly	**A**	0.086	0.175	0.018	0.086	0.176	-0.003
	B	0.095	0.2	0.022	0.095	0.195	0.032
	C	0.072	0.188	0.013	0.072	0.188	0.022
	D	0.097	0.199	0.03	0.097	0.207	0.032
	E-F	44.860	44.982	44.746	44.860	44.979	44.752
	G-H	44.902	45.042	44.769	44.902	45.049	44.786

The maximum difference between any of the data measured with the scanner and the Sigmund simulations in table 6 was 0.021mm, which allows us to confirm the validity of the Sigmund simulation procedure.

In table 6, it may be seen that the maximum diameter of the hole measured with the scanners was 10.770mm (dimension 4 of the base plate) and the minimum measurement of the rivet pin axis was 10.376mm (dimension 2 of the rivet pin), such that the maximum clearance was 0.394mm. The real data of the assembly should never be over this pre-set maximum clearance. However, when the point clouds of the component parts were merged with the point cloud of the assembly with CATIA, it could generate values slightly outside the maximum clearances obtained with the scanners, because of the randomness of the data in Sigmund. Figure 18 shows us: 1) the position of all the points at the two extreme ends of axes 1 and 2 in CAD, while the centres of the circles represent the extreme points of the axes of the holes in the base plate; 2) the maximum circle where all the points are found; and, 3) the angular and radial position of the centre of points A, B, C and D.

As may be seen, rather than being perfectly centred, the riveting process is off-centre by an average of 0.104mm, as opposed to a maximum permitted deviation of 0.197mm. The average angular deviation is 193º. The deviation of 0.104mm is 53% of the maximum value.

We think that the deviation position may be due either to the riveter that is not in a perpendicular position with respect to the supporting surface of the lock, or to the way that the rivets are inserted that causes a slight off-centre deviation and shifts the rivets towards one side of the hole.

The value of this deviation is only applicable to our example and we have been informed that there may be a deviation in some or many of the production processes that should be taken into account. In the CAT 3D simulation, therefore, prior to serial production, we should take an average deviation into account, in our case approximately 50% of the maximum value.

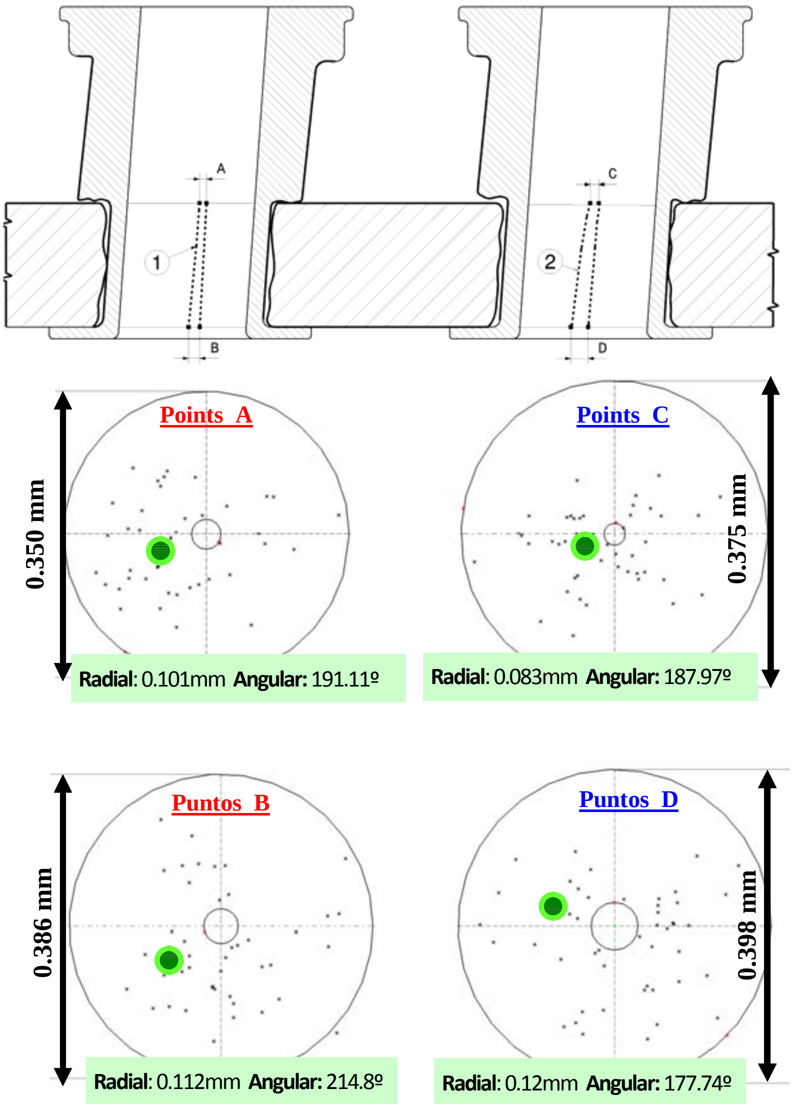


Fig. 18. Projection of the points A, B, C y D of the rivet pins.

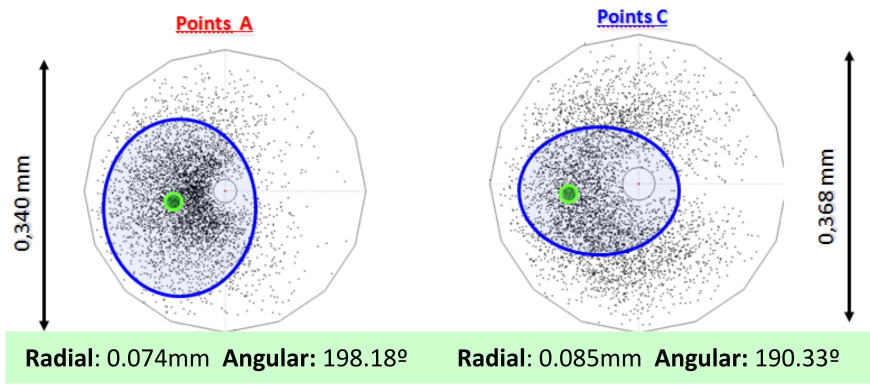


Fig. 19. Projection of the simulated points of the axis centres in Sigmund

Following the simulation with the Sigmund CAT software, the positions of the rivet pin centres at points A and C are represented in figure 19 in order to compare with the measured results, where the central point is the hole axis and the point cloud represents the endpoints of the pin axis. The parameters of the statistical distributions of table 2 have been introduced and the radial and angular deviations of figure 18. At first sight, it may be seen that they fit the positions measured on the scanner. In figure 19, deviation to the left and the diameters of the maximum deviations, which vary by 0.01 mm with regard to those in figure 18, may be seen for a sample of 5000.

The FR data from the scanner (distance between the mid-height cylinder axes of rivet pins E-F and G-H of figure 14) were compared with those generated in Sigmund (Fig. 20 and 21), in order to confirm the validity of the simulation process.

In the bar diagrams of figures 20 and 21, it may be seen that: 1) there are 10 DGTs that affect the functional requirements E-F and G-H, and 2) the dimension that most affects the FR in each riveting operation is the lateral radial clearance between the pin and the hole, with an average value at each end of 20%. Therefore, identification of the position of the rivet pin in the hole is fundamental, as the radial clearance between the rivet pin and the hole contributes 40% to the measurement of the distance between the two axes of the two rivet pins (functional dimensions E-F and G-H).

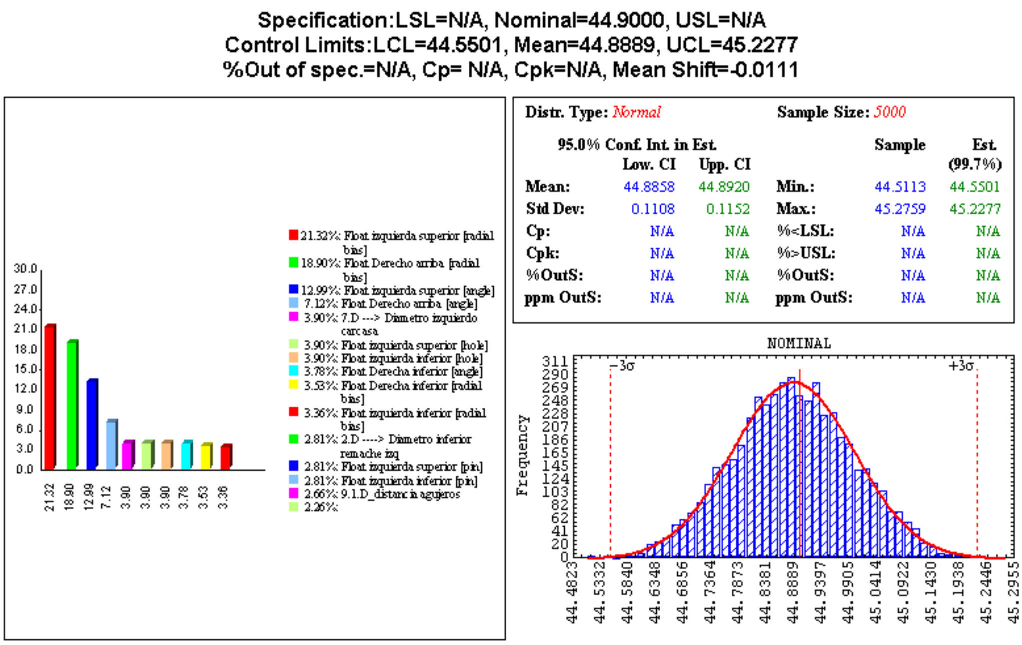


Fig. 20. Data obtained with Sigmund from the E-F dimension.

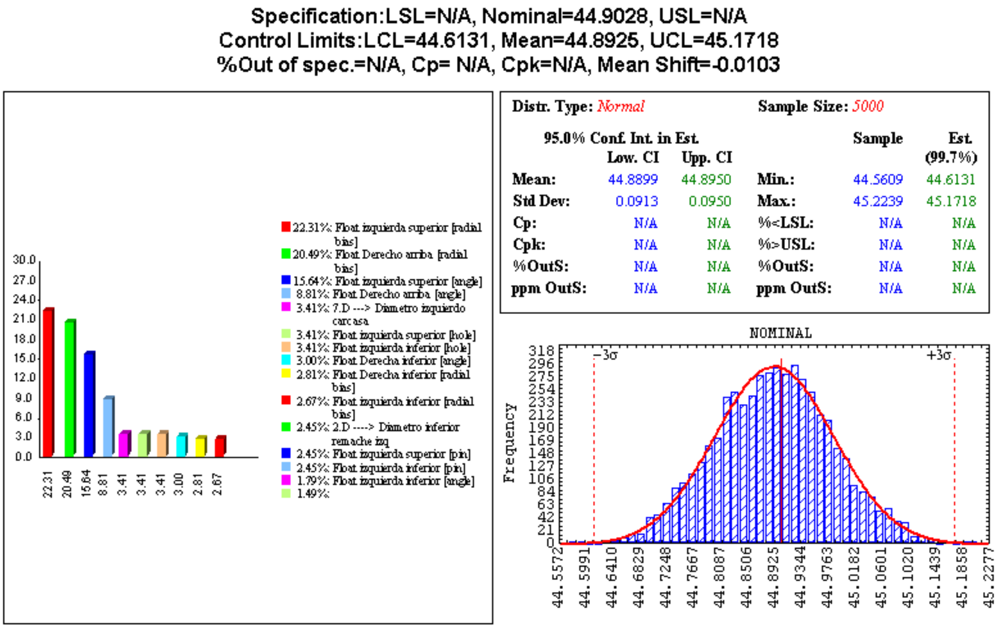


Fig. 21. Data obtained with Sigumund from the G-H dimension.

In table 7, the comparative data between the scanner measurements and those simulated with Sigmund are shown, from which we may observe that:

- All the comparative results both from the scanner and from the Sigmund simulation were within the tolerance limit values that appear in the blueprint (44.9±0.4mm).
- The average values of the distance between the centres of both measurements were practically coincident, with an average difference of 0.01mm.
- The minimum average values were 0.18mm greater in the scanner than in Sigmund and 0.19mm less than the average maximum values. These differences, although important, are because measurement with the scanner is with a sample of 50 and, in simulation, with 5000, which is why values further off Sigmund's appear, with a relative error of 4‰.
- The difference between the standard deviations (σ) measured in the comparitive study was 0.035mm.

Taking these earlier points into account, we may validate the simulation system in Sigmund for the functional requirement of the distance between the axes (Fig. 14). As a consequence, a link in that chain of dimensions may be validated, which is the deviation of the rivet pin in its hole (Fig. 13).

Table 7. Comparison of the measurements of distances E-F and G-H

	Scanner measurements				Sigmund simulation measurements			
	Average	σ	Minimum	Maximum	Average	σ	Minimum	Maximum
E-F	44.861	0.059	44.746	44.982	44.889	0.111	44.550	45.227
G-H	44.902	0.073	44.769	45.042	44.893	0.091	44.613	45.172
Average Values	**44.882**	**0.066**	**44.757**	**45.012**	**44.891**	**0.101**	**44.582**	**45.199**

It may also be confirmed in table 7 that the values of the distances E-F and G-H are within the extreme values of y in WC ($44.9 \pm 0,6$) and its limit values are close to the values given by the RSS method (44.9 ± 0.28). In the research the value obtained with the scanner is (44.88 ± 0.2) where σ=0,066 and the simulation software Sigmund (44.89 ± 0.3) where σ=0.101.

6. Conclusions

A simulation of tolerance analysis in assemblies using Sigmund CAT (Computer Aided Tolerancing) software has been validated through the example of an automobile locking device. Simulation with CAT, applying statistical distribution criteria and the position of the pin in the hole used in the example, means we can predict the tolerances of the functional dimensions in the assemblies, with greater accuracy and in the preliminary design phase. These tolerances will subsequently define the manufacturing specifications. This prediction allows us: 1) to establish the individual tolerances of the most critical pieces of the mechanism and their tolerances; 2) to adjust, in the drawing plans, the individual tolerance values to more optimal values; 3) to avoid errors in the assignation of tolerances that subsquently imply important costs in the redefinition of the design. In brief, tolerance behaviour prediction in a mechanism, by means of simulation, in the preliminary design phases, will imply important cost savings.

The main contributions of the study are:

- The position of the rivet pin in the hole is off-centre at an intermediate point between zero and half of the maximum clearance before riveting. In our lock, the average deviation of the rivet axis was 53% with regard to its maximum value.
- The statistical distribution that achieves the best fit in general, with the 21 tolerances each measured 50 times, in the case of the vehicle lock, was the triangular distribution followed by the normal distribution. If all the tolerances are divided between dimensional and geometric tolerances, the triangular distribution achieves the best fit with the dimensional tolerances, and the triangular and normal distributions with the geometric tolerances. Finally, if the tolerances are divided between the tolerances of each component part and of the assembly, the triangular distribution achieves the best fit for the assembly, and the triangular and normal distributions for the individual parts.

In summary, knowledge of the statistical distributions that best fit the tolerances and the axis of the rivet pin float position, has been shown to improve the methodology underlying the use of CAT software and to reduce simulation error with regard to reality.

The above conclusions are applicable to this example. So that they be generalized, it would be necessary for other researchers to continue these two lines of research:

- To confirm the position of the rivet pin axis through different dimensions of the rivet pin, hole, and radial clearance.
- To study the statistical distribution in other mechanisms and with other types of tolerances.

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