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Volatility forecasting with the wavelet transformation algorithm GARCH model: Evidence from African stock markets

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Abstract

The daily returns of four African countries' stock market indices for the period January 2, 2000, to December 31, 2014, were employed to compare the $GARCH_{(1,1)}$ model and a newly proposed Maximal Overlap Discreet Wavelet Transform (MODWT)-$GARCH_{(1,1)}$ model. The results showed that although both models fit the returns data well, the forecast produced by the $GARCH_{(1,1)}$ model underestimates the observed returns whereas the newly proposed MODWT-$GARCH_{(1,1)}$ model generates an accurate forecast value of the observed returns. The results generally showed that the newly proposed MODWT-$GARCH_{(1,1)}$ model best fits returns series for these African countries. Hence the proposed MODWT-GARCH should be applied on other context to further verify its validity.

Keywords: Volatility; Asset returns; MODWT; GARCH

1. Introduction

Stock market volatility is of paramount importance to both market practitioners and policy makers, particularly for emerging countries.[1,2] The practitioner is concerned about stock market volatility because it affects asset pricing and risk, whereas the policy maker attempts to curb excessive volatility to ensure financial and macroeconomic stability.[3] In both cases, an efficient quantitative tool for modeling stock market volatility is needed to minimize the risk of inaccurate measurement. In this regard, researchers continue to search for the best volatility model that is able to capture various stylized facts associated with market volatilities.

The volatility modeling of price returns was first performed in Ref. 4, wherein an autoregressive conditional heteroskedasticity model, the ARCH model, was used to predict UK inflation rate uncertainty. Engle noted that major changes tend to be followed by significant changes in either sign and that small changes tend to be followed by small

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changes. This phenomenon was designated volatility clustering. The author measured clustering effects based on an assumption of constant conditional return mean value.

However, other stylized volatility features could not be captured by the ARCH model. Bollerslev[5] generalized the ARCH model by creating the Generalized Conditionally Heteroskedasticity model (GARCH model). The model considerably extended the capacities of the ARCH model to account for stylized aspects of return volatility, given that it removed the excess kurtosis in returns series. However, the GARCH model, which is actually a linear model, could not address the fat-tailed distributions of financial time series.

The Exponential GARCH (EGARCH) model originated by Ref. 6, the Quadratic GARCH (QGARCH) model originated by Ref. 7, and other models such as the Glosten, Jogannathan, and Rankle (GJR) model,[8] are known as non-linear GARCH models, and they address the skewed distributions of financial time series, which are a very common characteristic of financial time series.

Furthermore, stock market returns are practically influenced by agent speculations and investor decisions over different time horizons that range from minutes to years. In such a situation, a useful tool of analysis may be wavelet analysis.[9]

Wavelets are particular types of function that are localized both in time and frequency domain that are utilized in the decomposition of time series into additional elementary functions containing various information relating to the time series. Within the numerous utilized statistical signal extraction and filtering methods, in addition to denoising methods, wavelets constitute just one tool. The ability to decompose macroeconomic time series into components of their time scale, is a major advantage of wavelet analysis. Haar (discreet), symmlets and coiflets (symmetric), daiblet (asymmetric), among others, make up the different categories of the available wavelets filters; they differ in their filter transfer function and filter lengths in terms of characteristics. This study is based on the Maximal Overlap Discreet Wavelet Transform (MODWT) tool. The MODWT represents an improvement on the Discreet Wavelet Transform (DWT). Through the simple modification of the pyramid algorithm utilized in computing DWT coefficient, the MODWT is obtained; and it is perceived as the DWT universal set. The MODWT, among other comparative advantages over the DWT, can accommodate any sample size; in addition, in terms of data filtering starting point of a time series, it is insensitive.[9]

The MODWT filtering method offers insights into the dynamics of financial time series beyond those revealed through existing methodologies.[10] A number of concepts, such as those of nonstationarity, multiresolution and approximate decorrelation, emerge from MODWT filters. Moreover, MODWT filters serve as a straightforward tool for studying the multiresolution properties of a process. They can also decompose a financial time series into different time scales, given that they reveal structural break and volatility clusters and identify the local and global dynamic properties of a process at such time scales. In addition, MODWT filters can conveniently dissolve the correlation structure of a process across time scales.

A book by Ref. 10 applied the DWT to daily IBM stock return series and found a large group of rapidly fluctuating returns between observations at certain intervals of the wavelet coefficients. They observed that, at the same frequency level, there were significant fluctuations in wavelet coefficient w_1 and a small increase in fluctuations of wavelet coefficient w_2 and that wavelet coefficients w_3 and w_4 were essentially zero. A study by Conejo et al[11] employed a time series analysis, a neural network and wavelet forecasting technique that predicts 24 market-clearing prices of a day-ahead electric energy market by using PJM Interconnection data. They exhaustively compared the forecasting errors generated from the techniques and recommended the study of combined wavelet transform and time series algorithms in future research. A study by Ref. 12 presented two hybrid forecasting frameworks, the Wavelet-Genetic Algorithm (GA)-Multilayer Perceptron (MLP) and the Wavelet-Particle Swarm Optimization (PSO)-Multilayer Perceptron (MLP), for predicting non-stationary wind speeds and for comparing the forecasting performance of the different algorithm combinations of the two hybrid frameworks. Their results, based on three experimental cases, show that among other results, in both of the hybrid frameworks, the contributions of the GA and PSO components to improving the MLP were not significant whereas those of the wavelet component were significant. A similar study by Ref. 13 proposed a novel price forecasting method based on wavelet transform approaches combined with ARIMA and GARCH models, and it was compared with some of the most recently published price forecasting techniques. The comparative results clearly showed that the proposed forecasting method was far more accurate than the other forecasting method.

Although several articles on the stock price volatility levels of developed capital markets have been published, scarce research has been conducted on this subject with respect to African markets. African stock markets are some of the important emerging markets now attracting global investors.[14] Despite offering attractive investment avenues, investors are wary of the volatility risks associated with these markets due to the differences between their risk and return characteristics and those of developed markets.[14] The risks of investing in these markets are more severe, and investors tend to lose money due to forecasting errors that arise from a lack of proper telecommunication and transportation resources and due to different accounting system characteristics.[14] Therefore, it is necessary to exhaustively and efficiently seek a methodology that will considerably reduce the amount of errors made in forecasting the returns of these stock markets, thereby increasing investor confidence and efficient portfolio management.[14]

In line with the discussions above, the objective of this study is to examine the capability of the Maximal Overlap Discreet Wavelet Transform (MODWT) algorithm in modeling and forecasting the volatility of African stock markets. Therefore, the study compares the forecast error results obtained using the $GARCH_{(1,1)}$ model. In doing so, the study uses the forecast error obtained using the MODWT-$GARCH_{(1,1)}$ model.

The paper is structured as follows. In section two, we describe the materials and methods used in this study. In section three, we discuss the approach to data analysis. Finally, section four presents the main conclusions of the study.

2. Materials and methods

The daily stock price indices of four African countries' stock markets, i.e., those of Kenya (NSE20), Nigeria (NIGERIA ALL SHARE), South Africa (FTSE/JSE100), and Tunisia (TUNINDEX), were collected from the data stream running from January 2, 2000, to December 31, 2014. To cope with the official holidays of these four African countries, the data are slightly different in length. The selection of these countries was based on market development, data availability and their ability to represent the four African regions (the North African region is represented by Tunisia, the South African region by South Africa, the West African region by Nigeria, and the East African region by Kenya).

To explore the return volatility levels, the return was calculated using the following formula:

$$r_t = \text{In}\, P_t - \text{In}\, P_{t-1} \tag{1}$$

where P_t is the share price at period t, P_{t-1} is the share price at period $t-1$, and t is the daily return. We utilize the time series $\{Y_t\}$ which has the $ARMA_{(p,q)}$ representation in (2) on the returns series to fit the $ARMA_{(p,q)}$ model by overfitting the (p,q) parameters. It is also utilized on the MODWT returns to fit the $ARMA_{(p,q)}$ model of the proposed MODWT by overfitting the (p,q) parameters. In doing so, we strictly adhere to the iterative procedures of Box−Jenkins ARMA modeling.

$$Y_t + \varphi_1 Y_{t-1} + \varphi_2 Y_{t-2} + \ldots + \varphi_p Y_{t-p} = C + E_t + \theta_1 E_{t-1} + \theta_2 E_{t-1} + \ldots + \theta_q E_{t-q} \tag{2}$$

where E_t is a white noise process with mean 0 and variance δ^2. The time series $\{Y_t\}$ is also normally distributed as white noise process with a mean of 0 and variance δ^2.

$\varphi(L)Y_t$is designated the AR component of the process $\{Y_t\}$. $\theta(L)E_t$ is the MA component of the process $\{Y_t\}$.

Again, the mathematical formulation for $GARCH_{(1,1)}$ model given in (3) is utilized on our returns series and also utilized on our MODWT returns series.

$$h_t = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \beta_1 h_{t-1} \tag{3}$$

where,

β_1 measures the distance to which a present volatility shock goes into the future volatility.
$(\alpha_1 + \beta_1)$ measures the rate at which this effect dies in the future; and h_{t-1} is the volatility at week $t-1$

The returns series were then decomposed and filtered using the Maximal Overlap Discreet Wavelet Transform, which decomposes the return series into time components (Gallegati, 2008). The MODWT wavelet and scaling coefficients $\widehat{h}_{j,l}$ and $\widehat{g}_{j,l}$ are given by

$$\widehat{h}_{j,l} = \frac{h_{j,l}}{2^{\frac{j}{2}}}$$

and

$$\widehat{g}_{j,l} = \frac{g_{j,l}}{2^{\frac{j}{2}}}$$

where $\widehat{h}_{j,l}$ and $\widehat{g}_{j,l}$ are the MODWT wavelet and scaling coefficients respectively.

The formula used in the algorithm to perform an order $w = 1$ MODWT returns is given in equation (5).

$$w_1 = (r_{2k} - r_{2k-1}) \Big/ \sqrt{2} \tag{5}$$

where w_1 is the MODWT and $r_1, \ldots, r_n$ is the return series for $i = 1$, $i \leq k$, $i{+}{+}$, $k = \frac{n}{2}$

For the purposes of evaluating the superiority of the proposed tool, the Mean Squared Error measure given in (6) is used.

$$MSE = \frac{\sum_{i=1}^{n} (r_t - \widehat{r}_t)^2}{n} \tag{6}$$

where r_t is the actual return at time t, and $\widehat{r}_t$ is the forecasted return at time t.

3. Data analysis approach

In Table 1, the descriptive statistics for the daily returns for the four African stock markets are presented. The sample size is the number of trading days for the data. The four countries' stock markets' mean returns range from 3.2×10^{-4}% (Nigeria) to 2.0×10^{-4}% (Kenya). The standard deviation, which measures volatility, ranges from 0.007% (Tunisia) to 0.018% (South Africa). The kurtosis for the standard normal distribution should be positive three. However, all four African countries' stock markets' returns kurtosis, and even the world stock market returns kurtosis, exceeds three, which suggests that they are leptokurtic in nature. Although the skewness for standard normal distribution should be zero, the four African stock markets display negative skewness. This finding is a clear indication that the lower tail of the distribution is thicker than the upper tail, which means that market losses are observed more often than market gains. Furthermore, in Table 1, the Ljung–Box Q-statistics at lag 24 show that for all four countries, the null hypothesis of no autocorrelation is rejected at the 5% significance level. In addition, according to the Jarque-Berra statistics, the null hypothesis of normality is rejected for all four countries' return series.

The time plots of the four countries' returns time series are shown in Figs. 1–4.

The plots of the returns in Figs. 1–4 reveal that some periods are riskier than others. Also the risky times are scattered randomly and there is some degree of autocorrelation in the riskiness of financial returns. The amplitudes of the returns vary over time as large or small changes are followed by large or small changes. This phenomenon is called volatility clustering and is one of the stylized facts of the financial times.

The result of the very basic model given in equation (2) for the returns series for each of the four countries and the corresponding results for the Maximal Overlap Discreet Wavelet Transform (MODWT) return series are presented in Tables 2–9. The optimal lag lengths for the best fitting $AR_{(p)}$, $MA_{(q)}$ or $ARMA_{(p,q)}$ models of the returns series and the MODWT transformed returns series for each of the four countries are determined by systematically over fitting the values of p and q. The Akaike Information Criterion (AIC) is used to determine the best fitting models. For the returns series, the AIC choose the $ARMA_{(1,0)}$ model as the best fitting model for Kenya, the $ARMA_{(1,2)}$ model as the best fitting model for Nigeria, the $ARMA_{(3,3)}$ model as the best fitting model for South Africa and, finally, the $ARMA_{(2,0)}$ model as the best fitting model for Tunisia. Similarly for the MODWT transformed returns series, the AIC chooses the $ARMA_{(0,1)}$ model as the best fitting model for Kenya, the $ARMA_{(1,0)}$ model as the best fitting model for Nigeria, the $ARMA_{(5,5),}$ model for South Africa, and the $ARMA_{(4,4)}$ model for Tunisia. Generally, the AIC values suggest that MODWT-$ARMA_{(p,q)}$ models best fit the four countries' stock market indices.

Table 1
Descriptive statistics.

Country	Sample size	Mean (%)	Standard deviation (%)	Skewness	Kurtosis	Q (24)[a]	Normality test[b]
Kenya	3913	2.0×10^{-4}	0.014	−0.22	686.20	141.79***	76081473***
Nigeria	3904	3.2×10^{-4}	0.014	−0.76	272.37	76.16***	11800377***
South Africa	3913	2.8×10^{-4}	0.018	−0.23	8.61	60.50***	5161.31***
Tunisia	3913	2.7×10^{-4}	0.007	−0.04	8.42	136.95***	4804.42***

[a] Ljung–Box Q statistics at lag 24. *** indicates significance at 1% level of significance.
[b] Normality of return series are tested by using Jarque-Berra statistics. *** indicates significance at 1% level of significance.

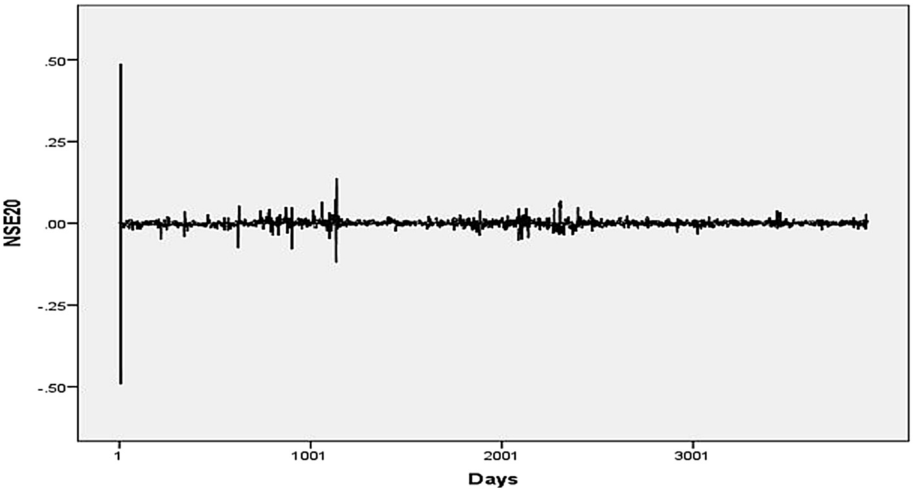

Fig. 1. Daily returns for Kenya.

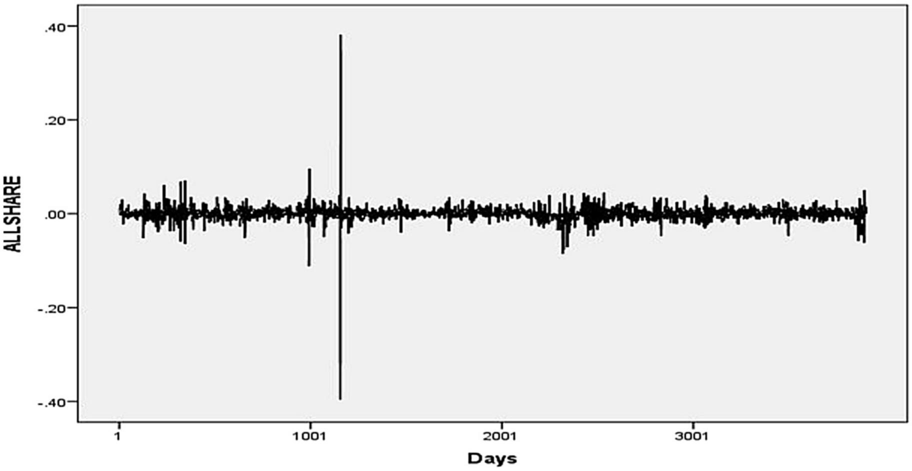

Fig. 2. Daily returns for Nigeria.

Furthermore, in Tables 2–9, the results of standard diagnostic residual checks for serial correlation and an ARCH effect are reported for all four countries' best fitting $ARMA_{(p,q)}$ return series models and best fitting MODWT-$ARMA_{(p,q)}$ returns series models. The results show that the null hypothesis of no serial correlation in the residuals of both models is accepted. In addition, the results show a remaining ARCH effect in the residuals of both models because the null hypothesis of no remaining ARCH effect in the residuals of both models is rejected. This finding is a strong confirmation of the GARCH type of heteroskedasticity. Hence, the famous $GARCH_{(1,1)}$ model is used to model the return series and also to model the MODWT transformed returns series. The summarized results for the best fitting models are presented in Table 10.

In Table 11 the parameter estimates of the $GARCH_{(1,1)}$ models for the four countries and the AIC values and the corresponding parameter estimates of the MODWT-$GARCH_{(1,1)}$ models for the four countries and the AIC values are reported and exhaustively compared. Furthermore, the β parameters of the two types of models for the four countries, which are all positive at even the 1% significance level, are of great importance. Additionally, in Table 11, the α and β parameters of the two types of models for the four countries, which are positive and whose sums are less than unity, warrant attention. When the AIC values of the $GARCH_{(1,1)}$ model are compared with those of the MODWT-$GARCH_{(1,1)}$ model, for all countries except South Africa, the MODWT-$GARCH_{(1,1)}$ model produces lower AIC

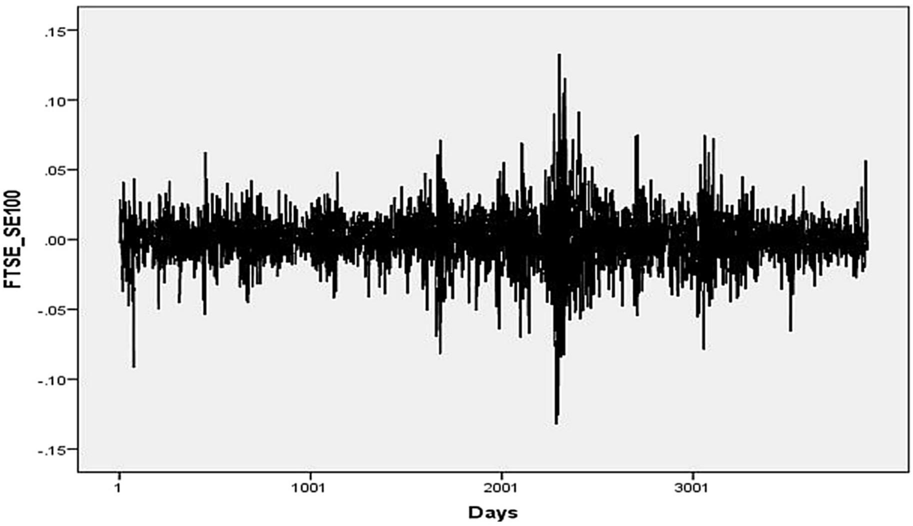

Fig. 3. Daily returns for South Africa.

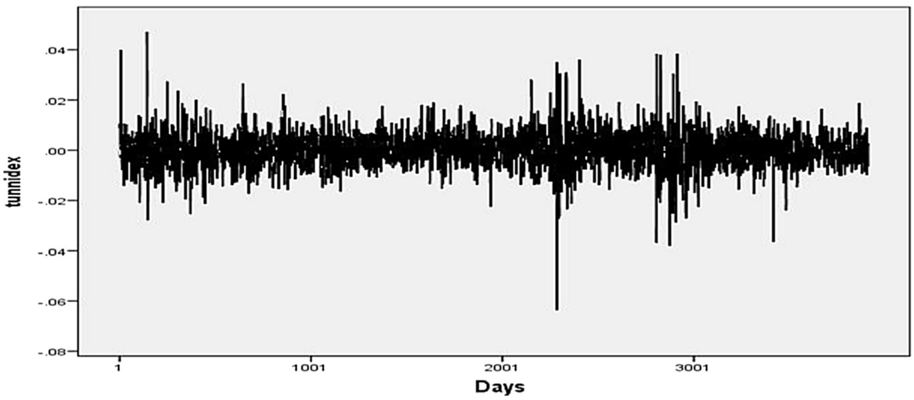

Fig. 4. Daily returns for Tunisia.

Table 2
Results of the estimated $ARMA_{(p,q)}$ for Kenya stock returns and their AIC values.

ARIMA	AIC	Serial correlation		ARCH-effect		Model significance
		F-stat	P-value	F-stat	P-value	
1,1,0	−5.82	3.94	0.00	5.26	0.00	Significant
2,1,0	−5.82	129.12	0.00	5.91	0.00	Not-significant
0,1,1	−5.83	0.49	0.98	6.06	0.00	Significant
0,1,2	−5.82	2.01	0.00	6.04	0.00	Significant
1,1,1	−5.83	9.93	0.00	6.44	0.00	Not-significant
1,1,2	−6.07	37.52	0.00	3.54	0.00	Significant
2,1,1	−5.87	123.37	0.00	6.66	0.00	Significant
2,1,2	−6.19	50.67	0.00	4.21	0.00	Significant
3,1,2	−6.70	8.00	0.00	2.06	0.00	Significant
2,1,3	−6.28	44.46	0.00	5.03	0.00	Significant
3,1,3	−6.74	4.52	0.00	2.51	0.00	Significant
4,1,3	−6.78	1.12	0.00	2.44	0.00	Significant
3,1,4	−6.75	4.13	0.00	2.79	0.00	Significant
4,1,4	−6.78	0.13	0.00	2.44	0.00	Significant

Table 3
Results of the estimated $ARMA_{(p,q)}$ for South Africa stock returns and their AIC values.

ARIMA	AIC	Serial correlation		ARCH-effect		Model significance
		F-stat	P-value	F-stat	P-value	
1,1,0	−5.17	2.37	0.00	61.59	0.00	Not-significant
2,1,0	−5.17	2.43	0.00	61.16	0.00	Not-significant
0,1,1	−5.17	2.37	0.00	61.58	0.00	Not-significant
0,1,2	−5.17	2.34	0.00	61.23	0.00	Not-significant
1,1,1	−5.17	2.36	0.00	61.51	0.00	Not-significant
1,1,2	−5.17	1.98	0.00	59.90	0.00	Significant
2,1,1	−5.18	2.07	0.00	59.93	0.00	Significant
2,1,2	−5.18	1.74	0.01	58.13	0.00	Significant
3,1,2	−5.18	1.88	0.00	60.02	0.00	Not-significant
2,1,3	−5.17	1.88	0.00	59.88	0.00	Not-significant
3,1,3	−5.18	1.46	0.07	56.98	0.00	Significant
4,1,3	−5.18	1.69	0.02	57.15	0.00	Not-significant
3,1,4	−5.18	1.42	0.08	57.09	0.00	Not-significant
4,1,4	−5.18	1.58	0.04	56.53	0.00	Significant

values than the $GARCH_{(1,1)}$ model. Generally, the $MODWT\text{-}GARCH_{(1,1)}$ model seems to be the most useful model for capturing the dynamic behavior of African emerging stock market returns.

3.1. Forecasting

According to (Day and Lewis, 1992; Pagan and Schwert, 1990; Franses and Van Dijk, 1996) as cited by Ref. 14, the true unconditional volatility in equation (7) should be calculated to ascertain the extent of the forecasting abilities of the $MODWT\text{-}GARCH_{(1,1)}$ model and the $GARCH_{(1,1)}$ model:

$$\gamma^2 = (r_t - r)^2 \tag{7}$$

where γ^2 is the unconditional volatility, r_t is the actual monthly return for month t, and r is the expected return for month t. The expected return is calculated by the method of moving average.[14]

Following Ref. 14, the one period ahead forecasting errors for the $MODWT\text{-}GARCH_{(1,1)}$ model and the $GARCH_{(1,1)}$ model are obtained from equation (8):

Table 4
Results of the estimated $ARMA_{(p,q)}$ Nigeria for South Africa stock returns and their AIC values.

ARIMA	AIC	Serial correlation		ARCH-effect		Model significance
		F-stat	P-value	F-stat	P-value	
1,1,0	−5.63	2.95	0.00	115.36	0.00	Not-significant
2,1,0	−5.64	1.65	0.02	113.24	0.00	Significant
0,1,1	−5.63	2.94	0.00	116.97	0.00	Not-significant
0,1,2	−5.64	1.65	0.02	112.30	0.00	Significant
1,1,1	−5.64	2.74	0.00	108.04	0.00	Significant
1,1,2	−5.64	1.48	0.06	112.70	0.00	Significant
2,1,1	−5.64	1.52	0.05	112.76	0.00	Not-significant
2,1,2	−5.64	1.43	0.08	114.09	0.00	Significant
3,1,2	−5.64	1.41	0.09	113.79	0.00	Not-significant
2,1,3	−5.64	1.40	0.09	113.91	0.00	Not-significant
3,1,3	−5.64	1.41	0.09	113.64	0.00	Not-significant
4,1,3	−5.64	1.44	0.08	113.86	0.00	Not-significant
3,1,4	−5.64	1.45	0.07	114.01	0.00	Not-significant
4,1,4	−5.64	1.33	0.13	113.51	0.00	Not-significant

Table 5
Results of the estimated $ARMA_{(p,q)}$ Tunisia for South Africa stock returns and their AIC values Tunisia.

ARIMA	AIC	Serial correlation		ARCH-effect		Model significance
		F-stat	P-value	F-stat	P-value	
1,1,0	−7.12	1.89	0.01	28.90	0.00	Significant
2,1,0	−7.12	1.51	0.05	27.67	0.00	Significant
0,1,1	−7.12	2.18	0.00	28.75	0.00	Significant
0,1,2	−7.12	1.54	0.04	27.80	0.00	Significant
1,1,1	−7.12	1.53	0.05	27.74	0.00	Significant
1,1,2	−7.12	1.52	0.05	27.65	0.00	Not-significant
2,1,1	−7.12	1.56	0.04	27.64	0.00	Not-significant
2,1,2	−7.12	2.65	0.00	27.68	0.00	Not-significant
3,1,2	−7.12	1.58	0.04	27.78	0.00	Not-significant
2,1,3	−7.12	1.93	0.00	27.81	0.00	Significant
3,1,3	−7.12	1.51	0.05	27.90	0.00	Not-significant
4,1,3	−7.12	2.10	0.00	27.47	0.00	Significant
3,1,4	−7.12	1.65	0.02	27.39	0.00	Not-significant
4,1,4	−7.12	2.13	0.00	27.49	0.00	Not-significant

Table 6
Results of the estimated $MODWT\text{-}ARMA_{(p,q)}$ for Kenya stock returns their AIC values.

ARIMA	AIC	Serial correlation		ARCH-effect		Model significance
		F-stat	P-value	F-stat	P-value	
1,1,0	−5.82	3.94	0.00	5.26	0.00	Significant
2,1,0	−5.82	129.12	0.00	5.91	0.00	Significant
0,1,1	−5.83	0.49	0.98	6.06	0.00	Significant
0,1,2	−5.82	2.01	0.00	6.04	0.00	Not-significant
1,1,1	−5.83	9.93	0.00	6.44	0.00	Not-significant
1,1,2	−6.07	37.52	0.00	3.54	0.00	Significant
2,1,1	−5.87	123.37	0.00	6.66	0.00	Not-significant
2,1,2	−6.19	50.67	0.00	4.21	0.00	Significant
3,1,2	−6.70	8.00	0.00	2.06	0.00	Significant
2,1,3	−6.28	44.46	0.00	5.03	0.00	Significant
3,1,3	−6.74	4.52	0.00	2.51	0.00	Significant
4,1,3	−6.78	1.12	0.31	2.44	0.00	Not-significant
3,1,4	−6.75	4.13	0.00	2.79	0.00	Significant
4,1,4	−6.78	0.13	0.88	2.44	0.00	Not-significant
5,1,4	−6.78	0.12	0.89	2.44	0.00	Not-significant
4,1,5	−6.78	0.13	0.88	2.44	0.00	Not-significant
5,1,5	−6.78	0.20	0.82	2.44	0.00	Not-significant

$$\eta_{t+1} = r^2 - h_{t+1} \tag{8}$$

where η_{t+1} is the forecasting error of the $MODWT\text{-}GARCH_{(1,1)}$ model and the $GARCH_{(1,1)}$ model, and h_{t+1} is the forecasted variance as generated by equation (7)

The one period-ahead forecast variance of the fifth to the last day of each of the two different samples would be found by running the regressions using the two models by employing the data from the first day in each of the two samples to the sixth day in each of the two samples and obtaining the constant parameters. The constant parameters are therefore entered into the equations of the two different models, and as a result, each of the forecast variances is found. The forecast variances for the fourth day in each of the two samples are obtained by running the regressions using the two models that employ the data from the second day in each of the two returns series to the fifth to the last day in each of the two returns series to obtain the constant parameters using the two different models. This procedure is followed in obtaining the forecast variances for the fifth to last day in each returns series until the forecast variance of the last day in each sample is obtained.[14]

Table 7
Results of the estimated MODWT-ARMA$_{(p,q)}$ for South Africa stock returns their AIC values.

ARIMA	AIC	Serial correlation		ARCH-effect		Model significance
		F-stat	P-value	F-stat	P-value	
1,1,0	−5.18	2.56	0.00	35.31	0.00	Not-significant
2,1,0	−5.18	2.30	0.00	33.00	0.00	Significant
0,1,1	−5.18	2.51	0.00	35.30	0.00	Not-significant
0,1,2	−5.18	2.29	0.00	32.79	0.00	Significant
1,1,1	−5.18	2.55	0.00	35.12	0.00	Not-significant
1,1,2	−5.19	1.95	0.00	32.49	0.00	Significant
2,1,1	−5.19	1.95	0.00	32.37	0.00	Significant
2,1,2	−5.19	1.82	0.01	32.36	0.00	Significant
3,1,2	−5.19	1.82	0.01	32.65	0.00	Not-significant
2,1,3	−5.19	1.89	0.01	32.87	0.00	Significant
3,1,3	−5.19	1.64	0.03	30.49	0.00	Significant
4,1,3	−5.19	1.66	0.02	30.59	0.00	Not-significant
3,1,4	−5.19	1.64	0.03	30.63	0.00	Not-significant
4,1,4	−5.18	1.81	0.01	32.18	0.00	Not-significant
5,1,4	−5.19	1.61	0.03	31.18	0.00	Significant
4,1,5	−5.19	1.81	0.01	32.71	0.00	Significant
5,1,5	−5.19	1.41	0.09	29.89	0.00	Significant

Table 8
Results of the estimated MODWT-ARMA$_{(p,q)}$ for Nigeria stock returns their AIC values.

ARIMA	AIC	Serial correlation		ARCH-effect		Model significance
		F-stat	P-value	F-stat	P-value	
1,1,0	−5.82	0.93	0.56	35.21	0.00	Significant
2,1,0	−5.78	3.77	0.00	54.00	0.00	Not-significant
0,1,1	−5.82	0.67	0.88	32.58	0.00	Significant
0,1,2	−5.78	3.71	0.00	54.02	0.00	Not-significant
1,1,1	−5.82	0.65	0.90	32.88	0.00	Significant
1,1,2	−5.82	0.60	0.94	32.98	0.00	Significant
2,1,1	−5.82	0.67	0.89	33.29	0.00	Significant
2,1,2	−5.82	0.65	0.90	32.97	0.00	Not-significant
3,1,2	−5.82	0.63	0.92	32.87	0.00	Not-significant
2,1,3	−5.82	0.66	0.89	33.11	0.00	Significant
3,1,3	−5.81	0.63	0.92	32.96	0.00	Not-significant
4,1,3	−5.82	0.48	0.99	32.56	0.00	Significant
3,1,4	−5.81	0.61	0.93	32.67	0.00	Not-significant
4,1,4	−5.82	0.47	0.99	32.44	0.00	Not-significant
5,1,4	−5.82	0.52	0.97	32.81	0.00	Significant
4,1,5	−5.82	0.45	0.99	32.97	0.00	Significant
5,1,5	−5.82	0.73	0.83	33.02	0.00	Significant

In Table 12, the mean squared errors are obtained from the GARCH$_{(1,1)}$ and MODWT-GARCH$_{(1,1)}$ models. The results indicate that for all four African countries except Kenya, the MODWT-GARCH$_{(1,1)}$ model produces smaller forecasting errors than the GARCH$_{(1,1)}$ model.

4. Conclusion

In this paper, both the MODWT-GARCH$_{(1,1)}$ model and the GARCH$_{(1,1)}$ model are applied to four African stock markets. The MODWT-GARCH$_{(1,1)}$ model has the advantage of filtering, denoising and decomposing macroeconomic time series data into their time scale components. In the comparisons in Tables 11 and 12 for all four African countries, the MODWT-GARCH$_{(1,1)}$ model produces better results than the GARCH$_{(1,1)}$ model. There is also within-sample evidence that the conditional estimates of the MODWT-GARCH$_{(1,1)}$ model outperform the conditional

Table 9
Results of the estimated MODWT-ARMA$_{(p,q)}$ for Tunisia stock returns their AIC values.

ARIMA	AIC	Serial correlation		ARCH-effect		Model significance
		F-stat	P-value	F-stat	P-value	
1,1,0	−7.22	1.51	0,05	5.37	0.00	Not-significant
2,1,0	−7.22	1.54	0.05	5.26	0.00	Not-significant
0,1,1	−7.22	1.44	0.08	5.38	0.00	Not-significant
0,1,2	−7.22	1.50	0.06	5.27	0.00	Not-significant
1,1,1	−7.22	1.40	0.08	5.21	0.00	Not-significant
1,1,2	−7.22	1.51	0.05	5.31	0.00	Not-significant
2,1,1	−7.22	1.53	0.05	5.37	0.00	Not-significant
2,1,2	−7.22	1.45	0.08	5.27	0.00	Not-significant
3,1,2	−7.22	1.43	0.08	5.33	0.00	Not-significant
2,1,3	−7.22	1.45	0.07	5.24	0.00	Not-significant
3,1,3	−7.22	1.37	0.11	5.50	0.00	Not-significant
4,1,3	−7.22	1.51	0.05	5.22	0.00	Not-significant
3,1,4	−7.22	1.48	0.06	5.25	0.00	Not-significant
4,1,4	−7.23	1.28	0.16	5.65	0.00	Significant
5,1,4	−7.22	1.25	0.18	5.44	0.00	Not-significant
4,1,5	−7.23	1.36	0.11	5.48	0.00	Not-significant
5,1,5	−7.22	1.55	0.04	5.43	0.00	Not-significant

Table 10
Summarized results of the estimated ARMA$_{(p,q)}$ and MODWT-ARMA$_{(p,q)}$ and their AIC values.

Country	ARMA$_{(p,q)}$				MODWT-ARMA$_{(p,q)}$			
	Model	AIC[a]	LM test[b]	ARCH test	Model	AIC[a]	LM test[b]	ARCH test
Kenya	ARMA$_{(1,0)}$	−5.67	1.00	78.71***	ARMA$_{(0,1)}$	−5.83	0.49	6.06***
Nigeria	ARMA$_{(1,2)}$	−5.64	1.48	112.70***	ARMA$_{(1,0)}$	−5.82	0.93	35.21***
South Africa	ARMA$_{(3,3)}$	−5.18	1.46	56.98***	ARMA$_{(5,5)}$	−5.19	1.41	29.89***
Tunisia	ARMA$_{(2,0)}$	−7.12	1.51	27.67***	ARMA$_{(4,4)}$	−7.23	1.28	5.68***

Note: *** denotes significance at 1% level.
[a] Akaike information criterion.
[b] Breusch−Godfrey serial correlation LM test.

Table 11
Results of the estimated GARCH$_{(1,1)}$ and MODWT-GARCH$_{(1,1)}$ parameters and their AIC values.

Country	Parameter estimates			Parameter estimates			AIC[a] values	
	GARCH$_{(1,1)}$			MODWT-GARCH$_{(1,1)}$			GARCH$_{(1,1)}$	MODWT-GARCH$_{(1,1)}$
	α_0	α_1	β_1	α_0	α_1	β_1		
Kenya	2.95×10^{-6} (23.20)***	0.15 (32.58)***	0.81 (331.62)***	1.41×10^{-5} (25.24)***	0.25 (18.03)***	0.49 (31.20)***	−6.93	−7.15
Nigeria	6.14×10^{-6} (15.82)***	0.33 (23.91)***	0.72 (85.64)***	8.90×10^{-6} (7.27)***	0.51 (13.17)***	0.62 (27.77)***	−6.25	−6.38
South Africa	4.07×10^{-6} (5.00)***	0.07 (11.56)***	0.91 (114.69)***	9.20×10^{-6} (4.76)***	0.10 (8.08)***	0.86 (49.23)***	−5.48	−5.47
Tunisia	2.15×10^{-6} (7.27)***	0.09 (15.72)***	0.85 (85.64)***	5.21×10^{-6} (3.19)***	0.05 (10.32)***	0.94 (123.96)***	−7.24	−7.34

Note: *** denotes significance at 1% level.
[a] Akaike Information Criterion.

Table 12
Mean squared error terms.

Country	MODWT-GARCH$_{(1,1)}$	GARCH$_{(1,1)}$
Kenya	4.7×10^{-4}	1.6×10^{-4}
Nigeria	3.2×10^{-4}	4.0×10^{-4}
South Africa	3.0×10^{-9}	3.1×10^{-4}
Tunisia	8.8×10^{-6}	2.8×10^{-4}

estimates of the GARCH$_{(1,1)}$ model. The out-of-sample evidence proves that daily volatilities are better predicted with the MODWT-GARCH$_{(1,1)}$ model.

Gokcan (2000) employed the linear and non-linear GARCH models to explain the volatility of the time series of seven emerging stock markets. Conversely, this study adopted the linear GARCH model as a MODWT-GARCH$_{(1,1)}$ model and utilized it as a based reference in comparison to the traditional linear GARCH$_{(1,1)}$ model in explaining the time series volatility of the four African countries' stock markets.

In his study Gokcan (2000) concluded based on empirical evidence that the return series though significantly skewed, the linear GARCH$_{(1,1)}$ model was still helpful in explaining the volatility of time series. This result resonates with that of this study in terms of the significance of skewness in return series. In the same vein, the MODWT-GARCH$_{(1,1)}$ model was still conveniently apt in explaining the volatility of the time series.

Furthermore, the findings in this study, proved that the MODWT-GARCH$_{(1,1)}$ model produced better result than that of the linear GARCH$_{(1,1)}$ model. This is in consonance with that of Gokcan (2000) when he unraveled that the linear GARCH$_{(1,1)}$ model outperformed that of the non-linear counterpart in terms of results. Thus, the within sample conditional estimates of both the linear GARCH$_{(1,1)}$ model and that of the MODWT-GARCH$_{(1,1)}$ model superseded the within sample conditional estimates of the non-linear GARCH$_{(1,1)}$ model and linear GARCH$_{(1,1)}$ model respectively.

Finally, the non-linear GARCH$_{(1,1)}$ model and the linear GARCH$_{(1,1)}$ model played a second fidel in terms of their predictive ability of the out-of-sample volatility forecast evidence to both the linear GARCH$_{(1,1)}$ model (Gokcan (2000)) and that of the MODWT-GARCH$_{(1,1)}$ model in this study respectively.

Based on the findings of this study and that of Gokcan (2000) there is a justification that the linear GARCH$_{(1,1)}$ model better predicts volatilities of emerging stock markets than the non-linear GARCH$_{(1,1)}$ model. Most importantly, the MODWT-GARCH$_{(1,1)}$ model also supersedes the linear GARCH$_{(1,1)}$ model in the wake of predicting the volatilities of the African countries' stock markets.

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