NON-LINEAR MECHANICAL DAMAGE MODELLING
FOR LONG FIBRE-REINFORCED LAMINATES

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ABSTRACT

A computational methodology is presented for modelling the material non-linear mechanical behaviour of composite structures made of FRP (Fibre-Reinforced Plastic) laminates.

The model is based on the appropriate combination of the constitutive models of component materials, considered to behave as isolated continua, together with additional 'closure equations' that characterize the micromechanics of the composite from a morphological point of view. To this end, any appropriate constitutive model may be selected for each phase. Each component is modelled separately and the global response is obtained by assembling all contributions taking into account the interactions between components in a general phenomenological way.

To model the behaviour of a single unidirectional (UD) composite lamina, a Serial-Parallel continuum approach has been developed assuming that components behave as parallel materials in the fibres alignment direction and as serial materials in orthogonal directions. Taking into account the internal morphology of the composite material, it is devised a strategy for decoupling and coupling component phases. This methodology [1], named "compounding of behaviours", allows to take into consideration local degradation phenomena (in the constituents materials), like plasticity, damage, fatigue [2], etc. in a coupled manner. It is based on the proper management of homogenous constitutive models, already available for each component. In this way, it is used all developments achieved in constitutive modelling of plain materials, what makes possible the transference of this technology to composites.

A lamination theory complemented with the proposed UD model is employed to describe the mechanical behaviour of multidirectional laminates. A specific solution strategy for the general non linear case is proposed. It provides quick local and global convergences, what makes the model suitable for large scale structures. The model brings answers on the non-linear behaviour of composites, where classical micro-mechanics formulas are restricted to their linear elastic part. The methodology is validated through several numerical analyses and contrasted against experimental data and benchmark tests.

Keywords: FRP composite laminates, Non-linear continuum damage modelling, Failure envelopes, FEA.

1   INTRODUCTION

The employment of Long Fibre Composites (LFC) has extensively developed in the automotive and aeronautical industry during the last 40 years, due, mainly, to the excellent mechanical properties of these types of materials. During the same period a great theoretical effort was spent in the analysis of LFC and in the construction of a mathematical basis for the description of their complex micro and macro mechanics; consequently, a large amount of literature devising

constitutive models for LFC has been produced. Despite these considerations, it is remarkable that confidence upon the effectiveness of the failure prediction theories and upon the constitutive models devised in the design of composite structures has not coupled to the reliance upon the structural properties of this class of materials.

In the last decade, this perspective has begun to change, since the pressing need of industry to reduce the time and cost of bringing new components to the market calls for improved design methods. Industrial design requires constitutive models which allow realistic structural analysis, failure predictions and the possibility of an efficient implementation in a finite element code. Such models would thus allow the transference of the large amount of already developed FE technology which is currently employed for metallic materials.

The design of a constitutive model with the above-mentioned features is not a simple task since, a realistic analysis of structural members made of fibre reinforced composites requires, both at the micro and macro-scales, a proper account of the non-linear stress-strain relationships. The lamination sequence obliges us to consider the behaviour of each lamina separately and even the modelling of a single lamina appears to be a complex task since phenomena like fracture, fibre-matrix debonding, microbuckling and large deformations have to be considered together with their mutual interactions. In addition, in the analysis of fibre reinforced laminates, it is essential to distinguish between fibre failure and matrix failure as well as between fibre degradation and matrix degradation. This constitutes a strong limitation for those models that treat the composite as a homogeneous material.

In the present paper, a formulation is developed to specifically model the non-linear material behaviour of unidirectional long fibre reinforced laminas. The aim of the model is making the composite behaviour dependent on the constitutive laws of component materials, according to their volume fractions and to their morphological distribution inside the composite.

This model was first sketched by Rastellini & co-workers [3-5] to account for components with additive plasticity and/or damage in elastic stiffness. Its generalization allows the compounding of materials with any non-linear constitutive model. It is based on the appropriate management of the constitutive models of component phases within a continuum framework. To this end, a multi-material approach denominated 'compounding of material models' is proposed. Two versions of the model are formulated which basically differ for the closure equations taken into account [1]. The first one, referred to as Basic Serial Parallel (BSP) model, inherits closure equations that consider isostrain hypothesis in fibre direction and isostress hypothesis in transversal directions; while the second, Enriched Serial Parallel (ESP) model, is devised to improve the transverse and shear stiffness underestimated by the BSP. The proposed model (for a single lamina) is combined with classical lamination theory to describe laminates consisting of unidirectional continuously reinforced layers.

2 CONSTITUTIVE MODEL

In this section we propose a constitutive model, for a biphasic composite, employing the constitutive laws of its component materials. The distinctive feature of long fibre composites is the well known strongly anisotropic mechanical behaviour. An appropriate closure equation devised for the specific problem of unidirectional LFC should have the following properties: 1) it should retain the essential axial constraint of the phases and maintain the transverse isotropy whenever component materials exhibit this property; 2) it should provide a correct tangent stiffness, which should be equal to the initial elastic one when no-inelastic phenomena have occurred; 3) it should retain a character of simplicity since the convenience of an approximate model, like the proposed one, stands in the possibility of avoiding the complex calculations required by a complete double-scale analysis; a too complex model would make void the motivation for a simplified analysis.

The equations governing the problem are:

1) the constitutive laws of both materials:

$$
\begin{aligned}
{}^{c}\dot{\boldsymbol{\sigma}} &= {}^{c}\boldsymbol{g}\left({}^{c}\boldsymbol{\varepsilon},\ {}^{c}\boldsymbol{\beta},\ {}^{c}\dot{\boldsymbol{\varepsilon}} \right) \\
{}^{c}\dot{\boldsymbol{\beta}} &= {}^{c}\boldsymbol{h}\left({}^{c}\boldsymbol{\varepsilon},\ {}^{c}\boldsymbol{\beta},\ {}^{c}\dot{\boldsymbol{\varepsilon}} \right)
\end{aligned}
\qquad \text{with} \quad c = f, m
\tag{1}
$$

Variables ${}^{c}\boldsymbol{\beta} := \left\{ {}^{c}\boldsymbol{\varepsilon}^{p},\, {}^{c}\mathbb{C}^{d} \right\}$ group all the set of internal variables corresponding to the components, like for example internal variables of damage and/or plasticity that define the state of the compounding materials.

2) the equation relating average strains and stresses:

$$
\boldsymbol{\varepsilon} = {}^{f}k\,{}^{f}\boldsymbol{\varepsilon} + {}^{m}k\,{}^{m}\boldsymbol{\varepsilon}
\tag{2}
$$

$$
\boldsymbol{\sigma} = {}^{f}k\,{}^{f}\boldsymbol{\sigma} + {}^{m}k\,{}^{m}\boldsymbol{\sigma}
\tag{3}
$$

${}^{f}k,\ {}^{m}k$ denote the volumetric fraction of fibres and matrix, evidently ${}^{f}k + {}^{m}k = 1$.

3) the BSP closure equations:

$$
{}^{m}\boldsymbol{\varepsilon}_{P} = {}^{f}\boldsymbol{\varepsilon}_{P},
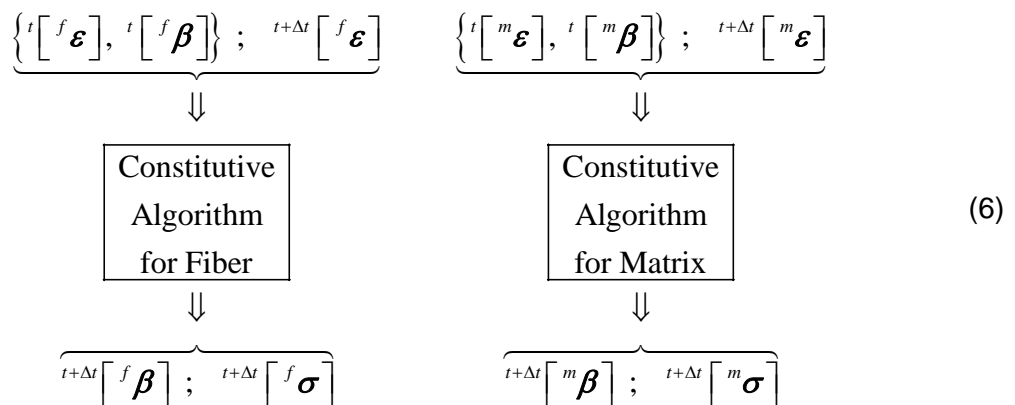\tag{4}
$$

$$
{}^{m}\boldsymbol{\sigma}_{S} = {}^{f}\boldsymbol{\sigma}_{S}.
\tag{5}
$$

Isostrain in parallel direction and isostress in serial directions are the usual and simpler assumptions when obtaining the properties of composite material.

The next chart shows the known and unknown variables of the problem:

Known variables:	${}^{t+\Delta t}[\boldsymbol{\varepsilon}]$,	free variable
	${}^{t}\left[{}^{m}\boldsymbol{\varepsilon}\right],\ {}^{t}\left[{}^{f}\boldsymbol{\varepsilon}\right],\ {}^{t}\left[{}^{m}\boldsymbol{\beta}\right],\ {}^{t}\left[{}^{f}\boldsymbol{\beta}\right]$.	internal variables
Unknown variables:	${}^{t+\Delta t}\left[{}^{m}\boldsymbol{\sigma}\right],\ {}^{t+\Delta t}\left[{}^{f}\boldsymbol{\sigma}\right],\ {}^{t+\Delta t}[\boldsymbol{\sigma}]$,	dependent variables
	${}^{t+\Delta t}\left[{}^{m}\boldsymbol{\varepsilon}\right],\ {}^{t+\Delta t}\left[{}^{f}\boldsymbol{\varepsilon}\right],\ {}^{t+\Delta t}\left[{}^{m}\boldsymbol{\beta}\right],\ {}^{t+\Delta t}\left[{}^{f}\boldsymbol{\beta}\right]$.	updated internal variables

The proposed algorithm is a general solver for composites that uses the constitutive models of component materials as 'black boxes'. This procedure allows using already developed algorithms for homogenous materials available in many FEM codes. That means, the algorithm for the local integration of the evolution equations is given for each component material:

$$
\underbrace{\left\{ {}^{t}\left[{}^{f}\boldsymbol{\varepsilon}\right],\ {}^{t}\left[{}^{f}\boldsymbol{\beta}\right] \right\} ;\quad {}^{t+\Delta t}\left[{}^{f}\boldsymbol{\varepsilon}\right]}
\qquad
\underbrace{\left\{ {}^{t}\left[{}^{m}\boldsymbol{\varepsilon}\right],\ {}^{t}\left[{}^{m}\boldsymbol{\beta}\right] \right\} ;\quad {}^{t+\Delta t}\left[{}^{m}\boldsymbol{\varepsilon}\right]}
$$

$$
\Downarrow \qquad\qquad\qquad\qquad \Downarrow
$$

$$
\boxed{\begin{array}{c} \text{Constitutive} \\ \text{Algorithm} \\ \text{for Fiber} \end{array}}
\qquad\qquad
\boxed{\begin{array}{c} \text{Constitutive} \\ \text{Algorithm} \\ \text{for Matrix} \end{array}}
\tag{6}
$$

$$
\Downarrow \qquad\qquad\qquad\qquad \Downarrow
$$

$$
\overbrace{{}^{t+\Delta t}\left[{}^{f}\boldsymbol{\beta}\right] ;\quad {}^{t+\Delta t}\left[{}^{f}\boldsymbol{\sigma}\right]}
\qquad\qquad
\overbrace{{}^{t+\Delta t}\left[{}^{m}\boldsymbol{\beta}\right] ;\quad {}^{t+\Delta t}\left[{}^{m}\boldsymbol{\sigma}\right]}
$$

The algorithms for the resolution of component materials' constitutive laws have been denoted by 'fibre/matrix constitutive algorithms' while the algorithm for the resolution of the whole BSP model denoted by 'composite algorithm'. Furthermore, each component material model must also provide the consistent tangent operators $^c\mathbb{C}$ for each component constitutive algorithm, sketched in (6). The composite algorithm employs the following Serial-Parallel decomposition of the tangent operators, related to fibre direction versor e_1.

$$^c\mathbb{C} = \begin{bmatrix} \dfrac{\partial\,^c\boldsymbol{\sigma}_P}{\partial\,^c\boldsymbol{\varepsilon}_P} & \dfrac{\partial\,^c\boldsymbol{\sigma}_P}{\partial\,^c\boldsymbol{\varepsilon}_S} \\[3mm] \dfrac{\partial\,^c\boldsymbol{\sigma}_S}{\partial\,^c\boldsymbol{\varepsilon}_P} & \dfrac{\partial\,^c\boldsymbol{\sigma}_S}{\partial\,^c\boldsymbol{\varepsilon}_S} \end{bmatrix} = \begin{bmatrix} ^c\mathbb{C}_{PP} & ^c\mathbb{C}_{PS} \\[2mm] ^c\mathbb{C}_{SP} & ^c\mathbb{C}_{SS} \end{bmatrix},$$ (7)

$$\text{where:} \begin{cases} ^c\mathbb{C}_{PP} = \mathbb{P}_P : ^c\mathbb{C} : \mathbb{P}_P \\ ^c\mathbb{C}_{PS} = \mathbb{P}_P : ^c\mathbb{C} : \mathbb{P}_S \\ ^c\mathbb{C}_{SP} = \mathbb{P}_S : ^c\mathbb{C} : \mathbb{P}_P \\ ^c\mathbb{C}_{SS} = \mathbb{P}_S : ^c\mathbb{C} : \mathbb{P}_S \end{cases} \text{with: } c = m,\ f.$$

$$\text{and} \qquad \mathbb{P}_P = \mathbf{N}_{11} \otimes \mathbf{N}_{11}, \qquad \mathbf{N}_{11} = e_1 \otimes e_1 \qquad \mathbb{P}_S = \mathbb{I} - \mathbb{P}_P.$$

The serial part of matrix strain $^m\boldsymbol{\varepsilon}_S$ is selected as the independent variable of the Newton-Raphson scheme to be adopted for the composite algorithm and the disequilibrium in the serial stresses ($\Delta\boldsymbol{\sigma}_S = {}^m\boldsymbol{\sigma}_S - {}^f\boldsymbol{\sigma}_S$) as the residue, in the sense that at the end of the algorithm it has to be set equal to zero. More details of the theoretical formulation can be found in previous work of the authors [1-3].

The Enriched SP (ESP) model.

It was pointed out in previous section, the assumption of equal stress in orthogonal directions to the fibre (pure serial behaviour in transverse directions) constitutes a lower bound for the transverse stiffness of the composite, and that for this reason, the BSP model needs to be enriched aiming to be able to predict the transversal behaviour more accurately.

In order not to resort to a more complex model and since the governing equations for the ESP model maintains the structure of the governing equations (1-5) for the BSP model, the new algorithm is similar to the one for the BSP. The only difference is the use of $\left(^m\boldsymbol{\varepsilon}\right)^*$, $\left(^m\boldsymbol{\sigma}\right)^*$, $\left(^m\mathbb{C}\right)^*$ instead of $^m\boldsymbol{\varepsilon}$, $^m\boldsymbol{\sigma}$, $^m\mathbb{C}$:

$$\left(^m\boldsymbol{\varepsilon}\right)^* = \left[^m\mathbb{K}\right]^{-1} : {}^m\boldsymbol{\varepsilon}$$ (8)

$$\left(^m\boldsymbol{\sigma}\right)^* = {}^m\mathbb{K} : {}^m\boldsymbol{\sigma}$$ (9)

$$\left(^m\mathbb{C}\right)^* = {}^m\mathbb{K} : {}^m\mathbb{C} : {}^m\mathbb{K}$$ (10)

where: $^m\mathbb{K} = \mathbb{P}_P : \mathbb{I} : \mathbb{P}_P + {}^m\gamma\ \mathbb{P}_S : \mathbb{I} : \mathbb{P}_S.$

The calculation of parameter ${}^{m}\gamma_{ij}$ which appears in the matrix ${}^{m}\mathbb{K}$ may be performed through the adoption of a simplified micro-mechanical model. By following the analysis done in [1], one obtains:

$$ {}^{m}\gamma = \frac{\sqrt{\eta + \omega^2 (1-\eta)}}{\eta + \omega(1-\eta)} \tag{11} $$

where: $\quad \omega = (R-1)\sqrt{{}^{f}k} + 1, \qquad R = \frac{{}^{f}E}{{}^{m}E}, \qquad$ and $\qquad \eta = \frac{\sqrt{{}^{f}k}}{1+{}^{f}k}.$

3 NUMERICAL VALIDATIONS

The purpose of this section is to show and discuss the results of several numerical analyses devised to validate the response of the proposed model (in its two versions: BSP and ESP).

This model was implemented in COMET [6], which is a general-purpose FEM code developed by CIMNE and RMEE. This software contains a collection of constitutive models that may be selected as material law for the components.

Since the proposed model is based on the proper management of the component constitutive models, it is essential to adopt the specific constitutive laws that better represent the mechanical behaviour of each component phase, in each particular analysis. To study the general response of the model, fictitious materials are used in order to be able to better appreciate the interaction between component phases. On the other hand, when the objective is to reproduce as accurately as possible the mechanical response of real composite materials, a non iso-resistent damage constitutive law is selected for both fibre and matrix.

3.1 Close equations fulfilment

The fulfilment of the close equations is verified as part of the validation process. With this aim in mind, stress and strain states are analysed not only in the components but also in the composite material. The unidirectional lamina under study is subjected to "transversal loading".

In the following, the mechanical properties of matrix and fibre are chosen in order to make the main features of the model clearly detectable. They do not correspond to real materials, and for this reason are called material 'M' and material 'F', respectively.

The test is performed by applying a load-unload transversal controlled deformation up to 5% strain. No other constrains are set. The constitutive law selected for the material 'F' is an isotropic damage model with softening, while for the material 'M', a J2 plasticity model with exponential hardening is chosen. The main mechanical properties of these materials are shown in the Table 2. Note that material 'M' is weaker and has a lower elastic limit than material 'F'.

	Material 'F'	Material 'M'
Constitutive law	Iso-damage with softening	J2 plasticity with hardening
Young modulus [MPa]	3000	2000
Elastic limit [MPa]	60	40
Poisson ratio	0.0	0.0
Volume fraction	0.5	0.5

Table 1 – Mechanical properties of constituents selected for testing the SP model under transversal loading (serial behaviour).

In Figure 1, the transversal stresses σ_S observed in all materials during the validation process are plotted against their respective transversal strains ε_S. The simulation shows that at each step of

the analysis the closure equation (iso-stress) is exactly fulfilled; this is denoted by the fact that at each step the serial stresses are identical for all materials. In the first elastic branch (O-A), the composite transversal stiffness, given by SP model, is in accordance with the inverse ROM. When material 'M' reaches the yielding threshold –point (A) in the composite– this material experiments plastic deformations, but keeps on incrementing its stress due to its hardening law. This fact also brings about a reduction in the composite stiffness along the branch (A-B), while material 'F' remains elastic up to point (B), when its damage begins. Material 'F' damages along branch (B-C), and thus causes all stresses to decrease; as a consequence, material 'M' experiments elastic unload. From point (C) on, the sign of the applied deformation is reversed (unloading), consequently all materials experiment elastic unload. Note that the material 'F' unloads with a reduced stiffness due to internal damage. Note also that the material 'M' unloads with the virgin elastic stiffness, and at complete unload retains residual plastic strains.
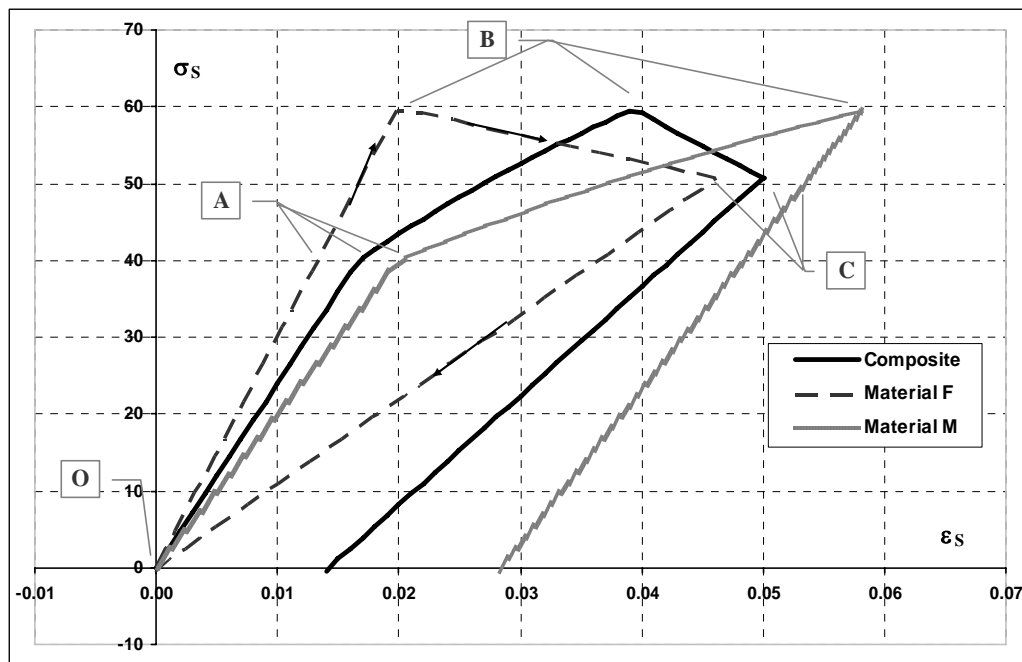
Figure 1 – Serial stress [MPa] vs. serial strain curves for the composite and component materials under transversal deformation-controlled load-unload testing.

It is worth remarking that the fulfilment of close equations is verified not only in the linear-elastic region but also in the material non-linear behaviour, including the softening process.

3.2   Transversal Stiffness Validation

In this validation, the predictive capacity of BSP and ESP models for the transversal stiffness of a glass-epoxy laminate with E_F/E_M=21.19, ν_F=0.22, ν_M=0.38 (see Table 2) is compared against experimental data and against the approximation given by broadly-used semi-empirical formulas. The validation consists in subjecting composite material to pure transversal loading at different fibre volume fractions V_f. Table 2 contains the mechanical properties of components.

	Fibre	Matrix
Material	glass	epoxy
Young modulus [MPa]	105950	5000
Poisson ratio	0.22	0.38
Volume fraction	0.6	0.4

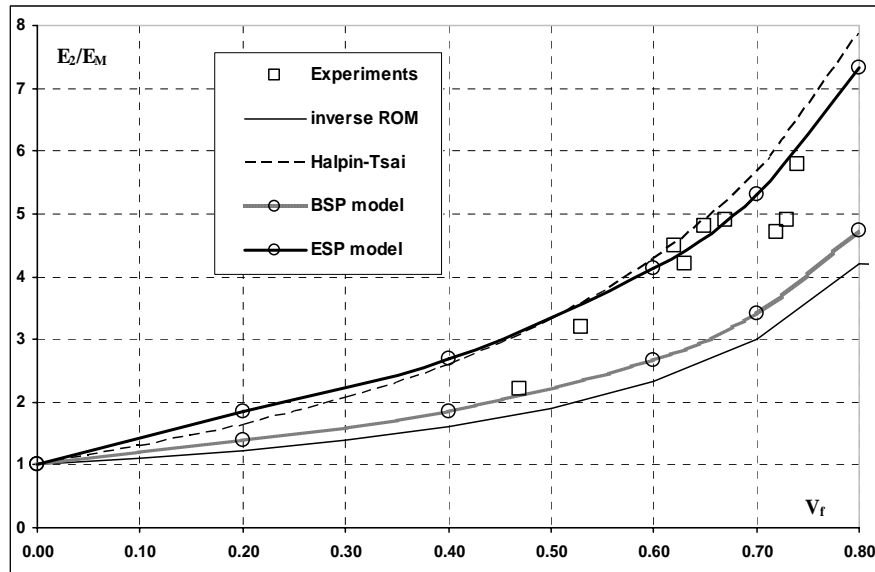Table 2 – Mechanical properties of constituents adopted to validate transversal stiffness.

Figure 2 – Relative transversal stiffness E_2/E_M versus fibre volume fraction V_f. Comparison between the result given by the proposed models, experimental data, inverse ROM and Halpin-Tsai equation.

In Figure 2, the adimensional curves E_f/E_m vs. V_f resulting from basic SP and Enriched SP simulations are reported together with experimental values taken from Barbero [7]. In the same figure, the curves resulting from perfect inverse ROM and from Halpin-Tsai equation [8] are also provided for comparison.

The transversal stiffness obtained by the simple SP model turns out to be slightly greater than the one given by inverse ROM, due to Poisson effects (fibre longitudinal constrain). The graph shows that the Basic SP model and the inverse ROM appreciably underestimate the experimental values. On the other hand, the Enriched SP model, with the adopted gamma evaluation, gives an approximation to experimental data as good as the one given by Halpin-Tsai equation, which is one of the formulae most frequently employed when only limited experimental information is available. It is important to remark that in the ESP model no experimental coefficient was introduced to fit experimental data, since this model is only based on micro-mechanical considerations.

3.3   Off axis strength validation

In this validation an hexahedral composite element, composed of the same materials used in previous strength validations for carbon-epoxy laminates and with $V_f=0.60$, is subjected to uniaxial stress (i.e. force-controlled loading) applied in a direction rotated by an angle θ with respect to fibre direction. The ultimate strength is given by the proposed model in function of the angle θ.

For moderate-to-large misalignment angles, the curve obtained by the ESP model almost superposes the one corresponding to the Tsai-Hill criterion given by:

$$X_{\theta TH} = \frac{1}{\sqrt{\dfrac{\cos^4\theta}{X_1^2} + \dfrac{\sin^4\theta}{X_2^2} - \dfrac{\sin^2\theta\cos^2\theta}{X_1^2} + \dfrac{\sin^2\theta\cos^2\theta}{X_{12}^2}}} \, .$$

But for small angles, the curve presents a non-smooth shape which makes it more similar to the maximum stress criterion. This is due to the fact that the SP model naturally distinguishes between fibre failure and matrix failure (see Figure 3).
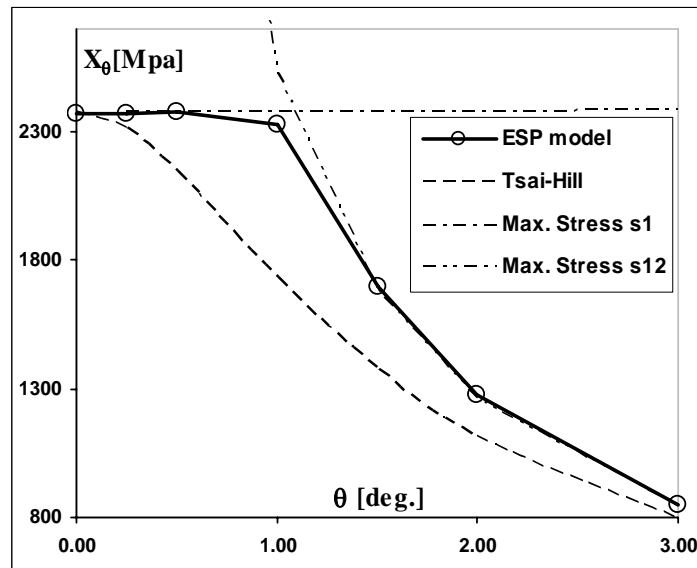
Figure 3 – Detail of the ultimate composite strength [MPa] curve at very low off-axis angles for a carbon-epoxy unidirectional lamina. Comparison with broadly-used failure formulas.

4   FAILURE ENVELOPES

In order to perform a reliable validation procedure, the experimental data provided by the "worldwide failure exercise" [9] have been used to validate the predictions of the ESP model. All voluminous data generated by the "failure exercise" is considered to be an appropriate benchmark procedure for any model that aims to predict the mechanical response of FRP laminates. Another main contribution is the exhaustive comparison of the predictive capabilities of current failure theories for composite laminates, as reported by Soden *et al.* [10] and Kaddour *et al.* [11].

All failure envelopes are obtained by testing a hexahedral composite element, whose material properties were calibrated employing components and lamina experimental data [12]. Incremental force-controlled loading scenarios are applied up to final failure.

4.1   Failure envelope for combined longitudinal & shear loading

Figure 1 shows the biaxial failure envelope obtained with the ESP model, under combined longitudinal and shear loading (σ_x vs. τ_{xy}) for the E-glass/epoxy LY556 unidirectional lamina. Several stress states with different ratios $\sigma_x{:}\tau_{xy}$ were applied.

As expected, rectangular envelope is obtained. Good accordance with experiments is achieved for tensile and compressive longitudinal strengths while shear response is slightly underestimated.

4.2   Failure envelope for combined transverse & shear loading

The biaxial failure envelope under combined transverse and shear loading (σ_y vs. τ_{xy}) for the LY556 unidirectional lamina is illustrated in Figure 2 as predicted by the ESP model, together with the experimental data provided by Soden and co-workers [13]. For comparison, the domain given by Puck & Schürmann [14-15] is also plotted.

Good accordance with experiments is achieved for the tensile and compressive transversal strengths. The shear response is underestimated in the first quadrant probably due to inappropriate selection of the matrix failure envelope (damage surface).
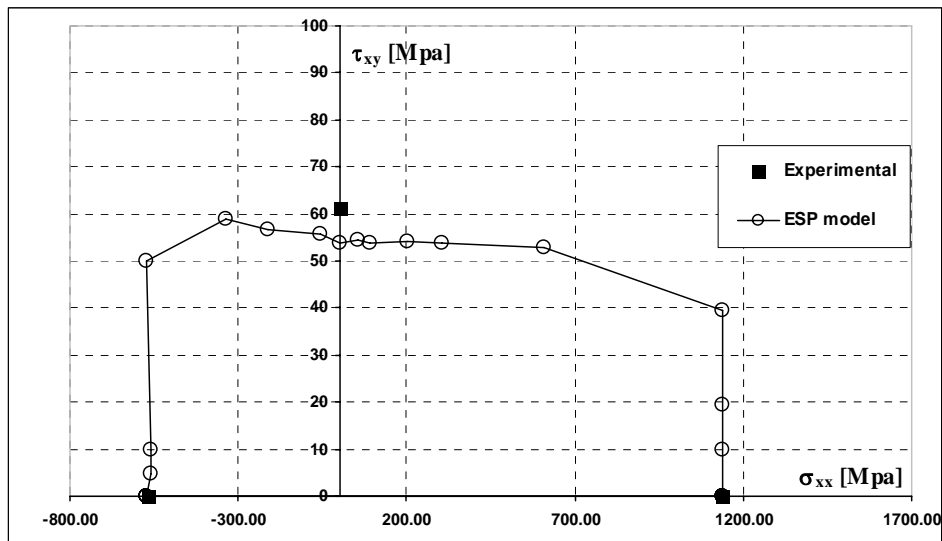
Figure 1 – Biaxial failure envelope for glass-epoxy LY556 unidirectional lamina under combined longitudinal & shear loading.
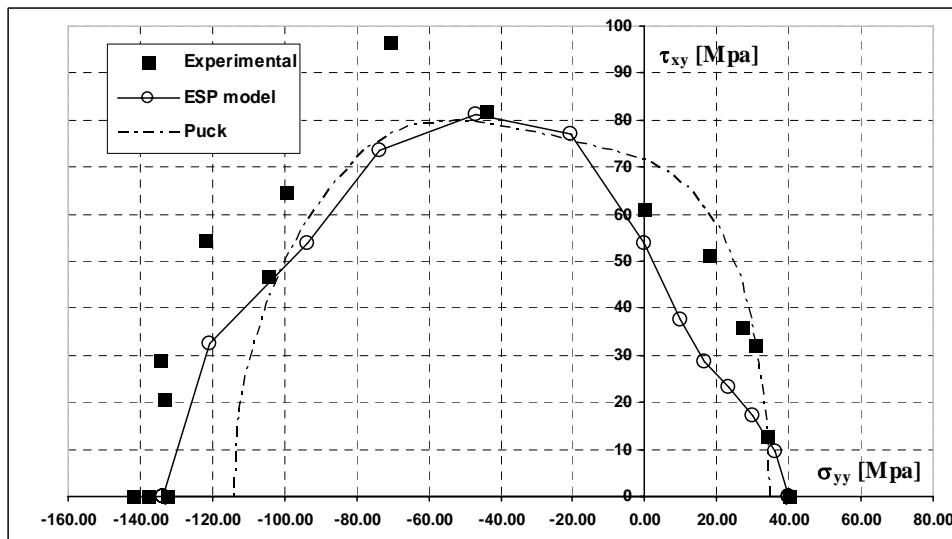

Figure 2 – Biaxial failure envelope for glass-epoxy LY556 unidirectional lamina under combined transversal & shear loading. Comparison with experimental data and Puck's estimation.

4.3 Failure envelope for [90°/±30°/90°] GFRP laminate

We now consider the failure envelope for a [90°/±30°/90°] laminate made of E-glass/epoxy LY556. The precise lay-up configuration is the following: [90°/+30°/-30°/-30°/+30°/90°].

In Figure 3, the experimental data points for failure under combined longitudinal and shear loading (σ_x vs. τ_{xy}) are reported, together with the failure envelope supplied by the ESP model prediction and Puck's estimation.
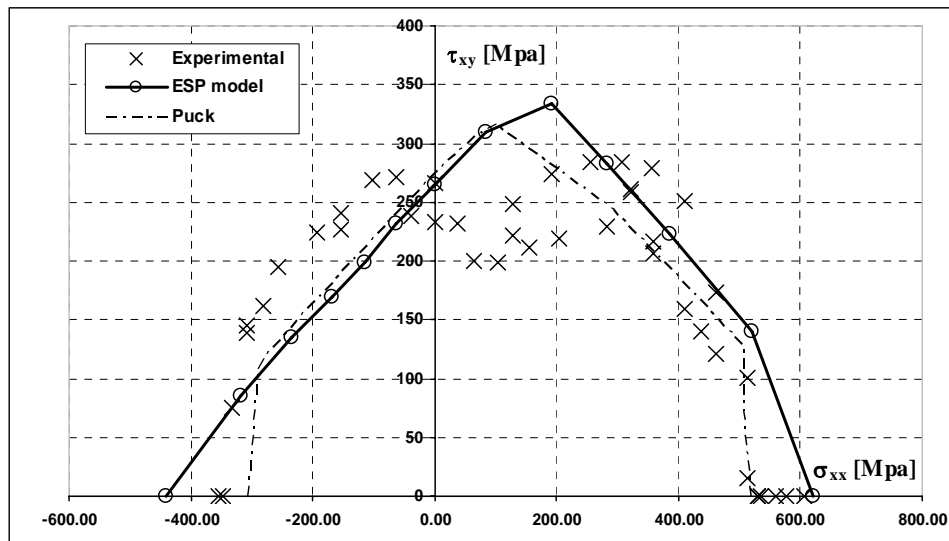
Figure 3 – Biaxial failure envelope for [90°/±30°/90°] glass-epoxy LY556 laminate under combined longitudinal & shear loading. Comparison with experimental data and Puck's estimation.

Acceptable agreement with the experimental data and Puck's envelope is achieved. We remark that in the zones where our model slightly overestimates the strength (i.e. for pure compressive σ_x and in the central zone of maximum τ_{xy}) the collapse is due to delamination and local buckling.

5   CONCLUDING REMARKS

The Serial-Parallel (SP) model is combined with classical lamination theory to describe laminates consisting of unidirectional continuously reinforced layers. Its relative simplicity and the resulting numerical efficiency make the SP approach well suited for implementation as a material model in Finite Element programs for studying the elastoplastic response of structures or components made of long fibre reinforced laminated composites. In addition, it requires relatively small computational resources when implemented into a structural FE code. Its initial drawback of underestimation of the transverse and shear stiffness is then improved upon with the Enriched SP model.
Validations show the capability of the ESP model to predict failure and post-failure behaviour of the composite based on appropriate constitutive models of components materials. Comparison between experimental and numerical testing enables us to state that the proposed methodology is very promising for material non-linear analysis of composite materials and structures.

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