Spatial competitive games with disingenuously delayed positions

Marzieh Soltanolkottabi, David Ben-Arieh, John (C-W) Wu
Department of Industrial and Manufacturing Systems Engineering
Kansas State University,
2069 Rathbone Hall
1701B Platt St., Manhattan, Kansas, Riley
Email: davidbe@ksu.edu

Abstract

During the last decade, spatial games have received great attention from researchers showing the behavior of populations of players over time in a spatial structure. One of the main factors which can greatly affect the destiny of such populations is the updating scheme used to apprise new strategies of players. Synchronous updating is the most common updating strategy in which all players update their strategy at the same time. In order to be able to describe the behavior of populations more realistically several asynchronous updating schemes have been proposed. Asynchronous game does not use a universal and players can update their strategy at different time steps during the play.

In this paper, we introduce a new type of asynchronous strategy updating in which some of the players hide their updated strategy from their neighbors for several time steps. It is shown that this behavior can change the behavior of populations but does not necessarily lead to a higher payoff for the dishonest players. The paper also shows that with dishonest players, the average payoff of players is less than what they think they get, while they are not aware of their neighbors' true strategy.

Keywords: Spatial games, Asynchronous updating, delayed positions

1. Introduction

Game theory is defined as the study of mathematical models to show conflict and cooperation between rational decision-makers [1]. It describes how rational players behave when interacting with each other and has been used as a method to model competition in competitive evolution which refers to as evolutionary game theory [2]. Evolutionary game theory is mainly the study of a population of interacting players in which players interact with each other and go through evolution [2]. Spatial games use evolutionary game dynamics on a spatial structure. This approach has brought together game theory and cellular automata

(CA). In these games, each player plays the game with its neighbors and based on the players' strategy the various positions will be occupied by the owner of the cell or a neighbor [2].

In most spatial games, cells are updated in synchrony. A synchronous method of updating indicates that at every time all players on the lattice play at the same time and all cells are updated simultaneously. In contrary, in the asynchronous approach not all cells are updated at the same time but they can be updated in different times. This approach can be used for modeling real-world problems in which there is not a universal clock to enforce a synchronized updating. Using an asynchronous updating method can result in different dynamic behavior in comparison with synchronous method of updating [3]. Several studies have used asynchronous updating methods and have probed the differences of their behavior from the synchronous ones. In the literature there are several types of asynchronous updating which are more commonly used [4]. These methods are as follows:

- Random updating: In this updating scheme, a player is selected at random from the population, and after playing against its neighbors its strategy is updated. This updating occurs in a time instance before the time that a general clock updates the value of all other cells. These time steps are usually called micro-time-steps. This method is the most used method of asynchronous updating which is in several studies used for probing the effect of asynchronous updating on stability of cellular automata or emerging cooperation [3, 5-11].
  This method is classified as Random asynchronous updating with replacement in which a player can be updated several times in micro-time-steps, or without replacement in which once a player is updated it cannot be updated again [5].
- Random order updating: In this updating method players are updated in a fixed random order in the entire simulation. In several studies this method has been used in analogy to random updating [3, 5, 6, 7, 10].
- Cyclic updating: in this updating method, cells are updated based on a fixed order. The method has been used in some studies in comparison with random updating [3, 10, 12]
- Clocked updating: In this updating method a clock or an oscillator is assigned to each player which moves along a period. In the starting position, each clock is set to a random starting position. The player updates its status when the clock reaches the top of its period [5, 7, 10]. A new methodology in this updating type is that the clock of each player can be influenced by the clocks of other players [5].
- α-synchronous updating: in this updating scheme an updating function is used to update a cell. Based on this function a cell updates its value with probability α, or remains unchanged with probability $1 - \alpha$. There are several studies in the literature of asynchronous updating in which a kind of probability based updating has been used in them [13-16].

In addition to the more common methods which have been used in asynchronous updating there are some studies that have used some novel methods of updating. Lee et al. [17]

presented a new cellular automaton in which each cell is consist of some sub-cells which are joined to each other and at each time step a random portion of cells on average are updated.

As we illustrate here, there are several methods that can be used to asynchronously update a game. Most of these methods choose players and update their value in a time instance before the general clock updates all cells. In this study we have proposed a method in which, in contrary to general asynchronous updating methods, there are some players do seem not to update their strategy for some time steps although they update their strategies on their own and after a defined number of time steps they reveal their true strategy to their neighbors. This updating scheme can be referred as an asynchronous strategy with dishonest players. The flow of information and decisions of the winning strategies in this method are not the same as in synchronous updating.

Moreover, there are some studies which has used a type of vague information shared between players. Bouré et al. [18] introduced a β-synchronism updating method. In this method, there is a probability to disrupt the transmission of information between cells. They showed that β-synchronous updating causes phase transitions. Chen et al. [19] analyzed players' long-run behavior in evolutionary coordination games in which there exists imperfect monitoring in a large population and players can observe signals of other players' unseen actions and extract information from the signals by using the proposed simple or maximum likelihood estimation algorithm. They showed that player's method of extracting information from the observed signal has a critical impact on the long-run behavior in evolutionary games. Zhang and Chen [20] have also considered a situation in which required payoff information does not exist. They built a model which is based on probability of switching for each player and showed that the number of neighbors consulted for updating and the relationship of the switching probabilities between competing strategies can intrinsically affect the evolutionary game. Wang et al. [21] considered a silence strategy in the prisoner's dilemma game, where players can either engage in the game as cooperators, defectors or silenced in which they gain no payoff. They showed that for the small payoff level, the silence strategy can increase the frequency of cooperation but for the large payoff level, great majority of players choose the silence strategy to avoid the high loss of engaging in games. They also presented an intermediate payoff level that could guarantee the optimal cooperation circumstance. Tanimito [22] proposed a mixed strategy system for spatial prisoner's dilemma in which the player stochastically shows different behaviors to its neighbors based on the agent's overall strategy. He showed that in this model, cooperation is increased in comparison with common mixed strategy models in which players offer strategies stochastically.

The current proposed method can be useful when in many real-world situations we intentionally delay information transmission such as between competing companies, political parties or even in disease transmission. In the following section the delayed updating method is described, then the game in which the model uses is introduced and finally results of delayed updating and consequent analysis are discussed.

2. Methodology

In our model we have considered that players are playing in a two-dimensional lattice in which each player in a center has 8 neighbors and the ones in the edges and in the corners have 5 and 3 neighbors respectively. The required steps to build a model that represents the behavior of a lattice in evolutionary spatial games with some dishonestly delayed players are as follows:

1. Generate a lattice of size n^2. Populate the lattice with players of type A and type B. Players of type A constitute *i*% of the players while type B are *(1-i)*% of the n^2 players. In our model players using strategy A are called Hawks and players with strategy B are referred to as Doves.
2. The honest and dishonest positions are randomly allocated to the cells in the lattice.
3. Each dishonest player is assigned a number between 1 and t and the player hides its strategy for that number of time steps. In our model an identical t is used for all dishonest players.
4. The payoff for all players is calculated. Variety of functions can be used to calculate the individual payoff for each player. In this model the payoff is defined as the sum of payoffs that each player can get from playing with all its neighbors based on a given payoff matrix. Payoff calculation is described in section 2.2. Each player then adopts the strategy with the highest payoff.
5. All the players update their true strategy in a synchronous manner but the dishonest players do not reveal their strategy for t time steps. The reader should note that the apparent and true strategies are the same for honest players.
6. Steps 4 and 5 repeat for a defined number of iterations or until the lattice reaches quasi-equilibrium.

2.1 Type of Game Used

Spatial evolutionary games have looked into a variety of well-defined games. In the populations with two strategies 2×2 games are used to represent the payoff of players of different strategies competing with each other. These games are normally described by a general payoff matrix as follows [2].

$$\begin{array}{cc} & \begin{array}{cc} A & B \end{array} \\ \begin{array}{c} A \\ B \end{array} & \begin{pmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{pmatrix} \end{array}$$

This payoff matrix shows that A gets payoff M_{11} when playing with A; A gets payoff M_{12} when playing with B; B gets payoff M_{21} when playing with A and B gets payoff M_{22} when playing with B.

Based on the payoff matrix, several types of games can be defined by changing the relations between the payoffs. There are three types of most commonly used games to study the behavior of players; these games are the Prisoners' Dilemma, Chicken (or Hawk and Dove) and Stag Hunt game.

In the Prisoners' Dilemma game, two persons are arrested because of a joint crime. Each of them can either cooperate with the other person and remain silent (strategy C), or can defect and confess (strategy D). if both cooperate then both get M_{22} points. If one cooperates while the other one defects, then the cooperator gets M_{21} points which is less than M_{22} and the defector gets M_{12} points which is more than M_{22}. If both remain silent, they both get M_{11} points which is more than M_{21} but less than M_{22}. So, the relation between the payoffs is $M_{12} > M_{22} > M_{11} > M_{21}$.

In the Chicken Game, there are two players fighting for a resource. In this game *b* is the value of the contested resource, and *C* is the cost of an escalated fight. It is also assumed that the value of the resource is less than the cost of a fight (C>b); if *b*>*C*, the game becomes the Prisoners' Dilemma game. There are two types of players defined in this game, termed Hawk and Dove, in which Hawks are strong fighters and Doves are appeasing. If two Hawks compete with each other, they both pay the cost of fighting and finally divide the resource between themselves ($M_{11} = (b - C)/2$). If one Hawk compete with a Dove, Hawk will get all resources ($M_{12} = b$) and Dove gets nothing ($M_{21} = 0$) and if two Doves compete with each other they will divide the resources between themselves without fighting $M_{22} = b/2$. So, the relation between the payoffs is $M_{12} > M_{22} > M_{21} > M_{11}$.

In the Stag Hunt game, two individuals are going on a hunt. Each person can choose either to hunt a stag or a hare. If someone decide to hunt a stag, he must have the cooperation of other person to succeed and the worth of a stag is more than a hare. In this game, if both individuals decide to hunt a hare each gets M_{11} points. If one of them decide to hunt a stag and the other one decide to hunt a hare, the one who has decided to hunt a hare gets M_{12} and the other person gets nothing ($M_{21} = 0$). Otherwise, they can hunt a stag together and each gets M_{22}. So, the relation between the payoffs is $M_{22} > M_{12} > M_{11} > M_{21}$.

In the evolutionary dynamic games, a game has a dilemma in it if the joint *ii* strategy with the highest payoff (M_{ii}) does not always have a non-negative replicator dynamic [23]. In a 2×2 game world, only the payoff matrix determines whether a dilemma occurs in a game or not [23]. In all the above presented games $M_{22} > M_{11}$ and the difference in their dilemma potential can be paraphrased as $DL_1 = M_{11} - M_{21}$ and $DL_2 = M_{12} - M_{22}$. We can say that if $DL_1 \leq 0$ or $DL_2 \leq 0$ is not satisfied, it is confirmed that a certain type of dilemma game arises. Prisoners' dilemma is a particular kind of game, because both conditions are not satisfied at the same time in it. Hawk-Dove game has just $DL_2 > 0$ and Stag-Hunt game has $DL_1 > 0$. In this study we just work with the second type of dilemma which can be seen in chicken type games.

2-2 payoff calculation

In every time step, each player plays against all of its immediate neighbors (Moore neighborhood). The payoff of each player (cell) is calculated as the sum all the payoffs that a player can get in playing with all its neighbors. The payoff matrix which shows the payoff that a player can get confronting any other player type is shown in Table 1.

It is worth mentioning that using the payoff matrix alone in calculating the game's outcome turns the game into a deterministic game, meaning that if we know the arrangement of the players in the lattice the result of the game is known with certainty.

Table 1- payoff matrix for chicken game

	Hawk	Dove
Hawk	(b–C)/2, (b–C)/2	b, 0
Dove	0, b	b/2, b/2

In this table, *b* is the value of the contested resource, and *C* is the cost of an escalated fight in the Hawk and Dove game.

2.3 updating rule

For updating a cell in general, we look for the player with the highest payoff in the neighborhood of a cell (center cell) and check if the payoff of that player is higher than the payoff of that center cell. If the payoff is higher, the center cell adopts the strategy of the cell with the highest payoff, otherwise it keeps its current strategy. There is a possibility that more than one neighbors have the same highest payoff. In this case if there is at least one wining neighbors whose strategy is the same as the center cell, the center cell will keep its strategy. Otherwise the center cell will adopt the winning strategy.

2.4 Hiding strategy

Hiding strategy is the behavior of players who are dishonest in showing their updated strategy to their neighbors. In other words, they present an outdated strategy but keep track of their real strategy, so their payoff calculations are correct, but their neighbors' are not. In the following, consider the center player as the only dishonest player with time delay of telling the truth equal to one period. Figure 1 shows the hiding strategy process and its effect on the final strategies. The numbers in the cells show the payoff for the center player and its neighbors, but the payoff of neighbors are calculated based on their 8 neighbors some of which are not shown here. The following payoff matrix is used to calculate the payoffs which is a Hawk and Dove game with C equal to 10 and b equal to 5.

$$\begin{array}{ccc} & H & D \\ \begin{array}{c} H \\ D \end{array} & \multicolumn{2}{c}{\begin{pmatrix} -2.5 & 5 \\ 0 & 2.5 \end{pmatrix}} \end{array}$$

In figure 1, it can be seen that the strategy of the central player after one step will be Hawk but the player displays its previous strategy which is Dove. All other players calculate their payoff based on the displayed strategy which is Dove other than the liar who knows its true strategy. So, the resulting payoff is what neighbors of a liar perceive and the honest payoff is the payoff that a liar calculates based on its true strategy and the perceived payoff information that its gets from its neighbors. In the next time step the center player reveals its true strategy which is Hawk which is calculated based on the honest payoff lattice and all other players reveal their strategy based on the perceived payoff lattice. We can see that the hiding strategy of the center player has changed the destiny of the lattice from all Hawks to more Doves.

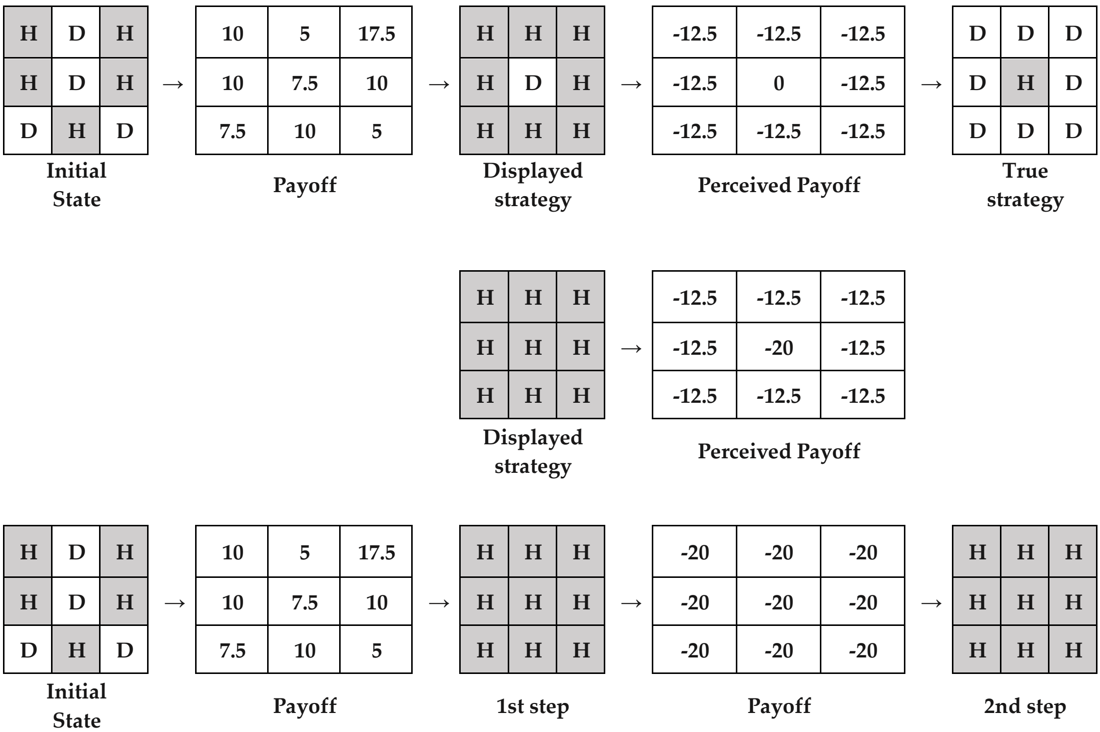


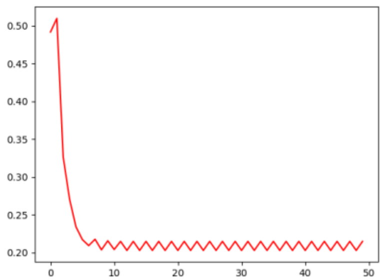
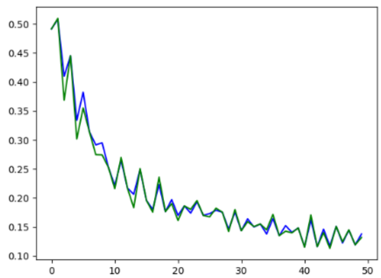
Figure 1- illustrating hiding strategy

In Figure 1, the first row shows the displayed strategy in asynchronous updating, the second row shows the true strategy of each player and the third row shows the strategy of the players using the regular synchronous updating.

3. Experimental Results

There are several factors to take into consideration when analyzing the results of our experiments. In brief, the variables in our models are the size of the lattice (size of population of players), distribution of players in the first lattice, payoff matrix, the distribution of dishonest players in the lattice, and the time steps to hide a strategy. Changing each of these factors can change the final result of the game.

In our experiments we have considered that after some iterations the population of each strategy in the final lattice becomes stable, so we can use the final lattice as a measure for our comparisons. Figure 2 shows this phenomenon that for both synchronous and asynchronous updating for different values of b, after some time steps the population will converge to an equilibrium or quasi-equilibrium state in which the percentage of players with the same strategy does not change or oscillates around the converged value. In the figure, the red line shows the percentage of hawks for synchronous updating, the blue line shows the percentage of players displaying a Hawk strategy in asynchronous updating, while the green line shows the true percentage of real Hawks in each time step.

	Synchronous updating	Asynchronous updating
b= 3		

b= 5

b= 7

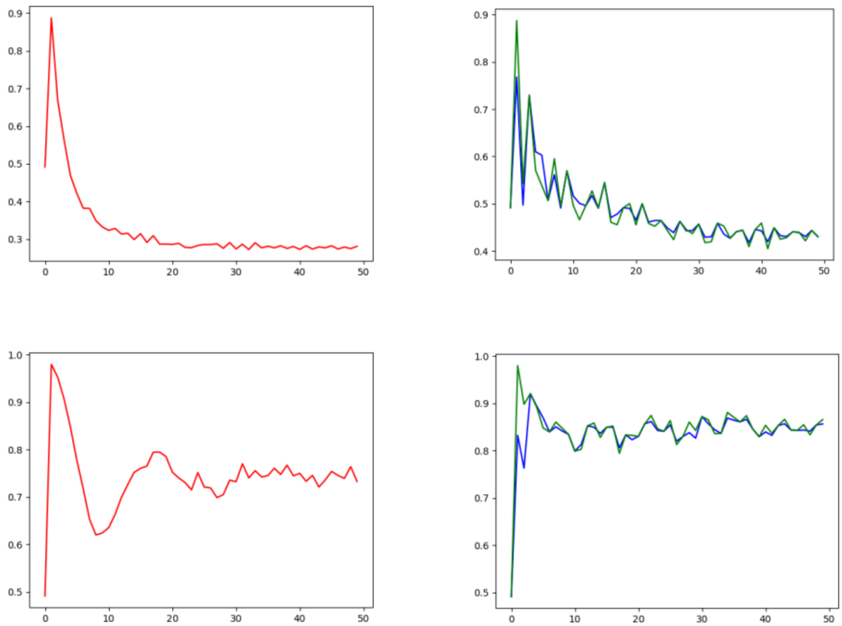


Figure 2- percentage of Hawks in the last lattice

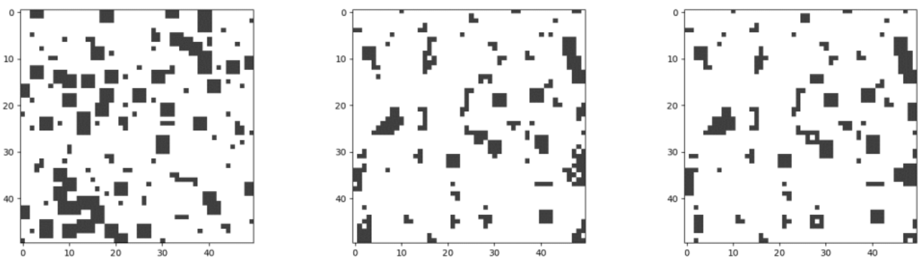
The following lattices in Figure 3 show the arrangement of players in the first and last lattice for the graphs in Figure 2. In these lattices, black cells show players with Hawk strategy and white cells show players with Dove strategy.

Synchronous updating

Asynchronous updating (displayed strategy)

Asynchronous updating (true strategy)

b=3



b=5

b=7

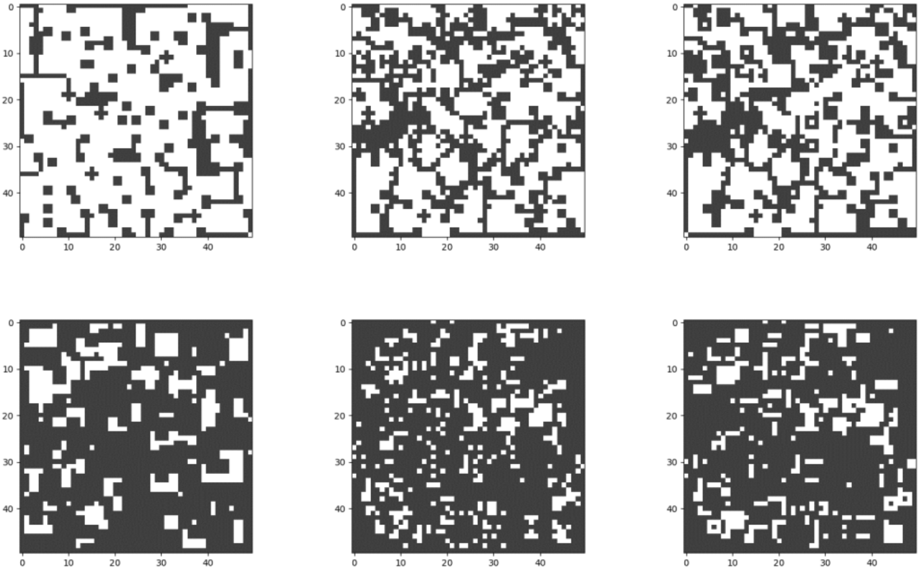


Figure 3- Last distribution of players in the lattice in Figure 2

Figure 3, reveals the obvious conclusion that regardless of the type of updating a payer's strategy, a high ***b*** value gives an advantage to the Hawk strategy as expected.

In order to probe the behavior of our methodology in comparison with synchronous updating we have defined several experiments which can lead to more comprehensive analysis. All experiments are done in a 50 by 50 lattices updated 50 times in which 50% of the initial population of players are Hawks. The following experiments (sections 3.1 to 3.3) have done for 5 different payoff matrices (b=1, 3, 5, 7, 9 and c=-10) and for 20 different random lattices in which 30% of players in the first lattice are dishonest. The time step for hiding the true strategy is 2.

3.1 Comparing Percentage of Hawks in the Final Lattice

This experiment shows the comparison in percentage of Hawks in the final lattice for synchronous and asynchronous updating. Table 2 shows the average percentage of Hawks in the last lattice using 20 experiments.

Table 2- - Average of percentage of hawks in the last lattice for different values of ***b***

	1	3	5	7	9
Synchronous updating	0.04922	0.19522	0.27369	0.73168	1
Asynchronous updating (displayed strategy)	0.01184	0.12838	0.41834	0.82762	1
Asynchronous updating (true strategy)	0.0121	0.12589	0.42632	0.82852	1

We can see in Table 2 that when b is small, the final number of players with hawk strategy is slightly lower than when we have dishonest players in the population of players, but as b increases, more players have tendency to choose the Hawk strategy when there are players who hide their true strategy in the population. Thus, existence of hiding strategy will increase the slope of the graph which shows changing in percentage of Hawks with regard to changing in b. The following graph (Figure 4) shows this result. While the values of percentage of Hawks for asynchronous updating using displayed strategy and true strategy is very close to each other, only the true strategy is used in plotting the values. In this graph red line shows the percentage of Hawks for synchronous updating while green line shows the percentage of true Hawks in asynchronous updating when dishonest players show their true strategy in the last lattice.

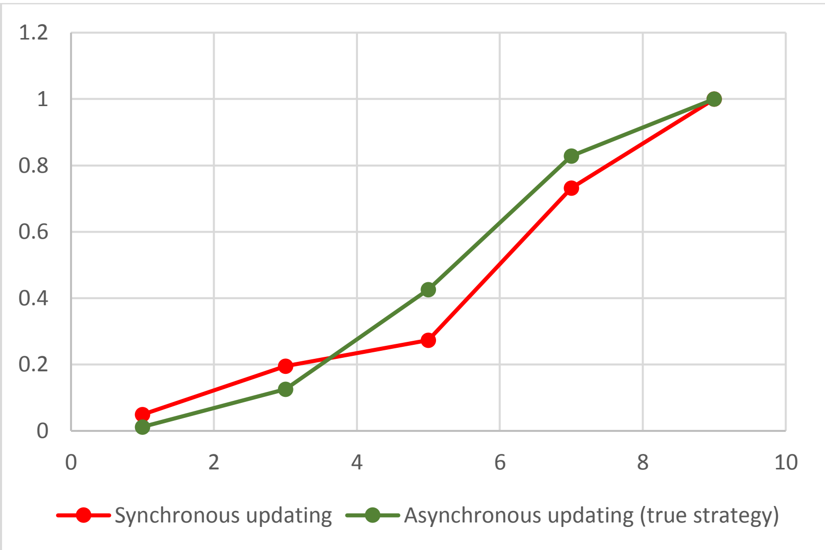


Figure 4- Average of percentage of hawks in the last lattice for different values of ***b***

Figure 4 implies that when there are players who hide their strategy and as *b* increases there will be more Hawks than if they all play synchronously without hiding their strategies.

3.2 Comparing Average Payoff in the Final Lattice between Synchronous and Asynchronous Updating

This analysis compares the average payoff of all players (in the entire lattice) in the final lattice when updating synchronously and asynchronously. In the following graph (Figure 5) red line shows the payoff of players for synchronous updating while blue line shows the payoff of players in asynchronous updating when dishonest players hide their true strategy and the green line shows the payoff of players for asynchronous updating when dishonest players show their true strategy in the last lattice.

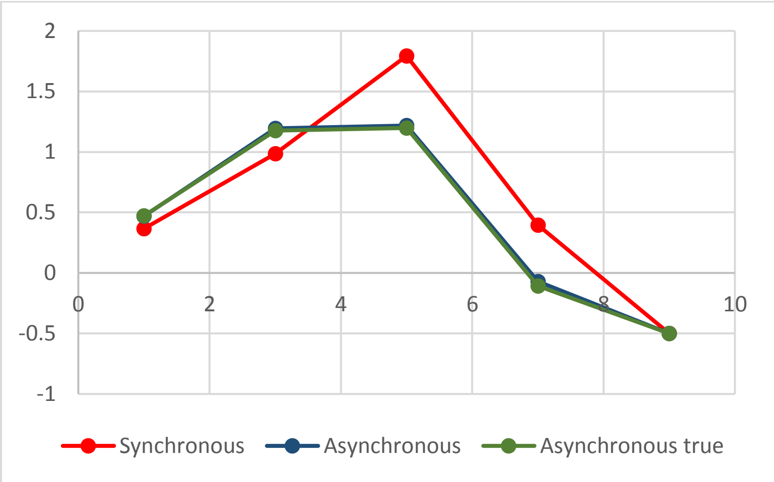


Figure 5- Average payoff of players in the last lattice using different *b* values

The graph shows that for a smaller *b*, the average payoff of the players in populations with dishonest players is more than the payoff in populations with no dishonest players, but as b increases the result will change to its opposite (rewarding more "honest" population). However, in both populations the average payoff has its highest amount when *b* is equal to C/2, but the changes in asynchronous updating is smoother.

Comparing the average payoff of asynchronous updating shows that average payoff of players is less than what they think they get, while they are not aware of their neighbors' true strategy. However, these two payoffs are still very close to each other.

3.3 Comparing Average Individual Payoff in the Final Lattice

This experiment shows the comparison in average individual payoff of dishonest players in the final lattice with average individual payoff of honest players in the same lattice when updating asynchronously. In the following graph (Figure 6) the solid blue line shows the average payoff of honest players considering the displayed strategy of each player's neighbors, the solid green line shows the payoff of honest players from the omniscient point of view in which the true strategy of neighbors is used in calculation, the dashed blue line shows the payoff of dishonest players from their own point of view which is calculated based on considering the true strategy of those players but using the displayed strategy of their neighbors and the dashed green line shows the payoff of dishonest players from the omniscient point of view calculated based on the true strategy of dishonest player and all its neighbors.

Figure 6- Average of payoff for various player types using different b values

This experiment shows that the average payoff that honest players get in playing in a dishonest society is more than the average payoff of liars, and so, lying has not led liars to higher payoff.

3.4 Result of Changing Time Steps for Updating

In this experiment we start with one random lattice and three payoff matrices (b=3, 5, 7), and check the behavior of asynchronous updating when we change time steps for updating from 1 to 20 with a fixed percentage of liars (30%). For comparing the behavior, we calculate the percentage of hawks in the last lattice. In the following graph (Figure 7) blue line shows the percentage of Hawks when dishonest players are hiding their true strategy and green line shows the percentage of Hawks when dishonest players show their true strategy in the last lattice.

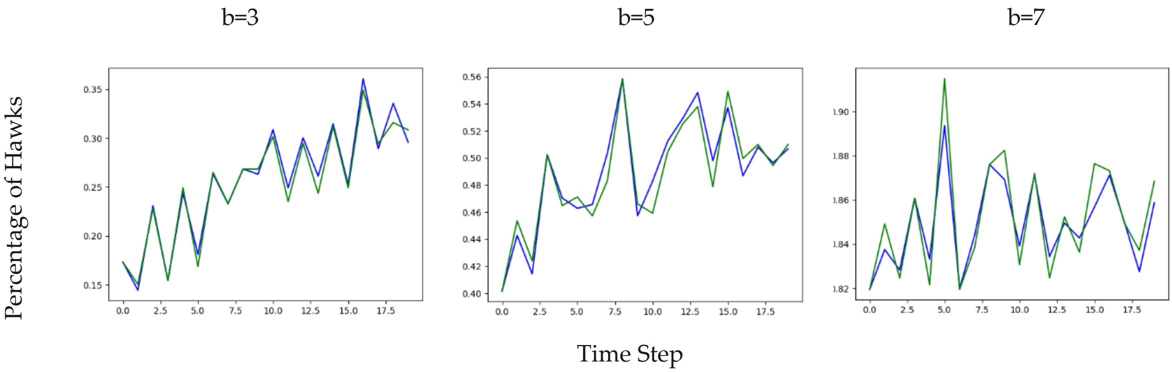


Figure 7- Percentage of Hawks using different time steps

We can see that increase of time delay can highly increase the percentage of hawks in the last lattice when b is small, but as b increases, the time step does not play an important role in the destiny of the lattice.

3.5 Result of Changing Percentage of Liars in the Population

In this experiment we start with one random lattice and three payoff matrices (b=3, 5, 7) and check the behavior of asynchronous updating when we change percentage of liars from 5% to 95% with a time step of 2, and a constant distribution of liars. For comparing the behavior, we calculate the percentage of hawks in the last lattice. In the following graph (Figure 8) blue line shows the percentage of Hawks when dishonest players are hiding their true strategy and green line shows the percentage of all real Hawks in the last lattice.

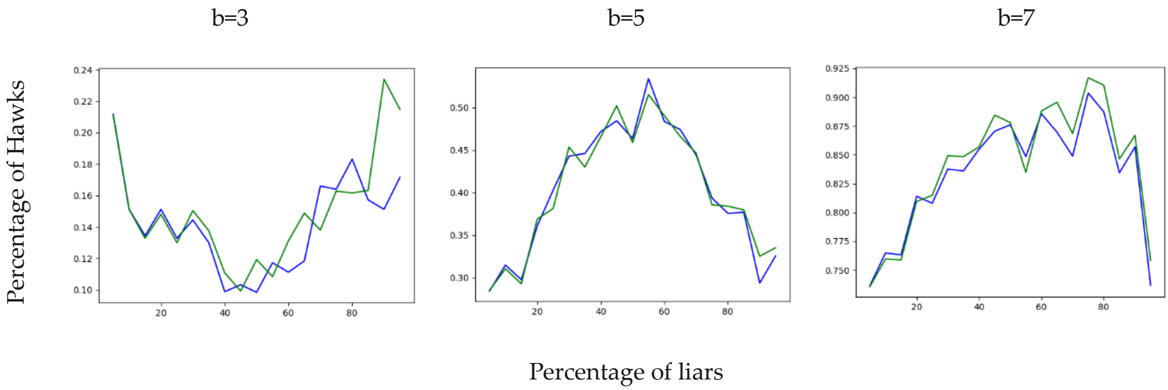


Figure 8- Percentage of Hawks using different percentage of dishonest players

We can see that increasing the percentage of liars when it is less than 50% can decrease percentage of Hawks in the final lattice for small b and increase the percentage of Hawks for large *b*, but when the percentage of liars goes more than 50% it has the reverse effect on the population of Hawks in the last lattice. Although the percentage of Hawks grows large in some combinations of *b* and percentage of liars, this value is still less than the percentage of Hawks in synchronous updating for small *b* and more for large *b* as we saw in Figure 4.

We can also show that when the number of liars is not large, what honest players get in reality is less than what they think they get and as the number of liars increase the honest players' true payoff grows larger than the displayed one, which can be viewed as the result of living in a dishonest society. It is also worth to mention that when the percentage of liars is large for small b, the displayed payoff of dishonest players is more than the honest players, but the true payoff of liars is always less than the true payoff of honest players.

4. Conclusion

In this study, we established a new methodology for updating spatial games when there are some players who hide their updated strategies from their neighbors for some time steps. Using the Hawk and Dove game, a series of numerical simulations shows that this method of updating in comparison with common synchronous game can result in higher number of Hawks and thus lower average payoff in the quasi-equilibrium state of the games when the value of the contested resource in the game (b) is large and lower number of Hawks and higher average payoff when b is small. Moreover, the results indicate that the true average payoff of honest players is more than the average payoff of dishonest players which shows that disingenuously delay of those players cannot result in higher payoff for them although it may seem that hiding strategy will lead to a higher payoff for these players. This unintuitive result can be the effect of false information that a player gets from its neighbors due to its own dishonest behavior. The sensitivity analysis on the number of time steps that a dishonest player hides its true strategy shows that the increase in number of time steps to hide the true strategy can increase the percentage of Hawks in the last lattice when b is small, but as b increase this effect is diminished. The sensitivity analysis on the percentage of dishonest players in the society also shows that when b is small, the increase in percentage of liars will decrease the percentage of Hawks when percentage of liars is not large and as the percentage of liars increase more than 50% the percentage of Hawks will start increasing again. But for large b we can see a reverse behavior.

Acknowledgement

The authors wish to acknowledge the financial support of the Kansas State Universiyu Open Access Publishing Fund towards publishing this chapter.

References

[1] Myerson, R. "Game Theory: Analysis of Conflict Harvard Univ." Press, Cambridge (1991).

[2] Nowak, Martin A. Evolutionary dynamics. Harvard University Press, 2006.

[3] Baetens, Jan M., Pieter Van der Weeën, and Bernard De Baets. "Effect of asynchronous updating on the stability of cellular automata." Chaos, Solitons & Fractals 45, no. 4 (2012): 383-394.

[4] Newth, David, and David Cornforth. "Asynchronous spatial evolutionary games." *BioSystems* 95.2 (2009): 120-129.

[5] Newth, David, and David Cornforth. "Asynchronous spatial evolutionary games." BioSystems 95, no. 2 (2009): 120-129.

[6] Radax, Wolfgang, and Bernhard Rengs. "Timing matters: lessons from the CA literature on updating." arXiv preprint arXiv:1008.0941 (2010).

[7] Valsecchi, Andrea, Leonardo Vanneschi, and Giancarlo Mauri. "A study on the automatic generation of asynchronous cellular automata rules by means of genetic algorithms." In Cellular Automata, pp. 429-438. Springer Berlin Heidelberg, 2010.

[8] Fates, Nazim. "Critical phenomena in cellular automata: perturbing the update, the transitions, the topology." Acta Physica Polonica B (2010).

[9] Yamauchi, Atsuo, Jun Tanimoto, and Aya Hagishima. "An analysis of network reciprocity in Prisoner's Dilemma games using Full Factorial Designs of Experiment." BioSystems 103, no. 1 (2011): 85-92.

[10] Bandini, Stefania, Andrea Bonomi, and Giuseppe Vizzari. "An analysis of different types and effects of asynchronicity in cellular automata update schemes." Natural Computing 11, no. 2 (2012): 277-287.

[11] Chan, Nat WH, C. Xu, Siew Kian Tey, Yee Jiun Yap, and P. M. Hui. "Evolutionary snowdrift game incorporating costly punishment in structured populations." Physica A: Statistical Mechanics and its Applications 392, no. 1 (2013): 168-176.

[12] Fatès, Nazim. "Quick Convergence to a Fixed Point: A Note on Asynchronous Elementary Cellular Automata." In Cellular Automata, pp. 586-595. Springer International Publishing, 2014.

[13] Peper, Ferdinand, Susumu Adachi, and Jia Lee. "Variations on the game of life." In Game of Life Cellular Automata, pp. 235-255. Springer London, 2010.

[14] Grilo, Carlos, and Luís Correia. "Effects of asynchronism on evolutionary games." Journal of Theoretical Biology 269, no. 1 (2011): 109-122.

[15] Bouré, Olivier, Nazim Fates, and Vincent Chevrier. "Probing robustness of cellular automata through variations of asynchronous updating." Natural Computing 11, no. 4 (2012): 553-564.

[16] Bouré, Olivier, Nazim Fates, and Vincent Chevrier. "First steps on asynchronous lattice-gas models with an application to a swarming rule."Natural Computing 12, no. 4 (2013): 551-560.

[17] Lee, Jia, Susumu Adachi, and Ferdinand Peper. "A partitioned cellular automaton approach for efficient implementation of asynchronous circuits."The Computer Journal (2010): bxq089.a
[18] Bouré, Olivier, Nazim Fates, and Vincent Chevrier. "Robustness of cellular automata in the light of asynchronous information transmission." InUnconventional Computation, pp. 52-63. Springer Berlin Heidelberg, 2011.
[19] Chen, Hsiao-Chi, and Yunshyong Chow. "Equilibrium selection in evolutionary games with imperfect monitoring." *Journal of Applied Probability*45, no. 2 (2008): 388-402.
[20] Zhang, Jianlei, and Zengqiang Chen. "Contact-based model for strategy updating and evolution of cooperation." *Physica D: Nonlinear Phenomena*323 (2016): 27-34.
[21] Wang, Xu-Wen, Zhen Wang, Sen Nie, Luo-Luo Jiang, and Bing-Hong Wang. "Impact of keeping silence on spatial reciprocity in spatial games." *Applied Mathematics and Computation* 250 (2015): 848-853.
[22] Tanimoto, Jun. "Correlated asynchronous behavior updating with a mixed strategy system in spatial prisoner's dilemma games enhances cooperation." *Chaos, Solitons & Fractals* 80 (2015): 39-46.
[23] Tanimoto, Jun, and Hiroki Sagara. "Relationship between dilemma occurrence and the existence of a weakly dominant strategy in a two-player symmetric game." *BioSystems* 90.1 (2007): 105-114.