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Asteroid observations at low phase angles. IV. Average parameters for the new H, G_1, G_2 magnitude system

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A B S T R A C T

We present new observational data for selected main-belt asteroids of different compositional types. The detailed magnitude–phase dependences including small phase angles (< 1°) were obtained for these asteroids, namely: (10) Hygiea (down to the phase angle of 0.3°, C-type), (176) Iduna (0.2°, G-type), (214) Aschera (0.2°, E-type), (218) Bianca (0.3°, S-type), (250) Bettina (0.3°, M-type), (419) Aurelia (0.1°, F-type), (596) Scheila (0.2°, D-type), (635) Vundtia (0.2°, B-type), (671) Carnegia (0.2°, P-type), (717) Wisibada (0.1°, T-type), (1021) Flammario (0.6°, B-type), and (1279) Uganda (0.5°, E-type). For several asteroids, the dependences of brightness on the phase angle were investigated in the BVRI bands. We found a great diversity in the opposition-effect behavior both in the magnitude and the width of the opposition surges, especially for low-albedo asteroids. Some low-albedo asteroids (e.g., (10) Hygiea) display a broad opposition effect with an amplitude of 0.15–0.20 mag relative to the extrapolation of the linear part of the phase curve. Other asteroids (e.g., (596) Scheila, (1021) Flammario) show linear magnitude–phase dependences down to small phase angles (0.1–0.2°). Using numerous data sets on the magnitude–phase dependences with extensive phase-angle coverage, we examined in more detail the new three-parameter H, G_1, G_2 magnitude system. We determined the values of the G_1 and G_2 parameters for magnitude phase dependences of individual asteroids and obtained the average parameters for main asteroid compositional types. The values obtained can be used for the estimation of the absolute magnitude of an asteroid from a single observed magnitude when the magnitude–phase dependency is unknown and/or to calculate a visible magnitude for the ephemerides.

1. Introduction

The present work is a continuation of the observational program devoted to the investigation of the brightness behavior of different types of asteroids in a wide range of phase angles including small phase angles (< 1°; Shevchenko et al., 1997, 2002, 2008). Knowledge of phase dependences is needed for understanding the mechanisms of light scattering for the surfaces of atmosphereless bodies, testing different light-scattering models, examining approximating functions that are used for an ephemeris provision, reconstruction of shape and size of the asteroids selected for space missions, and classification of asteroids by types (Belskaya and Shevchenko, 2000). Here we present new observational data for 12 main-belt asteroids of different compositional types, namely: (10) Hygiea (down to the phase angle of 0.3°, C-type), (176) Iduna (0.2°, G-type), (214) Aschera (0.2°, E-type), (218) Bianca (0.3°, S-type), (250) Bettina (0.3°, M-type), (419) Aurelia (0.1°, F-type), (596) Scheila (0.2°, D-type), (635) Vundtia (0.2°, B-type), (671) Carnegia (0.2°, P-type), (717) Wisibada (0.1°, T-type), (1021) Flammario (0.6°, B-type), and (1279) Uganda (0.5°, E-type). For nine of them, we have obtained the magnitude–phase relations in four BVRI standard bands for studying the opposition effect (OE) behavior with wavelength. These data are presented in Section 1.

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Table 1
Aspect data and measured magnitudes and colors of the observed asteroids. The columns present the date of observation, ecliptic coordinates at epoch 2000.0, the distance from the asteroid to the Sun and to the Earth (in AU), the phase angle, the reduced magnitudes corrected for distances from the Earth and the Sun and corresponding to the primary maxima of the asteroid lightcurves, and their errors.

UT date day	λ_{2000} (deg)	β_{2000} (deg)	r (au)	Δ (au)	α (deg)	$B_0(1,\alpha)$ (mag)		$V_0(1,\alpha)$ (mag)		R_0 $(1,\alpha)$ (mag)		I_0 $(1,\alpha)$ (mag)	
1	2	3	4	5	6	7	8	9	10	11	12	13	14
(10) Hygiea													
1993 Jan 02.90	116.354	−0.802	3.332	2.369	4.02	–	–	5.845	0.013	–	–	–	–
Jan 03.80	116.181	−0.816	3.331	2.365	3.71	–	–	5.810	0.014	–	–	–	–
Jan 13.80	114.171	−0.970	3.322	2.338	0.32	–	–	5.550	0.013	–	–	–	–
Jan 18.00	113.306	−1.032	3.318	2.336	1.42	–	–	5.665	0.018	–	–	–	–
Jan 19.90	112.918	−1.058	3.316	2.337	2.10	–	–	5.705	0.014	–	–	–	–
Jan 25.80	111.768	−1.148	3.310	2.347	4.17	–	–	5.875	0.013	–	–	–	–
Jan 30.80	110.808	−1.208	3.316	2.337	5.91	–	–	5.945	0.010	–	–	–	–
Feb 11.70	108.914	−1.331	3.293	2.429	9.70	–	–	6.105	0.013	–	–	–	–
Feb 14.70	108.535	−1.359	3.290	2.452	10.55	–	–	6.135	0.013	–	–	–	–
Feb 15.70	108.418	−1.368	3.289	2.460	10.83	–	–	6.165	0.013	–	–	–	–
Feb 16.70	108.306	−1.377	3.288	2.468	11.10	–	–	6.195	0.014	–	–	–	–
Feb 17.70	108.200	−1.386	3.287	2.476	11.37	–	–	6.220	0.014	–	–	–	–
Feb 21.80	107.817	−1.420	3.283	2.512	12.41	–	–	6.240	0.014	–	–	–	–
Mar 09.70	107.207	−1.523	3.266	2.680	15.58	–	–	6.420	0.016	–	–	–	–
Mar 13.70	107.272	−1.544	3.262	2.728	16.16	–	–	6.435	0.016	–	–	–	–
Mar 21.70	107.655	−1.580	3.253	2.827	17.08	–	–	6.465	0.016	–	–	–	–
(176) Iduna													
2005 Mar 30.02	203.858	0.435	3.712	2.737	3.87	9.045	0.015	8.325	0.011	7.993	0.012	7.665	0.013
Mar 30.96	203.677	0.373	3.712	2.733	3.58	9.001	0.015	8.281	0.012	7.940	0.012	7.618	0.012
Apr 08.94	201.896	0.219	3.713	2.713	0.75	8.883	0.012	8.162	0.011	7.819	0.011	7.497	0.013
Apr 09.92	201.707	0.281	3.713	2.712	0.44	8.817	0.013	8.084	0.011	7.745	0.012	7.437	0.011
Apr 10.90	201.497	0.349	3.714	2.711	0.15	8.799	0.012	8.050	0.012	7.717	0.012	7.402	0.012
Apr 13.86	200.896	0.545	3.714	2.712	0.84	8.883	0.015	8.151	0.011	7.831	0.010	7.495	0.010
Apr 14.99	200.669	0.620	3.714	2.713	1.19	–	0.013	8.186	0.011	–	0.011	–	0.011
Apr 27.82	198.185	1.449	3.715	2.751	5.17	9.148	0.015	8.420	0.012	8.100	0.012	7.758	0.013
Apr 28.97	197.978	1.522	3.715	2.757	5.51	–	0.012	8.394	0.012	–	0.012	–	0.012
May 18.88	195.127	2.668	3.716	2.910	10.67	9.399	0.015	8.667	0.013	8.337	0.013	8.007	0.014
Jun 03.86	194.077	3.433	3.716	3.093	13.57	9.562	0.017	8.822	0.014	8.482	0.013	8.154	0.014
Jun 09.84	193.993	3.685	3.715	3.172	14.35	9.588	0.018	8.848	0.014	8.505	0.013	8.180	0.014
(214) Aschera													
2004 Sep 02.99	343.434	0.021	2.661	1.653	1.01	–	–	9.555	0.008	9.133	0.008	–	–
Sep 03.97	343.207	0.042	2.660	1.652	0.57	–	–	–		–	–	–	–
Sep 04.81	343.016	0.060	2.660	1.652	0.19	–	–	9.420	0.010	9.025	0.010	–	–
Sep 05.98	342.748	0.086	2.660	1.652	0.35	10.181	0.021	9.461	0.012	9.033	0.012	8.654	0.020
Sep 07.01	342.511	0.108	2.660	1.652	0.81	10.242	0.018	9.530	0.009	9.104	0.009	8.730	0.017
Sep 11.98	341.380	0.215	2.658	1.658	3.06	–	–	9.650	0.008	–	–	–	–
Sep 12.96	341.162	0.236	2.658	1.660	3.50	10.325	0.019	9.670	0.010	9.270	0.010	8.850	0.018
Sep 16.79	340.329	0.317	2.657	1.670	5.20	10.385	0.020	9.730	0.011	9.351	0.011	8.934	0.019
Oct 06.85	336.977	0.698	2.652	1.784	13.07	10.619	0.017	9.915	0.008	9.566	0.008	9.177	0.016
Oct 08.74	336.778	0.729	2.651	1.799	13.70	10.661	0.017	9.950	0.008	9.582	0.008	9.227	0.016
Oct 13.80	336.361	0.808	2.650	1.844	15.26	–	0.019	9.989	0.010	–	–	–	–
Oct 20.76	336.070	0.908	2.648	1.912	17.13	10.726	0.017	10.026	0.008	9.666	0.008	9.252	0.016
(218) Bianca													
2005 Mar 10.02	172.357	0.141	2.486	1.494	1.16	9.511	0.021	8.687	0.011	8.257	0.014	7.868	0.016
Mar 10.82	172.162	0.230	2.485	1.492	0.76	9.477	0.021	8.657	0.011	8.217	0.014	7.832	0.016
Mar 11.78	171.922	0.340	2.484	1.491	0.31	9.366	0.021	8.550	0.011	8.130	0.014	7.726	0.016
Mar 15.72	170.941	0.788	2.480	1.487	1.72	9.603	0.022	8.777	0.012	8.337	0.015	7.938	0.017
Mar 16.82	170.669	0.912	2.479	1.487	2.27	9.623	0.023	8.791	0.013	8.351	0.016	7.959	0.018
Mar 29.87	167.640	2.356	2.465	1.509	8.61	9.989	0.024	9.143	0.014	8.685	0.017	8.290	0.019
Apr 04.90	166.478	2.977	2.459	1.535	11.30	10.072	0.024	9.232	0.014	8.769	0.017	8.391	0.019
Apr 10.78	165.553	3.543	2.453	1.567	13.73	10.163	0.025	9.307	0.015	8.864	0.018	8.457	0.020
May 19.81	165.736	6.227	2.417	1.923	23.62	10.479	0.026	9.619	0.016	9.181	0.019	8.776	0.021
(250) Bettina													
2000 Apr 11.90	201.713	0.745	3.194	2.192	0.30	–	–	–	–	7.030	0.018	–	–
Apr 13.91	201.299	0.634	3.196	2.194	0.95	–	–	–	–	7.155	0.018	–	–
Apr 14.85	201.106	0.582	3.198	2.196	1.29	–	–	–	–	7.177	0.017	–	–
Apr 27.81	198.596	−0.124	3.213	2.245	5.93	–	–	–	–	7.547	0.018	–	–
Apr 29.86	198.240	−0.233	3.215	2.257	6.62	–	–	–	–	7.551	0.018	–	–
Apr 30.87	198.069	−0.286	3.216	2.263	6.95	–	–	–	–	7.567	0.016	–	–
May 02.85	197.746	−0.389	3.219	2.277	7.60	–	–	–	–	7.575	0.017	–	–
May 06.88	197.138	−0.594	3.223	2.307	8.87	–	–	–	–	7.628	0.018	–	–
May 08.87	196.864	−0.692	3.226	2.323	9.47	–	–	–	–	7.644	0.018	–	–
May 12.86	196.371	−0.885	3.230	2.358	10.61	–	–	–	–	7.683	0.019	–	–
May 26.86	195.277	−1.502	3.246	2.508	13.96	–	–	–	–	7.703	0.019	–	–
May 27.84	195.239	−1.541	3.247	2.519	14.16	–	–	–	–	7.781	0.019	–	–
Jun 07.83	195.160	−1.953	3.260	2.660	15.99	–	–	–	–	7.791	0.019	–	–

Table 1(*continued*)

UT date day	λ_{2000} (deg)	β_{2000} (deg)	r (au)	Δ (au)	α (deg)	$B_0(1,\alpha)$ (mag)		$V_0(1,\alpha)$ (mag)		R_0 $(1,\alpha)$ (mag)		I_0 $(1,\alpha)$ (mag)	
1	2	3	4	5	6	7	8	9	10	11	12	13	14
(419) Aurelia													
2001 Oct 10.04	54.924	0.688	2.888	2.036	12.33	9.773	0.015	9.113	0.012	8.793	0.013	8.493	0.012
Oct 26.07	51.936	0.426	2.923	1.966	6.44	9.521	0.013	8.861	0.012	8.531	0.013	8.211	0.013
Nov 06.99	49.138	0.212	2.947	1.958	1.52	9.275	0.014	8.605	0.013	8.325	0.014	7.985	0.014
Nov 09.88	48.435	0.159	2.953	1.963	0.31	9.190	0.013	8.540	0.013	8.230	0.012	7.880	0.013
Nov 10.84	48.201	0.142	2.955	1.965	0.11	–	–	8.520	0.014	–	–	–	–
Nov 11.82	47.964	0.124	2.957	1.967	0.51	9.211	0.014	8.551	0.013	8.251	0.013	7.911	0.014
Nov 12.68	47.756	0.109	2.959	1.970	0.86	–	–	8.571	0.012	–	–	–	–
Dec 04.83	43.095	−0.271	3.001	2.105	9.34	–	–	8.953	0.012	–	–	–	–
Dec 06.78	42.786	−0.301	3.005	2.123	9.98	–	–	9.026	0.013	–	–	–	–
Dec 21.89	41.150	−0.511	3.032	2.293	14.16	–	–	9.198	0.014	–	–	–	–
(596) Scheila													
2012 Mar 08.10	251.022	6.821	2.465	2.144	23.57	–	–	10.163	0.015	9.693	0.013	–	–
Mar 09.07	251.250	6.789	2.465	2.132	23.54	–	–	10.158	0.015	9.688	0.013	–	–
Mar 10.06	251.478	6.756	2.464	2.119	23.51	–	–	10.144	0.014	9.674	0.013	–	–
Apr 14.01	256.011	5.030	2.450	1.700	18.79	–	–	9.916	0.014	9.444	0.013	–	–
Apr 29.99	255.377	3.755	2.446	1.556	13.89	–	–	9.682	0.016	9.214	0.012	–	–
May 01.97	255.174	3.571	2.446	1.542	13.16	–	–	9.620	0.013	9.162	0.012	–	–
May 10.97	253.949	2.669	2.445	1.486	9.51	–	–	9.527	0.013	9.067	0.013	–	–
May 12.98	253.613	2.454	2.445	1.476	8.63	–	–	9.497	0.012	9.037	0.012	–	–
May 19.94	252.308	1.675	2.445	1.449	5.45	10.025	0.015	9.319	0.012	8.879	0.011	–	–
May 22.97	251.685	1.323	2.445	1.441	4.00	–	–	9.231	0.011	8.771	0.012	–	–
May 30.88	249.978	0.381	2.445	1.432	0.20	–	–	9.090	0.011	8.630	0.010	–	–
May 31.83	249.770	0.267	2.446	1.432	0.35	–	–	9.095	0.011	8.635	0.010	–	–
Jun 01.88	249.541	0.141	2.446	1.432	0.85	–	–	9.148	0.012	8.688	0.011	–	–
Jun 02.87	249.323	0.021	2.446	1.432	1.33	–	–	9.170	0.013	8.710	0.012	–	–
1	2	3	4	5	6	7	8	9	10	11	12	13	14
(635) Vundtia													
2006 Mar 25.96	185.280	0.480	3.332	2.335	0.16	9.859	0.016	9.191	0.011	8.877	0.009	–	–
Mar 31.85	184.090	0.740	3.335	2.341	2.05	–	–	–	–	9.017	0.009	–	–
Apr 03.92	183.470	0.865	3.337	2.348	3.13	10.061	0.022	9.381	0.009	9.070	0.016	8.667	0.020
Apr 20.80	180.560	1.530	3.344	2.436	8.63	10.361	0.020	9.688	0.015	9.347	0.013	–	–
Apr 27.88	179.649	1.773	3.347	2.494	10.61	10.479	0.021	9.800	0.010	9.458	0.013	–	–
May 27.83	178.600	2.600	3.358	2.841	16.21	10.743	0.021	10.053	0.012	9.709	0.009	–	–
(671) Carnegia													
2009 Mar 20.87	181.861	−0.122	3.063	2.067	0.53	–	–	10.427	0.012	10.040	0.012	–	–
Mar 21.78	181.665	−0.157	3.063	2.067	0.17	–	–	10.385	0.012	10.012	0.012	–	–
Mar 22.88	181.434	−0.199	3.064	2.068	0.27	11.126	0.015	10.426	0.013	10.035	0.013	–	–
Mar 24.91	181.013	−0.276	3.066	2.069	1.06	–	–	10.478	0.013	10.080	0.013	–	–
Mar 26.80	180.622	−0.348	3.067	2.072	1.80	–	–	–	–	10.126	0.014	–	–
Mar 27.80	180.417	−0.386	3.067	2.074	2.18	–	–	10.551	0.013	10.148	0.013	–	–
Apr 03.87	179.021	−0.648	3.072	2.096	4.86	11.338	0.015	10.648	0.012	10.259	0.012	–	–
Apr 04.91	178.824	−0.686	3.073	2.100	5.25	11.367	0.016	10.668	0.012	10.273	0.012	–	–
Apr 05.77	178.666	−0.716	3.073	2.104	5.56	–	–	10.692	0.013	10.292	0.013	–	–
Apr 19.87	176.454	−1.193	3.083	2.194	10.32	11.589	0.016	10.889	0.014	10.482	0.014	–	–
Apr 24.87	175.890	−1.345	3.086	2.237	11.79	–	–	10.965	0.013	10.573	0.013	–	–
Apr 25.87	175.794	−1.375	3.087	2.246	12.06	11.675	0.016	10.988	0.013	10.586	0.013	–	–
May 22.85	175.278	−2.028	3.105	2.558	17.40	–	–	11.186	0.014	10.788	0.014		
(717) Wisibada													
2006 Aug 31.93	339.232	−0.251	2.404	1.395	0.41	11.945	0.019	11.263	0.012	10.815	0.011	10.357	0.013
Sep 01.87	339.047	−0.237	2.403	1.394	0.13	11.927	0.019	11.260	0.012	10.813	0.011	10.356	0.013
Sep 02.98	338.843	−0.222	2.401	1.393	0.58	11.965	0.019	11.270	0.012	10.829	0.011	10.365	0.013
Sep 03.89	338.651	−0.208	2.400	1.392	1.05	–	–	11.295	0.012	10.842	0.010	–	–
Sep 05.88	338.262	−0.179	2.398	1.392	2.02	12.045	0.019	11.343	0.012	10.899	0.010	–	–
Sep 10.91	337.302	−0.105	2.392	1.395	4.46	–	–	11.453	0.012	–	–	–	–
Sep 11.85	337.128	−0.092	2.391	1.396	4.91	12.169	0.019	11.450	0.012	11.062	0.011	10.591	0.014
Sep 15.91	336.405	−0.032	2.386	1.405	6.84	–	–	11.556	0.013	–	–	–	–
Sep 26.83	334.808	0.122	2.375	1.446	11.69	12.424	0.021	11.744	0.013	11.314	0.012	10.864	0.015
Sep 27.81	334.696	0.136	2.374	1.451	12.10	12.464	0.021	11.794	0.013	11.359	0.011	10.912	0.015
Oct 13.80	333.783	0.334	2.360	1.559	17.83	12.644	0.022	11.954	0.012	11.524	0.012	11.064	0.015
1	2	3	4	5	6					7	8	9	10
Oct 20.73	333.954	0.408	2.355	1.619	19.77	–	–	12.078	0.015	–	–	–	–
(1021) Flammario													
2004 Nov 18.08	123.310	−6.682	2.163	1.582	24.96	–	–	10.150	0.015	–	–	–	–
2005 Jan 14.97	116.239	−0.086	2.326	1.342	0.59	–	–	–	–	–	–	–	–
Jan 17.98	115.444	0.322	2.335	1.352	1.05	9.754	0.018	9.080	0.012	8.725	0.010	8.519	0.015
Jan 18.83	115.222	0.436	2.337	1.355	1.51	9.778	0.019	9.110	0.013	8.760	0.011	8.550	0.016
Jan 19.77	114.979	0.561	2.340	1.358	2.01	–	–	–	–	–	–	–	–
Feb 01.75	111.956	2.187	2.380	1.433	8.56	–	–	9.450	0.012	–	–	–	–
Feb 06.70	111.042	2.740	2.395	1.474	10.79	–	–	9.565	0.013	–	–	–	–
Feb 11.85	110.274	3.268	2.411	1.522	12.90	10.275	0.020	9.653	0.014	9.323	0.012	9.083	0.018

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Table 1 (*continued*)

UT date day	λ_{2000} (deg)	β_{2000} (deg)	r (au)	Δ (au)	α (deg)	$B_0(1,\alpha)$ (mag)		$V_0(1,\alpha)$ (mag)		R_0 $(1,\alpha)$ (mag)		I_0 $(1,\alpha)$ (mag)	
1	2	3	4	5	6	7	8	9	10	11	12	13	14
(1279) Uganda													
1999 Aug 23.04	343.392	0.162	2.076	1.080	6.72	–	–	12.703	0.023	–	–	–	–
Sep 01.87	340.957	0.807	2.097	1.088	1.03	–	–	12.522	0.018	–	–	–	–
Sep 02.90	340.698	0.873	2.099	1.091	0.54	–	–	12.483	0.015	–	–	–	–
Sep 03.90	340.447	0.936	2.102	1.093	0.51	–	–	12.480	0.015	–	–	–	–
Sep 04.85	340.210	0.996	2.104	1.095	0.93	–	–	12.513	0.018	–	–	–	–
Sep 10.75	338.781	1.354	2.117	1.116	4.23	–	–	12.651	0.021	–	–	–	–

In Section 2, we have used the new H, G_1, G_2 magnitude phase function (Muinonen et al., 2010) for the approximation of the magnitude–phase relations of observed asteroids and determined the parameters in all observed bands. The H, G_1, G_2 function was proposed by Muinonen et al. (2010) as the new magnitude system for asteroids, replacing the previously adopted H, G phase function (Bowell et al., 1989). Since the H, G phase function cannot accurately fit the phase curves of low-albedo and high-albedo asteroids (Belskaya and Shevchenko, 2000), the Division III of Commission 15 of IAU in August 2012 introduced a new magnitude system for asteroids. In this case, the reduced observed magnitudes $V(\alpha)$ can be obtained from

$$10^{-0.4V(a)} = a_1\Phi_1(a) + a_2\Phi_2(a) + a_3\Phi_3(a) = 10^{-0.4H}[G_1\Phi_1(a) + G_2\Phi_2(a) + (1 - G_1 - G_2)\Phi_3(a)],$$

where $\Phi_1(0) = \Phi_2(0) = \Phi_3(0) = 1$. The absolute magnitude H and the coefficients G_1 and G_2 are

$$H = -2.5\log(a_1 + a_2 + a_3), G_1 = a_1/(a_1 + a_2 + a_3), G_2 = a_2/(a_1 + a_2 + a_3).$$

The coefficients a_1, a_2, and a_3 can be estimated from the observations using the linear least-squares method in the flux value space $10^{-0.4V(\alpha)}$. The basis functions $\Phi_1(\alpha)$, $\Phi_2(\alpha)$, $\Phi_3(\alpha)$ are defined with cubic splines, a linear relation, or with a constant value, depending on the value of α. With linear least-squares fit, however, the possible values for coefficients, and thus for H, G_1, and G_2, are not limited in any way. Penttilä et al. (2015) suggest that in order to ensure a monotonic behavior of the phase–magnitude relation, the parameter values should be limited to $0 \le G_1$, G_2, $1 - G_1 - G_2 \le 1$. We will apply the suggested constrained non-linear least-squares method (Penttilä et al., 2015) directly in the magnitude value space to estimate the H, G_1, G_2 parameter values and their error estimates.In Section 3, we have presented results of determining the G_1 and G_2 parameters for magnitude–phase relations of 93 asteroids for which good-quality phase curves are available (see for example, Harris and Young, 1983, 1989; Harris et al., 1984, 1989a, 1989b, 1992; Shevchenko et al., 1996, 1997, 2002, 2008, 2010, 2012, and etc.). We have obtained the average parameters for the main asteroid compositional types as it was described earlier in Shevchenko and Lupishko (1998) for the H, G magnitude system, but using the H, G_1, G_2 system and a procedure described in Penttilä et al. (2015). We investigated also correlations of the G_1 and G_2 parameters among themselves and with albedo and wavelength.

2. Observations and results

Photometric observations of the selected asteroids were carried out at the Chuguevskaya Station of the Astronomical Institute of Kharkiv National University (70-cm reflector) and at the Simeiz Department of the Crimean Astrophysical Observatory (1-m reflector) using a single-channel photoelectric photometer in 1993 and CCD-cameras ST-6, IMG-1024, and IMG 47-10 in 1999–2012. Methods of the photoelectric data reduction were described by Shevchenko et al. (1992). The CCD observation and data reduction methods are explained in Krugly et al. (2002) and Shevchenko et al. (2012). The CCD-image data were reduced with the synthetic aperture photometry package (ASTPHOT) developed at the DLR by Mottola et al. (1995a). The absolute calibrations of the comparison stars were performed with standard star sequences from Landolt (1992) and Skiff (2007). The accuracy of absolute photometry is in the range of ± 0.01–0.02 mag.

High-quality magnitude–phase relations can be obtained only by taking into account magnitude changes due to rotation. We obtained lightcurves for each asteroid, and using long-term observations we were able to determine the rotation period for 10 out of 12 targets more precisely. For two asteroids, (717) and (1279), the lightcurve amplitude was low (0.03–0.05 mag) and their rotation periods were not determined. Our observations are presented as composite lightcurves which have been constructed according to the procedures described by Harris and Lupishko (1989) and Magnusson and Lagerkvist (1990). The data are combined with the period shown in the figures of the composite lightcurves. Data from individual nights, denoted by different symbols in the figures, were shifted along the magnitude axis in order to obtain the best fit. The values of these shifts are displayed in the figures. The accuracy of the measured magnitudes is not worse than 0.02 mag. Aspect data of the observed asteroids are given in Table 1. The magnitudes are given in the BVRI-bands of the standard Johnson–Cousins photometric system.

Table 2 contains the results of observations: rotation periods, lightcurve amplitudes, the measured color indices B–V, V–R, and R–I, and calculated values of the H, G_1, G_2 parameters of the magnitude–phase relation, where H is the absolute magnitude for the lightcurve primary maximum. Compositional type, albedo, and diameter, according to the references of the observed asteroids, are also presented in Table 2. We note that the H, G_1, G_2 function gives a good fit to magnitude–phase relation for all measured asteroid types with a small dispersion from the observed data. Next, short characteristics for the observed asteroids are given.

2.1. (10) Hygiea

The asteroid is one of the largest main-belt asteroids and was classified as C-type (Tholen, 1989) with low surface albedo and diameter about 450 km (Masiero et al., 2011; Usui et al., 2011; Tedesco et al., 2002). Photometric observations were performed by many researchers (Lagerkvist et al., 1987; Michalowski et al., 1991; López-González and Rodríguez, 2000, etc.). A correct value of the rotational period ($P = 27.63$ h) was determined for the first time by Michalowski et al. (1991), recent evaluations of pole coordinates and shape modeling were made by Hanuš et al. (2011), but the detailed magnitude–phase relation including small phase angles was not previously obtained. Our photoelectric observations were

Table 2
Physical parameters of observed asteroids.

Asteroid	Type	p_V[a]	D[a] (km)	P (h)	Amp. (mag)	$B-V$ (mag)	$V-R$ (mag)	$R-I$ (mag)	H (mag)	G_1	G_2
(10) Hygiea	C[b]	0.058	453.2	27.656 ± 0.003	0.11	–	–	–	$5.500^{+0.018}_{-0.022}$	$0.76^{+0.05}_{-0.06}$	$0.03^{+0.03}_{-0.03}$
(176) Iduna	P[b],X[c]	0.083	122.2	11.287 ± 0.003	0.30	0.73 ± 0.02	0.33 ± 0.02	0.33 ± 0.02	$8.046^{+0.012}_{-0.015}$	$0.84^{+0.01}_{-0.07}$	$0.00^{+0.04}_{-0.00}$
(214) Aschera	E[b],X[c]	0.42[d]	26.0[d]	6.8335 ± 0.0002	0.23	0.69 ± 0.03	0.39 ± 0.03	0.39 ± 0.03	$9.409^{+0.007}_{-0.007}$	$0.18^{+0.03}_{-0.03}$	$0.58^{+0.02}_{-0.02}$
(218) Bianca	S [b]	0.20	56.8	6.3355 ± 0.0005	0.18	0.84 ± 0.03	0.44 ± 0.02	0.40 ± 0.02	$8.474^{+0.010}_{-0.010}$	$0.31^{+0.04}_{-0.03}$	$0.31^{+0.02}_{-0.02}$
(250) Bettina	M[e]	0.11	121.3	5.054 ± 0.001	0.35	–	–	–	$6.925^{+0.017}_{-0.014}$	$0.06^{+0.05}_{-0.05}$	$0.48^{+0.03}_{-0.03}$
(419) Aurelia	F [b]	0.058	105.0	16.7865 ± 0.0005	0.07	0.66 ± 0.02	0.31 ± 0.02	0.33 ± 0.02	$8.509^{+0.007}_{-0.010}$	$0.87^{+0.05}_{-0.07}$	$0.04^{+0.05}_{-0.04}$
(596) Scheila	D [b]	0.037	114.1	15.8594 ± 0.0005	0.06	0.71 ± 0.02	0.46 ± 0.02	–	$9.079^{+0.016}_{-0.029}$	$0.96^{+0.01}_{-0.11}$	$0.00^{+0.05}_{-0.00}$
(635) Vundtia	B [e]	0.044	100.1	11.788 ± 0.003	0.15	0.68 ± 0.02	0.33 ± 0.02	0.40 ± 0.03	$9.164^{+0.013}_{-0.018}$	$0.84^{+0.01}_{-0.07}$	$0.00^{+0.04}_{-0.00}$
(671) Carnegia	X	0.028	69.3	8.331 ± 0.002	0.12	0.70 ± 0.02	0.39 ± 0.02	–	$10.385^{+0.007}_{-0.008}$	$0.80^{+0.04}_{-0.04}$	$0.10^{+0.02}_{-0.02}$
(717) Wisibada	T[b]	0.079	28.5	–	0.03	0.69 ± 0.02	0.44 ± 0.02	0.45 ± 0.02	$11.241^{+0.011}_{-0.011}$	$0.82^{+0.05}_{-0.05}$	$0.12^{+0.03}_{-0.03}$
(1021) Flammario	B[e]	0.047	98.0	12.151 ± 0.001	0.37	0.66 ± 0.03	0.35 ± 0.02	0.21 ± 0.02	$8.998^{+0.007}_{-0.016}$	$0.90^{+0.01}_{-0.04}$	$0.00^{+0.02}_{-0.00}$
(1279) Uganda	E	0.34	7.27	–	0.05	–	–	–	$12.407^{+0.004}_{-0.001}$	$0.00^{+0.04}_{-0.00}$	$0.79^{+0.00}_{-0.03}$

[a] Masiero et al. (2011).
[b] Tholen (1989).
[c] Bus and Binzel (2002).
[d] Usui et al. (2011).
[e] Alí-Lagoa et al. (2013).

carried out during 16 nights in the range of phase angles from 0.3° to 17° (see Table 1). The composite lightcurve with the rotational period of 27.656 h is shown in Fig. 1 and the magnitude–phase relation is presented in Fig. 2.

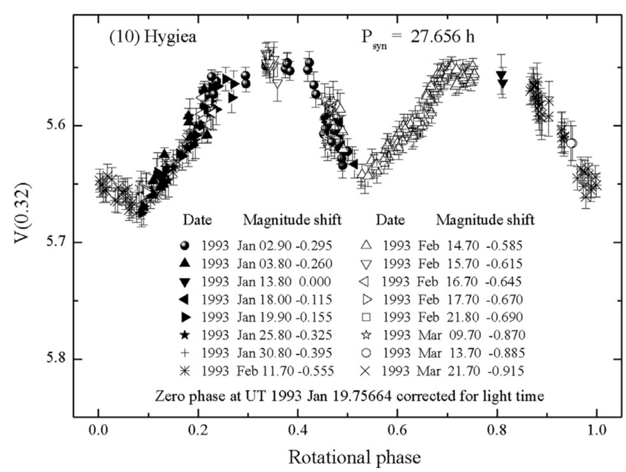

Fig. 1. Composite lightcurve of asteroid (10) Hygiea.

2.2. *(176) Iduna*

This low-albedo asteroid is classified as G-type (DeMeo et al., 2009; DeMeo and Carry, 2013). The rotational period was determined by Hansen and Arentoft (1997) and Riccioli et al. (2001). The detailed magnitude–phase relation including low phase angles was not previously obtained.

Our observations for 12 nights allowed us to determine more precisely the rotational period and to obtain the magnitude–phase relation in the BVRI-bands. The composite lightcurve with the rotation period of 11.287 h is shown in Fig. 3 and the magnitude–phase relations are given in Fig. 4.

2.3. *(214) Aschera*

The asteroid is classified as E-type (Tholen, 1989) with a high-albedo surface as it follows from the infrared data of IRAS (0.52, Tedesco et al., 2002) and AKARI (0.42, Usui et al., 2011) but the data of WISE give a moderate albedo (0.21, Masiero et al., 2011). The magnitude–phase relation was previously obtained by Harris and Young (1983) and by Belskaya et al. (2003) for the single V-band. We measured the magnitude–phase relation in the BVRI spectral bands to investigate the opposition-effect dependence on the wavelength. Our observations covered the phase-angle range from 0.2° to 17.1° (Figs. 5 and 6). Previously, the phase curves of two other E-types (44) Nysa and (64) Angelina were obtained in the same bands by Rosenbush et al. (2005, 2009). We plotted their data in Fig. 6 for the BRI bands and the data by Harris et al. (1989b) for Nysa in the V band using the magnitude shift to best fit our data for (214) Aschera. All data coincide within the accuracy of the observations, excluding the Nysa data in the B band at phase angles $<1°$ which show a larger opposition surge. We confirm the conclusion by Belskaya et al. (2003) that the phase curve behavior of Aschera is typical for the E-type asteroids.

2.4. *(218) Bianca*

There are many available photometric data for this S-type asteroid (see for example, Warner et al., 2013). The rotation period was determined for the first time by Harris and Young (1989) for the data obtained in the 1980 apparition. They obtained also the magnitude–phase relation in the range of phase angles from 4° to 14° in the V band. Our observations covered the phase angle range from 0.3° to 23° and were performed in the BVRI bands. A composite lightcurve with the maximum amplitude of 0.20 mag and the period of 6.3355 h is presented in Fig. 7. The magnitude–phase relations for the BVRI bands are plotted in Fig. 8. For

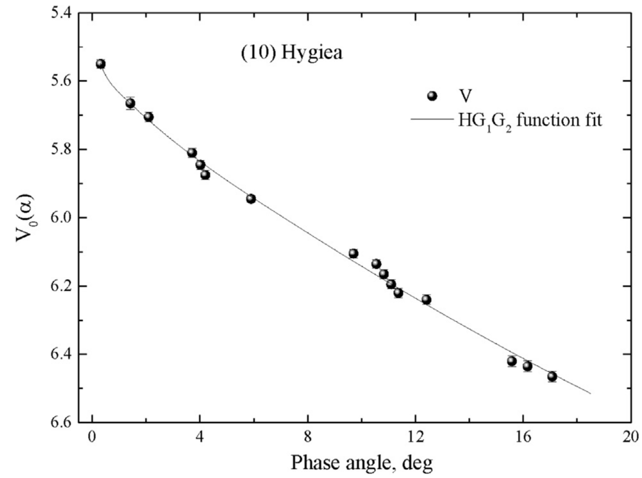

Fig. 2. Magnitude–phase relation of asteroid (10) Hygiea (solid line – best fit for the H, G_1, G_2 function).

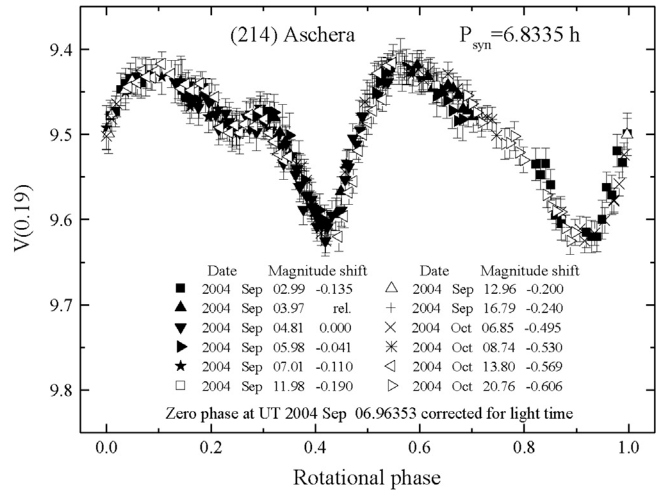

Fig. 5. Composite lightcurve of asteroid (214) Aschera.

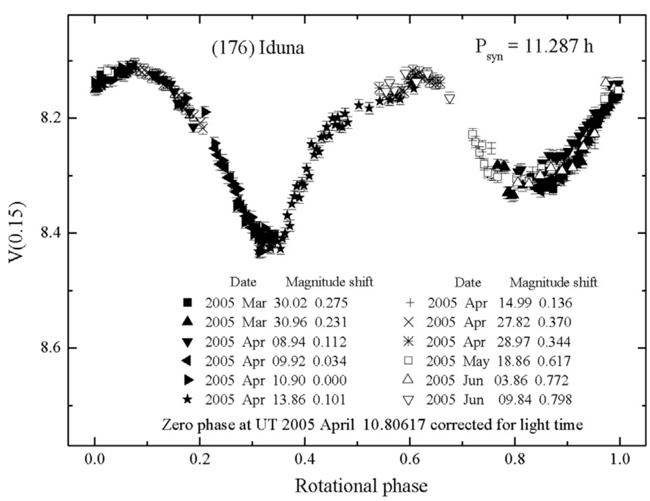

Fig. 3. Composite lightcurve of asteroid (176) Iduna.

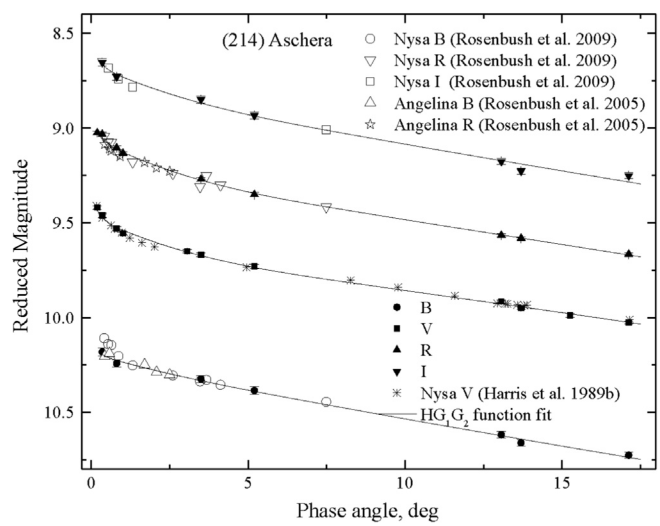

Fig. 6. Magnitude–phase relation of asteroid (214) Aschera.

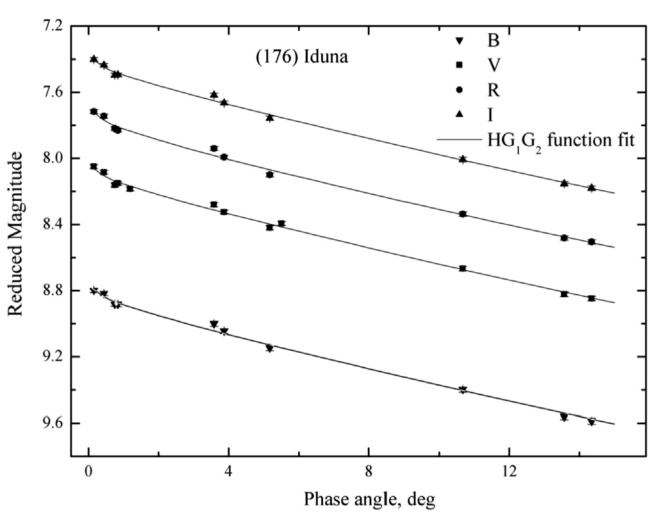

Fig. 4. Magnitude–phase relation of asteroid (176) Iduna.

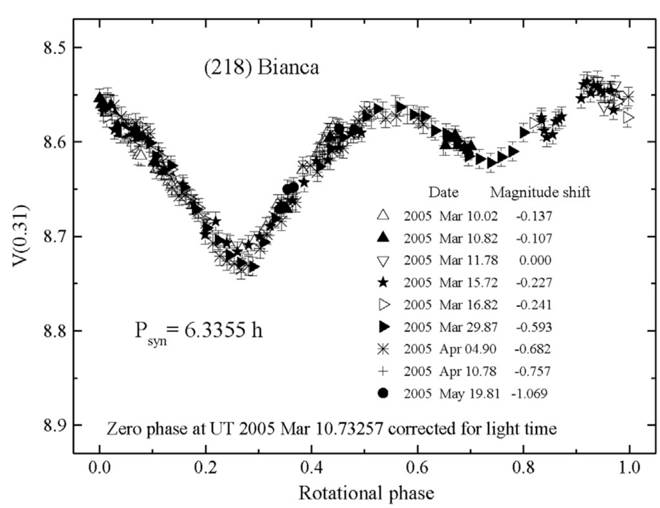

Fig. 7. Composite lightcurve of asteroid (218) Bianca.

comparison, the previous V-band observations (Harris and Young, 1989) of the asteroid are also given in Fig. 8. They were shifted to match our data, well coinciding with them.

2.5. (250) Bettina

This asteroid belongs to the M-types with the diameter of 120 km and albedo of 0.11 (Masiero et al., 2011). Although many

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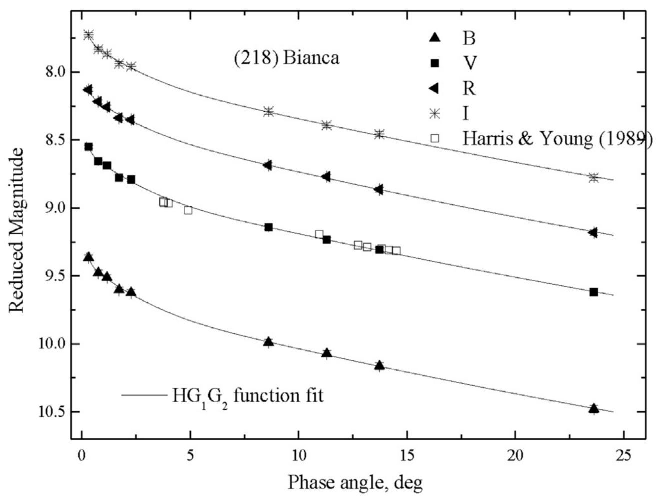

Fig. 8. Magnitude–phase relation of asteroid (218) Bianca.

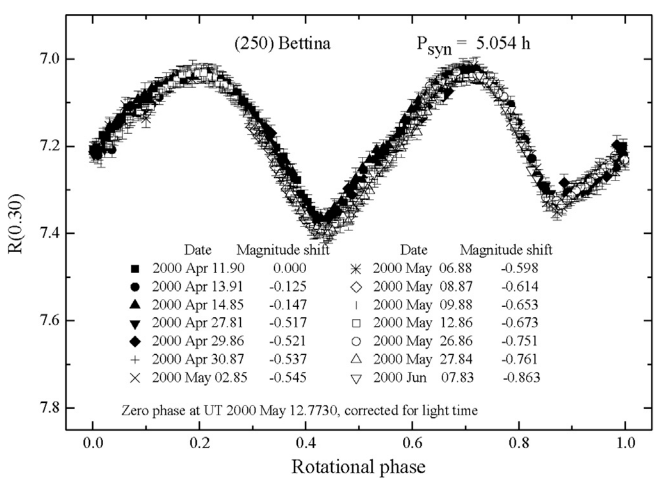

Fig. 9. Composite lightcurve of asteroid (250) Bettina.

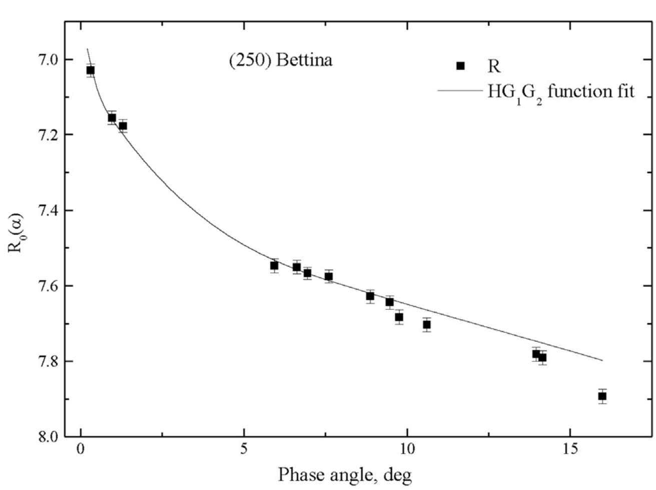

Fig. 10. Magnitude–phase relation of asteroid (250) Bettina.

photometric observations were made (see, for example, Warner et al., 2013), the magnitude–phase relation was not obtained. Our observations were performed in the R band for fourteen nights and covered the range of phase angles from 0.3° to 16°. The composite lightcurve is combined with a rotation period of 5.054 ± 0.001 h (Fig. 9). The magnitude–phase dependence typical for moderate-albedo asteroids is presented in Fig. 10.

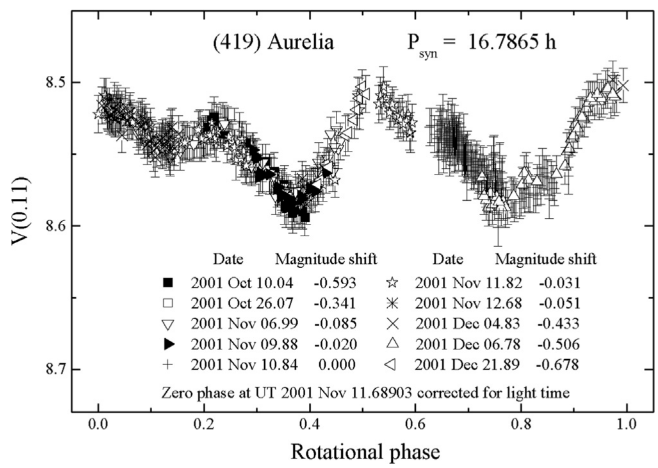

Fig. 11. Composite lightcurve of asteroid (419) Aurelia.

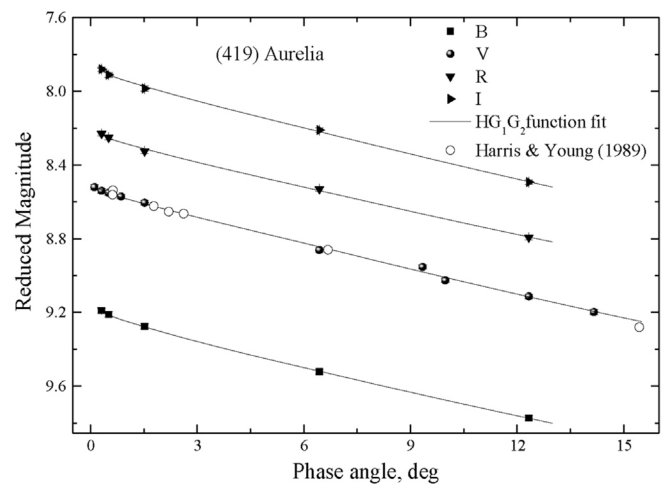

Fig. 12. Magnitude–phase relation of asteroid (419) Aurelia.

2.6. (419) Aurelia

This low-albedo asteroid was observed by Harris and Young (1989) in 1979 and 1980. They obtained a practically linear magnitude–phase relation down to the phase angle of 0.6°. We observed (419) Aurelia in the 2001 apparition, when it was a favorable opportunity to reach smaller phase angles down to 0.1°. Our preliminary results were published by Belskaya et al. (2002). We noted a possible opposition-effect surge close to opposition. To check the validity of the opposition surge at small phase angles, we used new standard star sequence from Landolt (1992) for standardization of the comparison stars used in our 2001 observations.

Here we present the revised observational data for magnitude–phase curves in the BVRI bands. The deviation from the linear behavior at small phase angles is weaker than that reported by Belskaya et al. (2002).

The composite lightcurve with a rotation period of 16.775 ± 0.005 h and an amplitude of 0.1 mag is shown in Fig. 11. The magnitude–phase relations for the BVRI bands are presented in Fig. 12. For comparison, the previous observations of the asteroid (Harris and Young, 1989) in the V band are also given in Fig. 12. They were obtained at similar aspects and are in a good agreement with our data. All observations were reduced to the primary lightcurve maximum.

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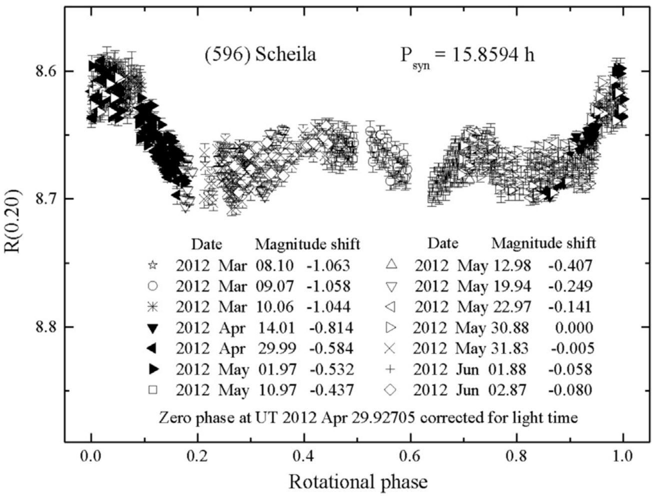

Fig. 13. Composite lightcurve of asteroid (596) Scheila.

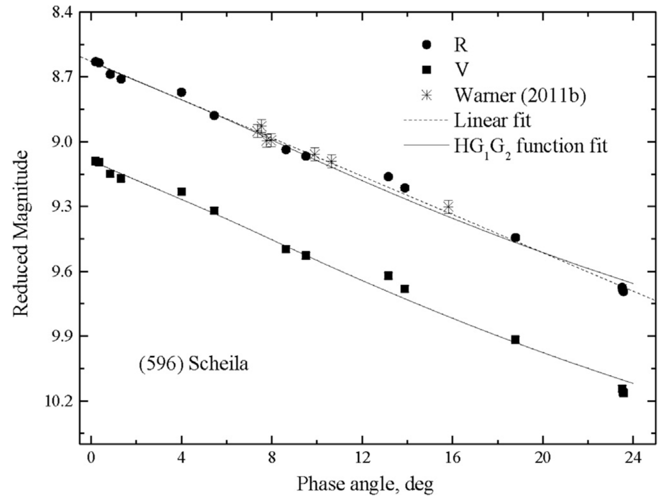

Fig. 14. Magnitude–phase relation of asteroid (596) Scheila.

2.7. (596) Scheila

Asteroid (596) Scheila is a middle size asteroid (about 115–120 km) having very dark surface with an albedo of 0.037 (Masiero et al., 2011; Tedesco et al., 2002). It was classified as a PD-type object (Tholen, 1989), that is a rather rare type for the main-belt asteroids but typical for the Hilda group asteroids and Jupiter Trojans. Previous photometric observations of Scheila were carried out by Warner (2006, 2011b) in 2006. Warner determined its rotation period (15.848 h) and measured the magnitude–phase slope in the range of phase angles from 7° to 16°.

An interest to this asteroid highly increased after Larson (2010) discovered its cometary-like activity in December 2010. Several groups of observers (Bodewits et al., 2011, 2014; Ishiguro et al., 2011; Jewitt et al., 2011; Husárik, 2012; Yang and Hsieh, 2011) performed observations of this asteroid in 2010–2011. Obtained images showed a dust shell around the asteroid, but the spectrum in the visible and infrared range did not show cometary bands of CO, CO_2, and/or other volatile substances. The authors assumed a collision of (596) Scheila with a small object of 20–100 m in diameter (Bodewits et al., 2014; Ishiguro et al., 2011; Jewitt et al., 2011). An impact could uncover the asteroid's subsurface layer of a higher albedo. Furthermore, if the asteroid is a solid body rather than a rubble pile, a seismic wave could shake the asteroid and globally change its regolith layer. These possible changes in the regolith layer may affect the phase relation of brightness of the asteroid. We carried out new observations of (596) Scheila in order to measure its magnitude–phase dependence and to check possible influence of the impact on the phase curve.

Our observations were performed for 14 nights in March–June 2012. We found that the rotational period is equal to 15.8594 ± 0.0015 h. A composite lightcurve with this value of the rotational period is shown in Fig. 13. The period is very close to the value obtained by Warner (2006, 2011b) before collision. Maximal amplitude of the lightcurve is 0.08 ± 0.02 mag which is also very close to the value obtained in Warner (2006), though the aspects of the observations differ by 100° in the ecliptic longitude. It should be noted that Bodewits et al. (2014) and Ishiguro et al. (2011), comparing the Scheila observations performed before and after the presumed collision, found small differences in the shape of the lightcurve produced by the collision but the rotation period did not change. Thus, the impact energy was not high enough to change the rotational parameters.

The obtained magnitude–phase relation is shown in Fig. 14. It has a linear behavior in the entire observed range of phase angles including extremely small phase angles. Such behavior is typical for Hilda asteroids and Jupiter Trojans, the majority of which belong to the P- and D-types (Shevchenko et al., 2012; Slyusarev et al., 2012). The linear phase coefficients are 0.045 ± 0.001 mag/° in the V-band and 0.044 ± 0.001 mag/° in the R band, similar to the magnitude phase angle dependences for the P- and D-types Hildas and Trojans. The data obtained by Warner (2011b) for the R band before collision are shown by asterisks in Fig. 14. One can see that our data are well consistent with Warner's data. A similarity in brightness behavior before and after collision points out that the global regolith properties have not changed. It can be considered as supporting evidence that this asteroid is not a solid body and that the seismic wave decays fairly quickly without disrupting the global structure of the regolith layer.

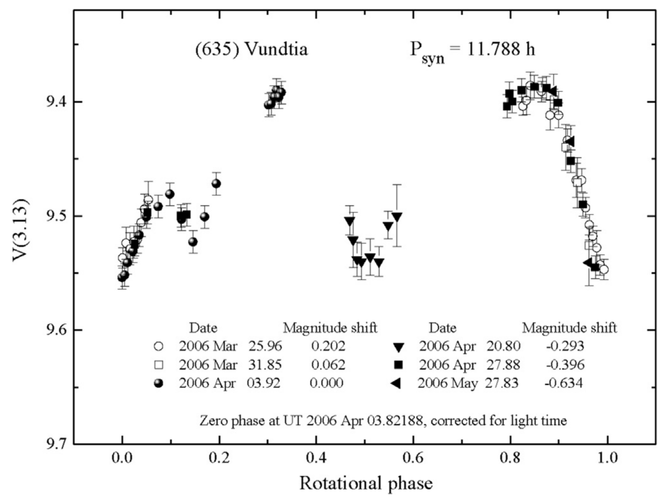

Fig. 15. Composite lightcurve of asteroid (637) Vundtia.

2.8. (635) Vundtia

This asteroid belongs to B-type objects that are characterized by slightly blue spectral slope in the visible range and by changes from negative to positive slopes in the 0.8–2.5 μm near-infrared region (Alí-Lagoa et al., 2013). Warner (2011a) observed Vundtia in June and July 2007 and found a rotation period of 11.816 h. Our observations were performed in 2006 from March to May and allowed us to define more precisely the period that is 11.788 ± 0.003 h but the lightcurve coverage during a full rotation

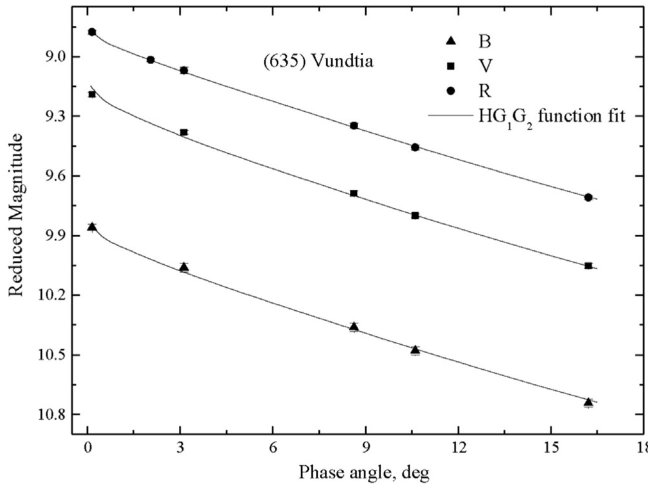

Fig. 16. Magnitude–phase relation of asteroid (635) Vundtia.

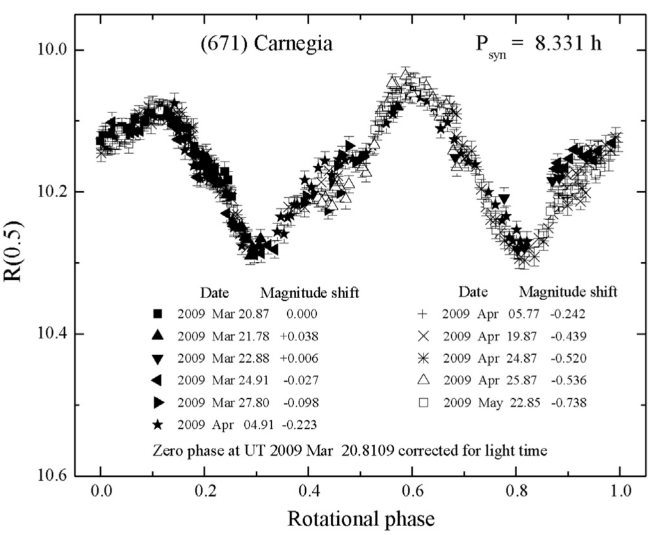

Fig. 17. Composite lightcurve of asteroid (671) Carnegia.

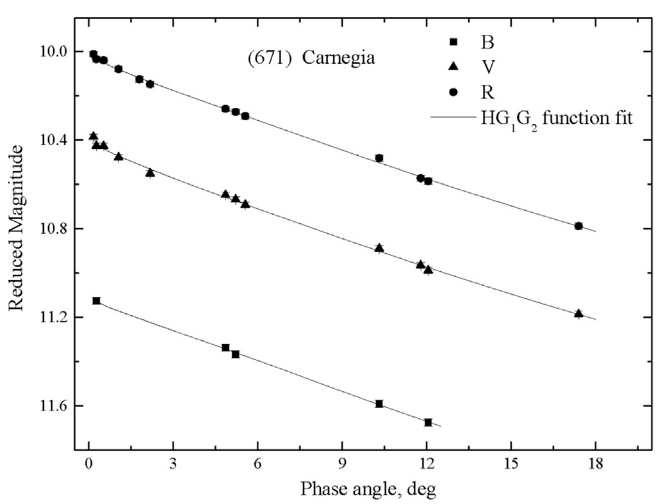

Fig. 18. Magnitude–phase relation of asteroid (671) Carnegia.

cycle was not obtained. The composite lightcurve with this period is presented in Fig. 15. The magnitude–phase relations in the BVR bands are shown in Fig. 16.

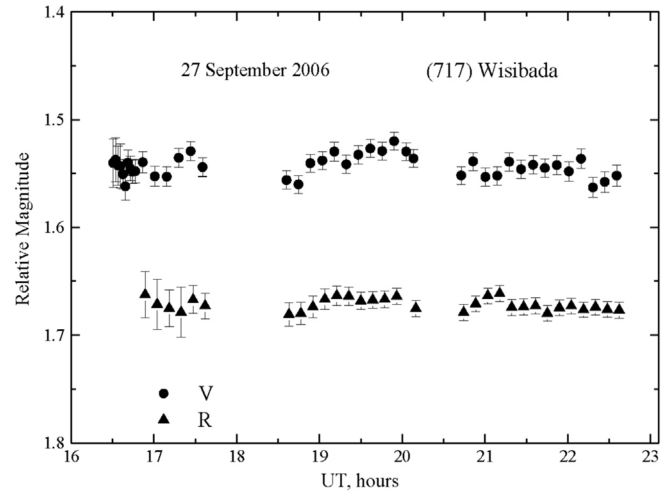

Fig. 19. Lightcurve of asteroid (717) Wisibada on 27 September 2006.

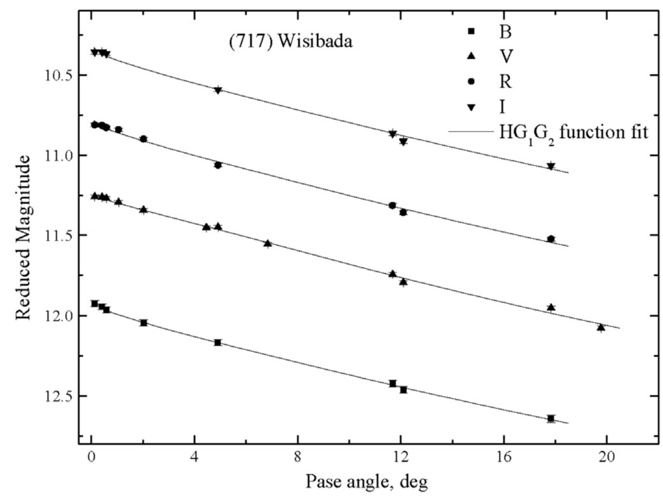

Fig. 20. Magnitude–phase relation of asteroid (717) Wisibada.

2.9. (671) Carnegia

This asteroid was classified by Bus and Binzel (2002) as Xk-type but later DeMeo and Carry (2013) associated it to the P-type asteroids. There are no published photometric observations of this asteroid. We observed Carnegia in the 2009 apparition for 13 nights and determined a rotation period of 8.331 h (Fig. 17). The magnitude phase relations in the BVR bands are presented in Fig. 18. The average absolute magnitude of this asteroid (taking into account the lightcurve amplitude of 0.20 mag) is 10.60 ± 0.02 mag and it differs from those adopted in the MPC data-base by 0.4 mag.

2.10. 717 Wisibada

This small low-albedo asteroid was previously observed by Piironen et al. (1994) for three nights. They reported a small amplitude of the lightcurve (0.05 mag) and impossibility to determine the rotational period. Our observations for 12 nights were also not successful to determine the rotational period due to the small lightcurve amplitude (<0.05 mag, Fig. 19). Our observations were performed at the ecliptic longitudes that are different from Piironen's data by about 80°. Small amplitudes in the two apparitions at different aspects suggest that the asteroid shape is close to spherical. The magnitude–phase relations in the BVRI

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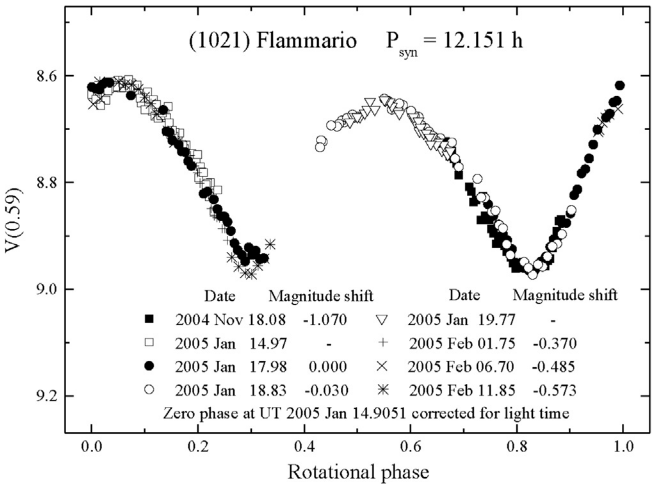

Fig. 21. Composite lightcurve of asteroid (1021) Flammario.

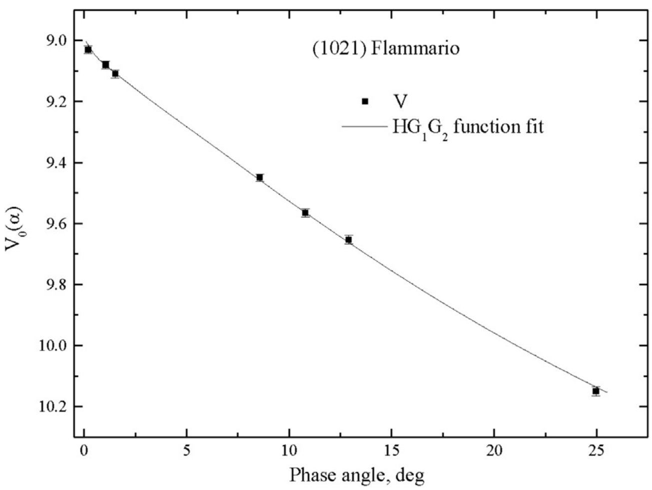

Fig. 22. Magnitude–phase relation of asteroid (1021) Flammario.

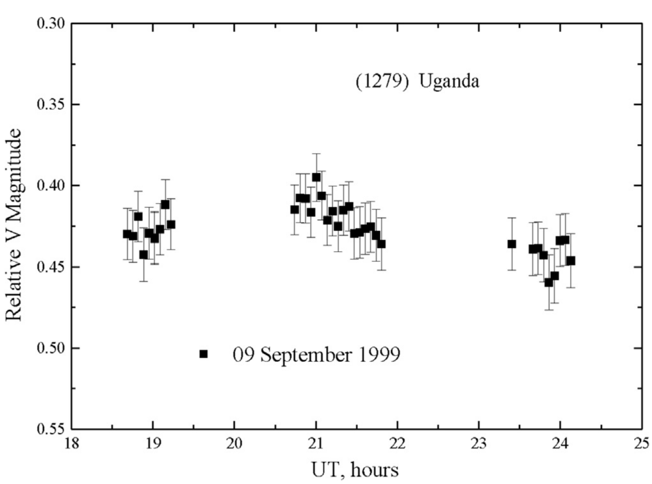

Fig. 23. Lightcurve of asteroid (1279) Uganda on 09 September 1999.

bands were constructed using an average brightness during each night (Fig. 20).

2.11. (1021) Flammario

The asteroid classified as F-type by Tholen (1989) has a diameter of about 100 km and an albedo of 0.047 (Tedesco et al., 2002; Usui et al., 2011; Masiero et al., 2011). The first estimations of a rotation period of 12.146 h were made by Schober et al. (1993) and Hainaut-Rouelle et al. (1995). Photometric observations were also performed by other authors (see, for example, Warner et al., 2013) but no magnitude–phase relation was obtained. Our observations were carried out for eight nights from November 2004 to February 2005. They allowed us to obtain the rotation period more precisely. The composite lightcurve with the period of 12.151 ± 0.001 h is shown in Fig. 21.

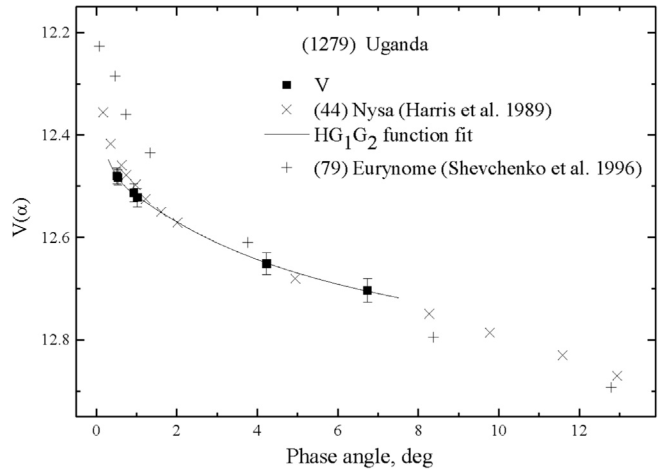

Fig. 24. Magnitude–phase relation of asteroid (1279) Uganda.

The lightcurve has large amplitude of 0.37 mag that is atypical for low-albedo asteroids. For the magnitude–phase relation we used also data obtained by R. Buchheim (2005) at Altimira Observatory on 16 January 2005. The magnitude–phase dependence of Flammario in the V band is presented in Fig. 22.

2.12. (1279) Uganda

This asteroid has a high-albedo surface (0.34) and a diameter of about 7 km (Masiero et al., 2011). Xu et al. (1995) pointed out a spectral similarity of this asteroid and the high-albedo E-type asteroid (44) Nysa. Photometric observations of Uganda were performed by Binzel (1987) in 1984 for three nights. They gave an estimate for the rotation period of about 23.2 h, with the lightcurve amplitude of 0.16 mag.

Our observations were carried out in 1999 for six nights. The amplitude of the lightcurve was less than 0.05 mag (Fig. 23). We did not determine the rotation period because of small amplitude of the lightcurve, but we found that the period determined by Binzel is inconsistent with our data. The magnitude–phase relation (Fig. 24) was constructed using the average magnitude for each night. The brightness behavior of this asteroid is very close to that of the E-type asteroid (44) Nysa (also shown in Fig. 24) and different from the moderate albedo S-type asteroid (79) Eurynome (also shown in Fig. 24). This favors the E-type classification of this asteroid.

3. Average parameters of the *H*, G_1, G_2 magnitude system

In order to determine the parameters of the *H*, G_1, G_2 magnitude system for the main asteroid types, we selected asteroid magnitude–phase dependences with a good phase angle coverage and small scatter within the observational data. The minimum phase angles were $<2°$ (for most of the asteroids $<1°$) for the selected magnitude–phase dependences in our sample. We used data both obtained by the authors and available in literature. The

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Table 3
H, G_1, G_2 parameters of selected asteroids.

Asteroid	Type	p_V[a]	D[a] (km)	H (mag)		G_1		G_2		Ref.
1	2	3	4	5	6	7	8	9	10	11
(1) Ceres	C	0.094[b]	933[b]	3.339	$^{+0.027}_{-0.029}$	0.63	$^{+0.06}_{-0.06}$	0.11	$^{+0.02}_{-0.03}$	Tedesco et al. (1983)
(2) Pallas	B	0.15	533	4.048	$^{+0.028}_{-0.027}$	0.36	$^{+0.06}_{-0.07}$	0.25	$^{+0.03}_{-0.04}$	Schroll et al. (1976)
(4) Vesta	V	0.37	503	3.140	$^{+0.013}_{-0.012}$	0.24	$^{+0.03}_{-0.02}$	0.46	$^{+0.01}_{-0.01}$	Gehrels (1967)
(5) Astraea	S	0.25	115	6.665	$^{+0.016}_{-0.013}$	0.09	$^{+0.07}_{-0.07}$	0.52	$^{+0.04}_{-0.05}$	Shevchenko et al. (2002)
(6) Hebe	S	0.27	185	5.617	$^{+0.019}_{-0.019}$	0.22	$^{+0.04}_{-0.03}$	0.40	$^{+0.01}_{-0.02}$	Gehrels and Taylor (1977)
(10) Hygiea	C	0.058	453	5.500	$^{+0.018}_{-0.022}$	0.76	$^{+0.05}_{-0.06}$	0.03	$^{+0.03}_{-0.03}$	This work
(16) Psyche	M	0.18[b]	207[b]	5.550	$^{+0.024}_{-0.021}$	0.10	$^{+0.04}_{-0.05}$	0.44	$^{+0.02}_{-0.02}$	Shevchenko et al. (2010)
(19) Fortuna	C	0.050	223	7.166	$^{+0.060}_{-0.075}$	0.73	$^{+0.12}_{-0.18}$	0.06	$^{+0.09}_{-0.06}$	Shevchenko et al. (2010)
(20) Massalia	S	0.26[b]	132[b]	6.336	$^{+0.028}_{-0.025}$	0.07	$^{+0.05}_{-0.05}$	0.49	$^{+0.03}_{-0.03}$	Belskaya et al. (2003)
(21) Lutetia	M	0.18[b]	108[b]	7.124	$^{+0.031}_{-0.033}$	0.40	$^{+0.06}_{-0.06}$	0.20	$^{+0.03}_{-0.03}$	Belskaya et al. (2010)
(22) Kalliope	M	0.17	167	6.317	$^{+0.016}_{-0.016}$	0.29	$^{+0.04}_{-0.04}$	0.34	$^{+0.02}_{-0.02}$	Scaltriti et al. (1978)
(24) Themis	CB	0.064	202	7.089	$^{+0.008}_{-0.008}$	0.63	$^{+0.04}_{-0.03}$	0.14	$^{+0.02}_{-0.02}$	Harris et al. (1989a)
(29) Amphitrite	S	0.16	227	5.742	$^{+0.016}_{-0.015}$	0.31	$^{+0.04}_{-0.04}$	0.32	$^{+0.02}_{-0.02}$	Shevchenko et al. (2010)
(33) Polyhymnia	S	0.23[b]	54[b]	8.367	$^{+0.019}_{-0.017}$	0.23	$^{+0.04}_{-0.05}$	0.44	$^{+0.03}_{-0.02}$	Zappala et al. (1982)
(44) Nysa	E	0.48[b]	75.7[b]	6.904	$^{+0.006}_{-0.006}$	0.05	$^{+0.03}_{-0.02}$	0.67	$^{+0.01}_{-0.01}$	Harris et al. (1989b)
(47) Aglaja	CB	0.060	138	8.001	$^{+0.012}_{-0.013}$	0.69	$^{+0.05}_{-0.05}$	0.13	$^{+0.04}_{-0.03}$	Shevchenko et al. (2010)
(50) Virginia	X	0.036	100	9.215	$^{+0.014}_{-0.014}$	0.82	$^{+0.04}_{-0.04}$	0.04	$^{+0.02}_{-0.03}$	Shevchenko et al. (2008)
(51) Nemausa	C	0.10	143	7.295	$^{+0.009}_{-0.010}$	0.49	$^{+0.03}_{-0.03}$	0.20	$^{+0.02}_{-0.02}$	Kaasalainen et al. (2003)
(55) Pandora	M	0.34[b]	63.3[b]	7.618	$^{+0.013}_{-0.014}$	0.33	$^{+0.05}_{-0.05}$	0.34	$^{+0.03}_{-0.03}$	Shevchenko et al. (2010)
(59) Elpis	CP	0.043	166	7.900	$^{+0.009}_{-0.012}$	0.88	$^{+0.04}_{-0.05}$	0.03	$^{+0.03}_{-0.03}$	Shevchenko et al. (1996)
(63) Ausonia	S	0.16	103	7.152	$^{+0.023}_{-0.025}$	0.26	$^{+0.05}_{-0.06}$	0.36	$^{+0.03}_{-0.03}$	Scaltriti and Zappala (1977)
(64) Angelina	E	0.47	50.3	7.575	$^{+0.014}_{-0.007}$	0.00	$^{+0.10}_{-0.00}$	0.69	$^{+0.01}_{-0.07}$	Harris et al. (1989b)
(69) Hesperia	M	0.16[b]	133[b]	6.926	$^{+0.024}_{-0.026}$	0.38	$^{+0.09}_{-0.08}$	0.28	$^{+0.05}_{-0.06}$	Poutanen et al. (1985)
(75) Eurydike	M	0.098	68.6	8.956	$^{+0.008}_{-0.009}$	0.41	$^{+0.04}_{-0.04}$	0.32	$^{+0.04}_{-0.03}$	Shevchenko et al. (2002)
(76) Freia	P	0.049	159	7.954	$^{+0.009}_{-0.013}$	0.84	$^{+0.01}_{-0.05}$	0.00	$^{+0.03}_{-0.00}$	Shevchenko et al. (2008)
(77) Frigga	M	0.15	67.2	8.560	$^{+0.019}_{-0.021}$	0.32	$^{+0.04}_{-0.04}$	0.32	$^{+0.02}_{-0.02}$	Shevchenko et al. (2002)
(79) Eurynome	S	0.22	72.6	7.801	$^{+0.024}_{-0.021}$	0.15	$^{+0.08}_{-0.08}$	0.48	$^{+0.05}_{-0.05}$	Shevchenko et al. (1996)
(83) Beatrix	X	0.086	84	8.663	$^{+0.020}_{-0.025}$	0.58	$^{+0.09}_{-0.11}$	0.16	$^{+0.07}_{-0.06}$	Krugly et al. (1994)
(91) Aegina	CP	0.041	104	8.768	$^{+0.013}_{-0.013}$	0.59	$^{+0.09}_{-0.09}$	0.19	$^{+0.05}_{-0.06}$	Shevchenko et al. (1997)
(102) Miriam	P	0.046	87	9.301	$^{+0.007}_{-0.008}$	0.87	$^{+0.01}_{-0.02}$	0.00	$^{+0.01}_{-0.00}$	Shevchenko et al. (1997)
(105) Artemis	C	0.047	119	8.555	$^{+0.017}_{-0.016}$	0.50	$^{+0.07}_{-0.07}$	0.22	$^{+0.04}_{-0.04}$	Shevchenko et al. (2008)
(110) Lydia	M	0.17	89	7.734	$^{+0.014}_{-0.016}$	0.38	$^{+0.04}_{-0.04}$	0.28	$^{+0.02}_{-0.03}$	Taylor et al. (1971)
(119) Althaea	S	0.20	61.1	8.136	$^{+0.012}_{-0.011}$	0.08	$^{+0.04}_{-0.03}$	0.52	$^{+0.02}_{-0.02}$	Shevchenko et al. (2002)
(124) Alkeste	S	0.15[b]	81.4[b]	8.018	$^{+0.007}_{-0.008}$	0.30	$^{+0.03}_{-0.03}$	0.38	$^{+0.02}_{-0.02}$	Shevchenko et al. (2002)
(126) Velleda	S	0.18[b]	43.9[b]	9.193	$^{+0.011}_{-0.012}$	0.15	$^{+0.04}_{-0.04}$	0.47	$^{+0.02}_{-0.02}$	Shevchenko et al. (2010)
(127) Johanna	C	0.065[b]	114[b]	8.459	$^{+0.016}_{-0.014}$	0.73	$^{+0.07}_{-0.06}$	0.05	$^{+0.03}_{-0.04}$	Toth (1997)
(130) Electra	C	0.071	199	6.925	$+0.016$	0.60	$+0.11$	0.13	$+0.07$	Shevchenko et al. (1997)

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Table 3(*continued*)

Asteroid	Type	p_V[a]	D[a] (km)	H (mag)		G_1		G_2		Ref.
1	2	3	4	5	6	7	8	9	10	11
					−0.017		−0.12		−0.07	
(133) Cyrene	S	0.18	80.5	7.826	+0.010 −0.010	0.16	+0.06 −0.06	0.42	+0.05 −0.05	Harris et al. (1984)
(144) Vibilia	C	0.060[b]	142[b]	7.967	+0.006 −0.017	0.83	+0.01 −0.07	0.00	+0.04 −0.00	Harris and Young (1989)
(146) Lucina	C	0.053	132	8.266	+0.006 −0.006	0.63	+0.02 −0.02	0.12	+0.01 −0.02	Harris and Young (1989)
(158) Koronis	S	0.14	47.7	8.974	+0.029 −0.019	0.03	+0.06 −0.03	0.54	+0.02 −0.03	Buchheim. (2011)
(160) Una	C	0.069[b]	77.7[b]	8.916	+0.025 −0.038	0.70	+0.02 −0.40	0.00	+0.33 −0.00	Warner (2007)
(165) Loreley	S	0.051[b]	174[b]	7.860	+0.014 −0.014	0.78	+0.05 −0.04	0.07	+0.03 −0.03	Harris et al. (1992)
(176) Iduna	C	0.083	122	8.046	+0.012 −0.015	0.84	+0.01 −0.07	0.00	+0.04 −0.00	This work
(190) Ismene	P	0.051[b]	180[b]	7.772	+0.011 −0.020	0.96	+0.01 −0.08	0.00	+0.05 −0.00	Shevchenko et al. (2008)
(201) Penelope	M	0.097	88	8.097	+0.011 −0.012	0.27	+0.03 −0.04	0.35	+0.02 −0.02	Shevchenko et al. (2002)
(211) Isolda	C	0.060	143	7.913	+0.015 −0.017	0.67	+0.04 −0.04	0.07	+0.03 −0.03	Harris and Young (1989)
(214) Aschera	E	0.42[b]	26.0[b]	9.409	+0.007 −0.007	0.18	+0.03 −0.03	0.58	+0.02 −0.02	This work
(218) Bianca	S	0.20	56.8	8.474	+0.010 −0.010	0.31	+0.04 −0.03	0.31	+0.02 −0.02	This work
(243) Ida	S	0.23[b]	29[b]	9.697	+0.014 −0.012	0.29	+0.05 −0.04	0.36	+0.02 −0.03	Binzel et al. (1993)
(250) Bettina	M	0.11	121	6.925	+0.017 −0.014	0.06	+0.05 −0.05	0.48	+0.03 −0.03	This work
(253) Mathilde	C	0.050[b]	54[b]	9.976	+0.003 −0.003	0.47	+0.01 −0.01	0.17	+0.00 −0.01	Mottola et al. (1995b)
(261) Prymno	B	0.10	54.3	9.384	+0.058 −0.124	0.68	+0.05 −0.29	0.00	+0.16 −0.00	Harris et al. (1989a)
(276) Adelheid	C	0.063	103	8.602	+0.003 −0.009	0.94	+0.00 −0.12	0.00	+0.10 −0.00	Piironen et al. (1994)
(303) Josephina	C	0.042	106	8.961	+0.013 −0.012	0.74	+0.05 −0.05	0.08	+0.03 −0.03	Shevchenko et al. (2008)
(309) Fraternitas	C	0.030	44.5	10.571	+0.009 −0.010	0.54	+0.04 −0.06	0.25	+0.03 −0.03	Shevchenko et al. (2008)
(313) Chaldaea	C	0.053	96	8.800	+0.012 −0.013	0.64	+0.06 −0.06	0.13	+0.03 −0.03	Shevchenko et al. (2008)
(317) Roxane	E	0.53	19.9	10.022	+0.014 −0.013	0.06	+0.07 −0.06	0.67	+0.03 −0.05	Harris et al. (1992)
(354) Eleonora	S	0.17	165	6.111	+0.093 −0.068	0.13	+0.15 −0.13	0.42	+0.05 −0.08	Zappala et al. (1979)
(379) Huenna	B	0.065	87.5	8.995	+0.026 −0.075	0.74	+0.11 −0.37	0.06	+0.19 −0.06	Harris et al. (1992)
(419) Aurelia	C	0.051[b]	122[b]	8.509	+0.007 −0.010	0.87	+0.05 −0.07	0.04	+0.05 −0.04	This work
(423) Diotima	C	0.066	177	7.512	+0.020 −0.034	0.84	+0.01 −0.07	0.00	+0.04 −0.07	Shevchenko et al. (2010)
(433) Eros	S	0.27[b]	15.3[b]	10.313	+0.016 −0.014	0.28	+0.03 −0.04	0.33	+0.02 −0.02	Shevchenko et al. (2010)
(444) Gyptis	C	0.048	163	7.859	+0.019 −0.018	0.67	+0.07 −0.08	0.13	+0.04 −0.04	Shevchenko et al. (2008)
(508) Princetonia	C	0.062	120	8.190	+0.003 −0.003	0.38	+0.02 −0.02	0.25	+0.01 −0.02	Piironen et al. (1994)
(588) Achilles	D	0.023	161	8.393	+0.005 −0.004	0.87	+0.04 −0.04	0.08	+0.03 −0.03	Shevchenko et al. (2012)
(596) Scheila	D	0.037	114	9.079	+0.016 −0.029	0.96	+0.01 −0.11	0.00	+0.05 −0.00	This work
(604) Teknessa	Xc	0.082	67.2	9.412	+0.022 −0.024	0.57	+0.05 −0.04	0.14	+0.02 −0.02	Baker & Warner (2011)
(615) Roswitha	C	0.052	51.6	10.290	+0.010 −0.015	0.82	+0.01 −0.06	0.00	+0.04 −0.00	Shevchenko et al. (2008)
(620) Drakonia	E	0.43	11.4	10.980	+0.018 −0.016	0.07	+0.04 −0.04	0.72	+0.02 −0.02	Belskaya et al. (2003)
(635) Vundtia	B	0.044	100	9.164	+0.013 −0.018	0.84	+0.01 −0.07	0.00	+0.04 −0.00	This work
(636) Erika	C	0.046	76.9	9.439	+0.042 −0.040	0.50	+0.13 −0.11	0.25	+0.07 −0.07	Piironen et al. (1994)
(671) Carnegia	X	0.028	69.3	10.385	+0.007 −0.008	0.80	+0.04 −0.04	0.10	+0.02 −0.02	This work
(695) Bella	S	0.25	41.2	9.022	+0.035	0.43	+0.15	0.26	+0.08	Harris et al. (1992)

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Table 3 (*continued*)

Asteroid	Type	p_V[a]	D[a] (km)	H (mag)		G_1		G_2		Ref.
1	2	3	4	5	6	7	8	9	10	11
					−0.035		−0.14		−0.08	
(717) Wisibada	T	0.079	28.5	11.241	+0.011	0.82	+0.05	0.12	+0.03	This work
					−0.011		−0.05		−0.03	
(776) Berbericia	C	0.066	151	7.499	+0.018	0.73	+0.04	0.02	+0.10	Harris et al. (1992)
					−0.059		−0.19		−0.02	
(811) Nauheima	S	0.25[b]	18.6	10.720	+0.008	0.32	+0.01	0.32	+0.01	Slivan et al. (2008)
					−0.008		−0.01		−0.01	
(884) Priamus	D	0.039	117	8.377	+0.007	0.78	+0.05	0.17	+0.03	Shevchenko et al. (2012)
					−0.008		−0.04		−0.03	
(951) Gaspra	S	0.25	12.2	11.500	+0.032	0.42	+0.08	0.28	+0.05	Wisniewski et al. (1993)
					−0.033		−0.09		−0.05	
(954) Li	C	0.055	52.1	10.194	+0.011	0.73	+0.06	0.08	+0.04	Shevchenko et al. (2008)
					−0.014		−0.06		−0.04	
(1021) Flammario	B	0.047	98.0	8.998	+0.007	0.90	+0.01	0.00	+0.02	This work
					−0.016		−0.04		−0.00	
(1029) La Plata	S	0.31[b]	16.5	10.689	+0.004	0.37	+0.01	0.26	+0.01	Slivan et al. (2008)
					−0.004		−0.01		−0.01	
1	2	3	4	5	6	7	8	9	10	11
(1079) Mimosa	S	0.17[b]	19.0[b]	11.078	+0.004	0.43	+0.02	0.26	+0.01	Slivan et al. (2008)
					−0.004		−0.004		−0.01	
(1130) Skuld	S	0.20	11.1	11.978	+0.015	0.30	+0.04	0.37	+0.03	Buchheim. (2010)
					−0.015		−0.05		−0.02	
(1143) Odysseus	D	0.072[b]	131[b]	8.018	+0.006	0.80	+0.04	0.12	+0.03	Shevchenko et al. (2012)
					−0.006		−0.05		−0.03	
(1336) Zeelandia	S	0.18	23.1	10.582	+0.009	0.37	+0.02	0.28	+0.01	Slivan et al. (2008)
					−0.009		−0.02		−0.01	
(1423) Jose	S	0.28	20.1	10.553	+0.005	0.43	+0.02	0.21	+0.01	Slivan et al. (2008)
					−0.005		−0.02		−0.01	
(1482) Sebastiana	S	0.21	17.6	10.838	+0.007	0.45	+0.04	0.24	+0.02	Slivan et al. (2008)
					−0.008		−0.04		−0.02	
(1618) Dawn	S	0.15	17.5	11.011	+0.007	0.34	+0.02	0.33	+0.01	Slivan et al. (2008)
					−0.006		−0.02		−0.01	
(1635) Bohrmann	S	0.21	17.5	10.756	+0.007	0.52	+0.03	0.21	+0.02	Slivan et al. (2008)
					−0.008		−0.03		−0.02	
(1748) Mauderli	D	0.037[b]	51.9[b]	10.759	+0.013	0.81	+0.06	0.09	+0.04	Slyusarev et al. (2012)
					−0.013		−0.06		−0.04	
(1862) Apollo	Q	0.26	1.5	16.250	+0.025	0.39	+0.03	0.35	+0.01	Harris et al. (1987)
					−0.027		−0.04		−0.01	
(2867) Steins	E	0.34	5.3	13.338	+0.007	0.16	+0.02	0.59	+0.01	Dotto et al. (2009)+A'Hearn et al. (2010)
					−0.007		−0.02		−0.01	

[a] Masiero et al. (2011).
[b] Usui et al., (2011).

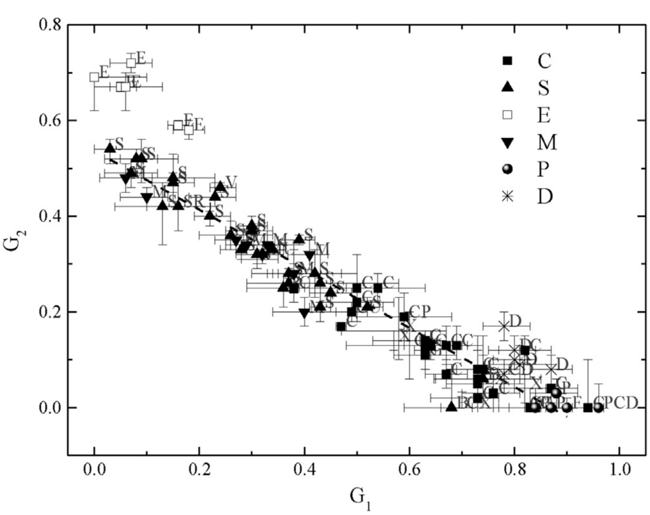

Fig. 25. Correlation of the G_1 and G_2 parameters.

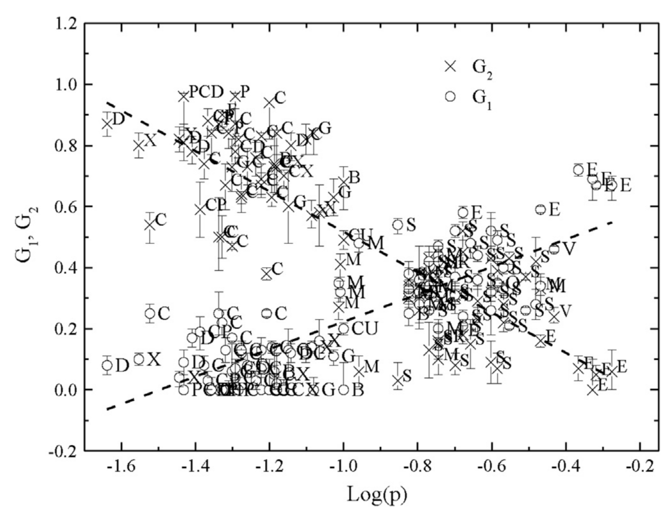

Fig. 26. Relationship of the G_1 and G_2 parameters with albedo.

detailed procedure of determining the H, G_1, G_2 parameters were described in Muinonen et al. (2010) and Oszkiewicz et al. (2011), and for determining the constrained parameters with a non-linear fit in Penttilä et al. (2015). The values of the G_1, G_2 parameters and the absolute magnitudes for the 93 asteroids are listed in Table 3. In Table 3, we present also composition types (data taken from DeMeo et al., 2009; DeMeo and Carry, 2013; Lazzaro et al., 2004; Tholen, 1989; Xu et al., 1995), as well as albedos and diameters of

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Table 4
Average G_1, G_2 parameters for main asteroid classes.

Type	N	p_V	G_1		G_2	
S	30	0.22 ± 0.05	0.26	$+0.01$ -0.01	0.38	$+0.01$ -0.01
M	10	0.17 ± 0.07	0.27	$+0.03$ -0.02	0.35	$+0.01$ -0.01
E	6	0.45 ± 0.07	0.15	$+0.02$ -0.02	0.60	$+0.01$ -0.01
C	34	0.061 ± 0.017	0.82	$+0.02$ -0.02	0.02	$+0.02$ -0.01
P	7	0.042 ± 0.008	0.83	$+0.02$ -0.03	0.05	$+0.02$ -0.01
D	6	0.049 ± 0.022	0.96	$+0.02$ -0.03	0.02	$+0.02$ -0.02
All types		0.17 ± 0.03	0.55 ± 0.06		0.24 ± 0.04	

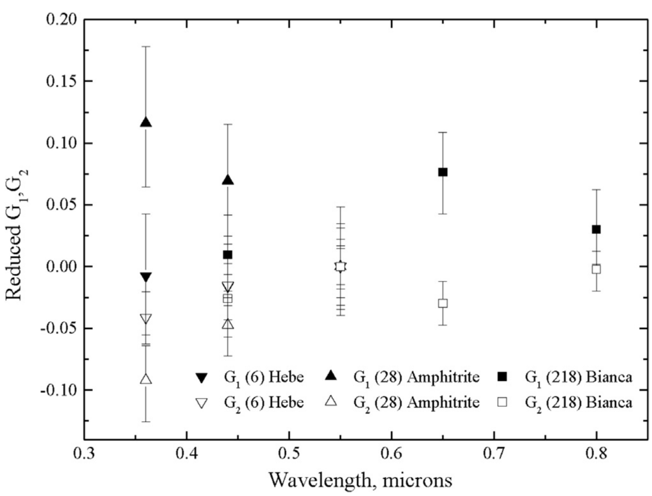

Fig. 27. Wavelength dependences for the reduced G_1 and G_2 parameters.

the asteroids (Masiero et al., 2011; Shevchenko and Tedesco, 2006; Usui et al., 2011; Tedesco et al., 2002). The range of asteroid sizes in our dataset (excluding large asteroids with sizes more than 400 km and small asteroids with sizes less than 10 km) varies from about 200 km to 11 km with a mean size of 94 km and standard deviation of 56 km.

We have investigated possible correlations of the G_1 and G_2 parameters and the albedo. A strong linear correlation (correlation coefficient of -0.95) was found between the G_1 and G_2 parameters. A relationship of the G_1 and G_2 parameters is shown in Fig. 25 where the dashed line is the linear fit $G_2 = 0.535$ $(\pm 0.011) - 0.635$ $(\pm 0.020) G_1$. The E- and D-types are excluded from the linear fit (see Penttilä et al., 2015). This correlation was noted by Muinonen et al. (2010), and the larger data-set presented in this article allowed Penttilä et al. (2015) to slightly refine the relationship. The strong relationship between the G_1 and G_2 parameters is further used when simplifying the H, G_1, G_2 phase function to the H, G_{12} phase function for targets with sparse data. It should be noted, that the H, G_{12} phase function for asteroids with sparse magnitude data was used by Oszkiewicz et al. (2012). The authors explored the correlation between an asteroid's taxonomy and parameter G_{12}, and concluded that this parameter cannot be used in determining taxonomic class for individual asteroids, but it can be utilized in the separation of asteroid families and different regions of the main asteroid belt.

The G_1 and G_2 parameters have a strong linear correlation with the logarithmic albedo from Table 3 (Fig. 26) with the correlation coefficients -0.87 and 0.84, respectively. The linear regressions are of form -0.107 $(\pm 0.048) - 0.643$ $(\pm 0.047) \log(p)$ and 0.644 $(\pm 0.035) + 0.433$ $(\pm 0.034) \log(p)$. The clusters of low, moderate and high-albedo asteroids are well-divided in Fig. 26.

Using the spectral classification by DeMeo and Carry (2013), we sorted our dataset into the six main compositional types: S, M, E, C, D, and P. The average values of the G_1, G_2 parameters and the average albedos of these types are presented in Table 4. As one can see from Table 4, the values of the parameters for the M and S asteroids cannot be distinguished within the errors of the estimations; this is also true for the C and P types. The obtained average values can be used for the estimation of the absolute magnitude of an asteroid from a single observed magnitude when the magnitude–phase dependency is unknown and/or to calculate a visible magnitude for the ephemerides.

The data available on the magnitude–phase relation in the BVRI bands for asteroids of different types are insufficient for the investigation of a possible detailed dependence of the G_1, G_2 parameters on the wavelength. As it was shown by Shevchenko et al. (2012), the linear coefficients practically did not change with the wavelength for the D-type asteroids.

In Fig. 27, we present the wavelength dependences of the reduced G_1 and G_2 parameters for three S-type asteroids (since a spectral gradient is maximal for spectra of S-asteroids), namely (6) Hebe, (29) Amphitrite, and (218) Bianca. We did not find statistically significant variations of the G_1, G_2 parameters with the wavelength (excluding the U band), and we recommend using the values of the G_1, G_2 parameters obtained in the V band for the BRI bands when necessary. But it is needed new observational data for more detail investigation of the G_1, G_2 parameters with the wavelength.

4. Conclusions

As a result of a series of photometric observations we have obtained the magnitude–phase relations for 12 asteroids of different compositional types. This is a substantial addition to the available dataset of asteroids for which the brightness behavior has been well-measured both in the region of the opposition effect (including phase angles $< 1°$) and in the linear part of phase curve.

We have determined the lightcurve amplitudes of the observed asteroids and obtained new or improved values of the rotation periods for some of them. New, more accurate values of the absolute magnitudes were determined and they can be used in more accurate estimation of the albedos or diameters for these asteroids.

We calculated the values of the G_1, G_2 parameters for 93 asteroids of different compositional types in the new H, G_1, G_2 magnitude system, and determined the average values of the G_1, G_2 parameters for six main asteroid types: S, M, E, C, D, and P. These values can be used for the estimation of asteroid absolute magnitudes. Finally, we recommend using the V-band values of the G_1, G_2 parameters for other bands, too.

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