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Mixture Latent Markov Modeling: Identifying and Predicting Unobserved Heterogeneity in Longitudinal Qualitative Status Change

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Abstract

There are many areas of organizational research where we may be concerned with subgroup differences in status change profiles. The purpose of this article is to illustrate, using a real data set on retirees' postretirement employment statuses (PES), how mixture latent Markov modeling may be applied to substantive research in organizational settings to identify population subgroups with varying status change profiles and examine their correlates, by modeling unobserved heterogeneity in longitudinal qualitative changes. Steps in the modeling process are highlighted and limitations, cautions, recommendations, and extensions of the technique are discussed.

Keywords

mixture latent Markov modeling, latent transition analysis, longitudinal analysis, qualitative status change

Many organizational research questions (e.g., newcomer adaptation, changes in organizational commitment, dynamic performance, skill acquisition, withdrawal behaviors, and changes in employment status) are concerned with, either explicitly or implicitly, phenomena that involve one or more facets of changes over time. To empirically examine these research questions, we need to be able to adequately model the data representing the substantive longitudinal processes of interest so that we can accurately conceptualize and assess the relevant changes over time. An adequate longitudinal model of the data presupposes the application of appropriate statistical techniques that accurately describe and predict the various facets of change over time. A longitudinal modeling technique is appropriate to the extent that its assumptions are valid for the data set to which it is applied and the conceptual questions of interest on the nature of the changes over time (e.g., qualitative vs. quantitative changes), the level of measurement corresponding to the unit of theory in the

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change process (e.g., categorical vs. continuous variables), and the unreliability of measurement are adequately incorporated in the technique (Chan, 1998).

Latent variable approaches are well suited for longitudinal modeling because they can explicitly take into account both cross-sectional and longitudinal measurement errors. Hence, the researcher is able to model a variety of error covariance structures and assess any distorting effects that cross-sectional or longitudinal measurement errors may have on the various parameter estimates of true change. In addition, longitudinal latent variable approaches are highly flexible and powerful because a variety of latent variable models can be fitted to the longitudinal data to describe, in alternative ways, the change over time (Chan, 2009).

Longitudinal latent variable approaches such as longitudinal factor analysis, longitudinal means and covariance structures analysis, and latent growth modeling are appropriate when the latent variables are continuous in nature (e.g., Chan, 1998; Raudenbush & Bryk, 2002). When the latent variables are discrete (i.e., categorical) in nature, latent class analysis is appropriate. When latent class modeling is applied to discrete longitudinal data, the analysis is known as latent transition analysis, which allows the researcher to specify dynamic latent class variables to test changes in discrete statuses over time (e.g., stage-sequential changes in child development, changes in employment status among retirees). An excellent introduction to latent class analysis and latent transition analysis is provided by Collins and Wugalter (1992). Recently, Muthén (2004) developed an inclusive framework known as general growth mixture modeling, which combines latent growth models and latent class models. This general framework allows the researcher to identify latent classes (i.e., subgroups of individuals in the population) characterized by different patterns of latent growth including changes in discrete statuses.

Within the general growth mixture modeling framework, it is possible to examine situations in which it is unclear whether the population of interest is homogenous or heterogeneous with respect to the longitudinal status change pattern, in the sense that whether a single status change pattern adequately characterizes the entire population or the population is more adequately characterized by subgroups with varying distinct status change patterns. Specific growth mixture modeling techniques may be applied to identify possible unobserved subgroups in the population distinguished by their distinct status change patterns, which we will label as status change profiles. If subgroups indeed exist, we may proceed to identify correlates of the subgroups in terms of predictors and criterion outcomes associated with the distinct subgroup differences in change profiles.

In many settings, including organizational situations, qualitative (i.e., discrete) changes in status over time may not follow the same pattern for the entire population of interest. In other words, subpopulations characterized by distinct status change profiles may exist and these subpopulation differences may have substantive implications for the theories or hypotheses being examined. If these subpopulations are observable subgroup membership variables that are known a priori such as groupings by demographics (e.g., sex), then the longitudinal analyses may be quite easily performed using some form of multiple-group analyses that separate the data set according to the subgroup membership variable. However, if these subpopulations are unobserved (latent) in the sense that subgroup membership is not known a priori but only empirically derived from the individual's values on a set of variables, then it is not possible to perform a straightforward multiple-group longitudinal analysis because there is no known grouping variable.

There are many areas of organizational research where we may be concerned with subgroup differences in status change profiles. For example, people's employment status may change over time (e.g., at each time point, it could simply be dichotomized into two discrete statuses: being employed vs. not being employed). For organizational researchers who are interested in studying long-term career change–related issues, it is important to statistically identify and summarize these employment status changes over time into meaningful trends and predict these trends to better understand the different status change profiles. In the field of retirement research, there is an increasing interest for researchers to identify and describe retirees' different change patterns in employment status after retirement and to profile the corresponding retiree subpopulations, including identifying time-invariant predictors or time-varying correlates of these change patterns (e.g., Shultz, 2003; Wang, Zhan, Liu, & Shultz, 2008). This type of research on unobserved subgroup populations has important conceptual and practical implications. In this example, the unobserved subgroup heterogeneity in longitudinal changes provides conceptual clarifications to the retirement process. Practically, it could help identify the factors that attract and motivate retirees to take postretirement employment. Another example is the area of employee withdrawal behaviors in which we could model subpopulations with different profiles of status changes in discrete withdrawal variables (e.g., present vs. absent at work) and correlates of these withdrawal (e.g., absenteeism) profiles. Other examples include modeling profiles of changes in recidivism status in addiction research, profiles of changes in pass–fail status in skills test in skill acquisition research, profiles of changes in performance award status (e.g., receiving vs. failing to receive excellence employee award) in customer service research, and profiles of changes in organizational status on the annual listing of "employers of choice" (in vs. out of the list) in organizational attractiveness research.

The purpose of this article is to illustrate how mixture latent Markov modeling may be applied to substantive research to identify population subgroups with varying status change profiles and examine their correlates, by modeling unobserved heterogeneity in longitudinal qualitative changes. A real data set on retirees' postretirement employment statuses (PESs) is used to illustrate the substantive organizational applications of the technique. Although the mixture latent Markov modeling technique described here is not new, the main contribution here is to provide a nontechnical introduction that serves as a useful interface between researchers in substantive organizational research and the abstract technical/mathematical work on this technique (e.g., Kaplan, 2008; Langeheine & Van de Pol, 2002), in the same spirit as Chan (1998, 2002) reviewed the latent growth modeling technique and its extensions to organizational research. Although some organizational researchers may now be familiar with the methodology and substantive applications of standard latent growth models for assessing changes in continuous variables, many are probably not familiar with using latent Markov modeling to model longitudinal changes in qualitative status. In addition, mixture latent Markov modeling helps address several important questions in organizational research regarding longitudinal change noted by Chan (1998) such as whether the change is best represented as proceeding through one single pathway or through multiple different pathways (p. 425) and whether there is invariance or difference across groups with respect to the specific facet of change over time under investigation (p. 428). As Chan as well as others (e.g., Wang & Bodner, 2007) noted, an adequate change assessment methodology should be able to identify subgroups of individuals, which follow different change patterns.

This article is organized as follows. First, conventional latent Markov modeling is briefly reviewed to set up the necessary background for introducing the mixture latent Markov model. Next, model specification, estimation, and selection in the mixture latent Markov modeling framework are introduced. A research scenario (i.e., identifying and predicting unobserved retiree subpopulations according to their different PES change patterns) is used throughout to help describe and explain the technique. A real data set on retirees' PESs is then used to demonstrate how to apply the mixture latent Markov modeling method. Finally, advantages, limitations, and recommendations of using this technique are discussed.

The Latent Markov Modeling

To understand mixture latent Markov modeling, we need to first understand the latent Markov modeling method, which was first developed by Wiggins (1973). Basically, the method consists of a single Markov chain, where the current state of an individual/subject is only predicted by the

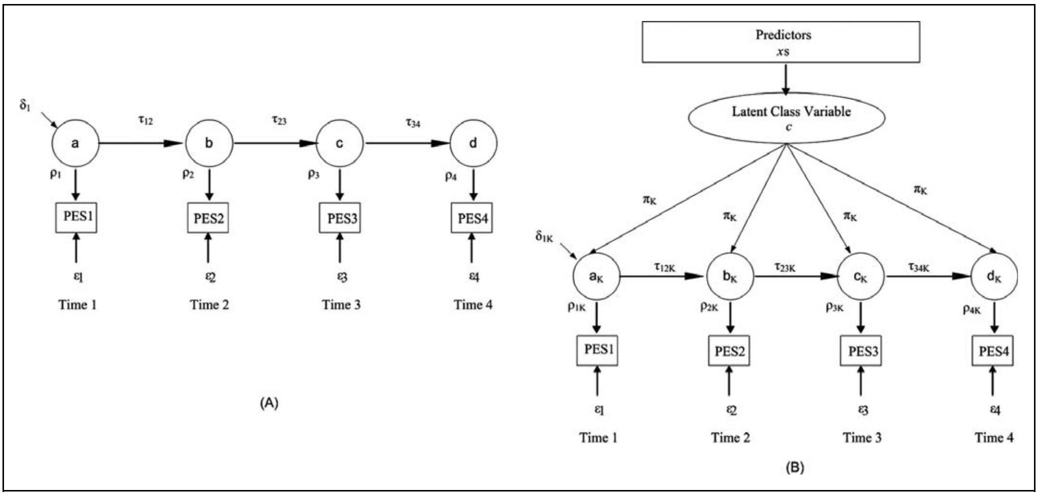

Figure 1. A, An illustration of the conventional latent Markov model for longitudinal categorical measures of employment status; B, An illustration of the mixture latent Markov model including predictors of the latent class variable. PES = postretirement employment status.

previous state of the same individual/subject. In addition, it takes into account the possible measurement error in observed qualitative statuses, assuming the observed qualitative statuses are indicators of unobservable latent states. Below, we illustrate the latent Markov modeling method using a research scenario of modeling retirees' PES change.

The model depicted in Figure 1A represents a conventional latent Markov model of longitudinal observations for retirees' PES, measured at four equally spaced time points (PES1, PES2, PES3, and PES4). Each observation of the PES takes a dichotomized value (i.e., 0 = *not being employed* vs. 1 = *being employed*). The latent Markov model can be written as

$$P_{ghij} = \delta_{1a} \cdot \rho_{1(g|a)} \cdot \tau_{12(b|a)} \cdot \rho_{2(h|b)} \cdot \tau_{23(c|b)} \cdot \rho_{3(i|c)} \cdot \tau_{34(d|c)} \cdot \rho_{4(j|d)}, \tag{1}$$

where P_{ghij} is the model-based expected proportion of individuals in the studied population (i.e., retirees) in cell (g, h, i, and j). The subscripts associated with P (i.e., g, h, i, and j) are the observed categorical values for Times 1, 2, 3, and 4, with $g = 1 \ldots G$, $h = 1 \ldots H$, $i = 1 \ldots I$, and $j = 1 \ldots J$. In the current research scenario, given that there are only two possible employment statuses at each time point, $G = H = I = J = 2$. Among the subscripts at the right side of the equation, a, b, c, and d denote the categorical latent employment states indicated by the observed employment statuses at Times 1, 2, 3, and 4, respectively, with $a = 1 \ldots A$, $b = 1 \ldots B$, $c = 1 \ldots C$, and $d = 1 \ldots D$. The parameter δ_{1a} represents the proportion of individuals at Time 1 corresponds to a latent distribution of A latent employment states. The linkage of the latent states to the observed categorical values is represented by the response probability (or reliability probability) ρ, indicating the extent to which observed qualitative statuses accurately reflect the unobservable latent employment state. The interpretation of ρ, thus, is analogous to that of factor loadings in factor analysis. Accordingly, $\rho_{1(g|a)}$ represents the response probability associated with the observed value g given membership in latent state a, linking the observed qualitative status to the latent state. The remaining response probabilities (i.e., $\rho_{2(h|b)}$, $\rho_{3(i|c)}$, and $\rho_{4(j|d)}$) are similarly interpreted. When the measurement reliability is perfect for PES measures at all time points, $\rho_{1(g|a)} = \rho_{2(h|b)} = \rho_{3(i|c)} = \rho_{4(j|d)} = 1$. In other words, when the PES measures enjoy perfect reliability, the latent employment states will take the exact same distribution of the observed employment statuses.

The parameters $\tau_{12(b|a)}$, $\tau_{23(c|b)}$, and $\tau_{34(d|c)}$ denote the transition probabilities between latent employment states. Specifically, the parameter $\tau_{12(b|a)}$ denotes the transition probability from Time 1 to Time 2 for those retirees in latent employment state *b* given they were in latent employment state *a* at Time 1. The parameter $\tau_{23(c|b)}$ denotes the transition probability from Time 2 to Time 3 for those retirees in latent employment state *c* given they were in latent employment state *b* at Time 2. Finally, the parameter $\tau_{34(d|c)}$ denotes the transition probability from Time 3 to Time 4 for those retirees in latent employment state *d* given they were in latent employment state *c* at Time 3. In applications of latent Markov modeling, these transition probabilities are usually the focus of interest, because they represent how likely individuals change their qualitative statuses from one time point to the next time point.

When estimating the latent Markov model, the only parameters that are important to specify are the response probabilities (i.e., ρs). Specifically, the response probabilities are typically constrained to be invariant over time, because the measurement error of the categorical status measure is considered to be equal over time (Langeheine & Van de Pol, 2002). All the rest of parameters (i.e., δ and τs) can be freely estimated using the maximum likelihood estimation method. More extensive mathematical introductions to the latent Markov modeling with examples can be found in Langeheine and Van de Pol (2002) and Mooijaart (1998).

Mixture Latent Markov Modeling

Mixture modeling generally refers to modeling with *categorical latent variables* that represent mixtures of subpopulations where population membership is not known but is empirically derived from the data. In mixture latent Markov modeling, unobserved heterogeneity in the change of a qualitative status over time is captured by a categorical latent variable. Specifically, mixture latent Markov modeling relaxes the single population assumption of conventional latent Markov modeling method to allow for simultaneously estimating several latent Markov chains that correspond to multiple unobserved subpopulations. Using the above research scenario example of modeling retirees' PES change, the application of mixture latent Markov modeling assumes that there exist multiple unobserved retiree subpopulations (the subpopulation membership is unknown for each retiree), which correspond to different PES change patterns. This analysis is accomplished by estimating latent Markov classes, which are represented by a categorical latent variable, for retirees. The following sections describe the model specification, model estimation, and model selection in mixture latent Markov modeling.

Mixture Latent Markov Model Specification

Consider a categorical latent variable *c* representing the unobserved subpopulation membership for each retiree. Assuming that *K* unobserved retiree subpopulations exist in the longitudinal data of PES, for a given retiree, *c* could be any number from 1 to *K*, representing the membership in the *K*th latent retiree subpopulation. Here, *c* is referred to as a latent class variable. As such, mixture latent Markov modeling specifies a separate latent Markov chain for each of the *K* latent retiree subpopulations simultaneously. This mixture latent Markov model can be written as

$$P_{ghij} = \pi_K \times \delta_{1a|K} \times \rho_{1(g|aK)} \times \tau_{12(b|aK)} \times \rho_{2(h|bK)} \times \tau_{23(c|bK)} \times \rho_{3(i|cK)} \times \tau_{34(d|cK)} \times \rho_{4(j|dK)} \tag{2}$$

where π_K denotes the proportion of retirees in the *K*th retiree subpopulation. The remaining parameters (i.e., δ, ρs, and τs) are interpreted in the same way as in Equation 1, with the exception that they are conditioned on membership in *K*th latent retiree subpopulations. In other words, all parameters other than π_K may differ across different latent retiree subpopulations. As a result, the proportions of individuals that belong to different latent employment states at Time 1 (denoted by δ_{1a})

may be different in different latent retiree subpopulations. For example, the majority of retirees may be working for pay at Time 1 in one subpopulation, whereas in another subpopulation, the majority of retirees may not be working at all at Time 1. Similarly, the probabilities for individuals to change their employment statuses from one time point to the next time point (denoted by τ parameters) may also depend on the retirees' subpopulation membership. In one subpopulation, retirees may remain being employed across all time points; whereas in another subpopulation, retirees may change their employment statuses frequently across different time points. An illustration of this mixture latent Markov model is shown in Figure 1B.

Furthermore, the above mixture latent Markov model can be extended to include time-invariant predictors. Consider incorporating a predictor x that influences c (i.e., unobserved retiree subpopulation membership) as illustrated in Figure 1B. To estimate the predictive effect of x on the latent retiree subpopulation membership c, a multinomial logistic regression model for K latent retiree subpopulations can be constructed

$$P(c_i = K|x_i) = \frac{e^{a_K + b_K x_K}}{\sum_{c=1}^{K} e^{a_c + b_c x_i}}, \tag{3}$$

where a denotes the logit intercept and b denotes the logit slope. Assuming the Kth latent retiree subpopulation is the reference class in this multinomial logistic regression model with coefficients standardized to 0 (i.e., $a_K = 0$, $b_K = 0$), it gives

$$P(c_i = 1|x_i) = \frac{1}{1 + e^{-(a_1 + b_1 x_i)}}, \tag{4}$$

where b_1 is the increase in the log odds of being in the first latent retiree subpopulation versus being in the Kth latent retiree subpopulation for a unit increase in x. Suppose that x is a dichotomous variable with 0 for female and 1 for male, it follows that e^{b_1} is the odds ratio for being in the first latent retiree subpopulation versus being in the Kth latent retiree subpopulation when comparing males to females. For example, $b_1 = 1$ implies that the odds of being in the first latent retiree subpopulation versus being in the Kth latent retiree subpopulation is $e^1 = 2.72$ times higher for males than females.

Mixture Latent Markov Model Estimation

As in most other types of latent variable modeling, the mixture latent Markov model can be estimated using the maximum likelihood approach. Specifically, an expectation maximization (EM) algorithm is used. In the E step, data on the latent class variable c (e.g., unobserved retiree subpopulation membership in the above example) are considered missing. Therefore, the conditional probability of individual i belonging to the latent class K (i.e., the posterior probability of group membership) can be estimated. In the M step, this posterior probability for each individual is inserted in the complete-data log likelihood function. Then, the M step maximizes this function with respect to the mixture parameter (i.e., π), the latent Markov parameters (i.e., δ, ρs, and τs), and the logistic regression coefficients of x on latent classes (i.e., a and b). Interested readers may refer to Muthén and Shedden (1999) for further technical aspects of this EM algorithm.

In mixture latent Markov modeling estimation, missing data in observed longitudinal categorical variables (e.g., PES measures) can be modeled using full information maximum likelihood (FIML) method with the assumption that the data are missing at random (MAR; Little & Rubin, 1987). Newman (2003) showed that FIML outperformed listwise deletion, pairwise deletion, and single imputation and provided better standard error estimates for longitudinal data modeling. However, the MAR assumption is rarely testable or tenable (Newman, 2003). There has also been little agreement on how to model the missing data when they exist in predictors of the latent class variable

(Wang & Bodner, 2007). One method is to apply the same FIML solution to model the missing values in the predictors. However, it imposes normality assumptions about the predictors that are often violated in empirical research, especially when the predictors are categorical (Muthén, Jo, & Brown, 2003). Furthermore, participants belonging to different latent or observed subgroups may show differential attrition rate that cannot be predicted (e.g., some subgroups of retirees in the current example may be more likely to die before the end of the study than others, leading to more missing values in Time 4). Therefore, modeling them with the same distribution assumptions as FIML does may be incorrect.

Mixture Latent Markov Model Selection

To evaluate the absolute fit between the mixture latent Markov model and the data, a classic Pearson chi-square test can be performed. It assesses the model fit by comparing the observed frequency distributions to the model-derived frequency distributions in each cell of the longitudinal qualitative status contingency table (i.e., the *GHIJ* contingency table). In the current research scenario of modeling retirees' PES change, this contingency table has 16 cells (i.e., 2^4). This classic Pearson chi-square test follows:

$$\chi^2 = \sum\nolimits_{ghij} \left[\left(O_{ghij} - M_{ghij} \right)^2 \Big/ M_{ghij} \right], \tag{5}$$

where O_{ghij} are the observed frequencies in the cells in the longitudinal qualitative status contingency table and M_{ghij} are the model-based frequencies in those cells. A nonsignificant chi-square statistic suggests that the model-based frequency distributions are not significantly different from the observed frequency distributions in the contingency table, indicating that the model fits to the data very well. However, when missing values are modeled with likelihood-based estimation procedures (e.g., FIML) the Pearson chi-square statistic will not be available, because the observed frequencies in the cells cannot be determined.

To select the optimal mixture latent Markov model that best fits the observed longitudinal categorical data, the number of latent classes needs to be determined. In this case, using the conventional likelihood ratio test (i.e., the restricted chi-square test) comparing a *K*-1 and a *K*-class model is not appropriate, because the likelihood ratio statistic does not follow a chi-square distribution when the null hypothesis of K-1 classes is true (McLachlan & Peel, 2000). Therefore, the generally accepted approach to determine the optimal number of latent classes is to compare the information criteria among mixture latent Markov models with different number of latent classes. These information criteria may include Akaike's information criterion (AIC; Akaike, 1974), Bayesian information criterion (BIC; Schwartz, 1978), and sample size–adjusted BIC (SSABIC; Sclove, 1987). Usually, the smaller the information criteria, the better is the model fitting to the data. It should be noted that recent research in the area of latent mixture modeling has made significant progress in developing decision rules in selecting latent mixture models with optimal number of latent classes, such as the Lo-Mendell-Rubin test (Lo, Mendell, & Rubin, 2001) and the parametric bootstrapped likelihood ratio test (Nylund, Asparouhov, & Muthén, 2007). However, at this point, these tests can only be applied to latent mixture models containing only one latent class variable. Their utility with latent mixture models containing more than one latent class variables (e.g., mixture latent Markov models) is still unclear.

Same model selection can also be informed by the latent classification accuracy as measured by entropy (Jedidi, Ramaswamy, & Desarbo, 1993), which ranges from 0.00 to 1.00 with higher values indicating better classification quality. In essence, this index represents how clearly separable different latent classes are based on individuals' posterior probabilities for different classes. According to Muthén and Muthén (2000), when entropy is high, it means that the average posterior probability for each class for individuals whose highest probability is for that class is considerably higher than the average posterior probabilities for the other classes for those individuals, indicating unambiguous classification of each individual. When entropy is low, it means that the average posterior probability for each class for individuals whose highest probability is for that class is similar to the average posterior probabilities for the other classes for those individuals, indicating high levels of uncertainty in the classification of each individual. In previous research, entropy values higher than 0.80 have been viewed as suggesting good classification (e.g., Muthén, 2004; Wang, 2007).

However, it should be cautioned that most of the above criteria (especially AIC and BIC) are extremely sensitive to sample size and asymptotically these indices favor very highly parameterized (i.e., low parsimony) models (e.g., Hu & Bentler, 1998, 1999). In addition, when the sample size is large, researchers may want to use model selection criteria that are less affected by the sample size (e.g., entropy; SSABIC) than the sample-dependent information criteria (i.e., AIC and BIC) for deciding among competing models with different numbers of latent classes (Yang, 1998). As such, we recommend researchers to pay attention to all these indices as well as the substantive research context in which the mixture latent Markov modeling technique is applied when selecting the best mixture latent Markov model. Specifically, it is important to decide whether a particular model makes theoretical sense by explicating the conceptual meanings of the latent Markov chains that correspond to the subpopulations identified and evaluating the empirical evidence of convergent validity for the subpopulations by examining the covariates of these subpopulations. For example, in a research study on employee withdrawal behaviors identifying distinct subpopulations of employees with varying status change profiles in absenteeism, the subpopulations should be empirically related to relevant covariates (e.g., supervisory ratings of job performance) so that subpopulations with more "severe" absenteeism profiles should also be associated with more negative supervisory ratings. Several researchers (e.g., Dayton & Macready, 2002; Muthén, 2004) have pointed out that incorporating additional variables (e.g., covariates) into a mixture model has great potential to alter conditional probabilities of class membership. We recommend that researchers incorporate additional variables (covariates/predictors) into their models throughout the process of class enumeration to warrant accurate estimation of conditional probabilities for class membership.

In summary, the mixture latent Markov modeling relaxes the single population assumption of conventional latent Markov modeling by estimating a latent class variable for different longitudinal change patterns in categorical statuses corresponding to unobserved subpopulations. Furthermore, predictors of this latent class variable can be incorporated simultaneously in the model estimation to predict the unobserved heterogeneity in the population, which provides a good way to test theories regarding the unobserved subpopulations. Empirically, using mixture latent Markov modeling could efficiently summarize the large longitudinal qualitative change contingency table (e.g., in the current research scenario there are 16 possible transition profiles) into several major transition patterns, which provides us parsimonious description and explanation to the data (Cudeck & Henly, 2003). It also accounts for the measurement errors associated with observed measures, which may reveal that two retirees with different observed transition patterns may indeed have high probabilities to follow the same latent transition patterns. In addition, the observed patterns of qualitative transitions are rarely evenly distributed, rendering sparse observations for certain observed transition patterns (Magidson & Vermunt, 2004). It is particularly useful to apply mixture latent Markov modeling to this situation to identify whether the sparse observations represents a unique transition pattern or merely just an artificial due to lack of measurement reliability. Furthermore, when the research goal is to examine predictors of different transition patterns, using mixture latent Markov modeling is better than conducting discriminant analysis for all the observed transition patterns. This is because, first, when there are a lot of different observed transition patterns, large number of orthogonal discriminant functions will be needed to sufficiently discriminate these patterns. Subsequently, the direct relationships between the predictive variables included in the discriminant functions and the observed transition patterns will be hard to identify, because these variables are included in all discriminant functions. Second, the discriminant analysis does not perform well when only a small number of observations exist in some categories. Previous Monte Carlo research has shown that at least 40–70 observations are needed in each observed group/class to warrant achieving reliable discriminant functions (e.g., Barcikowski & Stevens, 1975). Third, discriminant analysis is not able to take into account the measurement error in the observed qualitative status. Therefore, using mixture latent Markov modeling will yield more parsimonious and direct profiling of the subgroups (e.g., retiree subgroups), which correspond to different latent transition patterns.

Numerical Demonstration

In this section, the mixture latent Markov model is applied to a real data set to numerically demonstrate the analytical framework delineated above. Specifically, we used the data from Waves 1–5 (contains data collected in 1992, 1994, 1996, 1998, and 2000) of the Health and Retirement Study (HRS; Juster & Suzman, 1995). Related to the current research question, we selected a sample of retirees who were not retired at the Wave 1 data collection, but considered themselves as retired in Wave 2 and later data collection periods, resulting in a sample size of 994. The time interval between successive waves was 2 years. Retirees' PESs from Wave 2 to Wave 5 (i.e., Time 1 to Time 4 in Figure 1B) were used as the observed longitudinal categorical variables that manifested the qualitative status changes across different time points. Retirees' years of education (measured at Wave 1 of HRS) were included as the potential predictor of the unobserved subpopulations, which may exist in corresponding to different longitudinal change patterns in qualitative employment status.

To guide the search for the latent subpopulations and the transition patterns of the latent change in retirees' PESs, we rely on theory and previous findings in the retirement research literature. The dynamic perspective of postretirement employment has suggested that heterogeneity exists in retirees' postretirement employment patterns that are influenced by retirees' individual attributes, previous experience, and socioeconomic context (e.g., Wang, Adams, Beehr, & Shultz, 2009; Wang et al., 2008). Specifically, this literature has suggested three qualitative change patterns of retirees' PES change. Accordingly, retirees may be distinguished into three unobserved subgroups. The first subgroup of retirees may never be employed after their retirement (e.g., Shultz, 2003). The second subgroup of retirees is always employed after their retirement (e.g., Kim & Feldman, 2000). Finally, the third subgroup of retirees may transition in or out employment across different time points after their retirement (e.g., Wang et al., 2008), but the general tendency for them is to transition from being employed to not employed in conforming to the social norm of retirement (Wang et al., 2009). Furthermore, previous studies have found that education is an important variable that predicts retirees' PES (e.g., Shultz, 2003). Specifically, after retirement, educated individuals have better preparation to provide further contribution to their organization or industry because of their professional knowledge and/or skills. In addition, they might continue to work in their career field by engaging in consulting roles. Therefore, we hypothesize that retirees with higher education may be more likely to be employed consistently after their retirement, whereas retirees with lower education may be more likely to stay unemployed after their retirement. As such, in this numeric demonstration, we also included retirees' years of education as a covariate to predict the latent subgroup membership.

It should be noted that the latent qualitative change patterns we hypothesized in this numeric demonstration were quite similar to the qualitative change patterns specified in "mover–stayer" models that were typically examined in developmental psychology research (e.g., Blumen, Kogan, & McCarthy, 1955; Goodman, 1961; Mooijaart, 1998; Vermunt, Tran, & Magidson, 2008). In the mover–stayer model, there exists a latent class of individuals who transition across different development stages over time (movers) and a latent class that does not transition across stages (stayers). For example, in the context of reading development, the stayers are those who never move beyond mastery of letter recognition (Kaplan, 2008). The key difference between our hypothesized model and the typical "mover–stayer" model is that instead of hypothesizing only one "stayer" class, we hypothesized two "stayer" classes: one contained retirees who were employed at all time points after retirement (i.e., "stayers" who were always employed) and the other contained retirees who were never employed at all time points after retirement (i.e., "stayers" who were never employed). This is due to the difference of our theoretical foundation in comparison to the typical developmental psychology research that uses "mover–stayer" model. In developmental psychology research, when using "mover–stayer" model, the typical assumption is that all participants start in the same qualitative status (i.e., lack of certain type of development). Thus, the latent transition estimated is unidirectional (i.e., from lack of certain type of development to achieve that type of development), which could not be reversed. However, in our research scenario, the employment status could change from "not employed" to "employed," and vice versa, thus manifesting two possible qualitative statuses at the starting point of the longitudinal observation. In this sense, our hypothesized three-class model is more general than the traditional "mover–stayer" model in terms of accommodating different transition directions as well as multiple qualitative statuses at the starting point.

Mplus 5.2 (Muthén & Muthén, 2008) was used to analyze the real data set. The Mplus program for the selected three-class mixture latent Markov model with the x variable (i.e., years of education) incorporated is included in Appendix (Table A1) to provide an idea on how mixture latent Markov modeling is executed in actual programs. Another program that is also good to estimate these kinds of models is the Latent GOLD program (Vermunt & Magidson, 2005). As we recommended earlier, we included the predictor of latent subpopulations (i.e., years of education) in all mixture latent Markov models we tested throughout the process of class enumeration to warrant accurate estimation of conditional probabilities for class membership.

Selecting Mixture Latent Markov Model With Optimal Number of Latent Classes

Before we started the mixture latent Markov modeling, we first inspected the frequencies of observed longitudinal qualitative change patterns. These frequencies are presented in Table 1. Specifically, 366 retirees (36.82%) were never employed after they retired, 121 retirees (12.17%) were always employed after they retired, and 255 retirees (25.55%) transitioned in or out employment at least once during the four time points (i.e., movers). Among all potential "mover" patterns, some had very sparse observations (0.20%–3.02%). Therefore, it will be useful to use the mixture latent Markov modeling to summarize these observed transition patterns. Furthermore, it is difficult to directly interpret these observed frequencies, because 253 retirees (25.45%) did not provide their employment status at least once during the four data collection time points. To model these missing values, we used FIML method with the MAR assumption to conduct subsequent mixture latent Markov modeling. As we mentioned earlier, the Pearson chi-square statistics are not available to evaluate the model fit when likelihood-based estimation procedures are used to model missing values. In addition, because we did not have specific hypotheses regarding the response probability change across time, we only specified one time-invariant response probability for each observed employment status. In other words, we specified that the probability for the observed "not employed" response to accurately reflect the unobserved employment status might be different from the probability for the observed "employed" response to accurately reflect the unobserved employment status.

We tested our current hypotheses by comparing a series of mixture latent Markov models to our hypothesized three-class mixture latent Markov model. Information criteria and entropy indices for these models are presented in Table 2. Specifically, we first fit one-class latent Markov model (i.e., the conventional latent Markov model) to the data. Then, a two-class "mover–stayer" mixture latent

Table 1. Observed Employment Status Change Patterns in the Current Sample

Longitudinal Employment Pattern	*N*	Percentage of the Sample
0000	366	36.82
0001	15	1.51
0010	19	1.91
0100	22	2.21
1000	46	4.63
0011	20	2.01
0101	2	0.20
0110	10	1.01
1010	3	0.30
1100	30	3.02
1001	4	0.40
0111	42	4.23
1011	11	1.11
1101	10	1.01
1110	20	2.01
1111	121	12.17
Total	741	74.55

Note: "0" = *not employed*, "1" = *employed* (e.g., "0000" denotes the longitudinal employment pattern that the retirees never worked after their retirement; "0101" denotes the longitudinal employment pattern that the retiree did not work at time 1 and time 3 data collection, but worked at time 2 and time 4 data collection). 253 retirees (25.45%) were not included in this table because they had missing employment status in at least one of the four time points.

Table 2. Fit Indices, Entropy, and Model Comparisons for Estimated Mixture Latent Markov Model

Mixture Latent Markov Models	Log Likelihood	Number of Free Parameters	AIC	BIC	SSABIC	Entropy
One class	−4,154.36	26	8,360.71	8,488.16	8,405.58	–
Two class	−1,639.92	15	3,309.84	3,389.43	3,341.78	0.77
Three class	−1,637.06	13	3,300.12	3,363.84	3,322.55	0.77
Four class	−1,633.12	18	3,302.24	3,397.75	3,340.56	0.74
Five class	−1,629.54	23	3,305.08	3,417.82	3,344.71	0.72

Note: *N* = 994. AIC = Akaike information criterion; BIC = Bayesian information criteria; SSABIC = sample size–adjusted Bayesian information criteria. Years of education were included in each of these models as a predictor.

Markov model was estimated with one latent class being specified as a "stayer" class and the other latent class being specified as a "mover" class. Specifically, for retirees who belong to the "stayer" class, their employment statuses were modeled as not changing across different time points (i.e., the transition probabilities were fixed to 0). Nevertheless, their employment statuses at the starting time point were allowed to be freely estimated. In other words, these "stayers" were either employed at all time points or never employed at all time points. For retirees who belong to the "mover" class, their employment statuses were modeled to freely transition in or out employment across different time points. This two-class model resulted in smaller information criteria compared to the one-class model (see, Table 1). In addition, the entropy (0.77) indicates acceptable classification accuracy based on this two-class model.

The hypothesized three-class mixture latent Markov model was then estimated by including two latent "stayer" classes and one "mover" class. One "stayer" class was modeled as containing retirees

Table 3. Transition Probabilities for the Three-Class Mixture Latent Markov Model

Retiree Subpopulations (Rows) by Time 1 Employment Status (Columns)			Proportion of Total Sample ($N = 994$)
	Not employed at Time 1	Employed at Time 1	
Stayers—never employed	100.0%	0.0%	43.1%
Stayers—always employed	0.0%	100.0%	14.8%
Movers	63.7%	36.3%	42.2%
Transition probabilities for movers			
	Not employed at Time 2	Employed at Time 2	
Not employed at Time 1	85.6%	14.4%	
Employed at Time 1	22.0%	78.0%	
	Not employed at Time 3	Employed at Time 3	
Not employed at Time 2	92.9%	7.1%	
Employed at Time 2	19.8%	80.2%	
	Not employed at Time 4	Employed at Time 4	
Not employed at Time 3	97.6%	2.4%	
Employed at Time 3	17.4%	82.6%	

Note: Transition probabilities for the two "stayer" classes are not presented because they are all zeros.

who were employed at all time points after retirement (i.e., stayers who were always employed). This was done by specifying retirees' latent employment statuses at Time 1 to be "employed" and fixing the transition probabilities to be zero. The other "stayer" class was modeled as containing retirees who were never employed at all time points after retirement (i.e., stayers who were never employed). This was done by specifying retirees' latent employment statuses at Time 1 to be "not employed" and fixing the transition probabilities to be 0. The "mover" class was specified as the same in the previous two-class model. This three-class mixture latent Markov model resulted in smaller information criteria compared to the one-class model and the two-class "mover–stayer" model (see, Table 2), indicating good model fit improvement over the two-class model. In addition, the entropy (0.77) of this three-class model did not differ from the entropy in the two-class model, indicating similar level of classification accuracy was achieved by this three-class model.

Table 3 presents the probability estimates of the hypothesized three-class mixture latent Markov model. Specifically, three retiree subpopulations were identified according to longitudinal change patterns of retirees' PES: 43.1% of retirees ($N = 428$) in the sample were classified into the latent class for "stayers" who were never employed after their retirement; 14.8% of retirees ($N = 147$) were classified into the latent class for "stayers" who were always employed after their retirement; and 42.2% of retirees ($N = 419$) were classified into the latent class for "movers" who changed their previous employment status at some point during the span of the study. Furthermore, by examining the transition probabilities for retirees who were classified into the "mover" class (see, Table 2), it seems that over time, retirees who were not employed at previous time point were more likely to stay not employed. For example, the probabilities for staying in the "not employed" status were quite high (i.e., 85.6%, 92.9%, and 97.6%) over the three successive transition periods (i.e., Time 1–Time 2, Time 2–Time 3, and Time 3–Time 4). However, there were also high probabilities for retirees who were employed at previous time point to transition out their employment. Specifically, the probabilities for retirees to transition out their employment were 78.0%, 80.2%, and 82.6% over the three successive transition periods. These findings support our hypothesis that the general transition tendency for retirees in the "mover" class is from being employed to not employed.

After fitting the three-class mixture latent Markov model, a four-class model was estimated by adding one latent class to the three-class model. Specifically, the added class was specified as another "mover" class, which takes the opposite transition tendency (i.e., transiting from not

Table 4. Logistic Coefficient Estimates for the Predictor on the Latent Class Variable

	Latent Classes			
	Stayers—Never Employed		Stayers—Always Employed	
	Estimates	*SE*	Estimates	*SE*
Years of education	−0.12*	0.05	0.10*	0.05

Note: $N = 994$. The reference class in this multinomial estimation is the "mover" class.
* $p < .05$.

employed to employed in postretirement) as we hypothesized for the "mover" class. This additional "mover" class was modeled as containing retirees who were not employed at the beginning of their retirement but transition to employment in the later time points. By estimating this four-class model and comparing it to the hypothesized three-class model, it helps us to determine whether the "mover" class in the three-class model successfully summarized the general transition tendency among the "movers." This four-class model resulted in larger information criteria compared to the three-class mixture latent Markov model and the entropy decreased a little to 0.74 (see, Table 2). In addition, by examining the classification results of retirees based on their most likely latent class patterns, very few retirees ($N = 7$; i.e., 0.7% of the sample) were classified to this newly added "mover" class. In other words, it seems that there was not a systematic transition pattern from "not employed" to "employed" status over time. Therefore, the hypothesized three-class model was preferred over this four-class model.

Previous authors (e.g., Magidson & Vermunt, 2004; Nyland et al., 2007) have pointed out the possibility that keeps adding additional latent classes may later reduce information criteria values further. Therefore, to make sure that the three-class model we hypothesized was the best-fitting model, we also estimated a five-class model by adding one more "mover" class, which allowed retirees to freely transition in or out employment across different time points to the previous four-class model. This five-class model yielded larger information criteria than the three-class model did and its entropy further decreased to 0.72. Inspecting classification results of retirees based on their most likely latent class patterns, no retirees were classified to this newly added "mover" class. Therefore, we conclude that this five-class model did not improve our explanation of the current data and it is reasonable to retain the three-class model as optimal mixture latent Markov model.

Predictive Effect of Retirees' Years of Education

Table 4 presents estimated logistic coefficients and corresponding standard errors of the years of education in predicting the latent class variable in the selected three-class mixture latent Markov model. Years of education were significantly and negatively related to the log odds of being in the "stayers who were never employed" class versus being in the "mover" class ($b_1 = -0.12, p < .05$). This suggests that when a retiree had longer years of education, there is a lower probability for that retiree to be classified into the "stayers who were never employed" class than to be classified into the "mover" class (the corresponding odds ratio was $e^{-0.12} = 0.88$). In addition, years of education were significantly and positively related to the log odds of being in the "stayers who were always employed" class versus being in the "mover" class ($b_2 = 0.10, p < .05$). This suggests that when a retiree had longer years of education, there is a higher probability for that retiree to be classified into the "stayers who were always employed" class than to be classified into the "mover" class (the corresponding odds ratio was $e^{0.10} = 1.10$). Correspondingly, among the three subpopulations, retirees who were classified into the "stayers who were always employed" class had the longest years of education (M = 13.92, SD = 2.81); retirees who were classified into the "mover" class had the second longest years of education (M = 12.47, SD = 3.15); whereas retirees who were classified into the "stayers who were never employed" had the shortest years of education (M = 11.14, SD = 2.72). Taken as a whole, these findings support a priori hypothesis, indicating that retirees' levels of education were related to the longitudinal patterns of their postretirement employment.

Response Probabilities of the Observed Employment Statuses

As we noted earlier, in the current mixture latent Markov modeling, we only specified one time-invariant response probability for each observed employment status, because we did not have specific hypotheses regarding the response probability change across time. In the fitted three-class mixture latent Markov model, the response probability of the observed "employed" status was 88.03% ($z = 4.46$, $p < .01$), and the response probability of the observed "not employed" status was 83.34% ($z = 2.48$, $p < .05$). These estimates suggest that both observed "employed" and "not employed" responses had good reliabilities in reflecting their corresponding latent employment statuses. Nevertheless, it should be noted that more flexible structures on the response probabilities (e.g., linearly increasing or decreasing reliability over time) could be tested in latent mixture Markov models, as far as there is good theoretical basis to hypothesize those reliability structures.

Discussion

The purpose of this article was to introduce the mixture latent Markov modeling technique to organizational research and illustrate its substantive applications. The contribution of this technique is best understood in terms of how it helps answer important questions in longitudinal analysis of qualitative changes in latent status, which cannot be answered by standard latent growth modeling or other conventional techniques that presuppose longitudinal quantitative changes in a continuous variable.

As demonstrated in the numerical example from the HRS data set, mixture latent Markov modeling answers the question regarding longitudinal qualitative status change patterns by identifying distinct coexisting latent Markov chains in the sample. In the HRS data set example, there were three latent Markov chains identified, which corresponded to a subgroup of retirees (43.1% of the sample), who were never employed again after their retirement; a subgroup of retirees (14.8% of the sample), who were always employed after their retirement; and a subgroup of retirees (42.2% of the sample), who moved in and out employment after their retirement. This demonstration illustrates how the mixture latent Markov modeling technique may be used to identify unobserved subpopulations with different longitudinal qualitative status changes. The HRS data example also shows how mixture latent Markov modeling tests the predictors of subpopulation membership. Specifically, the individual's years of education was incorporated in the model estimation and was found to be a significant predictor of the odds of being a member in the subpopulation with a "stayer" pattern versus being a member in the subpopulation with a "mover" pattern. Correspondingly, on average, retirees who were classified into the "stayers who were always employed" class had the longest years of education, whereas retirees who were classified into the "stayers who were never employed" had the shortest years of education.

It should be noted that although we used a research scenario of modeling retirees' PES changes to illustrate how to conduct mixture latent Markov modeling, the technique can be easily applied to other substantive areas in the organizational research and the unit of observation in the status change may be at different levels of analysis such as the individual, group, or organization. Some examples include modeling status profiles (i.e., longitudinal changes in discrete status) with respect to employment within versus outside a specific industry throughout the individual's career trajectory, promotion versus no promotion throughout the employee's tenure in an organization, pass versus fail in a regular skills test throughout a practice period, award versus no award (or in vs. out of a select listing) throughout a period of time for a sample of individuals, groups, or organizations.

Limitations and Recommendations of Using Mixture Latent Markov Modeling

Despite its advantages, the mixture latent Markov modeling technique has several limitations and therefore cautions should be exercised in its application to substantive research. First, one statistical issue in its model estimation that warrants caution is that multiple local maxima may exist for the complete-data log likelihood function (McLachlan & Peel, 2000). In other words, with different starting values, different maximizing results may be obtained for the complete-data log likelihood function. Therefore, it is recommended that different sets of starting values are used to carry out all potential local maxima for a given data set and a given model. In Mplus 5.2, this can be accomplished by estimating the model with a large number of sets of random-generated starting values and then selecting the model with the starting values that have the highest log likelihood values as the best estimation solution.

Second, various caveats need to be highlighted in model selection. Although the modeling technique involves empirically deriving the unobserved subpopulations from the data, model selection needs to be theory driven as opposed to completely data driven in an atheoretical manner. Model selection that is solely based on empirical results (i.e., in a post hoc manner) may lead to selecting statistically fitting models that are in fact fitting the sample well due to capitalization on chance (Cudeck & Henly, 2003; MacCallum, Roznowski, & Necowitz, 1992; Muthén, 2003; Rindskopf, 2003). Therefore, although several methods and indices (e.g., Pearson chi-square, information criteria, and entropy) are available to compare mixture latent Markov models with different numbers of classes or different parameter values, the best way to guide the optimal model selection is to test different models following theory-based hypotheses (i.e., a priori hypothesis). In other words, researchers should form theory-based expectation regarding the number of latent classes and more importantly the specific transition pattern for each latent class. Following this approach, researchers could better understand the mixture latent Markov models generated in the model selection process and identify more meaningful solutions. This point has been made very clear in previous research that discussed model selection and class enumeration issues in growth mixture modeling (e.g., Bauer & Curran, 2003a, 2003b; Wang & Bodner, 2007). Both Bauer and Curran (2003b) and Muthén (2004) argued that the selection of models in mixture modeling needs to have adequate theoretical grounding or conceptual bases and it is not simply a matter of statistical significance of the increment model fit in nested model comparisons. They also argued that with theoretical guidance, even when several models have similar statistical results, researchers would still be able to retain the model that is most meaningful and logical. Therefore, we are biased in favor of using mixture latent Markov model in a confirmatory manner rather than an exploratory manner, such that theories of coexisting latent Markov chains can be evaluated by systematically testing different theory-derived hypotheses about specific mixture latent Markov models representing competing ways of describing and explaining the longitudinal data. Our numeric demonstration provides a useful example that incorporates theoretical-driven hypotheses in comparing and selecting optimal models.

Furthermore, in addition to the lack of a gold standard in determining the number of latent subpopulations, another issue in model selection in mixture latent Markov modeling is that there is no indication of the stability of the identified latent subpopulations, including the number of subpopulations, the relative proportions, and the strength and pattern of prediction by covariates. As such, models with different numbers of subpopulations need to be examined to establish the sensitivity of the latent classification (e.g., the robustness of the identified latent subpopulations). In addition, inspection of the estimation accuracy by examining standard error estimates of the parameter estimates may be helpful as well. It is also important to replicate the findings obtained in the latent mixture Markov model in a hold-out sample for purpose of cross-validation, especially when the research goal is to make inferences beyond the sample at hand (Wang & Bodner, 2007).

Finally, similar to structural equation modeling, mixture latent Markov modeling is a statistical procedure that is based on large samples and should not be employed with small samples. This is because having sparse cells (i.e., few observations) in the longitudinal contingency table would yield biased estimation of the model parameter. In general, as the number of latent classes increases, larger sample sizes will be needed. In addition, it is also important to consider the relative size of the latent groups. As research in latent class analysis (i.e., Bayesian cluster analysis; McLachlan & Peel, 2000) has shown, if a hypothesized latent subgroup has a small relative frequency in the population of interest, the overall sample size must be large enough for the sample to include a sufficient number of these subgroup members and for the statistical power to detect their presence. At present, given the recency of the mixture latent Markov modeling technique, there are no established general guidelines for sample sizes in application of the technique. Clearly, future research on the effect of sample sizes on the accuracy of parameter estimation in mixture latent Markov modeling is needed.

Extensions of Mixture Latent Markov Modeling and Future Statistical Advances

The basic technique of mixture latent Markov modeling may be extended to address other important substantive research questions concerning qualitative changes over time. One extension is to incorporate into the model outcome variables that are predicted by the latent class variable representing the unobserved subpopulations. Given that different latent qualitative change patterns are succinctly summarized by the latent class variable, the latent class variable may be used to predict the outcomes of the qualitative change process. The outcomes may be static criterion variables that occur at a time point that is at the end or after the end of the longitudinal period of status change. Alternatively, the outcomes may be time-varying criterion variables that are also undergoing changes during the longitudinal period of study.

Mixture latent Markov modeling may also be extended to fit data simultaneously to multiple observed (a priori known) groups. For example, to examine whether the unobserved latent subpopulations are the same across gender groups, a multiple-group analysis can be conducted to estimate mixture latent Markov models simultaneously for both male and female participants. The logic of the procedures for this multiple-group mixture latent Markov modeling analysis is similar to the conventional multiple-group analysis of longitudinal latent variable analysis in which specific parameters may be fixed or freely estimated across the observed groups to produce different multiple-group longitudinal latent models for nested model comparisons to determine the "best" model (e.g., see, Chan, 1998).

Finally, we end with several suggestions for future methodological research directions on the mixture latent Markov modeling technique. First, more research is needed to adapt the technique to go beyond dichotomous status variables to include polytomous status variables, so that more than two possible discrete statuses at any single time point may be incorporated in the analysis. In this respect, it is important to distinguish multiple statuses that are nominal categories versus those that are ordered categories. When polytomous status variables are modeled, a challenging but interesting task for researchers concerns how the technique can be adapted to specify and test different models representing change trajectories or developmental processes that exhibits distinctive patterns of status changes over time such as an equifinity pattern (i.e., trajectories that started from different statuses but ended at the same status) and a multifinality pattern (i.e., trajectories that started from the same status but ended at different statuses). Another methodological research direction concerns how we can incorporate in the mixture latent Markov model a time-specific intervention during the longitudinal period of study so that we can examine the effect of the intervention on longitudinal qualitative status change and how this intervention effect may differ across the different unobserved subpopulations. Finally, a future methodological research direction is adapting the technique to apply to multilevel contexts in which the data sets have nested hierarchical structures where observations are nested under higher order units of analysis. Multilevel latent variable modeling would provide the statistical basis necessary for such adaptations.

In summary, mixture latent Markov modeling provides a promising approach for modeling longitudinal qualitative status changes across unobserved subpopulations. We end with the same cautionary note in Chan (1998) concerning powerful and flexible approaches to model longitudinal changes—the search for these unobserved subpopulations and the different functional forms of qualitative changes in status should be guided by adequate theories and relevant previous empirical findings.

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Appendix

Table A1. Mplus 5.2 Program Specifications for the Mixture Latent Markov Model With *x* Variable

Syntax	Comments
DATA: FILE IS Sample.DAT;	Specify the data file; Sample.dat contains the raw data
VARIABLE: NAMES ARE W1-W4 x;	Specify the variable names
CLASSES = C(3) C1(2) C2(2) C3(2) C4(2);	Specify the number of latent classes to be estimated; C corresponds to the latent class variable for three unobserved retiree subgroups (C#1-C#3); C1-C4 correspond to the retiree's employment status at four time points, each having two latent classes: not employed (#1) vs. employed (#2)
MISSING ARE ALL (999);	Specify that all "999" values in the data set represent missing values
ANALYSIS: TYPE = MIXTURE MISSING;	Specify the analysis type as mixture modeling with FIML missing value modeling
MODEL: %OVERALL%	Specify the overall model so that three latent longitudinal qualitative change patterns are modeled and predicted by x
C1#1 on C#1 @20; C2#1 on C#1 @20; C3#1 on C#1 @-20; C4#1 on C#1 @20;	Specify that in the first latent longitudinal qualitative change pattern (i.e., C#1), retirees were never employed (i.e., C1#1-C4#1 regress on C#1 with logistic regression coefficients of 20, representing 100% of nonemployment)
C1#1 on C#2 @-20; C2#1 on C#2 @-20; C3#1 on C#2 @-20; C4#1 on C#2 @-20;	Specify that in the second latent longitudinal qualitative change pattern (i.e., C#2), retirees were always employed (i.e., C1#1-C4#1 regress on C#2 with logistic regression coefficients of −20, representing 0% of nonemployment)
C#1 C#2 on x;	Regress the membership of the latent retiree subgroups on x
MODEL C:	Model for transition coefficients in each latent retiree subgroups
%C#1%	Model retirees who were never employed after retirement

(continued)

Table A1 (continued)

Syntax	Comments
C4#1 on C3#1 @20; C3#1 on C2#1 @20; C2#1 on C1#1 @20;	Specify same status across four time points
%C#2%	Model retirees who were always employed after retirement
C4#1 on C3#1 @20; C3#1 on C2#1 @20; C2#1 on C1#1 @20;	Specify same status across four time points
%C#3%	Model retirees who transition in and out in employment after retirement)
C4#1 on C3#1; C3#1 on C2#1; C2#1 on C1#1;	Freely estimate the transition probabilities across time in employment status
MODEL C.C1:	Model for employment status at Time 1 (i.e., C1) by latent retiree subgroups
%C#1.C1#1% [W1$1] (1); %C#1.C1#2% [W1$1] (2);	Latent employment status at Time 1 by three latent retiree subgroups were estimated freely based on the Time 1 observed employment status
%C#2.C1#1% [W1$1] (1); %C#2.C1#2% [W1$1] (2);	When estimating the latent class "not employed," the reliability coefficient for observed employment status, that is, parameter (1), is set to be the same across all latent retiree subgroups and across all time points throughout the syntax;
%C#3.C1#1% [W1$1] (1); %C#3.C1#2% [W1$1] (2);	When estimating the latent class "employed," the reliability coefficient for observed employment status, that is, parameter (2), is also set to be the same across all latent retiree subgroups and across all time points through the syntax
MODEL C.C2:	Model for employment status at Time 2 (i.e., C2) by latent retiree subgroups
%C#1.C2#1% [W2$1] (1); %C#1.C2#2% [W2$1] (2); %C#2.C2#1% [W2$1] (1); %C#2.C2#2% [W2$1] (2); %C#3.C2#1% [W2$1] (1); %C#3.C2#2% [W2$1] (2);	Latent employment status at Time 2 by three latent retiree subgroups were estimated freely based on the Time 2 observed employment status;
MODEL C.C3:	Model for employment status at Time 3 (i.e., C3) by latent retiree subgroups
%C#1.C3#1% [W3$1] (1); %C#1.C3#2% [W3$1] (2); %C#2.C3#1% [W3$1] (1); %C#2.C3#2% [W3$1] (2); %C#3.C3#1% [W3$1] (1); %C#3.C3#2% [W3$1] (2);	Latent employment status at Time 3 by three latent retiree subgroups were estimated freely based on the Time 3 observed employment status;

(continued)

Table A1 (continued)

Syntax	Comments
MODEL C.C4:	Model for employment status at Time 4 (i.e., C4) by latent retiree subgroups
%C#1.C4#1% [W4$1] (1); %C#1.C4#2% [W4$1] (2); %C#2.C4#1% [W4$1] (1); %C#2.C4#2% [W4$1] (2); %C#3.C4#1% [W4$1] (1); %C#3.C4#2% [W4$1] (2);	Latent employment status at Time 4 by three latent retiree subgroups were estimated freely based on the Time 4 observed employment status;
OUTPUT: TECH11;	Generate output

Note: The @ symbol is used to fix the values of parameters. The # symbol is used to label a latent class.

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