

Low-Cost Precise Positioning Using a National GNSS Network

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BIOGRAPHY

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ABSTRACT

This article presents an implementation of Real-Time Kinematics (RTK) using a low-cost GNSS receiver and a sparse countrywide reference station network. Furthermore, we assess the feasibility of implementing RTK on a smartphone by comparing the raw GNSS measurements of a commercial smartphone's internal GNSS receiver with a commercial off-the-shelf (COTS) low-cost GNSS receiver. The RTK implementation presented in this paper utilizes the Finnish national GNSS network FinnRef as the RTK base station, either in single-base or network RTK mode; although the main purpose of FinnRef is to maintain the national coordinate system, it is also capable of delivering DGNSS and network RTK data over the NTRIP protocol.

The test results show that despite the sparseness of FinnRef, a horizontal position accuracy of 0.5 meters or better was achieved for more than 90 % of the time with the COTS receiver both in a dynamic single-base test and in a network RTK experiment using GPS signals only. Obtaining such a positioning performance with low-cost and small-size devices is expected to be useful in various applications, particularly in the field of intelligent transportation systems. Furthermore, the results indicate that the smartphone's GNSS measurements are less precise than those of the COTS receiver and suffer from frequent outliers, making them less favorable for use in precise positioning applications as such.

INTRODUCTION

Current consumer-grade GNSS solutions routinely offer a positioning accuracy in the order of 5 meters, and satellite-based augmentation systems (SBAS) such as WAAS and EGNOS can be used to improve the accuracy to the order of 1 m. However, this is not adequate for all use cases; in particular, intelligent transportation systems (ITS) require a better positioning performance. For instance, a horizontal accuracy of 0.5 meters would be needed in order to reliably identify the lane on which a vehicle is driving. As another example, maintaining inventory of, e.g., machines and road signs would benefit from sub-meter accuracy. In addition to the sub-optimal accuracy, the visibility of geostationary augmentation satellites cannot be guaranteed at high latitudes.

Sub-meter or even sub-decimeter positioning accuracies can be attained with a relatively good reliability in real time if a dual-frequency GNSS receiver and a physical or virtual base station are available. However, such receivers and virtual base station services are currently too expensive to gain popularity in the mass market. In recent years it has been demonstrated that comparable accuracies can be attained without a base station using real-time precise correction data [1], but the typical drawback of this approach, usually referred to as precise point positioning (PPP), is a long convergence time. In contrast, differential methods utilizing raw base station observations, such as real-time kinematics (RTK), converge much faster.

This paper presents new results from the P3-Service (Public Precise Positioning) project [2] whose goal is to achieve a horizontal positioning accuracy of 0.5 meters using low-cost equipment. The project hinges on the utilization of the Finnish national GNSS network FinnRef which has been recently modernized [3]. With inter-station baselines in the order of 200 km, the FinnRef network is relatively sparse in comparison with commercial RTK networks. This makes the modeling of atmospheric errors challenging, especially from a single-frequency user's perspective. However, the purpose of FinnRef is to maintain the national coordinate system and to offer a half-meter positioning accuracy, not to compete with commercial RTK networks.

Implementing precise positioning on low-cost GNSS receivers has been an active research topic for years, and recent research projects related to P3-Service exists. Guo et al. [4] described a precise positioning service developed for China. The service is based on proprietary correction data formats and can achieve a lane-level accuracy for ITS applications. Lovas et al. [5] investigated the use of SBAS signals and RTK on ITS. Pesyna et al. [6] investigated the performance of the GNSS antenna of a smartphone by connecting the antenna to an external radio frontend and pro-

cessing the signals using a software GNSS receiver, concluding that integer ambiguity resolution was possible even though the signals were received by the smartphone antenna.

The contribution of this paper is twofold. First, we show that the goal of achieving a horizontal RTK accuracy of 0.5 meters is feasible with a low-cost GNSS receiver using standard RTCM data formats [7] even with a sparse GNSS network and without ambiguity resolution at the receiver. Then, we present raw GNSS pseudorange and carrier phase measurement data from the actual internal GNSS receiver of a commercial smartphone (Nokia Lumia 1520) and assess its suitability for precise positioning applications. The paper is organized as follows. First, the basics of RTK positioning are reviewed. Then, the FinnRef network is presented in more detail, after which experimental results are presented.

REAL-TIME KINEMATIC POSITIONING

RTK is a method of *relative* positioning, i.e., it can only resolve the user's location with respect to a known reference point. The three-dimensional vector from the reference (base station) to the user, often referred to as the *rover*, is called the *baseline* and constitutes the unknowns to be estimated, along with the carrier phase ambiguities. The key to the high precision in RTK is to mitigate measurement errors by forming measurement differences where the errors are significantly decreased or canceled out totally. In the following sections, we first describe the double difference model which RTK hinges on, and then outline the RTK processing workflow. Finally, we briefly describe the concept of network RTK.

Differential Measurements

Let us model the carrier phase measurement ϕ to satellite i in units of meters as

$$\phi_i = \|\mathbf{p} - \mathbf{p}_i - \delta\mathbf{p}_i\| + c(\delta t - \delta t_i) - I_i + T_i + \lambda N_i + \varepsilon_i \quad (1)$$

where \mathbf{p} and \mathbf{p}_i are the positions of the user and satellite i , respectively, and $\delta\mathbf{p}_i$ denotes the error in the broadcast satellite position; δt and δt_i are the clock biases of the receiver and the satellite, respectively; c is the speed of light; I_i and T_i are the ionospheric and tropospheric delays for satellite i , respectively; λ denotes the signal wavelength and N_i is the carrier phase cycle ambiguity; and finally, ε_i contains all unmodeled error sources, such as measurement noise, multipath, antenna phase center variations, etc. Note that the cycle ambiguity N_i contains fractional phase biases caused both in the satellite and in the receiver and, therefore, is *not* an integer by nature.

We assume that the noise term ε_i can be modeled as an uncorrelated zero-mean random variable having a Gaussian distribution. The pseudorange measurement can be modeled in a way similar to (1), with the differences being the sign of the ionospheric delay I_i (the carrier wave is subject to a phase advance, i.e., a negative delay, in the dispersive ionosphere while the ranging code modulation is subject to a group delay of equal magnitude), the absence of the ambiguity term N_i , and the variance of the noise ε_i .

Now suppose that a base station exists close to the user and that the base station is equipped with a GNSS receiver that makes measurements perfectly synchronized with the user's receiver. Then, we can form the receiver–receiver carrier phase single difference $\Delta\phi_i$ as

$$\begin{aligned}\Delta\phi_i &= \phi_i^{\text{user}} - \phi_i^{\text{base}} \\ &= \|\mathbf{b} + \mathbf{r} - \mathbf{p}_i\| - \|\mathbf{r} - \mathbf{p}_i\| + \Delta\delta t + \lambda\Delta N_i + \Delta\varepsilon_i\end{aligned}\quad (2)$$

where the new terms \mathbf{b} and \mathbf{r} denote the baseline and the position of the base station, respectively. Satellite clock errors have been canceled out and atmospheric errors have been mitigated; the amount of residual atmospheric errors depends on the length of the baseline. Also note that the difference essentially cancels out satellite ephemeris errors. The receiver clock bias and the cycle ambiguity were re-defined as the differences of those of the two receivers; note that single-differenced cycle ambiguity ΔN_i is still not an integer because of receiver-dependent fractional phase biases. Assuming that the measurement noise variance is equal for both receivers, the variance of the differenced noise term $\Delta\varepsilon_i$ is twice as large as the non-differenced measurement variance.

In order to cancel out the receiver clock biases and the rest of the fractional phase biases, compute the difference of the single differences to satellites i and j , widely known as the *double difference*:

$$\begin{aligned}\Delta\phi_{i-j} &= \Delta\phi_i - \Delta\phi_j \\ &= \|\mathbf{b} + \mathbf{r} - \mathbf{p}_i\| - \|\mathbf{r} - \mathbf{p}_i\| - \|\mathbf{b} + \mathbf{r} - \mathbf{p}_j\| + \|\mathbf{r} - \mathbf{p}_j\| \\ &\quad + \lambda\Delta N_{i-j} + \Delta\varepsilon_{i-j}.\end{aligned}\quad (3)$$

The only remaining unknowns are the baseline and the cycle ambiguity which is now free of fractional biases; note that this *integer ambiguity* is constant in time as long as both receivers maintain a phase lock on the signal. The variance of the noise term is again doubled, therefore, the standard deviation of the double difference noise is twice the standard deviation of the noise in the original, non-differenced observable (1).

RTK Workflow

The process of baseline estimation in RTK consists of three main steps:

1. Float solution: use double-differenced pseudoranges and carrier phases to estimate the baseline \mathbf{b} and the ambiguities ΔN_{i-j} without any integer constraints
2. Ambiguity resolution
3. Fixed solution: if the ambiguity resolution succeeded, substitute the integer-valued estimates to the carrier phase double differences to obtain a precise baseline estimate.

Typically, the float solution is filtered using an Extended Kalman Filter (EKF). This way, a covariance estimate is obtained for the float ambiguity estimates. During the float estimation process, cycle slip detection and other quality control methods are applied. Moreover, if the rover receiver's measurements are not synchronized with the base station, the measurements are interpolated to a common time tag to satisfy the underlying assumption of (2).

The ambiguity resolution step involves solving an integer programming problem. Nowadays, the most popular ambiguity resolution method is LAMBDA [8] which implements integer least squares, i.e., finds the integer vector that is closest to the float ambiguity estimate in terms of the Euclidian norm weighted by the inverse of the covariance of the float ambiguities. Acceptance or rejection of the ambiguities is based on some kind of a test. Often, the ratio of the residuals of the two closest integer vectors is compared to a fixed threshold; however, more sophisticated statistical tests exist, e.g., [9]. If accepted, the fixed baseline solution can be computed by conditioning the float estimate on the integers [8].

Network RTK

Since the double-differencing approach assumes that the user is located so close to the base station that residual atmospheric (and other) errors are negligible, the admissible baseline lengths are very short; for instance, it is often stated that single-frequency systems are limited to baselines of a few kilometers [10] because they cannot directly estimate the ionospheric errors; dual-frequency systems can successfully resolve the ambiguities with somewhat longer baselines.

Network RTK [11] is a technique to avoid the baseline length limitation by utilizing multiple base stations. Having such a network makes it possible to model the atmo-

spheric errors over the entire area of the network and use the model to compensate for the double-differencing residuals at the user's location. Currently, three network RTK approaches have been standardized: Flächenkorrekturparameter (FKP), Master–Auxiliary Concept (MAC), and non-physical reference station [7]. In FKP and MAC, the network provides the user with error models that are applied to the base station's measurements by the user; in contrast, in the non-physical reference station approach, the network sends the user artificial GNSS measurements that would be made by a base station in his or her vicinity.

In terms of computational load, FKP and MAC impose the burden of computing the corrections on the user's side whereas the creation of a non-physical reference station is entirely carried out on the network server side. In this article, we choose to use the non-physical reference station approach because from an RTK processing flow point of view, the algorithms are no different from those of single-base RTK.

THE FINNISH NATIONAL GNSS NETWORK FINNREF

The FinnRef GNSS network was modernized between 2012 and 2014 for the maintenance of the national coordinate system; the network is shown in Fig. 1. A typical station was set up on solid bedrock on the top of a three-meter-high steel grid mast with a narrowed top [3]. Any surrounding trees were felled to approximately ten degrees from the antenna level to ensure good visibility to the satellites. The station data have shown very low near-field effects, which confirms a successful station configuration.

The positioning services offered by FinnRef are based on the GNSMART software developed by Geo++ GmbH, Germany. GNSS data from 20 reference stations are transferred in real time to the processing center where the errors affecting the positioning at the reference stations are modeled. Then, real-time observation and correction data are available for users via the NTRIP protocol in various RTCM formats. Freely available DGNSS pseudorange corrections may be either received from a single station or tailored for the user's location through error modeling of the network, which enables a 0.5 m static positioning accuracy with adequate GIS receivers.

More precise network RTK corrections intended for carrier phase measurements, i.e., Pseudo-Reference Station (PRS), MAC, and FKP, are currently available for test users only. Our initial tests have shown that network RTK with better than 5 cm (95 %) accuracy and high fixing rate is possible even with this sparse reference network (see Fig. 1) when high-end dual-frequency GNSS receivers are used.

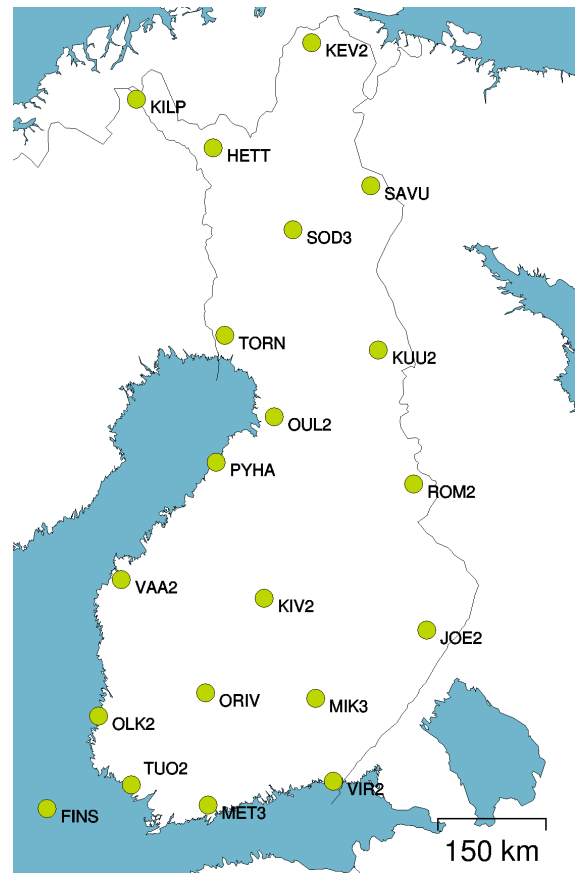


Figure 1. The FinnRef permanent GNSS network

FinnRef also supports state-space representation (SSR) error models for PPP. Although SSR is still under development as an RTCM standard, it is available because we are running a tailored version of GNSMART offered for the P3-Service project by Geo++ GmbH.

TEST RESULTS

In this section, we present field test results for single-frequency RTK using FinnRef as the base station. First, a single physical FinnRef station is used as the base; then, another experiment is carried out with the base station information retrieved in the form of network RTK. Finally, we analyze the raw GNSS measurements, i.e., pseudorange and carrier phase, obtained from the positioning module of a real commercial smartphone to assess the feasibility of RTK on a cell phone.

Test Equipment

The positioning performance was evaluated with a post-processing test setup where a consumer-grade U-Blox EVK-6T GNSS receiver and a geodetic-grade dual-frequency No-

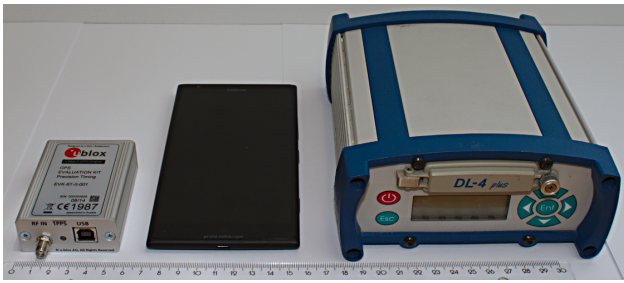


Figure 2. GNSS receivers used in the tests: U-Blox EVK-6T (left), Nokia Lumia 1520 smartphone (middle), and NovAtel DL-4 plus (right)

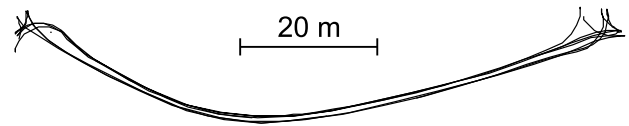
vAtel DL-4 plus GPS receiver were connected to the same antenna through a signal splitter; the purpose of the NovAtel receiver was to provide a reference position solution. FinnRef data were logged on site over NTRIP using a cellular network connection. Note that the accuracy of the reference position obtained this way depends on the quality of the RTK network data, but its error can be assumed to be negligible in comparison with the low-cost single-frequency solution.

The smartphone measurements were made by a Nokia Lumia 1520 running a custom firmware, courtesy of Microsoft Mobile, that allows access to the raw GNSS measurements from the phone's internal GNSS receiver (Qualcomm integrated receiver). The three receivers used in the tests are shown in Fig. 2. Satellites below 10° elevation were ignored in the RTK computations in each test, and ambiguity validation was based on the residual ratio test with threshold value 3. Furthermore, only GPS signals were considered because the U-Blox receiver has no GLONASS support.

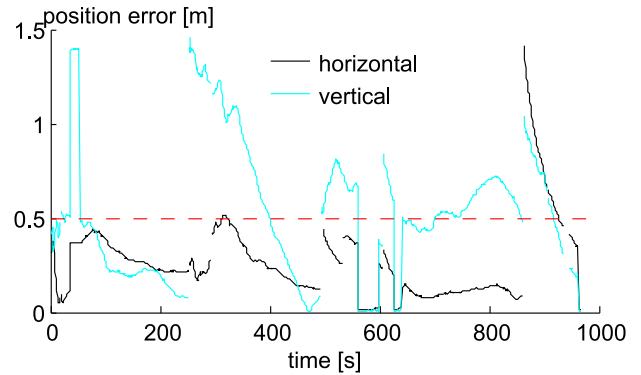
Single-Base RTK Test

In the single-base test, the antenna was mounted on the roof of a vehicle driving back and forth on a short road, see Fig. 3a. The test site was located 2.7 kilometers from a physical FinnRef station which was used as the RTK base station. The resulting position errors are shown in Fig. 3b. The curves are partly discontinuous because of two reasons. First, the RTCM stream reception seemed to suffer from occasional outages and, therefore, the base station observations were missing for certain epochs (see Fig. 3c). Second, there were a handful of epochs where the reference dual-frequency solution was float-level only; these epochs were ignored in the analysis.

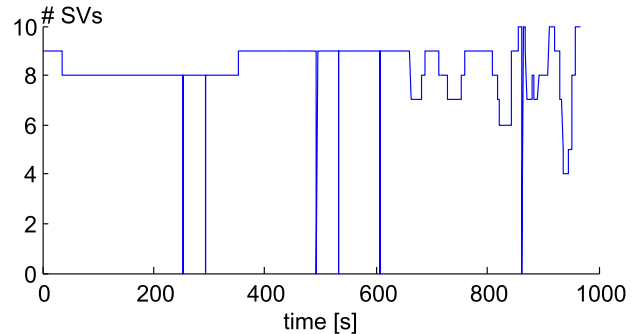
In 92.6 % of the epochs where an ambiguity-fixed reference solution was available, the horizontal positioning accuracy was better than 0.5 meters, and it seems evident



(a) Reference trajectory from dual-frequency receiver



(b) Resulting position estimation errors using the U-Blox receiver



(c) Number of satellites used in the U-Blox RTK solution

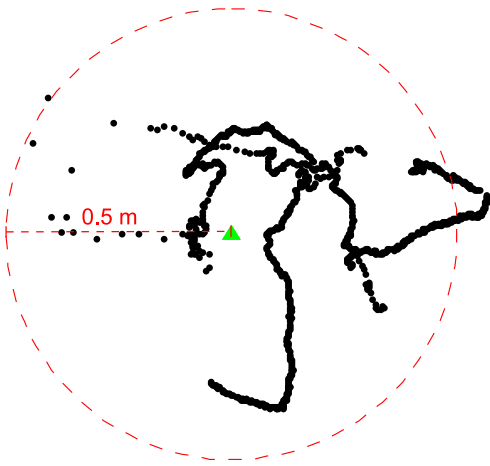
Figure 3. Dynamic single-base RTK field test results

that significant a part of the estimates with error exceeding the goal were caused by error accumulation during a data gap. Moreover, in 7.3 % of all epochs, the ambiguities were accepted by the residual ratio test. The same figure also shows the absolute vertical position errors even though they are not of primary interest in the P3-Service project; it can be seen that the vertical has a larger variance but stays below one meter for most of the time.

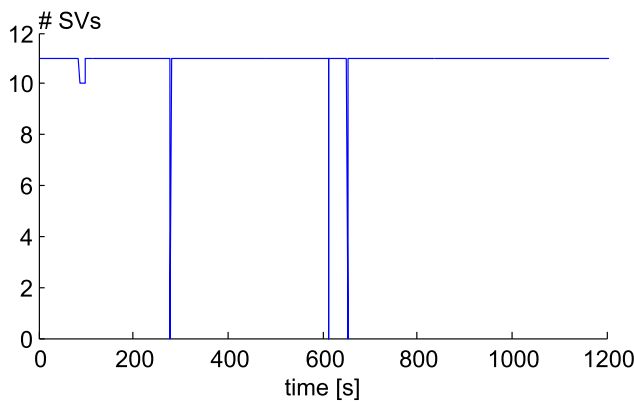
Network RTK Test

The network RTK performance was tested in a static scenario with the closest physical base station being located approximately 63 kilometers from the rover receiver. The network corrections were delivered in the PRS representation, and data were logged for 20 minutes at a rate of 1 Hz.

The resulting horizontal position errors, as referred to the



(a) Horizontal position estimation results; the green triangle marks the reference position solution



(b) Number of satellites used in the RTK solution

Figure 4. Static network RTK field test results using the U-Blox receiver

average of the dual-frequency solution, are shown in Fig. 4. The dashed red circle with radius 0.5 meters centered at the reference location (green triangle) contains 90.4 % of the position estimates. Ambiguity fixing did not succeed in this test, which is not surprising given the long distance to the nearest physical base stations: When using non-physical reference stations, the users are expected to perform (residual) ionospheric error estimation—the network cannot be assumed to construct perfect atmospheric models. For instance, the PRS concept utilized by FinnRef attempts to enforce receiver-level residual error estimation by creating the non-physical reference station nominally 5 kilometers away from the receiver [12]. However, it is well known that only multi-frequency receivers can observe the ionospheric errors directly; therefore, single-frequency users should not expect to obtain the same performance with non-physical reference station as when using a physical base station even if the virtual baseline is short.

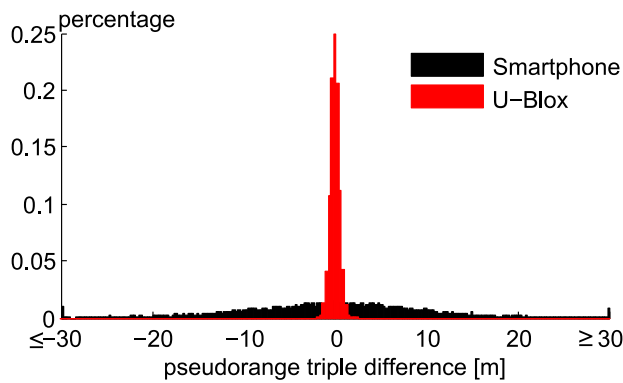
Smartphone Test

As smartphones in general do not have connectors for external GNSS antennas, a rigorous zero-baseline test cannot be conducted to measure the noise levels. Thus, to assess the measurement quality of the phone's internal GNSS receiver, the test setup was as follows. Again, a physical FinnRef station was used as the base station for double differencing (baseline approx. 930 m), and a Lumia phone and a U-Blox receiver's patch antenna were stationary next to each other (distance approximately 30 cm) for 20 minutes. The exact baseline between the antenna base centers was not known.

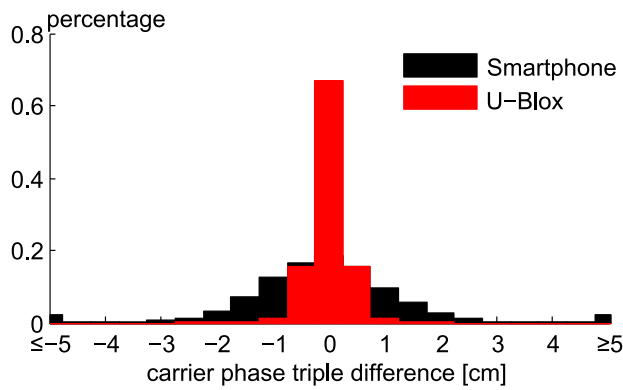
The measurement noise levels were estimated as follows. First, double-differenced measurements were computed for both the smartphone and the U-Blox receiver with respect to the FinnRef station; the base station's measurement noise can be expected to be negligible in comparison with the low-cost receivers. Then in order to cancel biases such as integer ambiguities, triple differences were computed by differencing consecutive double differences over time. Finally, a linear polynomial was fitted to each measurement channel and subtracted from the triple differences in order to obtain zero-mean residuals. Note that the noise variance of the differenced measurements is larger than that of the original measurements; this amplification was not compensated for, and thus, the residuals do not directly represent the measurement noise.

The distributions of the resulting triple difference residuals for all channels are shown in Fig. 5. Note that the histograms are truncated, i.e., the minimum and maximum bins have no lower and upper bound, respectively. It can be seen that for both observables, the smartphone measurements are much less precise. In particular, the smartphone pseudorange noise is in the order of tens of meters, but the pseudorange residuals cannot be regarded as directly comparable because it is likely that the receivers are applying different types of filtering on the pseudoranges. Notwithstanding, the smartphone's histograms are heavy-tailed as opposed to the U-Blox: both the smartphone's pseudorange and carrier phase distributions have distinctive minimum and maximum bins, indicating a substantial amount of outliers (or cycle slips in the case of carrier phase) in the data.

Fig. 6a demonstrates how the quality of the measurements reflected in Fig. 5—note that the noise level analysis excludes many factors such as the antenna phase center variations—translates to the position domain. The measurement noise variances of the EKF were set to reflect the precision of each receiver, but otherwise, the processing parameters were identical. It can be seen that the variations in the smartphone's float estimates are much larger than the



(a) Pseudorange



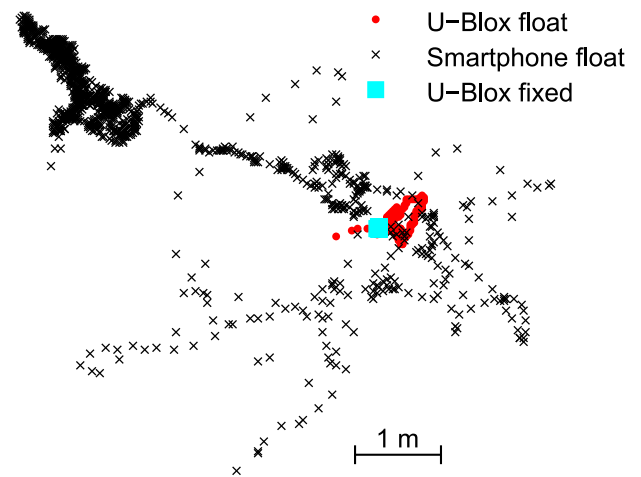
(b) Carrier phase

Figure 5. Noise level comparison of U-Blox and the smartphone. Constant biases and linear slopes were eliminated from the triple differences before computing the histograms, and the bin counts were normalized by the total sample count

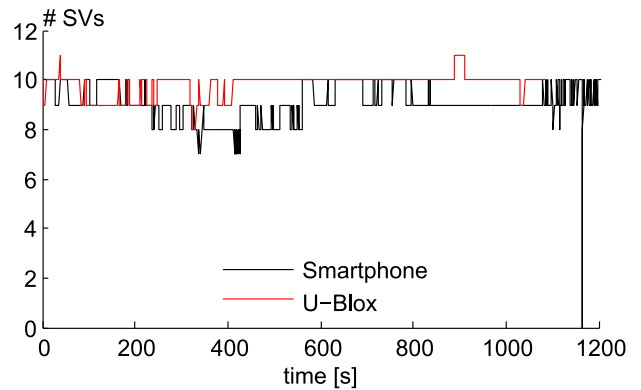
distance between the smartphone and the U-Blox' antenna. In contrast, the U-Blox solution is less scattered and good enough for ambiguity fixing; also note that it made use of more satellites as shown in Fig. 6b.

CONCLUSIONS AND FUTURE WORK

This paper presented results from the P3-Service project where the goal is to achieve a 0.5 m horizontal position accuracy in Finland using a low-cost GNSS receiver and the national GNSS network FinnRef. It was seen that the goal could be met for 90 % of the time using a U-Blox GNSS receiver both in a short-baseline kinematic test and in a static network RTK test 63 km away from the nearest physical base station. It is noteworthy that integer ambiguity fixing was not necessary to achieve this performance; however, successful ambiguity resolution is expected to improve the accuracy by an order of magnitude. Furthermore, the accuracy was achieved despite the sparsity of FinnRef.



(a) Horizontal position estimation results



(b) Number of satellites used in the RTK solution

Figure 6. Comparison of RTK results with smartphone and U-Blox

Moreover, we compared the raw GNSS pseudorange and carrier phase measurements of the internal GNSS receiver of a commercial smartphone (Nokia Lumia 1520) with those of the U-Blox receiver. It was seen that the smartphone measurements are noisy and suffer from a significant amount of outliers in comparison with the U-Blox. The difference in measurement quality becomes evident by computing the RTK solutions: the U-Blox was precise enough to allow ambiguity fixing while the smartphone-based float RTK solution suffered from meter-level errors.

The results presented in this paper were based on GPS only. An obvious way to improving the performance would be to increase the amount of satellites used by incorporating GLONASS, BeiDou, and Galileo measurements into the RTK processing. In particular, a large amount of observations would help to exclude the frequent outliers and cycle slips in the smartphone data, which might be a door opener for high-precision mobile positioning; it has been shown that the quality of a smartphone's GNSS antenna alone does not prevent ambiguity fixing [6]. A horizontal accuracy of

0.5 meters for small and low-cost devices would be useful in several applications, e.g., inventory management and various ITS use cases such as detecting the lane or parking slot occupied by the vehicle.

An important objective for P3-Service is to use the RTK solution to initialize a PPP filter. The concept has earlier been shown to be feasible [13]; however, in P3-Service, the purpose is to use SSR ionosphere corrections from the network instead of combining the pseudorange and carrier phase measurement to cancel the ionospheric error. SSR correction data [7] for PPP need not be updated as frequently as base station measurements for RTK, therefore, using PPP instead of RTK would decrease the necessary communications bandwidth and, on the other hand, make the system more robust against temporary network connection outages.

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