7-1979

Efficient Employment of Cohorts of Labor in the U.S. Economy: An Illustration of a Method

A. E. BOARDMAN
University of British Columbia

Steven M. MILLER
Singapore Management University, stevenmiller@smu.edu.sg

A. P. SCHINNAR
University of Pennsylvania

DOI: https://doi.org/10.1016/0038-0121(79)90010-7

Published in Socio-Economic Planning Sciences
Volume 13, Issue 6, 1979, Pages 297-302.
https://doi.org/10.1016/0038-0121(79)90010-7

EFFICIENT EMPLOYMENT OF COHORTS OF LABOR IN THE U.S. ECONOMY: AN ILLUSTRATION OF A METHOD

A. E. BOARDMAN
Faculty of Commerce and Business Adminsitration, University of British Columbia, Vancouver, B.C., Canada V6T 1W5

S. MILLER
Department of Engineering and Public Policy, Carnegie–Mellon University, Pittsburgh, PA 15213, USA

and

A. P. SCHINNAR
School of Public and Urban Policy, University of Pennsylvania, Philadelphia, PA 19104, USA

Abstract—This paper presents a model for the analysis of efficient labor force participation in the U.S. Economy. Ridge regression estimates of the elasticities of cohorts of labor, classified by sex and occupation, are used in conjunction with wage data to illustrate the derivation of efficient allocation of labor cohorts in five economic sectors. These efficient constructs are compared with actual census data for 1960 and 1970. The results, while tentative, show a trend toward more efficient utilization of labor and greater participation by women in the work force.

1. INTRODUCTION

Past studies of labor economics typically aggregate labor into a few broadly defined categories. In production function economics this classification usually applies to skilled and unskilled labor[1–4]. Recently, many sectors of the economy have come under increasing pressure to hire higher proportions of minority groups and women, particularly in more senior positions, and to avoid discrimination on the basis of ethnic and racial background, sex and age. One source of resistance to such change stems from the belief that any change organizations are forced to make would result in less efficient labor utilization and costly adjustments.

This paper has two purposes: to offer a more disaggregated analysis of labor factor inputs than previous research and to derive efficient factor shares of each cohort of labor within each sector of the economy. We analyze the structure of the 1960 U.S. labor force in terms of its occupational and sex composition within five economic sectors. Our approach is economy-wide, analogous to studies on economic-demographic accounting interactions[5–7] and draws upon economic concepts of production, profit maximization and efficiency. The assumption of efficient production by industrial sectors and the associated first order optimality conditions are used to construct a matrix of the efficient allocation of cohorts in each sector. Given this matrix and the existing levels of employment in each sector, we compute the efficient occupational and sex composition of the total labor force in the economy. The efficient employment matrix and the associated labor force mix vectors are then compared with the actual 1960 employment patterns. This analysis, while tentative, shows that females were underemployed in most sectors and that most sectors employed a too specialized labor force, i.e. the labor force in each sector should be less concentrated and more heterogeneous.

The approach taken in this paper differs markedly from previous research in this area. First our method, which draws on work by Cooper and Schinnar[8], is an economy-wide approach and necessarily examines all sectors of the economy. Second we prefer to use an extended Cobb–Douglas production function[9] rather than a CES[4], translog[3], CRESH or HCDE[10] production function. Third, we are more interested in the efficient employment matrix of cohorts in all sectors and possible substitutions among different types of labor than substitutions between each type of labor and capital.

Section 2 describes the assumptions and theoretical considerations leading to the model we estimate. Section 3 describes our data, which are based on 1960 and 1963 census information, and presents illustrative empirical results. Since we expect that the allocation of labor types will tend to move away from the actual allocation of labor in 1960 and towards our efficient employment matrix, we also examine the actual allocation matrix based on 1970 data to see if the anticipated shifts actually occurred.

2. NOTATION AND MODEL FORMULATION

Suppose that the population of workers can be partitioned into m groups and that each employee works in one of n sectors. In a general model the cohort groups of labor may be defined on the basis of age, sex, race and ethnicity, marital status, skill level and education. For example, one group might refer to young, male, black, single professionals with a college education. Sectors may be broadly defined in terms of the standard industrial classification, such as, agriculture, mining, construction, etc. or they may be more narrowly defined in terms of particular types of industries or firms.

Let $\mathbf{L} = \{L_{ij}\}$ be an $m \times n$ matrix where L_{ij} denotes the number of employees of demographic group i who work in sector j. Let $\mathbf{P} = L\mathbf{1}_n$ denote an $m \times 1$ vector whose elements are formed by summing across the columns of $\mathbf{L}$, and $\mathbf{1}_n$ denotes an $n \times 1$ vector whose elements equal unity. $\mathbf{P} = \{P_i\}$ is the labor force mix vector which describes the demographic structure of the economically active population; P_i denotes the total number of employees who belong to the ith cohort group. Let $\mathbf{l} = \mathbf{1}_m'\mathbf{L}$ denote an $n \times 1$ vector whose elements are formed by summing down the rows of $\mathbf{L}$. $\mathbf{1} = \{l_j\}$ is the labor component of the primary factor inputs vector; l_j denotes the number of employees in sector j.

Let $\hat{\mathbf{l}}$ denote a diagonal matrix constructed from the components of $\mathbf{l}$. Now, we define an $m \times n$ matrix

$$\mathbf{E} = \mathbf{L}\hat{\mathbf{l}}^{-1}. \quad (1)$$

$\mathbf{E} = \{E_{ij}\}$ is described as the employment matrix; E_{ij} equals the number of employees of cohort group i in sector j divided by the total number of employees in sector j, that is, E_{ij} denotes the proportion of the labor force in sector j who belong to the ith cohort group. Clearly, $E_{ij} \geq 0$ and $\sum_{i=1}^{m} E_{ij} = 1$ for all j. With these definitions we can write

$$\mathbf{P} = \mathbf{E}\mathbf{l}. \quad (2)$$

Thus, $\mathbf{E}$ provides a mapping from the labor component of the primary factor inputs vector to the labor force mix vector. Fox (1973) uses similar constructs to study manpower training programs.

As formulated above $\mathbf{P}$, $\mathbf{E}$ and $\mathbf{l}$ represent actual data. One of our purposes is to compute the optimal composition of the labor force mix vector $\mathbf{P}^*$, which requires derivation of an efficient employment matrix $\mathbf{E}^*$. For a given primary input vector $\mathbf{I}$ we write

$$\mathbf{P}^* = \mathbf{E}^*\mathbf{l}. \quad (3)$$

Later in this paper we compare $\mathbf{E}^*$ with $\mathbf{E}$ and $\mathbf{P}^*$ with $\mathbf{P}$. In the remainder of the present section we focuss on the computation of the entries of the efficient employment matrix $\mathbf{E}^*$.

Assume that each sector maximizes profit subject to a budget constraint, and let w_{ij} dentoe the wage of the ith cohort group in sector j. We also assume that these wages are determined exogenously by, for example, labor unions, competitive markets or socio-political processes. The budget constraint for each sector can be written:

$$\sum_{i=1}^{m} w_{ij}L_{ij} + w_{0j}K_j \leq S_j \quad (4)$$

where, w_{0j} dentoes the price of capital, K_j denotes the amount of capital used in the jth sector, and S_j equals the budget for all factor inputs.

For each sector we postulate a homogeneous production function expressible in extended Cobb–Douglas form:

$$Q(x_j) = C_jA(x_j)K_j^{\beta_0(x_j)} \prod_{i=1}^{m} L_{ij}^{\beta_i(x_j)} \quad j = 1, \ldots, n \quad (5)$$

where, $x = (K_j, L_{1j}, L_{2j}, \ldots, L_{mj})$, $Q(x_j/j)$ = the output of sector j measured in dollar values, $\beta_0(x_j)$ = the relative size of output elasticity of capital in sector j, $\beta_i(x_j)$ = the relative size of output elasticity of labor cohort type i in sector j, $\sum_{i=0}^{m} \beta_i(x_j) = 1$, $A(x_j)$ = a mean-aggregate productivity measure, and C_j = unit price of output.†

We assume in the present paper that $A_j = A(x_j)$ and $\beta_{ij} = \beta_i(x_j)$ are constant parameters. In order to determine the efficient employment patterns, i.e. the optimal values of $L_{1j}^*, L_{2j}^*, \ldots, L_{mj}^*$, we assume that each sector attempts to maximize the output value (5) subject to the budget constraint (4). The resulting first order optimality conditions give the following relations

$$\beta_{0j}w_{ij}L_{ij}^* - \beta_{ij}w_{0j}K_j^* = 0 \quad (6a)$$

$$\beta_{kj}w_{ij}L_{ij}^* - \beta_{ij}w_{kj}L_{kj}^* = 0 \quad i, k = 1, 2, \ldots, m. \quad (6b)$$

Alternatively, one could obtain the same first order conditions by minimizing costs subject to a stipulated output level.

From eqns (6b) we have $m-1$ linearly independent equations, and since $\sum_{i=1}^{m} L_{ij}^* = l_j^*$ we can set up the following nonsingular system of m simultaneous equations:

$$\begin{bmatrix} \beta_{2j}w_{1j} & -\beta_{1j}w_{2j} & & & \\ \beta_{3j}w_{1j} & & -\beta_{1j}w_{3j} & & \\ \vdots & & & \ddots & \\ \beta_{mj}w_{1j} & & & & -\beta_{1j}w_m \\ 1 & 1 & 1 & \cdots & 1 \end{bmatrix} \begin{bmatrix} L_{1j}^* \\ L_{2j}^* \\ \vdots \\ L_{m-1,j}^* \\ L_{mj}^* \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ l_j^* \end{bmatrix} \quad (7)$$

The solution gives $L_{ij}^* = \beta_{ij}\omega_i l_j^* (\sum_{k=1}^{m} \beta_{kj}\omega_k)^{-1}$. Because $E_{ij}^* = L_{ij}^*/l_j^*$ we can obtain a simple expression for each element of $\mathbf{E}^*$:

$$E_{ij}^* = \frac{\beta_{ij}\omega_{ij}}{\sum_{k=1}^{m} \beta_{kj}\omega_{kj}} \quad i = 1, 2, \ldots, m; \quad j = 1, 2, \ldots, n \quad (8)$$

where, $\omega_{ij} = \prod_{k=1}^{m} w_{kj}, k \neq i$.

Thus, given the wages of each cohort group and the elasticities of each cohort group in each sector, we can easily compute the efficient employment matrix. This matrix contains the efficient employment patterns for each sector based on the optimality conditions, that is, E_{ij}^* represents the optimal amount of cohort type i in industry j necessary to attain optimal levels of economic activity (profit) in industry j. Note that the computation of E_{ij}^* does not involve l_j^*. Given any values for the labor component of the primary input vector, $\mathbf{l}$, we can compute the efficient labor force vector $\mathbf{P}^*$ via eqn (3). In this paper we use the actual 1960 $\mathbf{l}$ vector to compute$\mathbf{P}^*$.

$\mathbf{P}^*$ can be interpreted as the optimal size of the cohort groups necessary to maximize profit. Bear in mind, however, that estimation of $\mathbf{E}^*$ does not account for social, policical, economic and technological constraints. $\mathbf{E}^*$ inherently allows for perfect (unit) elasticity of substitution among the participation rates of different labor cohorts. Further properties of the $\mathbf{E}^*$ matrix are discussed in Cooper and Schinnar[8].

†Assuming $hQ(x_j) > 0$, where h is the degree of homogeneity of $Q(x_j)$ and $h\beta_i(x_j)$ is the output elasticity of the ith input factor. When $A(x_j)$ and $\beta_i(x_j)$ are assumed to be constant parameters β_{ij} becomes the output elasticity and $h_j = \sum_{i=0}^{m} \beta_{ij}$ [4].

3. RESULTS OF AN ILLUSTRATIVE ANALYSIS

An empirical analysis is conducted with reference to a five-sector economy: agriculture, mining, construction, manufacturing and other; the latter sector represents primarily the service sector, and also transportation and utilities. Ideally, it would be desirable to classify labor by occupation and such personal characteristics as sex, age, race, ethnicity, experience (skill and seniority) and education. Because of data limitations and estimation problems, such a disaggregated breakdown is infeasible. Thus, similar to Weiss[10], we limit the analysis to ten demographic groups which are classified by sex and occupation alone.

Calculation of the efficient employment matrix requires wage data for each demographic group and estimates of the elasticities of each group in each sector. Median annual wages for each group were taken from the U.S. Summary of the 1960 Census of Population. Estimation of the elasticities requires data on labor and capital inputs and output. Data on the number of males and females in each occupational group in each sector were obtained for each state from the State Reports of the 1960 Census of Population. Economic data for capital inputs and output for each state were obtained from the 1963 Census of Manufacturing, the 1963 Census of Mineral Industries and Polenske *et al.* [11].

A number of caveats pertain to these data. Problems arise due to the level of aggregation. Each of our five sectors contains many diverse activities; for example, fishery, forestry, livestock, vegetable, fruit and grain products are all contained within agriculture. Within each sector, all types of output are aggregated into a single dollar measure using producers prices. Also, within each sector and wherever possible, all types of capital are aggregated into a single value. In general, its construction varies from one sector to another. For example, in agriculture we use annual expenditures on farm machinery and equipment; data on new construction, physical plant additions, maintenance and repair expenditures were unavailable. Within each sector, labor is also highly aggregated; e.g. professionals, technicians, managers, officials and proprietors are grouped into one category. Furthermore, the construction of this group differs across sectors; e.g. in manufacturing, male professionals are predominantly engineers, designers, technicians, accountants and others, while in the mining sector male professionals are predominantly engineers and scientists. Wages are taken from national data and are assumed to be constant for each cohort group across all sectors.

For these reasons and due to the inherent unreliability of census data, we emphasize that out empirical results should be treated with caution. They are intended primarily to supply an illustration of a method of analysis and not to test hypotheses or to effect forecasts of the sectoral structure of the U.S. labor force.

The initial stage of the analysis requires estimation of the elasticities of each labor group in the five sectors, that is, we wish to estimate the parameters of eqn (5). For estimation purposes we make the simplifying assumption that the production function is of a simple Cobb–Douglas from, rather than a more general, extended function. By assumption, the elasticities are non-negative and sum to unity. First, we take logarithms of eqn (5) and estimate the elasticities using ordinary least squares. Because of the high degree of multicollinearity among the factor inputs, some of the OLS estimates are clearly incorrect.† For this reason, we use ridge regression estimation[12]. The ridge parameter was increased until all the estimated elasticities were non-negative; the resultant ridge parameters varied among the sectors because of different degrees of multicollinearity. It is highest for agriculture, next highest for mining and is relatively low for construction, manufacturing and the other sector.‡

The OLS and ridge parameter estimates appear in Table 1. These estimates are based on 50 observations, one for each state. Labor is classified according to professionals and technicians, managers and officials and proprietors (PROF), clerical and sales (CLERKS), craftsmen, foreman and operatives (CRAFTS), service (SERVICE) and laborers or not-reported (LABOR) and the prefix M and F represents males and females, respectively. Because eqn (8) is homogeneous of degree zero, we need concern ourselves only with the relative sizes of the estimated elasticities. Therefore, the ridge parameter estimates are normalized so that $\sum_{i=0}^{m} \hat{\beta}_{ij} = 1 (j = 1, 2, \ldots, n)$ and the resultant parameter estimates for the labor inputs appear in Table 2.

In general, males appear to be slightly more productive at the margin than females for each occupation type, particularly in manufacturing and mining. Male managers

†Some elasticities are negative and many positive elasticities are too big.

‡An alternative approach involves a constrained least-squares estimation procedure; see Liew[13] and Liew and Shim[14].

Table 1. Ordinary least squares and ridge regression estimates of the elasticities

	Agric.		Mining		Constr.		Manuf.		Other	
Ridge parameter	0	2.7	0	1.9	0	0.35	0	0.15	0	0.20
Capital	0.2424	0.0777	1.0334	0.3251	0.2551	0.1588	−0.1635	0.1231	−0.0561	0.0048
Mprof	−0.0739	0.1674	−0.2545	0.1528	0.5183	0.1462	−0.2213	0.1023	0.1183	0.0978
Mclerks	−1.0847	0.0629	0.1651	0.1128	−0.2966	0.0663	1.3958	0.1344	−0.6127	0.0757
Mcrafts	0.3956	0.1982	0.2940	0.1710	0.0914	0.0690	0.6477	0.1360	−0.0467	0.0627
Mservice	−0.7235	0.0000	−0.0070	0.1207	−0.0331	0.0974	−0.0705	0.0776	0.3118	0.1504
Mlabor	2.3988	0.2060	−0.0879	0.0560	−0.0302	0.0320	−0.1639	0.1338	0.4520	0.1245
Fprof	0.8144	0.1525	−0.2949	0.0000	0.2686	0.1662	−0.3824	0.0213	0.1678	0.0857
Fclerks	−1.0960	0.0424	−0.0484	0.0809	0.3708	0.1199	−0.4449	0.0793	1.0180	0.1287
Fcrafts	1.0552	0.1443	−0.0041	0.0999	−0.2529	0.0050	−0.1820	0.0263	−0.0325	0.0870
Fservice	0.6728	0.2074	0.2188	0.0840	0.1364	0.0796	−0.0012	0.0675	−0.1109	0.1218
Flabor	−0.9027	0.1383	0.0598	0.0455	−0.0264	0.0301	0.2150	0.0551	−0.0256	0.1310

Table 2. Normalized elasticity coefficients and wages.

	Agric.	Mining	Constr.	Manuf.	Other	Wages($)
Mprof	0.1198	0.1224	0.1506	0.1069	0.1009	5760
Mclerks	0.0450	0.0903	0.0683	0.1405	0.0780	4885
Mcrafts	0.1419	0.1369	0.0711	0.1422	0.0646	4765
Mservice	0.0000	0.0967	0.1004	0.0811	0.1550	3261
Mlabor	0.1474	0.0448	0.0330	0.1398	0.1283	2996
Fprof	0.1092	0.0000	0.1713	0.0223	0.0883	3499
Fclerks	0.0303	0.0648	0.1235	0.0828	0.1327	2700
Fcrafts	0.1033	0.0800	0.0052	0.0275	0.0898	2363
Fservice	0.1485	0.0673	0.0820	0.0706	0.0225	1128
Flabor	0.0990	0.0364	0.0310	0.0567	0.1350	1956

and professionals and male craftsmen and operatives have the highest productivities, in general, but in construction and manufacturing, male craftsmen and operatives have quite low productivities. The groups with the lowest productivities appear to be female craftsmen and foremen and female laborers, except that the latter group has a high productivity in the service area. However, no single group clearly dominates over any of the other groups within any one sector.

The last column of Table 2 contains the median annual wages for each of these groups in 1960. We use these wages and the estimated elasticities to construct the efficient employment matrix from eqn (8). Notice that while the wages and the estimates of the elasticities may contain some error, errors in scale will cancel out.† The resulting efficient employment matrix, which appears in Table 3, can be compared with the 1960 actual employment matrix, which appears in Table 4. The last columns in Tables 3 and 4 contain the efficient and actual labor force mix vectors, respectively, which were calculated using eqns (3) and (2).

A comparison of **P** with **P*** shows that the efficient system contains more females at all occupation levels than under the actual 1960 system. Furthermore, in the efficient system there are more females than males at each occupation level. The most noticeable increase comes about at the low end of the socioeconomic scale: the number of female laborers increases nearly ten-fold and the number of female service workers almost doubles. The shift to a predominantly female work force can be attributed primarily to the difference in the wages

†While the median wages in Table 2 appear low, only the relative wages are required for our analysis.

Table 3. The efficient employment matrix E^* and the efficient labor force mix vector P^*

			E^*			P^*
	Agric.	Mining	Constr.	Manuf.	Other	($\times 10^6$)
Mprof	0.0551	0.0853	0.0926	0.0642	0.0517	3.755
Mclerks	0.0244	0.0742	0.0495	0.0995	0.0471	3.893
Mcrafts	0.0789	0.1153	0.0529	0.1033	0.0400	3.962
Mservice	0.0000	0.1190	0.1091	0.0861	0.1403	7.376
Mlabor	0.1304	0.0500	0.0390	0.1615	0.1264	8.426
Eprof	0.0827	0.0000	0.1734	0.0221	0.0745	4.261
Fclerks	0.0297	0.0963	0.1620	0.1061	0.1451	8.227
Fcrafts	0.1158	0.1359	0.0078	0.0403	0.1122	5.625
Fservice	0.3488	0.2394	0.2575	0.2166	0.0589	8.706
Flabor	0.1341	0.0747	0.0561	0.1003	0.2037	10.408

Table 4. The 1960 actual employment matrix E, the actual labor force mix vector P, and the actual primary factor input l

			E			P
	Agric.	Mining	Constr.	Manuf.	Other	($\times 10^6$)
Mprof	0.5661	0.1301	0.1404	0.1163	0.1664	11.497
Mclerks	0.0030	0.0403	0.0181	0.0829	0.1157	5.993
Mcrafts	0.0267	0.7616	0.6210	0.4660	0.1563	17.130
Mservice	0.0017	0.0087	0.0040	0.0134	0.0626	2.660
Mlabor	0.3064	0.0105	0.1762	0.0700	0.0770	6.187
Fprof	0.028	0.0046	0.0034	0.0104	0.0870	3.651
Fclerks	0.0062	0.0396	0.0310	0.0742	0.1692	7.953
Fcrafts	0.0038	0.0026	0.0033	0.1565	0.0192	3.508
Fservice	0.0011	0.0013	0.0011	0.0028	0.1162	4.511
Flabor	0.0567	0.0006	0.0015	0.0075	0.0304	1.549
l	4349884	654006	3815942	17513086	38306338	

between males and females within the same occupations. Since our model assumes that firms maximize profit subject to a fixed budget constraint, the lower wages of females encourages their increased employment, unless their productivities are very low. While we have seen that in some sectors males are more productive than females, these differences are insufficient to overcome the wage differences.

A comparison of E and E* shows that the proportion of female laborers increases in every sector and the proportion of female service workers increases in every sector except the other sector, in which they have the lowest productivity of any group. In fact the proportion of female workers increases in every occupation in every sector except the other sector and excluding managers and professionals in mining and craftsmen and foreman in manufacturing. For both exceptions the productivities of these groups are very low.

As the share of females increases, the share of males necessarily decreases. However, this decrease does not occur uniformly. Under 1960 actual employment patterns most male workers are either in the managerial and professional category or the craftsmen and foreman category. These two groups experience the most dramatic reductions. The number of male clerical workers also declines, but to a lesser extent, and the number of male service workers and laborers actually increases. Thus, if we consider skill levels without regard to sex, we see that by far the biggest increases occur in the low socioeconomic positions. Suppose now that we classify managers, professionals, clerical and sales workers as white-collar workers, and classify craftsmen, foremen and laborers as blue-collar workers. In this case, as we shift from the 1960 employment patterns toward a more efficient employment pattern, the number of white-collar workers decreases, the number of blue-collar workers remains the same and the number of service workers increases.

Another tentative conclusion is that the distribution of the labor force under the efficient allocation is much less concentrated than it is under the actual allocation, except for the other sector. Under the present allocation 86% of the agricultural work force is composed of male managers or professionals and male laborers, 89% of the mining work force is composed of male managers or professionals and male craftsmen or foremen, 94% of the construction work force is composed of male managers or professionals, male craftsmen or foremen and male laborers and 74% of the manufacturing work force is composed of male managers or professionals, male craftsmen or foremen and female craftsmen or foremen. In each case over 70% of the work force in each sector comes from only two or three of the ten cohort groups. In order to obtain the same percentages under the efficient allocation it is necessary to sum over at least six groups in all sectors.

Since we suppose that firms behave efficiently, we expect that the employment patterns would move away from the actual 1960 employment matrix, E, towards the efficient employment matrix, E^*. As a partial test of this hypothesis we examine the actual employment matrix based on 1970 data, which appear in Table 5. A comparison of Tables 3, 4 and 5 shows that during the decade between 1960 and 1970, 64% of the entries in Table 4 changed in the direction suggested by the efficient employment matrix, Table 3. An element by element comparison is presented in Table 6 where (+) denotes a change between 1960 and 1970 in the direction suggested by E^* and (−) denotes the converse. The employment patterns in agriculture, mining and construction clearly move towards efficiency, while the manufacturing and other sector has mixed results.

Table 7 provides summary statistics for comparing Tables 3–5. The statistic, called the Khichin–Kullback–Leibler measure ([15], chap. 3), is calculated thus:

$$I = \sum_{i=1}^{m} \delta_i \ln \frac{\delta_i}{f_i}. \tag{9}$$

In the first two rows of Table 7, it measures the mean information for discriminating in favor of the hypothesis given by the distribution of the columns of E^*(δ_i in eqn 9) against the hypothesis given for the actual employment pattern for 1960 and 1970 (f_i in eqn 9). The results

Table 6. Direction of shifts between actual employment in 1960 and actual employment 1970, in accordance with the efficient matrix

	Agric.	Mining	Constr.	Manuf.	Other
Mprof	+	−	+	−	−
Mclerks	+	+	+	−	+
Mcrafts	+	+	−	+	+
Mservice	−	+	+	+	+
Mlabor	−	+	+	−	−
Fprof	+	−	+	+	−
Fclerks	+	+	+	+	−
Fcrafts	+	+	+	−	+
Fservice	+	+	+	+	−
Flabor	+	−	+	−	−

Table 5. The 1970 actual employment matrix

	E					*P*
	Agric.	Mining	Constr.	Manuf.	Other	($\times 10^6$)
Mprof	0.5086	0.1583	0.1341	0.1364	0.1767	13.464
Mclerks	0.0074	0.0446	0.0292	0.0701	0.1104	6.946
Mcrafts	0.0315	0.6622	0.6394	0.4475	0.1457	19.398
Mservice	0.0039	0.0142	0.0085	0.0195	0.0704	3.873
Mlabor	0.3362	0.0410	0.1304	0.0410	0.0319	3.942
Fprof	0.0286	0.0087	0.0051	0.0147	0.1084	5.677
Fclerks	0.0165	0.0588	0.0418	0.0814	0.2128	12.247
Fcrafts	0.0046	0.0100	0.0083	0.1809	0.0210	4.668
Fservice	0.0024	0.0018	0.0013	0.0033	0.1196	5.900
Flabor	0.0603	0.0005	0.0020	0.0052	0.0032	0.438
l	2842488	630788	4572235	19837208	48670880	

Table 7: Mean-information for discrimination among columns of Tables 3–5

Comparison of:	Agric.	Mining	Sectors: Constr.	Manuf.	Other
Tables 3 and 4 (E^* and E_{1960})	2.552	2.413	2.747	1.322	0.530
Tables 3 and 5 (E^* and E_{1970})	2.178	1.993	2.471	1.444	1.019
Table 4 and 5 (E_{1970} and E_{1960})	0.015	0.049	0.017	0.015	0.046

show that between 1960 and 1970 agriculture, mining and construction moved toward more efficient utilization of labor cohorts, while manufacturing and the other sector moved toward less efficient utilization of labor. The third row compares E for 1970 (δ_i) with E for 1960 (f_i) and shows that mining and the other sector have undergone the largest degree of realignment.

Again, we stress that Tables 6 and 7 merely illustrate the usefulness of our method. They do not provide conclusive support for using our empirical results for policy purposes.

4. CONCLUSION

In this paper we have provided an empirical illustration of a general approach for studying the efficiency of labor utilization in a multiple sector economy. Labor was disaggregated by sex and occupation into ten cohort groups. The empirical results, though preliminary, point to greater participation of women in the labor force, particularly in low socioeconomic occupations.

Acknowledgements—This research was supported in part by National Science Foundation grant No. SOC76-15876 and Department of Health, Education and Welfare grant No. G0077-02044. The authors wish to acknowledge the helpful comments of an anonymous referee.

REFERENCES

1. Z. Griliches, Capital-Skill complementarity. *Rev. of Econ. and Statis.* **51**, 465–468 (1969).
2. F. Welch, Education in production. *J. Political Economy*, **78**, 35–59 (1970).
3. E. R. Berndt and L. R. Christensen, Testing for the existance of a consistent aggregate index of labor inputs. *Am. Econ. Rev.* **64**, 391–404 (1974).
4. P. R. Fallon and G. Layard, Capital-skill complementarity, income distribution and output accounting. *J. Political Economy*, **83**, 279–302 (1975).
5. K. A. Fox, *Social Indicators and Scoial Theory*, Chap. X. Wiley, New York (1974).
6. R. Stone, Economic and demographic accounts and the distribution of income. *Acta Oeconomica*, **11**, 165–176 (1973).
7. A. P. Schinnar, An eco-demographic accounting-type multiplier analysis of Hungary. *Environment and Planning A*, **9**, 373–384 (1977).
8. W. W. Cooper and A. P. Schinnar, A model for demographic mobility analysis under patterns of efficient employment. *Economics of Planning*, **13**, 139–173 (1977).
9. A. Charnes, W. W. Cooper and A. P. Schinnar, A theorem on homogeneous functions and extended Cobb–Douglas forms. *Proc. of the National Academy of Sciences, U.S.A.*, **73**, 3747–3748 (1976).
10. R. D. Weiss, Elasticities of substitution among capital and occupations in U.S. manufacturing. *J. Am. Statist. Assoc.* **72**, 764–771 (1977).
11. K. R. Polenske *et al. State Estimates of Technology*, 1963, Vol. 4, Lexington Books, Lexington Mass. (1972).
12. H. D. Vinod, A survey of ridge regression and related techniques for improvements over ordinary least squares. *Rev. of Econom. and Statist.* **60**, 121–131 (1978).
13. C. K. Liew, Inequality constrained least-squares estimation. *J. Am. Statist. Assoc.* **71**, 746–751 (1976).
14. C. K. Liew and J. K. Shim, A computer program for inequality constrained least-squares estimation. *Econometrica*, **46**, 237 (1978).
15. S. Kullback, *Information Theory and Statistics*. Wiley, New York (1959).