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Parity Retransmission Hybrid ARQ Using Rate 1/2 Convolutional Codes on a Nonstationary Channel

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Abstract—**Hybrid automatic repeat request (ARQ) error control coding makes use of both error detection and error correction in order to achieve high throughputs and low undetected error probabilities on two-way channels. For nonstationary channels where the channel bit error rate (BER) varies over time, the technique of parity retransmission allows the error control strategy to adapt to the state of the channel.**

In this paper, we propose a parity retransmission hybrid ARQ scheme which uses rate 1/2 convolutional codes and Viterbi decoding. The performance analysis is based on a two-state Markov model of a nonstationary channel. Throughput efficiency is shown to improve as the channel becomes more bursty in nature.

I. Introduction

Automatic repeat request (ARQ) strategies have long been utilized to control errors on two-way digital transmission links. Most of the work in this area has been done using block codes with error detection only, due to the packetized nature of the messages and the relatively low coding overhead allowed in many systems. However, in systems where the packet lengths are relatively large, say on the order of 1000 bits or more, and where the noise and/or interference levels are high, error detection only results in a low throughput due to the large number of retransmissions required. Satellite networks [1] and packet radio [2] are examples of such systems. In these instances, a combination of error correction and error detection can offer significant advantages over an error detection only system. This is called *hybrid ARQ error control.*

Two basic types of hybrid ARQ error control strategies have been considered. The first type includes parity bits for both error detection and error correction in each transmitted packet. The decoder will correct those received packets within the error-correcting capability of the code, while requesting a retransmission of those packets with detectable but uncorrectable errors. Since bits for error correction are sent with every packet, the code rate places an upper limit on the throughput efficiency of the system. For this reason, this strategy is best suited for systems in which a fairly constant level of noise and interference is anticipated on the channel. In this case, enough error correction overhead can be designed into the system to correct the vast majority of packets, thereby greatly reducing the number of retransmissions compared to an error detection only scheme and enhancing the system throughput. On the other hand, if the channel is quiet most of the time and noisy only occasionally (a *nonstationary channel*), designing a code to correct the occasional noisy bursts will reduce throughput compared to an error detection only scheme because the error correction overhead is wasted during the quiet periods.

In the second type of strategy, bits for error detection only are sent on the first transmission. If errors are detected and a retransmission requested, parity bits on the original information packet are sent along with some bits for error detection. If no errors are detected on the second transmission, the parity bits are inverted to recover the original information. If errors are detected, the two received packets are treated together as a code word in a rate 1/2 code. If the error-correcting-capability of the code is exceeded, and decoding is unreliable, the original transmission is repeated. This process continues, alternating transmissions between the original data packet and the parity packet, until either an error-free packet is received or error correction is possible. This strategy is referred to as *parity retransmission* [3].

Since parity bits for error detection only are sent on the first transmission, the upper limit on throughput efficiency is near 1. Throughput suffers only when retransmissions are required, since it is only then that parity bits for error correction are sent. In other words, parity bits for error correction are transmitted only when they are needed. It is this feature which gives the parity retransmission strategy the ability to adapt to changing channel conditions. When the channel is quiet, parity bits for error detection only are transmitted, and a high throughput is maintained. Only when noise or interference cause packets to be received incorrectly are parity bits for error correction transmitted, resulting in a reduced throughput. This adaptive capability of the parity retransmission strategy is particularly useful in applications such as satellite communication and packet radio, where fluctuating channel conditions due to fading and interference are commonly encountered.

Either block or convolutional codes can be used with both types of strategy. Schemes using the first strategy and block codes have been in existence for quite some time [4]–[9]. Several of these schemes using convolutional codes have also appeared in the literature [2], [10]–[12]. Parity retransmission using block codes was introduced more recently [3], [13]–[15]. Several similar schemes using convolutional codes have also been presented [16]–[19]. Although all the parity retransmission schemes that have appeared in the literature have been proposed for use on nonstationary channels, the analysis in each case used a stationary channel model.

In this paper, the performance of a parity retransmission hybrid ARQ scheme using rate 1/2 convolutional codes on a nonstationary channel is analyzed. In Section II, a protocol is described which is capable of achieving higher throughputs than previously proposed parity retransmission schemes. In Section III, a two-state Markov chain channel model is defined. This model constitutes a first approximation to a non-

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stationary channel. In Sections IV and V, the two-state channel model is used to analyze the throughput and undetected error probability of the protocol presented in Section II, when the receiver has both an infinite and a finite buffer size. It is shown that the throughput improves as the channel becomes more bursty. This corresponds with our intuitive notion that parity retransmission schemes are best suited for nonstationary or bursty channels. In Section VI, performance curves are calculated for a particular example.

II. Description of the Protocol

The parity retransmission hybrid ARQ scheme with a rate 1/2 convolutional code employs two codes, C_0 and C_1. C_1 is a $(2, 1, m)$ convolutional code with two generator polynomials, $G_1(x)$ and $G_2(x)$, and it is used for error detection and error correction. C_0 is a high rate $(n - m, n - m - r)$ binary block code used for error detection only where r is the number of parity bits in C_0.

When an $(n - m - r)$ bit information sequence $I(x)$ is generated, it is first encoded into a code vector, denoted by $J(x)$, in the $(n - m, n - m - r)$ block code C_0. Then the $(n - m)$ bit vector $J(x)$ is encoded into an n bit code vector $V_1(x) = J(x) \cdot G_1(x)$, and $V_1(x)$ is transmitted over the channel.

Let $\hat{V}_1(x)$ be the noisy version of $V_1(x)$ arriving at the receiver. The syndrome of $\hat{V}_1(x)$ is checked in two steps. First, $\hat{V}_1(x)$ is considered as a noisy version of a codeword in the $(n, n - m)$ shortened cyclic code generated by $G_1(x)$. The syndrome of the shortened cyclic code is checked; if it is zero, we have an estimate $\hat{J}(x)$ of $J(x)$. Next the syndrome of $\hat{J}(x)$ in the high rate $(n - m, n - m - r)$ block code is checked; if it is zero, we have an estimate $\hat{I}(x)$ of $I(x)$. The estimate $\hat{I}(x)$ is assumed to be error-free and is delivered to the data sink. In this case, we call $\hat{V}_1(x)$ a *zero syndrome vector* (ZSV). If, however, the first or the second syndrome check is negative, i.e., $\hat{V}_1(x)$ is a *nonzero syndrome vector* (NSV), then a NACK signal is sent to the transmitter and $\hat{V}_1(x)$ is stored in a receiver buffer. The transmitter then sends a second vector $V_2(x) = J(x) \cdot G_2(x)$, and $\hat{V}_2(x)$ is received after a roundtrip delay. Its syndrome is checked in two steps in the same way as $\hat{V}_1(x)$. If both syndromes are zero, $\hat{V}_2(x)$ is assumed to be error-free and an estimate $\hat{I}(x)$ of $I(x)$ is recovered directly; otherwise, $\hat{V}_1(x)$ and $\hat{V}_2(x)$ are decoded using the Viterbi algorithm, producing an estimate $\hat{J}(x)$. The syndrome of $\hat{J}(x)$ is then checked using the $(n - m, n - m - r)$ block code. If it is zero, $\hat{I}(x)$ is recovered and delivered to the data sink; if it is nonzero, a second NACK is sent requesting the retransmission of $V_1(x)$. The previously received version of $V_1(x)$, $\hat{V}_1(x)$, is discarded at the receiver and replaced by the new one. The receiver continues checking the syndrome of each received vector, trying to decode using the Viterbi algorithm if the syndrome check is negative, and requesting a retransmission of $V_1(x)$ or $V_2(x)$ in alternating order if the decoding is unsuccessful, until the information vector is delivered to the data sink.

III. Description of the Channel Model

A. Two-State Model

Let us model the channel as a Markov chain (see Fig. 1). State 0 is the *quiet* state where the bit error rate (BER) is ϵ_0. State 1 is the *noisy* state where the BER is $\epsilon_1 \gg \epsilon_0$. p is the transition probability from state 0 to state 1 and p' is the transition probability from state 1 to itself. To simplify the model's treatment, we assume that one time frame in the model corresponds to the transmission of one data vector, i.e., the noisy bursts last for a multiple of the transmission time of a data vector. This type of model was first introduced by Gilbert [20].

Straightforward calculations (see [21] for details) from the Markov chain model show that *the average burst length,* i.e.,

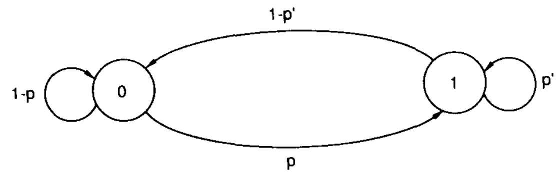

Fig. 1. A two-state Markov chain nonstationary channel model.

the average number of data vectors transmitted while in state 1 is

$$\bar{b} = \frac{1}{1-p'}, \tag{1}$$

the average BER is

$$\bar{\epsilon} = \frac{(1-p')\epsilon_0 + p\epsilon_1}{1-p'+p}, \tag{2}$$

and the *duty cycle* of the noisy bursts, or the probability of being in the noisy state is

$$p_1 = \frac{p}{1-p'+p}, \tag{3}$$

or

$$p_1 = \frac{\bar{\epsilon}-\epsilon_0}{\epsilon_1-\epsilon_0}. \tag{4}$$

Four parameters govern the two-state channel model. They can be chosen to be $\bar{b}$, $\bar{\epsilon}$, p_1, and the high-to-low BER ratio $\rho \triangleq \epsilon_1/\epsilon_0$.

B. Burst Noise Model

Of the four parameters selected, one is the average channel BER and the other three characterize the burstiness of the channel. We shall reduce the number of degrees of freedom by proposing a model for the noise bursts which can be *dense* (low duty cycle p_1 and high intensity, i.e., large high-to-low BER ratio ρ) or *diffuse* (large duty cycle and low intensity). These terms were first introduced by Massey [22]. The conditions that we impose on a burst channel model are

$$\lim_{p_1 \to 0} \epsilon_0 = 0, \tag{5.1}$$

$$\lim_{p_1 \to 0} \epsilon_1 = 1/2, \tag{5.2}$$

and

$$\lim_{p_1 \to 1} \epsilon_0 = \bar{\epsilon}, \tag{6.1}$$

$$\lim_{p_1 \to 1} \epsilon_1 = \bar{\epsilon}. \tag{6.2}$$

Conditions (5) represent the limiting case of a dense burst channel, i.e., $p_1 \to 0$ and $\rho \to \infty$, while conditions (6) represent the limiting case of a diffuse burst channel, i.e., $p_1 \to 1$ and $\rho \to 1$, which is equivalent to a *binary symmetric channel* (BSC).

The two-state channel model described by (1)–(4) does not meet these conditions. In fact, from (4) we see that only condition (6.2) is satisfied. We now modify the two-state channel model such that conditions (5) and (6) are met.

Let

$$\epsilon_0 = \bar{\epsilon} p_1. \tag{7}$$

From (4) and (7), we have

$$\epsilon_1 = \frac{\bar{\epsilon}}{p_1} - (1-p_1)\bar{\epsilon}, \text{ for } p_1 > \frac{\bar{\epsilon} + 1/2 - \sqrt{(1/2-\bar{\epsilon})(1/2+3\bar{\epsilon})}}{2\bar{\epsilon}} \tag{8.1}$$

where the inequality ensures that $\epsilon_1 < 1/2$, and

$$\epsilon_1 = 1/2, \text{ otherwise.} \tag{8.2}$$

From (7), we see that (5.1) and (6.1) are satisfied. From (8.1) and (8.2), we see that (5.2) and (6.2) are also satisfied. Now the burst channel model is completely described by $\bar{\epsilon}$, p_1, and $\bar{b}$, for if these three parameters are known, p', p, ϵ_0, and ϵ_1 can be determined from (1), (3), (7), and (8). Before leaving this section, we note that the special case $p_1 = p = p'$ corresponds to the two-state *block interference* (BI) channel model proposed by McEliece and Stark [23]. The BI channel model is completely determined by p_1 and $\bar{\epsilon}$.

IV. System Throughput and Undetected Error Probability Analysis with an Infinite Receiver Buffer

In the next two sections, we analyze the throughput and the undetected error probability of the parity retransmission hybrid ARQ scheme in the *selective-repeat* mode for both an infinite receiver buffer and a finite receiver buffer. Our analysis is based on the assumption that the feedback channel is error-free. In order to carry out the analysis, we first model the receiver's decoding status as a Markov chain.

A. Receiver's Decoding Status

Consider a Markov chain with N possible states: $1, 2, \cdots, N$ where changes in states can occur only at discrete times $t_1, t_2, \cdots, t_n, \cdots$. Let us denote the transition probability from state i to state j after k time units by $p_{ij}(k)$. $p_{ij}(k)$ is called the *k step transition probability,* and it can be determined from the one-step transition probabilities, namely, $p_{ij}(1)$, or simply p_{ij}, between all pairs of states. These transition probabilities can be summarized by an $N \times N$ matrix, called the one-step *transition probability matrix:*

$$P = [p_{ij}] = \begin{bmatrix} p_{1,1} & p_{1,2} & \cdots & p_{1,N} \\ p_{2,1} & p_{2,2} & \cdots & p_{2,N} \\ \vdots & & & \\ p_{N,1} & p_{N,2} & \cdots & p_{N,N} \end{bmatrix}. \tag{9}$$

The probabilities in each row of P add to 1. The k-step transition probabilities can then be determined from the k-step transition probability matrix, which is given by

$$[p_{ij}(k)] = P^k. \tag{10}$$

The states in the Markov chain can be divided into several categories. The one of most interest to us is the *absorbing state.* A state i in a Markov chain is an absorbing state if the k step transition probability $p_{ii}(k) = 1$, $k = 1, 2, \cdots$. Thus, an absorbing state is a state that cannot reach any other state in the chain except itself. A Markov chain may contain more than one absorbing state.

For the parity retransmission hybrid ARQ scheme described in Section II, decoding is said to be *successful* if the decoded information vector is accepted by the user (it may be decoded correctly or contain undetected errors). Suppose that an initial NSV has been received. Then the receiver's decoding status can be modeled by the Markov chain shown in Fig. 2. In Fig. 2, states 00, 01, 10, and 11 mean that the decoder has received two NSV's, sent while the channel was in states 0 and 0, 0 and 1, etc., and that decoding has been unsuccessful. State e indicates an undetected error pattern either before or after convolutional decoding. State c corresponds to error-free decoding. Once the system is in state e or state c, successful decoding results and retransmissions will therefore terminate. Hence, both state e and state c are absorbing states and we assign $p_{e,e} = p_{c,c} = 1$. On the other hand, if upon the reception of a retransmission, the decoder cannot recover the data vector, retransmissions will continue (the Markov chain will remain in state 00, 01, 10, or 11) until successful decoding occurs (the Markov chain reaches state e or state c).

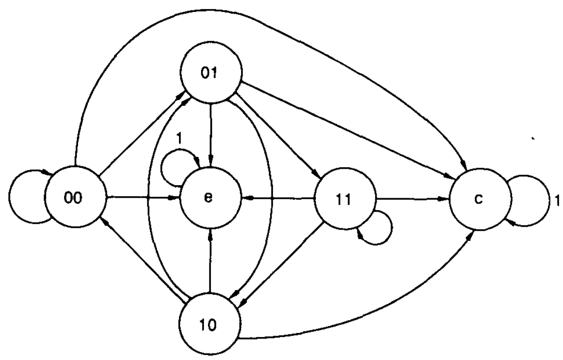

Fig. 2. Receiver's decoding status after receiving an initial NSV.

To derive the transition probabilities of the Markov chain, we first consider the undetected error probability. Undetected errors occur at two levels: the syndrome check before convolutional decoding and the syndrome check after convolutional decoding. First, we consider the former case. Let Q_i, $i = 0, 1$, be the probability of receiving a ZSV with an undetected error pattern while the channel is in state i. Q_i can be upper bounded by [24]

$$Q_i \le 2^{-(m+r)}[1 - 2(1-\epsilon_i)^n + (1-2\epsilon_i)^n] \tag{11}$$

where $m + r$ is the sum of the memory order of the convolutional code and of the number of parity checks in the block code (the two codes are treated at this stage as a single block code with $m + r$ parity checks).

To find the probability of undetected error after convolutional decoding, we need to evaluate the BER at the output of the convolutional decoder. The state ij in Fig. 2 means that a NSV transmitted over channel state i has been held in the receiver buffer, another NSV sent over channel state j after one roundtrip delay is received, and the convolutional decoding of the two NSV's fails. The average BER in the two received NSV's before convolutional decoding is then

$$\epsilon(i, j) = (\epsilon_i' + \epsilon_j')/2 \tag{12}$$

where

$$\epsilon_i' = \frac{\epsilon_i}{P_{Ni}} \tag{13}$$

is the conditional BER given that the syndrome is nonzero, and

$$P_{Ni} = 1 - (1-\epsilon_i)^n - Q_i \tag{14}$$

is the probability of receiving a NSV sent while the channel was in state i. The BER at the output of the convolutional decoder is upper bounded by [25]

$$P_b(i, j) \le \frac{1}{2} \frac{\partial T(X, Y)}{\partial Y}\Big|_{X = 2\sqrt{\epsilon(i,j)[1-\epsilon(i,j)]},\ Y=1} \tag{15}$$

where $T(X, Y)$ is the generating function of the convolutional code.

The output of the convolutional decoder is then checked by the $(n - m, n - m - r)$ block code C_0. We assume that the errors at the output of the convolutional decoder are independent.[1] The probability of undetected error after convolutional decoding in state ij is then upper bounded by

$$Q_{ij} \leq 2^{-r}\{1-2[1-P_b(i,j)]^{n-m}+[1-2P_b(i,j)]^{n-m}\}. \quad (16)$$

The probability of decoding failure is the probability of a nonzero syndrome in the block code C_0, and it is given by

$$P_{ij}=1-[1-P_b(i,j)]^{n-m}-Q_{ij}. \quad (17)$$

By arranging the states of Fig. 2 in the order: 00, 01, 10, 11, e, c, the transition probabilities of the Markov chain are found in Appendix A and are summarized in the following transition probability matrix:

$$P=\begin{bmatrix} p_{00,00} & p_{00,01} & 0 & 0 & p_{00,e} & p_{00,c} \\ 0 & 0 & p_{01,10} & p_{01,11} & p_{01,e} & p_{01,c} \\ p_{10,00} & p_{10,01} & 0 & 0 & p_{10,e} & p_{10,c} \\ 0 & 0 & p_{11,10} & p_{11,11} & p_{11,e} & p_{11,c} \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \quad (18)$$

where, from (A.2), (A.3), and (A.4),

$$\begin{aligned} p_{00,00}&=[1-p_{01}(d)]P_{N0}P_{00} \\ p_{00,01}&=p_{01}(d)P_{N1}P_{01} \\ p_{01,10}&=[1-p_{11}(d)]P_{N0}P_{10} \\ p_{01,11}&=p_{11}(d)P_{N1}P_{11} \\ p_{10,00}&=[1-p_{01}(d)]P_{N0}P_{00} \\ p_{10,01}&=p_{01}(d)P_{N1}P_{01} \\ p_{11,10}&=[1-p_{11}(d)]P_{N0}P_{10} \\ p_{11,11}&=p_{11}(d)P_{N1}P_{11} \\ p_{00,e}&=[1-p_{01}(d)][Q_0+P_{N0}Q_{00}]+p_{01}(d)[Q_1+P_{N1}Q_{01}] \\ p_{01,e}&=[1-p_{11}(d)][Q_0+P_{N0}Q_{10}]+p_{11}(d)[Q_1+P_{N1}Q_{11}] \\ p_{10,e}&=[1-p_{01}(d)][Q_0+P_{N0}Q_{00}]+p_{01}(d)[Q_1+P_{N1}Q_{01}] \\ p_{11,e}&=[1-p_{11}(d)][Q_0+P_{N0}Q_{10}]+p_{11}(d)[Q_1+P_{N1}Q_{11}] \\ p_{00,c}&=1-p_{00,00}-p_{00,01}-p_{00,e} \\ p_{01,c}&=1-p_{01,10}-p_{01,11}-p_{01,e} \\ p_{10,c}&=1-p_{10,00}-p_{10,01}-p_{10,e} \\ p_{11,c}&=1-p_{11,10}-p_{11,11}-p_{11,e}, \end{aligned} \quad (19)$$

$p_{01}(d)$ and $p_{11}(d)$ are the d step channel transition probabilities given by (A.1), and d is the number of code vectors that can be transmitted during one channel round trip delay period.

B. Throughput Efficiency Calculation

Throughput efficiency is defined as the ratio of the average number of data vectors accepted by the receiver and delivered to the user per unit time to the total number of vectors that can be transmitted per unit time [27]. Let $E[N]$ be the expected total number of transmissions (including the initial transmission and all retransmissions) required for a data vector to be successfully accepted by the receiver. Then the throughput of selective-repeat parity retransmission hybrid ARQ is given by [27]

$$\eta=\frac{1}{E[N]}\frac{n-m-r}{n}. \quad (20)$$

In practice, $m \ll n$ and $r \ll n$ and hence $(n-m-r)/n \approx 1$. Let T be the expected number of retransmissions required for a data vector to be successfully decoded by the receiver. Then $E[N]$ can be expressed as

$$E[N]=1+T=1+\sum_{i=0}^{1} T_i P_{Ni} p_i \quad (21)$$

where T_i, $i = 0, 1$, is the expected number of retransmissions required for a data vector to be successfully decoded by the receiver, given that the initial transmission was a NSV and was sent over channel state i, and $p_0 = 1 - p_1$ is the probability that the channel starts in state 0. Thus, only T_i, $i = 0, 1$, must be determined to find η.

Referring to the Markov chain shown in Fig. 2, and assuming that an initial NSV has been received, we see that retransmissions will be needed to recover the information sequence associated with the initial NSV. As soon as an undetected error or error-free decoding occurs (i.e., the Markov chain reaches state e or state c), retransmissions will terminate, and the estimated information sequence will be delivered to the data sink. Therefore, the expected number of retransmissions is the *infinite-step chain mean absorption time* defined in Appendix B, and (B.9) can be applied to find T_i, $i = 0, 1$.

Suppose that the initial NSV was sent over channel state 0. To determine T_0, either state 00 or state 10 in Fig. 2 can be used as our initial state, since both states assume that a NSV sent over channel state 0 is held in the receiver buffer and waiting to be processed upon reception of the first retransmission. Hence, the infinite-step chain mean absorption time (or equivalently, the expected number of retransmissions) starting from states 00 and 10 should be the same, i.e.,

$$T_0=M_{00}=M_{10}. \quad (22.1)$$

Similarly,

$$T_1=M_{01}=M_{11}. \quad (22.2)$$

Substituting the transition probabilities of (19) into (B.9), we obtain the following equations:

$$M_{00}=1+M_{00}p_{00,00}+M_{01}p_{00,01}$$

$$M_{01}=1+M_{10}p_{01,10}+M_{11}p_{01,11}.$$

Using (22.1) and (22.2), the above equations become

$$T_0(p_{00,00}-1)+T_1 p_{00,01}=-1 \quad (23.1)$$

$$T_0 p_{01,10}+T_1(p_{01,11}-1)=-1 \quad (23.2)$$

and their solutions are

$$T_0=\frac{1+p_{00,01}-p_{01,11}}{(1-p_{00,00})(1-p_{01,11})-p_{00,01}p_{01,10}} \quad (24.1)$$

and

$$T_1=\frac{1+p_{01,10}-p_{00,00}}{(1-p_{00,00})(1-p_{01,11})-p_{00,01}p_{01,10}}. \quad (24.2)$$

[1] The bit errors at the output of a convolutional decoder are not independent. However, we assume that an interleaver can be placed between the inner and the outer encoder and a deinterleaver between the inner and the outer decoder to make them independent.

Therefore, from (21) and (24) we obtain

$$E[N]=1+\frac{[1+p_{00,01}-p_{01,11}]P_{N0}p_0+[1+p_{01,10}-p_{00,00}]P_{N1}p_1}{(1-p_{00,00})(1-p_{01,11})-p_{00,01}p_{01,10}}. \tag{25}$$

The throughput for selective-repeat parity retransmission hybrid ARQ can then be obtained by substituting (25) into (20).

For the special case when the channel is a BSC with BER ϵ, (25) can be simplified, after some calculations, to

$$E[N]=\frac{1+P_N(1-P_f)}{1-P_NP_f} \tag{26}$$

where

$$P_N\approx 1-(1-\epsilon)^n \tag{27}$$

is the probability of receiving a NSV and

$$P_f\approx 1-(1-p_b)^{n-m} \tag{28}$$

is the probability of a convolutional decoding failure on two NSV's where

$$p_b\leq\frac{1}{2}\frac{\partial T(X,Y)}{\partial Y}\bigg|_{X=2\sqrt{\epsilon'(1-\epsilon')},\,Y=1}$$

and

$$\epsilon'\approx\frac{\epsilon}{1-(1-\epsilon)^n}.$$

The throughput on a BSC is then

$$\eta=\frac{1-P_NP_f}{1+P_N(1-P_f)}\cdot\frac{n-m-r}{n}. \tag{29}$$

In [18], only a lower bound on n was obtained for a parity retransmission hybrid ARQ scheme on a BSC.

C. Undetected Error Probability Calculation

The *undetected error probability* is the average probability of decoding error, given that the receiver accepts a data vector. We denote this average probability by P_{ud}. It is easy to see that

$$P_{ud}=\sum_{i=0}^{1}\{Q_i+U_iP_{Ni}\}p_i \tag{30}$$

where U_i, $i = 0, 1$, is the average probability of an undetected error pattern in the data vector accepted by the receiver, given that the initial transmission was a NSV and sent over channel state i, and p_i is the probability of being in state i.

Referring to Fig. 2, given that an initial NSV was received, undetected errors will occur if and only if the Markov chain enters state e. Therefore, finding U_i is equivalent to finding the *infinite-step absorption probability* of the Markov chain (see Appendix B.) Let $A_{00,e}$, $A_{01,e}$, $A_{10,e}$, and $A_{11,e}$ be the probabilities that the chain of Fig. 2, starting at states 00, 01, 10, and 11, respectively, will eventually be absorbed by state e. By an argument similar to the throughput calculation, we can show that

$$U_0=A_{00,e}=A_{10,e} \tag{31.1}$$

$$U_1=A_{01,e}=A_{11,e}. \tag{31.2}$$

Substituting the transition probabilities of (19) into (B.8), it follows that

$$A_{00,e}=A_{00,e}p_{00,00}+A_{01,e}p_{00,01}+p_{00,e}$$

$$A_{01,e}=A_{10,e}p_{01,10}+A_{11,e}p_{01,11}+p_{01,e}.$$

From (31), the above equations reduce to

$$U_0(1-p_{00,00})-U_1p_{00,01}=p_{00,e} \tag{32.1}$$

$$-U_0p_{01,10}+U_1(1-p_{01,11})=p_{01,e}. \tag{32.2}$$

Solving (32) we obtain

$$U_0=\frac{p_{00,e}(1-p_{01,11})+p_{01,e}p_{00,01}}{(1-p_{00,00})(1-p_{01,11})-p_{00,01}p_{01,10}} \tag{33.1}$$

$$U_1=\frac{p_{01,e}(1-p_{00,00})+p_{00,e}p_{01,10}}{(1-p_{00,00})(1-p_{01,11})-p_{00,01}p_{01,10}}. \tag{33.2}$$

Thus, from (30) and (33), the probability of undetected error for selective-repeat parity retransmission hybrid ARQ with an infinite receiver buffer is given by

$$P_{ud}=\left[Q_0+\frac{p_{00,e}(1-p_{01,11})+p_{01,e}p_{00,01}}{(1-p_{00,00})(1-p_{01,11})-p_{00,01}p_{01,10}}P_{N0}\right]p_0+\left[Q_1+\frac{p_{01,e}(1-p_{00,00})+p_{00,e}p_{01,10}}{(1-p_{00,00})(1-p_{01,11})-p_{00,01}p_{01,10}}P_{N1}\right]p_1. \tag{34}$$

If the channel is a BSC with BER ϵ, (34) can be simplified to

$$P_{ud}=Q+\frac{Q+P_NP_e}{1-P_NP_f} \tag{35}$$

where P_N and P_f are given by (27) and (28), respectively,

$$Q\leq 2^{-(m+r)}[1-2(1-\epsilon)^n+(1-2\epsilon)^n] \tag{36}$$

is the probability of receiving a ZSV with an undetected error pattern, and

$$P_e\leq 2^{-r}[1-2(1-p_b)^{n-m}+(1-2p_b)^{n-m}] \tag{37}$$

is the probability of undetected error after convolutional decoding of two NSV's.

V. System Throughput and Undetected Error Probability with a Finite Receiver Buffer

In this section, we analyze the throughput and the probability of undetected error for the selective-repeat parity retransmission hybrid ARQ with a finite receiver buffer. Let B be the number of code vectors that the receiver buffer can store. The performance is analyzed for the case when

$$B=l\cdot d,\quad l=1,2,3,\cdots. \tag{38}$$

The system with receiver buffer size B operates as follows. Normally, a transmitter operating in the selective-repeat mode sends code vectors continuously to the receiver. The receiver checks the syndrome of each received code vector. If the syndrome is zero, the received vector is assumed to be error-free and is delivered to the user, and an ACK signal is sent to the transmitter. When the channel is quiet, data transmission proceeds smoothly; error-free vectors are delivered to the user in consecutive order and the receiver buffer is empty. The receiver is said to be in the *normal phase* if the receiver buffer is empty.

When a received code vector is detected in error (NSV) while the receiver is in the normal phase, the receiver enters the *blocked phase* and sends a NACK to the transmitter. The NSV is then stored in the receiver buffer for error correction at a later time. In the blocked phase, the receiver continues to check the syndrome of each incoming received code vector, sends an ACK to the transmitter for each received ZSV, and sends a NACK to the transmitter for each received NSV. The received vectors, no matter whether they are ZSV's or NSV's, are stored in the receiver buffer until they are ready to be released to the data sink. In the blocked phase, no vector is delivered to the data sink until the earliest received NSV is accepted by the data sink.

If the earliest NSV is recovered within l retransmissions, the receiver then starts to deliver this vector and the subsequent ZSV's (which are held in the receiver buffer) to the data sink in order until the next NSV is encountered. This vector then becomes the earliest NSV. If all the vectors held in the receiver buffer are released to the data sink after the earliest NSV has been recovered, the receiver buffer becomes empty again and the receiver returns to the normal phase.

If the receiver fails to recover the earliest NSV after l retransmissions, no further retransmissions are allowed. The receiver simply delivers the erroneous decoded vector to the data sink and sends an ACK to the transmitter. Because the receiver has a buffer size $B = l \cdot d$, buffer overflow will never occur, and the system operates in the same way as though the receiver had an infinite buffer size, except for the *forced* vector delivery upon receiving the lth retransmission. The tradeoff in system performance by limiting the number of retransmissions to l will be an increased throughput and a decreased system reliability compared with systems without this limitation.

A. Throughput Efficiency Calculation

Let $E[N(l)]$ be the expected number of transmissions needed to deliver a code vector to the user within $l + 1$ transmissions (including the initial transmission and up to l retransmissions). The throughput of the selective-repeat parity retransmission hybrid ARQ with receiver buffer size $B = l \cdot d$ is then

$$\eta = \frac{1}{E[N(l)]} \frac{n-m-r}{n}. \tag{39}$$

By a similar argument as in the infinite receiver buffer case, we obtain

$$E[N(l)] = 1 + T_0(l) P_{N0} p_0 + T_1(l) P_{N1} p_1 \tag{40}$$

where $T_i(l)$, $i = 0, 1$, is the expected number of retransmissions required for a data vector to be successfully decoded within l retransmissions, given that the initial vector was a NSV and sent over channel state i.

We readily recognize that finding $T_i(l)$ is equivalent to finding the Markov chain mean absorption time within l transitions (see Appendix B), so that (B.4) and (B.6) can be used. Note that the computation can be reduced by combining state e and state c in Fig. 2 into a single state, called state s, since both states e and c are absorbing states. Thus, state s is simply the state corresponding successful decoding. The new transition probability matrix, obtained by merging states e and c into state s, is given by [see (18)]

$$P' = \begin{bmatrix} p_{00,00} & p_{00,01} & 0 & 0 & p_{00,s} \\ 0 & 0 & p_{01,10} & p_{01,11} & p_{01,s} \\ p_{10,00} & p_{10,01} & 0 & 0 & p_{10,s} \\ 0 & 0 & p_{11,10} & p_{11,11} & p_{11,s} \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \tag{41.1}$$

where

$$p_{ij,s} = p_{ij,e} + p_{ij,c}. \tag{41.2}$$

Let $M_{00}(l)$ and $M_{01}(l)$ be the mean absorption time within l transitions of the Markov chain described by (41), conditioned on the chain starting in state 00 and state 01, respectively. From (B.4) we have

$$T_0(l) = M_{00}(l) = p_{00,s} + \sum_{n=2}^{l} n[p_{00,s}(n) - p_{00,s}(n-1)], \tag{42.1}$$

$$T_1(l) = M_{01}(l) = p_{01,s} + \sum_{n=2}^{l} n[p_{01,s}(n) - p_{01,s}(n-1)] \tag{42.2}$$

where $p_{00,s}(n)$ and $p_{01,s}(n)$ are the n step transition probabilities for state 00 and state 01, respectively, to state s, and they can be determined from (41), (19), and (10). The system throughput of selective-repeat parity retransmission hybrid ARQ with receiver buffer size $B = l \cdot d$, from (39), (40), and (42), is given by

$$\eta = \frac{(n-m-r)/n}{1 + \sum_{i=0}^{1} \left\{ p_{0i,s} + \sum_{n=2}^{l} n[p_{0i,s}(n) - p_{0i,s}(n-1)] \right\} P_{Ni} p_i}. \tag{43}$$

B. Undetected Error Probability Calculation

The probability of undetected error for selective-repeat parity retransmission hybrid ARQ with an infinite receiver buffer is given in (30). The probability of undetected error for the system with receiver buffer size $B = l \cdot d$ can be derived from (30) with a slight modification. It is given by

$$P_{ud} = \sum_{i=0}^{1} \{ Q_i + [U_i(l) + E_i(l)] P_{Ni} \} p_i \tag{44}$$

where $U_i(l)$ is the probability of undetected error within l retransmissions, given that the initial transmission was a NSV and sent over channel state i. $E_i(l)$ is the probability of unsuccessful decoding on the lth retransmission, given that the initial transmission was a NSV and sent over channel state i. Since only up to l retransmissions are permitted in the system, if the lth retransmission results in a decoding failure, the erroneously decoded data vector will be delivered to the data sink. Hence, it is reasonable to regard this decoding failure as an undetected error, and its probability $E_i(l)$ should be included in (44).

Let $A_{00,e}(l)$ and $A_{01,e}(l)$ be the probabilities that the Markov chain, starting at states 00 and 01, respectively, will be absorbed by state e within l transitions. Clearly, $U_0(l) = A_{00,e}(l)$ and $U_1(l) = A_{01,e}(l)$. It follows from (B.2) that

$$U_0(l) = A_{00,e}(l) = p_{00,e}(l) \tag{45.1}$$

and

$$U_1(l) = A_{01,e}(l) = p_{01,e}(l) \tag{45.2}$$

where $p_{0i,e}(l)$, $i = 0, 1$, is the l step transition probability from state $0i$ to state e. We also have

$$\begin{aligned} E_i(l) &= \Pr\{\text{The chain reaches 00, 01, 10, or 11 after } l \text{ steps} \mid \text{The chain starts in } 0i\} \\ &= p_{0i,00}(l) + p_{0i,01}(l) + p_{0i,10}(l) + p_{0i,11}(l). \end{aligned} \tag{46}$$

Combining (45) and (46), we obtain

$$U_i(l)+E_i(l)=1-p_{0i,c}(l), \tag{47}$$

which is the probability that the chain starting in state $0i$, i = 0, 1, will not reach state c within l transitions. In other words, $U_i(l) + E_i(l)$ is simply the probability that the initial NSV will not be decoded correctly within l retransmissions. Thus, the probability of undetected error for selective-repeat parity retransmission hybrid ARQ with receiver buffer size $B = l \cdot d$, from (44) and (47), is given by

$$P_{ud}=\sum_{i=0}^{1} \{Q_i+[1-p_{0i,c}(l)]P_{Ni}\}p_i. \tag{48}$$

VI. Examples

In this section, we plot the performance of selective-repeat parity retransmission hybrid ARQ for the following parameters:

C_0 : (1024, 1000) binary code

C_1 : (2, 1, 6)d_f= 10 convolutional code

Channel roundtrip delay: d= 128.

Four curves, numbered 0 through 3, appear in each plot. They correspond to four values of the burst duty cycle, which was defined in Section III as the parameter which determines the burstiness of the channel:

Curve 0: $p_1 = 1$ (Stationary channel)

Curve 1: $p_1 = 0.25$ (Diffuse burst channel)

Curve 2: $p_1 = 0.1$

Curve 3: $p_1 = 0.05$ (Dense burst channel).

In the performance calculation, we found that, for a given p_1 and $\bar{\epsilon}$, the values of the average burst length $\bar{b}$ have a very small influence on the throughput and undetected error probability. Therefore, we considered only the case when $p = p' = p_1$, i.e., a BI channel. Figs. 3 and 4 show the throughput and the probability of undetected error for selective-repeat parity retransmission hybrid ARQ with infinite receiver buffer. Fig. 5 shows the probability of undetected error with receiver buffer size $B = 5d$. The system throughput is essentially the same as in the infinite receiver buffer case.

From Fig. 3 we observe that the system throughput is much better for a given average BER if the errors occur in bursts, since the errors are then concentrated in fewer vectors and the convolutional code is still powerful enough to decode them. From Fig. 4 we see that the burstiness of the channel has a limited influence on the undetected error probability, except in the range of average BERs $4 \times 10^{-4} \leq \bar{\epsilon} \leq 10^{-2}$. In this range, and for a bursty channel, most of the errors are concentrated in the vectors received while the channel is noisy. The convolutional code is powerful enough to decode most of these noisy vectors reliably. On the other hand, if the channel is stationary, the vectors which are received during the quiet state contain more errors, and there are more undetected errors than in the nonstationary case, because of the limited power of the block code.

Fig. 5 shows the degradation in system reliability when the receiver buffer size is limited to five times the channel roundtrip delay ($B = l \cdot d = 5d$). The degradation becomes obvious for average BER's in the range $\bar{\epsilon} \geq 4 \times 10^{-3}$. For small l and high $\bar{\epsilon}$, although most errors can be corrected within l retransmissions, a small percentage of error vectors cannot be corrected, and those error vectors will have a serious effect on the probability of undetected error. Obvi-

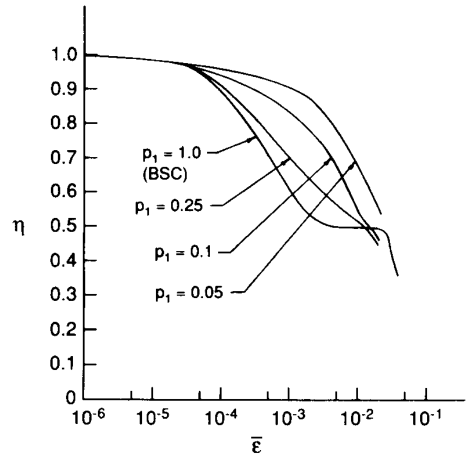

Fig. 3. Throughput performance of rate 1/2 selective-repeat ARQ with infinite receiver buffer for various values of the burst density p_1.

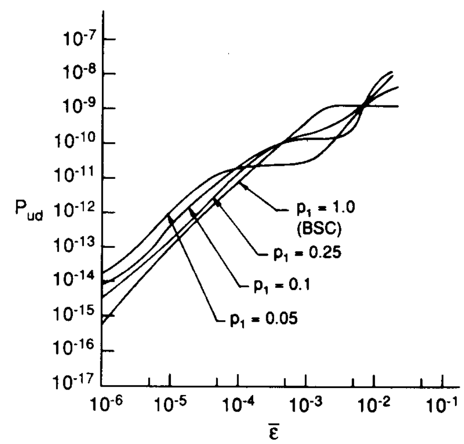

Fig. 4. Probability of undetected error of rate 1/2 selective-repeat ARQ with infinite receiver buffer for various values of the burst density p_1.

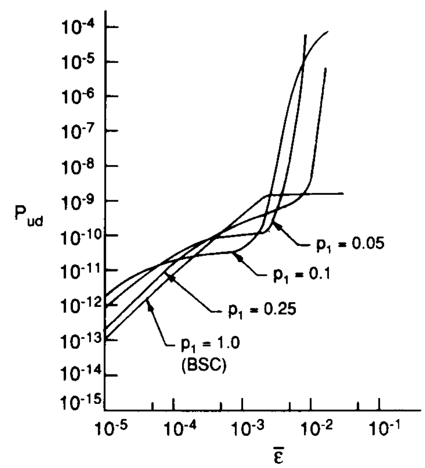

Fig. 5. Probability of undetected error of rate 1/2 selective-repeat ARQ with receiver buffer size $B = 5d$ for various values of the burst density p_1.

ously, as l becomes large, the system becomes more and more reliable.

VII. Discussion and Conclusions

Wang and Lin proposed a hybrid ARQ scheme with parity retransmission using two block codes C_0 and C_1 [18]. C_0 is an (n, k) high-rate error-detecting code and C_1 is a half-rate $(2n, n)$ code which is designed for error correction only. However, C_1 must be invertible and an inverse operation is required at the decoder to recover the information data vector from the retransmitted parity check vector. In [18], the performance of hybrid ARQ schemes using both a rate 1/2 invertible block code and a rate 1/2 convolutional code were analyzed for a BSC. The results indicate that convolutional codes provide a higher throughput than block codes, especially on very noisy channels (BERs around 10^{-2}). However, since these results were obtained for a BSC, and since parity retransmission hybrid ARQ schemes are designed for use on nonstationary channels, a more detailed analysis was needed. In this paper, we have presented a thorough analysis of a parity retransmission hybrid ARQ scheme using convolutional codes for a nonstationary channel, assuming both infinite and finite receiver buffers. Results show that high throughput efficiencies and low undetected error probabilities can be maintained over a wide range of channel parameters and that the throughput efficiency improves as the channel becomes burstier in nature. In addition, the undetected error probability performance is better for burstier channels. These results substantiate the claim that parity retransmission hybrid ARQ schemes using convolutional codes are particularly well suited for use on nonstationary channels.

The rate 1/2 convolutional code hybrid ARQ scheme presented in this paper can be extended to rate $(b-1)/b$, $b > 2$, schemes (see [21] for details). The generator matrix of the rate $(b-1)/b$ convolutional code can be written as

$$G(x)=\begin{bmatrix} G_1(x) & & & G_{1,b}(x) \\ & G_2(x) & & G_{2,b}(x) \\ & & \ddots & \vdots \\ & & G_{b-1}(x) & G_{b-1,b}(x) \end{bmatrix}. \quad (49)$$

Without loss of generality, consider the hybrid ARQ scheme using a rate 2/3 convolutional code. The generator matrix of the convolutional code is then given by

$$G(x)=\begin{bmatrix} G_1(x) & 0 & G_{13}(x) \\ 0 & G_2(x) & G_{23}(x) \end{bmatrix}. \quad (50)$$

In this scheme, a high rate block code C_o and the rate 2/3 convolutional code are concatenated in a manner similar to the rate 1/2 scheme. Information sequences are first encoded into code vectors in C_o, then into code vectors using $G_1(x)$ and $G_2(x)$ in an alternating fashion. Suppose an information sequence $I_1(x)$ is encoded into a code vector $J(I_1(x))$ in the block code C_o and then into $V_1(x) = J(I_1(x))\cdot G_1(x)$. Let $V_1'(x)$ be the noisy received version of $V_1(x)$. Let $\hat{J}(I_1(x))$ be the estimate of $J(I_1(x))$ if the syndrome of $V_1'(x)$ is zero. The receiver now checks the syndrome of $\hat{J}(I_1(x))$ in the outer block code C_o. If this second syndrome is also zero, the estimated information vector $\hat{I}_1(x)$ is assumed to be error free and is delivered to the user. If either syndrome is nonzero, the receiver then stores all succeeding vectors in a buffer until a vector $V_2'(x)$ [corresponding to a new information sequence $I_2(x)$] generated by $G_2(x)$ is received whose syndrome check is negative. Once two vectors $V_1'(x)$ and $V_2'(x)$ with negative syndrome checks have been received, a NACK is sent to the transmitter.

When the transmitter receives the NACK signal, it generates a vector $V_3(x) = J(I_1(x))\cdot G_{13}(x) + J(I_2(x))\cdot G_{23}(x)$ and sends it to the receiver. Let $V_3'(x)$ be the noisy version of $V_3(x)$. When $V_3'(x)$ is received, the receiver decodes $V_1'(x)$, $V_2'(x)$, and $V_3'(x)$ using the Viterbi algorithm for the rate 2/3 convolutional code with generator matrix given by (50). Let $\hat{J}(I_1(x))$ and $\hat{J}(I_2(x))$ be the resulting estimates of $J(I_1(x))$ and $J(I_2(x))$, respectively. The syndromes of $\hat{J}(I_1(x))$ and $\hat{J}(I_2(x))$ are now checked. If both syndromes are zero, the estimated information sequences $\hat{I}_1(x)$ and $\hat{I}_2(x)$ are assumed to be error free and delivered to the user. All the information sequences which had been stored in the buffer since the reception of the initial NSV $V_1'(x)$ are also delivered to the user. If either syndrome check is negative, a NACK signal is sent to the transmitter, which then sends $V_1(x)$ a second time. The receiver then discards the old version of $V_1(x)$, and attempts another convolutional decoding of the three vectors that it holds. If the decoding is successful, the information sequences are delivered to the user; if not, retransmissions of $V_2(x)$, $V_3(x)$, and $V_1(x)$ are requested in order until decoding is successful.

The high rate schemes provide a much better throughput than the rate 1/2 scheme at the cost of a more complex Viterbi decoder, a larger buffer, and a more complex buffer management strategy. For example, assuming the duty cycle of the noisy burst is $p_1 = 0.05$, the average channel BER is $\bar{\epsilon} = 10^{-2}$, and the memory order of the convolutional code is 3, the throughput is equal to 0.6 for the rate 1/2 scheme, 0.74 for the rate 2/3 scheme, and 0.8 for the rate 3/4 scheme [21]. These higher rate hybrid ARQ schemes are very attractive for use on high speed nonstationary channels, such as satellite communication channels.

Appendix A

Transition Probability from State kj to State ji, $p_{kj,ji}$

In this appendix, we derive the transition probabilities of the Markov chain shown in Fig. 2.

Let d be the number of code vectors that can be transmitted during one channel roundtrip delay period. Since the retransmitted vector is received after one channel roundtrip delay, in the following calculations we need to know the channel d step transition probabilities. The channel d step transition probabilities $p_{01}(d)$ and $p_{11}(d)$ of being in state 1 d time frames after being in state 0 and state 1, respectively, are given by [26]

$$p_{01}(d)=\frac{p}{1+p-p'}-\frac{p}{1+p-p'}(p'-p)^d, \quad (A.1.1)$$

and

$$p_{11}(d)=\frac{p}{1+p-p'}+\frac{(1-p')}{1+p-p'}(p'-p)^d. \quad (A.1.2)$$

Let us consider the transition from state 01 to state 10. State 01 means that two NSV's $\hat{V}_1(x)$ and $\hat{V}_2(x)$, sent over channel states 0 and 1, respectively, have been received and that decoding has failed. $\hat{V}_1(x)$ is then discarded and $\hat{V}_2(x)$ is held in the receiver buffer. The transition from state 01 to state 10 means that a new NSV $\hat{V}_3(x)$, sent over channel state 0 after a rountrip delay, has been received with probability $[1 - p_{11}(d)]P_{N0}$, and that convolutional decoding of the two NSV's, $\hat{V}_2(x)$ and $\hat{V}_3(x)$, has failed with probability P_{10}. The transition probability from state 01 to state 10 is therefore given by

$$p_{01,10}=[1-p_{11}(d)]P_{N0}P_{10}.$$

By a similar argument we obtain

$$p_{00,00} = [1 - p_{01}(d)] P_{N0} P_{00}$$
$$p_{00,01} = p_{01}(d) P_{N1} P_{01}$$
$$p_{01,10} = [1 - p_{11}(d)] P_{N0} P_{10}$$
$$p_{01,11} = p_{11}(d) P_{N1} P_{11}$$
$$p_{10,00} = [1 - p_{01}(d)] P_{N0} P_{00}$$
$$p_{10,01} = p_{01}(d) P_{N1} P_{01}$$
$$p_{11,10} = [1 - p_{11}(d)] P_{N0} P_{10}$$
$$p_{11,11} = p_{11}(d) P_{N1} P_{11}. \quad \text{(A.2)}$$

A. Transition Probability from State ij to State e, $p_{ij,e}$

Consider the transition from state 01 to state e. In state 01, the decoding of the two NSV's $\hat{V}_1(x)$ and $\hat{V}_2(x)$ sent over channel states 0 and 1, respectively, has failed. $\hat{V}_1(x)$ has been discarded and replaced by a retransmitted vector $\hat{V}_3(x)$, which was received one roundtrip delay after the reception of $\hat{V}_2(x)$. The transition from state 01 to state e means that there is an undetected error pattern in $\hat{V}_3(x)$, or if $\hat{V}_3(x)$ is a NSV, in the decoding of $\hat{V}_2(x)$ and $\hat{V}_3(x)$. Note that $\hat{V}_3(x)$ may be sent over channel state 0 or 1. Consider the case when the channel is in state 0. The probability of such an event is the d step transition probability in the channel model from state 1 to state 0, $[1 - p_{11}(d)]$. There is an undetected error pattern in $\hat{V}_3(x)$ with probability Q_0. If the syndrome check on $\hat{V}_3(x)$ is nonzero (with probability P_{N0}), the output of the convolutional decoding of $\hat{V}_2(x)$ and $\hat{V}_3(x)$ contains an undetected error pattern with probability Q_{10}. The undetected error probability, given that $\hat{V}_3(x)$ is received while the channel is in state 0, is therefore,

$$E_0 = Q_0 + P_{N0} Q_{10}.$$

Similarly, the undetected error probability, given that $\hat{V}_3(x)$ is received when the channel is in state 1, is

$$E_1 = Q_1 + P_{N1} Q_{11}.$$

The transition probability from state 01 to state e is then obtained by averaging E_0 and E_1:

$$p_{01,e} = [1 - p_{11}(d)] E_0 + p_{11}(d) E_1.$$

The transition probabilities from the three other states to state e are obtained similarly, and we have

$$p_{00,e} = [1 - p_{01}(d)](Q_0 + P_{N0} Q_{00}) + p_{01}(d)(Q_1 + P_{N1} Q_{01})$$
$$p_{01,e} = [1 - p_{11}(d)](Q_0 + P_{N0} Q_{10}) + p_{11}(d)(Q_1 + P_{N1} Q_{11})$$
$$p_{10,e} = [1 - p_{01}(d)](Q_0 + P_{N0} Q_{00}) + p_{01}(d)(Q_1 + P_{N1} Q_{01})$$
$$p_{11,e} = [1 - p_{11}(d)](Q_0 + P_{N0} Q_{10}) + p_{11}(d)(Q_1 + P_{N1} Q_{11}). \quad \text{(A.3)}$$

B. Transition Probability from State ij to State c, $p_{ij,c}$

Realizing that the transitions from state ij to states jk, e, and c are mutually exclusive and collectively exhaustive, the sum of the corresponding transition probabilities must add up to 1. Therefore,

$$p_{00,c} = 1 - p_{00,00} - p_{00,01} - p_{00,e}$$
$$p_{01,c} = 1 - p_{01,10} - p_{01,11} - p_{01,e}$$
$$p_{10,c} = 1 - p_{10,00} - p_{10,01} - p_{10,e}$$
$$p_{11,c} = 1 - p_{11,10} - p_{11,11} - p_{11,e}. \quad \text{(A.4)}$$

Appendix B

Some Results About Markov Chains

In this Appendix, we present some results concerning Markov chains which are used throughout the paper.

A. First Passage Probability

The *n step first passage probability,* denoted by $f_{j,b}(n)$, is defined as the probability that a Markov chain starting from state j will be in state b for the first time after n transitions. If b is an absorbing state, these probabilities can be found directly from the n step transition probabilities as follows:

$$f_{j,b}(1) = p_{j,b}(1) = p_{j,b}$$
$$f_{j,b}(2) = p_{j,b}(2) - p_{j,b}(1)$$
$$f_{j,b}(3) = p_{j,b}(3) - p_{j,b}(2)$$
$$\vdots$$
$$f_{j,b}(n) = p_{j,b}(n) - p_{j,b}(n-1). \quad \text{(B.1)}$$

To prove (B.1), we observe that the one-step first passage probability is the same as the one step transition probability. For $n = 2$, the two-step transition probability $p_{j,b}(2)$ contains the probability of visiting state b immediately after the first transition and remaining in state b during the second transition. Hence, the probability of this event, $p_{j,b}(1) \cdot p_{b,b}(1)$, should be subtracted from $p_{j,b}(2)$ to obtain the two-step first passage probability, i.e., $f_{j,b}(2) = p_{j,b}(2) - p_{j,b}(1) \cdot p_{b,b}(1)$. Since b is an absorbing state (i.e., $p_{b,b}(1) = 1$), we have $f_{j,b}(2) = p_{j,b}(2) - p_{j,b}(1)$. The rest of (B.1) is based on similar reasoning.

Once the first passage probabilities have been determined, the following important quantities can be evaluated.

B. Absorption Probability and Mean Absorption Time

The *l step absorption probability,* denoted by $A_{j,b}(l)$, is the probability that a Markov chain initially in state j will be absorbed by absorbing state b *within l transitions.* It follows from (B.1) that

$$A_{j,b}(l) = \sum_{n=1}^{l} f_{j,b}(n) = p_{j,b}(l). \quad \text{(B.2)}$$

The *infinite-step absorption probability,* denoted by $A_{j,b}$, is the probability that a chain starting in state j will *eventually* be absorbed by absorbing state b. Thus, from (B.2),

$$A_{j,b} = A_{j,b}(\infty) = p_{j,b}(\infty). \quad \text{(B.3)}$$

Note that if the Markov chain contains only one absorbing state, $A_{j,b} = 1$, since the chain will eventually reach b and be trapped there.

We denote the *l step mean absorption time* by $M_{j,b}(l)$. It is defined as the mean time that a Markov chain starting in state j will be absorbed by absorbing state b *within l transitions.* From (B.1), we obtain

$$M_{j,b}(l) = \sum_{n=1}^{l} n f_{j,b}(n) = p_{j,b} + \sum_{n=2}^{l} n[p_{j,b}(n) - p_{j,b}(n-1)]. \quad \text{(B.4)}$$

The *infinite-step mean absorption time,* denoted $M_{j,b}$, is then

$$M_{j,b} = M_{j,b}(\infty) = p_{j,b} + \sum_{n=2}^{\infty} n[p_{j,b}(n) - p_{j,b}(n-1)] \quad \text{(B.5)}$$

$M_{j,b}$ is the mean period of time after the chain leaves state j until it is eventually absorbed by state b.

Let S be the set of absorbing states in a Markov chain. Define the *l step chain mean absorption time* as

$$M_j(l) = \sum_{b \in S} M_{j,b}(l), \tag{B.6}$$

and the *infinite-step chain mean absorption time* as

$$M_j = \sum_{b \in S} M_{j,b}. \tag{B.7}$$

Obviously, $M_j(l)$ is the mean time of absorption within l transitions if the chain starts from a given initial state j, and M_j is the mean time required for a Markov chain to be eventually absorbed if it starts in state j.

C. Easy Ways of Finding $A_{j,b}$ and M_j

Because the calculations of $A_{j,b}$ and M_j involve the infinite-step transition probability, $p_{j,b}(\infty)$, direct evaluation of $A_{j,b}$ and M_j from (B.3) and (B.7) become impractical. However, they can be determined in alternative ways. Let N be the number of states in the chain. The $A_{j,b}$'s are related by the following set of equations [28]:

$$A_{j,b} = \sum_{i=1}^{N} A_{i,b} p_{j,i} \quad \text{for all } j \bar{\in} S. \tag{B.8}$$

Note that, when the initial state is b, it is already absorbed in b, and hence $A_{b,b} = 1$, whereas when the initial state is some other absorbing state, say a, then it will never be absorbed by b, and hence $A_{a,b} = 0$. For all other initial states, a set of simultaneous equations may be written from (B.8) whose solution is $A_{j,b}$.

The infinite-step chain mean absorption times M_j can be determined by solving the following set of equations [28]:

$$M_j = 1 + \sum_{i=1}^{N} M_i p_{j,i} \quad \text{for all } j \bar{\in} S. \tag{B.9}$$

Observe that if state i is an absorbing state, the chain is already in an absorbing state, and $M_i = 0$.

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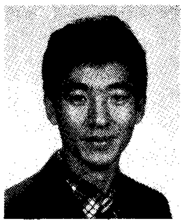

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