12-1980

The Role of Technological Change

Robert U. AYRES
Carnegie Mellon University

Steven M. MILLER
Singapore Management University, stevenmiller@smu.edu.sg

DOI: https://doi.org/10.1016/0095-0696(80)90027-3

Published in Journal of Environmental Economics and Management
Volume 7, Issue 4, December 1980, Pages 353-371.
https://doi.org/10.1016/0095-0696(80)90027-3

The Role of Technological Change

ROBERT U. AYRES AND STEVEN M. MILLER

Carnegie–Mellon University, Department of Engineering and Public Policy, Schenley Park, Pittsburgh, Pennsylvania 15213

We develop an optimal growth model that includes several important new features. First, technological change is endogenously related to the growth of "knowledge." Investment may be directed either towards physical capital or knowledge (or both). Knowledge becomes an effective substitute for scarce resources by increasing the technical efficiency of resource utilization both for consumption and in capital. Nevertheless, a finite quantity resource must be embodied in capital and a finite flow is required for depreciation. Thus, there is an upper limit to technical efficiency and economic growth is thus ultimately limited by the availability of renewable resources. For a simple aggregate production function it is shown that technical efficiency never approaches unity on an optimal path.

THERMODYNAMICS AND ECONOMICS

It is generally accepted that, in the absence of technological progress, resource constraints would impose ultimate limits on economic growth [1–5]. Even if continuing technological progress is assumed, the existence and relevance of such constraints has seemed clearly evident to a number of observers, particularly natural scientists. Economists, on the other hand, have traditionally argued that, since resource availability is normally a function of both prices and the current state of exploration, extraction and conversion technology, the notion of rigid "limits" is inappropriate and should be discarded [6]. The neoclassical "economists' view" of the resource problem is that impending scarcity casts a shadow, so to speak, in the form of rising prices. This, in turn, automatically triggers technological substitution of capital—or of other resources—for the scarce resource on the supply side, and altered consumption patterns on the demand side. Increased supply, coupled with decreased demand, brings supply and demand back into balance and the market clears. Thus, an actual shortage never occurs in a perfectly free market, provided the elasticity of substitution between reproducible capital and exhaustible resources is sufficiently large [7, 8]. Stated another way, in the classical paradigm of economics, scarce resources are infinitely substitutable and economic growth can, in principle, go on forever. Certainly there are well-documented historical examples of technological substitutions that have "come to the rescue" in the above sense. Moreover plausible "nonscarce" substitutes for most—if not all—so-called "scarce" resources can be identified without much difficulty by any competent technologist [9, 10].

The neoclassical view of the resource problem noted above, is not shared by all economists. For instance, it has been noted by Cummings and Schultze [11] that substitution of capital for scarce resources has limits if scarce resources (either mass or energy) must be embodied in the capital itself. They show that, if energy is embodied in capital, the first law of thermodynamics—conservation of mass energy—precludes the possibility of boundless economic growth unless a limitless supply of renewable energy is available. On the other hand, if the production function is constrained by the availability of some scarce material species, even constant consumption is impossible unless the critical scarce materials can be recovered and recycled with 100% efficiency. The latter possibility appears, at first glance, to be ruled out by the Second Law of Thermodynamics (the "entropy" law). Indeed, it is on this basis that Georgescu-Roegen [12] denies even the possibility of a steady-state economy—still less a perpetually growing one [12].

While remaining strong, advocates of the application of physical principles—especially the laws of thermodynamics—to economics [13], we do not think Georgescu-Roegen's arguments for this position are convincing. Georgescu-Roegen's reasoning is based on the law of increasing entropy—i.e., dispersion and consequent unavailability—of scarce materials, coupled with an assertion that intrinsically scarce materials *cannot* be recovered (regardless of energy expenditure) from "average" rocks in the Earth's crust [14] or presumably equivalent sources like the ocean. As to the second assertion, we argue the contrary. In fact, it is a consequence of the First Law of Thermodynamics (and the law of gravity) that human activity, no matter how wasteful, can never change the total amounts, or the average concentration, of each element in the Earth's crust (barring minor amounts expended in extraterrestrial activities). Since some parts of the Earth will always have concentrations of any given element—say gold—*below* the average crustal abundance, other parts must always have concentrations *above* the average. It follows that physical dissipation (i.e., dispersion) of scarce elements *can never result in a distribution worse* (from the standpoint of recovery) *than a hypothetical homogeneous regolith in which every element is present in exactly its average crustal abundance*.

The question arises: can all elements be extracted from such a regolith? The answer is, trivially, yes—*provided there is enough available energy*. It is not necessary to provide a detailed "recipe" for the technology of extracting scarce elements from such material. It is sufficient to demonstrate that such a technology is possible. Indeed, a candidate has been named and described: it is the so-called "fusion torch" [15] which uses an extremely hot ionized gas or plasma (e.g., from a thermonuclear reaction) as a "universal solvent" to vaporize, dissociate and ionize other materials.[1]

To recapitulate, the limiting factor in recycling or in finding substitutes for materials is energy or more precisely that component of energy that can be converted by a "prime mover" into kinetic energy of motion or some other form of "useful work." This fraction is called available work [16]. By expending enough energy one can either find a substitute for any material, or obtain the material itself from low-quality sources. For instance, as native copper and high-quality copper ores were exhausted in the 19th century, means were developed to extract copper from ores of much lower quality (now averaging about 0.3% in the U.S.). But there is no substitute for energy *per se*. And, though the total amount of energy in a system is conserved through any process or transformation, the useful component is not. In fact, the well-known second law of thermodynamics is essentially a mathematical statement of the fact that available work is *not* conserved. On the

[1]About 10,000 MW of fusion energy suffice to process 1.67×10^5 moles of waste material per minute. Both chemical recombination and electromagnetic means can be used to separate individual elements (and even isotopes).

contrary, available work is converted into forms of energy (i.e., low-temperature heat) which cannot be utilized to do work. The increase of entropy is a measure of increasing energy unavailability. For the rest of this paper we use the term energy as shorthand for available work.

If there is any natural resource which might constitute an ultimate limiting factor for economic growth, then, it is the primary (renewable) supply derived from the sun, plus the stock of available reserves in the form of fossil or nuclear fuels in the Earth's crust. Of course, the amount of available work that can be extracted over a long period of time from fossil fuels or uranium is obviously a function of the state of technology. As technology progresses, new deposits are discovered, subeconomic deposits become available reserves, and completely new energy sources become feasible for practical exploitation.

The circumstances of accelerating use, and possible near or medium term exhaustion, of *known* energy resource, together with major uncertainties as to the feasibility, cost and timing of downstream substitutes, constitute a challenge for economic analysis. Several frameworks are possible. The case where technology offers no substitution possibility was examined many years ago by Hotelling [17] and in greater detail more recently by Herfindahl [18] and others. The basic results need not be recapitulated here. Nordhaus [19] considered a variant case where the supply curve becomes infinite at some finite price, where the so-called "backstop" technology takes over and provides unlimited energy availability. Stiglitz [7] assumes "technological progress" occurs at a constant rate, regardless of policy indefinitely. Dasgupta and Heal [20] have introduced a different twist: the new technology eliminates the *need* for the resource, and arrives exogenously and costlessly at some uncertain time in the future. In more recent work these authors [21], as well as Kamien and Schwartz [22] (and others) have examined variations where the new development itself becomes endogenous and costly. In the context of energy analysis, these models retain the backstop concept, the focus being on a single millennial "breakthrough" technology and on optimal policy during the interim period.

For Nordhaus the problem is to compute the price at which available energy supply becomes effectively infinite and to allocate supplies from scarcer and increasingly costly resources during the interim in such a way as to minimize an appropriate objective function. In effect, choice of interim technology is the control variable in the Nordhaus formulation. For Dasgupta and Heal the problem is similar, with the added complexity of uncertainty; in their 1974 paper the arrival of the millennium is independent of the price of exhaustible resources during the interim, but optimal policy choice is complicated by stochastic elements. Introducing a link between expenditures on R & D and the time required to attain the millennium (Dasgupta *et al.* [21], Kamien and Schwartz [22]) adds another dimension to the problem and another control variable.

The framework adopted in the present paper differs from those cited above in several ways. First, no technological millennium, in the Nordhaus sense, is envisaged. On the contrary, technical progress is reflected in economic terms is identified with a concave function of "technological knowledge," which is taken to be an explicit, endogenously determined factor of production. The term "knowledge," as we use it, will be defined later. While knowledge may increase without limit, its effect on production is assumed to have limits. Both the assumption of concavity—or declining returns—and the assumption that technological knowledge is endogenous to the productive system, are in contrast to the prevailing views in the neoclassical literature. An important notion underlying our entire approach is that natural resources, physical capital and knowledge are all mutually substitutable (within limits, to be discussed later) and therefore equivalent, at least in an information-theoretic sense. This notion is similar to that suggested by Tribus *et al.* [23] and Marchetti [24]. In fact, we assume that all the above forms of capital are equivalent to, and therefore measurable in terms of available work or "negentropy."[2] As noted already, this view differs from that of Georgescu-Roegen [14].

STATE (OR STOCK) VARIABLES

To develop an explicit optimal policy choice model it is convenient at this stage to introduce several state variables, as follows:

population N; labor force $L = bN$,
constant-vintage invested capital $K = kN$,
stored available energy reserves $S = sN$,
technological knowledge T.

Here k and s are per capita stocks. As mentioned, K, S, T are all regarded as alternate forms of negentropy. A Hamiltonian model will be constructed in which these variables appear as factors of production, or via constraints. We consider the behavior of each state variable, in the above order.

It is usual in the literature to assume that population N grows exponentially over time, at a constant rate n. We find this assumption simplistic (on biological grounds) and mathematically unnecessary. A more reasonable assumption seems to be that humans can, and eventually will, regulate their population to the level that can be supported by the natural environment [25]. A simple differential equation

$$\dot{N} = gN(1 - N/\bar{N}), \qquad N \geq 0, \tag{1}$$

where $\bar{N}$ is the maximum population theoretically sustainable by conventional agriculture, given existing world soil characteristics, rainfall, insolation and topographic conditions. We need not concern ourselves unduly with the numerical value of $\bar{N}$.[3] However, since population growth is exogenous by assumption, we do not concern ourselves with the notion of optimizing N. Note that N is nondecreasing. We will focus attention, subsequently, on *per capita* measures of production and consumption.

[2]Negentropy is a measure of intrinsic order, or nonrandomness. A natural resource stock is economically valuable precisely because the useful elements are presorted and easy to separate, rather than being randomly distributed. Forms of energy (e.g., electricity or high-temperature heat) with very high available work content are "presorted" in a similar way. Electric energy is a direct manifestation of the sorting and separation of negative charges from positive charges. The higher the electric potential, the more separation occurs. Similarly, high temperature reflects a preselection of high velocity molecular states from the ensemble of all possible states. Finally, knowledge can also be regarded as a kind of negentropy in that knowledge of possibilities is a necessary condition for selection. Indeed the act of nonrandom selection is an exercise of knowledge. See Ayres [13].

[3]See Buringh *et al.* [5]. Obviously, if humans were able to colonize other planets or grow food in orbiting space colonies(!) this limitation would not apply.

Next consider fixed (constant vintage) invested capital K. The usual assumed accumulation law is

$$\dot{K} = I - dK, \qquad I \geq 0, \tag{2}$$

where I is the current level of investment and d is the rate of physical depreciation, assumed to be exponential for convenience. In per capita terms, using (1), we obtain

$$\dot{k} = i - \big(d + g(1 - N/\overline{N})\big)k, \qquad i \geq 0. \tag{3}$$

The nonnegativity of investment implies that fixed capital cannot be consumed. We emphasize that K and k measure the quantity of constant vintage capital referred to a given vintage year (e.g., 1980), it being understood that technological improvements over time will tend to increase the capabilities of current machines and/or structures built at later times. Thus a given quantity of constant capital will be equivalent in productive capability to a smaller quantity of current capital, at any future time. This performance improvement reflects the embodiment of new technological knowledge.

Traditionally aggregated capital is measured in dollar terms, for obvious empirical reasons. However, the above conceptual distinction between constant capital and technological knowledge embodied in capital, undermines the presumption that capital of different vintages can be measured meaningfully in dollars. We therefore fall back on the notion of negentropic equivalence, introduced above. In this paper, both capital K and knowledge T will be measured in common units, viz., available work.

The third state variable to be considered in the model is the stock of available energy (negentropy) S, either renewable or exhaustible. There is a straightforward relationship between stock and flow, namely

$$R = \overline{R} - \dot{S}, \qquad \dot{S} \leq 0, \tag{4}$$

where R is the renewable flow (e.g., sunlight) and $(-S)$ is the rate of extraction of the exhaustible resource from the Earth's crust. In per capita terms, we obtain

$$\dot{s} = \bar{r} - r - g(1 - N/\overline{N})s, \qquad \dot{s} \leq 0. \tag{5}$$

The inequality states that the stock of exhaustible reserves can never increase. Another constraint on the problem is that total extraction of exhaustible resources over time is limited to the amount of the original stock S_0, viz.,

$$S_0 + \int_0^\infty S\,dt = S_0 + \int_0^\infty (\overline{R} - R)\,dt = 0$$

or in per capita terms,

$$S_0 + \int_0^\infty (\bar{r} - r)N\,dt = 0. \tag{6}$$

The fourth stock is technological knowledge, T. The increase of technological knowledge can be equated roughly with technological progress, as the term is used in the literature.[4] Knowledge makes its contribution to productivity indirectly, either through embodiment in capital, or improvement in the skills of the labor force, or by increasing the resource base through discoveries. There is no direct measure of knowledge, however. For this reason, perhaps, one does not normally associate a rental value or a "shadow price" for knowledge *per se*.

What, then is knowledge? There is no generally accepted definition—the word connotes many things—but in the present context the following definition seems to be reasonably satisfactory: *Knowledge is the capability to copy or reproduce an "object" given the appropriate tools and materials.* The "object" being reproduced need not be tangible; it may be a behavior pattern, a picture, a design, a piece of music or a set of data points. Based on the foregoing general definition, the intuitive definition of knowledge as an understanding of "how things work" is valid, if somewhat limited.[5] Theoretical knowledge does not contribute to productivity unless it is incorporated in machine capabilities, labor skills, or product design.

It is not too misleading to think of knowledge manifest in labor skills as a set of "programs" for a self-programming biological computer (the brain) operating a general-purpose machine (the body). General education starting with infancy provides the comprehensive internal "executive monitor" for the human brain, while job-related training provides the specific "operating programs" for controlling process equipment, handling tools, driving vehicles, or carrying out other functions. Managerial skills, including allocation of resources, scheduling, motivation of employees, etc., are also forms of knowledge. Productivity in labor is an aggregate measure of these skills.

Evidently, in a similar manner, the productivity of capital—or, roughly speaking, its quality—is related to the knowledge "embodied" in it. Successive generations of capital equipment are typically more and more productive because they embody an accumulation of knowledge, based on research and practical experience. Over successive generations, skills initially learned by the workers are gradually shifted to the machines—permitting them to be operated by less skilled workers, or to perform more specialized and intricate tasks. Clerical and some managerial functions, too, can be shifted from humans to machines (e.g., computers).

It follows that only productive facilities of the same vintage can be compared meaningfully in terms of scale of output. Returns-to-scale are only well defined when output is measured per unit of capital of the same intrinsic productivity or quality—although the caveat is seldom explicitly stated. However, facilities producing at the same scale of output, but built at different times will generally utilize different technologies. Greater output per unit of constant capital or constant labor input by the more recent of the two will generally be attributable to "technological progress"—which is another term for increased skills and/or embodied knowledge.

As noted, knowledge can be embodied in product design or in services. However, for reasons that will emerge, we choose to ignore this last form of embodied knowledge in the formulations that follow. The growth of technological knowledge

[4] The concept of "technological knowledge" is obviously not original with us. It has been treated in the economics literature, so far as we have been able to ascertain, largely as an exogenous prior. Denison's discussion is fairly typical [26].

[5] Each possible "object" has a description with characteristic minimum information content (to distinguish it from all other possible objects). Knowledge, on the other hand, is a property not of passive objects, but of transformation processes, each of which also has a characteristic information content.

can be presumed, for purposes of our model, to follow a simple law, such as

$$\dot{T} = J = Nj, \qquad j \geq 0, \tag{7}$$

where J is the current rate of embodiment (diffusion) of theoretical knowledge in machines or labor skills. It may be noted that (7) is similar to the formulation used by Dasgupta *et. al.* [21].

Again, the restriction $j \geq 0$ must hold, reflecting the fact that investment in knowledge is irreversible. The rate of embodiments (or diffusion) is doubtless related to the rate of aquisition of new knowledge due to *R & D* over some prior period. We do not explore this linkage. It is important to note that both T and j are not directly measurable.

TECHNICAL EFFICIENCY

It is convenient at this point to introduce a new variable E which is a function of T and which can be easily interpreted and measured. Let

$$E = (1 + \exp(T_0 - T))^{-1}, \tag{8}$$

where T_0 is an accumulation of knowledge such that $E = 0.5$ when $T = T_0$. This function asymptotically approaches unity. Solving,

$$T = T_0 + \ln(E/1 - E). \tag{9}$$

The variable E satisfies a nonlinear differential equation similar to (2), vis.,

$$\dot{E} = E(1 - E)J = E(1 - E)Nj \tag{10}$$

It can be seen that E is an elongated S-shaped curve: it is convex near the origin, but after passing a point of inflection it enters a concave region of saturation, asymptotically approaching unity. This behavior is characteristic of any efficiency measure over time. Since T is, by assumption, a stock of knowledge pertaining to production, we can interpret E more precisely as the *technical efficiency* with which the economic system converts basic resource inputs—notably available energy—into a given fixed set of goods and services for consumers. This usage of the term of efficiency is broader than the most familiar one in economics, where an "efficient" allocation of resources is one that yields a given final product with the minimum resource input that is consistent with the current state of technology. Our use of the term implies that, as technology changes there will be a progression of (static) optima, eventually approaching a limit. This progression need not be monotonic, however, because utilization of existing labor and capital resources may fluctuate below the optimum level, in the short run, due to the vagaries of the business cycle.

It is important to emphasize here, that ultimate technological efficiency is inherently limited, even though knowledge *per se* may be accumulated indefinitely. There are definite and well-known limits on physical performance in almost every field (see Ayres [27]). For instance, there is a definite lower limit to the amount of electricity required to produce a horsepower of mechanical work. There is a lower limit, similarly, to the amount of electricity required to produce a given amount of illumination. And, of course, there is a lower limit to the amount of available work derived from fossil fuels that is needed to generate a given amount of electricity. There are limits to the capacity of information channels or computers. There are upper limits to the strength of materials. Temperatures and pressures cannot be less than zero. Velocity cannot exceed the speed of light. And so on.

PRODUCTION FUNCTION

The next step is to introduce an aggregate production function of some or all of the "state" variables introduced above. It is standard to assume that output F is an explicit function of the labor supply (stock) L, and the capital stock K. The function itself essentially specifies how these two factors jointly contribute and how they can be substituted for each other.

Output is also commonly assumed to be proportional to an exogenous "technological progress" term. It is traditional, at least in the literature of resource economics (e.g., Stiglitz [28]), to assume technological progress is an exogenously given, exponentially increasing function of time. Technology is, in effect, regarded as a "deus ex machina" which requires only time—no investment of capital or labor—to produce more and more outputs from the same inputs, forever. Stated thus, the inherent absurdity of this proposition hardly needs comment. It violates ordinary common sense, as well as the laws of conservation of matter and energy, and it is really no wonder that, with the help of such an assumption, Stiglitz [7] and Solow [8] have suggested that economic growth can—in principle—continue without limit.

A more technical, but still critical objection to the simplistic treatment of technological progress is that it does not permit substitution between technological knowledge and other factors of production. On the contrary, experience suggests that such substitutions can and do occur, although they tend to be unidirectional (i.e., increasing embodied knowledge reduce the need for capital or labor, but apparently not vice versa). In this context it might be noted that the observed substitutions between capital and labor have also been largely unidirectional.

Our approach to defining a production function attempts to overcome the above objections by treating the stock of knowledge, T, as a factor of production. However, to eliminate the possibility of unlimited exponential productivity increase, we introduce an efficiency term E as explicit function of knowledge T. Technical efficiency is, therefore, an implicit function of the historical path of economic development. This developmental path is, as we shall show, a function of investments in (constant vintage) capital, in the creation and diffusion of technological knowledge, and of the rate of extraction and use of exhaustible resources.

So far we have discussed the relation of output to "state" (or stock) variables. However, it is really the renewable service flows generated by the stocks in question that determine maximum output over time. Implicitly, we assume something like full utilization of capital, labor and knowledge, so that the magnitudes of the stocks and the resulting flows are proportional and measures of the former can be regarded as a surrogate for measures of the latter. The situation is more complex, however, for the resource flow R, since it *cannot* be assumed that the flow R is proportional to the existing stock S of exhaustible resources. Though S is a state variable, output is not a function of S, but of R. The question to be resolved is: should R itself be regarded as a state variable?

The assumption adopted in most of the recent literature is to treat R as a state variable (analogous to K) and thus as a factor of production. This is compatible with the common view that the use of available energy (the "ultimate" resource) is essentially proportional (in the short run) to the output F of goods and services, viz.,

$$R \propto F.$$

Having already substituted an embodied energy measure for constant-vintage capital K, it is logically consistent and convenient to do the same for total output F. Given that F measures the quantity of available energy "embodied" in the output of the economy and R is, by definition, the input of available energy, one is tempted to assert that efficiency E is necessarily equal to the ratio F/R:

$$E = \frac{F(K, L, R)}{R} \tag{11}$$

Equation (11) can be rewritten in reduced form:

$$E = \frac{f}{r} \quad \text{or} \quad r = \frac{f}{E}, \tag{12}$$

where f is per capita output

$$f = F/N. \tag{13}$$

As noted previously, we assume $L = bN$. We must emphasize, here, that substitution possibilities between resources and capital or labor do exist but are intrinsically limited. Resources destined for physical embodiment in final products cannot be substituted for except by design changes which we have ruled out from the beginning. Only intermediate uses of resources can be replaced by capital or labor services—or conversely—and all such substitutions are captured by changes in technical efficiency E.[6]

It follows from these considerations that the use of traditional or neoclassical (Cobb–Douglas or CES) production functions which permit "infinite" substitutability between resources and other factors of production is simply incorrect on physical grounds. This has been pointed out by Ayres [13], Cummings and Schultze [11], and implicitly by Georgescu-Roegen [32]. It immediately follows that the neoclassical long-run criterion for perpetually continued economic growth proposed by Stiglitz [7] and endorsed by Solow [8] cannot be satisfied. Such production functions clearly fail to satisfy the "essentiality" criteria set forth by Dasgupta and Heal [20], viz.,

$$\lim_{R \to 0} F(R) = 0; \qquad \lim_{R \to 0} \frac{\partial F(R)}{\partial R} = \infty.$$

[6]See, for example, Herfindahl [18].

[7]There is an ongoing controversey in the econometric literature as to whether energy and capital are substitutes or complements. (See Berndt and Jorgenson [29], Berndt and Wood [30], Griffen and Gregory [31]). An obvious rationale for complementarity is that space-heating requirements depend on the technology embodied in buildings at the time of their construction, transport energy requirements depend on the technology embodied in automobiles, railroad cars or aircraft as and when produced, and so forth.

While substitution between capital and resource flows (available energy) is perfectly feasible in the short run, it is only the use of *nonembodied* energy that can be reduced by capital investment. When this substitution has gone as to the ultimate limit ($E = 1$) there remains a irreducible requirement for available energy that must still be externally supplied. The elasticity of substitution at this point must be identically zero. To escape the dilemma noted above, a nontraditional production function is needed. Cummings and Schultze have suggested the so-called Bergstrom function

$$F(K,R) = [1 - \exp(-\gamma K/R)]R.$$

This may be generalized to

$$F(K,L,R) = [1 - \exp(-\Pi(K,L)/R)]R, \tag{14}$$

where (K,L) is a conventional production function of the Cobb–Douglas or CES type. It can be verified easily that the isoquants of (14) have the desired properties. We shall use (14) later.

In reduced form

$$f(k,b,r) = [1 - \exp(-\pi(k,b)/r)]r, \tag{15}$$

where b is the fraction of the population N belonging to the labor force L,

$$N\pi(k,b) = \Pi(K,L) \tag{16}$$

and from (12)

$$E = 1 - \exp(-\pi(k,b)/r). \tag{17}$$

Solving for r

$$r = \frac{-\pi(k,b)}{\ln(1-E)}. \tag{18}$$

Substituting (18) back into (15) $f(k)$ can be rewritten in terms of k, b, E, viz.,

$$f(k,b,E) = \frac{-\pi(k.b)E}{\ln(1-E)}. \tag{19}$$

It is useful to note that, for the Bergstrom function

$$f_k = \frac{-\pi_k(k,b)E}{\ln(1-E)} = \frac{\pi k}{\pi} f, \tag{20}$$

$$f_E = \left[\frac{1}{E} + \frac{1}{(1-E)\ln(1-E)}\right] f. \tag{21}$$

OPTIMAL POLICY PATHWAY

We proceed, now, to define a utility function U of the conventional sort. Suppose U is a function of net per capita consumption y, where

$$Y = yN. \tag{22}$$

Thus

$$Y = F - I - J \tag{23}$$

and

$$y = f - i - j \tag{24}$$

As usual we assume $U(y)$ is strictly concave and twice differentiable. An optimal policy requires that we maximize an integral over time

$$\max W = \int_0^z \exp(-\delta t) U(y)\, dt + a_1 k(z) + a_2 S(z) + a_3 T(z), \tag{25}$$

where δ is the assumed intertemporal discount rate and z, fixed in advance, is the end of the "planning period." The constants a_1, a_2, and a_3 are chosen to guarantee that the terminal conditions for an optimal solution will be satisfied. They are chosen to put a prohibitively high penalty on negative values of the state variables at the terminal point. Having said this, the a_i need not be specified further (see Arrow [33]). This formulation permits us to consider a range of possible social discount rates, including zero. The integral W must be maximized subject to a number of restrictions, including the first-order constraints on "state variables" (Eqs. (4), (6), (7)) and the nonnegativity conditions

$$i \geq 0, \tag{26}$$

$$j \geq 0, \tag{27}$$

$$-\dot{S} \geq 0, \tag{28}$$

$$S_0 + \int_0^\infty (r - r) N\, dt \geq 0. \tag{29}$$

To solve this problem we define a Hamiltonian system, following Takayama [34]

$$\begin{aligned} H = e^{-\delta t}\Big\{ & U(f - i - j) + P_k\Big[i - \big(d + g(1 - N/\bar{N})\big)k\Big] \\ & - P_s\Big[\bar{r} - r - g(1 - N/\bar{N})s\Big] + P_T N j \\ & + q_k i - q_s\Big[\bar{r} - r - g(1 - N/\bar{N})s\Big] + q_T N j\Big\} + \lambda(\bar{r} - r). \end{aligned} \tag{30}$$

We eliminate the per capita resource flow r (not a state variable in our formulation) by using (12), which yields

$$H = e^{-\delta t}\Big\{U(f-i-j) + P_k\Big[i - \big(d + g(1-N/\bar{N})\big)k\Big]$$
$$- P_s\Big[\bar{r} - \frac{f}{E} - g(1-N/N)s\Big] + P_T Nj$$
$$+ q_k i - q_s\Big[\bar{r} - \frac{f}{E} - g\big(1 - N/\bar{N}\big)s\Big] + q_T Nj\Big\} + \lambda\Big(r - \frac{f}{E}\Big) \tag{31}$$

with further requirements as follows:

$$q_k i = 0, \qquad q_k \geq 0, \tag{32}$$

$$q_s\Big[\bar{r} - \frac{f}{E} - g\big(1 - N/\bar{N}\big)s\Big] = 0, \qquad q_s \geq 0, \tag{33}$$

$$q_T Nj = 0, \qquad q_T \geq 0, \tag{34}$$

and

$$\lambda\Big(S_0 + \int_0^\infty (\bar{r} - r)N\,dt\Big) = 0, \qquad \lambda \geq 0. \tag{35}$$

We note that the co-state variables P_k, P_s and P_T are shadow prices corresponding to flows of services from constant vintage capital stock, resource stock and knowledge stock, respectively. The constant can be interpreted as the present value of the shadow price of the exhaustible resource.

Initial conditions are simply

$$N(0) = N_0,$$
$$k(0) = k_0 = K_0/N_0,$$
$$s(0) = s_0 = S_0/N_0,$$
$$T(0) = T_0.$$

Terminal conditions (at the point $t = z$) are as follows:

$$P_k(z)k(z) = 0, \qquad P_k(z) \geq 0,$$
$$P_s(z)s(z) = 0, \qquad P_x(z) \geq 0,$$
$$P_T(z)T(z) = 0, \qquad P_T(z) \geq 0.$$

But $k(z) > 0$ and $T(z) > 0$, by assumption, whence $P_k(z) = P_T(z) = 0$. On the other hand, it is convenient to specify $s(z) = 0$, whence $P_s(z) \geq 0$.

Assuming the usual constraint qualifications hold, the first two Euler–Lagrange equations for an optimal path are

$$\frac{\partial H}{\partial i} = 0 = e^{-\delta t}[-U'(y) + P_k + q_k], \tag{36}$$

$$\frac{\partial H}{\partial j} = 0 = e^{-\delta t}[-U'(y) + P_T + q_T], \tag{37}$$

whence

$$U'(y) = P_k + q_k \tag{38}$$

and

$$P_T + q_T = P_k + q_k, \tag{39}$$

where $U'(y)$ is the marginal utility of consumption.

Upon differentiating H with respect to P_k, P_s, P_T respectively, we obtain

$$\frac{\partial H}{\partial P_k} = \dot{k}, \tag{40}$$

$$\frac{\partial H}{\partial P_s} = \dot{s}, \tag{41}$$

$$\frac{\partial H}{\partial P_T} = \dot{T}, \tag{42}$$

which yield the required first-order conditions (3), (5), and (7), respectively. Finally, we differentiate with respect to the state variables k, s, T.

$$\frac{\partial H}{\partial k} = -\left(\dot{P}_k - \delta P_k\right)e^{-\delta t}, \tag{43}$$

$$\frac{\partial H}{\partial s} = -\left(\dot{P}_s - \delta P_s\right)e^{-\delta t}, \tag{44}$$

$$\frac{\partial H}{\partial T} = -\left(\dot{P}_T - \partial P_T\right)e^{-\delta t}. \tag{45}$$

Using (38) and the fact that

$$\frac{\partial}{\partial T} = E(1 - E)\frac{\partial}{\partial E}$$

we obtain from (43)

$$0 = \dot{P}_k - P_k\delta + (P_k + q_k)f_k - P_k\left(d + g\left(1 - N/\bar{N}\right)\right) + \left(P_s + q_s - \lambda e^{\delta t}\right)\frac{f_k}{E}, \tag{46}$$

from (44)

$$0 = \dot{P}_s - P_s\delta + (P_k + q_k)f_s + (P_s + q_s)g\left(1 - N/\bar{N}\right) + \left(P_s + q_s - \lambda e^{\delta t}\right)\frac{f_s}{E}, \tag{47}$$

and from (45)

$$0 = \dot{P}_T - P_T\delta + (P_k + q_k)E(1 - E)f_E + \left(P_s + q_s - \lambda e^{\delta t}\right)E(1 - E)\left[f_E/E - f/E^2\right]. \tag{48}$$

Clearly, from (33), there are two distinct phases (or eras) to consider, viz.,

$$\text{pre-exhaustion era } \le 0, t \le hz, \ge 0, \qquad q_s = 0,$$

$$\text{post-exhaustion era} \le hz, t \le z, s = 0, \qquad q_s \ge 0.$$

Equations (46)–(48) take different forms in the two eras. Within each era there are four different types of solutions, depending on whether $i,j = 0$, $q_k = 0$, $q_T = 0$. When i,j are both nonzero (a "free interval), $q_k = q_T = 0$. The solution for this case is unconstrained by the nonnegativity conditions [18, 19]. If such constraints were eliminated altogether, there would be occasionally periods of disinvestment ($i < 0$ or $j < 0$), resulting in a fluctuating time path for per capita capital accumulation and knowledge accumulation. Imposition of the nonnegativity constraints on i,j results in a monotonically increasing or time path that coincides with the unconstrained solution during "free intervals," but departs from it during intervals when the unconstrained solution results in $i < 0$ or $j < 0$. The relation between the two solutions is illustrated in Fig. 1.

From the perspective of short-term economic management, the optimizer (presumably a central planner) must monitor the unconstrained solution, called the "myopic" solution, by Arrow. At a point where it is still rising, but is about to turn over and fall again in the near future, the planner will cease investing. The myopic solution may continue, for a while, to rise above the "corrected" ($i = 0$) solution; later it will fall and drop below. Still later it reverses and rises again. When it reaches the same level where the two originally diverged, the planner reopens the investment spigot ($i > 0$) and the two solutions coincide for the next interval.

Hereafter, we concern ourselves with the unconstrained solution ($qk = q_T = 0$). Substituting in (47) and eliminating P_k between (45) and (47) yields an algebraic expression for λ, viz.,

$$\lambda e^{\delta t} = P_s + q_s + P_k E\left[\frac{f_k - \left(d + g\left(1 - N/\bar{N}\right)\right) - E(1 - E)}{f_k - (1 - E)(Ef_E - f)} f_E\right] \tag{49}$$

Substituting (49) back into (46) yields a differential equation for P_k, the shadow price of a unit of service provided by constant-vintage capital. This equation, which is valid for both $s \neq 0$ and $s = 0$ (hence for all t) is as follows:

$$\frac{\dot{P}_k}{P_k} = \delta - (1 - E)f\left[\frac{f_k + \left(d + g\left(1 - N/\bar{N}\right)\right)(Ef_E/f - 1)}{f_k - (1 - E)(Ef_E/f - 1)}\right] \tag{50}$$

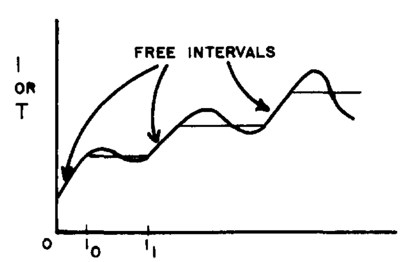

FIG. 1.

For a production function with the convenient property $Ef_E = f$, (which is not true for the Bergstrom production function), (49) and (50) become

$$\lambda e^{\delta t} = P_s + q_s + P_k E\left[1 - \frac{d + g(1 - N/N)}{f_k} - (1 - E)\frac{f}{f_k}\right], \tag{51}$$

$$\frac{\dot{P}_k}{P_k} = \delta - (1 - E)f \tag{52}$$

In the case $f_s = 0$, which holds for our choice of production function, (47) simplifies to

$$\frac{\dot{P}_s}{P_s} = \delta - g\left(1 - N/\bar{N}\right) - \frac{q_s}{P_s} g\left(1 - N/\bar{N}\right), \tag{53}$$

where the last term is zero for $t \leq hz$ (where $s \geq 0$) but is nonzero for times $t \geq hz$. Equation (50) is a generalization of the Ramsay condition for optimal capital accumulation.

Equation (53) is, essentially, a variant of the well-known Hotelling model, where the shadow price of the exhaustible resource rises at the social discount (i.e., interest) rate minus the current rate of population growth. In the absence of any constraints on resources, the multipliers P_s and λ must vanish resulting in a simple relationship between marginal productivities of constant vintage capital and knowledge on any free interval along the optimal growth path, namely,

$$f_k - \left(d + g\left(1 - N/\bar{N}\right)\right) = E(1 - E)f_E = f_T. \tag{54}$$

For the case of an unconstrained (free) interval utilizing (38), one has

$$P_k = U'(y),$$
$$\dot{P}_k = U''(y)\dot{y}. \tag{55}$$

Introducing the elasticity of marginal per capita utility η, defined as

$$\eta(y) = -y\frac{U''(y)}{U'(y)} \tag{56}$$

one obtains

$$\frac{\dot{P}_k}{P_k} = -\eta\frac{\dot{y}}{y}, \tag{57}$$

which can be substituted into (50) or (52) to obtain equations for the time path of consumption.

LIMITING CASES

Assume the Bergstrom production function (15), where $\Pi(K,L)$ has the conventional Cobb–Douglas form

$$\Pi(K,L) = AK^{\alpha}L^{1-\alpha}, \tag{58}$$

$$\pi(k,b) = Ak^{\alpha}b^{1-\alpha}. \tag{59}$$

In the post-exhaustion era, the resource flow is limited by r (a constant), whence (from (15))

$$f \to 1 - \exp(-\pi/\bar{r})F = Er,$$

$$E \to 1 - \exp(-\pi/\bar{r}),$$

$$f_k = \frac{\pi k}{\pi} \qquad f = \frac{\alpha}{k} \qquad f = \frac{\alpha \bar{r}}{k}E,$$

$$\frac{\dot{P}_k}{P_k} = \delta - (1-E)f\left[\frac{f_k + \left(d + g\left(1 - N/\bar{N}\right)\right)(Ef_E/f - 1)}{f_k - (1-E)(Ef_E/f - 1)}\right] \tag{60}$$

$$= \delta - r\left[\frac{\frac{\alpha r}{k}(1-E) + \left(d + g\left(1 - N/\bar{N}\right)\right)E/\ln(1-E)}{\frac{\alpha r}{k} - 1/\ln(1-E)}\right].$$

From $P_k \geq 0$ and the terminal condition $P_k(z) = 0$ it follows that $P_k \leq 0$ as $t \to z$, which implies that the term in square brackets must be positive and greater than $\delta/\bar{r}$. That is,

$$\frac{\frac{\alpha \bar{r}}{k}(1-E) + \left(d + g\left(1 - N/\bar{N}\right)\right)E/\ln(1-E)}{(\alpha\bar{r}/k)1/\ln(1-E)} \geq \delta\bar{r}$$

whence

$$\frac{\alpha\bar{r}}{k}(1-E) + \left(d + g\left(1 - N/\bar{N}\right)\right)E/\ln(1-E) \geq \left(\frac{\alpha\bar{r}}{k} - 1/\ln(1-E)\right)(\delta/\bar{r}),$$

$$\frac{-\alpha}{k}\ln(1-E)\big((r(1-E) - \delta\big) \geq \left(d + g\left(1 - N/\bar{N}\right)\right)E + \delta/\bar{r}.$$

Clearly this condition *cannot* be satisfied unless

$$\bar{r}(1-E) > \delta$$

or

$$E \leq 1 - \delta/\bar{r}$$

and

$$\frac{k}{\alpha} \le -\left[\frac{(\bar{r}(1-E)-\delta)\ln(1-E)}{d+g\left(1-N/\bar{N}\right)E+\delta/\bar{r}}\right].$$

In the limit as $\delta \to 0$ (which implies a truly "stationary" economy).

$$\frac{k}{\alpha} \le \left[\frac{\bar{r}(1-E)\ln(1-E)}{d+g\left(1-N/\bar{N}\right)E}\right] \tag{61}$$

and if we also specify that population has reached its limit ($N = \bar{N}$) then

$$\frac{k}{\alpha} \le -\frac{\bar{r}}{d}(1-E)\ln(1-E). \tag{62}$$

It is clear that his condition cannot be satisfied for any finite k if $E \to 1$. Equation (62) tells us that k/α has an upper limit that is proportional to the renewable energy flow $\bar{r}$ and inversely proportional to the rate of capital depreciation d. This result is intuitively reasonable. In the absence of depreciation, capital can be accumulated without limit. Maximum sustainable output occurs when k is as large as possible. Maximum k occurs at

$$\max[-(1-E)\ln(1-E)] \approx 0.37,$$

which corresponds to

$$E \approx 0.6. \tag{63}$$

Thus, along the optimal path (for $\delta = 0, N = \bar{N}$)

$$\bar{k} = \max k \approx 0.37\left(\frac{\alpha\bar{r}}{d}\right) \tag{64}$$

and

$$\bar{E} = \max E \approx 0.6. \tag{65}$$

For $\delta \neq 0$ the results are qualitatively similar, but somewhat messier. It is also clear that the limiting (stationary) case must be approached gradually. Evidently, in the long run (as $k \to \bar{k}, E \to \bar{E}$)

$$i \to d\bar{k} \tag{66}$$

and

$$j \to 0. \tag{67}$$

It should be noted that (67) is not incompatible with our assumption of an unconstrained solution ($q_k = 0$), provided j approaches zero from the positive side as some function of $\bar{E} - E$.

CONCLUSIONS

In this paper we have criticized some of the key assumptions made in earlier long-range optimal path models. The most insupportable assumption in the literature appears to be that economic goods and services (including capital goods) are intangible, requiring no physical embodiment of resources or available energy, and permitting unlimited substitution of fixed capital for resource input flows. This assumption is directly contrary to the laws of thermodynamics.

Another set of assumptions, that is built into some influential prior work is, essentially, that technological progress is automatic, exogenous, requiring no effort on investment, and is subject to no limits. To be sure some of these assumptions have been challenged elsewhere and more palatable treatments of technological progress have been presented, notably by Dasgupta *et. al.* and by Kamien and Schwartz. However, these authors do not treat knowledge as a factor of production, as we do. In particular, our treatment implies that accumulation of knowledge requires finite resource inputs, and reaches finite limits.

We have formulated and presented an optimal consumption/investment model which avoids some of the more unfortunate assumptions, and examined some of its general properties. So far as we can determine without numerical solutions the behavior of the model is reasonable. The most significant theoretical result is that for a production function that is consistent with conservation of matter/energy, and subject to the assumption that both capital and consumption goods embody energy (the ultimate resource), the optimal path leads to a stationary state with finite capital and finite technical knowledge, resulting in maximum technical efficiency less than unity. We hope, in the near future, to undertake simulation studies using the model.

REFERENCES

1. P. R. Ehrlich and A. H. Ehrlich, "Population, Resources, Environment: Issues in Human Ecology," Freeman, San Francisco (1970).
2. J. W. Forrester, "World Dynamics," Wright–Allen, Cambridge, Mass. (1971).
3. D. Meadows *et. al.*, "The Limits to Growth: A Report for the Club of Rome's Project on the Predicament of Mankind," Universe Books, New York (1972).
4. M. Mesarovic and E. Pestel, "Mankind at the Turning Point–The Second Report to the Club of Rome," Dutton/Readers Digest Press, New York (1974).
5. P. Buringh, H. Van Heemst, and G. Staring, "Computation of the Absolute Maximum Food Production of the World," Agricultural University, Wageningen, Netherlands (January 1975).
6. H. J. Barrett and E. Morse, "Scarcity and Growth," John Hopkins Press, Baltimore (1963).
7. J. Stiglitz. Growth with exhaustible natural resources. Efficient and optimal growth paths, *Rev. Econ. Studies* (1974).
8. R. Solow, The economics of resources or the resources of economics, *Amer. Econ. Rev.* **64** (1974).
9. H. Goeller and A. Weinberg, The age of sustitutability, *Science* **191** (February 1976).
10. H. Kahn, W. Brown, and L. Martel, "The Next 200 Years—A Scenario for America and the World." New York 1976.
11. R. Cummings and W. Schulze, Does conservation of mass–energy matter for economic growth? preprint.
12. N. Georgescu-Roegen, "*Energy and Economic Myths*: *Institutional and Analytical Economic Essays*," Pergamon, New York 1976.
13. R. U. Ayres, "Resources, Environment and Economics," Chapter 3, Wiley–Interscience, New York 1978.
14. N. Georgescu-Roegen, Energy analysis and economic valuation, *Southern Econ. J.* **4** (April 1979).

15. W. C. Gough and B. J. Eastlund, "The Fusion Torch—Closing the Cycle from Use to Reuse," U.S. Atomic Energy Commission (June 14, 1968).
16. M. Ross, R. Socolow, *et. al.*, "Effective Use of Energy: A Physics Perspective," Report of the Summer Study of Technical Aspects of Efficient Energy Utilization, *Amer. Phys. Soc.* (January 1975).
17. H. Hotelling, The economics of exhaustible resources, *J. Pol. Economy* **39** (1931).
18. O. Herfindahl, Depletion and economic theory, *in* "Extractive Resources and Taxation" (M. Gaffney, Ed.), Univ. of Wisconsin Press, Madison (1967).
19. W. P. Nordhaus, The allocation of energy resources, *Brookings Papers on Economic Activity* 3 (1973).
20. P. Dasgupta and G. Heal, The optimal depletion of exhaustible resources, Rev. Econ. Studies, Symposium on the Economics of Exhaustible Resources (1974).
21. P. Dasgupta, G. Heal and M. Majundar, Resource depletion and research and development, *in* "Frontiers of Quantitative Economics" (M. Intriligator, Ed.), North-Holland, Amsterdam (1977).
22. M. Kamien and N. Schwartz, Optimal exhaustible resource depletion with endogeneous technical change, *Rev. Econ. Studies* **65** (1978).
23. M. Tribus, P. T. Shannon, and R. B. Evans, Why thermodynamics is a logical consequence of information theory, *Amer. Inst. Chem. Eng. J.* **244** (March 1966).
24. C. Marchetti, Energy systems—The broader context, *Technol. Forecasting and Social Change* **14** (1979).
25. E. C. Pielou, "An Introduction to Mathematical Ecology," Wiley-Interscience, New York (1969).
26. E. F. Denison, "Why Growth Rates Differ." Brookings Institution, Washington D.C. (1967).
27. R. U. Ayres, "Technological Forecasting and Long-Range Planning," McGraw-Hill, New York (1969).
28. J. Stiglitz, A neoclassical analysis of the economics of natural resources, *in* "Scarcity and Growth Reconsidered" (V. K. Smith, Ed.), Resources for the Future, Washington, D.C. (1979).
29. E. R. Berndt and D. W. Jorgenson, Production structure, *in* "U.S. Energy Resources and Economic Growth," Ford Foundation Energy Policy Project (October 1973).
30. E. R. Berndt and D. O. Wood, "Engineering and Econometric Approaches to Industrial Energy Conservation and Capital Formation: A reconciliation," MIT Energy Laboratory Working Paper MIT-EL 77-040WP (1977).
31. J. M. Griffen and P. R. Gregory, An intercountry translog model of energy substitution responses, *Amer. Econ. Rev.* **66**, 845–857.
32. N. Georgescu-Roegen, Comments, *in* "Scarcity and Growth" (H. J. Barnett and E. Morse, Eds.) Johns Hopkins Press, Baltimore (1973); "Scarcity and Growth Reconsidered" (V. K. Smith, Ed.) Resources for the Future, Washington, D.C. (1979).
33. K. Arrow, Applications of control theory to economic growth, *in* "Mathematics of the Decision Sciences" Amer. Math. Soc., Providence, R.I., (1968).
34. A. Takayama, "Mathematical Economics," Dryden Press, Hinsdale, Ill. (1974).
35. A. C. Fisher, J. V. Krutilla, and C. J. Cicchetti, Alternative uses of natural environments: The economics of environmental modification, *in* "Natural Environments: Studies in Theoretical and Applied Analysis" (J. V. Krutilla, Ed.), Resources for the Future, Washington, D.C. (1972).