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Hybrid ARQ Schemes for Point-to-Multipoint Communication Over Nonstationary Broadcast Channels

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***Abstract*—Hybrid automatic-repeat-request (ARQ) error control schemes make use of both error detection and error correction in order to achieve high throughput and low undetected error probabilities on two way channels. This paper proposes two hybrid ARQ schemes, termed hybrid go-back-*N* (HGB-*N*) and hybrid selective-repeat (HSR), for point-to-multipoint communications over broadcast channels. Both schemes incorporate a concatenated code for error correction and error detection.**

The performance study of the hybrid schemes is based on a two-state Markov model of a burst noise channel. An analytic solution is derived for the throughput efficiency of the HSR scheme, while approximations and computer simulation are used to evaluate the throughput efficiency of the HGB-N scheme. It is shown that the proposed point-to-multipoint hybrid ARQ schemes perform considerably better than the corresponding pure ARQ schemes in which a block code is used for error detection only, especially in environments with a large number of receivers and large channel roundtrip delays, such as satellite broadcast links.

I. Introduction

Automatic-repeat-request (ARQ) strategies have long been utilized to control errors on two-way digital transmission links. Most of the work in this area has been done for point-to-point communications (see [1] for an excellent survey of this field by Lin, Costello, and Miller). There are two types of ARQ schemes, the *pure* ARQ schemes and the *hybrid* ARQ schemes. In pure ARQ schemes, a block code is used for error detection only, due to the simplicity of implementation of the error-detection decoding operations and the relatively low coding overhead allowed in many systems. However, in systems where the packet lengths are relatively large, and where the noise and/or interference levels are high, error detection only results in a low throughput due to the large number of retransmissions required. Satellite networks [2] and packet radio [3] are examples of such systems. In these instances, a combination of error correction and error detection can offer significant advantages over an error detection only system. This is called hybrid ARQ schemes.

Recently, we have observed increasing applications of point-to-multipoint communications over broadcast links, such as file distribution, video text systems, and teleconferencing. This trend will continue in future communication environments, especially with the introduction and deployment of national and international *integrated services digital networks* (ISDN's), linked worldwide by satellite communication systems [4]. In ISDN's where various information service centers provide information services to a large number of public users, the volume of point-to-multipoint data traffic will be significant, and the need for efficient point-to-multipoint ARQ schemes is obvious.

In contrast to the study of point-to-point ARQ schemes, only a limited amount of work has been accomplished for point-to-multipoint ARQ schemes [5]–[8]. A stop-and-wait ARQ scheme was proposed and analyzed by Calo and Easton [5]. Go-back-N schemes were studied independently by Mase *et al.* [6], Gopal and Jaffe [7], and Sabnani and Schwartz [8]. Sabnani and Schwartz also proposed and studied some selective-repeat schemes [8]. In all these schemes, a linear block code is used for error detection only and for this reason we call them point-to-multipoint pure ARQ schemes. In systems where the channel round trip delay is large or where there are a large number of receivers, such as satellite broadcast channels, the throughput efficiencies of the *pure stop-and-wait* scheme and the *pure go-back-N* (PGB-N) schemes become too poor to be acceptable [8]. Although the *pure selective-repeat* (PSR) scheme provides a better throughput, additional price must be paid for its implementation complexity and large buffering at the receivers.

In this paper we propose two hybrid ARQ schemes for use in point-to-multipoint communications over broadcast links: a *hybrid selective-repeat* (HSR) scheme with infinite receiver buffers, which serves as an "ideal" scheme for point-to-multipoint link protocol, and a *hybrid go-back-N* (HGB-N) scheme, which can be implemented with only slightly more complexity than the corresponding PGB-N scheme. The performances of the schemes are studied by assuming a nonstationary broadcast channel.

In Section II, we describe the nonstationary broadcast channel. A two-state Markov chain channel model is defined, which constitutes a first approximation to a nonstationary channel. In Section III, the point-to-multipoint HSR and HGB-N schemes are introduced. Both schemes incorporate a concatenated coding system for error correction and error detection. Throughput

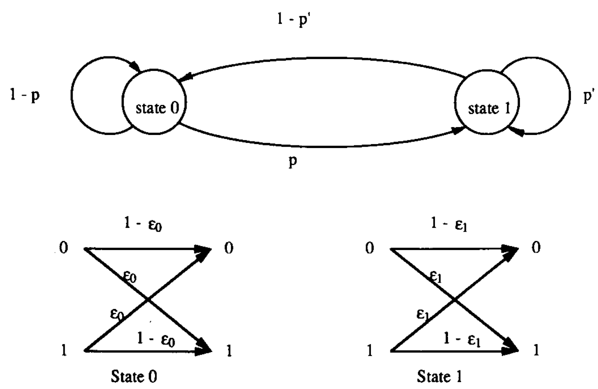

Fig. 1. A two-state Markov chain nonstationary channel model.

performance of the hybrid schemes over the nonstationary broadcast channel is analyzed in Section IV. Numerical results on performance of the proposed hybrid schemes are presented in Section V. Finally, Section VI contains our discussions and conclusions.

II. Modeling of a Nonstationary Broadcast Channel

A. Two-State Burst Noise Model

We follow the approach of [9] in modeling a nonstationary channel. The channel is modeled as a two-state Markov chain (see Fig. 1). State 0 is the quiet state where the bit error rate (BER) is ε_0. State 1 is the noisy state where the BER is $\varepsilon_1 >> \varepsilon_0$. p is the transition probability from state 0 to state 1 and p' is the transition probability from state 1 to itself. To simplify the treatment of the model, we assume that one time frame (i.e., one state transition period) in the model corresponds to the transmission of one data packet, i.e., the noisy bursts last for a multiple of the transmission time of a packet.

Let p_1 denote the duty cycle of the noisy bursts, or the probability of being in the noisy state. We are interested in a model for the noisy bursts which can be *dense* (low duty cycle p_1, and high intensity, i.e., large high-to-low BER ratio $\varepsilon_1/\varepsilon_0$) or *diffuse* (large duty cycle and low intensity). Stated mathematically, we have

$$\lim_{p_1 \to 0} \varepsilon_0 = 0, \tag{1.1}$$

$$\lim_{p_1 \to 0} \varepsilon_1 = \frac{1}{2}, \tag{1.2}$$

and

$$\lim_{p_1 \to 1} \varepsilon_0 = \bar{\varepsilon}, \tag{2.1}$$

$$\lim_{p_1 \to 1} \varepsilon_1 = \bar{\varepsilon} \tag{2.2}$$

where $\bar{\varepsilon} = (1-p_1)\varepsilon_0 + p_1\varepsilon_1$ is the average BER of the channel. Equations (1.1)–(2.2) can be satisfied by imposing the following constraint on the two-state Markov model [9]

$$\varepsilon_0 = \bar{\varepsilon} p_1 . \tag{3}$$

We call this modified version the *burst noise channel* (BNC) model. For the BNC, it can be shown that the *average burst length,* i.e., the average number of packets transmitted while in state 1 is

$$\bar{b} = \frac{1}{(1-p')}, \tag{4}$$

the *duty cycle* of the noisy bursts is

$$p_1 = \frac{p}{(1-p'+p)}, \tag{5}$$

and

$$\varepsilon_1 = \frac{\bar{\varepsilon}}{p_1} - (1-p_1)\bar{\varepsilon}, \quad \text{for } p_1 > \frac{\bar{\varepsilon} + \frac{1}{2} - \sqrt{\left(\frac{1}{2} - \bar{\varepsilon}\right)\left(\frac{1}{2} + 3\bar{\varepsilon}\right)}}{2\bar{\varepsilon}} \tag{6.1}$$

where the inequality ensures that $\varepsilon_1 < 1/2$, and

$$\varepsilon_1 = \frac{1}{2}, \text{ otherwise.} \tag{6.2}$$

The BNC is completely described by $\bar{\varepsilon}$, p_1, and $\bar{b}$, for if these three parameters are known, p', p, ε_0, and ε_1 can be determined from (3)–(6).

A special case of the BNC model is when $p_1 = p = p'$, which corresponds to the two state *block interference channel* (BIC) proposed by McEliece and Stark [10]. The BIC is completely specified by p_1 and $\bar{\varepsilon}$.

B. Nonstationary Broadcast Channel

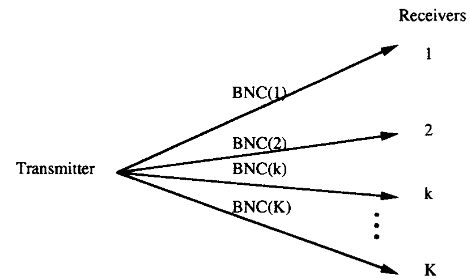

Fig. 2. Channel model of the nonstationary broadcast channel.

The broadcast communication environment we consider consists of $K+1$ stations, one being the transmitter and the other K being receivers. The transmitter continuously broadcasts data packets to all K receivers and constantly listens for acknowledgments of the sending packets from all receivers. We call the transmission path from the transmitter to the kth receiver, the kth *component channel* of the broadcast channel, $k = 1, 2, \cdots, K$. The component channels are assumed to produce independent noise processes and are modeled by the burst noise channels (BNC(k)'s) described in Section II-A, with parameters $\bar{\varepsilon}_k$, $p_{1,k}$, and $\bar{b}_k$, and $k = 1, 2, \cdots, K$. The nonstationary broadcast channel is depicted Fig. 2. To facilitate the analysis in Section IV, we assume that $\bar{\varepsilon}_k = \bar{\varepsilon}$, $p_{1,k} = p_1$, and $\bar{b}_k = \bar{b}$, $k = 1, 2, \cdots, K$.

Hereafter, we refer the broadcast channel with BNC's as its component channels the *burst noise broadcast channel*

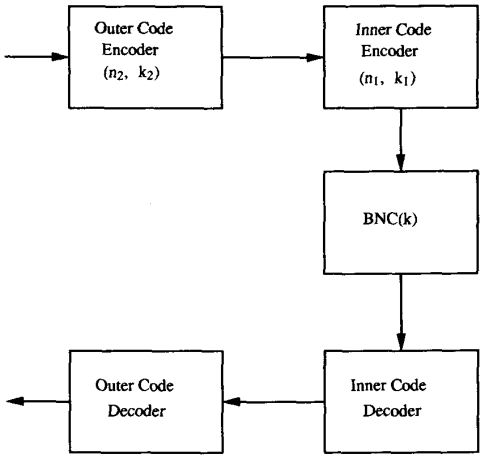

Fig. 3. A concatenated coding system.

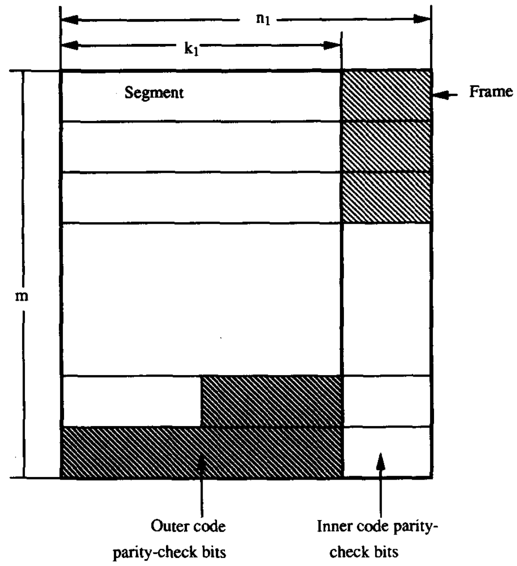

Fig. 4. Packet format.

(BNBC). For the special but important case where the component channels are BIC's, the broadcast channel is referred to as *block interference broadcast channel* (BIBC).

III. THE POINT-TO-MULITPOINT HYBRID ARQ SCHEMES

A. A Concatenated Coding System

Both the HSR ARQ scheme and the HGB-N ARQ scheme implement a concatenated coding system for error control. The concatenated code uses two linear systematic block codes C_1 and C_2 as shown in Fig. 3. The inner code is an (n_1, k_1) code with minimum Hamming distance d_1. The inner code is designed to correct t or fewer errors where $t = \lfloor (d_1 - 1)/2 \rfloor$ [11]. The outer code C_2 is an (n_2, k_2) code with minimum Hamming distance d_2 and code length

$$n_2 = mk_1$$

where m is a positive integer. The outer code is designed for error detection only.

The encoding of the concatenated code is achieved in two stages. A message of k_2 bits is first encoded into a code word of n_2 bits in the outer code C_2. Then the n_2-bit code word is divided into m k_1-bit *segments.* Each k_1-bit segment is then encoded into an n_1-bit code word in the inner code C_1. This n_1-bit word is called a *frame.* Thus, corresponding to each k_2-bit message at the input on the outer code encoder is a sequence of m frames output by the inner code encoder. This sequence of m frames is a code word in the concatenated code and is called a *packet.* A two-dimensional packet format is given in Fig. 4.

Decoding of a received m-frame packet consists of error correction on each frame based on the inner code C_1 and error detection on the m decoded k_1-bit segments based on the outer code C_2. When a frame in a packet is received, it is decoded based on the inner code C_1. The $n_1 - k_1$ parity bits are then removed from the decoded frame, and the k_1-bit decoded segment is stored in a buffer. If there are t or fewer transmission errors in a received frame, the errors will be corrected and the decoded segment is error free. If there are $t + 1$ or more transmission errors in a received frame, the errors will not be corrected and the decoded segment contains decoding errors. After the m frames of a packet have been decoded based on the inner code C_1, error detection is performed on the m decoded segments based on the outer code C_2. If no errors are detected, the m decoded segments are assumed to be error free and are accepted by the receiver. If the presence of errors is detected, the m decoded segments are discarded, and the receiver requests a retransmission of the rejected packet. The retransmission schemes are described in the next subsection.

The concatenated coding system presented above is a variation of the coding system studied by Kasami *et al.* [12] and by Deng and Costello [13], where the inner code is used for both error detection and error correction.

B. The Hybrid ARQ Schemes

1) Transmitter Operation: The transmitter transmits packets (the concatenated code words) continuously and constantly listens for acknowledgments (ACK's) of previous transmissions. Each transmitted packet carries a sequence number in the transmitted packet sequence. In the following, we assume that the sequence number is strictly increasing and that two consecutive packets in the transmitted sequence always differ in sequence number by 1.

When a message of k_2-bit is ready for transmission, it is numbered and is encoded into a packet by the concatenated code encoder. Immediately upon transmission of a packet, the transmitter sets a time-out counter for that packet and waits for ACK's of the transmitted packet from all receivers. If the transmitter has not received the ACK's from all the receivers before the time-out counter expires, its operation proceeds in the following manners, depending on which scheme it is using: in the HGB-N scheme, the transmitter goes back to the

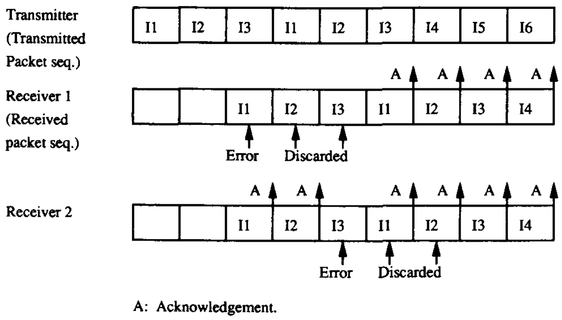

Fig. 5. HGB-N with $N = 3$ and $K = 2$.

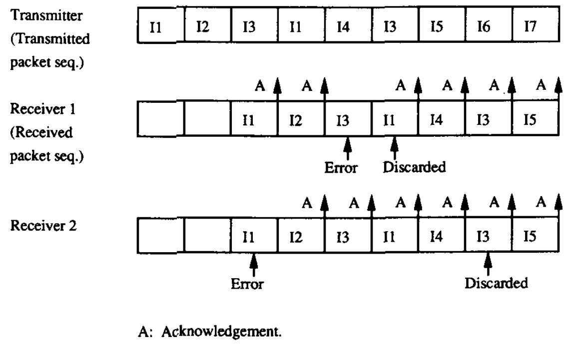

Fig. 6. HSR with $N = 3$ and $K = 2$.

packet in error and retransmits the packet and all the following packets; in the HSR scheme, the transmitter retransmits only the packet in error. In both schemes, the transmitter retains a memory of the outcomes of all previous transmissions and retransmissions. That is, the transmitter has to remember which packet has been acknowledged by which receivers. Thus, the next time around, ACK's are expected only from receivers that have not sent a previous ACK. Retransmission of a packet continues until the transmitter has received ACK's from all receivers.

2) Receiver Operation: Let s_0 be the sequence number of the last packet accepted and delivered by a receiver. Define the forward index f_r of the current received packet as

$$f_r = s - s_0$$

where s is the sequence number of the current received packet. Obviously, if $f_r = 1$, the current received packet is the one immediately following the last accepted and delivered packet in the transmission sequence.

Normally, a receiver receives error-free packets and the receiver buffer is empty. When a packet is received, it is decoded by the concatenated decoder and then its forward index is computed. If the received packet is decoded successfully (which includes both error-free decoding and undetected-error decoding) and if its $f_r = 1$, it is accepted and delivered to the user; if $f_r < 1$, the received packet is regarded as a packet that was previously accepted and delivered and it is discarded by the receiver. In both cases an ACK is sent to the transmitter. If the received packet is not decoded successfully (i.e., errors are detected by the outer code), it is simply discarded by the receiver. Finally, if a received packet is decoded successfully but with $f_r > 1$, it is handled in the following ways, according to which scheme the system is using. In the HGB-N scheme, the packet is simply discarded. In the HSR scheme, the packet is stored in the receiver buffer at the proper position. This packet will not be delivered to the user until all packets preceding it have been accepted and delivered to the user by the receiver.

Figs. 5 and 6 show examples of the operations of the HGB-N and HSR ARQ schemes, respectively, where N is the number of packets that can be transmitted during a channel roundtrip delay period.

IV. Throughput Analysis

In this section we evaluate the throughput efficiencies of the point-to-multipoint HSR ARQ scheme and the point-to-multipoint HGB-N ARQ scheme over the nonstationary broadcast channel described in Section II. We make the following assumptions in our analysis: 1) there is always a new packet waiting for transmission at the transmitter and as a result, the throughput results provide the maximum possible throughput achievable for the given ARQ scheme, 2) the feedback channel is noiseless and that all the ACK's from the various receivers for a particular packet transmission arrive at the transmitter before the time-out counter for that packet expires, and 3) the time-out counter is set to expire after the transmission of exactly N packets.

A. Performance of the Concatenated Coding System

Assume that a packet is transmitted over a binary symmetric channel (BSC) with BER ε. Let $P_f(\varepsilon)$ denote the probability of correct decoding for the inner code C_1. Suppose that a bounded-distance decoding algorithm [11] is used for the decoding of the frames. Bounded-distance decoding corrects all received n_1-bit frames with t or fewer errors. For an n_1-bit frame with more than t errors, no attempt is made to correct the errors. Since there are $\binom{n_1}{i}$ distinct ways in which i errors may occur among n_1 bits, we have

$$P_f(\varepsilon) = \sum_{i=1}^{t} \binom{n_1}{i} \varepsilon^i (1-\varepsilon)^{n_1 - i} \tag{7}$$

for bounded-distance decoding.

Let $P_c(\varepsilon)$ be the probability of correct decoding of a transmitted packet. A transmitted packet will be decoded correctly if and only if all m frames are decoded correctly,

$$P_c(\varepsilon) = P_f(\varepsilon)^m. \tag{8}$$

Let $P_d(\varepsilon)$ and $P_u(\varepsilon)$ be the probabilities of detected error and undetected error, respectively, for the concatenated code. Obviously,

$$P_c(\varepsilon) + P_d(\varepsilon) + P_u(\varepsilon) = 1.$$

Note that $1 - P_d(\varepsilon) = P_c(\varepsilon) + P_u(\varepsilon)$ is the probability of successfully decoding, i.e., the probability of either a correct decoding or the decoded packet containing undetected errors. Usually, $P_u(\varepsilon) << P_c(\varepsilon)$ and the effect of $P_u(\varepsilon)$ on the throughput calculation is negligible. Hereafter, we will use $P_c(\varepsilon)$ as an approximation to the probability of successfully decoding of a received packet.

B. Throughput of the HSR Scheme

For point-to-multipoint ARQ schemes we define the throughput efficiency as the ratio of the average number of information bits successfully accepted by all receivers per unit time to the total number of bits that can be transmitted per unit time. Let $E[T]$ denote the average number of transmission and retransmissions required for a packet to be accepted (i.e., to be decoded successfully) and delivered to users by all K receivers. Since the receiver buffer size is infinite and the retransmission scheme is selective-repeat, the throughput efficiency of the HSR scheme, denoted by η_{HSR}, is then

$$\eta_{\text{HSR}} = \frac{k_2}{n}\frac{1}{E[T]} \tag{9}$$

where $n = mn_1$ is the packet size and k_2/n is the rate of the concatenated code.

Define X_k, $k = 1, 2, \cdots, K$, to be the number of transmission and retransmissions of a packet required until it is accepted and delivered to the user by the kth receiver. Because the broadcast channel from the transmitter to each individual receiver is assumed to produce independent identically distributed (i.i.d.) noise processes, the X_k, $k = 1, 2, \cdots, K$, are i.i.d. random variables. Then it follows that

$$\begin{aligned} E[T] &= \sum_{i=1}^{\infty} i \Pr\{T = i\} = \sum_{i=1}^{\infty} i \Pr\left\{\max_k [X_k] = i\right\} \\ &= \sum_{i=0}^{\infty} \left[1 - \Pr\{X_k \le i\}^K\right] \end{aligned} \tag{10}$$

where the last term is obtained by using a formula for finding the average of the maximum of several i.i.d. integer-valued random variables [14]. Now the problem reduces to the evaluation of the probability $P_r\{X_k \le i\}$ for an arbitrary single receiver, say the kth receiver. To this end, in the following we model a receiver's decoding status as a Markov chain.

We introduce the notion of *outstanding error packet* (OEP). We say that a received packet is an OEP if at the time of its decoding at the kth receiver it contains uncorrectable but detectable errors and that the packet has not been accepted by the receiver before. Suppose that an initial OEP has been received. Then the kth receiver's decoding status can be modeled by the simple Markov chain shown in Fig. 7. In Fig. 7, states 0 and 1 mean that the receiver has received an OEP, sent while the BNC(k) (the transmission path from the transmitter to the kth receiver) was in state 0 and 1, respectively. State s corresponds to successful decoding. Once the system is in state s, retransmissions will terminate and the packet will be accepted and delivered to the user. Hence, state s is an absorbing state and we assign $p_{\text{ss}} = 1$. On the other hand, if upon the reception of a retransmission, the

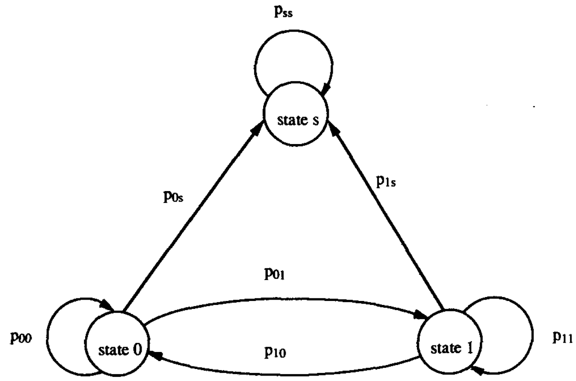

Fig. 7. A receiver's decoding status after receiving an initial OEP.

decoder results in an unsuccessful decoding, retransmissions will continue (the Markov chain will remain in state 0 or 1) until successful decoding occurs. By arranging the states of Fig. 7 in the order: 0, 1, s, the transition probabilities of the Markov chain are derived in Appendix A and are summarized in the following (one-step) transition probability matrix

$$\boldsymbol{P} = \begin{bmatrix} p_{00} & p_{01} & p_{0s} \\ p_{10} & p_{11} & p_{1s} \\ 0 & 0 & 1 \end{bmatrix} \tag{11}$$

where, from Appendix A,

$$\begin{aligned} p_{00} &= [1 - q_{01}(N)][1 - P_c(\varepsilon_0)], \\ p_{01} &= q_{01}(N)[1 - P_c(\varepsilon_1)], \\ p_{0s} &= 1 - p_{00} - p_{01}, \\ p_{10} &= [1 - q_{11}(N)][1 - P_c(\varepsilon_0)], \\ p_{11} &= q_{11}(N)[1 - P_c(\varepsilon_1)], \\ p_{1s} &= 1 - p_{10} - p_{11}. \end{aligned}$$

In the above transition probabilities, $q_{01}(N)$ and $q_{11}(N)$ are the N-step BNC(k) transition probabilities given by (A.1) and (A.2) in the Appendix, and N is the number of packets that can be transmitted during one roundtrip delay period of the BNC(k).

Let $p_{ij}(l)$ be the l-step transition probability from state i to state j for the Markov chain in Fig. 7, $i, j = 0, 1, s$. $p_{ij}(l)$ can be obtained from $\boldsymbol{P}^l = [p_{ij}(l)]$. Let $a_{js}(l)$, $j = 0, 1$, be the l-step absorption probability, which is defined as the probability that a Markov chain initially in state j will be absorbed by absorbing state s within l transitions [15]. Then it follows that

$$\Pr\{X_k \le 1\} = P_c \tag{12.1}$$

and for $i > 1$,

$$\begin{aligned} \Pr\{X_k \le i\} = &\ P_c + (1 - p_1)[1 - P_c(\varepsilon_0)]a_{0s}(i-1) \\ &+ p_1[1 - P_c(\varepsilon_1)]a_{1s}(i-1) \\ = &\ P_c + (1 - p_1)[1 - P_c(\varepsilon_0)]p_{0s}(i-1) \\ &+ p_1[1 - P_c(\varepsilon_1)]p_{1s}(i-1) \end{aligned} \tag{12.2}$$

where $P_c(\varepsilon)$ is given by (8) and where

$$P_c = (1 - p_1)P_c(\varepsilon_0) + p_1 P(\varepsilon_1) \tag{13}$$

is the average probability that a received packet will be decoded correctly. In (12.2), we have used the fact that $a_{js}(l) = p_{js}(l)$, $j = 0, 1$ (see Appendix B). The throughput of the HSR ARQ scheme over the BNBC can then be obtained by substituting (10) and (12) into (9).

For the special case when the broadcast channel is a BIBC, an explicit expression can be derived for the throughput. Since there is no memory between packets sent over a BIC, we have

$$\Pr\{X_k \le i\} = P_c \sum_{j=1}^{i} (1 - P_c)^{j-1} = 1 - (1 - P_c)^i, \quad \text{for } i = 1, 2, \cdots. \tag{14}$$

Substituting (10) and (14) into (9), the throughput efficiency of the HSR ARQ scheme over the BIBC is given by

$$\eta_{\text{HSR}} = \frac{k_2}{n} \frac{1}{\sum_{i=0}^{\infty} \left\{1 - \left[1 - (1 - P_c)^i\right]^K\right\}}. \tag{15}$$

C. Throughput of the HGB-N Scheme

The throughput analysis of the HGB-N ARQ scheme over the BNBC is a very difficult problem if not impossible. So far there are no analytical results available in the literature even for memoryless broadcast channels, except an approximation formula given by Gopal and Jaffe [7]. The throughput of the HGB-N over the BNBC will be studied by computer simulations in the next section. However, since there is no memory between packets transmitted over a BIC, the result presented in [7] can be used to approximate the throughput of the HBG-N ARQ scheme over a BIBC,

$$\eta_{\text{HGB-}N} = \frac{QP_c^K + (1 - Q)P_c^R}{Q[P_c^K + N(1 - P_c^K)] + (1 - Q)[P_c^R + N(1 - P_c^R)]} \tag{16.1}$$

where

$$R = 1 + (K - 2)(1 - P_c), \tag{16.2}$$

$$Q = \frac{P_c^R}{P_c^R + N\left[1 - (1 - P_c)^K - P_c^K\right]} \tag{16.3}$$

and where P_c is given by (13).

V. Numerical Results

In this section, some typical values for throughput efficiency are calculated and compared for each of the ARQ schemes. For comparison, values of the throughput efficiency for the PGB-N and PSR point-to-multipoint ARQ schemes are also included.

To keep the encoding/decoding process as simple as possible, in the following we consider one of the simplest concatenated codes in our throughput calculations. In this concatenated code, the inner code C_1 is a shortened distance-3 Hamming code with generator polynomial

$$g_1(x) = x^6 + x + 1 \tag{17}$$

where $x^6 + x + 1$ is a primitive polynomial of degree 6. The full-length code generated by $g_1(x)$ is a $(63, 57)$ cyclic Hamming code. By deleting 1 b from the full-length code, the inner code C_1 becomes a $(62, 56)$ shortened Hamming code. The 56 information bits form seven 8 b information bytes. Since the code has minimum Hamming distance 3, it is used to correct a single bit error in a received frame (i.e., $t = 1$).

The outer code C_2 is a shortened distance-4 Hamming code with generator polynomial

$$\begin{aligned} g_2(x) = {} & (x + 1)(x^{15} + x^{14} + x^{13} + x^{12} \\ & + x^4 + x^3 + x^2 + x + 1) \\ = {} & x^{16} + x^{12} + x^5 + 1 \end{aligned} \tag{18}$$

where $x^{15} + x^{14} + x^{13} + x^{12} + x^4 + x^3 + x^2 + x + 1$ is a primitive polynomial of degree 15. This code is the X.25 standard for packet-switched public data networks [17]. The full-length of this code is $2^{15} - 1 = 32767$ b. In our example, however, we choose the number of frames in a packet $m = 16$, so that the outer code length $n_2 = mk_1 = 16 \times 56 = 896$ b, and the packet size $n = mn_1 = 16 \times 62 = 992$ b.

Throughout the following examples, the above concatenated code is used in hybrid ARQ schemes. In pure ARQ schemes, we use a $(992, 976)$ shortened Hamming code with generator polynomial of (18) for error detection. Numerical results on pure ARQ are shown as dotted curves and results on hybrid ARQ schemes are plotted as solid curves.

Throughput performances of selective-repeat ARQ schemes, calculated according to the formulas derived in Section IV-B, are depicted in Figs. 8–10. Fig. 8 shows how throughput varies as a function of the average channel BER $\bar{\varepsilon}$ and the average burst length $\bar{b}$. For small to medium values of $\bar{\varepsilon}$, the throughput of the HSR is not sensitive to $\bar{b}$. However, as $\bar{\varepsilon}$ becomes large, the performance of the HSR ARQ degrades very rapidly as the value of $\bar{b}$ increases. In Fig. 9, for $N = 50$, $K = 20$, and $\bar{b} = 10$, we let the burst duty cycle $p_1 = 1$ (stationary channel), 0.25 (diffuse channel), and 0.05 (dense channel). For small to medium values of $\bar{\varepsilon}$, the throughput is worse for a given average BER if the errors occur in bursts. As the average BER gets large, however, the throughput becomes much better over dense channels. This is because, for large BER, packets sent over the noisy state of the channel have a high probability of being retransmitted due to the limited error correcting capability of the concatenated code implemented; and for dense channels errors are concentrated in fewer packets and therefore results in fewer retransmissions. Fig. 10 shows the effects of the number of receivers on the throughput. We see from Fig. 10 that, for average BER of practical interest, the HSR ARQ provides a very satisfactory throughput even when the number of receivers is large.

The throughput efficiency of the go-back-N ARQ schemes are given in Figs. 11 to 13. Fig. 11, obtained from computer simulations, shows the throughput as a function of the values of $\bar{b}$. It is seen that the values of $\bar{b}$ have a small influence on the

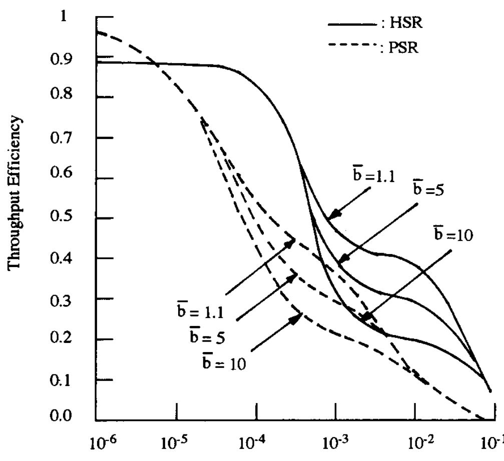

Fig. 8. Throughput of selective-repeat ARQ schemes with $N = 5$, $K = 20$, $p_1 = 0.1$ and various values of $\bar{b}$.

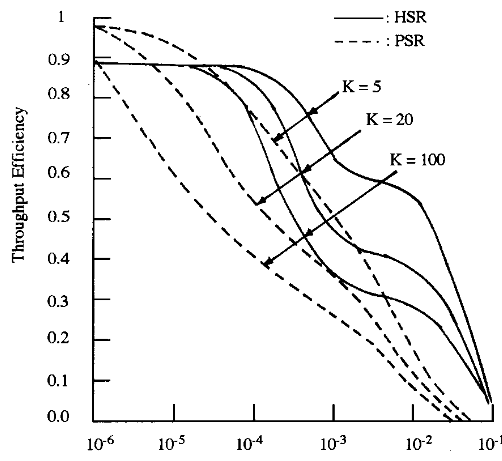

Fig. 10. Throughput of selective-repeat ARQ schemes with $N = 50$, $\bar{b} = 10$, $p_1 = 0.1$, and various values of K.

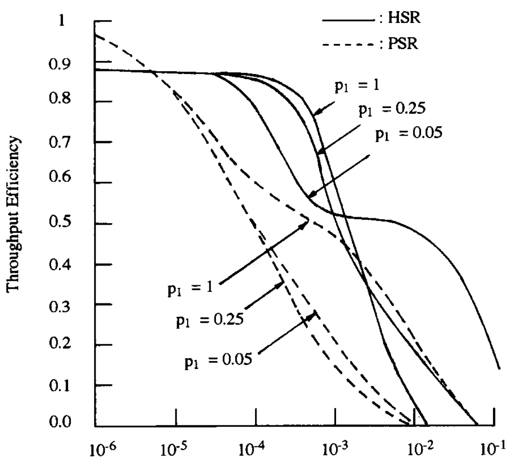

Fig. 9. Throughput of selective-repeat ARQ schemes with $N = 5$, $K = 20$, $\bar{b} = 10$, and various values of p_1.

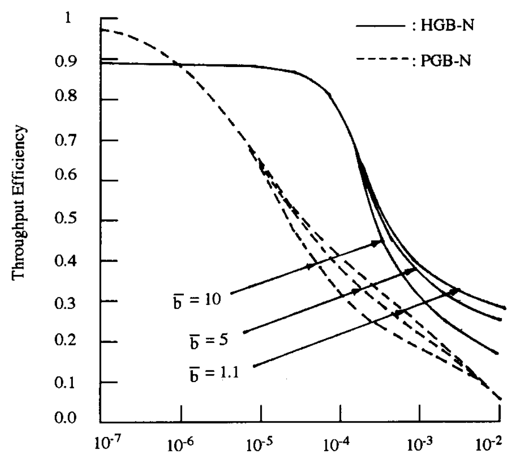

Fig. 11. Throughput of go-back-N ARQ schemes with $N = 5$, $K = 20$, $p_1 = 0.1$ and various values of $\bar{b}$.

throughput of go-back-N ARQ, due to the go-back N nature of the scheme. In the rest of the numerical examples we assume that the broadcast channel is a BIBC (i.e., $p = p' = p_1$) so that eqn. (16) instead of computer simulations are used in the throughput calculations. Figs. 12 and 13 show how throughput varies as functions of the duty cycle p_1 and the number of receivers K, respectively.

From these examples we observe that the hybrid ARQ schemes are very robust against burst channel errors and perform considerably better than the corresponding pure ARQ schemes at only slightly increased system complexity. The additional system complexity mainly comes from the encoding and decoding operations of the inner code C_1. For the shortened Hamming code with the generator polynomial of (17), the encoding and decoding can be performed with a simple 6-stage shift register [11]. The throughput improvement is more evident for go-back-N schemes, especially when the roundtrip delay or the number of receivers is larger. As examples, consider Fig. 13 for the case with $N = 50$ and $K = 100$. For $\bar{\varepsilon} = 10^{-5}$, PGB-$N$ gives a throughput of only 0.3, whereas HGB-N provides a throughput within 90% of its maximum achievable values; for $\bar{\varepsilon} = 10^{-4}$, PGB-$N$ virtually fails to yield any throughput while HBG-N still provides a remarkable throughput of about 0.45.

The HSR ARQ with infinite receiver buffers is an "ideal" scheme for point-to-multipoint communications and its throughput performance is always superior to any other hybrid ARQ schemes. Comparing the results in Figures 8 to 10 with those in Figs. 11–13, we observe that under channel conditions of practical interest, the throughput of HGB-N and HSR come very close to each other even the roundtrip delay or the number of receivers is large. This suggests that the HGB-N ARQ is able to provide a throughput efficiency close to that provided by the ideal HSR ARQ for practical channel error conditions with much reduced system complexity.

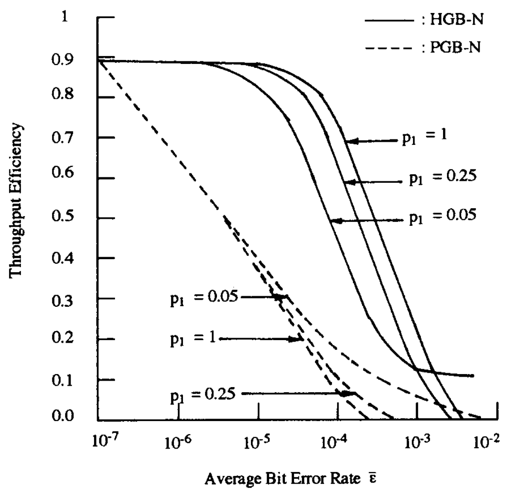

Fig. 12. Throughput of go-back-N ARQ schemes with $N = 50$, $K = 20$, and various values of p_1.

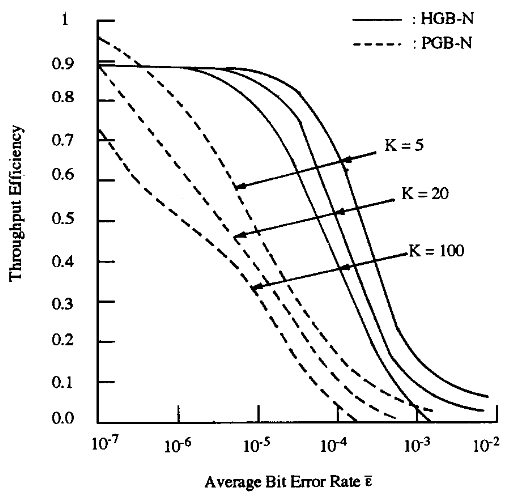

Fig. 13. Throughput of go-back-N ARQ schemes with $N = 50$, $p_1 = 0.1$, and various values of K.

VI. Conclusion

In this paper we have considered point-to-multipoint ARQ schemes for data transmission over nonstationary broadcast channels. Specifically, we proposed an HSR ARQ scheme and an HGB-N ARQ scheme, both employ a concatenated coding system for error control.

The broadcast channel is based on a two-state Markov model which constitutes a first approximation to a nonstationary channel. An analytical solution on the throughput of the HSR ARQ was derived. Computer simulations and approximations were used to study the performance of the HGB-N ARQ. Numerical results indicated that, with the use of a simple concatenated code, the hybrid schemes are very robust against burst channel errors and are able to provide very high throughput in a wide range of communication environments.

In communication environments where the channel roundtrip delay is large and where there is a larger number of receivers, the throughput of PGB-N ARQ becomes too poor to be acceptable (see Fig. 13 for the case of $K = 100$ and $\bar{\varepsilon} = 10^{-5}$). Although the PSR ARQ provides a useful throughput (see Fig. 10 for the case of $K = 100$ and $\bar{\varepsilon} = 10^{-5}$), additional complexity and much larger buffering at the receiver are required. In many real environments all stations may be receiving and transmitting simultaneously. If each transmitter broadcasts simultaneously to K receivers, each receiver will on average receive from K separate stations. Thus, the buffer requirements at each receiver are multiplied by a factor of K. In many practical situations, K could be 50 or more, thus making the receiver buffer requirements of the selective-repeat scheme impractically large [7]. In this paper we showed that with the use of a simple concatenated code, the HGB-N ARQ achieves a throughput close to that achieved by the ideal HSR ARQ for practical channel error conditions with much reduced system complexity, even for environments with large channel round-trip delay or large number of receivers.

Appendix A

In this Appendix, we derive the transition probabilities of the Markov chain shown in Fig. 7. Let N be the number of packets that can be transmitted during one roundtrip delay period of the BNC(k), $k = 1, 2, \cdots, K$. The BNC(k) N-step transition probability $q_{01}(N)$ and $q_{11}(N)$ of being in state 1 N time frames after being in state 0 and 1, respectively, are given by [16]

$$q_{01}(N) = \frac{p}{1+p-p'} - \frac{p}{1+p-p'}(p'-p)^N, \tag{A.1}$$

and

$$q_{11}(N) = \frac{p}{1+p-p'} + \frac{(1-p')}{1+p-p'}(p'-p)^N. \tag{A.2}$$

Let us consider the transition from state 0 to state 1. State 0 means that the OEP, sent over state 0 of the BNC(k) has been received and the decoding has failed. The OEP is then discarded by receiver k. The transition from state 0 to state 1 means that a retransmission of the packet, sent over state 1 of the BNC(k) after a roundtrip delay (with probability $q_{01}(N)$), has been received and that the decoding of the retransmitted packet has again failed (with probability $[1 - P_c(\varepsilon_1)]$. The transition probability from state 0 to state 1 is therefore given by

$$p_{01} = q_{01}(N)[1 - P_c(\varepsilon_1)].$$

By a similar argument we obtain

$$p_{00} = [1 - q_{01}(N)][1 - P_c(\varepsilon_0)],$$
$$p_{10} = [1 - q_{11}(N)][1 - P_c(\varepsilon_0)],$$
$$p_{11} = q_{11}(N)[1 - P_c(\varepsilon_1)].$$

Finally, realizing that the transition probabilities from state i, $i = 0, 1$, to states 0, 1, and s must add up to 1, we have

$$p_{0s} = 1 - p_{00} - p_{01},$$
$$p_{1s} = 1 - p_{10} - p_{11}.$$

APPENDIX B

In this Appendix, we show that the l-step absorption probability $a_{js}(l)$ equals the l-step transition probability $p_{js}(l)$. First we define the l-step first passage probability, denoted by $f_{js}(l)$, as the probability that a Markov chain starting from state j will be in state s for the first time after l transitions. For an absorbing state s, we have

$$f_{js}(1) = p_{js}(1) = p_{js}$$
$$f_{js}(2) = p_{js}(2) - p_{js}(1)$$
$$f_{js}(3) = p_{js}(3) - p_{js}(2)$$

$$f_{js}(l) = p_{js}(l) - p_{js}(l-1)\,. \tag{B.1}$$

To prove (B.1), we observe that the one-step first passage probability is the same as the one-step transition probability. For $l = 2$, the two-step transition probability $p_{js}(2)$ contains the probability of visiting state s immediately after the first transition and remaining in state s during the second transition. Hence, the probability of this event, $p_{js}(1) \cdot p_{ss}(1)$, should be subtracted from $p_{js}(2)$ to obtain the two-step first passage probability, i.e., $f_{js}(2) = p_{js}(2) - p_{js}(1) \cdot p_{ss}(1)$. Since s is an absorbing state, we have $f_{js}(2) = p_{js}(2) - p_{js}(1)$. The rest of (B.1) is based on the similar reasoning.

By the definition of $a_{js}(l)$ and using (B.1), we have

$$a_{js}(l) = \sum_{n=1}^{l} f_{js}(n) = p_{js}(l)\,. \tag{B.2}$$

This completes the proof.

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