11-1993

An adaptive coding scheme with code combining for mobile radio systems

Robert H. DENG
Singapore Management University, robertdeng@smu.edu.sg

Huafei Zhou

DOI: https://doi.org/10.1109/25.260765

An Adaptive Coding Scheme with Code Combining for Mobile Radio Systems

Robert H. Deng, *Member, IEEE,* and Hong Zhou, *Member, IEEE*

Abstract—**In this paper, we propose and study an adaptive error-control coding scheme for binary digital FM (BFM) mobile radio transmission. The scheme employs code combining through packet retransmissions. The number of transmissions of a packet is in proportion to the channel fading/noise levels, which is in contrast to time diversity techniques where a fixed number of repetitions of a data packet is performed even in the absence of channel errors. Furthermore, the receiver uses received signal envelopes as channel state information (CSI), which significantly improves the throughput and bit error rate (BER) performance. Performance of the proposed scheme is analyzed for frequency-flat Rayleigh fading channels with additive white Gaussian noise (AWGN), co-channel interference (CI), and random FM noise.**

I. Introduction

In land digital mobile radio transmission, multipath Rayleigh fading can severely degrade the signal transmission performance [1]. In the case of relatively low-speed digital transmission, the spread of various propagation delays can be neglected in comparison to the duration of symbol transmission, and the transmitted signal is said to suffer from frequency-flat fading. In frequency-flat fading, degradation of transmission performance is mainly caused by AWGN, random FM noise, and co-channel interference (CI) [2]. The two most effective techniques to mitigate multipath fading effects are diversity reception and error-control coding. Various diversity combining schemes have been proposed and studied in the literature for mobile radio communication [1]–[4]. Among the many diversity schemes, time diversity is most attractive in the sense that it does not require multiple antennas at the receiving site. However, it is not an adaptive scheme since it repeats the same data packet a fixed number of times even in the absence of channel errors.

Many forward error correction (FEC) systems employing block or convolutional codes have been investigated, and their performance has been analyzed for fading channels [5]–[11]. Recently, FEC systems for fading channels employing trellis-coded modulation have also been proposed and studied [12]–[15]. In FEC systems, the code rate must be carefully chosen to match the channel noise conditions in order to meet the user's reliability requirement. Unfortunately, there are some channels where the initial steps of estimating the channel noise level and determining the code rate to match this channel condition are, at best, an educated guess. Mobile radio channel represents one such example. In mobile radio, channels are characterized by rapid multipath Rayleigh fading superimposed on slow shadow fading [1]. Rayleigh fading is caused by interference between multipath waves from scatterers surrounding the mobile station, while shadowing is due to gross variation in the terrain between the base and mobile stations. Shadow fading can significantly reduce the average signal-to-noise ratio. If the code rate is selected to match the multipath Rayleigh fading condition, long decoded error bursts become inevitable during the shadowing period.

Another commonly used error-control technique for noisy channels is hybrid automatic-repeat request (ARQ) [16]. Most of the hybrid ARQ schemes were analyzed for channels with AWGN [17]–[23]. Relatively less work has been done on ARQ schemes for fading channels. Several ARQ schemes were studied and compared in [24] for fading channels. An ARQ scheme with rate-compatible punctured convolutional codes (PCC) for mobile radio was analyzed in [25]. An ARQ scheme with time diversity combining but without coding was investigated in [26].

In this paper, we introduce an adaptive coding scheme using high rate PCC's [27] and code combining for binary digital FM (BFM) mobile radio system. Code combining is achieved by automatic packet repetitions. The receiver constructs increasingly reliable estimates of the transmitted packet by code combining all the repetitions of a packet. The number of repetitions varies in proportion to the channel noise and interference levels. In other words, the proposed scheme automatically adapts to the channel noise conditions. This is in contrast to a time diversity system where a fixed number of repetitions is maintained for each data packet, regardless of the channel noise conditions. Moreover, the receiver uses channel state information (CSI) to perform error-and-erasure correction Viterbi decoding. The objective is to improve transmission performance without significantly increasing the receiver complexity.

The organization of the paper is as follows. Section II describes a model of the fading channel and the mathematical representation of the frequency demodulator (FD) output. The adaptive coding scheme with code combining is presented in Section III, together with its performance analysis. Section IV reports some numerical performance examples of the proposed scheme taking into account of channel fading, CI, and random FM noise. Finally, Section V contains our concluding remarks.

Manuscript received January 13, 1992; revised May 27, 1992, November 10, 1992, and May 10, 1993.
The authors are with the Department of Electrical Engineering, National University of Singapore, Kent Ridge, Singapore 0511, Republic of Singapore.
IEEE Log Number 9211590.

II. The Fading Channel Model and Representation of FD Output

We consider BFM with frequency demodulation throughout this paper. In this section, we present the fading channel model and briefly review the mathematical representation of FD output as given in [28].

At the transmitter of the communication system, a binary sequence $[x_i]$, $x_i = \pm 1$, is fed to the input of the BFM modulator. The modulator output signal at the angular frequency ω_c can be represented as

$$v(t) = \text{Re}\,[A \exp j(\omega_c t + \Phi_S(t))] \tag{1}$$

where Re$[\cdots]$ denotes the real part of a complex value, A is the amplitude, and $\Phi_S(t)$ is the modulating phase whose time derivative is given by

$$\Phi'_S(t) = \frac{d}{dt}\Phi_S(t) = \beta \sum_{i=-\infty}^{\infty} x_i g(t - iT) \tag{2}$$

with β the modulating index and $g(t)$ a unit pulse of duration T centered at $t = 0$. $v(t)$ is then transmitted over Rayleigh fading channel with AWGN and CI. The input signal to the FD at the receiver can be written as

$$e(t) = \text{Re}\,[w(t) \exp(j\omega_c t)] = R(t) \cos[\omega_c t + \Psi(t)] \tag{3}$$

where $R(t)$ and $\Psi(t)$ are the time-varying envelope and signal phase plus noise contributed by CI and AWGN, respectively. We assume that both the desired signal and CI have a bandwidth less than the coherence bandwidth of the multipath channel. The received signal is then subject to multiplicative fading. The complex envelope $w(t)$ is given by

$$w(t) = w_S(t) \exp[j\Phi_S(t)] + w_I(t) \exp[j\Phi_I(t)] + w_N(t). \tag{4}$$

In the above expression, the subscripts S, I, and N denote desired signal, CI, and AWGN, respectively; $\Phi_I(t)$ is the modulation phase of CI; $w_S(t)$, $w_I(t)$, and $w_N(t)$ are mutually independent zero-mean complex Gaussian processes with average powers of σ_S^2, σ_I^2, and σ_N^2, respectively. The received signal $e(t)$ is frequency demodulated and then sampled at iT. The FD output may be expressed as [28]

$$y_i = \text{Im}\left\{\frac{w_i(-w'_i)^*}{|w_i|^2}\right\} \tag{5}$$

where w_i and w'_i are the values of $w(t)$ and of its time derivative at the sample instant iT.

III. Adaptive Coding with Code Combining

A. Description of the Scheme

Code combining, originally suggested in [29], is a maximum-likelihood decoding technique for combining an arbitrary number of noisy received code words so that reliable communication is achieved even for channels with high noise levels. Denote a code word of a rate R code by $\boldsymbol{X} = [x_i]$. A code combined word of degree 2 is defined as $\boldsymbol{X_2} = [x_{2i}]$, where $x_{2i} = (x_i, x_i)$. That is, $\boldsymbol{X_2}$ is obtained by interleaving $\boldsymbol{X}$ with itself once. In general, we can obtain a code combined word of degree $J > 1$ by interleaving $\boldsymbol{X}$ with itself $(J-1)$ times, i.e., $\boldsymbol{X_J} = [x_{Ji}] = [(x_i, \cdots, x_i)]$, where in the parentheses the symbol x_i repeats itself J times. The rate of $\boldsymbol{X_J}$ is R/J. One important fact to note is that, $\boldsymbol{X}$ and $\boldsymbol{X_J}$ share the same code structure (i.e., the same trellis structure); therefore, they can be decoded by the same decoder.

Code combining in the proposed scheme is achieved via packet repetitions. Two linear codes are concatenated in the system to control channel distortions on transmitted code words. The outer code, denoted C_0, is an (n_0, k) CRC code used for error-detection. The inner code, denoted C_1, is a rate $R = b/(b+1)$ PCC produced from a rate 1/2, memory v $(2, 1, v)$ mother code. The inner code is used for error-and-erasure correction. At the transmitter, an information packet of k bits is first encoded into a code word of n_0 bits in C_0, and then encoded into a code word of n_1 bits in C_1, where $n_1 = (n_0 + v)/R$. Denote a code word of C_1 by $\boldsymbol{X} = [x_i]$, $i = 1, 2, \cdots, n_1$, where $x_i = 1$ for an inner encoder output bit 1 and $x_i = -1$ for an output bit 0. Code words produced by the inner encoder are then block-interleaved symbol by symbol. The coded symbol sequence after interleaving is then binary frequency modulated and transmitted over the fading channel. In the following analysis we assume that ideal interleaving with infinite interleaver size is used. The effect of finite interleaver sizes on the system performance will be discussed in Section IV.

At the receiver, the received FM symbol sequence is frequency demodulated and then block-deinterleaved. Suppose that the Jth transmission of $\boldsymbol{X}$ has been made. Let $\boldsymbol{Y_J} = [y_{Ji}]$ and $\boldsymbol{R_J} = [r_{Ji}]$, $i = 1, \cdots, n_1$, be the FD output vector and the CSI vector, respectively, obtained at the deinterleaver output. From (5), the y_{Ji} is given by

$$y_{Ji} = \text{Im}\left\{\frac{w_{Ji}(-w'_{Ji})^*}{|w_{Ji}|^2}\right\}. \tag{6}$$

For fading channels, the most useful CSI is the demodulator input envelope $|w_{Ji}|$, which can be easily obtained by sampling the output of an envelope detector at the FD sampling instant. Hence, we let $r_{Ji} = |w_{Ji}|$ in our system.

To improve decoding performance, the proposed scheme uses $\boldsymbol{R_J}$ to erase unreliable FD output symbols. Specifically, the receiver makers the following decision to produce a vector $\boldsymbol{U_J} = [u_{Ji}]$, $i = 1, \cdots, n_1$, where

$$u_{Ji} = \begin{cases} 1, \; y_{Ji} \geq 0 \quad \text{and} & r_{Ji} > \Omega \\ \text{erasure}, & r_{Ji} \leq \Omega \\ -1, \; y_{Ji} < 0 \quad \text{and} & r_{Ji} > \Omega \end{cases} \tag{7}$$

where Ω is the erasure threshold. Since a small r_{Ji} implies a deep fade and y_{Ji} is noisy and unreliable. The adaptive scheme then code combines all the received vectors $\boldsymbol{U}_1, \boldsymbol{U}_2, \cdots, \boldsymbol{U_J}$, to form a more reliable vector $\boldsymbol{Z_J} = [z_{Ji}]$, $i = 1, \cdots, n_1$. Specifically, upon the Jth reception of $\boldsymbol{X}$, the code combiner updates the combiner output vector $\boldsymbol{Z}_{\boldsymbol{J}-1} = [z_{(J-1)i}]$ that has been obtained at the $(J-1)$th reception using $\boldsymbol{U_J} = [u_{Ji}]$, i.e.,

$$z_{Ji} = (z_{(J-1)i}, u_{Ji}) = (u_{1i}, u_{2i}, \cdots, u_{Ji}), \quad i = 1, \cdots, n_1 \quad (8)$$

The newly constructed vector $\boldsymbol{Z_J}$ is the noisy received version of the code word $\boldsymbol{X_J}$ in a rate R/J code, which is more powerful than the rate R code C_1. The vector $\boldsymbol{Z_J}$ is then decoded by the Viterbi decoder of the inner code and CRC checked by the outer code decoder. If the decoded sequence is checked error-free, the information packet is accepted and delivered to the data sink; otherwise, the $(J+1)$th transmission of $\boldsymbol{X}$ is requested and retransmitted. This transmission, code combining, and decoding process continues until either final recovery of the information packet occurs or a pre-specified maximum number of allowable transmissions, $\boldsymbol{M}$, is reached. In the latter situation, the decoded data will be delivered to the data sink regardless of whether it is error-free or not.

B. Performance Analysis

In this section, throughput and decoded BER of the proposed adaptive coding scheme are analyzed. For simplicity, throughout the paper we assume that the feedback channel is noise-free.

Assume that a combined code word $\boldsymbol{X_J}$ is transmitted and that $\boldsymbol{Z_J}$ is received. The sequence $\boldsymbol{Z_J}$ is first decoded by the Viterbi decoder based on C_1 and then CRC checked by the decoder of C_0. There are two types of errors in the Viterbi decoder output: detectable errors and undetectable errors, with corresponding probabilities denoted by $P(D_J)$ and $P(U_J)$, respectively. With appropriate selection of the (n_0, k) error detection code C_0, the probability of undetected error can be made very small. It has been shown in [24] $P(U_J) \le P(D_J) \times 2^{-(n_0-k)}$. For example, $P(U_J) \le 6\times10^{-8}$ for $n_0 - k = 24$. Therefore, we will assume that the probability of undetected error $P(U_J)$ is negligible in the subsequent analysis.

An exact analysis of code combining represents a difficult task because received vectors that result in erroneous decoding are noisier on the average than those that are decoded error-free. Let $T(M)$ be the average number of transmissions that must be made before a packet is accepted by the receiver (whether correctly or incorrectly). It has been shown in [22] that $T(M)$ can be bounded by

$$1 + \sum_{J=1}^{M-1} \prod_{j=1}^{J} P(D_J) \le T(M) \le 1 + \sum_{J=1}^{M-1} P(D_J) \quad (9)$$

where $P(D_J)$ is the probability of detected error in decoding the vector $\boldsymbol{Z_J}$. Using (9), bounds on the throughput $\eta(M)$ of the proposed scheme with selective-repeat mode can be obtained from $\eta(M) = (k/n_0)R/T(M)$.

Let $P(B_M)$ be the decoded bit error rate (BER) in the decoding of $\boldsymbol{Z_M}$, and let P_b be the overall decoded BER of the adaptive coding scheme, following the approach of [22] it can be show that

$$P(B_M) \prod_{J=1}^{M-1} P(D_J) \le P_b \le P(B_M). \quad (10)$$

In the following we evaluate $P(D_J)$ and $P(B_M)$ by analyzing the performance of the Viterbi decoder. Consider two code words $\boldsymbol{X}$ and $\boldsymbol{X}'$ of C_1, with $\boldsymbol{X}$ being the transmitted one and the two corresponding combined code words $\boldsymbol{X_J}$ and $\boldsymbol{X'_J}$. Assuming that the Hamming distance between $\boldsymbol{X}$ and $\boldsymbol{X}'$ is Jd. Then the Hamming distance between $\boldsymbol{X_J}$ and $\boldsymbol{X'_J}$ is d. With maximum likelihood decoding, the combiner output $\boldsymbol{Z_J}$ will be decoded into $\boldsymbol{X}'$ if $\boldsymbol{Z_J}$ is closer to $\boldsymbol{X'_J}$ than to $\boldsymbol{X_J}$ in Hamming distance. Suppose that there are l erasures among the Jd positions in which $\boldsymbol{X_J}$ and $\boldsymbol{X'_J}$ differ. The probability of this event is $\binom{Jd}{l} e^l(1-e)^{Jd-l}$, where e is the erasure rate. Then the pair-wise probability of $\boldsymbol{Z_J}$ being decoded into $\boldsymbol{X}'$ is

$$p_J(d) = \sum_{l=0}^{Jd} \binom{Jd}{l} e^l (1-e)^{Jd-l} p_J(d, l) \quad (11)$$

where

$$\begin{aligned} p_J(d, l) &= \sum_{i=(Jd-l+1)/2}^{Jd-1} \binom{Jd-l}{i} \epsilon^i (1-\epsilon)^{Jd-l-i}, \\ &\qquad Jd-l \quad \text{odd} \\ &= \frac{1}{2} \binom{Jd-l}{\frac{(Jd-l)}{2}} \epsilon^{(Jd-l)/2} (1-\epsilon)^{(Jd-l)/2} \\ &\quad + \sum_{i=(Jd-l)/2+1}^{Jd-l} \binom{Jd-l}{i} \epsilon^i (1-\epsilon)^{Jd-l-i}, \\ &\qquad Jd-l \quad \text{even} \end{aligned} \quad (12)$$

where ϵ is the raw BER in the vector $\boldsymbol{Z_J}$. Then the error-event probability and decoded BER of the Viterbi decoding in decoding $\boldsymbol{Z_J}$ is bounded by [30]

$$P(E_J) \le \sum_{d=d_{\text{free}}}^{\infty} a_d p_J(d) \quad (13)$$

and

$$P(B_J) \le \frac{1}{b} \sum_{d=d_{\text{free}}}^{\infty} c_d p_J(d) \quad (14)$$

respectively. In these expressions, d_{free} is the free distance of C_1, a_d is the number of code words with Hamming weight d, and c_d is the information error weight on all code words with weight d. The probability of erroneous decoding of $\boldsymbol{Z_J}$ can be approximated by

$$P(D_J) \le 1 - (1 - P(E_J))^{n_0}. \quad (15)$$

To complete the performance analysis, we next derive the raw BER ϵ and the erasure rate e in the vector $\boldsymbol{Z_J}$. For the random variables w_{Ji} and w'_{Ji} in (6) we define the variances $\sigma_1^2 = \langle w_{Ji} \cdot w_{Ji}^* \rangle/2$ and $\sigma_2^2 = \langle w'_{Ji} \cdot w'^*_{Ji} \rangle/2$, where x^* is the complex conjugate of x. We further define the correlation coefficient $\rho = \rho_{\mathrm{Re}} + j\rho_{\mathrm{Im}} = \langle w_{Ji} \cdot w'^{*}_{Ji}\rangle / 2\sigma_1\sigma_2$. Assuming that $x_i = -1$ is transmitted in the ith symbol time, ρ is given by [28] (see (16) shown at the bottom of the next page), where $\Gamma = \sigma_S^2/\sigma_N^2$ and $\Lambda = \sigma_S^2/\sigma_I^2$ are the average signal-to-noise ratio and average signal-to-interference ratio, respectively, and $\rho_S(\tau)$, $\rho_I(\tau)$, and $\rho_N(\tau)$ are the autocorrelation functions of $w_S(t)$, $w_I(t)$ and $w_N(t)$, respectively. Assuming that the fading channel is exponentially correlated [33] and that the receiver predetector filter has a rectangular bandpass characteristic with bandwidth-time product $BT = 1.0$. Then $\rho_S(\tau) = \rho_I(\tau) = \exp(-2\pi f_D \tau)$ and $\rho_N(\tau) = \sin(\pi B\tau)/(\pi B\tau)$, where f_D is the maximum Doppler frequency given by vehicle speed/carrier wavelength [1], [2]. We further assume that the CI signal is also binary frequency modulated with the same modulation index as the desired signal. The signal timing and binary data of the CI signal can be assumed to be independent of these of the desired signal.

Assuming that marks and spaces are sent with equal probability, the pdf of Φ_I' is then given by

$$p(\Phi_I') = \frac{1}{2}\delta(\Phi_I' - \beta) + \frac{1}{2}\delta(\Phi_I' + \beta). \tag{17}$$

Note that ρ is a function of Φ_I' and can be rewritten as (18), shown at the bottom of the page. The probability density function (pdf) of w'_{Ji} conditional on w_{Ji} is given by [28]

$$p(w'_{Ji}|w_{Ji}) = \frac{1}{2\pi\sigma_2^2(1-|\rho|^2)} \exp\left[-\frac{|w'_{Ji} - (\sigma_2/\sigma_1)\rho^* w_{Ji}|^2}{2\sigma_2^2(1-|\rho|^2)}\right] \tag{19}$$

Equation (19) indicates that with w_{Ji} given, w'_{Ji} is a complex Gaussian variable with mean $(\sigma_2/\sigma_1)\rho^* w_{Ji}$ and variance $\sigma_2^2(1-|\rho|^2) = \sigma_2^2(1-\rho_{\mathrm{Im}}^2)$. Therefore, $\mathrm{Im}\{w_{Ji}(-w'_{Ji})^*\}$ is a Gaussian variable with mean $-(\sigma_2/\sigma_1)\rho_{\mathrm{Im}} r_{Ji}^2$ and variance $\sigma_2^2(1-\rho_{\mathrm{Im}}^2) r_{Ji}^2$. Having assumed that $x_i = -1$ is transmitted, a decision error will occur if $y_{Ji} > 0$ and the corresponding probability is

$$\Pr\{y_{Ji} > 0 | r_{Ji}, \Phi_I'\} = \Pr\{\mathrm{Im}\{w_{Ji}(-w'_{Ji})^*\} > 0 | r_{Ji}, \Phi_I'\} = Q\left[\frac{\rho_{\mathrm{Im}}(\Phi_I') r_{Ji}}{\sigma_1\sqrt{1-\rho_{\mathrm{Im}}^2(\Phi_I')}}\right] \tag{20}$$

The random envelope r_{Ji} follows Rayleigh distribution with pdf

$$p(r_{Ji}) = \frac{r_{Ji}}{\sigma_1^2}\exp\left(-\frac{r_{Ji}^2}{2\sigma_1^2}\right). \tag{21}$$

Following (7), (17), (20) and (21) we obtain the raw BER

$$\begin{aligned}\epsilon &= \int_{-\infty}^{\infty}\int_{\Omega}^{\infty} \Pr\{y_{Ji} > 0 | r_{Ji}, \Phi_I'\} p(\Phi_I') p(r_{Ji})\, dr_{Ji}\, d\Phi_I' \\ &= \int_{-\infty}^{\infty}\left\{Q\left[\frac{\rho_{\mathrm{Im}}(\Phi_I')\Omega}{\sigma_1\sqrt{1-\rho_{\mathrm{Im}}^2(\Phi_I')}}\right]\exp\left(-\frac{\Omega^2}{2\sigma_1^2}\right) - \rho_{\mathrm{Im}}(\Phi_I') Q\left[\frac{\Omega}{\sigma_1\sqrt{1-\rho_{\mathrm{Im}}^2(\Phi_I')}}\right]\right\} p(\Phi_I')\, d\Phi_I' \\ &= \frac{1}{2}\sum_{i\in\{-1,1\}}\left\{Q\left[\frac{\rho_{\mathrm{Im}}(\beta i)\Omega}{\sigma_1\sqrt{1-\rho_{\mathrm{Im}}^2(\beta i)}}\right]\exp\left(-\frac{\Omega^2}{2\sigma_1^2}\right) - \rho_{\mathrm{Im}}(\beta i) Q\left[\frac{\Omega}{\sigma_1\sqrt{1-\rho_{\mathrm{Im}}^2(\beta i)}}\right]\right\}\end{aligned} \tag{22}$$

The erasure rate can be found from (7) and (21) as

$$e = \int_0^{\Omega}\frac{r_{Ji}}{\sigma_1^2}\exp\left(-\frac{r_{Ji}^2}{2\sigma_1^2}\right) dr_{Ji} = 1 - \exp\left(-\frac{\Omega^2}{2\sigma_1^2}\right). \tag{23}$$

IV. Performance Examples and Discussions

We now compare the performance of the proposed coding scheme with the basic ARQ scheme and the MRC ARQ scheme. The basic ARQ uses only CRC for error detection, while the MRC ARQ employs postdetection maximal-ratio diversity combining reception and CRC for error detection [26]. In all the examples, we assume that the BFM modulation index $\beta = 0.5$ (MSK), the CRC uses 24 parity-check bits and has a code length of $n_0 = 325$ bits. The adaptive scheme uses rate $R = 5/6$, $v = 5$, $d_{\mathrm{free}} = 4$ PCC [30] for error-and-erasure decoding. This is a relatively simple code which is amenable to fast Viterbi decoding. We also assume that the normalized maximum Doppler frequency $f_D T = 0.01$. This corresponds to carrier frequency $f_c = 900$ MHz, a 60 mph vehicle speed, and a symbol rate of $1/T = 8$ kHz [25].

A. Optimal Symbol Erasure Threshold

We first consider the effects of AWGN on the system performance. In the adaptive coding scheme, the receiver compares the envelope of each received symbol against a threshold value Ω to erase unreliable symbols. In order to determine the optimal threshold, we compute the channel cut-off rate R_0 for the error-and-erasure channel as a function of Ω, and select the value of Ω which maximizes the R_0 parameter.

$$\rho = \rho_{\mathrm{Re}} + j\rho_{\mathrm{Im}} = j\frac{\beta\Gamma + \Phi_I'\Gamma/\Lambda}{\sqrt{\Gamma + 1 + \Gamma/\Lambda}\sqrt{\Gamma\{\beta^2 - \rho_s''(0)\} + \{\Phi_I'^2 - \rho_I''(0)\}\Gamma/\Lambda - \rho_N''(0)}} \tag{16}$$

$$\begin{aligned}\rho(\Phi_I') \equiv \rho &= j\rho_{\mathrm{Im}}(\Phi_I') \\ &= j\frac{\beta\Gamma + \Phi_I'\Gamma/\Lambda}{\sqrt{\Gamma + 1 + \Gamma/\Lambda}\sqrt{\Gamma\{\beta^2 - (2f_DT)^2\} + \{\Phi_I'^2 - (2f_DT)^2\}\Gamma/\Lambda + 1/3}}\end{aligned} \tag{18}$$

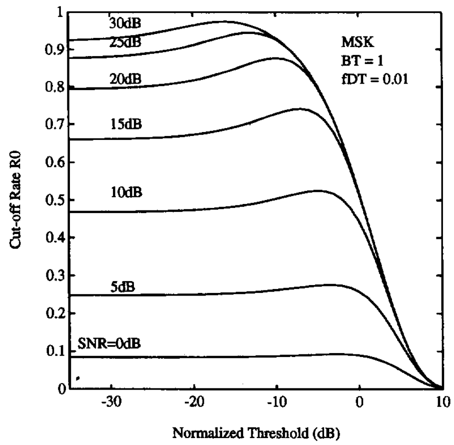


Fig. 1. Cutoff rate versus normalized threshold.

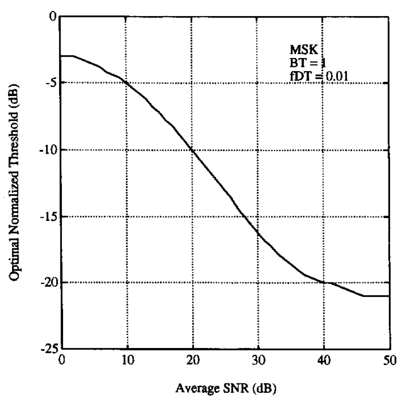


Fig. 2. The optimal normalized threshold versus SNR.

By definition, the cut-off rate is given by [31]

$$R_0 = 1 - \log_2 \left[1 + e + 2\sqrt{\epsilon(1 - e - \epsilon)}\right] \tag{24}$$

where ϵ and e are functions of Ω as given by (22) and (23), respectively. The R_0 for various values of SNR $= \Gamma/R$ (information bit signal-to-noise ratio) are plotted in Fig. 1 with respect to the normalized threshold $10\log_{10}[\Omega^2/\sigma_1^2]$. From Fig. 1, optimal threshold values are found and depicted in Fig. 2. As can be expected, the normalized optimal threshold decreases as SNR increases. The optimal threshold values in Fig. 2 are used in the system performance evaluation in the following.

B. Throughput and BER Performance

The system throughput and decoded BER can be evaluated using the bounds given in (9) and (10), respectively, assuming ideal interleaving/deinterleaving (effect of finite interleaver size is discussed in subsection IV.E). Our numerical calculations showed that the bounds are extremely tight. Therefore, in the following we will only show the lower bound on throughput and the upper bound on decoded BER. Fig. 3 depicts the throughput of the proposed scheme using the $R = 5/6$, $v = 5$ PCC with error-and-erasure decoding, where the optimal threshold values given in Fig. 2 have been

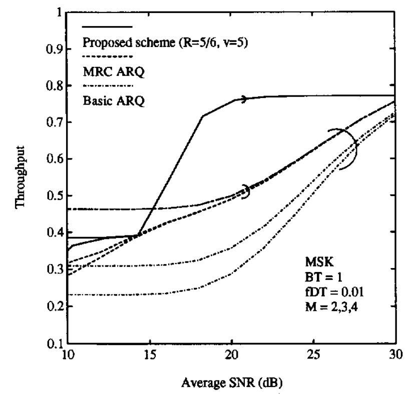


Fig. 3. Throughput versus SNR.

used. Also shown in Fig. 3 are the throughputs of the MRC and basic ARQ schemes. The maximum allowable number of transmissions M are assumed to be 2, 3, and 4 (note that the throughput curves for $M = 3$ and 4 merge into one for the proposed scheme and that the throughput for the MRC and basic ARQ schemes are identical for the case of $M = 2$). The corresponding decoded BER P_b of the various schemes are given in Fig. 4. From Fig. 3 we observe that the proposed scheme provides considerably higher throughput than the other two ARQ schemes for SNR > 15 dB. For example, at SNR $= 20$ dB, the throughputs are 0.82, 0.54, and 0.37 for the proposed scheme, the MRC ARQ, and the basic ARQ, respectively. Although the MRC and the basic ARQ achieve higher throughput than the proposed scheme at SNR < 15 dB and $M = 2$, the corresponding BER's are too high to be acceptable. From Fig. 4 we note that the BER of the proposed scheme is always less than 10^{-5} for SNR ≥ 15 dB, which is significantly lower than the BER of the other two ARQ schemes. As an example, for the case of $M = 2$, $P_b = 10^{-5}$ is achieved at SNR $=$ 15 dB, 25.5 dB, and 35 dB, respectively, for the proposed scheme, the MRC ARQ and the basic ARQ. That is, the coding gains of the proposed scheme over the MRC and the basic ARQ schemes are 10.5 dB and 20 dB, respectively. We remark that the 10.5 dB coding gain of the proposed scheme over the MRC ARQ scheme is attributed to the additional diversity that it provides through the use of $d_{\text{free}} = 4$ code in combination with adaptive retransmissions.

C. Effect of Symbol Erasure Threshold

Inspection of Fig. 1 reveals that (i), for a fixed SNR, the R_0 has a "flat-top" around its optimal threshold and that (ii), for some threshold values, say around -11 dB, the values of R_0 are at or around its maxima for practical values of SNR. These observations indicate that system performance is not too sensitive to the setting of the threshold value and is quite robust against certain variations in the SNR. Fig. 5 compares the throughputs of the proposed scheme with optimal threshold setting and with a normalized threshold fixed at -11 dB. The system performance degrades only slightly with fixed threshold than with the optimal threshold, which confirms the R_0 prediction. To show the effectiveness of the symbol erasure in improving the performance, also plotted in Fig. 5 is the

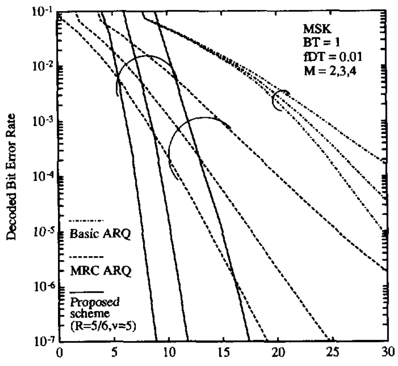


Fig. 4. Decoded BER versus SNR.

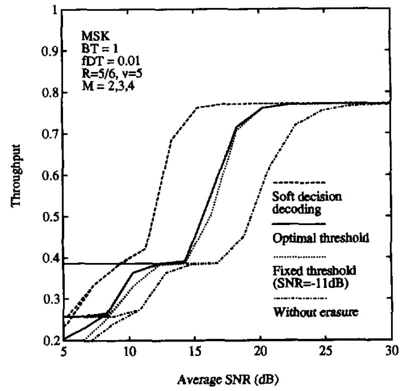


Fig. 5. Throughput versus SNR for the proposed scheme with optimal threshold, with fixed threshold, without erasure operation, and with soft-decision decoding.

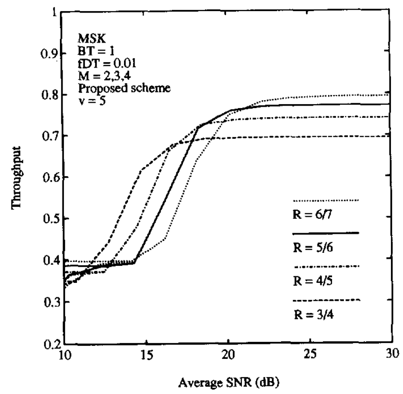


Fig. 6. Effects of code rate on the throughput of the proposed scheme.

throughput for the proposed scheme without symbol erasure operation. The advantage of performing symbol erasure is apparent. The scheme with symbol erasure operation gives 3 to 5 dB more coding gain than the scheme without symbol erasure operation. Although the present paper emphasizes on error-and-erasure decoding, the proposed adaptive coding scheme, i.e., the concept of convolutional coding, code combining, and adaptive retransmissions, is general and applies equally well with soft-decision decoding [32]. The throughput of the proposed scheme with soft-decision decoding and without demodulator output quantization is also shown in Fig. 5. Soft-decision decoding without quantization provides us the best possible performance. It is about 3 to 5 dB better than the scheme with error-and-erasure decoding; however, soft-decision decoding is more complex in implementation than the simple error-and-erasure decoding. The adaptive coding scheme with soft-decision Viterbi decoding uses $r_i y_i x_i$ as decoding metric, where y_i is the ith received symbol, r_i is the corresponding signal envelope, and x_i is a code symbol on the branch of the PCC trellis. There are two approaches in realization of the soft-decision Viterbi decoding. The first approach is to compute the bit metric $r_i y_i x_i$ without quantization. This approach gives the best performance but requires floating point operation. The second approach is to quantize r_i, say, to q_r bits and y_i, say, to q_y bits. The precalculated metric values can be stored in a read-only memory which can be addressed by $q_r + q_y + 1$ bits [25]. Quantization causes performance degradation. Effect of quantization to coding performance on AWGN channel is well known [16]; however, to the authors' knowledge, no parallel study has been done for fading channels.

D. Effect of Interleaver Size

Fading channels produce nonindependent errors as characterized by the channel correlation between adjacent received symbols. In all the above analysis and examples, we have assumed that ideal interleaving/deinterleaving is used to produce random erasures and errors at the decoder input. In practice, however, block interleaver of finite interleaver size is used. A block interleaver can be regarded as a buffer with d rows that represent the depth of interleaving and s columns that represent the span of interleaving. Thus the size of the interleaver is $d \times s$. Data are written into the buffer in successive rows and read out of the buffer in columns. At the receiver, a block deinterleaver performs the reverse operation. Thus two adjacent channel symbols are separated by $d - 1$ symbols during transmission. When the interleaving span s is chosen on the order of the Viterbi decoding delay, it can be shown that block interleaving has the same effect, from an error performance point of view, as transmitting at an increased symbol duration dT [33]. Thus if the channel has a maximum Doppler frequency $f_D T$, the channel including block interleaver/deinterleaver with interleaver size $d \times s$ will have an equivalent maximum Doppler frequency $df_D T$, as long as s is on the order of the Viterbi decoding delay. Performance degradation due to use of finite interleaver size compared with ideal interleaving can be estimated based on the equivalent maximum Doppler frequency $df_D T$ using the technique developed in [33].

In all the above numerical examples, we have assumed that the maximum Doppler frequency $f_D T = 0.01$, the CRC has a length $n_0 = 325$ bits, and that the PCC has code rate $R = 5/6$ and memory order $v = 5$. The packet is then $n_1 = (n_0 + v)/R = 396$ bits long. This PCC requires a decoding delay of about 66 bits [30]. Therefore, the interleaving span is chosen as $s = 66$ bits. If one packet is block interleaved, we have an interleaver size $d \times s = 396$ bits and interleaving depth

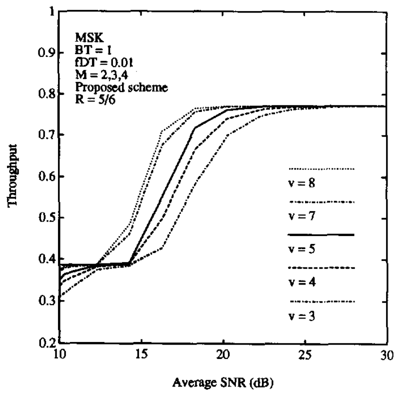


Fig. 7. Effects of code memory order on the throughput of the proposed scheme.

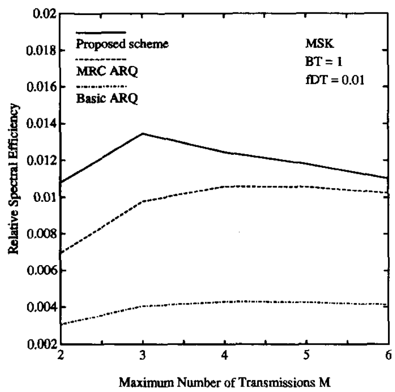


Fig. 8. Relative spectral efficiencies versus M.

$d = 396/s = 6$. Hence the equivalent maximum Doppler frequency $df_DT = 0.06$. In this case, the SNR degradation can be estimated to be 3 dB compared with the ideal interleaving case [33]. On the other hand, if two packets are block interleaved, we have an interleaver size $d \times s = 792$ bits and interleaving depth $d = 12$. Hence, the equivalent maximum Doppler frequency $df_DT = 0.12$, and the SNR degradation is reduced to 1 dB compared to the ideal interleaving case.

E. Effect of PCC Parameters

We next examine the effect of various PCC parameters on the system performance. Again, we assume $n_0 = 325$, $f_DT = 0.01$, and ideal interleaving. Fig. 6 shows the throughputs of the proposed scheme using PCC's with $v = 5$ but varying code rates. The maximum throughput is determined by the code rate. That is, a higher code rate gives a higher throughput at large average SNR's. As the channel becomes noisy, i.e., for average SNR < 20 dB, a lower code rate provides a better throughput. This is because that with v fixed, a lower code rate has a larger free distance and, therefore, a larger error-correcting capability. On the other hand, with code rate being fixed, codes with large v are more powerful in error correcting, and as a consequence, provide better throughputs. This is illustrated in Fig. 7 for the proposed system with PCC's of $R = 5/6$ and memory orders $v = 3$ to 8.

Cochannel interference has a similar influence on system performance as AWGN; hence, the performance curves as functions of SIR are not given in this paper in order to keep the presentation compact.

F. Spectral Efficiency

In a cellular land mobile radio system, the radio channels are divided into several channel sets, and the same channel sets are reused in spatially separated cells in order to make efficient utilization of the limited radio spectrum. Assuming hexagonal cell layout, the spectral efficiency ξ of the cellular system is given by [26]

$$\xi \propto \xi_0 = \frac{\eta(M)}{(1+\Lambda_m^{1/\alpha})^2} \tag{25}$$

where $\eta(M)$ is the system throughput, α is the propagation constant (typical value for urban area is $\alpha = 3.5$ [2]), and Λ_m is the required area average SIR determined by the outage probability due to shadowing. The outage probability, denoted Q, is defined as the probability that the average SIR Λ will fade below a required SIR Λ_0 for achieving the desired BER P_b. Assuming log-normal shadow fading with a standard derivation σ, Q is related with Λ_m and Λ_0 by

$$Q = \frac{1}{2}\,\text{erfc}\left(\frac{10\log_{10}(\Lambda_m/\Lambda_0)}{2\sigma}\right). \tag{26}$$

To evaluate the spectral efficiency, one first calculates the throughput and decoded BER as functions of SIR. Next one needs to find Λ_0 corresponding to the desired BER. Finally, with Q, Λ_0 and σ given, Λ_m can be determined using (26) and spectral efficiency follows from (25). Assuming that $Q = 0.1$, $\sigma = 6$ dB, $\alpha = 3.5$ and $P_b = 10^{-4}$, the spectral efficiencies of the proposed scheme using $R = 5/6$, $v = 5$ PCC, the MRC scheme, and the basic ARQ scheme were evaluated and plotted in Fig. 8. Although the proposed scheme employs an PCC of rate $R = 0.83$, it provides the highest spectral efficiency among the three schemes. This is because the SIR required to achieve a given BER for the proposed scheme is much smaller than that for the MRC and basic ARQ. As a result, the reuse distance of the channel sets becomes smaller and the usage of spectral becomes more efficient.

V. Conclusions

In this paper, we have proposed an adaptive coding scheme for mobile radio communications. The scheme employs code combining which is achieved by automatic retransmission of erroneous decoded packets. Code combining preserves the trellis structure of the original code; therefore, only a single Viterbi decoder is required to decode any received packet or any combination of the repetitions of the packet.

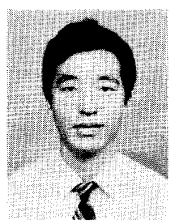
The received signal envelope is sampled and used as CSI, resulting in an effective error-and-erasure decoding operation at the receiver. Channel cut-off rate R_0 is believed to be the upper limit on the rate of error-control codes for a practically implementable system. For this reason, we have used R_0 to predict the optimal setting of the erasure threshold, and our numerical results have shown that the prediction is very accurate. The proposed scheme has been analyzed for BFM over frequency-flat fading channel with AWGN, CI, and random FM noise. Analytical expressions have been derived for the throughput, decoded BER, and spectral efficiency. Results have shown that the proposed scheme performs significantly better that the schemes without code combining. As a final remark, we note that the proposed scheme and its performance analysis can be generalized easily to other modulation/demodulation schemes and to fading channels with delay spread.

ACKNOWLEDGMENT

The authors wish to thank the anonymous reviewers for their valuable comments, which have helped improve the quality of the paper significantly.

REFERENCES

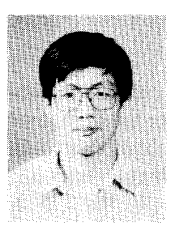
[1] R. H. Clarke, "A statistical theory of mobile-radio reception," *AT&T Bell Syst. Tech. J.*, vol. 47, pp. 957–1000, 1968.
[2] W. C. Jakes, *Microwave Mobile Communication.* New York: Wiley, 1974.
[3] J. D. Parsons, M. Henze, P. A. Ratliff, and M. J. Withers, "Diversity techniques for mobile radio," *Radio Electron. Eng.*, vol. 45, pp. 357–367, July 1975.
[4] H. Zhou, S. Yoshida, and T. Takeuchi, "Selection diversity reception based on cross-channel-interference measurement for digital mobile radio," *Trans. IEICE Japan*, vol. E73, pp. 658–664, May 1990.
[5] P. J. Mabey, "Mobile radio data transmission-coding for error control," *IEEE Trans. Veh. Technol.*, vol. 27, pp. 99–109, 1978.
[6] M. H. Kahn, Y. R. Shayan, T. Le-Ngoc, and U. K. Bhargava, "Forward error control coding for mobile communication systems," in *Proc. Conf. Rec. VTC'88*, Philadelphia, PA, p. 509, 1988.
[7] J. Hagenauer, "Viterbi decoding of convolutional codes for fading- and bursty," in *Proc. Conf. Rec. 1980 Zurich Seminar Digital Commun.*, pp. G2.1–G2.7, 1980.
[8] J. Hagenauer and E. Lutz, "Forward error correction coding for fading compensation in mobile satellite channels," *IEEE J. Select. Areas Commun.*, vol. 5, pp. 215–225, Feb. 1987.
[9] T. Matsumoto, "Soft decision decoding of block codes using received signal envelope in digital mobile radio," *IEEE J. Select. Areas Commun.*, vol. 7, pp. 107–113, Jan. 1989.
[10] T. Matsumoto, "Decoding performance of linear block codes using a trellis in digital mobile radio," *IEEE Trans. Veh. Technol.*, vol. 39, pp. 68–74, Feb. 1990.
[11] W. E. Ryan and S. G. Wilson, "Two classes of convolutional codes over $GF(q)$ for q-ary orthogonal signaling," *IEEE Trans. Commun.*, vol. 39, pp. 30–40, 1991.
[12] S. G. Wilson and Y. S. Leung, "Trellis-coded phase modulation on Rayleigh channels," in *Proc. Int. Conf. Commun.*, Seattle, WA, pp. 21.3.1–21.3.5, June 1987.
[13] D. Divsalar and M. K. Simon, "The design of trellis coded MPSK for fading channels: Performance criteria," *IEEE Trans. Commun.*, vol. 36, pp. 1004–1012, Sept. 1988.
[14] ——, "The design of trellis coded MPSK for fading channels: Set partitioning for optimum code design," *IEEE Trans. Commun.*, vol. 36, pp. 1013–1022, Sept. 1988.
[15] C. Schlegel and D. J. Costello, Jr., "Bandwidth efficient coding for fading channels: Code construction and performance analysis," *IEEE J. Select. Areas Commun.*, vol. 7, pp. 1356–1368, Dec. 1989.
[16] S. Lin and D. J. Costello, Jr., *Error Control Coding: Fundamentals and Applications.* Englewood Cliffs, NJ: Prentice-Hall, 1983.
[17] T. Kasami, T. Fujiwara, and S. Lin, "A concatenated coding scheme for error control," *IEEE Trans. Commun.*, vol. 34, pp. 481–488, May 1986.
[18] R. H. Deng and D. J. Costello, Jr., "Reliability and throughput analysis of a concatenated coding scheme," *IEEE Trans. Commun.*, vol. 35, pp. 698–705, July 1987.
[19] S. Lin and P. Y. Yu, "A hybrid ARQ scheme with parity retransmission for error control of satellite channels," *IEEE Trans. Commun.*, vol. 30, pp. 1701–1719, July 1982.
[20] Y. M. Wang and S. Lin, "A modified selective-repeat type-II hybrid ARQ system and its performance analysis," *IEEE Trans. Commun.*, vol. 31, pp. 593–608, May 1983.
[21] L. R. Lugand, D. J. Costello, Jr., and R. H. Deng, "Parity retransmission hybrid ARQ using rate 1/2 convolutional codes on a nonstationary channel," *IEEE Trans. Commun.*, vol. 37, pp. 755–765, July 1989.
[22] S. Kallel, "Analysis of a type-II hybrid ARQ scheme with code combining," *IEEE Trans. Commun.*, vol. 38, pp. 1133–1137, Aug. 1990.
[23] R. H. Deng, "Hybrid ARQ scheme using TCM and code combining," *Electron. Letters*, vol. 27, pp. 866–868, May 9, 1991.
[24] R. A. Comroe and D. J. Costello, Jr., "ARQ schemes for data transmission in mobile radio systems," *IEEE J. Select. Areas Commun.*, vol. 2, pp. 472–481, July 1984.
[25] J. Hagenauer, N. Seshadri, and C.-E. W. Sundberg, "The performance of rate-compatible punctured codes for digital mobile radio," *IEEE Trans. Commun.*, vol. 38, pp. 966–980, July 1990.
[26] F. Adachi, S. Ito, and K. Ohno, "Performance analysis of a time diversity ARQ in land mobile radio," *IEEE Trans. Commun.*, vol. 37, pp. 177–183, Feb. 1989.
[27] J. B. Cain, G. C. Clark, Jr., and J. M. Geist, "Punctured convolutional codes of rate $(n-1)/n$ and simplified maximum likelihood decoding," *IEEE Trans. Inform. Theory*, vol. 25, pp. 97–100, Jan. 1979.
[28] F. Adachi and J. D. Parsons, "Unified analysis of postdetection diversity for binary digital FM mobile radio," *IEEE Trans. Veh. Technol.*, vol. 37, pp. 189–198, Nov. 1988.
[29] D. Chase, "Code combining—A maximum-likelihood decoding approach for combining an arbitrary number of noisy packets," *IEEE Trans. Commun.*, vol. 33, pp. 385–393, May 1985.
[30] D. Haccoun and G. Begin, "High-rate punctured convolutional codes for Viterbi and sequential decoding," *IEEE Trans. Commun.*, vol. 37, pp. 1113–1125, Nov. 1989.
[31] J. M. Wozencraft and I. M. Jacobs, *Principles of Communication Engineering.* New York: Wiley, 1965.
[32] H. Zhou and R. H. Deng, "A hybrid ARQ scheme with diversity combining for land mobile radio," in *Proc. 42nd VTS Conf.*, May 10–13, 1992, Denver, CO, pp. 902–905.
[33] F. Gagnon and D. Haccoun, "Bounds on the error performance of coding for nonindependent Rician-fading channels," *IEEE Trans. Commun.*, vol. 40, pp. 351–360, Feb. 1992.



Robert H. Deng (M'86) received the B.E. degree in electrical engineering from Changsha Institute of Technology, China, in 1981 and the M.S. and Ph.D. degrees in electrical engineering from the Illinois Institute of Technology, Chicago, in 1983 and 1985, respectively.

From 1986 to 1987 he was a Postdoctorate Research Associate at the Department of Electrical and Computer Engineering, University of Notre Dame, Notre Dame, IN. Between 1987 and 1991 he was a Research Staff Member at the Institute of Systems Science, National University of Singapore, Singapore. Currently he is a Senior Lecturer at the Department of Electrical Engineering, National University of Singapore. His research interests include error-control coding for digital communication and data storage systems, coded modulations, mobile radio systems, network interconnection, local area networks, performance evaluation, and network fault management.

Dr. Deng is a member of the IEEE Information Theory and Communications Societies.



Hong Zhou (M'91) was born in Jiangsu, P. R. China on October 27, 1962. He received the B.E. degree in electrical engineering from Tsinghua University, Beijing, P. R. China in 1983, and the M.E. degree in information science and the Ph.D. degree in electrical engineering from Kyoto University, Kyoto, Japan, in 1987 and 1990, respectively.

He is currently a Research Scientist at National University of Singapore, Singapore. Since 1987, he has been researching mobile radio communications. His current research area is digital mobile/personal communications with the emphasis on modulation/demodulation, error control coding, and diversity reception techniques.

Dr. Zhou is a member of the Institute of Electronics, Information and Communication Engineers of Japan.