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A High-Throughput Routing Metric for Reliable Multicast in Multi-Rate Wireless Mesh Networks

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***Abstract*—We propose a routing metric for enabling high-throughput reliable multicast in multi-rate wireless mesh networks. This new multicast routing metric, called *expected multicast transmission time* (EMTT), captures the combined effects of 1) MAC-layer retransmission-based reliability, 2) transmission rate diversity, 3) wireless broadcast advantage, and 4) link quality awareness. The EMTT of one-hop transmission of a multicast packet minimizes the amount of expected transmission time (including that required for retransmissions). This is achieved by allowing the sender to adapt its bit-rate for each ongoing transmission/retransmission, optimized exclusively for its next-hop receivers that have not yet received the multicast packet. We model the rate adaptation process as a Markov decision process (MDP) and derive an efficient procedure for computing EMTT from the theory of MDP. We present receiver-initiated algorithms and describe protocol implementation for the EMTT-based multicast routing problem. Numerical results are presented to demonstrate the accuracy of the proposed algorithms against optimal solutions to the multicast routing problem. Simulation experiments confirm that, in comparison with single-rate multicast, multi-rate multicast using the EMTT metric effectively reduces the overall multicast transmission time while yielding higher packet delivery ratio and lower end-to-end latency.**

I. INTRODUCTION

Multi-rate radios are commonly defined in wireless networking standards. Using different modulation techniques, the widely deployed IEEE 802.11a/b/g standards support transmitters with up to ten different bit-rates, varying from 1 Mbps to 54 Mbps. The use of multi-rate is, however, restricted to unicast in current 802.11 standards. In the case of a base station multicasting/broadcasting to a multiple of nodes, the MAC-layer protocol chooses the lowest available transmission rate by default. This is tolerable for conventional wireless LANs, where MAC-layer broadcast is mainly used for control messages. In wireless mesh networks (WMNs), MAC-layer multicast may be desirable to support reliable data transmission. In this context, using MAC-layer multicast at lower transmission rate leads to longer channel occupancy of the multicast session and under-utilization of the network resource. This motivates the development of efficient multicast algorithms by exploiting the *rate diversity* of wireless links.

One salient feature of wireless communication is the so-called *wireless broadcast advantage*. A single multicast transmission using the wireless medium can reach multiple nodes within the transmission range of the sender. It is important to exploit the wireless broadcast advantage in the design of multicast algorithms for multicast routing in WMNs. However, since the wireless medium is inherently broadcast, it makes all nodes within the transmission range of the sender part of one collision domain. This effect is aggravated in multi-hop wireless networks because of the existence of hidden terminals. Since lower transmission rate always results in longer transmission time, it further increases the chance of packet collisions caused by hidden terminals. For this reason, multicast algorithms that can effectively reduce the channel occupancy time for multicast transmissions are desirable for multicast routing in multi-hop WMNs.

The wireless medium is by nature error-prone. In multi-hop WMNs, packet collisions caused by hidden terminals result in added packet loss. Such probabilistic packet loss may cause the end-to-end packet delivery ratio (PDR) to be unacceptably bad for many applications, including multicast. Although current IEEE 802.11 standards do not provide any MAC-layer recovery mechanism for multicast frames, a number of reliable MAC-layer multicast protocols have been proposed to offer hop-by-hop recovery on probabilistic packet loss [1]–[5]. Researchers have also explored the idea of *physical-layer network coding* and developed bandwidth efficient methods for link-layer acknowledgement for multicast transmissions in wireless networks [6]–[8]. It is challenging to design multicast routing protocols for multi-hop WMNs that can take advantage of MAC-layer retransmission-based reliability.

Mesh routers in WMNs are in general stationary and do not have power supply constraint typical of ad hoc networks. The objective of WMNs is to offer high-performance wireless connection to end users. This motivates the design of a robust routing metric that can find high-performance paths compared to the simple hop-count metric used in most ad hoc networks. The expected transmission count (ETX) metric [9] and the expected transmission time (ETT) metric [10] are two popular link-quality-aware routing metrics in WMNs. However, they are both designed for unicast. The success probability product (SPP) metric [11] is designed for multicast, but does not take MAC-layer retransmission-based reliability, transmission rate diversity and wireless broadcast advantage into account. The expected multicast transmissions (EMT) metric [12] is designed for high-throughput reliable multicast in single-rate WMNs. We discuss these related work in more detail in Section VII.

This paper addresses the challenges of reliable multicast routing in multi-rate multi-hop WMNs by proposing a more robust multicast routing metric. The key contributions of this paper are:

- We propose a rate adaptation scheme specifically designed for MAC-layer reliable multicast transmissions in WMNs to exploit the rate diversity.
- Based on the rate adaptation scheme, we propose a new routing metric for achieving high-throughput, low-latency and reliable multicast in multi-rate multi-hop WMNs. The metric, called *expected multicast transmission time* (EMTT), is explicitly designed to capture the combined effects of 1) MAC-layer retransmission-based reliability, 2) transmission rate diversity, 3) wireless broadcast advantage and 4) link quality awareness.
- We model the rate adaptation process as a Markov decision process (MDP), and apply the theory of MDP for computing the EMTT metric and determining the optimal rate adaptation policy.
- We formulate the EMTT-based multicast routing problem and prove its NP hardness. Integer linear programming (ILP) is provided to find an optimal solution for the problem. Given that this is an NP-hard problem, a polynomial-time greedy algorithm is proposed.
- We extend the centralized algorithm to a distributed algorithm, and also describe its protocol implementation. Numerical results are provided for demonstrating the effectiveness of EMTT-based multi-rate multicast routing.

The remainder of this paper is organized as follows. Section II provides the details of the rate adaptation scheme and the design of the EMTT metric. Section III formulates the EMTT-based multicast routing problem. The greedy algorithm and its distributed version are presented in Section IV and Section V, respectively. Description of protocol implementation and its simulation results are provided in Section VI. Section VII describes related work, and Section VIII draws the conclusion.

II. METRIC DESIGN

This section provides a detailed design of our EMTT metric for multicast routing in multi-rate multi-hop WMNs. As discussed, the EMTT metric is designed to capture the combined effects of 1) MAC-layer retransmission-based reliability, 2) transmission rate diversity, 3) wireless broadcast advantage and 4) link quality awareness. The computation of EMTT for one-hop multicast transmission takes as input the link PDRs at each transmission rate from the sender to its next-hop receivers. Note that the link PDR from node i to node j is a function of the transmission rate used (more accurately, a function of the modulation scheme used and its SNR sensitivity), and in this context, is defined as the probability that a multicast transmission (at the designated transmission rate) from node i is successfully received and acknowledged by the node j. The sender will retransmit a multicast packet to its next-hop receivers which have not acknowledged the packet successfully. The EMTT of one-hop transmission of a multicast packet is defined as the expected minimum transmission time needed for all next-hop recipients to receive the packet successfully including retransmissions.

For the purpose of this section, it suffices to consider one sending node i in the network and the set of its next-hop receivers $\mathcal{R}_i$ within its one-hop *neighborhood* N_i. The one-hop neighborhood of node i is defined as the set of nodes within the transmission range of node i. For node j to be within the transmission range of node i, we require that the link PDR from node i to node j is non-zero for at least one transmission rate. The set of transmission rates available in the network is denoted by $B = \{r_k : k = 1, 2, \ldots, K\}$, where the elements of B are arranged in an increasing order of the value of r_k. The link PDR from node i to node j at the k-th transmission rate is denoted by $p_{i,k,j}$.

A. Rate adaptation

With the link-layer acknowledgement mechanism (such as that of RMAC [4]) enabled for multicast transmissions, the sender is able to deduce from the acknowledgement whether any of its next-hop receivers has not yet received the multicast packet. Utilizing this information, our proposed rate adaptation scheme allows the sender to adapt its transmission rate for each ongoing retransmission. In particular, it retransmits the multicast packet at the best transmission rate in favor of EMTT, optimized exclusively for its next-hop receivers that have not yet received the multicast packet.

Formally, the rate adaptation process can be described as follows. Let us define a *state* of the rate adaptation process of the sending node i as a subset $\mathcal{S}$ of its next-hop receivers $\mathcal{R}_i$. Being in state $\mathcal{S}$ indicates that none of the nodes in the set $\mathcal{S}$ has received the multicast packet. Choosing different transmission rates in state $\mathcal{S}$ for an ongoing transmission/retransmission results in different amount of expected transmission time (including those expected for subsequent retransmissions if required). The rule of the rate adaptation scheme is for the sending node i to use the best transmission rate, denoted by $\Pi_{i,\mathcal{S}}$, that results in the minimum amount of expected transmission time, denoted by $\mathrm{EMTT}_{i,\mathcal{S}}$.

The rate adaptation process always starts in state $\mathcal{S} = \mathcal{R}_i$ before the first transmission of the multicast packet. After each transmission/retransmission, the process may remain in the same state $\mathcal{S}$ or move into a different state $\mathcal{S}'$.

- If the process remains in the same state $\mathcal{S}$, it indicates that the current transmission fails to deliver the multicast packet to any node in the set $\mathcal{S}$. In this case, the sender continues to use the same transmission rate $\Pi_{i,\mathcal{S}}$ for the next retransmission.
- If the process moves into a different state $\mathcal{S}'$, it indicates that at least one of the nodes in the set $\mathcal{S}$ has received the multicast packet due to the current transmission. Accordingly, the set $\mathcal{S}'$ must be a subset of $\mathcal{S}$, i.e., $\mathcal{S}' \subset \mathcal{S}$. In this case, since none of the nodes in the set $\mathcal{S}'$ has received the multicast packet, the sender uses the corresponding best transmission rate $\Pi_{i,\mathcal{S}'}$ for the next retransmission.

The process continues in this way until either the number of retransmissions of the multicast packet has reached the limit or the process moves into state $\emptyset$ (empty set). The latter indicates that all nodes in the set $\mathcal{R}_i$ have received the multicast packet.

Clearly, the rate adaptation process involves a policy for guiding which transmission rate the sender should choose when the process is in a particular state. We shall see next that the optimal policy of rate adaptation in different states and the corresponding best transmission rate in each state can be determined along with the computation of EMTT.

B. Computing EMTT

We model the rate adaptation process associated with the computation of EMTT for each forwarding node in the multicast session as a stationary infinite-horizon MDP [13]. MDPs are widely used for modeling sequential decision making in situations where outcomes are uncertain. An MDP model has five components: *decision epochs*, *states*, *actions*, *rewards*, and *transition probabilities*. At each decision epoch, the MDP is in some state $\mathcal{S}$. The decision maker may choose any action from the set of possible actions in state $\mathcal{S}$. As a result, the decision maker receives a corresponding reward, and the MDP moves into a new state $\mathcal{S}'$ at the next decision epoch with a certain transition probability. The process is Markov because the reward and the transition probability depend only on the current state and action, and are independent of any previous state or action.

The collection of actions that the decision maker may choose for each particular state of the MDP forms a *policy*. The goal of the MDP is to find an optimal policy for the decision maker such that certain optimality criterion is satisfied. Specifically, if the reward is in the form of *revenue*, the optimality criterion is to maximize the expected total revenue. If the reward is in the form of *cost*, the optimality criterion is to minimize the expected total cost. The optimal value of a state of the MDP is the expected total reward that the decision maker will receive if it starts the MDP in that state and executes the optimal policy.

For the multi-rate multicast routing problem considered in this paper, each forwarding node in the multicast session is a decision maker. Therefore, the rate adaptation process of the sending node i can be modeled as an MDP. A decision epoch corresponds to the beginning of one transmission/retransmission of the multicast packet. Since we allow an infinite number of retransmissions in computing the EMTT, the set of decision epochs is infinite, and the decision problem in this context is an infinite horizon problem [13].

The state space of the MDP for describing the rate adaptation process of the sending node i is represented by the power set of $\mathcal{R}_i$. Being in state $\mathcal{S} \subseteq \mathcal{R}_i$ indicates that none of the nodes in the set $\mathcal{S}$ has received the multicast packet from the sending node i. The MDP starts in state $\mathcal{R}_i$, and terminates whenever it moves into state $\emptyset$, meaning that all nodes in the set $\mathcal{R}_i$ have received the multicast packet. The set of possible actions in each state is the set of available transmission rates in the network. If the current state of the MDP is a non-empty set $\mathcal{S}$, it may move into any other state $\mathcal{S}'$ in the power set of $\mathcal{S}$ at the next decision epoch. In particular, if $\mathcal{S}'$ is a non-empty set, the MDP continues from $\mathcal{S}'$; otherwise, the MDP terminates there. Figure 1 illustrates the state transition diagram of the MDP for the sending node i with three next-hop receivers.

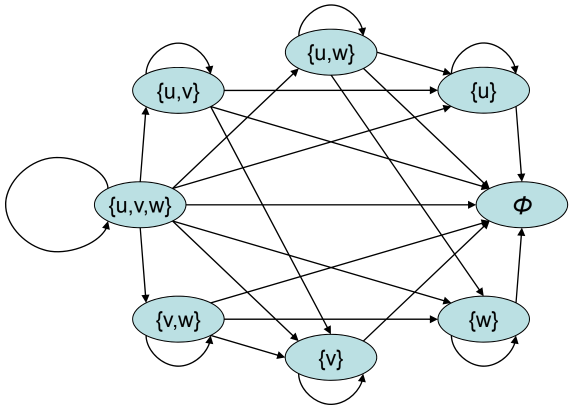


Fig. 1. State transition diagram of the MDP for computing $\mathrm{EMTT}_{i,\mathcal{R}_i}$ with $\mathcal{R}_i = \{u, v, w\}$.

We wish to know the optimal value of each state $\mathcal{S}$ of the MDP and the corresponding transmission rate $\Pi_{i,\mathcal{S}}$ the sending node i should use such that $\mathrm{EMTT}_{i,\mathcal{S}}$ is minimized. When such an optimal action is specified for each state, the optimal rate adaptation policy is determined for the sending node i.

For the MDP to move from state $\mathcal{S}$ into state $\mathcal{S}' \subseteq \mathcal{S}$ by choosing action k, i.e., using the k-th transmission rate, the transition probability is given by

$$P_{\mathcal{S},k,\mathcal{S}'} = \prod_{u \in \mathcal{S}-\mathcal{S}'} p_{i,k,u} \prod_{v \in \mathcal{S}'} (1 - p_{i,k,v})$$

assuming $p_{i,k,u}$ and $p_{i,k,v}$ are statistically independent. The cost of choosing action k in state $\mathcal{S}$ is the time required for one transmission of the multicast packet of size L using the k-th transmission rate, given by

$$C_{k,\mathcal{S}} = \frac{L}{r_k}.$$

Since none of the nodes in the set $\mathcal{S}'$ has received the multicast packet, the expected future cost of the MDP conditional on being next in state $\mathcal{S}'$ is $\mathrm{EMTT}_{i,\mathcal{S}'}$. Accordingly, for each element $\mathcal{S}$ in the power set of $\mathcal{R}_i$ excluding the empty set $\emptyset$, we have

$$\mathrm{EMTT}_{i,\mathcal{S}} = \min_{k} \Big(C_{k,\mathcal{S}} + \sum_{\mathcal{S}' \subseteq \mathcal{S}} P_{\mathcal{S},k,\mathcal{S}'} \mathrm{EMTT}_{i,\mathcal{S}'} \Big) \tag{1}$$

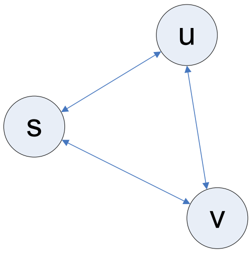
where $\mathrm{EMTT}_{i,\emptyset} = 0$ by definition.

We note that solving (1) is equivalent to solving the optimization problem

$$\max \ \mathrm{EMTT}_{i,\mathcal{S}}$$

subject to

$$\mathrm{EMTT}_{i,\mathcal{S}} \leq C_{k,\mathcal{S}} + \sum_{\mathcal{S}' \subseteq \mathcal{S}} P_{\mathcal{S},k,\mathcal{S}'} \mathrm{EMTT}_{i,\mathcal{S}'}, \ \forall k$$



Link	PDR	
	1 Mbps	2 Mbps
$\langle s,u\rangle$	0.8	0.6
$\langle s,v\rangle$	0.7	0.3
$\langle u,v\rangle$	0.6	0.4
$\langle v,u\rangle$	0.7	0.3

Fig. 2. Topology of a three-node multi-rate WMN and a snapshot of its link PDRs at two transmission rates.

which, in turn, is equivalent to solving

$$\max\ \mathrm{EMTT}_{i,\mathcal{S}}$$

subject to

$$\mathrm{EMTT}_{i,\mathcal{S}} \le \frac{C_{k,\mathcal{S}} + \sum_{\mathcal{S}' \subset \mathcal{S}} P_{\mathcal{S},k,\mathcal{S}'} \mathrm{EMTT}_{i,\mathcal{S}'}}{1 - P_{\mathcal{S},k,\mathcal{S}}},\ \forall k$$

given that $0 \le P_{\mathcal{S},k,\mathcal{S}} < 1$ for all k in this context. Thus, the solution to (1) is

$$\mathrm{EMTT}_{i,\mathcal{S}} = \min_k \frac{C_{k,\mathcal{S}} + \sum_{\mathcal{S}' \subset \mathcal{S}} P_{\mathcal{S},k,\mathcal{S}'} \mathrm{EMTT}_{i,\mathcal{S}'}}{1 - P_{\mathcal{S},k,\mathcal{S}}} \tag{2}$$

and we have

$$\Pi_{i,\mathcal{S}} = r_{k^*} \tag{3}$$

where

$$k^* = \arg\min_k \frac{C_{k,\mathcal{S}} + \sum_{\mathcal{S}' \subset \mathcal{S}} P_{\mathcal{S},k,\mathcal{S}'} \mathrm{EMTT}_{i,\mathcal{S}'}}{1 - P_{\mathcal{S},k,\mathcal{S}}}.$$

From (2), we apply dynamic programming to obtain the $\mathrm{EMTT}_{i,\mathcal{S}}$ value for each state $\mathcal{S} \subseteq \mathcal{R}_i$, which involves an $|\mathcal{R}_i|$-level iterative computation process with the boundary condition $\mathrm{EMTT}_{i,\emptyset} = 0$. Specifically, in level 1, we compute the $\mathrm{EMTT}_{i,\mathcal{S}}$ value for each element $\mathcal{S}$ in the power set of $\mathcal{R}_i$ with $|\mathcal{S}| = 1$. Then, in each subsequent level n, $n = 2, 3, \ldots, |\mathcal{R}_i|$, we compute the $\mathrm{EMTT}_{i,\mathcal{S}}$ value for each element $\mathcal{S}$ in the power set of $\mathcal{R}_i$ with $|\mathcal{S}| = n$, given that all $\mathrm{EMTT}_{i,\mathcal{S}}$ values for $|\mathcal{S}| = 1, 2, \ldots, n-1$ have been computed from the previous levels. By (3), we obtain the optimal rate adaptation policy as a byproduct of the iterative computation process.

Note that the EMTT metric is also applicable to multicast routing in single-rate WMNs. This is obtained by simply dropping the $\min$ operator in (2). For the three-node multi-rate WMN example provided in Fig. 2, let us consider node s sending a multicast packet of size $L = 1000$ bytes to both node u and node v. Based on the link PDRs provided in Fig. 2, the calculation of EMTT using (2) shows that $\mathrm{EMTT}_{s,\{u,v\}} = 12.42$ ms. The optimal rate adaptation policy obtained from (3) is: $\Pi_{s,\{u,v\}} = 1$ Mbps, $\Pi_{s,\{u\}} = 2$ Mbps, $\Pi_{s,\{v\}} = 1$ Mbps. However, in the case of single-rate multicast transmission, $\mathrm{EMTT}_{s,\{u,v\}}$ becomes 12.92 ms at 1 Mbps and 14.44 ms at 2 Mbps. Both of them result in longer transmission time than if we exploit the rate diversity.

III. EMTT-BASED MULTICAST ROUTING PROBLEM

In this section, we formulate the EMTT-based multicast routing problem. We begin by describing the model of the multi-rate multi-hop WMN considered in this paper, and then provide the definition of the EMTT-based multicast routing problem. We prove the NP hardness of the problem, and then present a mathematical formulation that can be used to find the optimal solutions to the problem.

A. Network model

The multi-rate multi-hop WMN considered in this paper supports link-layer acknowledgement for multicast transmissions. The network is represented by a directed graph $G = (V, E)$, where V is the set of mesh nodes and E is the set of directed links. A directed link $\langle i,j\rangle$ from node i to node j exists if node j is within the transmission range of node i. As defined in Section II, this requires the PDR of link $\langle i,j\rangle$ to be non-zero for at least one transmission rate in the set B, where $B = \{r_k : k = 1, 2, \ldots, K\}$. By definition, the PDR of link $\langle i,j\rangle$ at the k-th transmission rate is given by $p_{i,k,j} = \overrightarrow{d_{i,k,j}} \times \overleftarrow{d_{i,k,j}}$, where $\overrightarrow{d_{i,k,j}}$ and $\overleftarrow{d_{i,k,j}}$ are the forward delivery ratio and the reverse delivery ratio of link $\langle i,j\rangle$ at the k-th transmission rate, respectively. The set of nodes $\{j : \langle i,j\rangle \in E\}$ forms the one-hop neighborhood N_i of node i. Each node is equipped with one radio, with all radios tuned to a common channel. All nodes use the same transmission power for all transmission rates.

B. Problem definition

The EMTT-based multicast routing problem is defined for one single multicast session in the multi-rate multi-hop WMN. The members of the *multicast group* include one source node s and a set of destination nodes $\mathcal{D}$. The problem requires to establish a directed multicast tree T of G rooted at the source node and connecting all destination nodes in the multicast group. Since it is a multicast session, extra nodes may be selected from the set $V - \{s\} - \mathcal{D}$ and included in T as forwarding nodes, for ensuring end-to-end connectivity and for achieving the specified optimality criterion. In graph representation, all forwarding nodes of the multicast session (including the source node s) form the set of internal nodes of T. Note that the internal nodes of T may include certain destination nodes if they are also selected as forwarding nodes in the multicast session, but the leaf nodes of T are exclusively composed of destination nodes. For convenience, we let $\mathcal{I}$ denote the set of internal nodes of T.

We recall that the EMTT of one-hop transmission from each particular forwarding node in the multicast session minimizes the amount of expected transmission time required for delivering the multicast packet successfully to all next-hop receivers of the sender. The objective of the EMTT-based multicast routing problem is to find the optimal T for the multicast session that yields the minimum sum of EMTT over all forwarding nodes in the set $\mathcal{I}$ of T. By optimizing multicast routing in this way, we expect to reduce the channel occupancy time of the multicast session and thus increase the network throughput, while at the same time we ensure high end-to-end PDR of the multi-hop multicast transmission.

Theorem 1: The EMTT-based multicast routing problem is NP-hard.

Proof: A special case of the problem is where $p_{i,k,j} = 1$ for all $\langle i,j \rangle \in E$ and all k. Each node in V of this special case requires no more than one multicast transmission to deliver the multicast packet successfully to all its next-hop receivers, if the node is included in T as an internal node. Thus, the EMTT value of each sending node i is the same regardless of the set $\mathcal{S}$, i.e., $\mathrm{EMTT}_{i,\mathcal{S}} = L/r_K$ for all i and for all $\mathcal{S} \subseteq \mathcal{R}_i$. Assuming $L/r_K = 1$ and using the technique of [14], one can show that a polynomial-time transformation reduces the well-known *set cover* problem to this special case of the EMTT-based multicast routing problem. Since the decision version of the set cover problem is NP-complete [15], the EMTT-based multicast routing problem is NP-hard. ■

C. Mathematical formulation

Here, we provide an ILP formulation of the EMTT-based multicast routing problem. This mathematical formulation can be used by an ILP solver, e.g. CPLEX [16], to find optimal solutions to problem instances of reasonable size.

Define:

- The binary variables $e_{v,i,j}$, $\langle i,j \rangle \in E$, $v \in \mathcal{D}$, given by

$$e_{v,i,j} = \begin{cases} 1 & \text{if the directed link } \langle i,j \rangle \text{ is used by} \\ & \text{the path from the source node to} \\ & \text{the destination node } v, \\ 0 & \text{otherwise.} \end{cases}$$

- The binary variables $t_{i,j}$, $\langle i,j \rangle \in E$, given by

$$t_{i,j} = \begin{cases} 1 & \text{if the directed link } \langle i,j \rangle \text{ is included in } T, \\ 0 & \text{otherwise.} \end{cases}$$

- The binary variables $x_{i,\mathcal{R}}$, $i \in V$, $\mathcal{R} \subseteq N_i$, given by

$$x_{i,\mathcal{R}} = \begin{cases} 1 & \text{if node } i \text{ is selected as a forwarding node} \\ & \text{and } \mathcal{R} \text{ is the set of next-hop receivers} \\ & \text{selected for node } i, \\ 0 & \text{otherwise.} \end{cases}$$

Then, the ILP formulation of the EMTT-based multicast routing problem is:

$$Z = \min \sum_{i \in V} \sum_{\mathcal{R} \subseteq N_i} \mathrm{EMTT}_{i,\mathcal{R}} \cdot x_{i,\mathcal{R}} \tag{4}$$

subject to

$$\sum_{j:\langle s,j \rangle \in E} e_{v,s,j} - \sum_{j:\langle j,s \rangle \in E} e_{v,j,s} = 1, \ \forall \ v \in \mathcal{D} \tag{5}$$

$$\sum_{j:\langle v,j \rangle \in E} e_{v,v,j} - \sum_{j:\langle j,v \rangle \in E} e_{v,j,v} = -1, \ \forall \ v \in \mathcal{D} \tag{6}$$

$$\sum_{j:\langle i,j \rangle \in E} e_{v,i,j} - \sum_{j:\langle j,i \rangle \in E} e_{v,j,i} = 0, \ \forall \ v \in \mathcal{D}, \ i \in V - \{s,v\} \tag{7}$$

$$e_{v,i,j} \leq t_{i,j}, \ \forall \ v \in \mathcal{D}, \ \langle i,j \rangle \in E \tag{8}$$

$$t_{i,j} \leq \sum_{\mathcal{R}:\mathcal{R} \subseteq N_i, j \in \mathcal{R}} x_{i,\mathcal{R}}, \ \forall \ \langle i,j \rangle \in E \tag{9}$$

$$\sum_{\mathcal{R} \subseteq N_i} x_{i,\mathcal{R}} \leq 1, \ \forall \ i \in V \tag{10}$$

Constraints in (5), (6) and (7) enforce one set of *flow conservation* constraints along the path from the source node s to each destination node v in the set $\mathcal{D}$. Constraints in (8) ensure that the directed link $\langle i,j \rangle$ is included in the multicast tree if it is used by at least one of the end-to-end paths. Constraints in (9) ensure that, if the directed link $\langle i,j \rangle$ is included in the multicast tree, node i is selected as a forwarding node, and node j is one of the next-hop receivers of node i. Constraints in (10) ensure that, if node i is selected as a forwarding node, $\mathcal{R}$ identifies the (unique) set of next-hop receivers of node i in the multicast tree. These constraints together with the objective function in (4) jointly ensure that the optimal solution is a directed multicast tree rooted at the source node, connecting all destination nodes in the multicast group, and minimizing the sum of EMTT over all internal nodes of the tree.

IV. CENTRALIZED ALGORITHM

It is known that the set cover problem cannot be approximated to within less than a logarithmic factor [17]. The fact that the set cover problem is polynomial-time reducible to the EMTT-based multicast routing problem implies that we cannot expect to solve our problem in polynomial time with an approximation ratio better than $\mathcal{O}(\ln |\mathcal{D}|)$. The EMTT-based multicast routing problem can be transformed into a node-weighted directed Steiner tree problem, which yields a polynomial-time solution with an approximation ratio of $\mathcal{O}(4 \ln |\mathcal{D}|)$ [18]. This approach, however, requires transformation of the network graph G into an auxiliary graph, and therefore makes it impossible to be implemented in a distributed fashion.

In this paper, we propose a greedy algorithm for tackling the EMTT-based multicast routing problem. The algorithm starts with an initial tree T including only the source node s. At every step of the tree-building process, for each destination node v in the set $\mathcal{D}$ that is not yet included in T, we find the directed path requiring minimum cost among all shortest paths from nodes in T to the destination node v. We identify among all v the destination node v^* whose corresponding path has the smallest cost, where ties can be broken arbitrarily. Then, we add node v^* and its associated path to T, and, for each directed link $\langle i,j \rangle$ in the path, we add node j as a next-hop receiver of node i in T. When these are done, we update the set $\mathcal{D}$ by removing v^* from $\mathcal{D}$. The process continues until $\mathcal{D} = \emptyset$, meaning all destination nodes have been included in T and we have obtained a complete T based on the greedy algorithm.

For the purpose of EMTT-based multicast routing, we define the cost of a path in this context as the sum of *additional*

EMTT required by the forwarding nodes in the sequence of directed links along the path. The concept of additional EMTT can be conveniently explained by using the multi-rate WMN example provided in Fig. 2.

Consider node s as the source node, both node u and node v as members of the multicast group. Since the initial T includes node s only, adding node u to T would incur an additional EMTT at the sending node s given by $\mathrm{EMTT}_{s,\{u\}} = 6.67$ ms for a multicast packet of size $L = 1000$ bytes, while for node v the additional EMTT required at node s would be $\mathrm{EMTT}_{s,\{v\}} = 11.43$ ms. The greedy algorithm thus chooses node u as the first destination node to be included in T. Now, for node v, it has two choices:

1) Using the directed path formed by link $\langle u, v\rangle$ would incur an additional EMTT at the sending node u given by $\mathrm{EMTT}_{u,\{v\}} = 10$ ms.
2) Using the directed path formed by link $\langle s, v\rangle$ would incur an additional EMTT at the sending node s given by $\mathrm{EMTT}_{s,\{u,v\}} - \mathrm{EMTT}_{s,\{u\}} = 12.42 - 6.67 = 5.75$ ms. The calculation of the additional EMTT in this form for this choice is simply because node u has already been included in T as a next-hop receiver of node s. Thus, by exploiting the wireless broadcast advantage, a multicast transmission from node s to both node u and node v requires no more than an EMTT of 12.42 ms.

The greedy algorithm thus chooses the directed path $\{\langle s, v\rangle\}$ for node v, and it turns out to be the optimal solution to this particular problem instance.

For ease of calculating the additional EMTT for path selection, at the beginning of the algorithm, we initialize the weight of each directed link $\langle i, j\rangle$ to $\mathrm{EMTT}_{i,\{j\}}$. Then, at every step after the selected destination node and its associated path are included in T, we dynamically adjust the weight of each relevant directed link. Specifically, for each directed link $\langle i, j\rangle$ in the path, node i is included in T as a forwarding node, and node j is included in T as a next-hop receiver of node i. Thus, for each node n in the one-hop neighborhood N_i of node i but not in T, we adjust the weight of the directed link $\langle i, n\rangle$ to $\mathrm{EMTT}_{i,\mathcal{R}_i+\{n\}} - \mathrm{EMTT}_{i,\mathcal{R}_i}$, where $\mathcal{R}_i$ is the set of next-hop receivers of node i currently in T. This is due to the fact that for any sending node i in T, at every step of the greedy algorithm, at most one additional node in its one-hop neighborhood can be added as its next-hop receiver.

Let $\mathrm{WEIGHT}_{i,j}$ denote the weight of the directed link $\langle i, j\rangle$. Let $\mathrm{MACP}(T, v)$ denote the directed path requiring minimum cost among all shortest paths from nodes in T to the destination node v not in T. Let $\mathrm{COST}(T, v)$ denote the cost of $\mathrm{MACP}(T, v)$. By definition, we have

$$\mathrm{COST}(T, v) = \sum_{\langle i,j\rangle \in \mathrm{MACP}(T,v)} \mathrm{WEIGHT}_{i,j}.$$

A pseudo-code of the algorithm is provided in Fig. 3. We note that this algorithm requires at most $\mathcal{O}(|\mathcal{D}||V|^3)$ time, since finding $\mathrm{MACP}(T, v)$ for all v can be completed in at most $\mathcal{O}(|V|^3)$ time by applying Dijkstra's shortest path algorithm [19] at each origin in T and hence for up to $|V|$ times.

```
Input: $G = (V, E)$, $s$, $\mathcal{D}$, $B$, $\{p_{i,k,j}\}$
$T \leftarrow \{s\}$
for all $i \in V$ do
    $\mathcal{R}_i \leftarrow \emptyset$
end for
for all $\langle i, j\rangle \in E$ do
    $\mathrm{WEIGHT}_{i,j} \leftarrow \mathrm{EMTT}_{i,\{j\}}$
end for
while $\mathcal{D} \neq \emptyset$ do
    for all $v \in \mathcal{D}$ do
        Find $\mathrm{MACP}(T, v)$ and $\mathrm{COST}(T, v)$
    end for
    Find $v^* = \arg\min_v \mathrm{COST}(T, v)$
    $T \leftarrow T + \mathrm{MACP}(T, v^*)$
    for all $\langle i, j\rangle \in \mathrm{MACP}(T, v^*)$ do
        $\mathcal{R}_i \leftarrow \mathcal{R}_i + \{j\}$
        for all $n \in N_i - \mathcal{R}_i$ do
            $\mathrm{WEIGHT}_{i,n} \leftarrow \mathrm{EMTT}_{i,\mathcal{R}_i+\{n\}} - \mathrm{EMTT}_{i,\mathcal{R}_i}$
        end for
    end for
    $\mathcal{D} \leftarrow \mathcal{D} - \{v^*\}$
end while
```

Fig. 3. Pseudo-code of the greedy algorithm.

V. DISTRIBUTED ALGORITHM

Our proposed greedy algorithm for EMTT-based multicast routing can be easily extended to a distributed algorithm. In practice, members of the multicast group are likely to join the multicast session at different time. The principle of our design of the distributed algorithm is thus for each new member of the multicast group to initiate the procedure for finding a directed path from the existing tree. In particular, the new destination node chooses the path with the minimum sum of additional EMTT required. Again, the algorithm exploits the wireless broadcast advantage at the point where the branch to the node is extended from the existing tree. For convenience, we call any node in the existing tree as a *session member*. Below, we explain the details of the algorithm.

A. Node join

When node v wishes to join the multicast session as a destination node, it broadcasts a `Join_Req` message. The `Join_Req` message contains the information about the multicast group address, the IP address of node v, the sequence number, and the path cost (initially set to zero).

If a node that is not a session member receives the `Join_Req` message for node v, it broadcasts the `Join_Req` message to its one-hop neighbors. Before broadcasting the message, the node updates the path cost by adding the additional EMTT of its link to the incoming node. The node then marks the incoming node as its reverse entry to node v. In cases where the node receives multiple `Join_Req` messages for node v from its one-hop neighbors, it broadcasts each such message and updates its reverse entry accordingly so long as the updated cost indicates a shorter path to node v.

If a session member receives the `Join_Req` message for node v, it instead replies with a `Join_Reply` message. The `Join_Reply` message contains the path cost from the session member to node v, obtained by updating the path cost retrieved from the `Join_Req` message. Since the session member may receive multiple `Join_Req` messages for node v from its one-hop neighbors, it replies only after a timeout period (500 msec in our implementation) and chooses the incoming node with the smallest updated path cost as its reverse entry to node v. The `Join_Reply` message is unicast all the way back towards node v, using the reverse entry kept at each intermediate node along the path.

Each node that is not a session member may receive multiple `Join_Reply` messages for node v. In such cases, it forwards each such message so long as the message indicates a shorter path to node v. It also updates the incoming node as its forward entry to the corresponding session member that initiates the `Join_Reply` message. When node v receives multiple `Join_Reply` messages, it chooses the one that indicates the shortest path. Then, it unicasts a `Route_Activate` message all the way towards the nominated session member, using the forward entry kept at each intermediate node along the path. The route is activated by setting the intermediate nodes as forwarding nodes in the updated multicast tree.

B. Node departure

When the destination node v wishes to leave the multicast session, it is required to check whether it is currently a forwarding node in the multicast tree. If so, node v will stay in the multicast session; otherwise, it sends a `Prune` message to its upstream node and removes itself from the multicast tree. When a forwarding node receives a `Prune` message from a next-hop receiver, it simply deletes the node from its next-hop receiver table and remains in the multicast tree, given that it is a destination node or has multiple next-hop receivers in the multicast tree; otherwise, it forwards the `Prune` message to its upstream node and removes itself from the multicast tree.

VI. EVALUATION

This section provides detailed numerical results that we have obtained for evaluating the performance of EMTT as a routing metric for reliable multicast in multi-rate multi-hop WMNs.

A. Protocol implementation

We use RMAC [4] as the MAC-layer multicast protocol. The ARQ mechanism of RMAC uses *busy tone* to realize MAC-layer multicast reliability. Using a variable-length control frame, RMAC stipulates the response order of receivers to resolve feedback collision. The original RMAC is designed for single-rate transmissions. We have modified RMAC to address the need of multi-rate transmissions in our context. Each forwarding node in the multicast tree keeps a table of its next-hop receivers and establishes an optimal rate adaptation policy. For each transmission/retransmission of a multicast packet, the sender chooses the best transmission rate from the rate adaptation policy based on the current state.

We use the probing technique of [9] to estimate link PDRs required for EMTT calculation. For each available transmission rate, node i broadcasts a probe that contains 134 bytes of payload at every one second. Each probe sent by node i also contains the number of probes received by node i from each of its one-hop neighbors during the last ten seconds. For every $\langle i, j\rangle$ pair, this technique allows node i to estimate the forward delivery ratio $\overrightarrow{d_{i,k,j}}$ for data packets and the reverse delivery ratio $\overleftarrow{d_{i,k,j}}$ for ACKs at the k-th transmission rate.

B. Simulation configuration

We use QualNet [20] to simulate a network with 50 mesh nodes. The nodes are uniformly distributed in an area of size $1500 \text{ m} \times 1500 \text{ m}$. Each node has one interface, working in IEEE 802.11b. All experiments use the two-ray propagation path loss model, with free space path loss exponent of 2 for near sight and plane earth path loss exponent of 4 for far sight. The physical layer uses PHY802.11b, where the available transmission rates are 2/5.5/11 Mbps.

In each experiment, we set up one multicast constant bit rate (MCBR) session from the source node s to the set of destination nodes $\mathcal{D}$. The size of each multicast packet is 1100 bytes, including 1032 bytes of payload and 68 bytes of header length. The source node is configured to send the MCBR traffic at a rate of 50 packets per second, equivalent to a bit rate of 440 kbps. Background traffic is randomly generated to increase the chance of packet collision. Ten different topology maps are generated from QualNet for the purpose of simulation. The size of the multicast group varies from 5 to 49. For a given group size and topology, ten different $(s, \mathcal{D})$ pairs are considered, each with ten different joining sequences for the destination nodes.

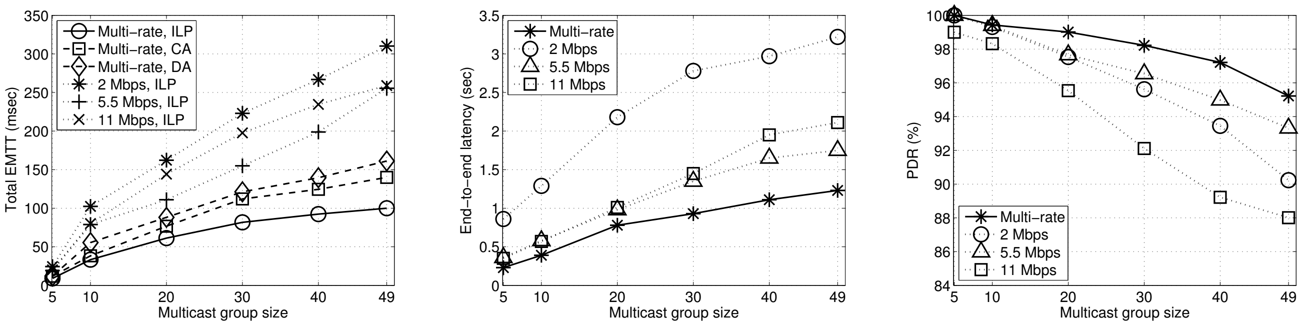
We investigate the performance of EMTT-based multi-rate multicast routing, and compare it against the state-of-the-art EMT-based single-rate multicast routing [12] using each of the three different transmission rates. The study is focused on three important performance measures:

- *Total EMTT*, defined as the sum of EMTT over all forwarding nodes in the multicast tree. A small amount of total EMTT implies less channel occupancy time.
- *Average end-to-end latency*, defined as the average end-to-end latency experienced by packets that are successfully received by all destination nodes.
- *Worst-case end-to-end PDR*, defined as the worst-case end-to-end PDR among the destination nodes.

The results are presented as the average over all experiments for each particular multicast group size.

C. Results

Figure 4(a) presents the total EMTT results for demonstrating the accuracy of the centralized algorithm (CA for short in the figure) and the distributed algorithm (DA for short in the figure) against the optimal solutions to instances of the EMTT-based multi-rate multicast routing problem. The optimal solutions are obtained by solving the ILP formulation using CPLEX [16]. We also include the optimal solutions



(a) (b) (c)

Fig. 4. Performance comparison: (a) total EMTT, (b) average end-to-end latency, (c) worst-case end-to-end PDR.

to the same problem instances but with the revised EMTT metric considering single rate only. Note that the multicast tree obtained in this way is equivalent to the one using the EMT metric [12] based on the set of link PDRs at each particular transmission rate.

The results in Fig. 4(a) confirm the effectiveness of both the centralized algorithm and the distributed algorithm. On average, the two algorithms require only up to 40% and 60% more transmission time than what is required by the optimal solutions. The distance between the two algorithms is as expected since the distributed algorithm allows an arbitrary joining sequence for the destination nodes. The results also show that, by exploiting the rate diversity for multicast transmission/retransmissions, multi-rate multicast requires much less transmission time than single-rate multicast. The total EMTT of single-rate multicast is higher by a factor of 2 compared to multi-rate multicast. It is interesting to see that, within the single-rate scenarios, 5.5 Mbps requires in general the least amount of transmission time, even though one single transmission indeed consumes half of the time at 11 Mbps. This phenomenon is caused by the different link PDRs at different transmission rates, so that 11 Mbps sometimes needs more than double the number of transmissions for a successful packet delivery compared with 5.5 Mbps.

Figure 4(b) provides the results of the protocol performance in average end-to-end latency. By exploiting the rate diversity for multicast transmission/retransmissions, each forwarding node always chooses the best transmission rate for a given set of next-hop receivers in multi-rate multicast. Therefore, the multicast tree formed by multi-rate multicast always achieves a smaller average end-to-end latency than the single-rate multicast solution. In particular, the results in Fig. 4(b) show that multi-rate multicast reduces end-to-end latency by up to 70%, 30% and 35% compared with single-rate multicast at 2 Mbps, 5.5Mbps and 11Mbps, respectively.

Figure 4(c) compares the protocol performance in worst-case end-to-end PDR. Although ARQ-based retransmission is applied to achieve reliability in one-hop multicast transmission, a packet may be dropped when the number of transmission attempts has exceeded the retry limit. The results show that transmissions at the highest rate (11 Mbps) result in the worst performance. This is because some nodes are too far away from each other, resulting in poor PDR at 11 Mbps. Moreover, although 2 Mbps in general has higher PDR than 5.5 Mbps when the channel is clear, in the presence of hidden nodes, the actual PDR of 5.5 Mbps is likely to be higher than 2 Mbps due to the combined effects of channel errors and channel contentions. Multi-rate multicast outperforms single-rate multicast at all rates because it more effectively takes advantage of the rate diversity, which is generally more reliable and less susceptible to collisions.

VII. RELATED WORK

Routing metric: The ETX metric [9] aims to find high-performance paths that minimize the expected number of MAC-layer transmissions including retransmissions. ETT and WCETT [10] are delay-based metrics, which are *bandwidth-enhanced* ETX considering both link quality and rate diversity. The design of ETX/ETT/WCETT is for unicast. The SPP metric proposed in [11] is designed for multicast and aims to achieve the maximum benefit for each individual destination with respect to end-to-end PDR. It is based on standard 802.11 MAC-layer multicast, and does not take into account MAC-layer retransmission-based reliability, transmission rate diversity and wireless broadcast advantage. The metric considered in [21] aims to form a multicast tree that minimizes the number of wireless transmissions by exploiting the wireless broadcast advantage. It is based on a binary packet reception model, which may be impractical in wireless scenarios. EMT [12] is designed for reliable multicast in single-rate WMNs. The EMTT metric proposed in this paper generalizes EMT and is more robust for multi-rate multicast in WMNs.

Reliable MAC-layer multicast: Current IEEE 802.11 standards simply transmit the data frame once without any recovery for multicast traffic. Therefore, the reliability of multicast is reduced due to the increased probability of lost frames resulting from interference or collisions. To address this deficiency, a number of ARQ-based reliable MAC-layer multicast protocols have been proposed. In LBP [1], one *leader* is selected by the multicast receivers to take the responsibility of sending ACKs, which avoids the multiple acknowledgements. The HIMAC solution proposed in [2] uses UCF and UNF to address two shortcomings in 802.11 multicast: channel-state indifference and demand ignorance. Both 802.11MX [3] and RMAC [4] use the busy tone mechanism to offer reliable MAC-layer multicast. RMAC uses a positive acknowledgement mechanism, where each recipient needs to acknowledge data receiving. We have modified RMAC in this paper to provide reliable MAC-layer multicast for EMTT-based multi-rate multicast routing.

Rate adaptation: A number of rate adaptation schemes [22]–[24] have been proposed for unicast, which can improve the performance for a given recipient. Authors in [25], [26] use the consecutive successes/losses metric to adapt the transmission rate. In the multicast area, [2], [27], [28] studied different rate adaption schemes. They simply use the SNR threshold or packet loss threshold in choosing the transmission rate without the comprehensive consideration for all recipients which may have different link quality. In [29], the total receiver utility is maximized for multicast sessions to address the rate control problem in multi-rate wireless networks. In [30], the authors considered the problem of maximising the total amount of broadcast/multicast traffic load that the network may simultaneously transport. Their rate and contention aware multicast algorithm can significantly enhance the amount of admissible multicast traffic in a WMN by exploiting both the rate diversity and the contention-free time division at individual nodes. None of the work takes the link quality as a function of the transmission rate. Moreover, all of them assume a binary packet reception model in link-layer multicast. Therefore, they are not suitable when ARQ-based reliable multicast is applied in the MAC layer.

VIII. CONCLUSION

In this paper, we have proposed EMTT as a robust metric for achieving high-performance multicast routing in multi-rate WMNs. EMTT captures the combined effects of 1) MAC-layer retransmission-based reliability, 2) transmission rate diversity, 3) wireless broadcast advantage and 4) link quality awareness. Simulation experiments have confirmed that multicast routing in multi-rate WMNs using the EMTT metric can effectively reduce the overall multicast transmission time and thereby yielding higher packet delivery ratio and lower end-to-end latency than if we do not exploit the rate diversity.

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