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Control-theoretic Utility Maximization in Multi-hop Wireless Networks under Mission Dynamics

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Control-theoretic Optimization of Utility over Mission Lifetimes in Multi-hop Wireless Networks

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***Abstract*—Both bandwidth and energy become important resource constraints when multi-hop wireless networks are used to transport relatively high data rate sensor flows. A particularly challenging problem involves the selection of flow data rates that maximize application (or mission) utilities over a *time horizon*, especially when different missions are active over different time intervals. Prior works on utility driven adaptation of flow data rates typically focus only on instantaneous utility maximization and are unable to address this temporal variation in mission durations. In this work, we derive an optimal control-based Network Utility Maximization (NUM) framework that is able to maximize the system utility over a lifetime that is known either deterministically or statistically. We first consider a static setup in which all the missions are continuously active for a deterministic duration, and show how the rates can be optimally adapted, via a distributed protocol, to maximize the total utility. Next, we develop adaptive protocols for the dynamic cases when we have (i) complete knowledge about the mission utilities and their arrivals and departures, and (ii) a varying amount of statistical information about the missions. Our simulation results indicate that our protocols are robust, efficient and close to the optimal.**

I. Introduction

Tactical and streaming applications, such as vehicle tracking, building surveillance and gunfire localization, often employ an adhoc wireless network as a transport substrate to receive data streams from a set of field-deployed, sophisticated, relatively high-data rate sources (e.g. video sensors and short-aperture radars). In many instances, these applications, which we refer to as missions, are operational for relatively well-defined, but potentially dissimilar, durations, such that it is possible to declare in advance, either *deterministically* or *statistically*, the time periods over which an individual mission is *active*. In these scenarios, the wireless transport network is often both *bandwidth* and *energy* constrained, and the data rates of the sensor streams must be controlled so as to balance the dual objectives of providing high instantaneous data rates and ensuring an adequately long network lifetime. Efficient resource management for such applications is a challenging task: besides considering the limited transmission capacity on the wireless links and the finite energy reserves on the battery-powered forwarding nodes, algorithms must consider when and how long missions are likely to consume the sensor data streams.

Utility maximization techniques, which focus on rate-dependent application utility as the principal metric, have been widely investigated for congestion control and bandwidth sharing across multiple traffic flows in both wired and wireless environments. In particular, the Network Utility

Maximization (**NUM**) approach, pioneered in [1], [2], casts resource sharing as a problem of constrained utility optimization; the principles of *decomposable optimization* [3] are then used to develop distributed, iterative solutions for maximizing the system utility. In its basic form, NUM focuses on the problem of *instantaneous utility maximization*, as opposed to our desire to ensure that the system utility remains "appropriately" high over a time horizon with fluctuating instantaneous demands. Limited work on lifetime-aware utility maximization has simplified the problem by either morphing lifetime objectives into instantaneous, time-independent power constraints [5] or by creating a single, linearized objective function through the use of arbitrary 'weights' [4]. These approaches implicitly assume a fixed set of 'always-active' missions, and are unable to capture the typical temporal mission dynamics that are central to our work.

In this paper, we consider the problem of *'utility maximization over a time horizon'* from a more fundamental perspective and show how *optimal control* theory can help derive the appropriate system adaptation behavior for a rich set of mission dynamics. As a mathematical framework for determining the optimal behavior of dynamical systems over time, optimal control has been widely used in a wide variety of engineering and economics problems [8], [9]. Optimal control-theoretic approaches for transport-layer protocol adaptation are, however, conspicuously absent. To our knowledge, we are the first to explicitly exploit knowledge of mission durations to perform time-varying congestion control and optimization of source rates.

More specifically, this paper demonstrates how optimal control-based solutions can be embedded into NUM-based adaptation strategies for two different application scenarios:

1) In the first scenario, all the missions are assumed to be static and continuously active over the operational lifetime of the network, T, which is pre-defined.
2) In the second scenario, we consider a dynamic network

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where missions arrive intermittently and remain active for variable, discrete time periods. In particular, we consider three different cases: (a) We have *deterministic* knowledge about individual mission arrival/departure times; (b) We have *stochastic* information about the missions, with the likelihood of new missions decreasing over time, and (c) Missions can arrive with constant or variable probability at any time during the life of the network, and we only have stochastic knowledge about the arrival and departure rates.

Figure 1 illustrates the motivation behind our mission-oriented optimization problem. There are three applications: a surveillance application consumes data from a video sensor and a short range radar, an 'asset verification' mission utilizes data from two video sensors, while a 'personnel identification' mission utilizes data from one video and one audio sensor. We assume that each mission maps to a specific 'sink' node in the wireless network, with the sink responsible for receiving the relevant data streams and then forwarding (over a wired network) to a backend server where the application actually runs. *Thus, a sink receives data from a set of data sources only when the corresponding mission utilizing those data sources is 'active'*. Each of these missions is active for different durations of time. We shall demonstrate how optimal control results can be used to model and solve progressively-sophisticated scenarios, via appropriate modifications to the basic NUM protocols developed in [6].

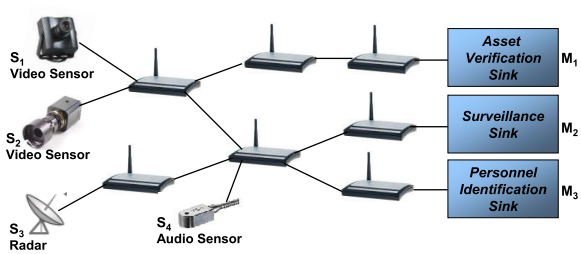


Fig. 1: An illustration of data sharing by multiple, intermittently-active missions

The rest of the paper is organized as follows. Section II surveys related prior work in NUM-based resource management and lifetime optimization, and provides a brief overview of the basic principles of optimal control. Section III develops and evaluates the optimal control-based adaptations for the static scenario. Subsequently, Section IV extends the approach to dynamic missions. We show how the protocol can be modified for such cases and provide a simulation-based evaluation of the protocols. Finally, Section V concludes the paper along with some discussion of future work.

II. Related Work and Introduction to Optimal Control

A. Prior NUM-based Work

The related work in this area can be categorized according to its focus either on the unique characteristics of mission-oriented wireless networks or on the problem of joint utility and lifetime optimization. Recent work (e.g., [6]) has extended the basic Network Utility Maximization (NUM) framework (originally proposed for unicast-oriented wireline [2], [3] and wireless [7] environments) to consider two unique aspects of mission-oriented WSNs: *a) shared consumption of a single sensor data stream by multiple sinks* (each corresponding to a mission with a different utility function), and *b) the specification of a mission's utility as a joint function of multiple source rates*. Let M be the set of all missions (data sinks) and S the set of source (sensor) nodes. Furthermore, for any mission m, let $set(m)$ be the set of sensors to which mission m subscribes; conversely, let $Miss(s)$ be the set of missions subscribing to the data stream from sensor s. Also, let K denote the set of all (source, sink and forwarding) nodes in the network. For this mission-centric model, where data is forwarded using link-layer multicast transmissions, the problem of *maximizing the instantaneous data rates* can be formulated as: $SENSOR(U)$:

$$\textbf{maximize} \sum_{m \in M} U_m(X_{s:s \in set(m)})$$
$$\textbf{subject to} \sum_{\forall (k,s) \in q} \frac{x_s}{c_{ks}} \leq 1, \forall q \in Q,$$

where $U_m(X_{s:s \in set(m)})$ represents the utility function of mission m, c_{ks} is the transmission rate used by node k during the link-layer broadcast of data from sensor s, and Q is the set of all maximal cliques in the transmission-based conflict graph (CG) defined over the wireless network. Based on this objective, it is shown (see [6]) that if a sensor (source) s now adapts its rate according to:

$$\frac{d}{dt} x_s(t) = \kappa(\sum_{m \in Miss(s)} w_{ms}(t) - x_s(t) * \sum_{\forall q \in flow(s)} \mu_q(t) \sum_{\forall (k,s) \in q} \frac{1}{c_{ks}}) \quad (1)$$

where μ_q is the 'cost' charged per bit by each forwarding clique, and is given by:

$$\mu_q(t) = (\sum_{\forall (k,s) \in q} \frac{x_s(t)}{c_{ks}} - 1 + \epsilon)^+ / \epsilon^2 \quad (2)$$

and each mission (sink) adapts its 'willingness to pay' terms w_{ms} for sensor s based on the source rates and its own utility function $U_m(.)$, according to the equation:

$$w_{ms}(t) = x_s(t) \frac{\partial U_m}{\partial x_s}$$

then the system will converge to an optimal solution of a relaxation of the problem $SENSOR(U)$. We shall extend this basic mission-oriented NUM model, that maximizes instantaneous utility, to consider the optimal behavior over a time horizon, for a dynamically varying set of missions.

The limited prior work on joint optimization of both mission utility and network lifetime includes [4], which maximizes a jointly linearized objective function, $\gamma U(.) - (1-\gamma) \sum F(.)$, where $U(.)$ is the utility function and $F(.)$ is the lifetime penalty function. The value of γ is a design choice and

determines the relative weightage given to utility and lifetime. A more simplified form of joint optimization was solved in [5], where lifetime constraints were assumed to be implicitly modeled by a constant upper bound on the instantaneous power consumption at any node k. Both of these approaches assume a pre-specified set of missions that are active throughout the entire operational duration; in contrast, we shall explicitly factor in the time-dependent evolution of an individual mission's activity state.

B. Overview of Optimal Control

We now present a brief review of the optimal control framework, which focuses on the problem of determining the best adaptive behavior of a system *over time*. The general form of the objective function of an inter-temporal optimization problem can be written as:

$$\begin{aligned} &\text{maximize} \int_{t_0}^{T} f(s(t), c(t), t)dt \\ &\text{subject to } \frac{ds}{dt} = g(s(t), c(t), t) \\ &\text{with } s(t_0) = s_0 \text{ and } s(T) = s_T \end{aligned} \tag{3}$$

Here $s(t)$ is a state variable and $c(t)$ is a control variable. The constraint in Eq.(3) denotes the rate of change of the state of the system as a function of the current state, control value and time. By taking Lagrangian and simplifying using integration by parts, we get

$$\begin{aligned} L = \int_{t_0}^{T} [f(s,c,t) + \lambda(t)g(s,c,t) + s\frac{d\lambda}{dt}]dt \\ -\lambda(T)s(T) + \lambda(t_0)s(t_0) \end{aligned} \tag{4}$$

Here, $\lambda(t)$ is a Lagrangian multiplier that represents the marginal valuation, i.e., the shadow cost, of the state variable. This means that if there is a unit increment in the state variable, then the increment in the objective value of the optimal solution will be at rate $\lambda(t)$ [9]. Differentiating Eq. (4) results in the following first-order necessary condition:

$$\begin{aligned} dL = \int_{t_0}^{T} [\frac{\partial f}{\partial c} + \lambda\frac{\partial g}{\partial c}dc + (\frac{\partial f}{\partial s} + \lambda\frac{\partial g}{\partial s} + \frac{d\lambda}{dt})ds]dt \\ -\lambda(T)ds(T) + \lambda(t_0)ds(t_0) = 0 \end{aligned} \tag{5}$$

Because of the additive nature of Eq. (5), we get the following conditions that the optimal solution satisfies:

$$\frac{\partial f}{\partial c} + \lambda\frac{\partial g}{\partial c} = 0 \tag{6}$$

$$(\frac{\partial f}{\partial s} + \lambda\frac{\partial g}{\partial s} + \frac{d\lambda}{dt}) = 0 \tag{7}$$

Transversality Conditions: The initial values of the state variables are usually fixed in such problems, making $ds(t_0) = 0$. If the terminal values are fixed too, then $ds(T) = 0$. However, if the terminal value of the state variable is left free, i.e., if it can take any value, then the condition that $\lambda(T) = 0$ must be satisfied at optimum. In some problems, there may be an endpoint condition of the form $s(T) \geq s_T$, instead of the terminal value being fixed or free. In such cases, the transversality condition: $\lambda(T)[s(T) - s_T] = 0$ must be satisfied [8].

Pontryagin's Maximum Principle: The term $f(s,c,t) + \lambda(t)g(s,c,t)$ in Eq. (4) is referred to as the Hamiltonian H. According to Pontryagin's Maximum Principle, the necessary and sufficient conditions for optimization over time are given as (rewriting Eq. (6), (7)):

$$(i) H_c = \frac{\partial f}{\partial c} + \lambda\frac{\partial g}{\partial c} = 0, \text{ and} \tag{8}$$

$$(ii) \frac{d\lambda}{dt} = -H_s = -(\frac{\partial f}{\partial s} + \lambda\frac{\partial g}{\partial s}) \tag{9}$$

where $H_c = \frac{\partial H}{\partial c}$ and $H_s = \frac{\partial H}{\partial s}$.

III. Optimal Control-Based Adaptation for Static Missions

We first apply the principles of optimal control to determine the best allocation of sensor data rates, and the resultant depletion of energy reserves on forwarding nodes, for the case where all missions are active continuously, throughout the lifetime of the network, i.e., all missions start at time 0 and end at time T, where T is the target lifetime of the network.

Before proceeding further, let us define a few mathematical symbols in addition to the ones specified in Section II-A. In our problem, the "state variables" correspond to the residual battery level at each node in the network. Let $p_k(t)$, $\forall k \in K$ denote the residual battery level of node k at time t. The "control variables" are the source rates of sensors, denoted as x_s, $\forall s \in S$. We use an energy model similar to that in [7], where the reception of data at a rate x depletes the receiver k's battery by $\alpha_{\mathbf{r}}^k x$, and the transmission of data at a rate x drains the transmitter k's battery by $\alpha_{\mathbf{t}}^k x$. Table (I) details the other commonly used symbols in the rest of this paper.

A. Utility Optimization Model

The goal of this problem is to maximize the cumulative utility of the network over its lifetime T. Note that we do not impose any additional constraints on individual node behavior– i.e., we permit individual nodes to 'die' at arbitrary time instants (although, we shall prove that premature node death before T never leads to an optimal solution.) The global network utility is the sum of the utilities of all active missions. The utility is a concave, non-decreasing function of the source rate, such as a logarithmic function. Accordingly, the objective function, J, for this optimization problem is as shown below:

$SENSOR - LIFE1(U; L)$:

$$\textbf{maximize } J = \int_0^T \sum_{\forall m \in M} U_m(x_{s:s \in set(m)}(t))dt \tag{10}$$

subject to

i) Energy Constraint: $$\frac{dp_k}{dt} = -(\sum_{\forall i \in InFlows(k)} \alpha_{\mathbf{r}}^k x_i(t) + \sum_{\forall i \in OutFlows(k)} \alpha_{\mathbf{t}}^k x_i(t)) = -\sum_{\forall i \in Flows(k)} \alpha_i^k x_i(t), \forall\, k \in K, \tag{11}$$

TABLE I: Most Common Mathematical Symbols

M	Set of all missions in the network	S	Total number of sources (sensors)
$set(m)$	Set of sources that are used by mission m	$Miss(s)$	Set of missions subscribing to flow s, i.e.,data from sensor s
$U_m(.)$	Joint utility function of mission m	c_{ks}	Transmission rate used by node k during the link-layer
Q	Set of all maximal cliques in conflict graph [6]	$x_s(t)$	The data rate of a flow from source s at time t
$\alpha_{\mathbf{r}}^k$	Power consumed per bit of received data at node k	$\alpha_{\mathbf{t}}^k$	Power consumed per bit of transmitted data at node k
$p_k(t)$	Residual battery at node k at time t	(k,s)	Transmission of the flow that originated from s by a node k
$Inflows(k)$	Set of all flows received at node k	$Outflows(k)$	Set of all transmitted flows at node k
$Flows(k)$	Since $Inflows(k) = Outflows(k)\ \forall k$,	K	Set of all nodes in the network
	this is simply written as $Flows(k)$	α_i^k	Power consumed per bit of communication at node k for flow i.
μ_q	Shadow cost associated with congestion in clique q		If k is a source for flow i, $\alpha_i^k = \alpha_{\mathbf{t}}^k$; if
λ_k	Shadow cost associated with state p_k		k is a forwarding node for flow i, $\alpha_i^k = (\alpha_{\mathbf{r}}^k + \alpha_{\mathbf{t}}^k)$
η_k	Energy cost associated with node k	κ	step-size in the gradient ascent algorithm used in NUM
$Path(s)$	Set of all nodes in the forwarding tree of source s	$Path(s,m)$	Set of all nodes in the route from source s to mission m

ii) Capacity Constraint: $$\sum_{\forall(k,s)\in q} \frac{x_s(t)}{c_{ks}} \leq 1 \quad \forall t,\ \forall\ q \in Q \tag{12}$$

iii) Terminal Constraint: $$p_k(T) \geq 0 \ \ \forall k \in K \tag{13}$$

The energy constraint in Eq. (11) defines the rate of depletion of residual battery power at each node k. The amount of energy consumed at each time instance at node k is equal to the total energy expended in receiving and transmitting all the flows that the node forwards. In other words, $p_k(t+1) = p_k(t) - \sum_{\forall i \in Flows(k)} \alpha_i^k x_i(t)$. The capacity constraint in Eq. (12) which is the same as in [6], states that the total air-time fractions of all interfering transmissions (i.e., all transmissions in a maximal clique of the conflict graph) must not exceed unity. Eq. (13) defines the terminal condition that the residual energy at each node at the end of network lifetime must be at least 0 (residual energy cannot ever become negative). In practice, because of the multicast distribution of flows and the potential for multiple bottlenecks, it is likely that the non-bottleneck nodes nodes will not be fully drained at T.

The Hamiltonian for the system of equations (10)-(13) is shown below:

$$H = \sum_{\forall m \in M} U_m(x_{s:s\in set(m)}) - \sum_{\forall q\in Q} \mu_q \sum_{\forall(k,s)\in q} (\frac{x_s}{c_{ks}} - 1) - \sum_{\forall k\in K} \lambda_k \sum_{\forall i \in Flows(k)} \alpha_i^k x_i(t) \tag{14}$$

where $\mu_q(t)$ and $\lambda_k(t)$ are the Lagrangian multipliers associated with the capacity and energy constraints, respectively.

The necessary and sufficient conditions according to Eq. (8),(9) are given as:

$$(i) H_{x_s} = \sum_{\forall m\in M} \frac{\partial U_m}{\partial x_s} - \sum_{\forall q \in Path(s)} \mu_q \sum_{(k,s)\in q} \frac{1}{c_{ks}} - \sum_{\forall k \in Path(s)} \lambda_k \alpha_k = 0 \text{ and} \tag{15}$$

$$(ii) \frac{d\lambda_k}{dt} = -H_{p_k} = 0 \tag{16}$$

Since we have inequality constraints for terminal values, the condition $\lambda_k(T)p_k(T) = 0$ must be satisfied as discussed in Section II-B. $\lambda_k(0)$ is the marginal valuation of p_k at $t = 0$, i.e., $\frac{\partial J^*(0)}{\partial p_k}$ and is equal to 0, where $J^*(t)$ is the optimal value of J at time t. This, along with Eq. (16) implies that $\lambda_k(t) = 0,\ \forall k$.

Differentiating Eq. (15) with respect to time, we get:

$$\frac{dH_{x_s}}{dt} = \sum_{\forall m\in M} \frac{\partial^2 U_m}{\partial x_s^2} * \frac{dx_s}{dt} - \sum_{\forall k in Path(s)} \frac{d\lambda_k}{dt}\alpha_s^k = 0$$
$$\Rightarrow \frac{dx_s}{dt} = 0 \tag{17}$$

The terminal contraint in Eq.(13) and the recursive equations in Eq.(11) and (17) can be simplified to obtain:

$$\sum_{\forall i \in Flows(k)} \alpha_i^k x_i(t) \leq \frac{p_k(t)}{(T-t)} \tag{18}$$

Taken together, these relationships indicate that the optimal strategy consists of the following:

1) From an energy perspective, the sensors transmit data at a constant rate (over the entire duration T) that maximizes Eq. (14) while satisfying Eq. (18) at each time instance.
2) The maximum amount of energy that is allowed to be consumed at time t at node k is $\frac{p_k(t)}{T-t}$, i.e, the residual battery at any time is shared evenly over the remaining time, as expected intuitively.
3) As the sensor rates must also satisfy the capacity constraints, the sensor data rates are constrained by *both* the power and capacity constraints, i.e., at any instant, the more restrictive of the two constraints will apply.

B. Protocol Details

It can be observed that Eq. (15) is similar to the gradient that is used in the WSN-NUM protocol [6]. Thus, from the protocol perspective, this problem can be solved by running the WSN-NUM protocol according to the following source rate adaptation algorithm:

$$\frac{dx_s}{dt} = \kappa(\sum_{\forall m\in M} w_{ms} - x_s(\sum_{\forall q\in Path(s)} \mu_q \sum_{(k,s)\in q} \frac{1}{c_{ks}} + \sum_{\forall k\in Path(s)} \eta_k \alpha_s^k)) \tag{19}$$

where, w_{ms} is the willingness-to-pay of mission m for flow s and is given as $x_s \frac{\partial U_m}{\partial x_s}$. μ_q is as given in Eq. (2); η_k is the Lagrangian multiplier corresponding to constraint in Eq. (18), and is defined as:

$$\eta_k = (\frac{(T-t)\sum_{\forall i\in Flows(k)} \alpha_i^k x_i(t)}{p_k(t)} - 1 + \varepsilon)^+ / \varepsilon^2 \tag{20}$$

where ε plays the same role as ϵ in Eq. (2).

In summary, the *STATIC* protocol follows the steps shown below iteratively throughout the lifetime of the network:

1) Each source s transmits data at current rate $x_s(t)$
2) Each forwarding node computes the clique cost according to Eq. (2) and the energy cost according to Eq. (20) and transmits this along with the data. Please note that the split shadow cost technique introduced in [6] is used here too. Each node cumulatively adds its costs to the received value before forwarding the data, until the mission is reached. Since T may not be known globally, and is typically known only to the missions and sources, this value may be piggy-backed along with data initially so that the forwarding nodes learn the value of T as well.
3) Each mission computes its willingness to pay and forwards this along with the received costs to the corresponding source as feedback.
4) Each source, on receiving this feedback, computes its rate for the next iteration according to Eq. (19).

Given that the optimal data rates are constant over time, our iterative technique can ensure that the overall total utility is infinitesmally close to the optimal value, as long as the initial adaptation transient is much smaller, compared to the overall lifetime T.

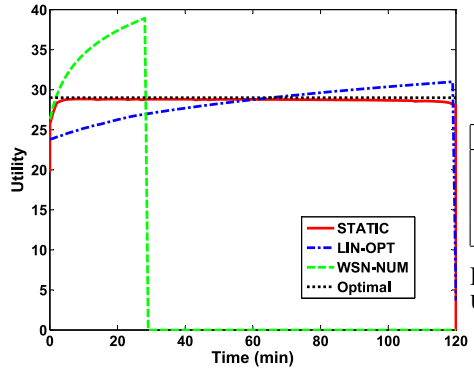
C. Evaluation

In this section, we quantitatively analyze the performance of the *STATIC* protocol. An 802.11-based ad hoc network with 30 nodes is simulated using a discrete-event simulator, Qualnet. The topology of the network is taken to be uniformly distributed. There are 10 sources and 5 missions, with each mission subscribing data from one or more sources. The utility of a mission subscribing to flows $s_1, s_2, ...s_n$ is given as $\sum_{i=1}^{n} ln(s_i)$. The target lifetime the network is 2 $hours$.

In Fig. (2), we compare the global network utilities when the rates are adapted according to (i) *STATIC* protocol described in Section III-B, (ii) the linearly weighted *LIN-OPT* protocol developed in [4], (iii) the *WSN-NUM* protocol developed in [6] and (iv) the numerical optimal value computed using the centralized solver, GAMS. For the *LIN-OPT* protocol, the value of γ (the weightage given to utility vs. network lifetime) is determined by trial-and-error such that the network lasts for exactly two hours.

We observe that our protocol performs close to the optimal. Although the instantaneous utilities of *WSN-NUM* protocol are initially high, the network dies well before the target lifetime (lasts only for 25% of the target duration T) due to greedy utilization of battery resources. The cumulative utility over the network lifetime (i.e., $\int_0^T \sum_m U_m(\tau)d\tau$) for each of these techniques is listed in Figure (3).

While the *LIN-OPT* protocol achieves an overall utility value reasonably close to ours, it does not provide a way to directly express a desired lifetime objective T. For *LIN-OPT*, γ had to be set using trial and error; our optimal control based approach is able to embed T directly inside the NUM control loop.



Method	Utility
Optimal	3444.12
STATIC	3443.65
WSN-NUM	1014.55
LIN-OPT	3406.93

Fig. 3: Cumulative Utility

Fig. 2: Evolution of network utility for static missions.

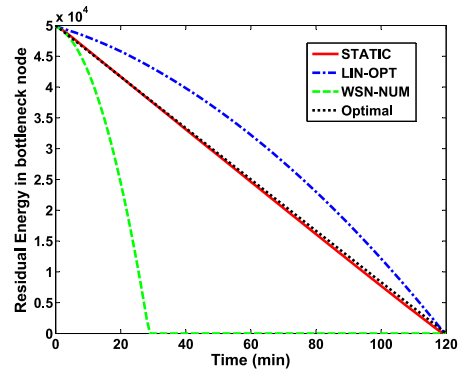


Fig. 4: Depletion of residual energy in a bottle-neck node.

Fig. (4) shows how the battery drains over time at a bottleneck node in the network. We see that our optimal-control based approach results in a linear depletion of the residual battery energy, which is optimal, while the other methods are either too aggressive or tardy; furthermore, the residual battery in the bottleneck node becomes 0 at time T, implying that there is no under-utilization of the resource.

IV. Optimal Control-Based Adaptation for Dynamic Missions

In many sensor network applications, missions are transient in nature. When missions arrive and leave at different times and last for variable durations, the optimal strategy would be to expend less energy when fewer or low utility missions are active, conserving energy for use in future time instants when more or higher utility missions are active. Hence, the uniform-rate approach in Section III is not sufficient to handle the dynamic case. In this section, we develop optimal control-based adaptation framework for dynamic missions. In particular, we consider the following three, progressively-complex scenarios:

- *Case A. Deterministic knowledge of variable duration missions:* In this case we assume that we have perfect *a-priori* knowledge about the dynamics of the mission, i.e., when each mission starts and terminates. Examples of such a scenario include military applications such as vehicle tracking and battlefield monitoring that operate based on a specific war plan; or a surveillance network that monitors various parts of a building at specific times.

- *Case B. Statistical knowledge of missions, which arrive early:* In this case, we model a scenario where the sensors are deployed with the expectation that a large number of missions will be active in the near-term. As time passes, the likelihood of missions arriving decreases, but we do not know the target lifetime of the network. A typical example of this might be an emergency chemical hazard monitoring scenario, where an individual mission uses specific sensors to detect the continued presence of specific contaminants is activated at $t = 0$. Once the contaminant disappears, the mission may be deactivated.
- *Case C. Statistical knowledge of missions, which arrive at any time:* In this case, we model a scenario in which we have a target lifetime for a sensor network, but missions may arrive or leave *at anytime* during the life of the network, with some probability. We know the statistics of the event, i.e., the distribution of their arrivals and departures, and their expected lifetime, but unlike case A, do not know the exact arrival and departure times in advance.

As a preview, we shall see that these more complex scenarios require an important departure from the base NUM protocol (where forwarding nodes adjust link shadow prices without any knowledge of missions): *we shall now require the intermediate nodes to possess some (albeit limited) knowledge of the characteristics of the missions.*

A. Case A. Deterministic knowledge of variable duration missions - Model and Protocol

In this case, the network has a target lifetime of T and each mission m is associated with a set of non-overlapping $TI(m)$ time intervals $(t_m^{s_i}, t_m^{e_i}), i = 1, 2, \ldots, TI(m)$ within which it is active. Formally, a mission's active duration is represented by $Act(m)$, where

$$Act(m) = \{(t_m^{s_i}, t_m^{e_i})\}; i = 1, 2, \ldots, TI(m); \quad \text{such that } t_m^{e_i} < t_m^{s_{i+1}} \forall i = 0, \ldots, TI(m) - 1$$

.

*1) **Utility Optimization Model**:* To model this scenario, we set the objective function to be:

$SENSOR - LIFE2(U; L)$:

$$\text{maximize} \int_0^T \sum_{\forall m \in M} f_m(t) U_m(x_{s:s \in set(m)}(t)) dt \quad \text{subject to Eq. (11, 12, 13)}$$

Here, $f_m(t)$ has value 1 if $t_m^{si} \leq t \leq t_m^{li}$, for any activation cycle i and 0 otherwise.

If we assume the utility of a mission m to be of the form $\sum_{s \in Set(m)} a_{ms} ln(x_s)$, where a_{ms} is a scalar[1], the necessary and sufficient condition (derived from the Hamiltonian as before) simplifies to:

$$\frac{dx_s}{dt} = \frac{x_s \sum_{\forall m \in Miss(x_s)} \frac{df_m}{dt} * a_{ms}}{\sum_{\forall m \in Miss(x_s)} a_{ms} f_m(t)} = x_s A_s(t) \tag{21}$$

where $A_s(t) = \frac{\sum_{\forall m \in Miss(x_s)} \frac{df_m}{dt} a_{ms}}{\sum_{\forall m \in Miss(x_s)} a_{ms} f_m(t)}$. While $f_m(t)$ is itself a non-continuous, 'step' function, our analysis assumes a continuous function. We achieve this goal by approximating the 'staggered heaviside step function' using a smooth differentiable function as:

$$f_m(t) \approx \sum_{i=1,2,\ldots} \frac{1}{1 + e^{-2k(t - t_m^{si})}} - \frac{1}{1 + e^{-2k(t - t_m^{li})}} k \to \infty$$

Using Eq. (11), (13) and (21), we obtain the following energy-consumtpion constraint that must hold at each node k:

$$\sum_{\forall i \in Flows(k)} \alpha_i^k x_i(t)[1 + (1 + A_i(t)) + \ldots + \\ +(1 + A_i(t))\ldots(1 + A_i(T - 1))] \leq p_k(t) \tag{22}$$

It can be noted that since the schedules are known in advance and $f_m(t)$ is continuous, the value of $A_i(t)$ can be computed for all values of t. We observe from Eq. (22) that the energy constraint each node takes into account not only the current rates, but also future dynamics of missions that affect that node.

*2) **Protocol Details**:* The protocol corresponding to this model, *DYN-DET*, is similar to *STATIC* and the rate is adapted according to Eq. (19). However, the willingness-to-pay is computed as $w_{ms} = f_m(t) x_s(t) \frac{\partial U_m}{\partial x_s}$ and η_k is given from Eq. (22) as $\eta_k =$:

$$\left(\frac{\sum_{\forall i \in Flows(k)} \alpha_i^k x_i(t)[1 + \sum_{j=t}^{T-1} \prod_{l=t}^{j} (1 + A_i(l))]}{p_k(t)} - 1 + \epsilon\right)^+ / \epsilon^2 \tag{23}$$

For the forwarding nodes to compute $A_i(t)$ in a distributed and independent way, we allow each source s to compute the values of $e^{t_m^{si}}$ and $e^{t_m^{li}}$ for all its missions m and transmit this along with the data to the forwarding nodes. This needs to be done only initially and this information is sufficient for the forwarding nodes to computed $\eta_k(t)$. The actions taken by the source, sink and forwarding nodes in this protocol are listed in Table (II).

B. Case B. Statistical knowledge of missions, which arrive early - Model and Protocol

In this case, we jettison the need for deterministic knowledge and perform adaptation based on statistical information about missions, for the scenario where the number of active missions is likely to diminish over time. For such scenarios, we want the network to last as long as possible, but also have the likelihood of utility demanded by a mission degrade over time. Alternately, the utility itself could degrade over time–e.g., an application using a camera to monitor the identities of vehicles arriving at a meeting might have less utility for the camera feed as the meeting proceeds, as new vehicle arrivals are less likely.

[1]This particular form of utility function is assumed here only for expositional convenience, see [10] for the case of more generic utilities and detailed derivation.

*1) **Utility Optimization Model**:* For modeling such scenarios, we assume that the probability of a mission being active decreases exponentially over time (alternative models of decay can also be accommodated in identical fashion). Hence, the network is most valuable initially, and the utility of a mission m degrades exponentially over time at a rate ρ_m. Since we do not know the lifetime of the network and want the network to last as long as possible, we use an infinite time horizon. The objective function for this scenario is modeled as:

$SENSOR-LIFE3(U;L)$:

$$\text{maximize} \int_0^\infty \sum_{\forall m \in M} e^{-\rho_m t} U_m(x_{s:s \in set(m)}(t))dt$$

$$\text{subject to Eq. (11), Eq. (12) and} p_k(\infty) \geq 0 \ \ \forall k, \quad (24)$$

Differentiating the marginal Hamiltonian, $\frac{\partial H}{\partial x_s}$ over time and using the necessary and sufficient conditions given by the Maximum Principle, we get the following, if we assume logarithmic utility of the form $\sum_{s \in Set(m)} a_{ms} ln(x_s)$ (please see [10] for generic utilities):

$$\frac{dx_s}{dt} = \frac{-x_s \sum_{\forall m \in Miss(x_s)} \rho_m a_{ms}}{\sum_{\forall m \in Miss(x_s)} a_{ms}} \quad (25)$$

Eq. (25) implies that, as expected intuitively, the optimal rate control strategy is to reduce the rate over time proportional to the discount rate. If all missions have the same discount rate ρ, then $\frac{dx_s}{dt} = -x_s \rho$.

Using Eq. (25), (11) and the terminal constraint in Eq. (24), we get

$$\sum_{\forall i \in Flows(k)} \alpha_i^k x_i(t) \leq p_k(t) r(i)$$

$$\text{where, } r(i) = \frac{\sum_{\forall m \in Miss(x_i)} \rho_m a_{ms}}{\sum_{\forall m \in Miss(x_i)} a_{ms}} \quad (26)$$

*2) **Protocol Details**:* The *DYN-DECAY* protocol can also be obtained by an extension of the base *WSN-NUM* protocol, with the source rate adjusted according to Eq. (19), where

$$w_{ms} = e^{-\rho_m t} x_s(t) \frac{\partial U_m}{\partial x_s}, \text{ and from Eq. (26),}$$

$$\eta_k = [\frac{\sum_{\forall i \in Flows(k)} \alpha_i^k x_i(t)}{r(i) p_k(t)} - 1 + \epsilon]^+ / \epsilon^2 \quad (27)$$

Th steps involved in this protocol are listed in Table (II). For smooth distribution, the sources propagate the values of ρ_m to the forwarding nodes during the initial data transmissions.

C. Case C. Statistical knowledge of missions, which arrive at anytime - Model and Protocol

In this case, we consider a more sophisticated and interesting scenario in which we have a target lifetime for a sensor network, but events may occur at *at anytime, with any probability* during the life of the network. We know the statistics of the event, i.e., the distribution of their arrivals and their expected lifetime, but do not know the specific arrival and departure times in advance.

*1) **Utility Optimization Model**:* In this model, missions can enter or leave the network at any time and we refer to this *change in mission configuration* as an 'event'. This event may result in an increase or a decrease in the number of missions. Let τ be the probability that this event occurs over an unit interval of time. τ is assumed to be conditionally independent of past events, so that the fact that the event has not occurred in the past does not affect the likelihood of its occurring in the present[8]. Let us assume that we know the set of all possible missions, and let $|M|$ be the cardinality of this universal set; at any time, the group of active missions, g, will be one of the $2^{|M|}$ possible subsets. Let $\phi(g)$ be the probability that group g is active; this can be of any probability distribution and typically depends on the probability of each mission in the group being active.

The objective of this problem is to maximize the *expected* utility over time:

$SENSOR-LIFE4(U;L)$:

$$\text{maximize } E[\int_0^T \sum_{\forall m \in M} U_m(x_{s:s \in set(m)}(t))dt]$$

subject to Eq.(11,12,13)

The necessary and sufficient conditions are given as:

$$(i) H_{x_s} = \sum_{\forall m \in M} E[\frac{\partial U_m}{\partial x_s}] - \sum_{\forall q \in Path(s)} \mu_q \sum_{(k,s) \in q} \frac{1}{c_{ks}} - \sum_{\forall k \in Path(s)} \lambda_k \alpha_s^k = 0, \text{ and} \quad (28)$$

$$(ii) \frac{d\lambda_k}{dt} = -H_{p_k} = 0 \quad (29)$$

Differentiating Eq. (28) w.r.t time must obey the *Generalized Ito's Lemma* for stochastic calculus [9], [8]. Accordingly, we get:

$$\frac{E_t[dH_{x_s}]}{dt} = \frac{dH_{x_s}}{dt} + \tau \sum_{g=0}^{2^{|M|}} \phi(g) * [\sum_{\forall m \in g} \frac{\partial U_m}{\partial x_s} - \sum_{\forall m \in g_c(t)} \frac{\partial U_m}{\partial x_s}]$$

where $g_c(t)$ is the group of missions currently active. If we assume logarithmic utility and simplify the above equation, we get:

$$\frac{dx_s}{dt} = x_s \tau (\frac{E_t[\sum a_{ms}]}{\sum_{\forall m \in g_c(t)} a_{ms}} - 1) \quad (30)$$

where $E_t[\sum a_{ms}] = \sum_{g=0}^{2^{|M|}} \phi(g) \sum_{\forall m \in g} a_{ms}$, conditional on the information available at t. Equations (30),(13),(11) give rise to the following condition at each node k:

$$\sum_{\forall i \in Flows(k)} \alpha_i^k x_i(t)[1 + A_i(t) + ... + A_i(t)...A_i(T-1)] \leq p_k(t) \quad (31)$$

where $A_i(t) = 1 + \tau(t+1)(\frac{E_t[\sum a_{mi}]}{\sum_{\forall m \in g_c(t)} a_{mi}} - 1)$. At any instance of time, $A_i(t)$ has to be computed for all values of t from the current time to $T-1$. This is possible since τ and the

(possibly time-varying) probability of each mission being active are known a priori, and the unknown quantity, $\sum_{\forall m \in g_c(t)} a_{mi}$ (weight of missions active at time t) can be substituted with the expected value (i.e., $E_{t-1}[\sum a_{mi}]$).

2) ***Protocol Details****:* For this protocol *DYN-RANDOM*, the basic rate adaptation algorithm remains the same as in Eq. (19). However the willingness to pay and energy cost (based on Eq. (31) are computed differently, as shown as below:

$$w_{ms} = x_s(t)\frac{\partial U_m}{\partial x_s}, \text{ and,}$$
$$\eta_k = [\frac{\sum_{\forall i \in Flows(k)} \alpha_i^k x_i(t)[1 + A_t + ... + A_t ... A_{T-1}]}{p_k(t)} - 1 + \epsilon]^+/\epsilon^2 \quad (32)$$

The actions taken by the source nodes, missions and the forwarding nodes for this protocol are shown in Table (II).

D. Evaluation

We use the same simulation set-up as in Section III-C.

(i) DYN-DET protocol for Case A:

For Case A, we simulate the scenario where initially only four missions are active, and at time $t = 50\ min$, six more missions enter the network, and at $t = 110\ min$, two of the missions leave. The network lifetime is 2 $hours$ and there are 10 sources. The resulting network utility is shown in Fig. (5) for (i) *DYN-DET* protocol, (ii) *STATIC* protocol and (iii) the numerical optimal obtained from GAMS. We see that the protocol developed exclusively for handling dynamics is more conservative initially, in anticipation of higher utility at later time (which is also the optimal strategy).

(ii) DYN-DECAY protocol for Case B:

For testing Case B, we simulated the following scenario: Missions arrive according to Poisson distribution, and the arrival rate of the missions decreases exponentially over time, with a decay factor of $\rho_m = 0.05$. All missions have the same average active lifetime of 15 minutes. This scenario depicts our case where the network is most busy initially. The resulting utility is shown in Fig. (6). The optimal utility becomes zero around $t = 70\ min$, by which time all missions leave the network. We see that our protocol also regulates the network such that the maximum utility is achieved before $t = 70\ min$. The utility of the network approaches zero by this time, since all the battery in the bottleneck nodes is used by then.

To evaluate how well the *DYN-DECAY*protocol performs with respect to Case A in which perfect knowledge is available, we recorded the mission dynamics from the simulation and used this information as input to the *DYN-DET* protocol. The results show us the utility we could receive if we had perfect knowledge of the mission dynamics for Case B. The results are shown in Figure 6. We see that with perfect knowledge, we achieve only slightly higher utility than using the*DYN-DECAY*protocol without a priori knowledge. We simulated this scenario 50 times with different mission lifetimes and arrival rates and found that *DYN-DECAY* achieves within 4.5% of the optimal utility on average, and within 3.1% of the *DYN-DET* which operates with perfect knowledge. Thus, the cost in terms of utility for not having perfect knowledge is small.

(iii) DYN-RANDOM protocol for Case C:

We simulated the following scenario for evaluating this protocol: The number of missions in the network is a Poisson process with mean 2 and the network lifetime is 2 hours. The maximum number of possible missions is 10, and a subset of this is active at any given time. Of these, 5 missions have utility weight $a_m = 1$ (for all its sources), and the probability of these missions being active is 0.9 in the first hour and 0.1 in the second. The other five missions are more valuable with $a_m = 20$. The probability of these missions being active is 0.1 in the first hour and 0.9 in the second. To evaluate how well the *DYN-RANDOM* protocol operates compared to the case of perfect knowledge, we recorded the mission dynamics of Case C and used them as input to the *DYN-DET* protocol. We also ran the scenario with the *STATIC* protocol. The results are shown in Figure 7.

Fig. (7) shows the evolution of network utility over time for this scenario for *STATIC*, *DYN-DET* and *DYN-RANDOM* protocols, along with the optimal strategy. We see that the utility is closest to the optimal when we have deterministic information. With stochastic information, we still perform close to the optimal. The *STATIC* protocol performs the worst due to lack of any information about the missions. Fig. (8) shows the natural logarithm of the rate (i.e., $ln(rate)$) of one source that is common to all the missions, along with the number of high utility missions in the network. This further illustrates the difference in the rate adaptation strategies of the protocols. The energy-consumption for these protocols are shown in Fig. (9). We observe that *DYN-RANDOM* conserves the energy until the higher utility missions are more probable, which is the optimal strategy, while *STATIC* consumes energy linearly. We simulated this scenario for different random seeds. The average % deviation from optimal, and the 95% confidence intervals are shown in Fig. (10).

V. Conclusion and Future Work

In this work, we developed a novel and flexible framework for optimally adapting the resource usage in a wireless network according to the lifetimes associated with the applications (missions) that receive data over the network. We developed different optimal control-based NUM protocols for both static missions and also more-realistic, dynamic mission scenarios; the resulting protocols require different amounts of mission-related information to be available at the intermediate forwarding nodes. We show via quantitative evaluation that our protocols perform close to the optimal values. *Most importantly, the framework is generic enough to capture many other interesting scenarios (e.g., when batteries at different nodes are renewed with different periodicity.)* In future, we will extend our stochastic models to accommodate alternative mission arrival and departure patterns, and *renewable* energy scenarios.

Node	*STATIC*	*DYN-DET*	*DYN-DECAY)*	*DYN-RANDOM*
Source Node, s	(i) Performs rate adaptation as: $x_s(t+1) = x_s(t) + \kappa(TotalWillToPay(t) - x_s(t)(TotalCliqueCost(t) + TotalEnergyCost(t)))$ (ii) Transmits T (once) and data at current rate.	(i) Performs rate adaptation as in *STATIC*. (ii) Transmits $\{e^{2kt_m^{s_i}}, e^{2kt_m^{s_i}}\}$, $\forall i$ and T (once), and data at current rate.	(i) Performs rate adaptation as in *STATIC*. (ii) Transmits ρ_m (once) and data at current rate.	(i) Performs rate adaptation as in *STATIC*. (ii) Transmits T, $E[n]$ (once) and data at current rate.
Forwarding Node, k	(i) Computes $CliqueCost_k$ for cliques whose costs have not yet been computed. (ii) Computes $EnergyCost_k$ according to Eq.(20). (iii) Forwards data, total split cost and list of cliques whose costs have been computed.	Same as in *STATIC* except that $EnergyCost_k$ is computed according to Eq. (23)	Same as in *STATIC* except that $EnergyCost_k$ is computed according to Eq. (27)	Same as in *STATIC* except that $EnergyCost_k$ is computed according to Eq. (32)
Sink Node	(i) Computes $WillToPay = x_s(t)\frac{\partial U_m}{\partial x_s}$ (ii) Echoes back total cost, $WillToPay$ to each source	(i) Computes $WillToPay = x_s(t) f_m(t)\frac{\partial U_m}{\partial x_s}$ (ii) Echoes back total cost, $WillToPay$ to each source	(i) Computes $WillToPay = x_s(t) e^{-\rho_m t}\frac{\partial U_m}{\partial x_s}$ (ii) Echoes back total cost, $WillToPay$ to each source	(i) Computes $WillToPay = x_s(t)\frac{\partial U_m}{\partial x_s}$ (ii) Echoes back total cost, $WillToPay$ to each source

TABLE II: Distributed Rate Adaptation Procedure for all four protocols.

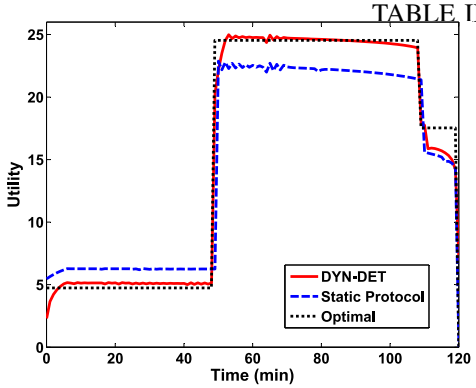


Fig. 5: Global utility for Case A.

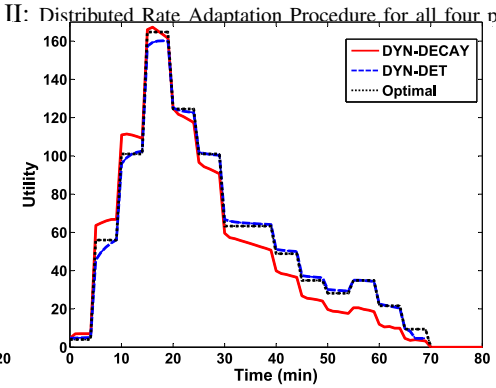


Fig. 6: Global utility for Case B.

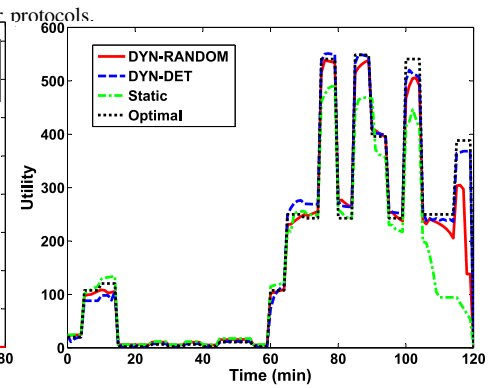


Fig. 7: Global Utility for Case C.

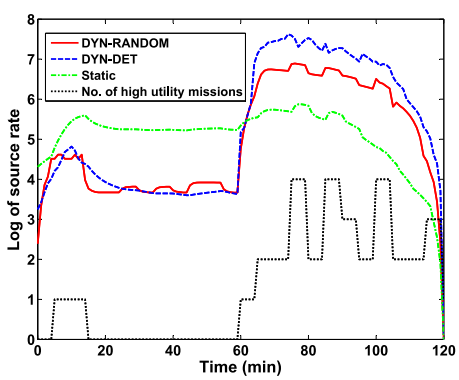


Fig. 8: Rate adaptation for Case C.

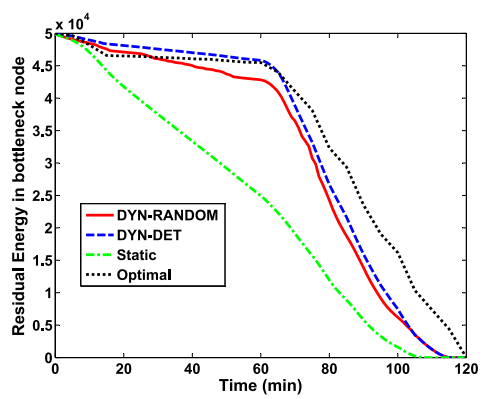


Fig. 9: Energy adaptation for Case C.

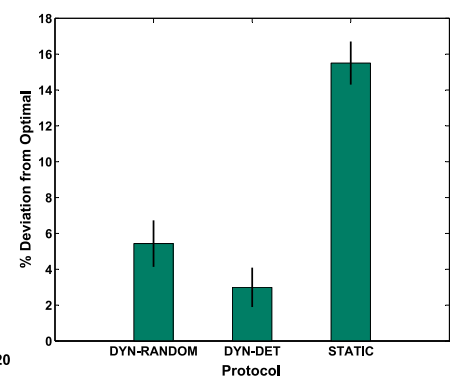


Fig. 10: % deviation from optimal for Case C.

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