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Risking Returns: Moving from Public to Private Equity

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Abstract

The notion of risk identifies a project that matches with the risk appetite of an entrepreneur not necessarily the investors. This can explain why entrepreneurs would start up companies but it cannot explain why *ex-post* the investors continue, given that a diversified portfolio of publicly traded assets could potentially generate similar return with lower risk. We re-evaluate the evidence through performance measures using relative probability distributions of public and private equity funds, and identify the nature of the deviations. We observe that the heterogeneity in different investor classes are greatly reduced using standard covariates to identify the choice between public and private equity funds.

JEL Classification: G11, G24, C14

Keywords: Probability integral transform, smooth test, fractile graphical analysis, risk premium, higher-order moments

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1 Introduction and Motivation

Financial theory predicts that investors in publicly traded securities would assume more risk if they are compensated with more returns. Naturally one has to presume that entrepreneurs who assume more risk in venturing into privately held companies are lured by the premium commanded by these inherently riskier assets. While there is substantial evidence for the conventional wisdom of risk-return trade-off in publicly traded assets (Fama, 1970; Fama and French, 1992, 1999), recent literature on returns and performance of private equity suggests that these assets, although riskier than their publicly traded counterpart, do not have sufficient return to justify the excess risk (Moskowitz and Vissing-Jorgensen, 2002; Kaplan and Schoar 2005; Gottschalg, Phalippou, Zollo, 2003).

Despite the sheer size of the private equity market (the market value of total private equity ranges from \$3.7 trillion in 1989 to \$5.7 trillion in 1998; the corresponding figures for public equity are \$1.6 trillion and \$7.3 trillion, see Moskowitz and Vissing-Jorgensen, 2002), it has not received much academic attention till very recently, at least in terms of the relative performances of private and public equity markets. In 2004 as much as 11.5% of households had some business equity with a median holding of 100,000 in 2004 dollars while nearly 47.4% households had some stocks in mutual funds or other pooled accounts that accounted for 24,300 in 2004 dollars (see Bucks, Kennickell, and Moore, 2004, Tables 5 and 6). According to National Venture Capital Association (NVCA, www.nvca.org), on Venture Capital's contribution "According to a 2004 Global Insight study, venture-backed companies accounted for 10.1 million jobs and \$1.8 trillion in revenue in the United States in 2003."

One of the continuing topics of interest is the apparent anomaly (referred to as the "private equity premium puzzle" by Moskowitz and Vissing-Jorgensen, 2002) in the risk-return tradeoff in private equity. In particular, one would like to answer, "What drives the entrepreneurs to assume so much risk in relatively undiversified and extremely concentrated portfolios with very little commensurate return?" Economic theory tell us that entrepreneurs assume the risk associated with a project driven by the prospect of abnormal returns, and most often they have high appetite for risk. While this can explain why entrepreneurs would start up companies or projects but it cannot explain why ex-post they (and their financier or investors) will continue given that a diversified portfolio of publicly traded assets will give them as much return with lower risk exposure. One possible answer that has been suggested is a large degree of non-pecuniary benefits of being self-employed and an expectation of large pecuniary benefits (Hamilton 2000). Others have pointed out that there is a high degree of heterogeneity in returns obtained by private equity investors and managers depending on their institution type and degree of sophistication (Lerner, Schoar and Wongsunwai, 2007).

There are several important issues in private equity research. First, the dearth of reliable data on returns and actual investment in private equity pose serious problems on the inference based on the data. The quality of the data unlike readily available public equity returns does indeed have an impact on the measure of the risk involved. The data available through SDC Platinum Venture Economics and Thomson Banker One databases from Thomson Reuters' is verified in each round by both the *Limited Partner* (or LP or the investor) and the *General Partners* (or GP or the manager). However, it continues to suffer from self-reporting biases of possibly badly performing projects or funds (Kaplan, Sensoy, Stromberg, 2002; Ljungqvist and Richardson, 2003). Second, the high volatility in returns and very high failure rates (Moskowitz and Vissing-Jorgensen, 2002, p. 746, notes "...survival rates of private firms are only around 34 percent over the first ten years of the firm's life...") make it very challenging to get a good estimate of measures of central tendency like the mean return to investment. There is often an upward bias of the measure owing mainly to survivorship and misperception of the risk of failure involved by entrepreneurs (Moskowitz and Vissing-Jorgensen, 2002, see public equity funds see Carhart, Carpenter, Lynch and Musto, 2002). Cochrane (2005) incorporated selection problems using a maximum likelihood procedure to evaluate the risk-return tradeoff of Venture Capital firms more accurately, and find that there is a significant effect of survivorship in both mean and volatility of returns. Hence, there might be a need for a more robust measure, possibly based on ranks or quantiles of the return or some covariates, to deal with this issue (see subsection 4.4). Third, the downside risk associated with private equity is pro-cyclical with business cycles, and is often positively correlated with public equity returns, and hence do not provide a good hedge against public equity holdings (Phalippou and Zollo, 2005). Phalippou and Zollo (2005) also find that private equity funds are also exposed to right tail-risks or those due to higher order moments. This implies that tests based on standard measures like the Sharpe Ratio would not accurately reflect the risk-return tradeoff. Finally, due to the lack of availability of accurate data the most frequently used measure of fund performance for private equity is the *internal rate of return* reported by Venture Economics, which is defined as the rate of return that makes the discounted net cashflow equal to zero for the private equity investment. However, the drawbacks of the internal rate of return, which independent of market timing, as a measure of performance has been pointed out by many researchers and experts, Jesse Reyes, Vice President of Venture Economics noted that "...private equity investment timing is totally under manager's control, timing decisions should be part of the performance measure so he can be penalized or rewarded for these timing decisions..."(LP corner reported by Lisa Bushrod, September 2004, http://www.evcj.com).

All these evidence suggests that standard risk-return analysis do not satisfactorily explain what motivates the entrepreneurs or investors of private equity (Moskowitz and Vissing-Jorgensen, 2002). One of the factors that have been proposed as an explanation lies in the higher moments of the return distribution (like right skewness and fatter tails; see for example, Harvey and Siddique 2000). Affinity for right or positive skewness implies that an entrepreneur might be willing to accept lower average returns if there is a positive probability of getting a very high return. However, in our literature survey, no formal test have been done to verify whether the return distribution of private equity is more or less skewed or have fatter tails than the return to publicly traded equity with the exception of very recent work in public equity on tests based Sharpe Ratio (Ledoit and Wolf, 2008).

In this paper our objective is simple. We want to re-evaluate the evidence of the dispersion between public and private equity returns. First, to make the public equity return for mutual funds comparable to the internal rate of return available for private equity funds available from Venture Economics, we only look at those mutual funds that reports yearly return with no dividend yield. Second, to focus more on the systematic differences we use private equity fund and publicly traded mutual fund data, rather than individual firm equity. We also do not compare the private and public equity indices as they are not in the choice set of individual investors. Third, to get a more accurate cash-flow information on private equity returns we restrict our attention to only mature (not necessarily liquidated) private equity funds that had inception before 1996 so that we have some actual return information rather than imputed ones. Finally, we picked the public equity mutual funds after 1996 to 2003 to control for the effect year of the inception of private equity funds (Kaplan and Schoar, 2005). Although there is still some degree of dependence left between the private and public equity funds, we reduce the impact by using both year and fund specific fixed effects (see subsection 4.3).

We compare the overall difference between the two return distributions in the panel data, and observe that incorporating both the year and fund specific fixed effects explains a substantial portion of the variation in private equity returns as well public equity. We also incorporate variables like lagged returns to see if there is persistence in returns in either public equity mutual funds or private equity ones, and include non-linear terms for size of the fund to investigate any evidence of convexity (or concavity) in the relationship between returns and fund size. We find persistence at least to two periods in private equity funds, however, public equity fund also shows some postive persistence, maybe due to momentum strategies, but quickly reverses to negative with two year lagged return maybe due to the size effect (Berk and Green, 2004). We also explore whether returns are indeed what entrepreneurs are after or is it just the size of the fund making "money chasing deals" using ranks of size as a covariate to compare the returns (Gompers and Lerner, 1998; Gompers and Lerner, 2000; Berk and Green, 2004; Jones and Rhodes-Kropf, 2002)?

The paper is organized as follows. We discuss the possible shortcoming of a standard measure of risk for mean-variance type analysis like the Sharpe Ratio in Section 2. In Section 3 we introduce the basic motivation of the two sample version of Neyman smooth test (Neyman, 1937, see also Bera and Ghosh, 2002). We also introduce the main theorems driving the two sample test and the sample selection criteria function procedure (discussed in details in Appendix A) proposed by Bera, Ghosh and Xiao (2007). The data and analysis part is discussed in details in different subsections in Section 4. We discuss the data and distributional comparison of the unadjusted private and public equity returns under different restrictions in subsection 4.1. This is followed by subsection 4.2 with an introduction to the nonparametric rank based graphical method called Fractile Graphical Analysis, and the ensuing bootstrap based hypothesis test using these methods (Mahalanobis, 1961; Bera and Ghosh, 2006). In subsection 4.3 we introduce the standard OLS regression analysis with fixed effects for years and funds for both public and private equity funds. We compare the private and public equity model residuals to test for departures in finite number of moment directions using the two sample version of smooth test (Bera, Ghosh and Xiao, 2007). In the final subsection 4.4 of Section 4, we apply rank (fractile) regression method to address possible non-linearity in a robust way and hence compare the residuals of this semiparametric model specification for private and public equity returns. In Section 5 we apply all the above techniques to compare Venture Capital and Buyout funds. We conclude with future directions in Section 6.

2 Measuring Risk in Return

One of the most common measures for risk adjusted average return is the Sharpe ratio (μ/σ), and different performance tests based on it have already been proposed (Jobson and Korkie, 1981; Memmel, 2003, Ledoit and Wolf, 2008). To test whether there is significant difference between the performances of public and private equity the following hypothesis can be tested,

$$H_0 : \frac{\mu_1}{\sigma_1} = \frac{\mu_2}{\sigma_2} \text{ against } H_1 : \frac{\mu_1}{\sigma_1} \neq \frac{\mu_2}{\sigma_2}$$

where the subscripts '1' and '2' refers to these two types of equity. There are many equivalent forms of this null hypothesis, such as, $H_0 : \frac{\sigma_1}{\mu_1} = \frac{\sigma_2}{\mu_2}$, $H_0 : \mu_1\sigma_2 = \mu_2\sigma_1$, $H_0 : \frac{\mu_1}{\mu_2} = \frac{\sigma_1}{\sigma_2}$ etc. And if we follow the standard approaches, each form of the same hypothesis could give different results. The tests that are standard among the practitioners were crucially based on the normality assumptions (Jobson and Korkie, 1981; Memmel, 2003). However, the standard tests are not valid when financial returns have tails heavier than the normal distribution or for time series and panel datasets (see Ledoit and Wolf, 2008 and references therein). Private equity funds can be leptokurtic or have fatter tails than normal distribution, hence standard tests based on Sharpe Ratios might be misleading (see Figure 1). To address the problem, Ledoit and Wolf (2008) suggested two different procedures. First, was to implement *Heteroscedasticity and Autocorrelation Consistent* (or *HAC*) standard errors for the difference of two Sharpe Ratios. Second, to apply *Studentized time series bootstrap* methods to construct confidence intervals with a corrected coverage probability from the actual data to do hypothesis testing. However, there are significant drawbacks of the procedure involving Sharpe ratios, and the jury is still out on tests solely based on the Sharpe ratio, as Ledoit and Wolf (2008, p. 851) themselves observe:

> "...It has been argued that for certain applications the Sharpe ratio is not the most appropriate performance measure; e.g., when the returns are far from normally distributed or autocorrelated (which happens for many hedge funds) or during bear markets. On the other hand, there is recent evidence that the Sharpe ratio can result in almost identical fund ranking compared to alternative performance measures..."

One first objective of this paper would be extend tests based on comparing Sharpe ratios from private and public equity to those comparing entire distributions (to a finite set of moments) rather than just a function of the first two moments. Moreover, testing individual factors like average risk, volatility measures, and higher moments like skewness and kurtosis should be done jointly to remove the effect of interrelationship among the individual tests. Extending it further, we should really look at the difference in the entire return distributions of public and private equity returns in Sections 4.

For example, we investigated the monthly return on private equity firms (obtained from Securities Data Corporation (SDC)-Platinum database on 370 new issues in the US in 2001 of private equity that did not give out dividends) and the monthly return on of a random sample of 1837 publicly traded companies from CRSP (December of 2001) given in Figures 1A and 1B.

Figures 1: Kernel probability density function of the monthly returns on 370 private equity firms from SDC-Plaitinum database new issues in the US in 2001 that did not give out any dividends. We have averaged out the monthly returns in Panel 1A. In Panel 1B we estimate the kernel probability density function of 1837 publicly traded firms from Center for Research on Security Prices (CRSP, December of 2001) and looked at the average monthly return for 2001. We have used the Gaussian kernel with an automatic bandwidth for a normal reference density method.

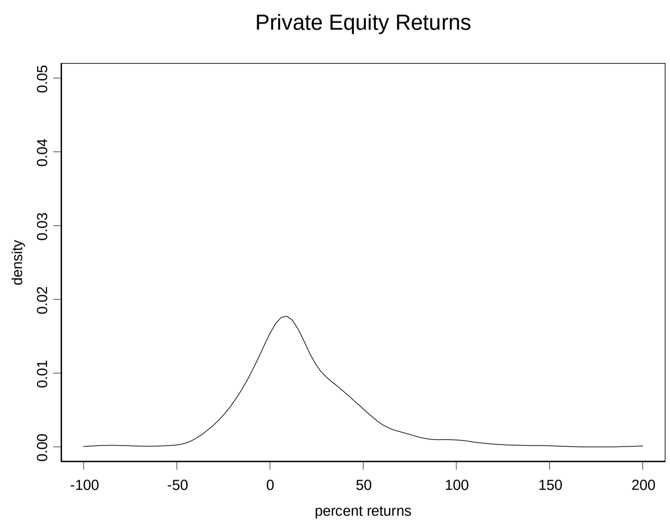


Figure 1A: Private Equity Return Distribution

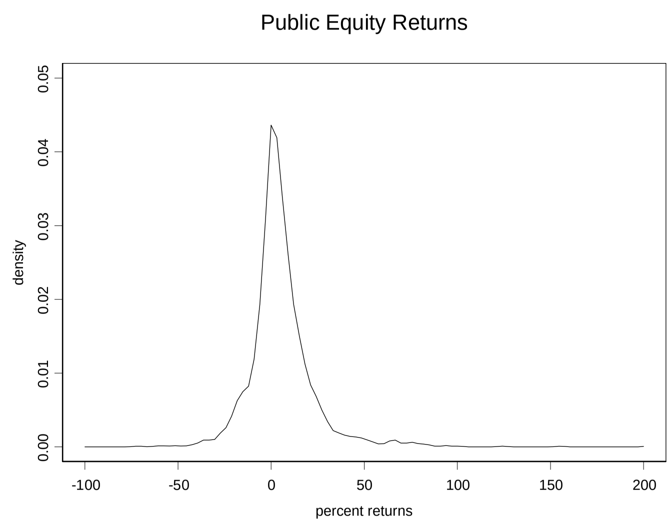


Figure 1B: Public Equity Return Distribution

Figures 1A and Figure 1B depict the differences between monthly private and public equity returns in 2001. While the average return seems to be similar, there could be distinguishable differences in terms of volatility, skewness, kurtosis or some higher order moments.

Our second objective is to identify the sources for the departure between the two return distributions. We can even select the exact number of moments we want to compare depending on the data. The major part of the paper would be to device and perform a statistical test that can compare the shapes of two return distribution and identify the exact order of moments [first (average), second (volatility), third (skewness), fourth (kurtosis or peakness) or even higher moments] where they might differ. Once that is achieved we will move forward to explain the plausible causes of the departure.

3 Smooth Test for Comparing Distributions

For performing this test of comparison of distributions of we use the two sample version of smooth test procedure as proposed in Bera, Ghosh and Xiao (2007). Neyman's smooth test for $\mathbf{H}_0$: $\mathbf{F} = \mathbf{F}_0$. was for the one sample case with completely specified distribution under null hypothesis $H_0 : f(x)$ is the true PDF (for a review, see Bera and Ghosh, 2002). This is equivalent to testing $H_0 : y = F(x) = \int_{-\infty}^{x} f(u)\,du \sim U(0,1)$ .Neyman considered the following smooth alternative to the uniform density:

$$h(y) = C(\theta)\exp\left[\sum_{j=1}^{k} \theta_j \pi_j(y)\right] \tag{1}$$

$\pi_j(.)$ are orthogonal normalized Legendre polynomials. For $H_0 : \theta_1 = \theta_2 = \cdots = \theta_k = 0$ has a test statistic

$$\Psi_k^2 = \sum_{j=1}^{k} \frac{1}{n}\left[\sum_{i=1}^{n} \pi_j(y_i)\right]^2 \sim \chi_k^2(0) \text{ under } H_0.$$

If we take the problem of testing $H_0 : F = G$. We need to modify the original smooth test since both F and G are unknown. If $F(.)$ were known, we can construct a new random variable $Z_j = F(Y_j)\,,\ j = 1, 2, ..., m.$

The CDF of Z is given by

$$\begin{aligned} H(z) &= \Pr(Z \leq z) = \Pr(F(Y) \leq z) \\ &= G\left(F^{-1}(z)\right) = G(Q(z)) \end{aligned}$$

where $Q(z) = F^{-1}(z)$ is the quantile function of Z.

The PDF of Z is given by

$$\begin{aligned} h(z) &= \frac{d}{dz} H(z) = g\left(F^{-1}(z)\right) \frac{d}{dz} F^{-1}(z) \\ &= g\left(F^{-1}(z)\right) \frac{1}{f\left(F^{-1}(z)\right)} \\ &= \frac{g(Q(z))}{f(Q(z))}, \quad 0 < z < 1. \end{aligned} \tag{2}$$

The main problem of comparing two distributions is to find a suitable measure of distance or norm between two distribution functions, i.e. to say, for any $x \in (-\infty, \infty)$,

$$\|G(x) - F(x)\|$$

If a density function exists over the support of F and G, then for any $t \in (0,1)$ this problem to be equivalent to the distance

$$\left|G \circ F^{-1}(t) - t\right|.$$

Under $H_0 : G = F$, $G \circ F^{-1}(t) = t$. In fact, the $h(z)$ in (2) is the corresponding PDF for the distribution function $G \circ F^{-1}$ defined over $(0,1)$. The PDF $h(z)$ is a ratio of two densities; and itself is a valid density function. Therefore, we will call it the *Ratio Density Function (RDF)* (Bera, Ghosh and Xiao, 2005).

When $H_0 : F = G$ is true (i.e. $f = g$) then from (2), $h(z) = \frac{g(Q(x))}{f(Q(x))} = 1, 0 < z < 1$. Z has the *Uniform* density in $(0,1)$.That means irrespective of what F and G are, the two-sample testing problems can be converted into testing only *one kind of hypothesis*; namely, *uniformity* of a transformed random variable.

For the two sample case with unknown F and G the Smooth test statistic is

$$
\begin{aligned}
\Psi_k^2 &= \sum_{l=1}^{k} u_l^2,\ u_l = \frac{1}{\sqrt{m}} \sum_{j=1}^{m} \pi_l\left(z_j\right), l = 1, 2, ..., k \\
z_j &= F\left(y_j\right) = \int_{-\infty}^{y_j} f\left(\omega\right) d\omega,\ j = 1, 2, ..., m.
\end{aligned}
$$

Under $H_0 : F = G, \Psi_k^2 \xrightarrow{D} \chi_k^2$.

The test has k components. Each component provides information regarding specific departures from $H_0 : F = G$.

However, in practice $F\left(.\right)$ is unknown. We use the Empirical Distribution Function (EDF),

$$
\begin{aligned}
F_n\left(x\right) &= \frac{1}{n} \sum_{i=1}^{n} I\left(X_i \leq x\right), \qquad \hat{z}_j = F_n\left(y_j\right) \\
\hat{\Psi}_k^2 &= \sum_{l=1}^{k} \frac{1}{m} \left[\sum_{j=1}^{m} \pi_l\left(\hat{z}_j\right)\right]^2
\end{aligned}
$$

The following two theorems [for proof and details see Bera, Ghosh and Xiao (2004)] provide some restrictions on relative sample sizes for consistent asymptotic χ^2 distribution of the test statistic, and also to minimize size distortion of the two sample smooth test of comparing two distributions.

Theorem 1 *If* $\frac{m \log \log n}{n} \to 0$ *as* $m, n \to \infty$ *then* $\hat{\Psi}_k^2 - \Psi_k^2 = o_p\left(1\right)$.
Proof. *See Bera, Ghosh, Xiao (2007)* ■

Theorem 2 *The optimal relative magnitude of* m *and* n *for minimum size distortion is given by* $m = O\left(\sqrt{n}\right)$.
Proof. *See Bera, Ghosh, Xiao (2007)* ■

4 Data and Analysis

4.1 Unadjusted Private and Public Equity Returns

The scarcity of reliable data on private equity investment and returns is further exacerbated by the bias that we only observe data on surviving firms or survivorship present in most private equity datasets (Kaplan, Sensoy, Stromberg, 2002; Ljungqvist and Richardson, 2003). We collect and refine the data on returns and investment in private equity available from several sources including SDC-Platinum, National Venture Capital Association (NVCA)* VentureExpert or Venture Economics database by Thomson Financial, and publicly traded equity returns from the CRSP database. We compare the average returns and risk, and measures like the Sharpe ratio of performance between the different types of assets. However, to get a fuller picture of the types of deviations between the two asset classes, we construct the distribution of returns of funds of private and publicly traded assets. Hence, we can perform a joint test to see if there are significant differences in moments like location (average return), scale or volatility (risk measures) and higher-order moments like skewness and kurtosis. The summary statistics reported in Tables 1A and 1B clearly shows that the distribution of the publicly traded and private equity returns are different from each other.

We have also collected data on publicly traded open-ended US equity mutual fund returns from Morningstar Principia database from January CDs 1997-2003 (data as of December from 1996-2002) to reduce survivorship issues. This data is compared with the private equity funds returns data from Venture Economics database of funds that had inception after 1980. For comparing the two we have used non-intersecting time points public equity funds from 1996 and private equity funds that started before 1996 particularly in light of recent research that private equity funds returns are influenced by the public equity market conditions in the year of its inception (Kaplan and Schoar, 2005). We find overwhelmingly that the unconditional distributions of the private and public equity fund returns are indeed distinctly different (see, Figure 2A) and the results in Tables 1A and 1B. In fact, we also report the *Empirical Distribution Function* or EDF for each of the two groups in Figure 3A.

To improve comparability we further filter to only those public equity funds that offer no dividend yield in that year for several reasons. First, it is more in line with private equity funds that tend to have similar characteristics in intermediate years between different rounds after inception. Second, it reduces the differential tax implications on the returns like capital gains and those on dividend that is treated almost like income in US tax codes (see Poterba, 1989; Bergstresser and Poterba, 2002; Ghosh, 2007). Finally, the *internal rate of return* that is reported as a standard for private equity funds can be compared directly to the annual rates of return for publicly traded mutual funds that reinvest in the stocks rather than giving out dividends. Even in this smaller group the unadjusted returns are widely different across the years without any year specific fixed effects (see Figures 2B and 3B).

Finally, given that the data on private equity is self reported between rounds (although it is verified from both the *Limited Partner* or *LP*, the investor of the private equity firm, and the *General Partner* or *GP*, the managing company of the private equity fund), the cashflow between different rounds might be affected by misreporting or underreporting of losses (or gains possibly for tax purposes) due to non-survival (or merger) of some funds with others (Moskowitz and Vissing-Jorgensen, 2002; Cochrane, 2005). Hence, we can expect those funds that have been liquidated would be more stable with more accurate report on the amount of the cash-flow generated. This, of course, among other things imply that our sample size gets reduced substantially (491 funds as compared to 1714 in the full sample). Even in this smaller group of liquidated private equity funds, we find that the distributions of public equity funds with no dividend yields and the liquidated private funds are substantially different.

Unfortunately, standard tests of goodness-of-fit like Kolmogorov-Smirnov(K-S) and Cramér-von Mises (C-vM) (reported in Table 1B) does not provide us with the exact nature of such departures from the null hypothesis of equality of two distributions. The data shows that not only is there a difference in both the location and scale of the distribution, but the shape parameters of the distribution might also be different. In order for us to numerically compare the returns distribution of private equity funds with public equity funds , we investigate the summary statistics of each of the groups. Table 1A provides a sample size of public equity fund to $n = 10103$ (full sample after 1996 till 2002) and $n = 5635$ for mutual funds with no yields. The size of the sample of private equity funds are $m = 1714$ (full sample) and $m = 491$ (for liquidated funds), respectively. As we apply the sample size selection methods for comparing distributions, we have restricted our sample for private equity to only the ones that are more mature or spent some time after inception. We restrict our attention to only those private equity funds with fund inception year before 1996 ($m = 840$). Our working assumption is that private funds that are mature will start to show some cash-flow from 6 years after inception (Kaplan and Schoar, 2005).

Figure 2-3: Kernel density functions (2A-2B) and Empirical Distribution Functions (3A-3B) of unadjusted annual public equity funds returns (1996-2002) and private equity internal rates of returns (inception before 1996). Figure 2C and 3B are the kernel density estimates of Venture Capital and Buyout funds.

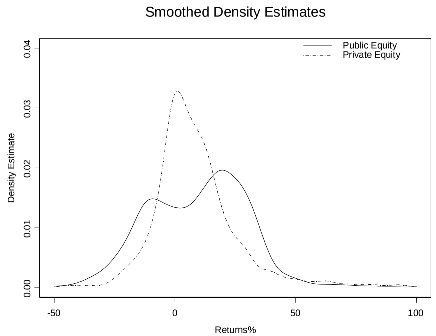


Figure 2A: Equity Returns.

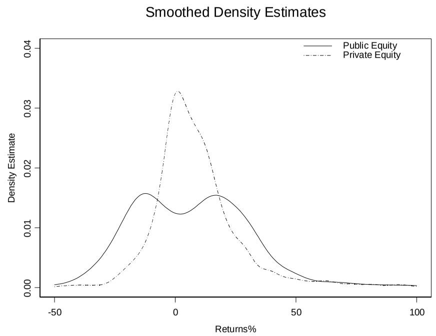


Figure 2B: No Yield Public and PE.

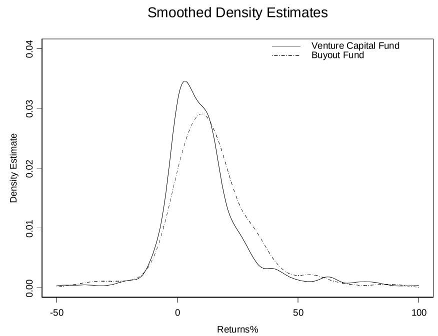


Figure 2C: VC and LBO.

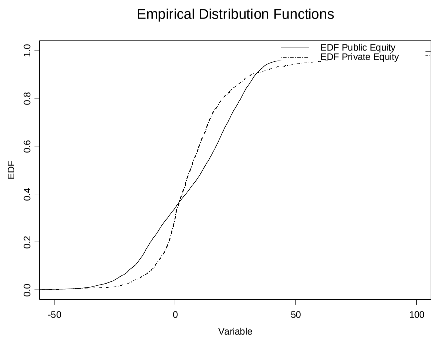


Figure 3A: Public and Private.

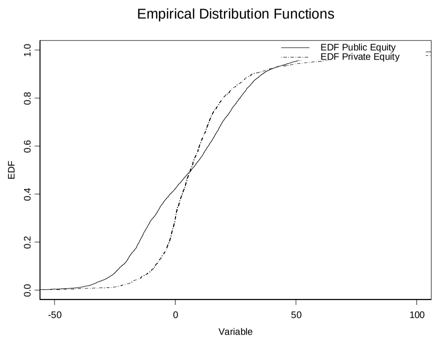


Figure 3B: No yield Public and PE.

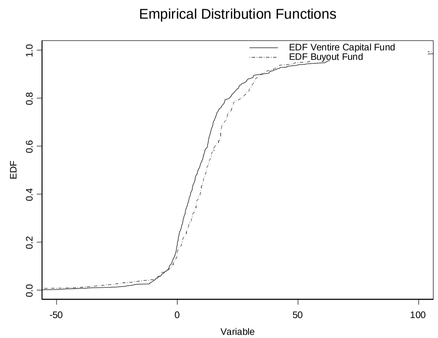


Figure 3C: VC and LBO.

Table 1: Summary Statistics (1A) of returns of Public and Private Equity funds with different restrictions. Table 1B shows the omnibus test statistic for distribution comparison (Modified Kolmogorov-Smirnov and Cramer-von Mises statistics) and corresponding 0.1% critical value reported in 'D'Agostino, R.B. and M.A. Stephens. Goodness-of-Fit Techniques. New York: Marcel Dekker, 1986.)

Variable	Obs.	Mean	Median	Std. Dev.	1^{st} Q.	3^{rd} Q.	coef. skew	excs.kurt	Sharpe Ratio
Public (full)	10103	10.19	11.51	22.69	-6.63	24.4	0.813	3.93	0.45
Public (No yield)	5635	7.59	6.65	26.58	-12.31	23.19	1.079	3.54	0.29
Private (full)	1714	13.36	6.28	42.25	-1.14	16.28	9.44	137.61	0.32
Private (liq.)	491	11.15	8.79	20.05	1.39	16.13	2.49	13.01	0.56
Private (pre96)	840	15.11	9.85	37.13	2.04	18.4	12.31	238.93	0.41
VC(pre96)	610	15.27	18.7	41.59	1.43	17.07	11.97	208.29	0.38
LBO (pre96)	206	15.27	12.17	22.32	4.225	22.43	1.65	9.13	0.68

Table 1A. Summary Statistic for Return Distributions

Test	Modified KS	Modified CvM
T* Statistic (All–all)	6.96	16.47
T* Statistic (NoYield–All)	9.12	21.85
T* Statistic (NoYield–liq.)	6.31	9.67
T* Statistic (NoYield–pre96.)	8.7	16.62
T* Statistic(VC-LBO)	2.04	2.38
Critical Upper 0.1%	1.95	1.167

Table 1B. Goodness-of-Fit Statistics based on the EDF.

Table 2: BGX Smooth Test of Goodness-of-Fit and corresponding p-values for the unadjusted returns for the full sample of public and private equity with corresponding sample sizes for the bigger (n) sample and the smaller (m) sample sizes.

	$\hat{\Psi}_6^2\,(\tilde{}\,\chi_6^2)$	$\hat{u}_1^2\,(\tilde{}\,\chi_1^2)$	$\hat{u}_2^2\,(\tilde{}\,\chi_1^2)$	$\hat{u}_3^2\,(\tilde{}\,\chi_1^2)$	$\hat{u}_4^2\,(\tilde{}\,\chi_1^2)$	$\hat{u}_5^2\,(\tilde{}\,\chi_1^2)$	$\hat{u}_6^2\,(\tilde{}\,\chi_1^2)$
Full	842.03***	10.79***	223.67***	330.87***	210.79***	46.15***	19.75***
($n = 10103, m = 1714$)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)
No Yield	990.02***	26.02***	445.33***	42.67***	425.39***	3.19*	47.42***
($n = 5635, m = 1714$)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.07)	(0.00)
PE Sample	262.06***	7.56**	134.94***	9.36***	96.93***	1.77	11.49***
($n = 5635, n_1 = 518$)	(0.00)	(0.01)	(0.00)	(0.00)	(0.00)	(0.18)	(0.00)
PE liquidated	417.49***	14.4***	189.26***	0.83	163.15***	0.35	49.48***
($n = 5635, n = 491$)	(0.00)	(0.00)	(0.00)	(0.36)	(0.00)	(0.55)	(0.00)
PE pre1996	604.5***	56.45***	235.53***	0.48***	260.12***	1.34***	50.59***
($n = 5635, m = 840$)	(0.00)	(0.00)	(0.00)	(0.49)	(0.00)	(0.246)	(0.00)
PE pre96-liq	382.55***	15.87***	169.59***	2.35	156.99***	1.11	36.64***
($n = 5635, m = 463$)	(0.00)	(0.00)	(0.00)	(0.13)	(0.00)	(0.29)	(0.00)
PE VC pre96	474.95***	32.57***	184.878***	0.767	218.7***	0.39	37.63***
($n = 5635, m = 610$)	(0.00)	(0.00)	(0.00)	(0.38)	(0.00)	(0.53)	(0.00)
PE BO pre96	109.7336***	25.729***	35.13***	5.588**	34.09***	0.41	8.78***
($n = 5635, m = 206$)	(0.00)	(0.00)	(0.00)	(0.02)	(0.00)	(0.52)	(0.00)
VC-BO pre96	25.2***	11.86***	0.35	5.66**	0.41	5.1**	1.82
($n = 610, m = 206$)	(0.00)	(0.00)	(0.55)	(0.02)	(0.52)	(0.02)	(0.18)
VC-BO pre96 (sample)	18.47**	8.26***	0.76	1.94	0.02	6.49**	1.00
($n = 610, m = 137$)	(0.01)	(0.00)	(0.38)	(0.16)	(0.89)	(0.01)	(0.32)

Table 2. Smooth test and p-values (distributions under $H_0 : F = G)^{***}$ significant at 1%, ** significant at 5%, * significant at 10%.

We perform a smooth test (see Bera and Ghosh, 2001 for an introduction to smooth tests) on the two return distributions from public equity funds and private equity funds. We note that the sample size for publicly traded funds is much larger than the privately traded funds. This differential nature of sample size can be exploited by the two sample version of the smooth test (Bera, Ghosh and Xiao, 2007, see Section 3). From the full sample version reported in Table 2, it is evident that the overall test $H_0 : F = G$ of equality of two distributions is overwhelmingly rejected ($\hat{\Psi}_6^2 = 842.03$) which under the null hypothesis has a central χ^2 distribution with 6 degrees of freedom, so chosen so as to focus on the higher order moments of the distribution of private equity returns. Further, under the null hypothesis each of the components should follow independent central χ_1^2.

The overall two-sample smooth goodness of fit test shows that one or more of the constituent elements must contribute to the directions of departure from the hypothesized distribution. As expected from the Figures 2A and 3A, the estimated components $\hat{u}_1^2$ through $\hat{u}_6^2$ are all strongly statistically significant, hence we can conclude that the private and public equity distributions are different in the directions of the first six moments of the distribution of the probability integral transform (or the *imputed ranks*). There are departures in the general directions of location, scale and shape parameters at least up to order 6. Furthermore, we can also conclude that the private equity return distribution is also different from the public equity returns in the directions of higher order moments, namely, the skewness and kurtosis related terms ($\hat{u}_3^2 = 330.87$ and $\hat{u}_4^2 = 210.79$, both are significant at 1% level). This implies that when testing jointly, the public and private equity returns differs in the first four moment directions. Furthermore, $\hat{u}_5^2$ and $\hat{u}_6^2$ are also both statistically significant at 1% level, hence, some higher order terms are different between the two distributions. Tests based on the Sharpe ratio might not reveal these details although it could be a convenient test procedure (Ledoit and Wolf, 2008).

In the applied literature, size distortion in finite sample is a common problem in Score (or LM) type tests. One way of reducing the finite sample size distortion is to have the smaller sample size (m) increase at a much slower pace than the bigger sample size (n) as discussed in the Section 3 (see also, Bera, Ghosh and Xiao, 2007). Hence, we select a sample size as small as 9.21% of n and perform the smooth test again with $m = \frac{n_2}{n_1} \times n$. instead of the original m (as per recommendation by the minimum criteria function analysis in the Appendix A). The results are qualitatively similar although results are less strongly significant compared to the full sample across the board for the smooth test. The first and third order terms are now marginally significant at 1% level

($\hat{u}_1^2 = 7.56$ and $\hat{u}_3^2 = 9.36$) while the second, fourth and fifth order term is more strongly significant (see line 3 in Table 2).

However, we should recognize that given the different covariates like size or year of inception or sequence might have a role to play, there might be predictable components explaining the internal rates of return from these funds. This is addressed in the following subsections 4.2 through 4.4.

4.2 Fractile Graphical Analysis of Equity Returns

Although we do reject $H_0 : F = G$ that the return distributions for private and public equity are the same with the BGX Smooth test but there is no indication of the nature of departure from H_0 using the traditional tests like Kolmogorov-Smirnov or Cramér-von Mises type tests (see Table 1B). We use a modified version of *Fractile Graphical Analysis* method (Mahalanobis, 1960, also see Bera and Ghosh, 2006 for an overview) to test the overall distribution of returns conditional on the size of the fund for private and public equity. We include size as a possible covariate as several studies found an impact of fund size on return distribution but not the sequence number (Gompers and Lerner, 1999; Kaplan and Schoar, 2005; Phalipou and Zollo, 2005). Figures 3(i),3(ii) and 3(iii) represent the *fractile graphs* with number of fractile groups $g = 10, 20$ and 50 and depicts the difference between private and public equity mutual funds. In the figures, the blue (top) solid line represents the private equity funds returns for each size fractile group. The shaded area around the line represents the estimation uncertainty or dispersion, i.e., the bootstrapped standard error at each fractile group mean. As we observe with higher number of fractile (or rank) groups of sizes, the separation area between the two graphs is more fragmented. This also make it increasingly difficult to conclude whether the distributions are different overall. Hence we would need some more tangible analytical or simulation based hypothesis testing methodology to test for separation of the two fractile graphs.

Figure 3: Fractile Graphs (Mahalanobis, 1960) for different number fractile groups (g) for public and private equity $(i)-(iii)$ and between Venture capital and Buyout Fund returns $(iv)-(vi)$.

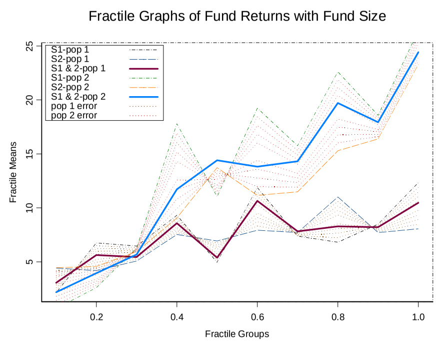


Figure 3 (i): $g = 10$ Size Fractiles

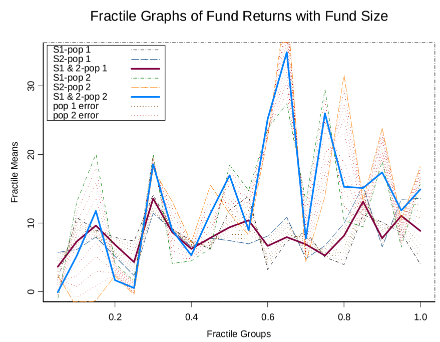


Figure 3 (ii): $g = 20$ Size Fractiles

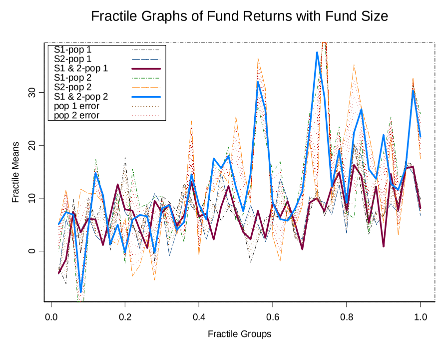


Figure 3 (iii): $g = 50$ Size Fractiles

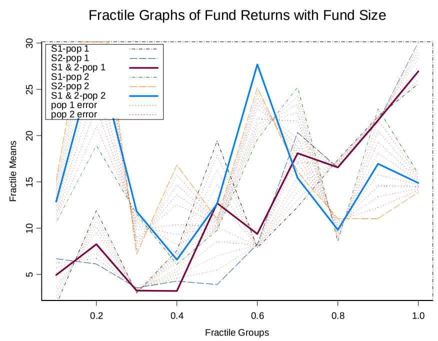


Figure 3 (iv): $g = 10$ Size Fractiles (VC-BO).

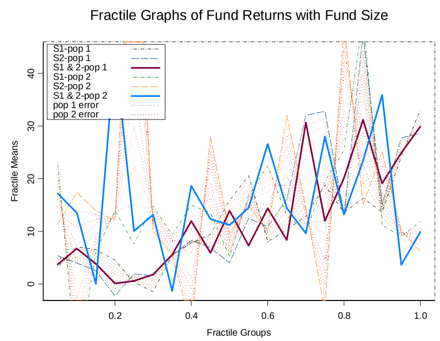


Figure 3 (v): $g = 20$ Size Fractiles (VC-BO)

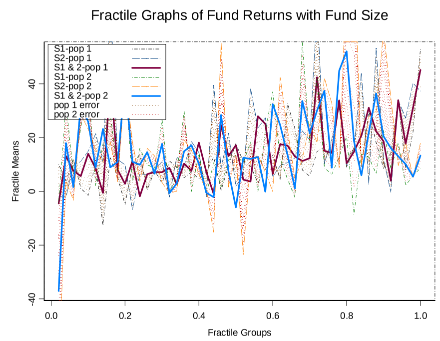


Figure 3 (vi): $g = 50$ size fractiles (VC-BO)

Following the notation of Bera and Ghosh (2006), we divide the data into m groups of size g each i.e. $n = mg$. The group means of the variables ranked with respect to X are

$$u_i = \frac{1}{m} \sum_{r=(i-1)m+1}^{im} x_{(r)}, i = 1, 2, ..., g \tag{3}$$

$$v_i = \frac{1}{m} \sum_{r=(i-1)m+1}^{im} y_{[r]}, i = 1, 2, ..., g. \tag{4}$$

Samples $\left(x_1^1, y_1^1\right)$, $\left(x_2^1, y_2^1\right)$, ..., $\left(x_n^1, y_n^1\right)$ and $\left(x_1^2, y_1^2\right)$, $\left(x_2^2, y_2^2\right)$, ..., $\left(x_n^2, y_n^2\right)$, are independently drawn from population P^{12}.

Let G^1,G^2 and G^{12} be the plots of the g group means $\left(v_1^1, v_2^1, ..., v_g^1\right)$, $\left(v_1^2, v_2^2, ..., v_g^2\right)$ and $\left(v_1^{12}, v_2^{12}, ..., v_g^{12}\right)$ against the group ranks $1/g$ through 1. Also define, for population P^{34}, G^3,G^4 and G^{34} be the plots of the group means $\left(v_1^3, v_2^3, ..., v_g^3\right)$, $\left(v_1^4, v_2^4, ..., v_g^4\right)$ and $\left(v_1^{34}, v_2^{34}, ..., v_g^{34}\right)$ against the covariate group ranks. Define A_{12} be the error area bounded by fractile graphs G^1 and G^2 between the rank points of the covariate x, 1 and g; A_{34} be the error area bounded by graphs G^3 and G^4 between the rank points of the covariate x, 1 and g; and A_* be the separation area bounded between the combined graphs G^{12} and G^{34}.

One way of addressing the problem of the difference between two fractile graphs G^1 and G^2 is to look at a norm in a $g-$dimensional Euclidean space. The L_2-norm can be defined as one way of addressing the problem of the difference between two fractile graphs G^1 and G^2 is to look at a norm in a $g-$dimensional Euclidean space. The L_2-norm can be defined as

$$\begin{aligned} \triangle_{12} &= \left\|G^1 - G^2\right\| \\ &= \left\|\left(v_1^1 - v_1^2, v_2^1 - v_2^2, ..., v_g^1 - v_g^2\right)\right\| \\ &= \sqrt{w_{1(12)}^2 + w_{2(12)}^2 + ... + w_{g(12)}^2} \end{aligned} \tag{5}$$

Similarly, define $\Delta_{34} = \sqrt{w_{1(34)}^2 + w_{2(34)}^2 + ... + w_{g(34)}^2}$ between G^3 and G^4, and finally, Δ_* between the combined graphs G^{12} and G^{34}.

Suppose, $B = ((b_{ij}))$ is a positive definite matrix like the covariance matrix, then a more general class of distance measure is

$$\Gamma_{12}^2 = \sum_{i=1}^{g}\sum_{j=1}^{g} w_{i(12)}w_{j(12)}b_{ij} = W_{(12)}^T B W_{(12)}. \tag{6}$$

Now, extending the result with size m fractile groups $m\Delta_{in}^2$ converges to a mixture of χ^2 variates where Δ_{in} is the error area of fractile graph $i = 1, 2$. If B is the inverse of the covariance matrix of W, $m\Gamma_{in}^2$ converges to χ^2 with g degrees of freedom. Furthermore if $m\Delta_{in}^2$,$i = 1, 2$ and $2m\Delta_{*n}^2$ are asymptotically independent,

$$\frac{2\Delta_{*n}^2}{\left(\Delta_{1n}^2 + \Delta_{2n}^2\right)} \to \text{Ratio of mixture of } \chi^2.$$

Similarly, for a suitable normalization matrix B, like the inverse of the bootstrapped variance covariance matrix,

$$\frac{2\Gamma_{*n}^2}{\left(\Gamma_{1n}^2 + \Gamma_{2n}^2\right)} \to F_{g,2g}.$$

We report the results of the individual and group F-tests in Table 3A, if we want to test all the conditional fractile means jointly. We observe from the results of the overall F-tests and tests for Error Areas of the two fractile graphs gives similar results for different values of g. Individually, after adjusting for the ranks of the fund size the adjusted Error Areas of fractile graphs of both private and public returns are distributed as χ^2 with g degrees of freedom. This signifies that that the FGA model is indeed a good fit for both public and private equity returns. The test for the Area of Separation though indicates that at 5% level of significance there is a difference between the two fractile graphs. The overall F-test for fractile graphs helps us to compare the conditional *fractile means* jointly, and infer that the at 5% level at least one of the size fractile means of returns is different between the groups. We can conclude that the public and private equity fund distributions are different using the F-test, or adjusting for the fractile groups of rank, private and public equity fund returns are different using 5% level of significance. This implies that there might be some abnormal returns at each size fractile, hence, size alone or "money chasing deals" cannot explain the difference of returns (Gompers and Lerner, 2000; Phalippou and Zollo, 2005).

However, although it adjusts for the fractiles of the covariate size, the overall F-test do not give us any indication of the directions of departures from the null hypothesis very much like the *omnibus* test (Kolmogorov-Smirnov and Cramer-von Mises tests). We further note that tests based on fractile graphs provides a non-parametric alternative to tests based on functions of the first two moments (or Sharpe Ratio). Unlike the tests based on moments we also adjust for the conditional fractile groups of the fund size (Jobson and Korkie, 1981, Memmel, 2003, Ledoit and Wolf, 2008).

We also compare the actual size of the tests of hypothesis using bootstrap covariance matrices to normalize the test statistic. We have simulated the test statistic by drawing the same first and second samples of X and Y variable and repeated it $r = 1000$ times, the bootstrap replication was $B = 10000$ to estimate the covariance matrix. We observe that the test size of all the tests are pretty close to the 5% nominal level test (minimum being 0.042 to maximum of 0.064), though there is some finite sample size distortions.

Table 3: Hypothesis tests based the Fractile Graphical Analysis using bootstrapped Standard Errors for Private and Public Equity and between Venture Capital and Buyout Funds with p-values in parenthesis for the asymptotic tests (3A). We also look at the actual size of nominal 5% level test to see possible size distortion with the distribution under H_0 on the column header.

#Fractile Groups Under $H_0 : m_1(x) = m_2(x)$	Private Equity $m\Gamma_{12}^2 \sim \chi_g^2$	Public Equity $m\Gamma_{34}^2 \sim \chi_g^2$	Area of Separation $2m\Gamma_*^2 \sim \chi_{2g}^2$	Overall F-test $\frac{2\Gamma_*^2}{(\Gamma_{12}^2+\Gamma_{34}^2)} \sim F_{2g,g}$	VC-Buyout F-test
$g = 10$	8.11	10.11	95.72***	10.51***	0.774
	(0.62)	(0.43)	(0.0)	(0.0)	(0.65)
$g = 20$	19.51	23.97	104.35***	4.8***	1.53
	(0.49)	(0.24)	(0.0)	(0.0)	(0.13)
$g = 50$	56.81	49.4	146.24***	2.75***	1.84***
	(0.24)	(0.5)	(0.0)	(0.0)	(0.00)

Table 3A. Asymptotic tests of normalized Error Area and the Area of Separation (Pvalues in parenthesis)

Size of the tests Under $H_0 : m_1(x) = m_2(x)$	Private Equity $m\Gamma_{12}^2 \sim \chi_g^2$	Public Equity $m\Gamma_{34}^2 \sim \chi_g^2$	Area of Separation $2m\Gamma_*^2 \sim \chi_{2g}^2$	Overall F-test $\frac{2\Gamma_*^2}{(\Gamma_{12}^2+\Gamma_{34}^2)} \sim F_{2g,g}$
$g = 10$	0.048	0.042	0.054	0.064
$g = 20$	0.054	0.046	0.06	0.04
$g = 50$	0.046	0.05	0.054	0.054

Table 3B: Actual Size of the 5% FGA tests using Boostrap Covariance matrix $(repl. = 500, Boots. = 5000)$

A problem with *omnibus test* methods like Kolmogorov-Smirnov and Cramér-von Mises type tests that have power in all directions is that they have weak power against more directional alternative. Hence, we might fail to reject a hypothesis that is indeed false. In our case here we do reject the null hypothesis of equality of the distributions of private and public equity returns. So we can believe beyond reasonable doubt the two distributions are indeed different overall. However, the same thing cannot be said about all parts of the distributions measured by subsets of fractile means or graphs (see Figures $3(i)$ $-3(iii)$). It appears that for $g = 10$ and $g = 20$ fractile groups, there is a difference between private and public equity returns after the 40^{th} percentile of net asset for public equity funds or total commitment size of private equity funds (or top 60% of fund sizes). It is however more difficult to separate out for the bottom 40% of the funds, or when $g = 50$ due to the wide variation of the fractile means.

4.3 Fixed Effects Adjusted Equity Returns

In previous subsections 4.2 we noted that there could be other covariates and possible fixed effects due to year and/or specific firms (or funds) that might have effects on the internal rates of return of private equity firms or returns of public equity funds. We first look at the effects of different covariates like the sequence number of the private equity firm or the year of existence of the public equity fund. We allow for linear dependence in both models shown in Tables 4-6. For the private equity funds we have used the model similar in Kaplan and Schoar (2005 p. 1803) but used the internal rate of return reported by Venture Economics,

$$IRR_{it} = \alpha_t + \beta(FundSize_{it}) + \lambda(Sequence_{it}) + \gamma VC + \varepsilon_{it}, \tag{7}$$

where for the i^{th} individual firm or partnership and the t^{th} period of time IRR_{it} is reported by Venture Economics, $FundSize_{it}$ is the logarithm of the capital committed to the fund, $Sequence_{it}$ is the logarithm of the sequence number of the fund (later funds of the same private equity partnership or firm), and VC is a dummy equal to 1 if the partnership is a venture capital firm and 0 otherwise. We have included non-linear terms of fund size and sequence number to account for the size of the fund to have some non-linear relationship with the rate of return reported in Kaplan and Schoar (2005). This in essence is similar to the findings of Ippolito (1992), Chevalier and Ellison (1997), Sirri and Tufano (1998) for public mutual fund returns, and Berk and Green (2004). We have further used fixed effects for both year and firm to filter out predictable information related to the investment environment in the year of inception and reputation of the firm. Lack of enough data points due to a short time dimension t could potentially be a problem in models with several independent variables. The results of the regressions are reported in the Table 4 in columns (1) – (8). In particular, from (1) we observe that allowing for a fixed year-specific effects and no non-linear (square) terms , both size ($\hat{\beta} = 3.38$) as well as venture capital status ($\hat{\gamma} = 6.79$) have significant positive effect on the internal rate of return, using 1% level with a sample of size $m = 840$ private equity returns with inception prior to 1996. If however, we include both year and firm specific effects in column (4), there is a reversal both fund size and venture capital status becomes statistically insignificant and economically negative. On the other hand, the sequence of number of the fund is now statistically significant at 5% level and negative, implying that after accounting for year and firm specific effects the returns to private equity reduces for follow-up funds. This has been mentioned in the literature, that one of the reasons of this could be the lack of performance of follow-up funds that starts in boom times besides follow-up funds might have a watering down of returns (Kaplan and Schoar, 2005). Given the argument that the returns self-reported by GP and LP might be misleading before the firm is completely liquidated and most cash-flows settled, we look at the funds that were liquidated after starting before 1996 (Kaplan, Sensoy, Stromberg, 2002; Ljungqvist and Richardson, 2003). With a smaller sample size of $m = 463$, it was evident in column (2) that accommodating for yearly fixed effect, size, sequence and whether the firm is a Venture Capital fund do not play any role in the internal rate of return. In column (5) ,the results are similar even if we include both year and fund specific fixed effects.

RESET test (not reported) shows that there is a possible non-linearity that has not been accounted for in the simple model, hence we include square terms for both size and sequence related variables in model (7). Using year specific fixed effects in column (3), venture capital fund status seem to be important, as is the size of the fund that positively affects the dependent variable, but is however concave in the second order term for size ($\hat{\beta}_{lin} = 11.6$ and $\hat{\beta}_{sq} = -0.96$) are both statistically significant at 5% level. If on the other hand, we compared to the results in the previous part with no non-linear terms, allowing for fixed yearly and fund specific, size and venture capital fund status are not significant. The linear expression for the sequence number seems to play an important negative role at 5% level ($\lambda_1 = -22.53$). From columns (7) – (8), for this full model with both linear and non-linear terms, internal rates of return of liquidated private equity firms do not show any dependence on size, sequence number or venture capital status of the funds unlike the findings in Kaplan and Schoar (2005) and Gompers and Lerner (2000).

We have also explored the Venture Capital only and Buyout only funds, and ran similar regression reported in Table 5. If we focus only on Venture Capital Funds (with $m = 610$ that outnumber Buyout funds which is $m = 206$) that had an inception prior to 1996 in column (1), we find that accounting for fixed effects for year and firm specific components, sequence play an important and negative role marginally statistically significant at 10% level. If on the other hand we only restrict ourselves to liquidated funds that started prior to 1996 with a reduced sample size of $m = 364$ in column (2), the effects are not statistically significant. The full model with non-linear terms for logarithms of size and sequences in column (3), the results remain qualitatively similar for VC funds starting before 1996, as does the insignificant results for liquidated funds. The LBO funds show an economically significant negative effects of size and sequence numbers in column (4) and (7), however neither of them are statistically significant after adjusting for fixed effect of years and firms. Liquidated LBO funds regression output reported in column (6) and (8), after adjusting for yearly fixed effects, do not show any statistically significant influence of size and sequence number.

We further investigate the persistence results that are reported for private equity fund returns based on internal rates of return of funds that are launched before or in earlier rounds to see returns are persistent as reported by Kaplan and Schoar (2005). The model

$$Performance_{it} = \alpha_t + \delta(Performance_{it-1}) + \gamma VC + \varepsilon_{it} \tag{8}$$

based on $IRRs$ reported by Venture Economics as a measure of performance for the i^{th} firm in the t^{th} period of time. The regression results are given in Table 4 in columns $(9) - (10)$. We observe that venture capital fund or not plays an important statistically significant positive role in the returns with both one and two lagged dependent variable using yearly fixed effects. From column (9), past sequence returns does have a positive impact on future returns$\left(\hat{\delta}_1 = 0.59\right)$, while past two sequence returns also seem to have a significant positive impact $\left(\hat{\delta}_1 = 0.78, \hat{\delta}_2 = 0.1\right)$. Similar results for Venture capital only private equity funds are reported in Table 5 columns $(9) - (10)$, we observe that past one periods return has a significant positive impact $\left(\hat{\delta}_1 = 0.85\right)$, and both of past two sequence returns show a positive impact $\left(\hat{\delta}_1 = 1.03, \hat{\delta}_1 = 0.1\right)$. All results accommodated for clustering with the funds for public equity (or firms for private equity).

We would like to compare similar models with public traded mutual funds reported by Morningstar Principia Mutual Fund Database. Given that Principia database is updated monthly and would potentially suffer from survivorship bias we use several years January releases (1997-2003) of previous year's data to mitigate the survivorship (see Bergstresser and Poterba, 2002; Carhart, Carpenter, Lynch, and Musto, 2002). It also enables us to use the panel data structure to take care of yearly fixed effects and in some cases fund specific fixed effects as well. We further incorporate the size effects and the effects of the total capitalization of the funds in terms of its net assets. Finally, to accommodate for comparing with the data available for private equity return we only look at open ended mutual funds that do not give out any dividend, this makes the internal rate of return (IRR_{it}) and the rate of return ($return_{it}$) of the fund the same. The overall regression model

$$return_{it} = \alpha_t + \beta(FundSize_{it}) + \lambda(Sequence_{it}) + \varepsilon_{it}, \tag{9}$$

where for the i^{th} individual fund and the t^{th} period of time $return_{it}$ is reported by Morningstar, $FundSize_{it}$ is the logarithm of the net assets and $Sequence_{it}$ is the logarithm of the year number of the fund (number of years of the same fund). As mentioned before we also incorporate non-linear terms for variables to account for non-linear effects of fund size and vintage of funds. In particular we are interested to incorporate effects of size reported in several works like Fama and French (1992, 1997, 1999), and effects of size and vintage that are implied in Berk and Green (2004, p. 1271):

> "...investments with active managers do not outperform passive benchmarks because investors competitively supply funds to managers and there are decreasing returns for managers in deploying their superior ability. Managers increase the size of their funds, and their own compensation, to the point at which expected returns to investors are competitive going forward."

From model in Table 6 column (1) with all publicly traded open-ended US mutual funds ($n = 5635$), we see that size has a positive impact on the returns $\left(\hat{\beta} = 0.38\right)$ but a negative impact on sequence $\left(\hat{\lambda} = -2.26\right)$ incorporating yearly fixed effects. In column (2) , we include square terms of log of size and see there is a significant negative impact $\left(\hat{\beta}_1 = 1.29, \hat{\beta}_2 = -0.1\right)$ meaning there is a convex relationship between returns and size, but not so much for returns and sequence for public equity $\left(\hat{\lambda}_1 = -3.59\right)$. However, if we accommodate for both year and fund specific fixed effects as in column (4), sequence is no longer statistically significant. On the other hand size now has a significant negative impact on returns $\left(\hat{\beta}_1 = -3.11\right)$. The results are not statistically significant if non-linear terms are included possibly due to multicollinearity.

To further illustrate the implications of the point we also use an alternate model to verify or accommodate for persistence using past returns

$$return_{it} = \alpha_t + \delta_1(return_{it-1}) + \delta_2(return_{it-2}) + \varepsilon_{it}. \tag{10}$$

As these are yearly returns and we accommodate for survivorship bias by including several years of return, we do observe in column (8) some persistence of returns with only yearly fixed effects $\left(\hat{\delta}_1 = 0.08, \hat{\delta}_2 = -0.28, n = 1930\right)$. However, this becomes both economically and statistically significant, and negative with both year and fund specific fixed effects ($\hat{\delta}_1 = -0.28, \hat{\delta}_2 = -0.6, n = 1930$ see Table 6 columns $(7) - (10)$). The best argument we can make is the one alluded to in the literature about the performance of managers is related to the amount of inflow and not necessarily the long term persistent return of the fund which becomes competitive with new fund inflow (Berk and Green, 2004). No such effect can be seen the private equity fund returns in Table 4 and Table 5.

The main focus of this paper is however different from the previous works on the determinants of persistence of returns or the effect of fund size or inflow on returns from private and public equity. We focus on the relative distributional structure, and hence risk-return structure of private and public equity markets in two panels. Kaplan and Schoar (2005, p. 1797) used proprietary data to calculate the *Public Market Equivalent* (PME) that:

> "...compares an investment in a private equity fund to an investment in the S&P 500. We implement the PME calculation by investing (or discounting) all cash outflows of the fund at the total return to the S&P 500 and comparing the resulting value to the value of the cash inflows (all net of fees) to the fund invested (discounted) using the total return to the S&P 500. A fund with a PME greater than 1 outperformed the S&P 500 (net of all fees).We (not VE) perform the PME calculations using fund cash flows."

We use the internal rates of return published by Venture Economics to evaluate the true choice made by an investor between publicly traded mutual funds and placements in funds holding private equity. Hence, we look at the distributional comparison using pre-tax public mutual fund returns rather than benchmarks like S&P 500 index that cannot be traded by individual investors. We have already applied a Neyman smooth type test for the two sample context introduced by Bera, Ghosh and Xiao (2007) in the unadjusted case. We would continue that on the residuals or abnormal returns under certain assumptions. First, the private and public equity returns independent. We have used the public equity returns from 1996 to 2003, while the inception of the private equity funds are set before 1996. Furthermore, we have used the year and fund specific fixed effects from both private and public equity returns to adjust for the market conditions in those years and for those funds. We have also seen that the number of publicly traded funds in the database are much larger than the number of privately traded funds that satisfies the condition given in Section 3 (also see Bera, Ghosh and Xiao, 2007 and simulation study in the Appendix and Figure 9). We also performed a Hausman-type test (not reported here) to verify that the time series and the cross-sectional terms can be pooled separately in both public and private equity funds.

The probability density function (PDF) estimates and the empirical distribution functions (EDF) of the residuals are plotted in Figures 5-8. It is evident from the plots of the PDFs of the residuals of private and public equity regression models (7) and (9), respectively, in Figure 5A using a yearly fixed effect that models have relatively few intersections in the middle, and would suit the Smooth test framework. If however we account for fixed effect for both year and fund specific effects as in Figure 5B, then even that difference is partially accounted for. Graphically, the EDF plots in Figure 6A or 6B also do not show distinct first order stochastic dominance i.e., either private or public equity consistently outperforming the other for all returns. We further test whether components of private equity funds like Venture capital funds (VC) or Buyout funds (LBO) show some consistent behavior with respect to the publicly traded mutual funds. We find that adjusting for both year and fund specific fixed effects the regression residuals are plotted in Figure 7A, and observe that the VC returns are similar in level to publicly traded funds but there are some differences possibly in higher order moments. Figure 8A also confirms the similarity of the EDFs, and no clear first order stochastic dominance patterns. The LBO funds residuals PDF do show some deviation from the PDF of the public equity residuals in Figure 7B, as does the EDF plot in Figure 8B. However, we should bear in mind that the sample size for the LBO funds are substantially smaller than the VC funds in the Venture Economics database. If we incorporate the lagged dependent variables then distributions of residuals of private and public equity models in (8) and (10), respectively, are indeed different as seen in Figure 5C, but there is no pattern in the EDFs in Figure 6C. The difference between the PDFs seem more pronounced for the Venture Capital funds (Figures 7C and 8C). One obvious reason for this is the introduction for lagged dependent terms in the model might make the model misspecified, and possibly autocorrelated. Having done the graphical analysis it is clear that while there are some differences in the distribution of residuals or abnormal returns, there could be differences in higher order moments that was not accounted for in the regression framework with essentially normally distributed residuals.

The results of the smooth test are reported in Tables 7 and 8. If we compare the models for the residuals of a public equity mutual fund with no yield ($n = 5635$) using a simple model with no non-linear terms (9) and private equity fund with a simple model with similarly no non-linear term for size and sequence number in (7), accounting for yearly fixed effects, the distributions are statistically significantly different overall (with $\hat{\Psi}_6^2 = 50.86$, $m = 840$, see Table 7). The test also shows departures in the first, fourth and sixth order terms ($\hat{u}_1^2 = 13.18$, $\hat{u}_4^2 = 13.01$ and $\hat{u}_6^2 = 20.75$). The third order term $\hat{u}_3^2 = 3.53$ is marginally insignificant at 5% level. However, if we chose the recommended sample size of $m = 518$ in the Appendix A so as to minimize finite sample size distortion of the test, it was significant overall ($\hat{\Psi}_6^2 = 37.23$) and also in the first and second order $\left(\hat{u}_1^2 = 4.97 \text{ and } \hat{u}_2^2 = 4.32\right)$ with p-values of 3% and 4%, respectively. To incorporate a second order term for findings of Kaplan and Schoar (2005), we indeed find the results are qualitatively similar with an overall $\hat{\Psi}_6^2 = 38.93$, with $\hat{u}_1^2 = 12.13$, $\hat{u}_4^2 = 12.91$ and $\hat{u}_6^2 = 12.43$ indicating deviations in the first, fourth and sixth orthogonal moment directions. Selecting a sample of size $n = 518$ also preserved the original results are 5% significance level. If we select only the liquidated private equity funds ($n = 463$) that had an inception year before 1996 (see Table 7 line 5), there is a statistically stronger difference between residuals for the public and private equity returns ($\hat{\Psi}_6^2 = 55.78$) with departures in the directions of the second, fourth, fifth and sixth moments ($\hat{u}_2^2 = 5.6, \hat{u}_4^2 = 29.97$, $\hat{u}_5^2 = 6.11$ and $\hat{u}_6^2 = 8.49$) allowing for 5% probability of Type I Error. We have used the bigger data set of all private equity funds that had an inception year before 1996 as qualitatively the results were similar with the liquidated funds although the data was almost twice as large. We also reckoned that funds that started before 1996 would begin to show some tangible returns by their sixth year of existence.

Now however to account for the variability of the funds and incorporate differences across different fund types (for similar partnerships or funds) we can include both a year and fund/firm specific fixed effects. The results change dramatically. For the simple model there is still statistically significant difference between the private and public residuals ($\hat{\Psi}_6^2 = 14.18$) at 5% level but not at 1%. The almost the entire difference is accounted for by the fourth order term ($\hat{u}_4^2 = 8.64$) or departure in the direction of the term related to excess kurtosis. The result is replicated if we take a sample, though at a slightly higher significance level. These results confirm the findings in previous literature that there is hardly any significant difference between private and public equity returns once you account for size and sequence, and reputation of the fund. If we use the full model with non-linear terms we also find that the overall statistical significance of the model ($\hat{\Psi}_6^2 = 16.4$) is coming from the fourth order or kurtosis related term ($\hat{u}_4^2 = 8.98$). Even this difference washes away if we take a smaller sample so as to reduce distortion of the size of the test of significance.

If we turn our attention to the components of private equity individually like Venture Capital and Leveraged Buyout, the results are even more striking (see Table 8). Using the panel structure of the model with yearly and individual fund specific fixed effects, comparing the residuals of the no yield publicly traded mutual funds models and the residuals from the regression using only Venture Capital Funds that were started before 1996 there is no difference between the distributions ($\hat{\Psi}_6^2 = 6.54$, Table 8 line 1). If we only include the Venture Capital Funds that were already liquidated, the results stay the same ($\hat{\Psi}_6^2 = 11.64$, Table 8 line 2).

If on the other hand we use LBO funds, for all funds ($m = 206$) there is substantial difference between the distributions of the residuals of the public equity funds with no yield and the LBO funds ($\hat{\Psi}_6^2 = 64.97$) that was mainly owing to differences in the second, fourth and sixth moment directions ($\hat{u}_2^2 = 32.05, \hat{u}_4^2 = 13.35$ and $\hat{u}_6^2 = 18.99$). However, if we look at only the liquidated LBO funds ($m = 85$), there are no differences between the two residual distributions ($\hat{\Psi}_6^2 = 5.41$). This anomaly might be due to data reporting error before the liquidation of the fund or due to the small sample size involved.

Kaplan and Schoar (2005) also pointed out that private equity funds show evidence of persistence over different sequence numbers, unlike the public equity market where stock picking ability to beat the market in the short run is often assumed to be fortuitous. From regression results in Table 4 columns 9 and 10, we observe that indeed their is a significant positive relationship between both a single lagged return as well as two lags of pre-tax returns, all as expected ($\bar{R}^2 = 0.27$). Returns for private equity show some degree of persistence in the data though the sample size is relatively small $m = 141$. Similar type of effect is also seen in Venture Capital Funds, but lack of data prevent us from corroborating that for LBO funds. If we turn to public equity with no yield and fit a model for the pretax returns with one and two previous periods returns we can fit a model similar to (10) without the dummy variable for VC funds. As compared to the private equity case whether with all private equity funds or a specific class of funds like the Venture Capital Funds, the estimation results are quite different. While we find more mean reversion rather than persistence in the significant negative coefficients of past returns using yearly fixed effects. We have used the residuals to run the BGX test and found that there is significant differences between the residual distributions both for one and two lags in the model. For one lagged term the overall statistics is $\hat{\Psi}_6^2 = 64.07$ that is chiefly caused by departures in the second, fourth and sixth orthogonal moment directions ($\hat{u}_2^2 = 15.86, \hat{u}_4^2 = 16.9$ and $\hat{u}_6^2 = 29.25$) with a sample of size $m = 275$. However, with a much smaller sample size of $m = 141$ we established that the overall $\hat{\Psi}_6^2 = 75.94$ and principal directions of departure are $\hat{u}_2^2 = 43.42$, $\hat{u}_4^2 = 19.63$ and $\hat{u}_6^2 = 6.77$, all statistically significant at 5% level. One of the serious issues of using the lagged dependent structure is that to exactly identify the number of lags, as we might introduce impure heteroscedasticity or autocorrelation by not selecting the right person. This might be introduced as an artifact if the lag structure is misspecified (see Figures 5C-6C).

For Venture Capital Funds alone we run a model with only two lagged dependent variable and get an overall $\hat{\Psi}_6^2 = 60.03$ though with a small sample size of $m = 106$..The departures of the distributions of the residuals after adjusting for fixed effects for the year between public equity funds and venture capital funds are coming from the second, fourth and sixth moment directions ($\hat{u}_2^2 = 28.05$, $\hat{u}_4^2 = 12.52$ and $\hat{u}_6^2 = 16.61$). However, this result might be affected by the small sample size of venture capital funds that has at least two previous returns data. The PDF and the EDF plots of public equity residuals and the private equity residuals in Figures 7C and 8C, respectively, also gives us an impression that the distributions might be different. We should also mention the caveat here, as before if the regression model is misspecified then including a lagged dependent variable might introduce dependence across the two groups as an artifact.

4.4 Robust Regression of Private and Public Equity Returns

The the use of OLS type residuals have several technical and methodological issues that challenges the robustness of the results. First, it can be argued that presence of sample selection issues like survivorship bias both for public and private equity makes the inference based on unadjusted OLS residuals problematic (see Carhart et. al.2002, Cochrane, 2005). Second, model selection to identify non-linear dependence of returns on fund size and sequence might have issues with data-snooping and omitted variables bias (Lo and MacKinlay, 1989). Third, self reporting in case of cashflows for non-liquidated funds and actual allocated (not committed) fund size for private equity throws into the mix possible measurement error issues (Kaplan, Sensoy, Stromberg, 2002; Ljungqvist and Richardson, 2003; Kaplan and Schoar, 2005). Fourth, the non-linear structure of the model and unknown error distribution specification for observational data makes it necessary to look for more robust alternative to ordinary least squares. Finally, as investors and funds in private and public equity are inherently different hence not quite comparable we use fractile or covariate-rank regression methods to make the two returns comparable.

We use robust rank regression or fractile regression methods to address the above problems for effects of size of the fund in the regression models of returns is reported in Table 9 (see Bera and Ghosh, 2006, Ghosh, 2006 and references therein). If we define the conditional expectation of the return distribution (Y) given the fractile of the covariate X as $m(x) = E[Y|X = x]$, and we have the rank or the CDF $F(.)$ of X and error term ε, our model is

$$\begin{aligned} y_{it} &= r(u_{it}) + \varepsilon_{it} = E(Y|F(X) = u_{it}) + \varepsilon_{it} \\ &= E\left(Y|X = F^{-1}(u_{it})\right) + \varepsilon_{it} = m\left(F^{-1}(u_{it})\right) + \varepsilon_{it}, \end{aligned} \tag{11}$$

where $F^{-1}(u) = \inf_x \{x|P(X \leq x) = u\}$ is the quantile function. We can use the linear function $m(x) = m(x; \boldsymbol{\beta}) = \beta_0 + \beta_1 x$ or keep it in the general form. This method is closely related to Quantile Regression method (see Koenker and Bassett, 1978).

For private equity funds the internal rate of return is affected positively by the rank of the fund size committed by the Limited Partner for funds that started before 1996 (here rank is the lowest for the smallest fund, column (1) $m = 840$) without any fixed effects. So bigger the fund size the higher is the internal rate of return, *ceteris paribus.* It is also strongly influenced by the type of the fund, i.e. Venture Capital based funds have a higher return. Even when yearly 'fixed effects were accounted for in model in (2), the effects of the rank of fund size and Venture Capital status remained strongly statistically significant and positive. However, both the effects vanished when fixed effects for firms were also introduced. In economic terms as seen before the coefficient for the size of the fund became negative but was not statistically insignificant for funds with inception before 1996. If we looked at funds that were already liquidated by 2003 and started before 1996, we have a slight positive significance of the rank of fund size at 5%, and statistically insignificant type of the private equity firm in column (4). Though with less strong significance, the results for $m = 463$ liquidated firms and those that started before 1996 are in essence similar.

Venture capital fund internal rates of return by themselves also statistically depend strongly positively on the rank of fund size ($\hat{\beta}_1 = 0.0405$) without any fixed effects and adjusted $R^2 = 0.03$. With the yearly fixed effects the results are essentially similar, both with sample size $m = 610$ (columns (9) and (10)). The results are not the same for

Buyouts funds that started before 1996, although the sample size $m = 206$ is smaller. There is no statistically significant effect on the internal rate of return if the yearly fixed effects were not considered. There was a slight significance at 10% level when yearly fixed effects are considered.

Public equity mutual fund returns when regressed on the rank of the size of the fund is similar, as there is a strong positive coefficient for larger funds without any fixed effects (Column (5)) with a sample size of $n = 5635$ and adjusted $R^2 = 0.004$. This implies that no matter whether yearly fixed effects are included in the model the rank of fund size does have an important role to play in determining the yearly returns (column (7), $\bar{R}^2 = 0.5$). If however, we included both year and fund specific fixed effects, the return is statistically significantly but negatively related to rank of fund size $\left(\hat{\beta}_1 = -0.0037\right)$ with 46% of the variation of returns explained by the variation of the rank of fund size and other fixed effect variables.

Our main emphasis in this paper has been to identify the moment directions of departure between the two distributions of private and public equity returns. In particular, we are interested in investigating how private equity returns are different from their public equity counterparts in a cross sectional sense (Table 10). We find with no adjustment for fixed effects overall smooth test indicates that the private and public equity returns after adjusting for the fractiles or ranks of the size are widely different for private and public equity $\left(\hat{\Psi}_6^2 = 567.35,\ \text{row } 1\right)$. The main directions of departure seems to be all moments except the first one $\left(\hat{u}_1^2 = 3.16,\ pvalue = 0.08,\ m = 840\right)$. The overall effect, however, reduces although still statistically significant once we take into account the fixed effect due to years $\left(\hat{\Psi}_6^2 = 47.99,\ m = 840,\ \text{row } 2\right)$. The main directions of departure of the residuals of the private equity model from the public equity after incorporating yearly fixed effects are towards the first, fourth and sixth moment directions ($\hat{u}_1^2 = 12.84$, $\hat{u}_4^2 = 18.3$ and $\hat{u}_6^2 = 13.79$). Further, if we include both fixed effects for years and firms (or funds), in the overall models the residuals from private and public equity are marginally significantly different at 5% level $\left(\hat{\Psi}_6^2 = 14.30,\ m = 840,\ \text{row } 3\right)$, that is almost entirely due to the fourth order term $\hat{u}_4^2 = 8.38$. The results are very similar if we only look at the Venture Capital funds by themselves. After adjusting for fixed effects for years alone (due to the lack of enough data points), the Venture Capital funds residuals are indeed different overall from public equity returns residuals ($\hat{\Psi}_6^2 = 53.67$, $m = 610$, row 4). The residuals are different in the directions of the first, fourth and sixth moment directions ($\hat{u}_1^2 = 5.33$, $\hat{u}_4^2 = 25.49$ and $\hat{u}_6^2 = 19.01$) using a 5% significance level. As opposed to that, if we look at Buyout funds alone at 5% level of significance there is no difference between the residuals of buyout funds and the public equity funds $\left(\hat{\Psi}_6^2 = 11.28,\, m = 206,\ \text{row } 5\right)$. The small sample size of the buyout funds in the sample period we looked at might have been one of the reasons for such a statistically insignificant difference.

5 Comparing Venture Capital and Buyout Funds

While it is evident that private and public equity returns might be different, LPs (or investors) are often faced with a choice between different types of private equity. We focus our attention on Venture Capital and Buyout Funds. Our sample of funds from Thomson Reuters SDC Platinum database with inception before 1996 has 610 Venture Capital Funds and 206 Buyout Funds. Figure 2C and 3C suggests that the distributions of internal rates of return (that is now more comparable across different private equity funds rather than between private and public equity funds) for the two unadjusted return distributions are similar with few intersections. Hence we apply the BGX smooth test to both the full sample with inception before 1996 and a recommended sample size selected from the full sample of Buyout funds that has a smaller sample size (see Appendix A). We observe that from Table 1, the mean returns of both VC and Buyout funds are numerically similar (15.27) but the medians are distinctly different (18.7 and 12.17 respectively), there is a substantial difference in dispersion between the absolute terms (standard deviation of 41.59 and 22.32, respectively) and relative terms (Sharpe Ratio of 0.38 and 0.68, respectively). These suggests that there might be some differences in the higher order moments of the return distributions. Furthermore, from Table 1B, traditional tests like the Kolmogorov-Smirnov and Cramer-von Mises type tests also show that the two distributions are statistically marginally different at 0.1% level.

We run a smooth test on the unconditional return distribution of Venture Capital Funds and Buyout Funds and find that in the full sample the two are different statistically at 5% level $\left(\hat{\Psi}_6^2 = 25.2\right)$. The main sources of departure are in the direction of the first, third and fifth order terms ($\hat{u}_1^2 = 11.86$, $\hat{u}_3^2 = 5.66$ and $\hat{u}_6^2 = 5.1$) that are all significant at 5% level. If under the recommendation of the BGX test to minimize size distortion of the asymptotic test we select a sample of size $m = 137$ ($m = 22.5\%$ of $n = 610$) of Buyout Funds, we get essentially a similar result with the directions of departure reducing to first and fifth moment directions $\left(\hat{u}_1^2 = 8.26,\ \hat{u}_5^2 = 6.49\right)$. We can infer from our results that the unadjusted returns from Venture Capital and Buyout funds are indeed different, and the main source of departure is coming from direction of location (and marginally, in the fifth order related terms).

Public and private equity are inherently different asset classes, hence the investors in the two groups might also be different with varying appetite for and tolerance of risk. So it is often worthwhile to look at different types of private equity funds like venture capital and buyout type funds to see how they compare with each other. While this was already addressed in subsection 4.1, we would like to extend the discussion when we take into account the size of the total commitment by the Limited Partner or investor. However, since we have seen that on an average Buyouts are larger than Venture Capital funds, we want to account for the size effect nonparametrically (or account for size without imposing any linear parametric structure). We observe from Fractile Graphs Figure 3 $(iv) - (vi)$ that for $g = 10, 20$ and 50 there doesn't seem to be clear evidence graphically to see that the two are different based on the fractiles of fund size. We performed the overall F-test of comparing g fractile means obtained using FGA techniques and the results are given in column 6 in Table FGA. We see that after accounting for size the two fractile graphs are indistinguishable for $g = 10$ and $g = 20$. Hence we can conclude that the returns of Venture Capital and Buyout funds are similar across all fractile groups. Hence, inherently investors of private equity are similar in their risk profile for average return conditional on the fractiles of the fund size. However, if we look at $g = 50$, we find that the VC and Buyout funds are distinctly different at 5% level. The main caveat of the conclusion is that the sample size of the buyout funds is not big enough to allow for 50 fractile groups, so the results in case with $g = 50$ might be misleading.

As we are looking at the internal rates of return of private equity firms that are partly based on self reported cash-flow data, it is probably more comparable with similarly reported internal rates of return rather than verifiable public equity returns. So to give a fair assessment within a similar asset class, we also compare returns to VC and Buyout funds using the residuals obtained from OLS with models similar to Kaplan and Schoar (2005). The results are reported in Table 8 rows 6-10. We first compare residuals from the estimation of model (7 without the dummy variable for VC) Venture Capital and Buyout funds separately. The residuals in the simple model with no non-linear terms in log of size or sequence, we see that the two residuals are marginally statistically different at 5% level ($\hat{\Psi}_6^2 = 15.51$, p-value=0.02). The main sources of departure are towards the first and fifth moment directions ($\hat{u}_1^2 = 7.56$ and $\hat{u}_5^2 = 7.17$). We observe that with fixed effects for years, the residuals from the returns regression with Venture Capital funds and those with Buyout funds using model (7) are statistically indistinguishable using the smooth test ($\hat{\Psi}_6^2 = 10.24$, $pvalue = 0.11$). Using the Kolmogorov-Smirnov (KS=0.08,p-value=0.16) the same result holds at 5% level of significance. Further, if we use the model with quadratic terms of log of size and sequence, there is no difference between the results with the simple model. However, if both fixed effects for year and firms are used the two residual distributions are different ($\hat{\Psi}_6^2 = 76.27$) overall owing to the second, fourth and sixth moment directions ($\hat{u}_2^2 = 41.88, \hat{u}_4^2 = 14.26$ and $\hat{u}_6^2 = 18.74$). Given the small sample size of buyout funds, fixed effects for both year and firms might reduce the degrees of freedom substantially and could lead to misleading results.

The size of the fund seems to play a crucial role in the internal rate of return of the fund. If we estimate a simple regression model with the fund size as the only explanatory variable, we see that the residuals from the Venture Capital Funds and Buyout Funds are very similar and marginally significant at 5% level ($\hat{\Psi}_6^2 = 14.18$, see row 6 in Table 8) with only significant departures in the direction of the first moment directions ($\hat{u}_1^2 = 7.97$). If we look at a more robust alternative regression using the rank of size as a covariate in Table 10, with no fixed effects we see that the two residual distributions from regression on the rank of the size of the fund are marginally significant using 5% level ($\hat{\Psi}_6^2 = 12.97$, see Table 10 row 6). The main departure in the direction of location $\left(\hat{u}_1^2 = 7.67\right)$. Using fixed effects for years once again we find that the distributions of residuals for Venture Capital and Buyout fund residuals are statistically not distinguishable at 5% level ($\hat{\Psi}_6^2 = 8.65$, p-value$= 0.19$). This implies that essentially the Buyout and Venture Capital funds residuals mainly differ in the first moment direction if size is not taken into account. The results imply that the size does play an important role in determining the returns on venture capital and buyout funds unlike what has been suggested in the literature (Metrick and Yasuda, 2007). In fact, after adjusting for the effects of fund size in a robust procedure, Buyout and Venture Capital fund return distributions are statistically similar using the smooth test.

6 Summary and Directions

Our findings help us explain how the return distributions are indeed different between private and public equity funds. However, the deviations between the two distributions reduces substantially when we take into account the size of the fund, and to a lesser extent the sequence. One of the major findings of this paper is that once year specific fixed effects are incorporated private and public equity returns are quite similar. This bolsters the argument in the literature that private equity returns are pro-cyclical with the business cycle and are positively correlated with public equity returns (Phalippou and Zollo, 2005). We also did find some non-linearity in fund size and return relationship, however, the sign of second order term was not as stable or statistically significant when fixed effect for year was included.

There is also strong evidence that private equity return is more persistent up to two previous sequences. However, the surprising result was for public equity as well there is a positive association with last year's returns for surviving funds that might be related to momentum based strategies. However, there is a strong negative relationship with second lagged term of returns. This could be due to the effect of inflow into the fund that waters down the return margin (Berk and Green, 2004).

We found evidence that Venture Capital Funds and Buyout (LBO) funds are inherently similar investment vehicles with a main source of departure in the first order or location term without any fixed effects. However, this also vanished when fixed effect for year was included. When we applied fractile (or rank) regression with and without fixed effects the same results were replicated. Hence the relationship between VC and LBO funds about their risk characteristic is pretty robust. The overall F-test with bootstrapped standard errors for the non-parametric FGA test also confirms that Venture Capital funds and LBO funds have similar risk exposure after adjusting for size.

Tests based on Fractile Graphs and bootstrapped standard errors provides an exciting non-parametric version of a distributional comparison test after adjusting for ranks of a conditioning variable like fund size. However, the power and size properties of such test hasn't been extensively tested for dependent or panel data, so this is a possible direction of future research.

With this evidence in the data we can suggest what motivates an entrepreneur or a general partner in a private equity firm or a venture capitalist or owner of private equity to hold assets with higher risk is not just an increased probability of higher returns but an affinity to some function of higher order moments in the return distribution. From the perspective of a Limited Parner, such a measure reflects the true risk-return tradeoff that often determined by the peculiarities of the different types of private equity investment instruments and institutions and their risk appetite (Lerner, Schoar and Wongsunwai, 2007).

Table 4: OLS estimation results for regression of private equity fund internal rates of returns under different restrictions. We look at data on mature funds with inception before 1996 (PE<96), data on private equity firms that are already liquidated (Liq<96), Size is the fund size and Seq is the sequence or round of the fund, VC is a dummy variable that takes 1 for a Venture Capital Fund and 0 otherwise, R_t is the IRR at time t,Fixed Effects for year (yr.) or funds (fnds).

	Private Equity Returns Static Regression Models								Lag Dep.Models	
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
	(PE<96)	(Liq<96)	(PE<96)	(PE<96)	(Liq<96)	(PE<96)	(Liq<96)	(Liq<96)	(PE<96)	(PE<96)
log(Size)	3.38***	2.3	11.6***	-1.55	-1.67	5.13	9.02*	-6.17		
	(0.98)	(1.5)	(3.15)	(2.88)	(2.96)	(9.15)	(5.17)	(19.66)		
log(Size)^2			-0.96***			-0.82	-0.82	0.56		
			(0.36)			(1.23)	(.65)	(2.47)		
log(Seq)	2.03	0.37	-1.95	-18.92**	-27.3	-22.53**	-4.63	-30.46		
	(1.33)	(2.12)	(4.34)	(9.62)	(19.38)	(9.77)	(6.2)	(19.58)		
log(Seq)^2			1.94			2.60	2.477	2.21		
			(1.89)			(3.23)	(2.67)	(4.69)		
VC	6.79**	0.92	5.73**	-0.17	-12.57	0.4	-0.22	-11.26	11.27***	19.96***
	(2.85)	(4.43)	(2.91)	(8.05)	(17.2)	(8.18)	(4.63)	(17.62)	(3.8)	(5.81)
R_{t-1}									0.59***	0.78***
									(0.18)	(0.25)
R_{t-2}										0.1***
										(.02)
F.E.	Yr	Yr	Yr	Yr, Frms	Yr, Frms	Yr, Frms	Yr	Yr, Frms	Yr	Yr
Adj. R^2	0.04	0.02	0.04	0.01	-0.17	0.01	0.02	-0.18	0.23	0.27
Obs.	840	463	840	840	463	840	463	463	275	141

Table 4: Estimation Results for all Private Equity Funds

Table 5: OLS estimation results for regression of different private equity fund internal rates of return under different restrictions. We look at data on mature funds with inception before 1996 (<96), data on private equity firms that are already liquidated (Liq), Type of fund (VC or BO), Size is the fund size and Seq is the sequence or round of the fund, R_t is the IRR at time t, F.E. Fixed Effects for year (yr.) or funds (fnds).

	Venture Capital (VC)				Buyout (LBO)				Lagged Models	
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
	(<96)	(Liq)	(<96)	(Liq)	(<96)	(liq)	(<96)	(Liq)	(VC)	(VC)
log(Size)	1.00	2.17	-14.33	6.52	-5.18	4.83	-9.14	9.85		
	(3.08)	(1.84)	(16.22)	(5.85)	(5.15)	(4.83)	(16.12)	(9.85)		
log(Size)^2			2.15	-0.59			0.38	-0.58		
			(2.44)	(.82)			(1.61)	(1.99)		
log(Seq)	-21.52*	-1.18	-23.71*	-5.75	-16.76	-1.28	-16.42	-11.93		
	(13.2)	(2.57)	(13.24)	(7.82)	(13.79)	(4.51)	(18.24)	(14.21)		
log(Seq)^2			2.71	2.2			-0.23	6.3		
			(3.38)	(3.18)			(6.81)	(7.34)		
R_{t-1}									0.85***	1.03***
									(0.28)	(0.36)
R_{t-2}										0.10***
										(.02)
F.E.	Yr,Frms	Yr	Yr,Frms	Yr	Yr,Frms	Yr	Yr,Frms	Yr	Yr	Yr
Adj. R^2	0.03	0.04	0.03	0.03	0.3	0.05	0.28	0.03	0.28	0.29
Obs.	610	364	610	364	206	85	206	85	207	102

Table 5: Estimation Results for Venture Capital and Buyout Funds

Table 6: OLS estimation results for regression of different public equity fund annual returns for mutual funds from Morningstar Principia database January returns with zero dividend yield, 1996-2003. Size is the fund size or net asset value (in \$MM) and Seq is the sequence or the year of existence of the fund in the sample, R_t is the *return* at time t, F.E. Fixed Effects for year (yr.) or funds (fnds).

	Public Equity Fund Returns: Static Models						Lagged Dependent Models			
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
log(Size)	0.38***	1.29***	1.2***	-3.11***	-1.07	-1.55				
	(0.13)	(0.46)	(.47)	(1.1)	(2.49)	(2.52)				
log(Size)^2		-0.1**	-0.11***		-.23	-.19				
		(0.05)	(0.05)		(.23)	(.23)				
log(Seq)	-2.26***	-3.59***		-3.79	-3.12					
	(.47)	(1.36)		(2.76)	(2.7)					
log(Seq)^2		0.94			5.72					
		(0.8)			(3.3)					
R_{t-1}							0.07***	0.08***	-0.06***	-0.28***
							(0.02)	(0.02)	(0.03)	(0.05)
R_{t-2}								-0.28***		-0.6***
								(.017)		(0.06)
F.E.	Yr	Yr	Yr	Yr,Fnds	Yr, Fnds	Yr, Fnds	Yr	Yr	Yr, Fnds	Yr, Fnds
Adj. R^2	0.51	0.51	0.5	0.46	0.46	0.46	0.51	0.61	0.51	0.68
Obs.	5635	5635	5635	5635	5635	5635	3376	1930	3376	1930

Table 7: BGX Smooth Test and p-values for the residuals for public and private equity.*** 1%, ** 5%, * 10%.

Public-Private	$\hat{\Psi}_6^2\,(\tilde{\ }\chi_6^2)$	$\hat{u}_1^2\,(\tilde{\ }\chi_1^2)$	$\hat{u}_2^2\,(\tilde{\ }\chi_1^2)$	$\hat{u}_3^2\,(\tilde{\ }\chi_1^2)$	$\hat{u}_4^2\,(\tilde{\ }\chi_1^2)$	$\hat{u}_5^2\,(\tilde{\ }\chi_1^2)$	$\hat{u}_6^2\,(\tilde{\ }\chi_1^2)$
No Yield-Pre96, simple FE(yr.)	50.88***	13.18***	0.005	3.53*	13.01***	0.39	20.75***
$(1)-(1)$ $(n=5635, m=840)$	(0.00)	(0.00)	(0.94)	(0.06)	(0.00)	(0.53)	(0.00)
No-yield -Sample Pre96, simple FE(Yr)	37.23***	4.97**	4.32**	0.85	10.28***	0.01	16.81***
$(n=5635, m=518)$	(0.00)	(0.03)	(0.04)	(0.36)	(0.00)	(0.91)	(0.00)
No Yield-Pre96, full FE(yr.)	38.93***	12.13***	0.06	1.00	12.91***	0.39	12.43***
$(2)-(3)$ $(m=5635, n=840)$	(0.00)	(0.00)	(0.80)	(0.32)	(0.00)	(0.53)	(0.00)
No Yield-Sample Pre96, full FE(yr.)	24.1***	4.51**	0.02	3.13*	6.3**	2.98*	7.16***
$(n=5635, m=518)$	(0.00)	(0.03)	(0.88)	(0.08)	(0.01)	(0.08)	(0.01)
No Yield-Pre96 liquidated, full FE(yr.)	55.78***	3.29*	5.60**	2.31	29.97***	6.11**	8.49***
$(n=5635, m=463)$	(0.00)	(0.07)	(0.02)	(0.12)	(0.00)	(0.01)	(0.00)
No Yield-Pre96, simple FE(yr.,firms)	14.18**	0.03	1.22	0.00	8.64***	0.25	4.04**
$(4)-(4)$ $(n=5635, m=840)$	(0.03)	(0.85)	(0.27)	(0.98)	(0.00)	(0.62)	(0.04)
No Yield-Pre96, sample simple FE(yr.,firms)	12.3*	1.37	0.21	0.24	3.09*	1.04	6.35**
$(n=5635, m=518)$	(0.06)	(0.24)	(0.65)	(0.62)	(0.08)	(0.31)	(0.01)
No Yield-Pre96, full FE(yr.,firms)	16.4**	0.05	1.43	0.02	8.98***	0.22	5.69**
$(5)-(6)$ $(n=5635, m=840)$	(0.01)	(0.83)	(0.23)	(0.88)	(0.00)	(0.64)	(0.02)
No Yield-Pre96, sample full FE(yr.,firms)	8.65	0.61	3.17*	0.18	2.28	0.08	2.33
$(n=5635, m=518)$	(0.19)	(0.43)	(0.07)	(0.67)	(0.13)	(0.78)	(0.13)
No Yield-Pre96, on 1 lagged returns, FE(yr.)	64.07***	1.78	15.86***	0.21	16.9***	0.08	29.25***
$(n=3376, m=275)$	(0.00)	(0.18)	(0.00)	(0.65)	(0.00)	(0.78)	(0.00)
No Yield-Pre96, on 2 lagged returns, FE(yr.)	75.94***	2.82*	43.42***	0.299	19.63***	2.99*	6.77**
$(n=1930, m=141)$	(0.00)	(0.09)	(0.00)	(0.58)	(0.00)	(0.08)	(0.01)

Table 8: BGX Smooth Test and p-values for the OLS residuals for the public and private equity.*** 1%, **5%, *10%.

Public-Different Private Equity	$\hat{\Psi}_6^2$ ($\tilde{}\chi_6^2$)	$\hat{u}_1^2$ ($\tilde{}\chi_1^2$)	$\hat{u}_2^2$ ($\tilde{}\chi_1^2$)	$\hat{u}_3^2$ ($\tilde{}\chi_1^2$)	$\hat{u}_4^2$ ($\tilde{}\chi_1^2$)	$\hat{u}_5^2$ ($\tilde{}\chi_1^2$)	$\hat{u}_6^2$ ($\tilde{}\chi_1^2$)
No Yield-Pre96, VC FE(yr., funds)	6.54	0.00	1.69	0.23	2.92*	0.25	1.45
$(5)-(3)$ ($n = 5635, m = 610$)	(0.37)	(0.96)	(0.19)	(0.63)	(0.09)	(0.62)	(0.23)
No-yield - Pre96 Liquidated, VC FE(Yr, funds)	11.64*	0.08	2.16	0.21	1.78	3.61*	3.78*
$(5)-(4)$ ($n = 5635, n_1 = 364$)	(0.07)	(0.77)	(0.64)	(0.36)	(0.18)	(0.06)	(0.05)
No Yield-Pre96-Liquidated, Buyout FE(yr.)	5.41	0.04	0.04	0.23	4.06**	0.13	0.9
$(5)-(8)$ ($n = 5635, m = 85$)	(0.49)	(0.85)	(0.84)	(0.63)	(0.04)	(0.71)	(0.34)
No Yield-Pre96 past two returns, FE(yr.)	60.03***	1.55	28.05***	1.31	12.52***	0.00	16.61***
$(8)-(10)$ ($n = 5635, m = 102$)	(0.00)	(0.21)	(0.00)	(0.25)	(0.00)	(0.97)	(0.00)
VC-Buyout-pre 1996 only size no FE	14.28**	7.97***	0.6	1.5	0.2	3.98*	0.03
($n = 610.m = 206$)	(0.03)	(0.00)	(0.04)	(0.22)	(0.66)	(0.05)	(0.86)
VC-Buyout-pre 1996 simple no FE	15.51**	7.56**	0.16	0.55	0.01	7.17**	0.06
($n = 610, m = 206$)	(0.02)	(0.01)	(0.69)	(0.46)	(0.94)	(0.01)	(0.8)
VC-Buyout-pre 1996 simple FE (yr)	10.24	2.00	0.6	1.82	2.18	1.62	2.02
($n = 610, m = 206$)	(0.11)	(0.15)	(0.43)	(0.18)	(0.14)	(0.20)	(0.15)
VC-Buyout-pre 1996 full FE(yr)	10.3	1.96	0.67	2.26	1.54	1.91	1.97
($n = 610, m = 206$)	(0.11)	(0.16)	(0.41)	(0.13)	(0.21)	(0.17)	(0.16)
VC-Buyout-pre 1996 full FE(yr,funds)	76.27***	0.00	41.88***	1.24	14.26***	0.13	18.74***
($n = 610, m = 206$)	(0.00)	(0.99)	(0.00)	(0.26)	(0.00)	(0.71)	(0.00)

Table 9: OLS estimation results for regression of different private equity fund internal rates of return from SDC-Platinum

and Thomson Banker databases. Rank_{Size} is the rank of fund size with lowest size having rank 1 and VC is Venture capital status, F.E. Fixed Effects for year (yr.) or funds (fnds).*** 1%, ** 5%, * 10%.

	Private				Public				VC		LBO	
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
	(PE<96)	(PE<96)	(PE<96)	(liq)	(>96)	(>96)	(>96)	(>96)	(<96)	(<96)	(<96)	(<96)
Rank_{size}	0.029***	0.022***	-0.004	0.031*	0.001***		0.0002	-0.004**	0.041***	0.025***	0.025	0.046*
	(.007)	(0.006)	(0.013)	(0.017)	(0.0002)		(0.0001)	(0.001)	(0.011)	(0.009)	(0.031)	(0.027)
VC	9.266***	7.983***	3.566	2.343								
	(2.872)	(2.95)	(8.956)	(4.043)								
F.E.	None	Yr	Yr, Frms	Yr	None	Yr	Yr	Yr, Frms	None	Yr	None	Yr
Adj. R^2	0.03	0.06	-0.01	0.01	0.004	0.5	0.5	0.46	0.03	0.07	0.004	0.151
Obs.	840	840	840	463	5635	5635	5635	5635	610	610	206	206

Table 10: BGX Smooth Test of Goodness-of-Fit and corresponding p-values for the rank regression residuals returns for the full sample of public and private equity with corresponding sample sizes for the bigger (n) sample and the smaller (m) sample sizes.*** 1%, ** 5%, * 10%.

Private Equity residuals for rank regression	$\hat{\Psi}_6^2\,(\tilde{\ }\chi_6^2)$	$\hat{u}_1^2\,(\tilde{\ }\chi_1^2)$	$\hat{u}_2^2\,(\tilde{\ }\chi_1^2)$	$\hat{u}_3^2\,(\tilde{\ }\chi_1^2)$	$\hat{u}_4^2\,(\tilde{\ }\chi_1^2)$	$\hat{u}_5^2\,(\tilde{\ }\chi_1^2)$	$\hat{u}_6^2\,(\tilde{\ }\chi_1^2)$
No Yield-Pre96 residuals, No FE	567.35***	3.16*	254.44***	79.79***	169.6***	53.87***	6.49**
(5) – (1) ($n = 5635, m = 840$)	(0.00)	(0.08)	(0.00)	(0.00)	(0.00)	(0.00)	(0.01)
No-yield - Pre96, FE(Yr)	47.99***	12.84***	0.01	2.73	18.3***	0.32	13.79***
(7) – (2) ($n = 5635, m = 840$)	(0.00)	(0.00)	(0.91)	(0.1)	(0.00)	(0.57)	(0.00)
No Yield-Pre96, FE(yr., funds)	14.30**	0.198	2.15	0.22	8.38***	0.00	3.349*
(8) – (3) ($n = 5635, m = 840$)	(0.03)	(0.66)	(0.14)	(0.64)	(0.00)	(0.97)	(0.07)
No Yield-Pre96, VC FE(yr.)	53.67***	5.33**	0.05	3.75*	25.49***	0.04	19.01***
(7) – (10) ($n = 5635, m = 610$)	(0.00)	(0.02)	(0.81)	(0.05)	(0.00)	(0.84)	(0.00)
No Yield-Pre96, LBO FE(yr.)	11.28*	0.00	1.63	0.11	0.47	0.00	9.07***
(7) – (12) ($n = 5635, m = 206$)	(0.08)	(0.96)	(0.2)	(0.74)	(0.49)	(0.98)	(0.00)
Venture Capital-Buyout Funds no FE	12.97**	7.67**	0.44	1.28	0.08	3.43*	0.07
(10) – (12) ($n = 610, m = 206$)	(0.04)	(0.01)	(0.51)	(0.26)	(0.78)	(0.06)	(0.79)
Venture Capital-Buyout Funds FE (yr)	8.65	2.07	0.55	1.22	2.2	0.97	1.63
(10) – (12) ($n = 610, m = 206$)	(0.19)	(0.15)	(0.46)	(0.27)	(0.14)	(0.32)	(0.2)

Figure 5-6: Kernel density functions (5A-5C) and Empirical Distribution Functions(6A-6C) of annual public equity funds returns residuals (1996-2002) and private equity internal rates of returns residuals (inception before 1996).

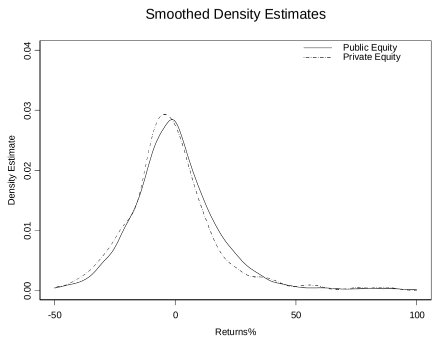


Figure 5A: Residuals (FE yr)

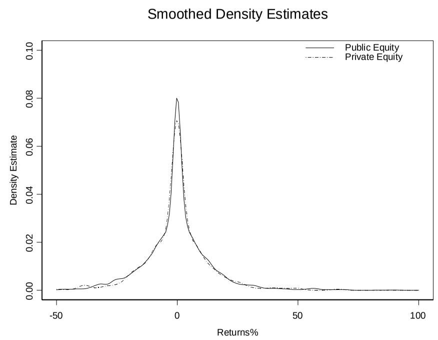
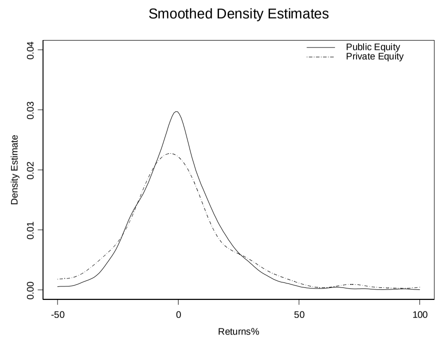


Figure 5B: Residuals (FE yr, fnds)



Fgure 5C: Lagged rtns residuals.

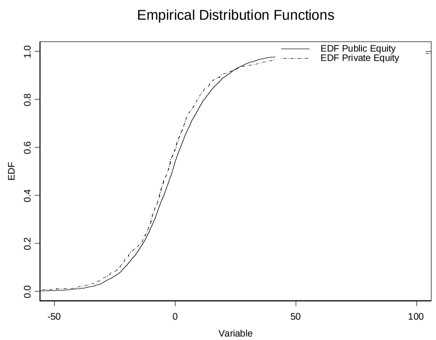


Figure 6A: Residuals, FE (yr)

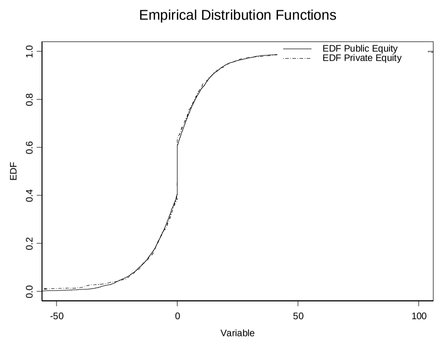


Figure 6B: Residuals, FE(yr, fnds).

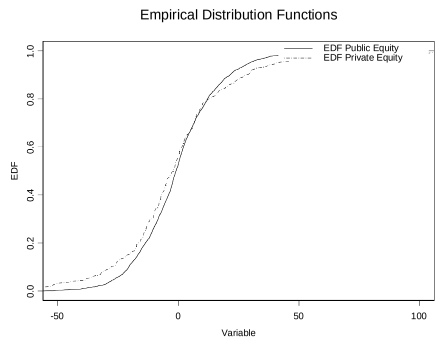


Figure 6C: Lagged rtns residuals.

Figure 7-8: Kernel density functions(7A-7C) and Empirical Distribution Functions (8A-8C) of annual public equity funds returns residuals (1996-2002) and different private equity internal rates of returns residuals (inception before 1996). All with fixed effect for years.

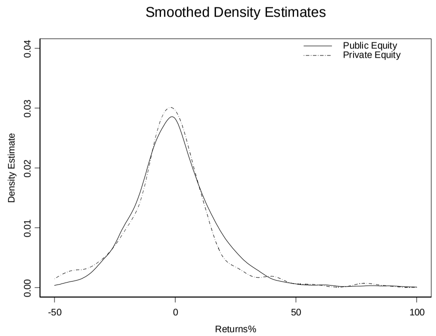


Figure 7A: VC Residuals.

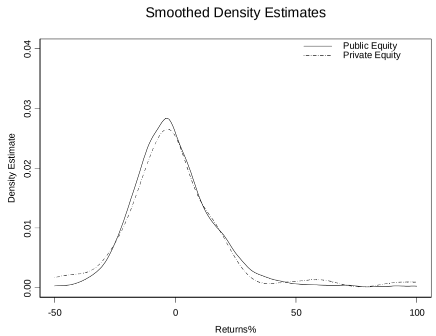


Figure 7B: LBO residuals.

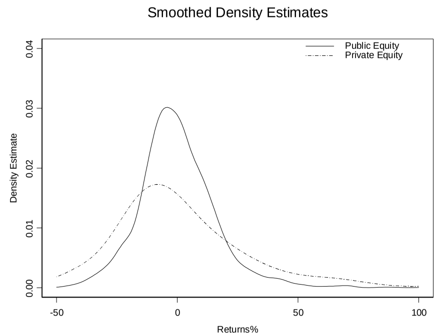


Figure 7C: Lagged VC residuals.

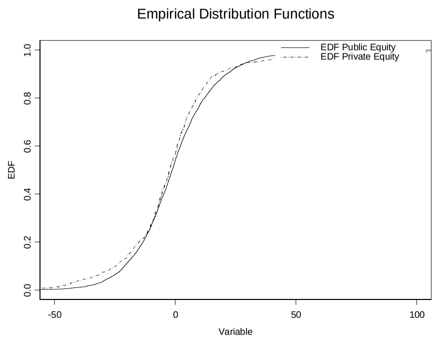


Figure 8A: EDF of VC residuals

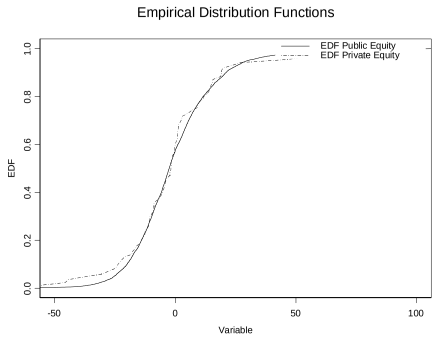


Figure 8B: LBO residuals (liq.)

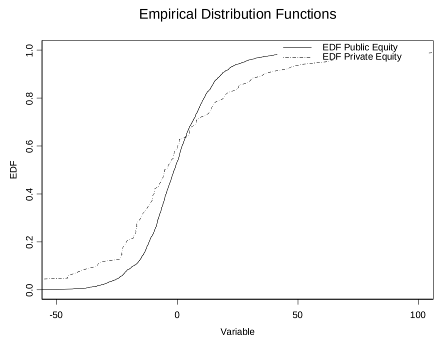


Figure 8C: VC lagged residuals

7 Appendix A (Sample Size Selection)

This appendix is following the method described in Bera, Ghosh and Xiao (2007). For finite sample, for each fixed n_2, we may divide the index set $\mathcal{N} = \{1, \ldots, n\}$ into two mutually exclusive and exhaustive (large) sets $\mathcal{N}_1$ and $\mathcal{N}_2$ with cardinalities n_1 and n_2, where $n_1 + n_2 = n$, and define the **training set**

$$\mathcal{Z}_1 = \{(X_j), j \in \mathcal{N}_1\}$$

and the testing set

$$\mathcal{Z}_2 = \{(X_j), j \in \mathcal{N}_2\}.$$

Then we can estimate $F(\cdot)$ using data $\mathcal{Z}_1$ and construct

$$F_{n_1}(X_i) = \frac{1}{n_1} \sum_{j \in \mathcal{N}_1} I(X_j \leq X_i), \text{ for } i \in \mathcal{N}_2.$$

$\mathcal{Z}_1$ and $\mathcal{Z}_2$ are from the same distribution F, $F(X_i)$ $(i \in \mathcal{N}_2)$ are uniformly distributed and $F_{n_1}(X_i)$ provides an estimator for the uniform distribution, we may compare it with the CDF of standard uniform, say, using some criterion function

$$\frac{1}{n_2} \sum_{i \in \mathcal{N}_2} d(F_{n_1}(X_i), U[0,1])$$

and take average over R replications

$$\frac{1}{R} \sum_{r=1}^{R} \left[\frac{1}{n_2} \sum_{i \in \mathcal{N}_2} d(F_{n_1}^{r}(X_i), U[0,1]) \right]$$

For each value of n_2, we can calculate the above criterion function. We may choose n_2 that minimizes the above criterion based on an Anderson-Darling type distance measure

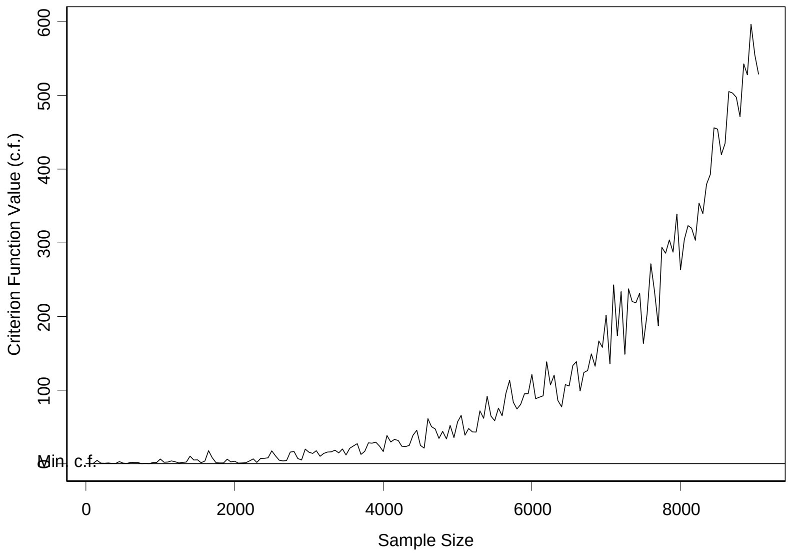


Figure 9: Plot of criterion function to choose finite sample size

Finally, we choose

$$m = \frac{n_2}{n_1} \times n.$$

The above method may have applications in more general settings. This is a cross-validation type procedure to select sample size. In the above problem the criterion function is showed in Figure 9, $\frac{n_2}{n_1} = 9.21\%$. In the data range, the sample size of public equity funds is 10090, we chose about 10% of the smallest sample size. We also note from the plot of criteria function and its values, a sample size of 2250 or one-fourth (22.3%) the size of the estimation sample also gives a reasonably small value of the criteria function. Any sample size of the range between 9.21% and 22.3% provides a reasonable maximal sample size for the correct nominal size of the test. Since our sample size for the private equity return is smaller than this range we would select the entire sample size of private equity return.

References

Bera, A. K., Ghosh, A., 2002,. Neyman's Smooth Test and Its Applications in Econometrics. In: Handbook of Applied Econometrics and Statistical Inference, Eds. Ullah, A., Wan, A., Chaturvedi, A., Marcel Dekker: New York, 177-249.

Bera, A.K., Ghosh, A., 2006,.Fractile Graphical Analysis and Non-parametric Regression in a new perspective. Working paper, School of Economics and Social Sciences, Singapore Management University.

Bera, A. K., Ghosh, A., Xiao, Z., 2007, Smooth Test for Copmaring Equality of Two Distributions. Working Paper, Singapore Management University.

Bergstresser, D., Poterba, J., 2002. Do after-tax returns affect mutual fund inflows? Journal of Financial Economics 63, 381-414.

Berk, J. B., Green, R.C., 2004, Mutual Fund Flows and Performance in Rational Markets. Journal of Political Economy, 114:6, 1269-1295.

B. K. Bucks, Kennickell, A.B., Moore, K.B., 2004, Recent Changes in U.S. Family Finances: Evidence from the 2001 and 2004 Survey of Consumer Finances, prepared for the Board of Federal Reserve Bank, Division of Research and Statistics.

Carhart, M. M., Carpenter, J. N., Lynch, A. W., Musto, D. K., 2002, Mutual fund survivorship, Review of Financial Studies 15, 1439–1463.

Chevalier, J., Ellison, G., 1997. Risk taking by mutual funds as a response to incentives. Journal of Political Economy 105, 1167-1200.

Cochrane, J. H., 2005. The Risk and Return of Venture Capital. Journal of Financial Economics 75, 3-52.

Fama, E.F., Efficient Capital Markets: A Review of Theory and Empirical Work. Journal of Finance, 25, pp. 383-417, 1970.

Fama, E.F., French, K.R.,1992, The Cross-Section of Expected Stock Returns. Journal of Finance, 47, pp. 427-465, 1992.

Fama, E. F. , French, K. R., 1999, The Corporate Cost of Capital and the Return on Corporate Investment. Journal of Finance, 54, pp.1939–1967.

Ghosh, A., 2007. Rank Chasing: How do fractiles of returns affect mutual fund flow? Working paper, Singapore Management University, Singapore.

Gottschalg, O., Phalippou, L. ,Zollo, M. G., 2003, Performance of private equity funds: Another puzzle? Working paper 2003/93/SM/ACGRD 3, INSEAD.

Gompers, P., Lerner, J., 1998, What drives venture capital fundraising? Brookings Papers on Economic Activity: Microeconomics 49–192.

Gompers, P., Lerner, J., 2000, Money chasing deals? The impact of fund inflows on private equity valuations, Journal of Financial Economics 55, 281–325.

Hamilton, B.H., 2000, Does Enterpreneurship Pay? An Empirical Analysis of the Returns to Self-Employment. Journal of Political Economy, 108, pp. 604-631.

Harvey, C. R., Siddique, A., 2000, Conditional Skewness in Asset Pricing Tests. Journal of Finance, 55, pp. 1263-1295.

Jobson, J.D., Korkie, B., 1981, Performance hypothesis testing with the Sharpe and Treynor Measures. Journal of Finance, 36, pp. 889-908.

Ippolito, R., 1992, Consumer reaction to measures of poor quality: evidence from the mutual fund industry. Journal of Law and Economics 35, 45-70.

Jones, C., Rhodes-Kropf, M., 2002, The price of diversifiable risk in venture capital and private equity, Working paper, Columbia Business School.

Kaplan, S. N., Schoar, A., 2005, Private Equity Performance: Returns, Persistence and Capital Flows. The Journal of Finance 60:4, 1791-1823.

Kaplan, S. N., Berk, A. S., Strömberg, P., 2002. How well do venture capital databases reflect actual investments? Working Paper, Graduate School of Business, University of Chicago.

Koenker, R., Bassett, G., 1978, Regression Quantiles. Econometrica 46, 33-50.

Ledoit, O., Wolf, M., 2008. Robust Performance Hypothesis Testing with the Sharpe Ratio.Journal of Empirical Finance 15, 850–859.

Lerner, J., Schoar, A.,2004, The illiquidity puzzle: Theory and evidence from private equity, Journal of Financial Economics 72, 3–40.

Lerner, J., Schoar, A., Wongsunwai,W., 2007, Smart Institutions, Foolish Choices: The Limited Partner Performance Puzzle. Journal of Finance 62:2, 731-764.

Ljungqvist, A., Richardson, M., 2003, The cash flow, return, and risk characteristics of private equity, Working paper, New York University.

Lo, A.W., MacKinlay, C., 1990, Data-Snooping Biases in Tests of Financial Asset Pricing Models. Review of Financial Studies 3:3, 431-467.

Memmel, C. Performance Hypothesis with the Sharpe Ratio, Working Paper, University of Cologne, Germany, 2003.

Metrick, A., Yasuda, A., 2007, The Economics of Private Equity Funds, Working Paper, Wharton School, University of Pennsylvania.

Moskowitz, T.J., Vissing-Jorgensen,A., 2002, The Return to Entrepreneurial Investment: A Private Equity Premium Puzzle? American Economic Review, 92, pp. 745-778.

Neyman, J., 1937, “Smooth test” for goodness of fit. Skandinaviske Aktuarietidskrift 20:150-199, 1937.

Phalippou, L., Zollo, M., 2005, What Drives Private Equity Fund Performance? Working Paper, R&D Group, INSEAD.

Poterba, J., 1989, Venture capital and capital gains taxation, in Lawrence Summers, Ed.: Tax Policy and the Economy, vol. 3 (MIT Press, Cambridge, MA), 47–68.

Sirri, E., Tufano, P., 1998. Costly search and mutual fund flows. Journal of Finance 3, 1589-1622.