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## Abstract

Most research on supply chain management deals with settings where firms do not voluntarily get rid of inventory in the system. Since voluntary inventory reductions are often observed in practice, in this paper, we propose a (multi-echelon) model where the firm can dispose of excess stock through sales in the secondary market at each stage in the supply chain. What are called *nested echelon base stock policies* are shown to be optimal. Secondary market sales complicate the structure of the system, so that the classical Clark and Scarf (1960) approach no longer applies. Nevertheless, we identify features of the optimal policy that significantly reduce the state space and simplify the computation, by providing conditions under which it is optimal for the firm not to both dispose of stock and order additional inventory. Finally, we formulate a class of disposal policies that achieves the Clark-Scarf decomposition, and reduces the optimal inventory policy to the classical echelon base stock policy. A numerical study demonstrates the performance of these policies as heuristics, illustrates the value of secondary markets to the supply chain, and provides managerial insights.

Traditional research on supply chain management tends to deals with settings where firms do not voluntarily get rid of excess inventory in the system. At the same time, because uncertainty in customer demand and other factors impacting the firm often results in a surplus of inventories throughout the supply chain, their reduction by means of secondary market sales can readily be observed in practice.

In this paper we propose a finite-horizon, multi-echelon model in which each stage in a series system can dispose of surplus stock through a secondary market mechanism. The decision to sell-off (i.e., dispose of) excess inventory results in the immediate (i.e., next period) and permanent removal of that inventory from the supply chain. Within the context of this model, our primary objectives are to establish the form (and special features) of the optimal policy, and to propose heuristic policies that capture the essential features of secondary-market sales while simplifying the structure of the problem. Our secondary objectives are to use numerical studies to both assess the performance of those heuristic policies, and to provide insights into the value of secondary markets to supply chains.

To the best of our knowledge, the only other paper to explore the impact of secondary markets on supply chain management is Lee and Whang (2002), which considers a two period, two-stage distribution system – one manufacturer and multiple retailers. In their paper, the secondary market exists only at the downstream stage, among the retailers themselves, which allows for the derivation of the secondary market equilibrium price, as well as the optimal order and market sale quantities for the retailers. By comparison, we address a pure series system, with multiple stages and periods, and allow secondary markets to exist at every stage in the system. Our secondary markets, on the other hand, are exogenous – market demand is abundant, and market prices are takes as given. Our focus is on optimal and practical heuristic policies.

Following Clark and Scarf (1960), we identify stages for the minimum number of periods that a product is away from reaching the (stochastic) customer demand at the most downstream stage. When a unit is first ordered, it is one period away from entering the supply chain. At the beginning of the next period, this unit is at the most upstream stage in the system. The unit can be moved down one stage each period, until it reaches the most downstream stage. The progress of a unit can be delayed (if necessary) at each

stage, by not moving it downstream. We also allow a unit to be disposed of at any stage in the system, through a sale in the secondary market at that stage.

We assume full backlogging of unsatisfied demand at the most downstream stage, with a unit backlogging cost, and inventory carryover, with a unit holding cost at each stage. Each unit ordered into a stage incurs an ordering cost, while each unit sold in the secondary market generates revenue. We allow all model parameters and stochastic demands to vary with time. Our model is a finite-horizon dynamic program, whose objective is to minimize the expected discounted net present value of costs (backlogging + inventory holding + ordering) less revenues incurred from the secondary market sales, over the time horizon.

In arriving at results, we first reformulate the model using echelon-type variables. Next, we show that the form of the optimal policy is a *bottom-up nested echelon base stock policy*: disposal decisions are made first, from the bottom-up, starting with the most downstream stage and moving upstream; order decisions, nested within adjacent sets of disposal decisions, are made next. Secondary market sales complicate the structure of the system (through the complexity of the boundary of the feasible region at each stage) so that the classical Clark and Scarf (1960) decomposition, and the additive convexity of the objective function, no longer hold, in general.

To help deal with the dimensionality inherent in the problem, we first identify features of the optimal policy that simplify the computation. In particular, we introduce some mild restrictions on cost and revenues parameters under which it is optimal for each stage in the system not to *both* dispose of excess inventory *and* order additional stock from the upstream stage; the dimensionality of the optimization space, and the corresponding calculation time, are thus (roughly) halved.

Next, we identify a special class of policies that renders our model more tractable, while embodying the features of secondary market sales observed in practice. In particular, we introduce a *disposal saturation policy*, which, for any stage in the system, considers disposing of inventory at that stage only if there is no stock available for disposal at any upstream stage. A disposal saturation policy thus "saturates" disposal at each stage, before initiating any disposals at the next stage downstream. We show that:

i) This class of policies achieves the Clark-Scarf decomposition and the additive convexity of the objective function;

ii) The optimal disposal policy is determined by only two critical numbers;

iii) The resulting optimal ordering policy follows the classical (state-independent) echelon basestock policy.

A disposal saturation policy thus allows disposal at each stage in the system, considerably simplifies the problem, and is straightforward to implement in practice. We conduct numerical studies for a two-stage and three-stage system to evaluate the performance of a disposal saturation policy as a heuristic for the original problem. Our results indicate that disposal saturation policies provide a very good approximation to the optimal policy (within 1% of the optimal objective function value) across a wide range of model parameters. Moreover, while the calculation of the optimal policy for a three-stage system and twenty periods took about 6 days to compute (for each model run), finding the optimal disposal saturation policy took only a couple of minutes.

Finally, we conduct a numerical study to assess the value of secondary markets to the supply chain performance (for a two- and a three-stage system). The results of the study indicate that this value is increasing in unit revenues (also shown analytically), and decreasing in unit order costs. Interestingly, the value of having secondary markets is neither uniformly increasing nor uniformly decreasing in inventory holding costs; instead, it appears to first increase, as unit holding costs increase, and then decrease, as those costs continue to grow. Additional research is needed to shed more light on this relationship, as well as provide more insight into the optimality of disposal saturation policies.

## References

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