Few-body multiple scattering calculations for ^{6}He on protons

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The elastic scattering of the halo nucleus ^{6}He from a proton target at 717 MeV/nucleon is investigated within three different multiple-scattering formulations of the total transition amplitude. The factorized impulse approximation (FIA) and the fixed scatterer approximation (FSA) of the multiple-scattering expansion are used to evaluate accurately the single-scattering terms and to test the validity of a few-body Glauber approach. The latter also includes terms beyond single scattering and the importance of these terms is investigated. The differential cross section is calculated for proton scattering from ^{6}He at 717 MeV in inverse kinematics and compared with recent data.

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I. INTRODUCTION

To understand fully the scattering of composite nuclei, consisting of A nucleons, from a structureless target such as a single proton, one ideally needs to solve the $A + 1$ scattering problem. Fortunately, for many nuclei, and halo nuclei in particular, this may not be necessary because such systems can be very well-described within a few-body picture of n constituents (where $n < A$). The strongly correlated n bodies can be individual nucleons or clusters of nucleons where each cluster is itself treated as a structureless, though not necessarily inert, body. Thus the scattering problem to be solved is a more manageable but nevertheless challenging $(n + 1)$-body one.

The ground-state wave function for the bound n-body system (where here n is typically 2, for the deuteron or ^{11}Be, or 3 for the Borromean nuclei ^{6}He and ^{11}Li) can be described via approximate solutions to the time-independent Schrödinger equation. This is then used as one of the ingredients, along with the optical potentials between each of the n constituents and the target, entering the fully $(n + 1)$-body scattering problem.

It is the aim of microscopic scattering theory to construct the total scattering amplitude in terms of well-defined dynamical and structural quantities. The many-body scattering framework is, however, a nontrivial one to deal with. Therefore, approximation schemes have been developed such as the continuum discretized coupled-channels (CDCC) method [1]. In this approach the $(N + 1)$-body Schrödinger equation is approximated as a set of effective two-body coupled-channel equations, which take into account bound as well as continuum states (something found to be of great importance when dealing with the scattering of weakly bound systems). However, this method is nontrivial to implement numerically and for the high scattering energies available at today's fragmentation facilities it becomes more practical to use a multiple-scattering expansion of the total transition amplitude (MST) [2–8].

In the MST approach the projectile-target transition amplitude is expanded in terms of off-shell transition amplitudes for projectile constituent-target subsystem scattering. Due to the complexity of the many-body operator suitable approximations need to be made to express in a convenient way the overall scattering amplitude in terms of these subsystem amplitudes. The MST approach therefore provides a clear and transparent interpretation of the composite system in terms of the free scattering of its constituents and is numerically advantageous. At the high energies and weak binding of the projectiles of interest here the expansion is expected to converge rapidly.

In this article, we use this framework to calculate the elastic scattering of the halo nucleus ^{6}He from a proton target at 717 MeV/nucleon where new experimental data are now available. However, the momentum transfers covered by the data still essentially probe only the part of the few-body dynamics of the halo system that is constrained by the rms matter radius. Thus a treatment of ^{6}He as a three-body system of α-particle core plus two valence neutrons is quite appropriate. We outline and compare three quite different approaches to evaluating the MST multiple-scattering expansion to show the validity of the different models and the importance of higher-order terms in the MST series. Specifically, we examine (i) the factorized impulse approximation (FIA); (ii) the fixed scatterer, or adiabatic, approximation (FSA); and (iii) the Glauber approximation [9,10]. The first two methods (described in detail in Ref. [8]), although making fewer simplifying assumptions than the Glauber model, are used here only to calculate the single-scattering terms of the full amplitude, whereas the Glauber approach includes higher-order terms. However, higher-order terms have been used elsewhere [5]. We first compare all three approaches at the single-scattering level and then include the higher order terms within the Glauber framework and compare with the experimental data. In all cases we use the same three-body ^{6}He ground-state wave function and similar amplitudes for the $\alpha - p$ and $n - p$ subsystem scattering.

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II. THE MULTIPLE-SCATTERING EXPANSION OF THE TOTAL TRANSITION AMPLITUDE

We consider the scattering of a projectile (labeled p) from a few-body target consisting of n subsystems weakly bound to each other (an $n + 1$ scattering problem). We refer to the composite system as the target although in practice many such scattering problems are carried out in inverse kinematics due to the short lifetime of the n-cluster system, requiring it in practice to be used as the projectile (such as the case of ^{6}He + p scattering investigated in Sec. VI). The n constituents of the target can themselves be either individual nucleons or clusters of nucleons. The total transition operator for the scattering is

$$T = V + VG_0T = \sum_{i=1}^{n} v_i + \sum_{i=1}^{n} v_i G_0 T, \qquad (1)$$

where v_i is the interaction between the projectile and the ith constituent of the target and G_0 is the free Green's operator (propagator) in the sense that it does not contain the projectile-target interactions

$$G_0 = (E + i\epsilon - K_p - H_0)^{-1}, \qquad (2)$$

where E is the total energy in the center-of-mass (c.m.) frame, K_p is the operator for the total kinetic energy of the projectile and target in the overall c.m. system, and H_0 the internal Hamiltonian of the n-body target. Within the multiple-scattering expansion of the total transition operator, one defines a projectile-ith constituent transition operator τ_i as

$$\tau_i = v_i + v_i G_0 \tau_i, \qquad (3)$$

where it is noted here that τ_i is not a two-body operator as it depends on the propagator G_0, which is a many-body operator. Equation (1) can then be written

$$T = \sum_{i=1}^{n} T_i, \qquad (4)$$

where T_i satisfies

$$T_i = \tau_i + \tau_i G_0 \sum_{j \neq i} T_j = \tau_i + \tau_i G_0 \sum_{j \neq i} \tau_j + \cdots, \qquad (5)$$

and thus

$$T = \sum_i \tau_i + \sum_i \tau_i G_0 \sum_{j \neq i} \tau_j + \cdots. \qquad (6)$$

In the limit when the target nucleus subsystems are weakly bound to each other, the multiple-scattering expansion to the P-T transition amplitude is expected to converge rapidly [3,4] and the multiple-scattering expansion Eq. (6) is useful. Often, the series in Eq. (6) is truncated and only the first term is retained, amounting to what is known as the single-scattering approximation, in which the projectile can scatter from each target constituent separately (to all orders in their interaction) but does not then proceed to scatter from any of the other constituents. Here, we retain higher-order terms in Eq. (6) but, instead, make a less drastic simplifying assumption: we retain only forward-scattering (FS) terms in the series. That is, once the projectile has scattered from constituent

i and moves on to constituent j it does not then return to constituent i because this would involve backscattering. With this assumption therefore—assumed valid at higher scattering energies—we see that Eq. (6) truncates naturally at the nth order. Thus for a two-body target ($n = 2$), for instance, we would have

$$T_{(n=2)} = \tau_1 + \tau_2 + \tau_1 G_0 \tau_2 + \tau_2 G_0 \tau_1, \qquad (7)$$

and for a three-body target ($n = 3$) we would have

$$T_{(n=3)} = \sum_{i=1}^{3} \tau_i + \sum_{i=1}^{3} \tau_i G_0 \sum_{j \neq i}^{3} \tau_j$$
$$+ \sum_{i=1}^{3} \tau_i G_0 \sum_{j \neq i}^{3} \tau_j G_0 \sum_{k \neq i \neq j}^{3} \tau_k. \qquad (8)$$

We define $\vec{k}, (\vec{k}')$ as the initial (final) momentum of the projectile in the projectile-target c.m. system and $\vec{q} = \vec{k}' - \vec{k}$ as the projectile momentum transfer. Note that we denote wave vectors with lowercase $\vec{k}$, whereas capital K denotes kinetic energy operators. The elastic scattering amplitude is written

$$F_{(n)}(q) = \frac{-\mu}{2\pi\hbar^2} \langle \vec{k}' \Phi^{(n)} | T_{(n)} | \vec{k} \Phi^{(n)} \rangle, \qquad (9)$$

where $\Phi^{(n)}$ is the n-body intrinsic wave function of the target and μ is the projectile-target reduced mass. Note that at this stage we use the nonrelativistic expression in the above equation. The appropriate relativistic kinematics factor can be introduced following Refs. [2,7,8]. Substituting for $T_{(n=2)}$ from Eq. (7) we have a sum of four scattering amplitudes, the two single-scattering ones and the two double scattering ones,

$$F_{(n=2)}(q) = F_1(q) + F_2(q) + F_{12}(q) + F_{21}(q). \qquad (10)$$

Similarly, for the case of a three-body target

$$F_{(n=3)}(q) = \sum_{i=1}^{3} F_i(q) + \sum_{i=1}^{3} \sum_{j \neq i}^{3} F_{ij}(q)$$
$$+ \sum_{i=1}^{3} \sum_{j \neq i}^{3} \sum_{k \neq i \neq j}^{3} F_{ijk}(q). \qquad (11)$$

III. GLAUBER MULTIPLE SCATTERING

According to Glauber's multiple-scattering theory the elastic amplitude for the scattering of a proton from a composite nucleus of mass A can be written as an integral over the proton impact parameter plane as [9,10]

$$F_{(n)}^{GL}(q) = \frac{ik}{2\pi} \int d^2\vec{b}\, e^{i\vec{q}\cdot\vec{b}}\, [1 - \mathcal{S}(b)]. \qquad (12)$$

The elastic S matrix, as a function of the proton-target c.m. impact parameter b, is

$$\mathcal{S}(b) = \langle \Phi^{(n)} | \prod_{j=1}^{n} S_{pj}(b_j) | \Phi^{(n)} \rangle, \qquad (13)$$

where the label j runs over each cluster in the composite target, with ground-state wave function $\Phi^{(n)}$. In the original

formulation by Glauber [11], this summation will of course run over all A nucleons in the target. The S matrices, $S_{pj}(b_j)$, describe the free scattering of the proton from each target constituent.

The Glauber model contains essentially two different approximations. As a first step, the adiabatic (or sudden) approximation is made [12] in which the internal degrees of freedom of the target (or projectile in the case of inverse kinematics) nucleus are frozen for the duration of the interaction time. The second approximation is the eikonal, or straight-line, assumption.

It must be stressed that the composite nucleus S matrix, $\mathcal{S}(b)$ in Eq. (13), is a many-body matrix element of the target ground-state n-body density $|\Phi^{(n)}|^2$ and, without considerable additional approximation, $\mathcal{S}(b)$ has no simple relationship with the composite nucleus one-body density $\rho_A(r)$.

To make a connection with multiple-scattering terms of the last section and to test the importance of terms in Eq. (6) beyond single scattering we consider the case of proton scattering from an $n = 3$ cluster target nucleus composed of a core and two valence nucleons as, for example, ^{6}He. (Note that the choice of the proton or the ^{6}He as the projectile does not alter the formulation of the scattering amplitude because we are working in the c.m. frame.) Thus the three-body S matrix of Eq. (13) becomes

$$\mathcal{S}(b) = \langle\Phi^{(n)}|S_c(b_c)S_1(b_1)S_2(b_2)|\Phi^{(n)}\rangle, \qquad (14)$$

where the constituent impact parameters, b_c, b_1 and b_2, are shown in Fig. 1 and depend on the c.m. impact parameter b and the projections of the target internal coordinates on the impact parameter plane weighted by the masses of the constituents relative to the whole projectile. We can write

$$1 - \mathcal{S}(b) = \langle\Phi^{(n)}|1 - S_c(b_c)S_1(b_1)S_2(b_2)|\Phi^{(n)}\rangle$$
$$= \langle\Phi^{(n)}|[(1 - S_c) + (1 - S_1) + (1 - S_2)$$
$$- (1 - S_c)(1 - S_1) - (1 - S_c)(1 - S_2)$$
$$- (1 - S_1)(1 - S_2) + (1 - S_c)(1 - S_1)$$
$$\times (1 - S_2)]|\Phi^{(n)}\rangle. \qquad (15)$$

Substituting the above expansion into the expression for the Glauber scattering amplitude of Eq. (12) we see that the terms correspond to just those retained in the forward-scattering
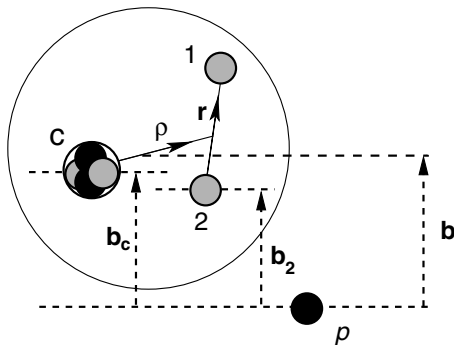

FIG. 1. Schematic representation of the collision of a proton on a three-cluster nucleus showing the various impact parameters.

approximation defined in Eq. (11). This is a natural consequence of the Glauber approximation involving the additivity of eikonal phases arising from its own forward-scattering assumption. Thus, the Glauber model provides us with a logical justification for our forward-scattering assumption of the previous section, provided of course that the scattering energy is high enough for those assumptions required the Glauber model to hold. The first three terms in the above expansion correspond to the single-scattering terms in Eq. (11). The above expansion also gives us a convenient way of testing the relative importance of single scattering and higher-order scattering terms when comparing with more precise multiple-scattering formulations in the next section. Note that in the multiple-scattering terms the few-body dynamics are properly taken into account because the S matrices are functions of both the c.m. impact parameter and the projections of the internal coordinates of the composite nucleus onto the impact parameter plane.

Note that even the single-scattering terms include effects of breakup of the composite nucleus. To reproduce the observables calculated in the single-scattering approximation within a coupled-channel framework one needs to include breakup channels.

IV. THE MST FACTORIZED IMPULSE APPROXIMATION

Within the impulse approximation (IA), the interaction between the clusters V_{ij} is assumed to have a negligible dynamical effect on the scattering of the projectile from the individual target subsystems and therefore can be neglected. The operator projectile-i target subsystem transition amplitude τ_i is then replaced by

$$\hat{t}_i = v_i + v_i\hat{G}_0\hat{t}_i, \qquad (16)$$

where $\hat{G}_0$ contains only the total kinetic energy operator K

$$\hat{G}_0 = (E^+ - K)^{-1}. \qquad (17)$$

The transition amplitude $\hat{t}_i$ is still a many-body operator, because the kinetic energy operator K has contributions from the projectile and all n target subsystems. In the approach followed by Crespo and Johnson [5] the initial relative momenta between the clusters are neglected in the transition matrix elements. One then obtains an FIA expression of a product of a transition operator and a target form factor

$$T_{(n=3)}^{\mathrm{FIA}} = \langle\vec{\mathcal{Q}}_1'|\hat{t}_1(\omega_1)|\vec{\mathcal{Q}}_1\rangle\rho_{12,3}\left(\frac{m_2}{M_{12}}\vec{q}, \frac{m_3}{M_{123}}\vec{q}\right)$$
$$+ \langle\vec{\mathcal{Q}}_2'|\hat{t}_2(\omega_2)|\vec{\mathcal{Q}}_2\rangle\rho_{12,3}\left(\frac{m_1}{M_{12}}\vec{q}, \frac{m_3}{M_{123}}\vec{q}\right)$$
$$+ \langle\vec{\mathcal{Q}}_3'\varphi_3|\hat{t}_3(\omega_3)|\varphi_3\vec{\mathcal{Q}}_3\rangle\rho_{12,3}\left(0, \frac{M_{12}}{M_{123}}\vec{q}\right) + (\cdots),$$
$$\qquad (18)$$

where, within the single-scattering approximation, only the first three terms are taken into account. In here, $M_{12} = m_1 + m_2$, $M_{123} = m_1 + m_2 + m_3$, φ_3 is the core internal wave

function and $\rho_{12,3}(\vec{q}_1, \vec{q}_2)$ is the target form factor

$$
\begin{aligned}
\rho_{12,3}(\vec{q}_1, \vec{q}_2) = \int d\vec{Q}_1 d\vec{Q}_2 \, \phi_{12,3}^{(n=3)*}(\vec{Q}_1 + \vec{q}_1, \vec{Q}_2 + \vec{q}_2) \\
\times \phi_{12,3}^{(n=3)}(\vec{Q}_1, \vec{Q}_2),
\end{aligned}
\tag{19}
$$

where $\phi_{12,3}^{(n=3)}(\vec{Q}_1, \vec{Q}_2)$ is the Fourier transform of the wave function of the two-body valence system relative to the core $\phi_{12,3}^{(n=3)}(\vec{r}, \vec{R})$.

Within this approach, the relative momenta $\vec{\mathcal{Q}}_i$ for the projectile scattering of each subsystem i are [8]

$$
\vec{\mathcal{Q}}_i = \frac{\mu_i}{\mu} \vec{k}, \quad \vec{\mathcal{Q}}_i' = \vec{\mathcal{Q}}_i + \vec{q},
\tag{20}
$$

with $\mu_i = m_p m_i / (m_p + m_i)$ the projectile-subsystem i reduced mass. In Eq. (18), the appropriate energy parameter ω_i is

$$
\omega_i = \frac{\mu_i}{\mu} E.
\tag{21}
$$

By construction, from Eqs. (20) and (21) the matrix elements of the transition amplitudes are on-shell. For projectile-target scattering the scattering amplitude is related to the transition amplitude according to Eq. (9). Equivalently, for projectile-subsystem i scattering

$$
|f_i(\omega_i)|^2 = -\frac{\mu_i}{2\pi\hbar^2} |\hat{t}_i(\omega_i)|^2.
\tag{22}
$$

It follows then that [8]

$$
\begin{aligned}
F^{\text{FIA}}(E) = \mathcal{N}_1^{1/2} \hat{f}_1(\omega_1, \vec{q}) \rho_{12,3}\left(\frac{m_2}{M_{12}}\vec{q}, \frac{m_3}{M_{123}}\vec{q}\right) \\
+ \mathcal{N}_2^{1/2} \hat{f}_2(\omega_2, \vec{q}) \rho_{12,3}\left(\frac{m_1}{M_{12}}\vec{q}, \frac{m_3}{M_{123}}\vec{q}\right) \\
+ \mathcal{N}_3^{1/2} \hat{f}_3(\omega_3, \vec{q}) \rho_{12,3}\left(0, \frac{m_3}{M_{123}}\vec{q}\right),
\end{aligned}
\tag{23}
$$

where the normalization factors are given by

$$
\mathcal{N}_i = \left[\frac{1}{d\omega_i/dE}\right]^2.
\tag{24}
$$

Equations (22)–(24) can be easily generalized to include relativistic kinematics following Refs. [2,7,8].

V. THE FIXED SCATTERER OR ADIABATIC APPROXIMATION (FSA)

Within the fixed scatterer or adiabatic approximation, the internal Hamiltonian between the clusters is taken to a constant $\bar{H}$, that is, the projectile-i target subsystem transition amplitude τ_i operator is replaced by

$$
\tilde{t}_i = v_i + v_i \tilde{G}_0 \tilde{t}_i,
\tag{25}
$$

where $\tilde{G}_0$

$$
\tilde{G}_0 = (E^+ - K_p - \bar{H})^{-1}.
\tag{26}
$$

Within the FSA framework the total transition amplitude takes the form [8]

$$
\begin{aligned}
T_{(n=3)}^{\text{FSA}} = \langle \vec{k}'|\tilde{t}_1(E)|\vec{k}\rangle \rho_{12,3}\left(\frac{m_2}{M_{12}}\vec{q}, \frac{m_3}{M_{123}}\vec{q}\right) \\
+ \langle \vec{k}'|\tilde{t}_2(E)|\vec{k}\rangle \rho_{12,3}\left(\frac{m_1}{M_{12}}\vec{q}, \frac{m_3}{M_{123}}\vec{q}\right) \\
+ \langle \vec{k}'\varphi_3|\tilde{t}_3(E)|\varphi_3\vec{k}\rangle \rho_{12,3}\left(0, \frac{M_{12}}{M_{123}}\vec{q}\right).
\end{aligned}
\tag{27}
$$

We note that in the FSA approach, the energy parameter is $E = \hbar k^2/2\mu$ and, thus, a distinctive feature of the FSA is that the two-body amplitudes are calculated with the projectile-target reduced-mass μ instead of the projectile-fragment reduced-mass μ_i. The scattering amplitudes are related to the transition amplitude

$$
|\tilde{f}_i(E)|^2 = -\frac{\mu}{2\pi\hbar^2} |\tilde{t}_i(E)|^2,
\tag{28}
$$

and at high energies one has

$$
\tilde{f}_i(E) \sim \frac{k}{k_i} \hat{f}_i(\omega_i),
\tag{29}
$$

an exact relation if the eikonal approximation is used. Notice that the Glauber multiple-scattering approach also makes use of the adiabatic approximation [10], and in this sense, it is closely related to the FSA rather than the FIA approximation.

VI. CALCULATIONS AND RESULTS

To calculate the elastic scattering cross sections for protons on weakly bound nuclei such as the Borromean halo ^{6}He within our models, we require as input the ground-state (three-body) wave function of relative motion. We therefore use a realistic (Faddeev) three-body wave function that gives the correct two-neutron separation energy and root-mean-square (rms) matter radii for ^{6}He. The wave function, when used within a few-body Glauber calculation, leads to total reaction cross sections that agree with experiment. The other inputs to our models are the on-shell matrix elements of the free elastic scattering amplitudes for the constituents from the proton target ($n + p$ and $\alpha + p$) evaluated at the appropriate energy and momentum transfer. Note also that all cross sections presented here were calculated using relativistic kinematics.

For the ^{6}He ground state, we use the FC model wave function that corresponds to a radius of 2.5 fm and that is described in Ref. [13]. We use the same two-body scattering amplitude for the $\alpha + p$ subsystem in the following way. We first compute the elastic $\alpha + p$ "Glauber" S matrix according to the prescription of Ref. [10], which is then used in Eq. (12) to obtain the Glauber scattering amplitude. For the FIA and FSA calculations, and to retain the same inputs, we take the $\alpha + p$ Glauber S matrix and associate this continuous function of impact parameter with partial-wave S matrix elements at particular ℓ values using the semiclassical relation $bk = \ell + 1/2$ [9,14]. The transition amplitude is then built from the sum of partial wave amplitudes.
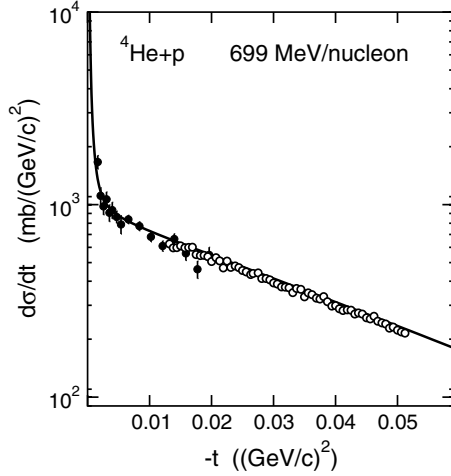
FIG. 2. Calculated and experimental $p+{}^4$He elastic differential cross-section angular distribution as a function of the square of the four-momentum transfer ($q^2 = -t$) at 699 MeV/nucleon. The data are from Refs. [16] (solid points) and [17] (open points).

Figure 2 shows the calculated elastic differential cross section at 699 MeV as a function of the square of the four-momentum transfer ($q^2 = -t$). It was found that very similar cross sections were obtained whether we used the Glauber S matrix directly in Eq. (12) or carried out a partial-wave sum of discrete partial S-matrix elements, S_ℓ. Therefore only one theoretical cross section is shown. Because the free NN-scattering parameters are taken from Ref. [15], without adjustment, and a simple microscopic α-particle wave function has been used, the only free parameter available was the assumed rms size of the α particle, through r_0. The level of agreement with the data for the physical rms matter radius of 1.49 fm is therefore very encouraging. No attempt was made to fine-tune the NN interaction parameters. In any case the experimental data also have a stated overall normalization uncertainty of order $\pm 2\%$ [16,17].

In the FIA and FSA calculations, the pn on-shell scattering amplitudes were obtained from a realistic NN Paris interaction, as in Ref. [5]. In the Glauber calculations, each pairwise NN-scattering operator (S matrix), such as the pnS matrices in Eq. (14), denoted by S_1 and S_2, is denoted by $S_{pj}(b_j) = 1 - \Gamma_{pj}(b_j)$, where b_j is the impact parameter of the incident proton relative to target nucleon j. The j label on Γ_{pj} also identifies the use of the pn or pp profile function, the two-dimensional transform of the free NN-scattering amplitudes

$$\Gamma_{pj}(b_j) = \frac{1}{2\pi i k} \int d^2\vec{q}\, e^{-i\vec{q}\cdot\vec{b}_j} f_{pj}(q). \qquad (30)$$

These profile functions are parametrized, as is usual, according to

$$\Gamma_{pj}(b) = \frac{\sigma_{pj}}{4i\pi\beta_{pj}}(\alpha_{pj} + i)\exp(-b^2/2\beta_{pj}), \quad (j = p, n), \qquad (31)$$

where σ_{pp} and σ_{pn} are the pp and pn total cross sections. The α_{pj} are the ratios of the real to imaginary parts of the
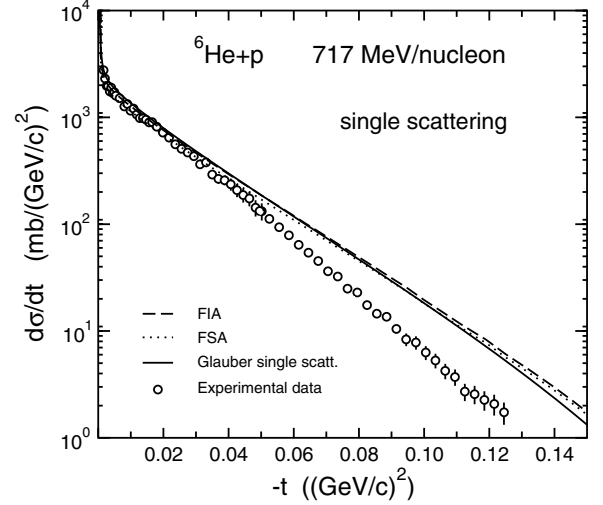

FIG. 3. Experimental and calculated $p+{}^6$He elastic scattering at 717 MeV using the FIA (dashed), the FSA (adiabatic) approximation (dotted), and Glauber (solid line). The data are taken from Refs. [19–21].

forward-scattering NN amplitudes and the β_{pj} are the range parameters. All parameters are deduced, e.g., Refs. [10,15,18], from fits to free pp- and pn-scattering data. In Fig. 3 we compare the calculated $p+{}^6$He cross section using the FIA (dashed), the FSA (adiabatic) approximation (dotted), and Glauber (solid line) as a function of the squared momentum transfer. In all the three cases, only the single-scattering contribution has been taken into account. The calculated differential cross sections using FIA and FSA are very similar, which might indicate that the same physics is retained in both the factorization and the adiabatic approximations. The calculated cross section using Glauber single scattering, which also makes use of the adiabatic assumption, is in very close agreement with the other two curves, underlining that the semiclassical approximation used by the Glauber scattering framework is valid at this energy.

In Fig. 4 the dashed line represents the calculated differential elastic cross section for $p+{}^6$He using the single-scattering contribution to the Glauber amplitude and is obtained from the first three terms in Eq. (15). The effect of including the higher-order scattering terms within the Glauber scattering framework defined in Eq. (15) is shown by the solid line.

Both calculations reproduce very well the data up to $0.05(\text{GeV}/c)^2$. However, at higher-momentum transfers the single-scattering cross section significantly overestimates the data, whereas the cross section including all higher-order contributions approaches the data. One then concludes that dynamical higher-order contributions are very significant at this high-momentum transfer and need to be included. This is particularly true when extracting rms radii from the elastic-scattering data. The full Glauber model cross section still slightly overestimates the data, however. We note that our few-body Glauber calculation has been compared [10] with one making making an optical limit (OL) Glauber approximation, which neglects few-body scattering effects. Although the difference between the cross sections calculated within the two
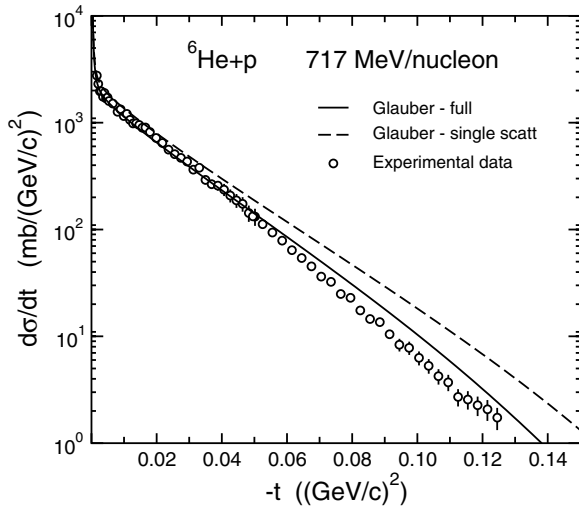
FIG. 4. Calculated p-^{6}He elastic scattering at 700 MeV using Glauber single scattering (dashed) and Glauber, including all orders (solid line). The data are taken from Refs. [19–21].

scattering frameworks is small at low-momentum transfers, they gradually deviate from the OL cross section, doing less well.

Because we have tighter control over the assumptions and approximations made in our scattering models, we feel confident that the remaining discrepancies between the full Glauber model and the data, although possibly due to short-range correlation effects important at the higher-momentum transfers and not treated here, could arise from the inputs to our models. And because the free scattering of the α core reproduces $\alpha + p$ scattering very well, although the full cross sections have little sensitivity to the exact form of the pnS matrices at the larger-momentum transfers [8], we can conclude that the remaining differences between theory and experiment are likely to be due to the inadequacy of the ^{6}He wave function. Nevertheless, the breakdown of the forward approximation used by the Glauber scattering framework cannot be ruled out and should be investigated due to the

implications on the accurate extraction of the rms radii from the elastic-scattering data.

VII. CONCLUSIONS

We have studied $p-^6$He elastic scattering at 717 MeV using the multiple-scattering expansion of the total transition amplitude by treating ^{6}He as a three-body system, hence having to solve a four-body scattering problem. We have discussed three different approaches. In the Glauber multiple-scattering model we make use of both the adiabatic and forward-scattering approximations. Although both these assumptions are expected to work well at such scattering energies, it is important, when comparing with experimental data, to delineate the different types of missing physics: those not treated in the scattering process or reaction mechanism and those coming from the nuclear structure (in this case of the ^{6}He).

We have, therefore, shown comparisons, at the level of the elastic differential cross section, between the Glauber approach and two other multiple-scattering methods: the factorized impulse approximation and the fixed scatterer approximation. We take only the single-scattering terms in this comparison and show that the Glauber model works remarkably well.

We have also shown that higher-order dynamical contributions are important in describing the data at high-momentum transfers $(q^2) 0.05 [\text{GeV}/c)^2]$. Moreover, the calculated cross section using Glauber to all orders still slightly overestimate the data and may indicate inadequacy of the structure model used to describe ^{6}He or the breakdown of the Glauber scattering framework.

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