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Global Financial Risks, CVaR and Contagion Management

Kian-Guan Lim*

The September 2008 collapse of Lehman Brothers was the 9/11 on Wall Street, and many articles had been written on the changes in the global risk landscape that followed. However, there is scarcity of rigorous studies using empirical data and advanced econometric methods to verify such a change and the nature of such a change. In this paper, we provide rigorous analyses of statistically significant changes in global financial risks and sharp increases in conditional Value-at-Risk after September 2008. We perform statistical analyses using conditional distributions on the tail losses of equity portfolios constructed from the stock indexes of six major global financial markets. Employing the generalized marginal Pareto distribution and multivariate copula method, we provide strong empirical evidence to assert the prevalence of heightened global financial risks and its contagion effect across the globe. An important implication arising out of these conclusions is that banks under BASEL II and BASEL III and financial institutions in the near-future should not underestimate its Conditional Value-at-Risk by using the normal distribution model since under stressed situations past September 2008, the portfolio return distributions have tails that simultaneously grow longer and thinner in the direction of the loss region. We also provide some thoughts for contagion management.

JEL Codes: C51, G15, G32

1. Introduction

The global financial crisis of 2008 saw a huge decline in the U.S. stock prices by a whopping 25% within ten days in the two weeks before the Black Friday of October 10th 2008 and within two weeks after the collapse of Lehman Brothers on September 15 2008. September 29 and October 9 saw the Dow Jones Industrial Average fell 6.98% and 7.33% respectively. There was another big fall on October 15 of 7.87%, although in-between there were some small re-bounces. On October 10, 2008, the day after the large DJIA fall, markets across the globe fell in contagion. FTSE in London was down by 6.6%, DAX in Germany by 8.2%, CAC-40 in France by 8.3%, N225 in Japan by 9.6%, and major stock indices in Asia countries ex-Japan mostly fell by about 8% with the exception of South Korea that fell a smaller 4.1% and the Shanghai Composite Index that fell only 2.8%. Since then till 2011, equity markets have become much more dependent and move more closely in tandem whenever shocks hit the market.

There are two significant observations in the above development. Firstly, country stock markets have since evidenced more frequent sudden sharp losses. Secondly, such losses appeared to be global in a contagion effect within a day across countries around the globe.

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Since the U.S. market is the largest capital market in the world, it is noticed that the contagion typically starts with the U.S. market movement as the lead. There is scarcity of rigorous studies using empirical data and advanced econometric methods to verify such changes and the nature of such changes in global financial risks. In this paper, we provide rigorous analyses of statistically significant changes in such global financial risks and the sharp increases in conditional Value-at-Risk after September 2008. Specifically we investigate if sharp drops are accompanied by structural changes in the loss tails of the conditional probability distribution of returns. We also study the contagion effect or the strong correlations of market stock return movements across countries by using the copula method. The Pareto distribution of the second kind (or type II) is used to model the conditional tail loss distribution. The tail loss is the area most critical to the risk-taking and risk management decisions made by banks and financial institutions.

In section 2, we provide a literature review and discussion of the motivation in using the Pareto distribution for modeling the tail loss. In section 3, we provide some description statistics of the sample data and an exploration of characteristics of the return loss tails. In section 4, we perform Pareto Type II estimation and testing on the returns of six major portfolios across the globe. We not only estimate and test the parameters using the maximum likelihood method, but also provide a statistic based on the tail characteristics, which is essentially the mean of the conditional distribution, to test for structural changes in the loss tails. This statistic is also the estimate of the conditional Value-at-Risk, subject to u being the VaR. Next we provide the econometric model for a multivariate copula method based on the univariate Pareto Type II distribution. The association or "correlation" of the various univariate country return losses are studied using the Clayton copula, and the parameter of association is also estimated and tested based on maximum likelihood theory. In the concluding section 5, we report the implications of these results and relate this to the underlying macroeconomics as well as provide a discussion on the measures of risk using VaR and conditional VaR or shortfalls.

2. Modeling Tail Distribution

Define any end-of-day t return to a country market as $r_t = \ln(P_t/P_{t-1})$ where P_t is the index level at day t. For the study of loss tails, we focus our attention on negative daily returns below a threshold u that we define to be at the 95% confidence level Value-at-Risk (VaR) measure. Thus the conditional negative returns $r_t < -u$, where $u>0$ is 1.645 standard deviations from the mean daily return. Let $x_t = -r_t$. Thus we study the conditional loss distribution of random variable x_t where $x_t>u>0$. The decumulative conditional distribution function of x_t is $\overline{F} = \left(1 + \frac{x_t - u}{\gamma}\right)^{-\alpha}$, where α, γ > 0, and $x_t > u$. The conditional distribution function (conditional cdf) of x_t is thus $F(x_t|x_t>u) = 1-\overline{F}$. This is a Pareto Type II distribution or is sometimes called the Lomax distribution.

To provide a motivation of the use of Pareto distribution in modeling the tail, we consider the tail area in the loss region $[x_t,\infty)$ that is given by $\overline{F}$. As γ value increases, holding α constant, the conditional loss tail becomes fatter and longer, producing higher probabilities of extreme loss events. The same effect is observed when we fix the value of γ while decreasing the value of α. By varying γ and α, different conditional tail distributions of different country market returns can be modeled. It can be shown that the above Pareto distribution is sufficiently flexible to model the typical shapes of loss tail distributions, and compares favourably with alternative popular models such as the Gumbel distribution and the half-normal distribution.

The Pareto distribution was first studied by Pareto (1897) to model income distribution. Multivariate pareto distributions (MPD) have also been a subject of rigorous studies such as in Yeh (2004) that showed the distributions have many mixture properties, and particular classes of multivariate Pareto distributions can be obtained as limit distribution under finite sample minima or under repeated geometric minimization. However, these classes of MPD have some rigidities with regard to the dependence or correlation amongst the univariate marginal Pareto random variables. For example, a MPD of the second kind has the restriction that for any pair of the univariate PD random variables, their correlation coefficient is a constant. Such a classical form of MPD first introduced by Mardia (1962) does not allow for variations across the returns of different countries.

Resnick (1987), Holger, Rootzen, and Nader Tajvidi (2006), and others also showed the close connection of slightly different forms of MPD to multivariate extreme distributions. The latter showed that modeling exceedances over a threshold, related to Peaks over Threshold (POT) methods, led asymptotically to generalizations of MPD. They also showed that the multivariate GPD in such a setup is the only one that is preserved under change of exceedance levels. The recognition of using MPD as a tool to model fat and long tails similar to extreme value distributions led to efforts to model the MPD differently, while keeping the marginal univariate distributions as Pareto-type. Asimit, Furman, and Vernic (2010) provided a more flexible type of MPD that allows for different correlations between different bivariate pairs of random variables, but the MPD is still restrictive and does not allow for the simultaneous occurrence of similar losses. The resulting correlation function is still restricted to a form that produces approximately the same correlation coefficients when the exponent α's of the univariate variables are nearly the same. A more flexible method would be to employ the copula function on the univariate density functions.

In a series of articles, viz. Li (2006), Li (2009), Li and Sun (2009), Joe and Li (2011), and Chana and Li (2008), the tail dependence of multivariate distributions using copula method has been characterized via methods of regular variations and mixtures of scales. Specific examples such as Burr distribution, t-distribution, and others in a multivariate setting are studied and some examples yield monotone properties of other parameters versus the tail index, which in the case of the Pareto Type II distribution, refers to the α parameter. However, there were no empirical studies to validate the models or to study situations of structural changes in the parameters in a time series setting.

There have been several recent applications of the multivariate Pareto distribution. Vernic (2011) provided an application of the multivariate Pareto distribution to computing the tail conditional expectation, or sometimes called conditional Value-at-Risk in risk measurement literature. They provided formulas for the conditional expectation, but there is no study of their statistical properties. Kim and Lee (2009) derived a cusum test of residuals from an autoregressive process to study change in the tail index or parameter of extreme value distributions such as the Burr distribution which is also called the Type IV MPD in Yeh (2004). The empirical study is restricted to test of the individual model parameters. Cazzulani, Meneguzzo, and Vecchiato (2001) provided one of the earliest study of country index return correlations using the copula method, but used the non-central t-distribution as univariate density instead, and it did not provide statistical tests of the parametric estimators nor employ the MPD.

3. Descriptive Statistics of Global Returns Risk

The daily stock index returns of U.S., U.K., Germany, France, Japan, and Hong Kong, respectively the S&P 500, FTSE, DAX, CDC-40, N225, and H.S.I., are collected over the total sampling period of January 2, 1991 till June 17, 2011. Assuming or ignoring negligible aggregate dividends, we compute the continuously compounded daily index returns as indicated in the introductory section. The log of the price relatives provides for theoretically infinite supports. However, we focus our attention on conditional distribution based on the condition of return losses exceeding the respective country index thresholds fixed at the 5% probability loss region, or 1.645 times the standard deviation away from the unconditional total sample mean.

Global financial markets have become more integrated and have behaved in relation to the macroeconomic development of the respective countries. Using U.S. as the lead country in terms of its global impact, we divide the total sampling period into 3 sub-periods of direct connection to the U.S. macroeconomic situation. The first sub-period covers January 2 1991 to March 10, 2000 when NASDAQ index peaked during intra-day trades, and was immediately followed by the DOT.COM crash of 2000/2001. During this first sub-period, apart from a small recession during 1991-1994 due to the Iraq war and rising oil prices, the U.S. economy saw rapid growth due to a number of reasons as elaborated in a best seller book, “Irrational Exuberance,” by Shiller (2000). The second sub-period covers March 11, 2000 to September 14, 2008, the day just before Lehman Brothers called “broke”. During this period, there was, by now well understood, an oversupply of cheap U.S. funding as a result of over-reaction to the DOT.COM crash and complacency on the part of relevant sectors of the U.S. government machinery. It led to a huge boom in housing market building excesses and bank mortgage loans as well as financial derivatives such as CDO's created to accentuate the get-rich-quick schemes underlying this economically unsustainable and mad boom. This of course led to the infamous U.S. mortgage lending fiasco and the subsequent global financial crisis (GFC). The third sub-period from September 15, 2008 till June 17, 2011 covers the period of the GFC and the incomplete and on-going struggles by the major world economies to recover. This sub-period saw U.S. Federal Reserve introducing huge liquidity to shore up their failing banks, essentially issuing more U.S. Treasuries, in programs called “quantitative easing”. Europe, and to some extent, Japan, followed the same. The 2011 European sovereign debt crisis in EU countries such as Greece, and potentially Spain, Portugal, Italy, and so on, due to excessive budget deficits, added to the woes confronting an already fragile inter-connected world economy with unprecedented financial market volatilities. The descriptive statistics of the returns in these sub-periods as well as of the total sampling period are reported in Table 1.

Table 1: Descriptive Statistics of Daily Returns of Country Equity Indices

Countries	Mean	Std. Dev.	Skewness	Kurtosis	Max	Min	% Negative
Sub-period 1	(Sample Size)	1644					
S&P500	0.000670	0.008906	-0.4136	10.3294	0.0498	-0.0711	47.3236
FTSE	0.000468	0.009071	0.0063	5.21360	0.0543	-0.0393	47.5060
DAX	0.000570	0.011519	-0.2534	6.68474	0.0728	-0.0609	46.0462
CDC-40	0.000546	0.011964	-0.0377	4.99927	0.0683	-0.0562	47.7493
N225	0.000422	0.014090	0.2068	5.13388	0.0727	-0.0595	48.7834
HSI	0.000946	0.017530	0.1266	14.3650	0.1724	-0.1473	47.2019
Sub-period 2	(Sample Size)	1582					
S&P500	0.000159	0.011517	0.1532	5.3670	0.0557	-0.0504	47.6310
FTSE	-0.000111	0.011387	-0.0214	5.5675	0.0590	-0.0491	49.1471
DAX	-0.000221	0.014859	0.0309	5.2708	0.0697	-0.0591	48.4523
CDC-40	-0.000144	0.013774	0.1412	5.3335	0.0700	-0.0554	49.5893
N225	-0.000179	0.013533	0.0283	4.1878	0.0722	-0.0556	51.1054
HSI	0.000221	0.013706	0.2472	6.4364	0.1018	-0.0559	49.1471
Sub-period 3	(Sample Size)	510					
S&P500	-0.000057	0.020255	-0.2614	9.3674	0.1095	-0.0946	45.6862
FTSE	-0.000703	0.015750	-0.3657	9.5134	0.0846	-0.0926	50.3921
DAX	-0.000450	0.017091	-0.1208	8.0676	0.1068	-0.0727	48.0392
CDC-40	-0.000635	0.017744	-0.2071	7.2634	0.0886	-0.0804	49.8039
N225	-0.000702	0.022193	-0.5507	10.9127	0.1323	-0.1211	48.8235
HSI	-0.000159	0.020157	0.5944	11.4345	0.1340	-0.0866	48.2352
Overall period	(Sample Size)	3736					
S&P500	0.000367	0.012108	-0.2086	13.1830	0.1095	-0.0946	47.2162
FTSE	0.000072	0.011169	-0.1871	8.7048	0.0846	-0.0926	48.5813
DAX	0.000100	0.013849	-0.1062	6.9210	0.1068	-0.0727	47.3233
CDC-40	0.000097	0.013650	-0.0377	6.4866	0.0886	-0.0804	21.0117
N225	0.000027	0.015215	-0.1686	9.5398	0.1323	-0.1211	49.7591
HSI	0.000503	0.016415	0.2905	13.0436	0.1724	-0.1473	48.1531

From Table 1, several cursory observations can be made. Firstly, the daily volatility of about 1% for almost all countries during sub-periods 1 and 2 (except Hong Kong that had a higher volatility in sub-period 1) increased to about 2% in sub-period 3 after the Lehman bankruptcy and during the GFC. Secondly, except for Hong Kong, in sub-period 3, all country index portfolio returns showed negative skewness at clearly more negative levels. Thirdly, during sub-period 3, the kurtosis of the index portfolio returns of all countries also displayed clear escalations.

Next we compute the cross-correlations of the country index returns during the sub-periods and during the total sampling period. It is important that we describe how the time series of the different country index returns are paired together in a joint distribution for the purpose of measuring their cross-moments. Since the indexes are reported based on stock prices in a broad portfolio in each of the respective country's major stock exchange, the daily indexes used for index return computations reflect the close-of-day broad portfolio price in each exchange. On a trading day t+1, at the close of trading in the London, Frankfurt and Paris bourses, the New York stock exchanges, where the S&P 500 stocks are traded, are still open. Thus the European country index returns at t+1 may reflect some on-going news in U.S. though it is not the close of trading day t+1 at U.S., but will reflect all of the news on trading day t, the previous day, in New York. However, for the Asian bourses such as Osaka, Tokyo, and Hong Kong, at trading day t, when the New York exchanges would have closed for trading, the Asian exchanges have not started trading, and would only start trading in a few hours at trading day t+1. In the above context, taking the U.S. as the lead country where news affecting its stock would be also transmitted to the other countries, we then pair the index closing prices at trading day t+1 of all the other bourses with the S&P 500 closing index prices at trading day t. The results are reported in Table 2.

Table 2: Pearson's Correlation Coefficients of Daily Country Equity Index Returns

Sub-period 1 ρ	S&P500	FTSE	DAX	CDC-40	N225	HSI
S&P500	1.00000000	0.30472524	0.34506933	0.30019452	0.26019675	0.34657629
FTSE	0.30472524	1.00000000	0.59539712	0.69468407	0.25944231	0.31920803
DAX	0.34506933	0.59539712	1.00000000	0.66087642	0.25217565	0.34808495
CDC-40	0.30019452	0.69468407	0.66087642	1.00000000	0.23671822	0.28867313
N225	0.26019675	0.25944231	0.25217565	0.23671822	1.00000000	0.28733699
HSI	0.34657629	0.31920803	0.34808495	0.28867313	0.28733699	1.00000000
Sub-period 2 ρ	S&P500	FTSE	DAX	CDC-40	N225	HSI
S&P500	1.00000000	0.29211145	0.16324004	0.28849644	0.38835940	0.44058516
FTSE	0.29211145	1.00000000	0.75410743	0.86900615	0.26057188	0.30746622
DAX	0.16324004	0.75410743	1.00000000	0.84883222	0.20603476	0.22589776
CDC-40	0.28849644	0.86900615	0.84883222	1.00000000	0.26301860	0.28964723
N225	0.38835940	0.26057188	0.20603476	0.26301860	1.00000000	0.52563259
HSI	0.44058516	0.30746622	0.22589776	0.28964723	0.52563259	1.00000000
Sub-period 3 ρ	S&P500	FTSE	DAX	CDC-40	N225	HSI
S&P500	1.00000000	0.29789829	0.20111335	0.29857096	0.64190933	0.40488450
FTSE	0.29789829	1.00000000	0.85931065	0.93820310	0.41316741	0.45793446
DAX	0.20111335	0.85931065	1.00000000	0.89646212	0.38684301	0.48286666
CDC-40	0.29857096	0.93820310	0.89646212	1.00000000	0.40647154	0.41872613
N225	0.64190933	0.41316741	0.38684301	0.40647154	1.00000000	0.65974820
HSI	0.40488450	0.45793446	0.48286666	0.41872613	0.65974820	1.00000000
Overall period ρ	S&P500	FTSE	DAX	CDC-40	N225	HSI
S&P500	1.00000000	0.29345898	0.21958716	0.29026468	0.42785592	0.37845653
FTSE	0.29345898	1.00000000	0.73012494	0.83048033	0.29987155	0.33873630
DAX	0.21958716	0.73012494	1.00000000	0.79767765	0.26333294	0.32113664
CDC-40	0.29026468	0.83048033	0.79767765	1.00000000	0.28907833	0.31280129
N225	0.42785592	0.29987155	0.26333294	0.28907833	1.00000000	0.44898950
HSI	0.37845653	0.33873630	0.32113664	0.31280129	0.44898950	1.00000000

From Table 2, it is significant to note that all correlations are positive. The results are basically similar when using alternative correlation measures such as the Spearman's rank correlation coefficients and Kendall's rank correlation coefficient. Thus our pairing scheme is plausible and there are also a number of previous studies that documented the next day impact by U.S. stock movements on financial markets in the rest of the world. See for example such a study involving interest rate financial instruments in Lim, How and Terry (1998). Moreover, we observe that the positive correlations are generally higher in sub-period 3 during the GFC era.

4. Econometric Estimation and Testing

Having motivated the important issue of understanding how sudden and large contagion stock price movements can occur, and explaining the usefulness of the generalized Pareto Distribution, we perform statistical analyses using conditional distributions on the tail losses of equity portfolios constructed from the stock indexes of six major global financial markets.

In this section we investigate the properties of the Pareto Type II multivariate distribution and derive consistent estimators of the parameters as well as their asymptotic test statistics. The estimates will enable us to capture descriptions of the fitted tail probabilities and then go on to model joint probabilities using the copula method. The latter will in turn enable estimation of the chance of occurrences of contagion.

Consider the Pareto Type II probability density function (pdf) of each country's conditional daily return distribution where returns are negative below thresholds denoted by $-u_i$ for country or country index i. For convenience, we shall work with $x_{t,i} = -r_{t,i} > u_i$ or the losses as positive numbers. We first analyze the conditional distributions for each country i. The conditional pdf of $x_{t,i}$ is given by:

$$f\left(x_{t,i} \mid x_{t,i} > u_i\right) = \frac{\alpha_i}{\gamma_i}\left(1 + \frac{x_{t,i} - u_i}{\gamma_i}\right)^{-(\alpha_i + 1)}. \tag{1}$$

Henceforth, unless otherwise stated, the country index i shall be respectively 1,2,3,4,5, and 6 for U.S. S&P500, U.K. FTSE, Germany DAX, French CDC-40, Japan N225, and Hong Kong H.S.I. The u_i's for the various countries are found as 0.0190, 0.0187, 0.0235, 0.0231, 0.0256, and 0.0277 respectively.

For T daily observations of county i's returns, conditional on $x_{t,i} > u_i$, within a particular period or sub-period, the log-likelihood function is obtained from equation (1) as:

$$\ln L_i\left(\{x_{t,i}\}\right) = T \ln\left(\frac{\alpha_i}{\gamma_i}\right) - \left(\alpha_i + 1\right)\sum_{t=1}^{T} \ln\left(1 + \frac{x_{t,i} - u_i}{\gamma_i}\right) \quad . \tag{2}$$

We maximize the log-likelihood function on the LHS, $\ln L_i\left(x_{t,i}\right)$, in equation (2), and obtain the following first order optimality conditions in equations (3) and (4). The second order conditions are also satisfied.

$$\frac{\partial}{\partial \gamma_i}: \quad \hat{\gamma}_i = \left(\hat{\alpha}_i + 1\right)\frac{1}{T}\sum_{t=1}^{T} \frac{x_{t,i} - u_i}{\left(1 + \frac{x_{t,i} - u_i}{\hat{\gamma}_i}\right)} \quad \forall i = 1,2,\ldots,N. \tag{3}$$

$$\frac{\partial}{\partial \alpha_i}: \quad \frac{1}{\hat{\alpha}_i} = \frac{1}{T}\sum_{t=1}^{T} \ln\left(1 + \frac{x_{t,i} - u_i}{\hat{\gamma}_i}\right) \qquad \forall i = 1,2,\ldots,N. \tag{4}$$

Equation (3) or (4) can be easily solved for the maximum likelihood estimates (MLE) of γ_i and α_i for i=1,2,…,6 given an auxiliary equation of the mean. The Fisher's information matrix can then be computed as $\hat{\Lambda}_i$. The covariance matrix of the estimators $\hat{\gamma}_i$ and $\hat{\alpha}_i$, $\forall i = 1,2,\ldots,6$, can be approximated via sampling estimates of $\hat{\Lambda}_i^{-1}$ for each i. The MLE $\hat{\gamma}_i$ and $\hat{\alpha}_i$ are found for each sub-period and the entire period. For each country i, the estimates are tested using the hull hypotheses H_0: $\gamma = 0$, H_A : $\gamma > 0$, and also H_0: $\alpha = 2$, H_A : $\alpha > 2$. The test statistic is the simple z-statistic based on the asymptotic theory that the MLE are asymptotically normal, consistent, and efficient. The null hypothesis asserts that the estimates are not significantly different from the minimum values and thus is a rejection of the model in such a case. The results are reported in Table 3.

Table 3: Maximum Likelihood Estimates and Tests of H_0: $\gamma = 0$, H_A : $\gamma > 0$, and H_0: $\alpha = 2$, H_A : $\alpha > 2$. Using one-tail test at 2.5% significance level, rejection of H_0 is indicated by a * beside the z-statistic. Using one-tail test at 1% significance level, rejection of H_0 is indicated by a ** beside the z-statistic.

	S&P500	FTSE	DAX	CDC-40	N225	HSI
Sub-period 1						
$\hat{\gamma}_i$	0.0350	0.0197	0.0308	0.0451	0.0591	0.0592
z-statistic	1.2529	3.3424**	1.8239	2.2810*	1.9221	2.7773**
$\hat{\alpha}_i$	5.4540	3.5370	4.0051	5.8882	7.9979	4.7492
z-statistic	0.9326	1.6355	1.1204	1.6925	1.5893	1.8797
Sub-period 2						
$\hat{\gamma}_i$	0.0660	0.0436	0.0484	0.0497	0.0884	0.0530
z-statistic	2.2453*	2.4761**	3.1322**	2.6585**	1.8133	1.5181
$\hat{\alpha}_i$	9.7790	5.9102	5.0380	5.6972	9.8814	5.7785
z-statistic	1.9185	1.8617	2.1660*	1.9589	1.5584	1.1509
Sub-period 3						
$\hat{\gamma}_i$	0.0650	0.0499	0.0240	0.0299	0.0657	0.0669
z-statistic	2.0196*	2.2154*	3.1877**	3.4122**	1.2303	1.4095
$\hat{\alpha}_i$	4.7610	4.2476	2.2017	2.5278	3.9914	4.9793
z-statistic	1.3761	1.3840	0.3644	0.8621	0.7583	0.9952
Overall period						
$\hat{\gamma}_i$	0.0460	0.0498	0.0385	0.0683	0.0415	0.0718
z-statistic	2.5801**	3.3353**	4.4987**	3.3295**	3.1555**	2.6352**
$\hat{\alpha}_i$	5.3460	5.9972	4.0637	6.9195	4.5162	5.9675
z-statistic	1.9012	2.5346**	2.7143**	2.6448**	2.1112*	2.0243*

From Table 3, it is seen that for the entire sampling period, the estimates are significantly different from the null, indicating that the conditional Pareto Type II distribution produces γ and α estimates that are plausible. Similar though weaker evidences occur in the sub-periods. However, it is significant to note that the high γ estimates indicate the fatness of the tail at high x values. The α estimates where high α's imply thinner tails at large x values, are mostly close to the minimum value of 2, which also indicates in this case, that the tail is fat.

4.1 Tests of Mean

Under the Pareto Type II distribution, the conditional distribution mean is $u + \frac{\gamma}{\alpha - 1}$. Its asymptotically consistent and efficient estimate is $u + \frac{\hat{\gamma}}{\hat{\alpha} - 1}$, where u is given. The asymptotic covariance matrix of this estimator is found as:

$$\begin{pmatrix} \frac{1}{\hat{\alpha}-1} & \frac{-\hat{\gamma}}{(\hat{\alpha}-1)^2} \end{pmatrix} \hat{\Lambda}^{-1} \begin{pmatrix} \frac{1}{\hat{\alpha}-1} \\ \frac{-\hat{\gamma}}{(\hat{\alpha}-1)^2} \end{pmatrix}.$$

This estimate is also the conditional Value-at-Risk measure given the Value-at-Risk can be interpreted as the threshold value u. Thus, we are able to report in Table 4 not only the asymptotic efficient estimates of the CVaR or the conditional mean over threshold u, but we can also test if the CVaR is significantly larger than zero. We test H_0: $\frac{\gamma}{\alpha-1}=0$, H_A: $\frac{\gamma}{\alpha-1}>0$, and also a 1% loss over and above VaR threshold, i.e. H_0: $\frac{\gamma}{\alpha-1}=0.01$, H_A: $\frac{\gamma}{\alpha-1}\neq 0.01$. The results are reported in Table 4.

Table 4: Maximum Likelihood Estimates of CVaR or Mean as $u+\frac{\gamma}{\alpha-1}$ and tests of H_0: $\frac{\gamma}{\alpha-1}=0$, H_A: $\frac{\gamma}{\alpha-1}>0$, using z-statistic z1 and also a 1% loss over and above VaR threshold, i.e. H_0: $\frac{\gamma}{\alpha-1}=0.01$, H_A: $\frac{\gamma}{\alpha-1}\neq 0.01$ using z-statistic z2. For tests at 2.5% significance level, rejection of H_0 is indicated by a * beside the z-statistic. For tests at 1% significance level, rejection of H_0 is indicated by a ** beside the z-statistic. (Note: $z_{0.9875}$=2.224 and $z_{0.995}$=2.57.)

	S&P500	FTSE	DAX	CDC-40	N225	HSI
Sub-period 1						
$u_i+\frac{\gamma_i}{\alpha_i-1}$	0.0269	0.0265	0.0337	0.0323	0.0340	0.0435
z1-statistic	7.935**	6.999**	8.395**	9.568**	11.915**	11.337**
z2-statistic	-2.163	-2.014	0.165	-0.832	-2.173	4.161**
Sub-period 2						
$u_i+\frac{\gamma_i}{\alpha_i-1}$	0.0265	0.0276	0.0355	0.0337	0.0356	0.0388
z1-statistic	12.600**	12.387**	12.408**	11.915**	11.073**	9.329**
z2-statistic	-4.160**	-1.562	2.068	0.674	-0.0556	0.925
Sub-period 3						
$u_i+\frac{\gamma_i}{\alpha_i-1}$	0.0363	0.0341	0.0435	0.0427	0.0476	0.0445
z1-statistic	9.167**	7.935**	4.612**	5.687**	6.818**	7.597**
z2-statistic	3.862**	2.772**	2.306*	2.785**	3.713**	3.075**
Overall period						
$u_i+\frac{\gamma_i}{\alpha_i-1}$	0.0296	0.0287	0.0361	0.0346	0.0374	0.0422
z1-statistic	16.677**	17.443**	15.888**	18.170**	15.707**	17.262**
z2-statistic	0.914	-0.061	3.278**	2.370*	2.396*	5.316**

Table 4 shows that all conditional means are significantly larger than the thresholds at 1% significance level. In sub-periods 1 and 2, i.e. prior to September 15, 2008, there was clearly no empirical evidence of conditional VaR exceeding 1% beyond the threshold, except for U.S. during the pre-GFC boom and for Hong Kong in the property boom period of the 1990's. However, what is most significant is that during sub-period 3, i.e. post September 15, 2008 till June 2011, the conditional VaR exceeded 1% significantly in all countries at the 2-tail 1% significance level. This is likely the reason why over the entire sampling period there is also significance in the number of exceedances. There is thus a clear evidence of significant changes in the riskiness of equity investment.

The mean formula that incorporates both the γ and the α parameters and thus capturing the shape of the tail, in this case, provides evidence of a change in the conditional tail distribution during the GFC. All the conditional loss tails had elongated and contained higher probabilities of larger losses.

To provide a comparison, we also compute the normal distribution CVaR $\hat{\sigma}\frac{\phi(v')}{\Phi(v')}-\bar{r}$, where $r_{t,i} \sim N(\mu,\sigma^2)$, and v' is the negative value on the standard normal distribution with a cdf of 5%. Hence v'=-1.645. ϕ and Φ are the respective pdf and cdf functions of the standard normal distribution. The comparisons are shown in Table 5 below.

Table 5: Comparison of CVaR using normal versus the Pareto Type II distributions

	Sub-period 1		Sub-period 2		Sub-period 3		Overall period	
	Pareto CVaR	Normal CVaR	Pareto CVaR	Normal CVaR	Pareto CVaR	Normal CVaR	Pareto CVaR	Normal CVaR
S&P 500	0.0269	0.0177	0.0265	0.0236	0.0363	0.0419	0.0296	0.0246
FTSE	0.0265	0.0182	0.0276	0.0236	0.0341	0.0332	0.0287	0.0230
DAX	0.0337	0.0232	0.0355	0.0309	0.0435	0.0357	0.0361	0.0285
CDC-40	0.0323	0.0241	0.0337	0.0285	0.0427	0.0372	0.0346	0.0281
N225	0.0340	0.0286	0.0356	0.0281	0.0476	0.0465	0.0374	0.0314
HSI	0.0435	0.0352	0.0388	0.0280	0.0445	0.0417	0.0422	0.0334

In Table 5, it is shown that the Pareto Type II CVaR, almost in all cases except one during sub-period 3 for S&P 500, provides a higher value and is thus more conservative in setting a higher capital requirement. This is consistent with the market experiences during the GFC, and is also consistent with criticisms over capital under-provisions using BASEL II computations based on an underlying normal distribution.

We also test if the change in the estimated mean is significant from one sub-period to the next. The standard deviation for the difference in means is estimated assuming that the statistics or de facto x values in the different sub-periods are not correlated. The results are reported in Table 6. Clearly all changes in CVaR from sub-period 2 to sub-period 3 and are significantly positive at 1% significance levels. Changes from sub-period 1 to 2 did not provide statistical evidence of significant changes except for the case of Hong Kong where there was a significant drop in CVaR likely due to the burst of the property bubble.

Table 6: Tests of Difference in the Conditional Means from one sub-period to the next. Using two-tail test at 2.5% significance level, rejection of H_0 is indicated by a * beside the z-statistic. Using one-tail test at 1% significance level, rejection of H_0 is indicated by a ** beside the z-statistic.

	S&P500	FTSE	DAX	CDC-40	N225	HSI
Sub-period 1 to Sub-period 2						
Change in Mean	-0.00034	0.00112	0.00180	0.00140	0.00151	-0.00470
z-statistic	-0.294	0.844	1.159	1.068	1.310	-2.565**
Sub-period 2 to Sub-period 3						
Change in Mean	0.00976	0.00649	0.00800	0.00900	0.01201	0.00570
z-statistic	4.937**	3.142**	1.801	2.528**	3.592**	2.270**

In the following sub-section we investigate the joint behavior of these tails or the conditional multivariate distribution of the global returns using the copula method. The copula method will be able to provide an analysis of the multivariate distribution of the equity returns.

4.2 Copula

The Type II or generalized Pareto distribution employed so far in the analyses for univariate return distributions can be extended to a multivariate setting for joint distributions of returns in different domains such as different country index returns or returns in different periods of different economic regimes. However the multivariate Pareto Type II distribution implies that for any bivariate situations, the correlation in the returns is constant. This is quite rigid as clearly the return correlations between different pairs of country returns are different, though they may usually be close if the countries are within the same geographical and economic grouping. Thus, instead of using the Pearson system of multivariate distributions, we employ the copula method to model multivariate distributions with univariate or marginal distributions as arguments in the copula function.

A copula is a function $C(u_1,u_2,\ldots,u_N)$ of standard uniform random variables $U_i = u_i \in [0,1]$, for i=1,2,….,N. It is assumed that:

(1) lower boundary conditions $C(u_1,u_2,\ldots,u_N) = 0$ for one or more variables $u_j=0$, for any j;
(2) upper boundary conditions $C(u_1,u_2,\ldots,u_N \mid u_j=1) \geq C(u_1,u_2,\ldots,u_N)$ for at least one variable u_j attaining its maximum;
(3) $\frac{\partial C}{\partial u_i} \geq 0 \quad \forall i.$

To avoid technical complications that would not alter the key results, we do not need to allow for possibly weaker conditions than (1), (2), and (3) above, and we assume that $C(\cdot,\cdot)$ is continuously differentiable in u_i for all i. The latter condition applies in most of the common copula functions. The above conditions intuitively are conditions that satisfy those of a multivariate cumulative distribution function.

Sklar's (1959) theorem states that if F(x) and G(y) are given marginal distribution functions of random variables X=x and Y=y, and F(·), G(·) are continuously differentiable, then a copula function C(F(x),G(y)) exists, that is a joint distribution of random variables F(X) ≡ U and G(Y) ≡ V, where U,V are standard uniform [0,1] random variables. Moreover, copula function C(F(x),G(y)) equals a joint distribution function H(x,y) in x and y. Conversely, any joint cdf H(x,y) in random variables X and Y can be represented by a copula function C(F(x),G(y)) where F(·) and G(·) are the marginal cdf's of X and Y. The implication is that given a copula function C(u,v) satisfying the regularity conditions (1), (2), and (3), if we substitute marginal cdf's F(x) for u, and G(y) for v, then we obtain a multivariate cdf H(x,y). The above bivariate case can obviously be generalized to the multivariate case of $C(F(x_1),F(x_2),\ldots,F(x_N))$ for multivariate distributions of random variables X_i for i=1,2,…,N.

This is a simple but very useful result to construct many different forms of multivariate cdf's and we use this technique to construct a multivariate Pareto distribution using the Clayton (1978) copula that is widely utilized for characterizing long tails. We define the Clayton N-Copula as the function:

$$C(u_1,u_2,\ldots,u_N)=\left(\sum_{i=1}^{N}u_i^{-\beta}-N+1\right)^{-\frac{1}{\beta}} \quad , \quad \beta>0 . \tag{5}$$

Since this function is de facto a multivariate distribution in x_i's where $F(x_i)=u_i$, for i=1,2,….,N, the multivariate pdf of the copula can be derived as:

$$f(x_1,x_2,\ldots,x_N)=\frac{\partial^N C}{\partial u_1 \partial u_2 \ldots \partial u_N}\times\prod_{i=1}^{N} f_i(x_i), \tag{6}$$

where $f_i(x_i)$ is the marginal univariate pdf of random variable X_i. The partial derivative term is obtained as

$$\frac{\partial^N C}{\partial u_1 \partial u_2 \ldots \partial u_N}=\left(\prod_{i=1}^{N}u_i\right)^{-(\beta+1)}\times\prod_{i=1}^{N-1}(1+i\beta)\times\left(\sum_{i=1}^{N}u_i^{-\beta}-N+1\right)^{-\left(\frac{1}{\beta}+N\right)} . \tag{7}$$

From equations (6) and (7), we therefore have an analytical function with which to maximize the log-likelihood function of:

$$\ln\prod_{t=1}^{T} f(x_{t,1},x_{t,2},\ldots,x_{t,N}),$$

given the data sample of N-variates $(x_{t,1},x_{t,2},\ldots,x_{t,N})$ for sample size T, i.e. for t=1,2,…..,T. As this log-likelihood function is separable into the sum of a term involving the log of expression (7) and the other involving the cross-sectional sum of log-likelihood functions ln L_i for i=1,2,….,N in equation (2), we obtain the ML estimates by a two-stage optimization. First, the ML estimates of univariate pdf parameters $\hat{\gamma}_i$ and $\hat{\alpha}_i$ for i=1,2,….,N are obtained according to equations (3) and (4) that we saw earlier.

Then in the second stage we maximize:

$$\max_{\beta} \quad \sum_{t=1}^{T}\left(-(\beta+1)\sum_{i=1}^{N}\ln u_{t,i}+\sum_{i=1}^{N-1}\ln(1+i\beta)-\left(\frac{1}{\beta}+N\right)\ln\left[\sum_{j=1}^{N}u_{t,j}^{-\beta}-N+1\right]\right), \tag{8}$$

where $u_{t,i} \equiv F(x_{t,i}) = 1-\left(1+\frac{x_{t,i}-u_i}{\hat{\gamma}_i}\right)^{-\hat{\alpha}_i}$.

As the condition of loss exceeding the threshold may not hold for all country index returns for every day, only a sub-sample within each sub-period provided for simultaneous occurrences of the conditional returns. We considered the associated return loss movements of U.S. and European stocks separately from those of U.S. versus Asian stocks due to the time difference as discussed earlier, in order to see if there is any difference in the results, and also in order to obtain sufficient data points for the statistical analyses. The various ML estimates $\hat{\beta}$ are thus obtained and are reported in Table 7.

In Table 7, we also performed 1000 simulations of each case, i.e. random generations of uniform $\{u_{t,i}\}$'s, assuming independence across country returns, and then obtain 1000 estimates of $\hat{\beta}$ for each of the 8 cases, viz. 4 sampling sub-periods and whole period times the 2 contexts of US-Europe linkage versus US-Asia linkage. The ML covariance matrix containing the variance of $\hat{\beta}$ is not used here because of the small sample sizes. The simulated small sample distribution allows the computation of the p-values for each estimate of $\hat{\beta}$ in each of the 8 cases. The reported p-values are the probabilities of occurrences exceeding the estimate given the null hypothesis that the x_i's or correspondingly the u_i's are independently distributed without any associations.

Table 7: Estimates of Clayton Copula parameter β, and a Test using Small Sample simulation (1000 simulated estimated values per case) of the null hypothesis of independence and zero association across return losses. Number of joint daily data points for US S&P500, FTSE, DAX, and CDC-40 are 3, 7, 8, 18 for sub-periods 1, 2, 3, and the overall period. Number of joint daily data points for US S&P500, N225, and H.S.I. are 3, 4, 16, 23 for sub-periods 1, 2, 3, and the overall period.

	US and Europe		US and Asia	
	ML Estimate of β	p-value	ML Estimate of β	p-value
Sub-period 1	0.0820	0.450	1.048	0.066
Sub-period 2	0.0473	0.090	0.784	0.144
Sub-period 3	4.687	0.001	1.783	0.001
Overall period	0.871	0.010	1.748	0.010

The Clayton copula belongs to the Archimedean family of copulas with the desirable property that the copula parameter can be related to "correlation" or association such as the Kendall's τ. See Genest and MacKay (1986) for such a discussion. For the Clayton N-copula we employ here, two properties are worth mentioning: (a) as $\beta \downarrow 0$, the joint cdf $C(u_1, u_2, \ldots, u_N) \downarrow 0$;(b) as $\beta \uparrow \infty$, the joint cdf $C(u_1, u_2, \ldots, u_N) \uparrow 1$. In the latter, the implication is that as β increases from 0, the probability of observing associated higher losses x_i, across most if not all i's, increases. Under the assumption of cross-sectional independence in the simulated distributions, it also implies that a rejection indicates that there exists dependence across return losses and thus positive associations of tail losses across the countries.

From Table 7, it is seen that we cannot reject the null of independence for US and Europe in sub-period 1 and for US and Asia in sub-period 2 at significance levels of 10%. There appears to be some associations of US and Asia during the 1990's at significance level (p-value) 7% likely due to common trending in US and Japan during the early part of the Japanese recession that coincides with the US recession in 1991 to 1994. There also appears to be some associations of US and Europe during the 2000's till 2008 at significance level (p-value) 9% likely due to similar booms in property markets and similar excesses in deficit funding that shaped the movements of their stock markets. However, the most interesting finding is in sub-period 3 during the GFC post September 15, 2011, when strong positive associations of losses cannot be rejected at a significance level of 0.1%. The latter is a very strong empirical result indeed. It is also seen that the copula parameter $\hat{\beta}$ increased to a high 4.687 in US-Europe and 1.783 in US-Asia during the GFC till June 2011.

5. Management Implications and Conclusion

Unlike most portfolio studies that consider the entire unconditional distributions, our study focuses investigation at the loss tails and thus conditional return distributions of major country indexes across the globe covering the leading U.S. financial markets, and including the European economies of U.K., Germany, France, and the Asian economies of Japan and Hong Kong. Employing the generalized marginal Pareto distribution and the multivariate Clayton copula, we provide rigorous tests of the changes in the shapes of the conditional tail losses over time, and also tests of the changes in the conditional Value-at-Risk or conditional expected losses. The Clayton copula indicates that whenever equity portfolio losses occurred after September 2008 till June 2011, a significant increase in associations of high losses across the major countries occurred in conjunction with significant higher loss in each country. Thus there is now strong empirical evidence supported by rigorous methodology to assert the prevalence of heightened global financial risks and its contagion effect across the globe. This is the period after the Lehman Brothers collapse and the beginning of the GFC of 2008.

Increase in the copula parameter coupled at the same time with structural changes of longer and fatter tails in the negative or return loss tail distributions of the major country financial stock markets have a very strong message of unprecedented risk peaking if and when the markets experience huge losses. The frequency of such occurrences has increased in recent years. An important implication arising out of these conclusions is that banks under BASEL II and BASEL III and financial institutions in the near future should not underestimate its Conditional Value-at-Risk by using the normal distribution model since under stressed situations past September 2008, the portfolio return distributions have tails that simultaneously grow longer and thinner in the direction of the loss region.

Our findings provide a better understanding of the dynamics of current global finance and the multivariate global markets return loss distributions, and indeed suggest the sheer importance of modeling and forecasting sudden losses, and not underestimating them. This is critical for effective risk management in banks or financial institutions in the new and less understood risk landscapes we are now treading. Moreover, in terms of managing and containing contagion risks, investment managers could do well to monitor closely the end-of-market price movements in the lead U.S. stock markets, and to execute contingent plans accordingly. We saw in Table 2 how the European markets move more in tandem with U.S. market movements than the Japanese market. Contingent measures could include hedging programs selling index futures and buying index put options on European equities when a major drop occurs in the U.S. market. There could also be some portfolio rebalancing tilting toward a heavier Asian and Japan market weightage when the global market becomes jittery with contagion fears.

Finally, we provide a couple of remarks on possible limitations in this study and how future study could proceed. Firstly, the time horizon could certainly be extended to allow for a larger sample. Some of the conditional samples used in this study could be improved with smaller sampling errors when the sampling period is lengthened. This is particularly an interesting proposition as the later part of 2011 and the beginning of 2012 saw the rapid development of the Eurozone debt crisis. Secondly, perhaps a more extensive study could include explorations of other extreme value copulas.

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