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Recent developments on the crossing number of the complete graph

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Introduction

There are two well defined periods in the history of the problem of finding the crossing number of the complete graph. Before 2004, the lack of tools to study the problem restricted the developments to the study configurations with a small number of points, the finding of conjectured optimal solutions for arbitrary n, and some fairly basic counting strategies. [4] is an excellent survey for the main results in this period, including the early history of the problem.

In 2004, and independently, Ábrego and Fernández-Merchant [1] and Lovász *et al* [5] discovered a very strong relation between the number of crossings of a rectilinear drawing of the complete graph and another well known object in combinatorial geometry: the number of j-edges of the set of vertices. This lead to a series of improvements on the known lower bounds and, although still only one very basic property of optimal configurations is known (the set of vertices has three extreme points), the gap in the leading term of the known lower and upper bounds has been greatly reduced. A very recent survey of all this developments can be found in [2].

Recently [3] the concept of j-edge has been generalized to topological drawings of the complete graph, and the relation between crossings and j-edges has emerged as a promising tool for the general problem. The conjectured bound has already proven to be optimal for some families of drawings, including 2-page drawings and monotone drawings.

References

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