COMPLEX TASKS IN DECISION MAKING VERSUS ECONOMIC THEORY

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Abstract:

Economics provides natural situations for complex tasks in decision making. A complex task starts with an empirical investigation, continues by creating an economic model using mathematical modelling and ends with a decision making applying economic theory. The paper presents an example from demand analysis showing that a close interplay is needed between constructing a model and making consequent decision analysis.

Keywords: demand function, approximation, price elasticity of demand, the normal case conditions, total revenue

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1 Introduction

Decision making in economics is a final stage of a process that usually starts with an empirical analysis and continues by developing a model. In the sequel, by a model we mean a mathematical model, ie.a model created by mathematical tools. For practical learning purposes we speak about a *complex task.* The crucial point is the quality of the model. Since modern economy is a very complicated place, the verification of the quality of economic models is an extremely demanding problem. An important aspect of scientific investigation is to sort out the „bad“ models from the „good“. For verifying economic models in this way two general methods are employed: (a) a direct approach, which tests the validity of the basic assumptions on which a model is based; and (b) an indirect approach, which attempts to confirm validity by showing that a model correctly describes real-world events. Our further considerations will concern a direct approach. It is quite convincing, that the necessary condition for a model to be „good“ is its agreement with economic theory. As a consequence of the complexity of economic processes, the validity of economic laws is related to so called „ *the normal case*“ conditions, that mostly result from economic feeling and reasoning. The normal case conditions must be permanently taken into account during creating the model. There is another „invisible“ source of „uncontrolled“ faults when constructing a model, namely the use of mathematical software to obtain results in terms of numbers or relevant mathematical structures (functions, vectors,…). For an economist to feel sure about the correctness of the model is necessary to possess firstly a good knowledge of economic theory and secondly an adequate command of fundamentals of mathematics „hidden“ in a software.

The aim of this paper is to present a typical example from demand analysis showing all the mentioned aspects. In the next section a current model of a market demand under imperfect competition is presented. We analyze discrete empirical data describing how the quantity demanded *Q* of a commodity depends on its price *P*. In a standard way, using the tools of approximation of functions we get a mathematical model of a demand in the form of a continuous function – a *demand function Q = D(P).* We will point to the importance of visual analysis to choose the suitable curve that simulates the trend of discrete data and also complies with the normal case conditions. We will refer to the theoretical results concerning the elasticity properties of resulting functions. Finally, we will perform decision analysis with a view to a total revenue.

2 Model of a demand under microscope of economic theory

From the investigation how the quantity demanded *Q* of a commodity depends on its price *P* the data were obtained as given in the Table 1:

Price *P*	15	16	17	18	19	20	21	22	23	24	25
Quantity Q	175	168	150	149	143	130	125	113	115	110	105

Table 1

By the table a functional dependence between price and quantity is given, more precisely the function *f* defined on discrete domain { 15, 16,…, 25 } is given. This function is reffered to as a *demand function.* For the primary stage of study of the problem the following graph of this function composed of isolated points (sometimes called a *scatter diagram*) is important (Fig. 1):

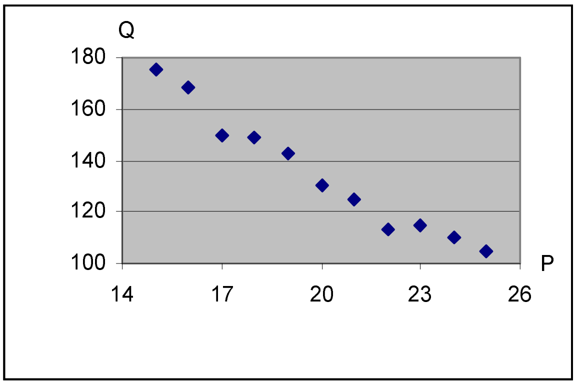

Fig. 1

Reflecting the basic economic law of decreasing demand any such demand function (under the normal case conditions) must be decreasing. It ought to be explained here that some irregularities(values increasing values 113, 115 corresponding to 22,23) are associated with a statistical nature of data. Now, to create a model suitable for further calculations, we approximate a given discrete function *f* by a continuous(elementary) new(demand) function *D, Q = D(P)* that provides a fit to these data; D is said to be a *regression function*. To find such function a software package is used. But there is a crucial stage before using a software – to determine the type of such function according to the allocation of points in scatter diagram depicting f. For this purpose a visual assessment of the diagram is required to select a „best" fitting function. In fact, the choice is not so large, but it claims a fundamental knowledge of the shapes of graphs of basic elementary functions. Mostly polynomials of lower degrees, exponencial or hyperbolic or logarithmic function are employed. An economists should be able to overcome this step. In our case, the selection of „well" fitting functions may contain:

(1) a linear function $Q = aP + b$ ($a<0, b>0$ due to down-sloping straight line)

(2) an exponencial function $Q = exp(aP + b)$ ($a<0$, $b>0$)

(3) a quadratic function $Q = aP^2 + bP + c$ ($a >0$ estimating a convex shape of a parabole, $c>0$, $b<0$)

(4) a hyperbolic function $Q = a + b/P$ ($a,b>0$).

At this stage, usually all selected functions are accepted as a „well" fitting functions and the choice one of them depends only on particular reasons and automatically software is used to find any of them setting concrete data. Regrettably such certainty lacks justification. Unfortunately not all gained functions may serve as demand functions. The reason rests with economic theory. When analysing demand elasticity, Allen [1] states that under the normal case conditions the price elasticity of demand is higher at higher prices. Precisely, under the normal case conditions, the *price elasticity of demand* $E_D(P)$, given by the formula

$$(5) \qquad E_D(P) = -\frac{P}{Q}\,Q',$$

where $Q = D(P)$ is a demand function and Q' denotes the derivative, must be an increasing function. It is known that not all decreasing functions satisfy this condition. The following Propositions 1 and 2 state(Mezník[6,7]), that linearity or concavity are sufficient to quarantee the desired property:

Proposition 1 *If a demand function is linear, then the price elasticity of demand is increasing.*
Proposition 2 I*f a demand function is concave, then the price elasticity of demand is increasing.*

Analogous property does not hold for convex functions. So, with respect to selected functions (1)-(4) only linear function is safe, the other functions (2)-(4) are convex and we are not sure whether they satisfy the mentioned normal case condition. Let us examine these functions:
For an exponencial function of the form (2), using formula (5), we get

$E_D(P) = \dfrac{-P}{e^{aP+b}} e^{aP+b} \cdot a = -aP$. From here it follows ($E_D(P))' = -a > 0$ (since $a<0$) and thus $E_D(P)$ is

increasing . We reach the conclusion, that <u>exponencial function</u> of the form (2) <u>is for demand analysis</u> in principle <u>applicable.</u> As to quadratic function (3), the situation is not unique and for convex-shaped scatter diagrams of demand is more safe to avoid using it(we may find regression quadratic function, but consequently we must verify, whether the normal case condition is satisfied) . For a hyperbolic function of the form (4) applying (5) we obtain

$E_D\ (P) = \frac{-P}{a+b/P} \cdot (\frac{-b}{P^2}) = \frac{b}{aP+b}$. Then ($E_D\ (P))' = \frac{-ab}{(aP+b)^2} < 0$ (since $a,b>0$) and from here it

follows that $E_D\ (P)$ is decreasing. We reach the conclusion, that hyperbolic function of the form (4) is for demand analysis in principle not applicable. Now, the choice narrows to functions (1) and (2).Using software(MAPLE, MATLAB, MICROSOFT EXCEL,…), by applying the least square method, the resulting regression function is found (under the stated conditions it is given uniquely in each case). We get the following solutions to the problem, demand functions D_1, D_2 :

(6) $Q = -6{,}874P + 272{,}677 = D_1(P)$

(7) $Q = \exp(-0{,}053P + 5.955) = D_2(P).$

Having a demand fuction, we can make various decisions. For simplicity we use demand function D_1.

I *Finding the price maximizing total revenue*

A total revenue TR is given by the formula $TR = P.D_1(P) = P.(\ -6{,}874P + 272{,}677).$ Using software or

the algorithm of finding extrema, we get that P_0 = 19,83 is the approximate price maximizing total revenue TR and the corresponding value of total revenue equals approximately 2704,12.

II *Decide about the change of price to increase total revenue*

The criterion for such decision provides the price elasticity. The following decision procedure is justified:

(a) if the price elasticity for some price is greater than 1(ie. demand is elastic), then reasonable small decrease in price results in an increase of total revenue

(b) if the price elasticity for some price is less than 1(ie. demand is inelastic), then reasonable small increase in price results in an increase of total revenue.

For this purpose we must compute the price elasticity for demand function D_1. Using (5) we have

$$E_{D_1}(P) = \frac{-P}{-6{,}874P + 272{,}677} \cdot (-6{,}874) = \frac{6{,}874P}{-6{,}874P + 272{,}677}.$$

Apparently $E_{D_1}(P) = 1$ for $P \approx 19{,}83$ (note, that it is simultaneously the point of maximum value of total revenue). Hence for prices P< 19,83 demand is inelastic, for prices P> 19,83 demand is elastic.

The following economic decision is straightforward:

(i) if the current market price is less than 19,83, then we slightly increase the price

(ii) if the current market price is greater than 19,83, then we slightly decrease the price.

Conclusion

We presented complex task on market demand starting from empirical study describing the relation between the price and the demanded quantity of a commodity and ending with mathematical model of the market, that enables making qualified economic decisions. The paper shows, that to solve such quite practical complex tasks an economist must be knowledgeable about economic theory and fundamentals of business mathematics with relevant mathematical software. This is in accordance with the requirements of modern economy to make sophisticated managerial decisions.

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