On the use of the Cauchy distribution to describe price fluctuations in R&D and other forms of real assets

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Abstract

An improved model for describing the returns of assets that result from R&D efforts is needed. Such a model may lead to better decision support tools to monetize the value of R&D activities for both public and private sector technology managers. Real option pricing methodologies are often used to gauge appropriate funding levels for assets such as R&D projects that contain large time-dependent uncertainties. A study of the commonly used Black-Scholes equation finds that the Gaussian distribution assumption used to describe the behaviour of the underlying assets' fluctuations is not appropriate for R&D. This conclusion is based on a study of 43 military R&D projects and 100 micro-cap technology intensive small firms. A power law, such as the Cauchy distribution, is shown to be more accurate in describing fluctuations in returns on R&D investments. Using historical data we find that the Cauchy distribution is a better representation of the underlying assets' behaviour in military R&D projects and in technology intensive firms with small market capitalization (i.e., single project firms that are commercializing R&D).

1. Introduction

The need for an accurate quantitative decision support tool in assessing the potential value of R&D projects is of critical importance to innovation management. Accurately monetizing the value and importance of R&D investments is a problem faced by managers of technology intensive firms seeking to maximize returns on investment in R&D efforts [1, 2, 3]. Alternatively, this is also a key issue facing managers of R&D funds (e.g., public and private R&D-focussed funds) [4, 6]. Fairly and transparently allocating these funds is a critical task for a public funding agency faced with the evaluation of several research proposals. Each proposal must be carefully assessed for its merit and should undergo some form of cost-benefit analysis moderated around some established risk profile. Using reliable quantitative data is a key step in building optimal R&D portfolios in both private and public sectors.

There are currently many tools at a manager's disposal to capitalize R&D activities - although the task is sometimes referred to as an intractable art [5]. The commonly used Net Present Value (NPV) technique is well documented [7]. However, NPV tends to be overly conservative in its estimate of the real value of performing R&D for a firm (i.e., the value in having the opportunity to pursue a certain course of action) [8, 9]. The technique is also quite sensitive in choosing a correct discount rate to accurately capture the proper behaviour of the technology during its maturation phase. In response, Real Options Pricing (ROP) techniques have been gaining traction in the past decade or so. The ROP methodology allows one to statistically model and account for the intractability inherent with decisions that rely heavily on managerial discretion and uncertainty of outcome, such as R&D project selection. This methodology has not been heavily adopted in practice as the underlying principles are not fully developed [10]. In this article, we concentrate on one of the main assumptions used in Real Options Pricing (ROP) methodologies that support decision making regarding the level of funding for R&D investments. This study specifically analyzes the behaviour of real assets' value fluctuations through the consideration of empirical evidence.

More precisely, we investigate one of the main assumptions that goes into the mathematical model used to price options. Namely, that the underlying asset's returns fluctuate according to a normal distribution. This article illustrates

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how monetary returns on investments in R&D do not strictly follow a normal distribution. We present actual data for real asset returns and show that these are more appropriately described by a power law distribution such as the Cauchy distribution function [11].

The article is presented as follows. The next section introduces the concept of options and the importance of the accuracy of its underlying assumptions. Data on technology intensive assets is then provided to illustrate that the Gaussian distribution of underlying price fluctuations does not always apply in assets where value is closely linked with R&D.

2. Background

Financial options belong to a class of financial instruments called derivatives. That is, instruments whose values depend on the values of other underlying variables. Options are used extensively in the financial field for stocks, bonds, and futures markets. For decades however, the purchase and sale of options (i.e., rights of purchase) was unusual and infrequent. The lack of popularity of options was largely due to the complexity and uncertainty of how to price them appropriately. Consequently, sellers tended to overprice options to ensure that they did not lose money and buyers tended to undervalue options to ensure that they did not overpay. Following the publication of a formula for determining the price or cost of an option, this problem of buyers and sellers disagreeing on the cost or price of an option was resolved because everyone could agree on the same economic value being offered by an option [12]. The resulting formula, known as the Black-Scholes model, was widely adopted to assign the value of an option. Due to the tremendous impact of this discovery to the field of finance and to financial markets, Scholes and Merton received the 1997 Nobel Prize in recognition of their contribution. However, an over-reliance on the Black-Scholes equation (and its assumptions) is blamed for the widespread program trading that led to the market crash of 1987. Results obtained from a ROP analysis should always be taken in the context of enhancing a more complete quantitative analysis concerning investments, such as R&D [13].

More recently, financial options were successfully applied to non-financial, or real assets. The term 'real option' was coined by Myers in 1977 who postulated that growth opportunities such as R&D expenditures could be viewed as call options [14]. Many have later applied models traditionally used to price financial options and extended them to quantify the economic value of research in terms of future implementation of derivative technology [15, 16, 17, 18].

There are some identified issues that are preventing the widespread use of this technique. Notably, the higher mathematical complexity of the model obscures the assumptions relevant to the model and turns this tool into a "black-box" that makes decision makers uneasy. One of these assumptions, related to the Black-Scholes equation, is that underlying asset fluctuations follow a geometric random walk (i.e., a stochastic process where the logarithm of the randomly varying quantity follows a Brownian motion or random walk). As we show, this is not the case for returns on R&D investments. Fluctuations that are larger than what would be permitted using a Gaussian distribution are commonly observed in returns obtained from commercial returns linked with R&D activities. We propose a simple methodology for testing the validity of the geometric random walk assumption. Finally, we model the behaviour of these underlying assets using a Cauchy distribution for single projects and R&D intensive small firms. The Cauchy distribution offers a better model describing real underlying asset returns.

3. Quest for sigma

We present data on fluctuations in real assets to show that returns are in fact better described by a Cauchy distribution function. In our first example, we use historical R&D data, as opposed to future projected values, in order to investigate the actual behaviour of these types of assets. The assumption is that the fluctuations in price seen in historical data should behave in a 'normal' way in comparison with future projections. We measure the historical accuracy of the real option pricing methodology by "backward engineering" solutions.

As mentioned earlier, the Black-Scholes equation gives a fair call price for an option with a strike price of x_S (i.e., the costs related to the commercialization of the asset) , a time to expiration of T, and an initial underlying asset price of x_0. The Black-Scholes equation is defined as

$$C = x_0 \Phi\left(\frac{y_-}{\sigma\sqrt{T}}\right) - x_S e^{-rT} \Phi\left(\frac{y_+}{\sigma\sqrt{T}}\right) , \tag{1}$$

where r is the risk-free annual interest rate (i.e., typically the five-year treasury bond rate), $y_{\pm} = \log(x_S/x_0) - rT \pm \sigma^2 T/2$, and $\Phi()$ is the normal cumulative distribution function. The fluctuation in the underlying asset, or return η in the value of the underlying asset at time i, is defined as

$$\eta_i = \frac{\delta x_i}{x_i} \approx \log x_{i+1} - \log x_i \, . \tag{2}$$

Performing a real option pricing analysis involves finding an appropriate value of σ. That is, being able to contain the fluctuations in the underlying asset as tightly as possible while adequately describing those fluctuations. Typical values of σ for real options tend to be quite large, $\sigma > 0.2$, since there are many unknown quantities in the underlying price estimates [19]. Due to the shape of the Gaussian distribution function however, increasing the value of σ in order to capture larger fluctuations occurs at the expense of accuracy in measuring the overwhelming bulk of returns centered around its mean value, μ.

The origin of σ comes from one of the basic assumptions embedded in the Black-Scholes equation. Namely, that the underlying price fluctuations are assumed to be distributed according to a normal distribution with a standard deviation of σ. This certainly seems plausible and a natural consequence of the central limit theorem. The normal probability distribution function $p(\eta)$ can be written as

$$p(\eta) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(\eta-\mu)^2}{2\sigma^2}} \, , \tag{3}$$

where μ is the mean value of the distribution and σ is a fitting parameter related to the broadness of the distribution. This latter parameter is meant to contain all the information pertaining to managerial discretion, technology breakthroughs or failures and market positions. Due to the exponentially decaying tails, the normal distribution does not allow for large fluctuations (approximately larger than 2σ) or those situations that are are outside of 'normality'. That is to say that normal processes have observables that do not behave far outside of their mean value. The central limit theorem only applies if the the underlying variables are independent - which is not always the case in price fluctuations. Therefore, we show that the Gaussian distribution is not always the most appropriate for use in a ROP methodology.

In the case of assets that are closely linked with R&D efforts, the price fluctuations are not normally distributed and are better characterized by the Cauchy distribution function. The Cauchy distribution has no defined mean nor standard deviation and depends on other parameters. It can be written as

$$p(\eta) = \frac{A}{\pi\left(A^2 + (\eta - \eta_0)^2\right)} \, , \tag{4}$$

where A is a scale parameter and η_0 describes the location of the peak.

The following short section describes the technique used in this article to both fit data and obtain quantitative measurements on the goodness of the fit. We start by obtaining the complementary distribution function (CDF)

$$P(\eta) = \int_{\eta}^{\infty} p(\eta')\, \mathrm{d}\eta' \, . \tag{5}$$

Distribution functions are fitted using the methodology presented by Clauset et al. [20]. First the scaling parameters of the distribution functions are estimated using the method of maximum likelihood, which is known to provide accurate estimations for these fitting parameters. Second, the goodness of the fit is calculated using a p-value calculated by counting the ratio of the number of times that the maximum distance between the empirical data and its corresponding best fit is smaller than versus a large number of bootstrap data sets and their corresponding best fits. The distance between the empirical or bootstrap data and their best corresponding fitting functions are obtained using the Kolmogorov-Smirnov method. This iterative process is used throughout this article to estimate best of fit parameters and evaluate the goodness of describing data using a distribution function over another.

4. Sources of data

Two types of real assets are presented that are best described by a power law. Although more sources may exist and further research in the use of power laws to describe fluctuations in real assets for use in a ROP scheme is desirable.

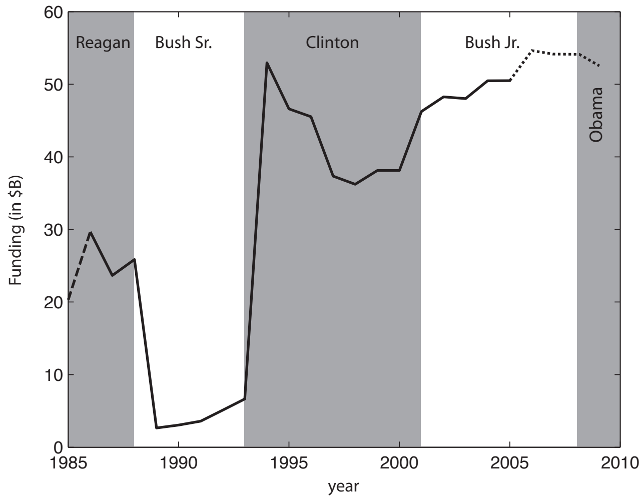

Figure 1: Historical funding for the V-22 Osprey military platform in the U.S. One can readily observe the political impact on funding as the election of a new President is strongly correlated with large changes in funding. The dashed line represents the product evaluation estimate, the solid line shows the development estimate and the dotted line shows the production estimate.

We start with a measure of 'return' on development investment for military procurement contracts in the US aerospace sector. Secondly, we present data on 100 small technology intensive firms whose market indices depend heavily on R&D efforts.

American Selected Acquisition Reports (SARs) were used to obtain data on the return on investment for military platforms [21]. These annual reports coincide with the budget of the Department of Defense and contain acquisition program costs and schedule changes. SARs contain summarized data on cost, schedule, and technical status for most platforms currently under development for the US military. These data embody the inherent managerial process (e.g., political changes in military priorities) and represent more accurately the types of fluctuations that are to be expected in situations where a manager decides to invest in R&D or not. Although the format of the report and the standards for reporting data have often changed throughout the years (e.g., inconsistency in applying inflation), it is nevertheless possible to obtain a relative difference in yearly cost estimate for each military program that is sufficient to show abnormal fluctuations in underlying asset prices [22]. The aggregated SAR reports forms a data set that spans roughly four decades and offers a rich account of military R&D costs (product development) together with the resulting platform costs (production estimates). This allows one to analyze fluctuations in cost and quantity in order to assess future risk exposure. The reports include procurement quantities as well as total costs. For this research, the total costs were used to measure fluctuations that include fluctuations in the number of acquired units. The distribution of the fluctuation therefore embodies managerial uncertainty and other inherently "political" aspects. The idea is that this justifiably adds to the uncertainty in the final value of the program. It is argued that SARs are nevertheless suitable for identifying broad trends and/or short time fluctuations across a number of programs.

Program costs are typically divided into product evaluation (PE), development (DE), and production estimate (PdE) categories. The latter representing the most accurate available data describing the actual cost (or value) of military programs in the United States. Probability distributions for the arithmetic returns of program values, $p(\eta)$, were compiled using equation 2. For this research, information on 43 military programs was used (e.g., AH-64 Apache attack helicopter, B-1 Lancer bomber, F-35 Lightning II fighter, and others) with each program being active for an average of 13 years between 1969 and 2007.

Using historical data on aerospace and defence sector procurement costs, it is possible to identify the general distribution of these fluctuations (see equation 2). It is essential to analyze past procurement information in order to construct a theoretical model that can replicate important characteristics observed in real data before making future predictions. For example, R&D projects can exhibit large variations due to managerial discretion (figure 1 where funding of the V-22 Osprey aircraft is shown). Large fluctuations in funding are associated with changes in administrations over the years of this particular procurement's funding. Since the ROP model requires one to estimate the variability of future values, it is important to understand how this variability behaves.

Second, data are presented on stock market returns for technology intensive firms whose main commercial output consists in R&D investments for a limited number of commercial platforms. For this, we first identified the 100 most research intensive companies with a market capitalization of less than $100 million using Standard & Poor's Capital IQ. Research intensity is defined as the ratio of revenues to R&D expenditures. It is assumed that the stock price is a proxy of the "value" that stems from current investments in innovative activities. This link has already been established between a firm's excess returns and investments in R&D [23]. We concentrate on firms with low market capitalization to better ensure that the R&D is directly linked with the main stream of revenue (e.g., technology intensive product or product line). An overview of the firms used in this study can be found in table 1.

Table 1: Description of the 100 firms used in this study. The majority of small firms with with R&D intensity are in the healthcare sector. These firms also occupy the top spots overall.

Sector	# firms	Top R&D intensive firm
Consumer Discretionary	3	Li-ion Motors Corp. (OTCBB:LMCO)
Consumer Staples	1	Health Enhancement Products Inc. (OTCBB:HEPI)
Energy	2	New Generation Biofuels Holdings, Inc. (OTCPK:NGBF)
Healthcare	60	Aastrom Biosciences, Inc. (NasdaqCM:ASTM)
Industrials	12	Beacon Power Corporation (NasdaqCM:BCON)
Information Technology	20	Parkervision Inc. (NasdaqCM:PRKR)
Telecommunication Services	1	Broadcast International Inc. (OTCBB:BCST)
Utilities	1	Raser Technologies, Inc. (OTCBB:RZTI)

5. R&D intensive assets are not quite normal

There is a strong theoretical foundation for having log-normally distributed underlying returns [24], however a growing number of researchers are considering the use of other distributions—such as power laws—to describe data for both financial markets and real assets [25, 26, 27]. Power laws are interesting, if anything, because of their ubiquitous appearance in the scaling parameters of physical systems near criticality (e.g., high level of connectivity between nodes in complex network) [28]. In some cases, the mere presence of emergent power law-type behaviour is used to indicate that a phenomenon is approaching criticality (e.g., long range order characteristic of phase transitions in physical systems). Therefore, it is perhaps not totally surprising that a firm's distribution of returns on commercial success (i.e., emergent behaviour) also scales as a power law. A technology intensive firm's success often depends on the complex collective behaviour of intricately-connected internal and external collaborators (i.e., networked nodes) engaged in R&D activities (i.e., collective behaviour) [29].

For example, the distribution of returns of indirect government investments in private R&D for specific military procurement projects. In the case of military procurement, the underlying asset's value is re-evaluated yearly. In this case, it is not clear a priori which distribution function will best describe the actual variations in the resulting commercial value of R&D activities – although we suspect large fluctuations instinctively. Consequently, it is important to understand the behaviour of the underlying assets used in an options analysis model that makes strict a priori assumptions on this behaviour (e.g., the Black-Scholes equation).

Figure 2 shows price fluctuations in the development and production estimate phases of aerospace-specific US military procurement contracts, both the probability density function and its respective complementary distribution function. The distribution is slightly asymmetric due to the fact that there are fluctuations greater than 1, $\eta > 1$. We present logarithmic returns thus there cannot be fluctuations smaller than -1. The Cauchy distribution (equation 4) is shown to offer a better fit for the data over the normal distribution (other popular distribution functions were also tested, but resulted in lower p-values). One can readily notice that a Gaussian distribution is inadequate in its

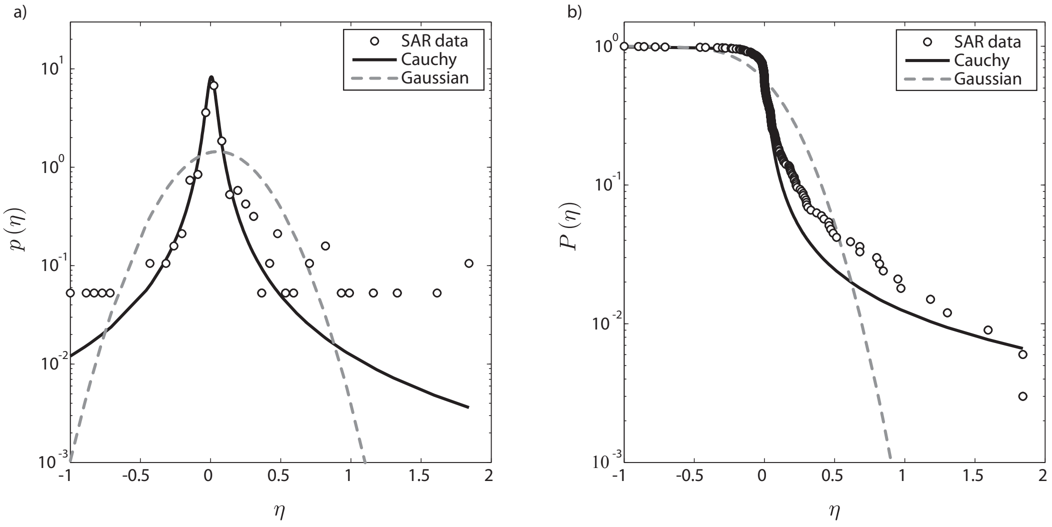

Figure 2: Probability a) and cumulative b) distribution functions showing a comparison between the Gaussian (μ = 0.052, σ = 0.28, and p-value= 0.00) and the Cauchy (A = 0.038, η_0 = 0.0056, and p-value= 0.11) distribution functions in describing the distribution of price fluctuation for all aerospace and defence projects requiring R&D in US military procurement contracts.

representation of the observed data – specifically near the tails of the distribution where large fluctuations are observed in the empirical dataset. Although the relatively small sample size is noticeable using only these data, the shape of the distribution remains characteristically similar between fluctuations observed in all three phases of procurement.

One can use the variability information on expenditures in R&D obtained during the product development estimation phase as a proxy of future technology market price. Ideally, one would use disaggregated commercialization figures from prime contractors to directly measure the capital resulting from individual R&D efforts, however, these data are not readily available on a large enough scale to be statistically significant. Regardless, it seems appropriate to conclude that managerial (or presidential) discretion has a large impact on the potential outcome of R&D efforts and that these fluctuations can not be modelled using the Gaussian distribution function.

Due to its availability, the SAR dataset is used to show that returns on real assets stemming from R&D activities do not follow a normal distribution. It is therefore suspected that other proxy measures that rely on R&D activities behave in a similar fashion. The following results consider the fluctuations in various market stocks that are closely tied to R&D activities. We show that a normal distribution is not the most appropriate way to model the stock price behaviour of these firms. Therefore, a Gaussian distribution to describe fluctuations, or using the Black-Scholes equation to price real options, is not appropriate in the case of these R&D intensive firms.

Weekly stock market data for the top 100 firms in research intensity with a market capitalization of less than $100 million between 2000 and 2010 was collected and analyzed. All of the analyses were conducted for the periods of 2000 to 2007 and 2000 to 2010 in order to identify and avoid potential biases due to the market crash of 2007. We did not observe significant discrepancies between the two dataset and therefore opted to present the 2000 to 2010 returns only. For each of these firms, we have fitted the returns using both Gaussian and Cauchy distribution functions. Figure 3a shows the best Gaussian fit out of all firms. Convera Corp.'s fluctuations in return are the most 'normal' out of all firms with a p-value= 0.11. However, in figure 3a, even the best Gaussian fit is slightly surpassed by a Cauchy fit with a p-value= 0.14 due to the presence of large positive returns. Figure 3b shows the best Cauchy fit in describing the fluctuations in return for Implant Sciences Corp with a p-value= 0.50. In this case, it is interesting to note that the

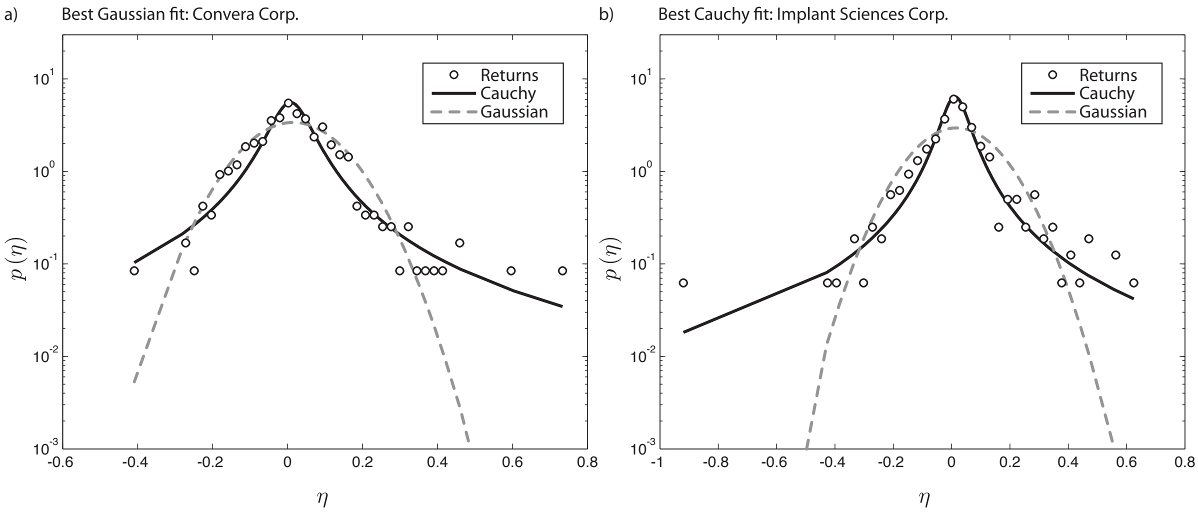

Figure 3: a) Best Gaussian fit out of the 100 top R&D intensive firms. Although this is the best Gaussian fit (μ = 0.015, σ = 0.12, and p = 0.11), the Cauchy fit (A = 0.057, η_0 = −0.0079, and p = 0.14) still offers a better description of the underlying asset. b) Best Cauchy fit (A = 0.050, η_0 = 0.012, and p = 0.50) shown together with a Gaussian fit (μ = 0.016, σ = 0.14, and p = 0.0013). There is a week where the firm's market price lost almost all of its value and one can see that the Cauchy distribution is able to capture that behaviour. These types of events are statistically impossible in a strict Gaussian model.

company suffered a large loss event. There is a week where the firm's market price lost almost all of its value and one can see that the Cauchy distribution is able to capture part of that behaviour. These types of events are statistically impossible in a strict Gaussian model, however, are assumed to occur relatively frequently in R&D development. Large variations are expected outcome (and part of the risk) in science and technology investments. For example, loss of value is assumed to correspond to situations where critical experiments fail or where technological barriers to improvement of a critical characteristic are encountered. The opposite is also true - market breakthroughs and revolutionary products/services resulting from R&D can also lead to large positive returns for a firm. In both cases, the Gaussian distribution does not capture the overall behaviour of the returns very well.

Practically speaking, p-values$\leq$ 0.1 are a good indication that the hypothesis that the data are taken from a particular fit can be ruled out [20]. By considering the p-values of all of the 100 firms' fits we see in figure 4 that only two firms have returns that can be said to be Gaussian according to our rule of thumb. This provides solid evidence that the Gaussian distribution simply is not correct in modelling the behaviour of the underlying asset for firms with high R&D intensity. There are far more large fluctuation events than what would be predicted by a Gaussian distribution. This is to be expected in technology intensive firms that are particularly sensitive to the commercial success (or failure) of their R&D efforts.

Similarly, figure 5 presents a histogram of the p-values obtained by fitting the returns of the same firms using a Cauchy distribution. In this case, 61 firms have fluctuations that can be adequately described using a Cauchy distribution. In fact, a large portion of the firms have p-values that are well above our somewhat conservative threshold of 0.1.

Clearly for R&D intensive assets, the Black-Scholes equation should not be used for decision support due to the lack of statistical significance of one of its main assumptions. While imperfect, the Cauchy distribution is clearly superior as a model of the underlying fluctuations that can be used in building a decision support tool for this sort of asset.

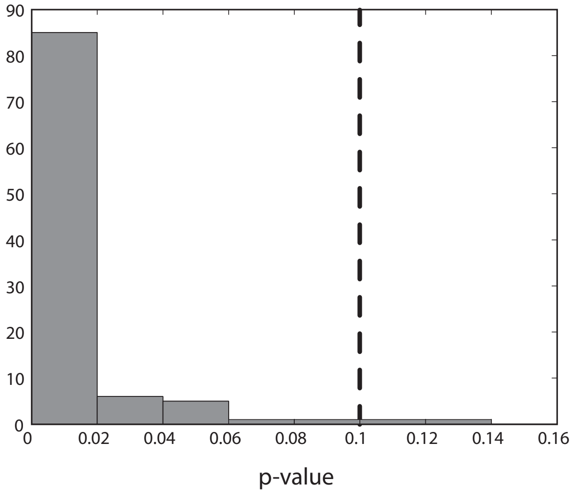

Figure 4: Distribution of the p-values obtained by fitting the market returns of the top 100 research intensive firms with a Gaussian distribution. One notices that in both cases, only two firms have a p-values> 0.1, shown by the dashed line.

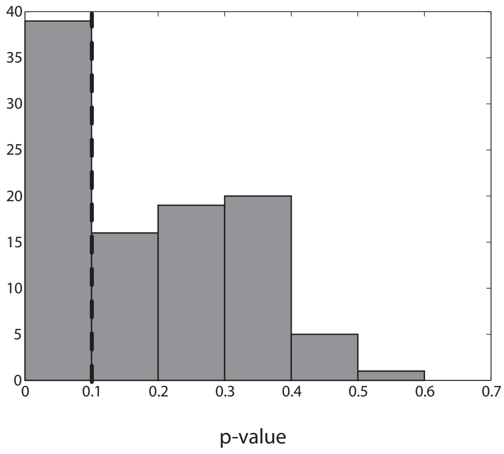

Figure 5: Distribution of the p-values obtained by fitting the market returns of the top 100 research intensive firms with a Cauchy distribution. This shows that 61 firms now have a p-values> 0.1 by fitting with a Cauchy distribution, shown by the dashed line.

6. Conclusion

The Cauchy distribution better represents returns for a number of real assets. For example, a Cauchy distribution was shown to be more accurate in describing the fluctuations in development phase procurement costs that relied on substantial government funded research and development. The same distribution function was also shown to be more accurate in describing the variations in indices from companies who rely heavily on R&D activities for their market value. A better model for the behaviour of the returns for R&D can lead to better decision support tools for public (and private) entities who fund R&D activities. A ROP methodology that is based on a more accurate model of R&D outcome returns can be a powerful quantitative tool to assist in the determination of the relative values of a number of potential R&D projects. We are not advocating for the replacement of expert judgement by a mathematical model; rather, we offer early development work on a new tool to inform managers on STI funding. More importantly, we have shown that technology managers should not rely on the Black-Scholes equation to make decisions on R&D activities.

Although the use of the Cauchy distribution in describing real asset fluctuations is perhaps not ideal for all cases where one uses a ROP methodology, this research underlines the importance of understanding the behaviour of the variations in the underlying assets before using a traditional Black-Scholes approach in a ROP methodology. The Cauchy distribution has no mean nor variance and does not converge, however, solutions can nevertheless be estimated using Monte Carlo simulation techniques.

The next step is to develop a generalized model of real option pricing that allows one to vary the behaviour of the underlying asset for these classes of derivatives that are known to undergo large fluctuations [11, 24]. This tool, combined with portfolio management tools could lead to more accurate quantitative assessments of technology valuation. A more reliable Real Options Theory would allow managers of private and public technology intensive funds to make better informed decisions on the relative priorities of specific R&D activities within their portfolio.

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