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# New supersymmetric quartet of nuclei in the $\boldsymbol{A} \boldsymbol{\sim} \mathbf{1 9 0}$ mass region 

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#### Abstract

We present evidence for a new supersymmetric quartet in the $A \sim 190$ region of the nuclear mass table. New experimental information on transfer and neutron capture reactions to the odd-odd nucleus ${ }^{194}$ Ir strongly suggests the existence of a new supersymmetric quartet, consisting of the ${ }^{192,193} \mathrm{Os}$ and ${ }^{193,194}$ Ir nuclei. We make explicit predictions for the odd-neutron nucleus ${ }^{193} \mathrm{Os}$ and suggest that its spectroscopic properties be measured in dedicated experiments.


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Nuclear supersymmetry is a composite particle phenomenon that should not be confused with fundamental supersymmetry, as used in particle physics and quantum field theory, where it is postulated as a generalization of the LorentzPoincaré invariance as a fundamental symmetry of nature and predicts the existence of supersymmetric particles, such as the photino and selectron, for which, however, experimental evidence is yet to be found. If experiments about to start at the CERN Large Hadron Collider (LHC) find evidence of supersymmetric particles, the supersymmetry would be badly broken, as their masses must be much higher than those of their normal partners. In contrast to particle physics, nuclear supersymmetry has been subjected to experimental verification.

Nuclear supersymmetry was proposed more than 25 years ago [1] in the context of the interacting boson model (IBM) and the interacting boson-fermion model (IBFM), which have proved remarkably successful in providing a unified framework of even-even [2] and odd-even [3] nuclei, respectively. One of its most attractive features is that it gives rise to a simple algebraic description, in which dynamical symmetries and supersymmetries play a central role, both as a way to improve our basic understanding of the importance of (super)symmetry in nuclear dynamics and as a starting point for more precise calculations. Nuclear supersymmetry provides a theoretical framework in which different nuclei are treated as members of the same supermultiplet and whose spectroscopic properties are described by a single Hamiltonian and a single set of transition and transfer operators.

Originally, nuclear supersymmetry was formulated as a symmetry among pairs of nuclei consisting of an even-even and an odd-even nucleus $[1,4,5]$. Subsequently, by including the neutron-proton degree of freedom, it was extended to quartets of nuclei, in which an even-even, an odd-proton, an

[^0]odd-neutron, and an odd-odd nucleus form a supermultiplet [6]. Supersymmetry imposes strong constraints on both the collective (bosonic) and the single-particle (fermionic) degrees of freedom. Nevertheless, various nuclei have been identified as examples. The odd-odd nucleus ${ }^{196} \mathrm{Au}$, together with the odd-neutron nucleus ${ }^{195} \mathrm{Pt}$, the odd-proton nucleus ${ }^{195} \mathrm{Au}$, and the even-even nucleus ${ }^{194} \mathrm{Pt}$, have been verified experimentally using state-of-the-art techniques [7-9] to closely fulfill the rules that define a supersymmetric quartet $[6,10]$. The interpretation of these four nuclei as members of a supersymmetric quartet made it possible to predict [6] the properties of ${ }^{196} \mathrm{Au}$ almost 15 years before they were measured experimentally [7,8].

It is the purpose of this paper to present evidence for the presence of a new quartet of supersymmetric nuclei in the mass $A \sim 190$ region, consisting of the ${ }^{192,193} \mathrm{Os}$ and ${ }^{193,194} \mathrm{Ir}$ nuclei. The evidence is based both on energies and on transfer strengths.

The $A \sim 190$ region of the nuclear mass table is a particularly complex one, displaying transitional behavior such as prolate-oblate deformed shapes, $\gamma$ instability, triaxial deformation, and coexistence of different configurations, which presents a daunting challenge to nuclear structure models. Despite this complexity, the $A \sim 190$ mass region has been a rich ore of empirical evidence for the existence of dynamical symmetries and supersymmetries in nuclei both for even-even, odd-proton, odd-neutron, and odd-odd nuclei, as well as for supersymmetric pairs $[1,4,5]$ and quartets of nuclei $[6-9,11]$. In addition to providing a unified description of collective nuclei, the dynamical (super)symmetries of the IBM and its extensions provide a powerful tool for unraveling and classifying the spectra of complex nuclei by means of a set of closed expressions for energies, electromagnetic transition rates, and spectroscopic factors for one- and two-nucleon transfer reactions, which can be used to analyze, classify, and interpret the experimental data. As an example, we mention the interpretation of the ${ }^{194,195} \mathrm{Pt}$ and ${ }^{195,196} \mathrm{Au}$ nuclei as members of a supersymmetric quartet with $U(6 / 12)_{\nu} \otimes U(6 / 4)_{\pi}$
supersymmetry [6,7], in which the odd proton is allowed to occupy the $2 d_{3 / 2}$ orbit of the $50-82$ shell, and the odd neutron the $3 p_{1 / 2}, 3 p_{3 / 2}$, and $2 f_{5 / 2}$ orbits of the $82-126$ shell. This supermultiplet is characterized by $\mathcal{N}_{\pi}=2$ and $\mathcal{N}_{\nu}=5$. In this scheme, the excitation spectra of the four nuclei that constitute a quartet are described simultaneously by a single energy formula

$$
\begin{align*}
E= & A\left[N_{1}\left(N_{1}+5\right)+N_{2}\left(N_{2}+3\right)+N_{3}\left(N_{3}+1\right)\right] \\
& +B\left[\Sigma_{1}\left(\Sigma_{1}+4\right)+\Sigma_{2}\left(\Sigma_{2}+2\right)+\Sigma_{3}^{2}\right] \\
& +B^{\prime}\left[\sigma_{1}\left(\sigma_{1}+4\right)+\sigma_{2}\left(\sigma_{2}+2\right)+\sigma_{3}^{2}\right] \\
& +C\left[\tau_{1}\left(\tau_{1}+3\right)+\tau_{2}\left(\tau_{2}+1\right)\right] \\
& +D L(L+1)+E J(J+1), \tag{1}
\end{align*}
$$

using the same values of the coefficients $A, B, B^{\prime}, C, D$, and $E$ for all four nuclei. The first three terms in Eq. (1) correspond to vibrational excitations and the final three to rotations.

Recently, the structure of the odd-odd nucleus ${ }^{194} \mathrm{Ir}$ was investigated by a series of transfer and neutron capture reactions [12]. The odd-odd nucleus ${ }^{194}$ Ir differs from ${ }^{196} \mathrm{Au}$ by two protons, the number of neutrons being the same. The latter is crucial, since the dominant interaction between the odd neutron and the core nucleus is of quadrupole type, which arises from a more general interaction in the IBFM for very special values of the occupation probabilities of the $3 p_{1 / 2}, 3 p_{3 / 2}$, and $2 f_{5 / 2}$ orbits, i.e., to the location of the Fermi surface for the neutron orbits [13]. This situation is satisfied to a good approximation by the ${ }^{195} \mathrm{Pt}$ and ${ }^{196} \mathrm{Au}$ nuclei, which both have the 117 neutrons. The same is expected to hold for the isotones ${ }^{193} \mathrm{Os}$ and ${ }^{194} \mathrm{Ir}$. In particular, the new data from the polarized $(\vec{d}, \alpha)$ transfer reaction provide crucial new information about and insight into the structure of the spectrum of ${ }^{194} \mathrm{Ir}$, which has led to significant changes in the assignment of levels from those in previous work [14]. Theoretically, the levels of this nucleus were interpreted successfully in terms of a dynamical symmetry in odd-odd nuclei [12].

The successful description of the odd-odd nucleus ${ }^{194} \mathrm{Ir}$ opens the possibility of identifying a second quartet of nuclei in the $A \sim 190$ mass region with $U(6 / 12)_{\nu} \otimes U(6 / 4)_{\pi}$ supersymmetry. The new quartet consists of the nuclei ${ }^{192,193} \mathrm{Os}$ and ${ }^{193,194} \mathrm{Ir}$ and is characterized by $\mathcal{N}_{\pi}=3$ and $\mathcal{N}_{\nu}=5$. The energy spectra of the quartet of nuclei are described simultaneously by the energy formula of Eq. (1) with a single set of parameters. The pair of nuclei ${ }^{192} \mathrm{Os}-{ }^{193} \mathrm{Ir}$ has been studied as an example of $U(6 / 4)$ supersymmetry by Balantekin, Bars, and Iachello [4], who found that the rotational levels in both nuclei can be described to a good approximation by $C=40 \mathrm{keV}$ and $D+E=10 \mathrm{keV}$. The $J^{P}=0^{+}$state at 1206 keV in the even-even nucleus ${ }^{192} \mathrm{Os}$ is interpreted as a vibrational excitation, leading to $B+B^{\prime}=-33.5 \mathrm{keV}$, whereas the $J^{P}=\frac{3}{2}^{+}$state at 460 keV in the odd-proton nucleus ${ }^{193}$ Ir has been identified as the bandhead of a vibrational excitation, which gives $B^{\prime}=-25.5 \mathrm{keV}$. More recently, the odd-odd nucleus ${ }^{194} \mathrm{Ir}$ was studied both experimentally and theoretically in Ref. [12]. The rotational levels were described by $C=35.1, D=6.3$, and $E=4.5 \mathrm{keV}$, in good agreement with the values determined previously for ${ }^{192} \mathrm{Os}-{ }^{193} \mathrm{Ir}$. The available experimental
information on ${ }^{194}$ Ir allowed the determination of two of the three vibrational terms, $A+B=35$ and $B^{\prime}=-33.6 \mathrm{keV}$. Finally, in the absence of detailed experimental information on the odd-neutron nucleus ${ }^{193} \mathrm{Os}$, this nucleus was not taken into account in the fit. A simultaneous fit to the energy levels in ${ }^{192} \mathrm{Os},{ }^{193} \mathrm{Ir}$, and ${ }^{194} \mathrm{Ir}$ with the energy formula of Eq. (1) gives $A=41.0, B=-6.0, B^{\prime}=-29.0, C=$ $38.0, D=6.3$, and $E=4.5$ (all in keV).

The main difference with the parametrization of Ref. [14] is due to the new experimental information on the odd-odd nucleus ${ }^{194} \mathrm{Ir}$, which has led to an interchange in the assignments of the ground band and the first excited bands in ${ }^{194} \operatorname{Ir}$ [12]. The fitted values of $B, B^{\prime}, C$, and $D+E$ are essentially the same, since in both studies they are determined from the energy spectra of the pair of nuclei ${ }^{192} \mathrm{Os}-{ }^{193} \mathrm{Ir}$. The difference in the values of $A$ and $D$ (or $E$ ) arises from the fact that in Ref. [14] they were extracted from the (scarce) experimental information on the odd-neutron nucleus ${ }^{193} \mathrm{Os}$, whereas in the present study we used the new detailed experimental data on the odd-odd nucleus ${ }^{194}$ Ir to determine their values. In addition, the present parameter set is closer to the parameter values determined for the quartet ${ }^{194,{ }^{195}} \mathrm{Pt}-{ }^{195,}{ }^{196} \mathrm{Au}$ [8] than in Ref. [14] (see Table I), indicating systematics in this zone of the nuclear chart.

In Figs. 1-4, we compare the experimental and theoretical spectra for the quartet of nuclei ${ }^{192,193} \mathrm{Os}$ - ${ }^{193,194} \mathrm{Ir}$ in the $U(6 / 12)_{\nu} \otimes U(6 / 4)_{\pi}$ supersymmetry scheme. Given the complex nature of the spectrum of heavy nuclei in the mass $A \sim 190$ region, and in particular that of the odd-odd nuclei, the agreement is remarkable. There is an almost one-to-one correlation between the experimental and theoretical level schemes. Whereas the even-even nucleus ${ }^{192}$ Os, the oddproton nucleus ${ }^{193} \mathrm{Ir}$, and the odd-odd nucleus ${ }^{194} \mathrm{Ir}$ are wellknown experimentally, the data on the odd-neutron nucleus ${ }^{193}$ Os are rather scarce. In Fig. 2, we show the predicted spectrum for ${ }^{193} \mathrm{Os}$ as obtained from Eq. (1) using the parameter set determined from a fit to the nuclei ${ }^{192} \mathrm{Os}$ and ${ }^{193,194}$ Ir.

The ground state of ${ }^{193}$ Os has spin and parity $J^{P}=\frac{3}{2}^{-}$, which implies that the second band with labels [7,1], $\langle 7,1,0\rangle$ is the ground state band, rather than $[8,0],\langle 8,0,0\rangle$. The assignment of levels in Fig. 2 is based in part on preliminary data from the ${ }^{192} \mathrm{Os}(\vec{d}, p){ }^{193} \mathrm{Os}$ one-neutron transfer reaction [15], which shows that the $j=\frac{1}{2}$ strength goes to the state at 234 keV , whereas the $j=\frac{3}{2}$ and $j=\frac{5}{2}$ transfers are predominantly to the states at 103 and 73 keV , respectively. The $J^{P}=\frac{3}{2}^{-}$ground state is populated very weakly, whereas the first excited state at 41 keV is not seen at all in this reaction.

Theoretically, these transfer reactions are described by the fermionic generators of the superalgebra which change

TABLE I. Values of the parameters in keV .

|  | $A$ | $B$ | $B^{\prime}$ | $C$ | $D$ | $E$ | Ref. |
| :--- | :---: | ---: | :---: | :---: | ---: | ---: | :---: |
| Os-Ir | 41.0 | -6.0 | -29.0 | 38.0 | 6.3 | 4.5 | Present |
| Os-Ir | 63.0 | -9.3 | -24.2 | 36.1 | -5.1 | 15.9 | $[14]$ |
| Pt-Au | 52.5 | 8.7 | -53.9 | 48.8 | 8.8 | 4.5 | $[8]$ |



FIG. 1. (Color online) Comparison between the theoretical and experimental spectra of the even-even nucleus ${ }^{192}$ Os. The theoretical spectrum is calculated for the $U_{\nu}(6 / 12) \otimes U_{\pi}(6 / 4)$ supersymmetry scheme with Eq. (1). The parameter values are given in Table I.
a boson into a fermion. In a study of the stripping reaction ${ }^{194} \mathrm{Pt} \rightarrow{ }^{195} \mathrm{Pt}$, in which the initial and final nuclei have the same number of neutrons as for ${ }^{192} \mathrm{Os} \rightarrow{ }^{193} \mathrm{Os}$, it was found that the intensities for $j=\frac{1}{2}$ transfers are described by the operator [16]

$$
\begin{equation*}
P_{v}^{\left(\frac{1}{2}\right) \dagger}=\frac{\alpha_{\frac{1}{2}}}{\sqrt{6}}\left[\left(\tilde{s}_{v} a_{v, \frac{1}{2}}^{\dagger}\right)^{\left(\frac{1}{2}\right)}-\sqrt{2}\left(\tilde{d}_{v} a_{v, \frac{3}{2}}^{\dagger}\right)^{\left(\frac{1}{2}\right)}+\sqrt{3}\left(\tilde{d}_{v} a_{v, \frac{5}{2}}^{\dagger}\right)^{\left(\frac{1}{2}\right)}\right] . \tag{2}
\end{equation*}
$$



FIG. 2. (Color online) As in Fig. 1, but for the odd-neutron nucleus ${ }^{193}$ Os.

According to the selection rules, only the $J^{P}=\frac{1}{2}^{-}$state with $\left(\tau_{1}, \tau_{2}\right)=(0,0), L=0$ of the symmetric band with $[8,0],(8,0,0)$ are populated. This suggests that we identify the energy level at 234 keV with this state.

The one-neutron $j=\frac{3}{2}, \frac{5}{2}$ transfer are described by the operators

$$
\begin{equation*}
P_{v}^{(j) \dagger}=\frac{\alpha_{j}}{\sqrt{2}}\left[\left(\tilde{s}_{\nu} a_{v, j}^{\dagger}\right)^{(j)}-\left(\tilde{d}_{\nu} a_{v, \frac{1}{2}}^{\dagger}\right)^{(j)}\right] . \tag{3}
\end{equation*}
$$

This operator can excite the $J^{P}=\frac{3}{2}^{-}, \frac{5}{2}^{-}$doublets with $\left(\tau_{1}, \tau_{2}\right)=(1,0), L=2$ belonging to the bands with $[8,0],(8,0,0)$ and $[7,1],(7,1,0)$ (see Figs. 1 and 2). Since ratios of intensities do not depend on the value of the coefficient $\alpha_{j}$ and provide a direct test of the wave functions, we consider the ratio $R$ for the excitation of the doublet of the band with $[\mathcal{N}-1,1],(\mathcal{N}-1,1,0)$ relative to that of the symmetric band $[\mathcal{N}, 0],(\mathcal{N}, 0,0)[16]$

$$
\begin{equation*}
R(\mathrm{ee} \rightarrow \mathrm{on})=\frac{(\mathcal{N}-1)(\mathcal{N}+1)(\mathcal{N}+4)}{2(\mathcal{N}+2)} \tag{4}
\end{equation*}
$$



FIG. 3. (Color online) As in Fig. 1, but for the odd-proton nucleus ${ }^{193}$ Ir.
which gives $R=37.8$ for the stripping reaction ${ }^{192} \mathrm{Os} \rightarrow{ }^{193} \mathrm{Os}$ (with $\mathcal{N}=\mathcal{N}_{\pi}+\mathcal{N}_{\pi}=8$ ), i.e., most of the strength goes to the doublet with $[7,1],(7,1,0)$. For this reason, we assign the states at 103 and 73 keV as the $\left(\tau_{1}, \tau_{2}\right)=(1,0), L=2$ doublet of the $[7,1],(7,1,0)$ band.

Finally, the ground state and the first excited state of ${ }^{193}$ Os are assigned as members of a $J^{P}=\frac{3}{2}^{-}, \frac{1}{2}^{-}$doublet with $\left(\tau_{1}, \tau_{2}\right)=(1,1), L=1$ of the $[7,1],(7,1,0)$ band, since neither of these states can be excited by the transfer operators of Eqs. (2) and (3).


FIG. 4. (Color online) As in Fig. 1, but for the odd-odd nucleus ${ }^{194} \mathrm{Ir}$.

To establish the assignments of the energy levels of ${ }^{193} \mathrm{Os}$ on a firmer basis and to test the predictions for their doublet structure of either nuclear supersymmetry and/or the particletriaxial rotor model [17], it is of great importance that the nucleus ${ }^{193}$ Os be studied in more detail experimentally.

In conclusion, symmetries and supersymmetries play an important role in analyzing, classifying, interpreting, and understanding the spectra of complex many-body quantum systems. In this manuscript, we presented evidence of the existence of a second quartet of nuclei in the mass $A \sim 190$
region with $U_{\nu}(6 / 12) \otimes U_{\pi}(6 / 4)$ supersymmetry, consisting of the ${ }^{192,193}$ Os and ${ }^{193,194}$ Ir nuclei. An analysis of the energy spectra of the four nuclei that make up the quartet shows that the parameter set obtained in 1981 for the pair ${ }^{192}$ Os- ${ }^{193}$ Ir [4] is very close to that of ${ }^{194} \operatorname{Ir}$ [12], which indicates that the nuclei ${ }^{192,193}$ Os and ${ }^{193,194}$ Ir may be interpreted in terms of a quartet of nuclei with $U(6 / 12)_{\nu} \otimes U(6 / 4)_{\pi}$ supersymmetry. The supersymmetry in this new quartet of nuclei is satisfied with an accuracy comparable to, if not better than, that found in the ${ }^{194,195} \mathrm{Pt}$ and ${ }^{195,196} \mathrm{Au}$ nuclei, which was theoretically predicted almost 25 years ago and confirmed in 1999.

Nuclear supersymmetry establishes precise links among the spectroscopic properties of different nuclei, a fact that has been used in this Rapid Communication to predict the energy spectrum of the odd-neutron nucleus ${ }^{193} \mathrm{Os}$ from the
known properties of the remaining three nuclei that make up the quartet. Similar relations hold for other observables, such as electromagnetic and transfer strengths. Since the wave functions of the members of a supermultiplet are connected by symmetry, there exists a high degree of correlation between different one- and two-nucleon transfer reactions between not only nuclei belonging to the same multiplet [18], but also nuclei from different quartets. As an example of the latter, we are currently considering a set of two-proton transfer experiments between different pairs of nuclei in the two quartets of the Os-Ir and Pt -Au nuclei [19].

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