Finding a door along a wall with an error afflicted robot

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Abstract

We consider the problem of finding a door in a wall with a blind robot, that does not know the distance to the door or whether the door is located left hand or right hand to its start point. This problem can be solved with the well-known doubling strategy yielding an optimal competitive factor of 9 with the assumption, that the robot does not make any errors during its movements. We study the case, that the robots movement is errorneous. We give upper bounds for the movement error, such that reaching the door is guaranteed. More precisely the error range δ has to be smaller than $\frac{1}{3}$. Additionally, the corresponding competitive factor is given by $1 + 8\frac{1+\delta}{1-3\delta}$.

Keywords: Online algorithm, Online motion planning, searching, competitive ratio, errors.

1. Introduction

Online motion planning in unknown environments is theoretically well-understood and practically solved in many settings. During the last decade many different objectives where discussed under several robot models. For a general overview of theoretical online motion planning problems and its analysis see the surveys [3,9,10,7].

Theoretical correctness results and performance guarantees often suffer from idealistic assumptions, therefore in the worst case a correct implementation is impossible. On the other hand practioners analyze correctness results and performance guarantees mainly statistically. Therefore it is useful to investigate, how online algorithms with idealistic assumptions behave, if those assumptions cannot be fulfilled. More precisely, can we incorporate assumptions of errors in sensors and motion directly into the theoretical analysis? We already successfully considered the behaviour of the well-known pledge algorithm, see Abelson and diSessa [1] and Hemmerling [6], in the presence of errors [8].

The task of finding a point on a line without knowing the direction or the distance to the start point was considered by Baeza-Yates et. al. [2] and independently by Gal [5] and lead to the so called doubling strategy, which gives a basic paradigma for other searching algorithms, i.e., searching for a point on m rays or approximating the optimal search path, see [4].

Under the competitive framework doubling with a factor of two is the optimal strategy for searching a point on the line. The competitive analysis compares the cost of the strategy with the cost of a solution which is computed under full information, see [2].

In this paper we investigate how the doubling strategy behaves in the presence of errors and how the error influences the correctness and the corresponding competitive factor of the strategy.

We assume that an error range for a single step is known in advance but the agent gets no further information about the cummulated error. Therefore for the agent it makes no sense not to use the doubling heuristic.

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2. The lost cow problem

The task is to find a door in a wall, respectively a point, t, on a line. The robot does not know whether t is located left hand or right hand to its start position, s, nor does it know the distance from s to t. Baeza-Yates et. al. [2] describe the strategy to solve this problem using a function f. $f(i)$ is the distance the robot walks in the ith step. If i is odd, the robot moves $f(i)$ steps from the start to the right and $f(i)$ steps back; if i is even, the robot moves to the left. It is assumed, that the movement is correct, so after moving $f(i)$ steps from the start point to the right and moving $f(i)$ steps to the left, the robot has reached its start point. Baeza-Yates et. al. showed, that a strategy with $f(i) = 2^i$ is 9-competitive and this is optimal.

An online strategy that produces a path of length $|\pi_{\text{onl}}|$ is called *C-competitive* if for all scenes

$$|\pi_{\text{onl}}| \leq C \cdot |\pi_{\text{opt}}| + A$$

holds, where $|\pi_{\text{opt}}|$ denotes the path length of the optimal strategy which makes use of full information.

3. Modelling the error

The robot moves straight line segments of a certain length from the start point alternately to the left and to the right. Every movement can be afflicted with an error, that causes the robot to move more or less far than expected. However, we require the robots error in every step is within a certain error bound, δ. More precisely if the strategy moves the robot a distance ℓ we require that the covered distance is in the range $[\ell \cdot (1-\delta), \ell \cdot (1+\delta)]$ with $\delta \in [0, 1[$.

4. Reaching the door

How large can the error bound δ become under the restiction, that the robot should be able to reach the door? W. l. o. g. we consider the case, that the door is located on the same side as the first step of the agent. We assume that the door is located at $d = 2^{2j} - \varepsilon$, so an error-free robot hits the door during the iteration with step width $f(j) = 2^{2j}$.

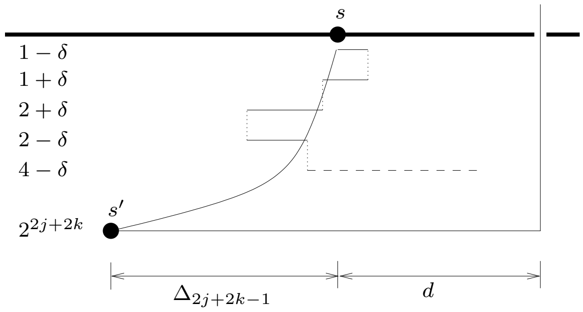

Fig. 1. In the worst case, the start point of every iteration drifts away from the door. The vertical path segments are to highlight the single iterations, the robot moves on horizontal segments only.

In the worst case, every step to the right of the error afflicted robot is too short and every step to the left is too long, so start point of the iterations drifts to the left, see Fig. 1. Let Δ_k denote the drift after k iterations.

$$\Delta_k = \sum_{i=0}^{k} \Big(\underbrace{2^i \cdot (1+\delta)}_{\text{Steps, that are too long}} - \underbrace{2^i \cdot (1-\delta)}_{\text{Steps, that are too short}} \Big)$$
$$= 2\delta \cdot \sum_{i=0}^{k} 2^i = 2\delta(2^{k+1} - 1).$$

Let the error afflicted robot miss the door during the iteration with step width $f(j) = 2^{2j}$ due to the drift to the left. Now we are interested in the number of additional steps, and whether the robot is able to reach the door at all. Obviously the reachability and the number of additional steps depends on the error δ. If the robot hits the door, it will hit in the iteration with a step with of 2^{2j+2k}, $k \in \mathbb{N}^{>0}$, because the door is located right hand to the start point. To ensure, that the robot hits the door, the length of the last straight path must be at least as large as the distance to door plus the overall drift to the left. The last straight path may be error afflicted again, but its length is a least the lower bound of the error range in the iteration 2^{2j+2k}. This yields

$$
\begin{aligned}
&\Delta_{2j+2k-1} + d \le 2^{2j+2k} \cdot (1-\delta) \\
\Leftrightarrow\, & 2\delta \cdot 2^{2j+2k} - 2\delta + 2^{2j} - \varepsilon \le 2^{2j+2k} - 2^{2j+2k}\delta \\
\Rightarrow\, & 2^{2j}\left[2\delta 2^{2k} + 1 - 2^{2k} + \delta 2^{2k}\right] - 2\delta \\
&< 2^{2j}\left[3\delta 2^{2k} + 1 - 2^{2k}\right] \le \varepsilon \\
\Leftrightarrow\, & \delta \le \frac{1}{3}\frac{2^{2k}-1}{2^{2k}} + \frac{\varepsilon}{3 \cdot 2^{2j+2k}} \\
\Rightarrow\, & \delta \le \frac{1}{3}\frac{2^{2k}-1}{2^{2k}}
\end{aligned}
$$

The last term converges for $k \to \infty$ to $\frac{1}{3}$. Thus

Corollary 1 *If the error δ is not greater than $\frac{1}{3}$ the robot will hit the door.*

With the given analysis it is also possible to present relations between the error range and the number of additional iterations. For example:

Corollary 2 *If the error δ is not greater than $\frac{1}{4}$ the robot will hit the door with only one iteration step more than the error-free robot.*

5. Analyzing the performance

W. l. o. g. for the competitive setting it would be the worst, if the door is hit during the iteration with step width 2^{2j+2}, but located just a litte bit further away than the rightmost point that was reached during the iteration with step width 2^{2j}, i. e.

$$d = 2^{2j}(1-\delta) - \Delta_{2j-1} + \varepsilon = 2^{2j}(1-3\delta) + 2\delta + \varepsilon.$$

This yields a factor of

$$
\begin{aligned}
\frac{|\pi_{\text{onl}}|}{d} &= \frac{1}{d}\left(2\sum_{i=1}^{2j+1} 2^i + \Delta_{2j+1} + d\right) \\
&= \frac{1}{d}\left(2 \cdot (2^{2j+2}-1) + 2\delta(2^{2j+2}-1) + d\right) \\
&= \frac{2^{2j}\left(8(1+\delta) - \frac{1+\delta+\varepsilon}{2^{2j}}\right)}{2^{2j}\left(1 - 3\delta + \frac{2\delta+\varepsilon}{2^{2j}}\right)} + 1 \\
&< 8\frac{1+\delta}{1-3\delta} + 1
\end{aligned}
$$

To show that this is the worst case, we show that the robots errors in the left and in the right direction are different. Furthermore the error in each step may be a different one. Let δ_i^+ be length of the movement to the right in the i-th step and δ_i^- be the length of the movement to the left. Now

$$\sum_{i=1}^{2j-1}(\delta_i^- - \delta_i^+)$$

denotes the deviation from the start point before the $2j$-th step and

$$\sum_{i=1}^{2j-1}(\delta_i^- + \delta_i^+)$$

is the path length up to this point.

As above the door is hit during the step $2j+2$, therefore its distance is

$$
\begin{aligned}
d &= \delta_{2j}^+ - \Delta_{2j-1} + \varepsilon \\
&= \delta_{2j}^+ - \sum_{i=1}^{2j-1}(\delta_i^- - \delta_i^+) + \varepsilon
\end{aligned}
$$

For the corresponding path length we have

$$|\pi_{\text{onl}}| = \sum_{i=1}^{2j+1}(\delta_i^- + \delta_i^+) + \sum_{i=1}^{2j+1}(\delta_i^- - \delta_i^+) + d$$

due to the fact that we will finally stop at distance d, we add the deviation to d.

With this we get

$$\frac{|\pi_{\text{onl}}|}{d} = 1 + \frac{\sum_{i=1}^{2j+1}(2\delta_i^-)}{\delta_{2j}^+ - \sum_{i=1}^{2j-1}(\delta_i^- - \delta_i^+) + \varepsilon}$$

We can conclude that the ratio achieves its maximum if we exceed every δ_i^- to the greatest extend, which is $2^i(1+\delta)$. Now we only have to fix δ_i^+ in order to maximize the ratio. Obviously the nominator gets its smallest value if every δ_i^+ is very small, therefore we use $\delta_i^+ = 2^i(1-\delta)$.

Theorem 3 *If the error δ is not greater than $\frac{1}{3}$ the robot will hit the door with the doubling strategy. The generated path is not longer than $8\frac{1+\delta}{1-3\delta}+1$ times the shortest path to the door.*

6. Summary

We have analyzed a simple doubling strategy to find a door in a wall respectively a point on a line in the presence of errors in movements. We showed that the robot is still able to reach the door if the error is less than $\frac{1}{3}$. Moreover, if the error is less than $\frac{1}{4}$ the robot will need at most two iteration steps more than the error-free robot. If the error in the movement is bound by $\delta < \frac{1}{3}$, the competitive ratio is

$$8\frac{1+\delta}{1-3\delta}+1.$$

Both error bounds are rather big, so it can be expected, that real robots will meet this error bounds.

The problem for m rays is a little bit different because the robot may detect the starting point if it changes from one corridor to another. Here, one may be interested in the probability of entering correctly the next corridor.

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