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# Resistance calculation of a masonry structure against a deformation due to an angular distortion

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**1. Introduction –** The present paper provides an analysis of the deformation capacity of a masonry structure caused by an angular distortion. We are going to examine the geometric characteristic of the deformation, stress increase associated with this movement, and the resistance front shear and deflection. We have used as a reference the studies from A. Tena y E. Miranda 2001 [1], and the technical norms CTE-DB-SE-F [2] and NBE-FL-90 [3].

## 2. Experimental.

2.1. Calculation of the structure deformation according to the cracks thickness.

When a differential deformation takes place between two points in a wall, cracks appear orthogonally to the direction with the highest tensile and lengthening as shown in Figure 1.

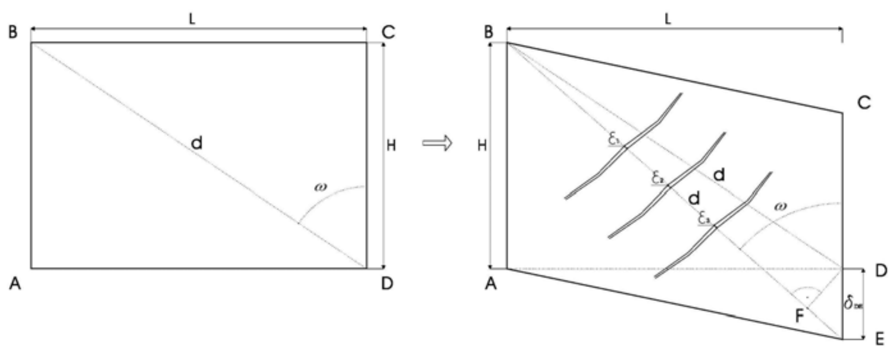


Figure 1. Geometric characteristic of the deformation

The sum of all the cracks widths can be equal to the increase of the diagonal length BD. Given that these are small deformations, we can suppose that the segment BF is equal to the segment BD, and that the angle BDC is equal to the angle BED and to $\omega$.

Therefore, the diagonal length increase is equivalent to the sum of the cracks thickness $\sum \varepsilon$, according to the following formula:

$$\sum \varepsilon = \delta_{DE} \cdot \cos \omega = \delta_{DE} \cdot \frac{CD}{BD} = \delta_{DE} \cdot \frac{H}{\sqrt{L^2 + H^2}} \rightarrow \delta_{DE} = \frac{\sum \varepsilon \sqrt{L^2 + H^2}}{H} \qquad (1)$$

If we show the previous formula through the three variables $\sum \varepsilon$, $L/H$ and $\delta_{DE}$, we can relate geometrically the cracks thickness and the structure deformation to the span L and height H in a wall, according to the following formula:

$$\boxed{\delta_{DE} = \sum \varepsilon \sqrt{1 + \left(\frac{L}{H}\right)^2}} \qquad (2)$$

2.2. Masonry structure strength against a deformation.

According to A. Tena and E. Miranda 2001 [1], the total deformation strengthened by a masonry element which is subjected to some imposed deformations, is the sum of the shear-strengthened and the bending deformation.

$$\delta_T = \delta_c + \delta_f \qquad (3)$$

Where

$$\delta_c = \alpha \frac{V_\delta \cdot L^3}{E \cdot I} \text{ Shear deformation} \quad (4) \qquad \delta_f = \frac{V_\delta \cdot L}{A \cdot G} \text{ Bending deformation} \quad (5)$$

Being

| | |
|---|---|
| $A$ | Bending area |
| $I$ | Wall section inertia |
| $G$ | Cross elasticity modulus |
| $E$ | Secant elasticity modulus |
| $V_\delta$ | Shear stress |
| $\alpha$ | Coefficient that depends on the restrictions to the deformations of the ends and which is considered equal to 1 as a perfectly fixed structure. |

The regulation NBE-FL-90 establishes that if we only observe the piece deformation, we can consider that the maximum tensile stress in the section is equal to the shear stress $\tau$. All this depends on the shear stress and the section area, according to the formula (6)

$$\sigma' = \tau = \frac{V_\delta}{A} \qquad (6)$$

Moreover, in order for the crack in a masonry element to happen, the shear stress $\tau$ has to be equal to or higher than the shear strength $f_{vk}$. As our objective is to know the structure strength against a deformation, we consider that $\tau = f_{vk}$.

This way, changing the value of the shear stress $V_\delta$ in the formulas (3) and (4) we will have:

$$\delta_c = 12 \cdot \frac{f_{vk} \cdot L^3}{E \cdot H^2} \text{ Shear deformation} \quad (7) \qquad \delta_f = \frac{f_{vk} \cdot L}{G} \text{ Bending deformation} \quad (8)$$

Considering that deformation depends on the L, we can show formula (3) in the following way:

$$\frac{\delta_T}{L} = \frac{\delta_c}{L} + \frac{\delta_f}{L} \qquad (9)$$

If we replace the variables (4) y (5) in to the equation (9) we obtain the following formula, where only there are two variables, the value dimensionless L/d and the shear strength $f_{vk}$.

$$\boxed{\frac{\delta_T}{L} = f_{vk}\left[\frac{1}{E}\cdot\left(\frac{L}{H}\right)^2 + \frac{1}{G}\right]} \qquad (10)$$

**3. Results and Discussion**.

3.1. Abacus for the calculation of the structure deformation according to crack thickness.

The following chart resolves the equation of two variables from the formula (2), and we obtain the following abacus, figure 2, which relates the sum of the crack thickness in millimetres $\sum \varepsilon$, and the dimensionless value L/H with the deformation also measured in millimetres.

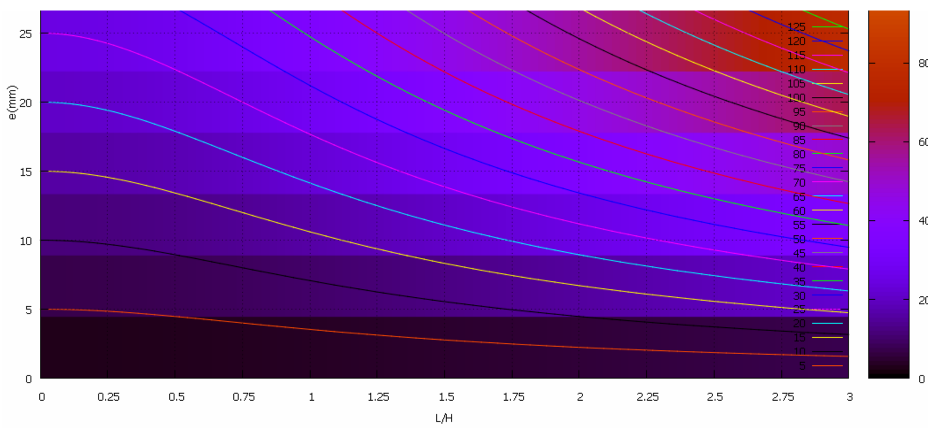


Figure 2. Abacus for the calculation of the structure deformation according to crack thickness

We deduce from the previous chart that a wall whose height H=3m and length L=3, that is a dimensionless value of L/H=1, and which shows a total crack thickness of 10mm, undergoes an approximate deformation of 15mm.

3.2. Abacus to calculate the masonry structure strength against a deformation.

In order to calculate the strength against a deformation, we define the constant corresponding to the secant elasticity modulus E, the cross elasticity modulus G, fixed level $\alpha$ and the shear strength $f_{vk}$.

According to the CTE-DB-SE-F, the secant elasticity modulus E of a masonry structure could be considered equal to the compression strength $f_k$ by 1000.

The table 4.4 from this regulation shows that the average compression strength in a masonry wall can vary from 2MPa for hollow brick to 11MPa for solid bricks with thin joints.

For this paper we estimate a average compression strength of 7MPa for the hollow brick, whose corresponding secant elasticity modulus E = 7000MPa

The CTE-DB-SE-F estimate that the cross elasticity modulus G can be considered as 40% os the secant elasticity modulus E, so we obtain an value G equal to 2800MPa.

If we replace the previous constant in to the equation (10) we obtain the following formula:

$$\frac{\delta_T}{L} = f_{vk}\left[\frac{1}{7000}\cdot\left(\frac{L}{H}\right)^2 + \frac{1}{2800}\right] \qquad (11)$$

The following chart, figure 3, resolves the equation of two variables (10), through an abacus which relates the shear strength for values between 0.25MPa y 1MPa, and the dimensionless value L/H for values between 0.5 to 3, with the maximum angular distortion from which the masonry structure will crack.

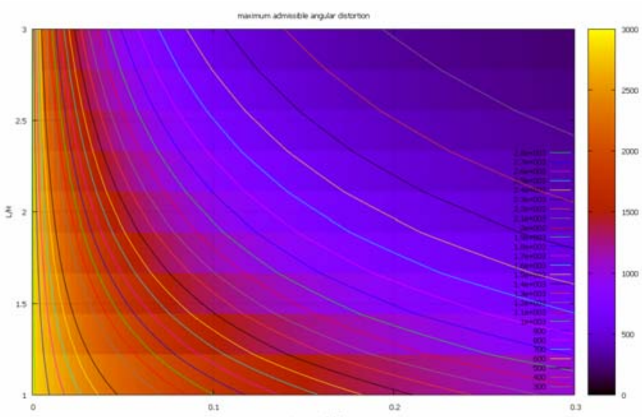


Figure 2. Abacus to calculate the masonry structure strength against a deformation

Using as an example a wall whose height H=3m and length L=6, that is a dimensionless value of L/H=1.5, whit a shear strength $f_{vk}$ = 0.2MPa, we can deduce from the previous chart that the maximum admissible angular distortion for the masonry structure will be L/1000. This will be the value from which the masonry structure will crack.

**4. Conclusions.**

The previous method of calculation allows us analyse what its deformation and structural damage are through the visual inspection of the cracks thickness.

Moreover, knowing the shear strength of the structure we can specify what the maximum admissible angular distortion is, and from which deformation the wall will crack because of a lack of structural stability in the section.

Finally, we confirm that the masonry structure shows such a high rigidity that when we have absolute distortion between L/1000 and L/2000 the structure will crack. This values will depend on the ratio L/H and therefore on the section rigidity.

**5. References**


[1] Comportamiento Mecánico De La Mampostería. A. Tena E. Miranda 2001.

[2] CTE-DB-SE-F, Código Técnico de la Edificación, Documento Básico de Seguridad Estructural, Fábrica de Ladrillo.

[3] NBE-FL-90. Norma Básica de la Edificación, fábrica de ladrillo.