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MANAGEMENT MATHEMATICS FOR EUROPEAN SCHOOLS

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Abstract. Beyond the topics, the Mathematics folklore says that pupils think that this field is a boring, non-practical mental exercise, far apart from daily life. This thought is influencing the entire education system and even society, motivating important lack of mathematical skills among students. This work presents experiences, developed within the MaMaEuSch project oriented to stimulate and enforce the study of Mathematics among the students in the High-School education level.

Keywords: Teaching, real world problems.

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§1. Introducción

Every summer when revising the statistics of the university access examination we get shocked with the poor marks of the students in scientific subjects: Mathematics, Physics, Chemistry,... It is also known that the same problem is happening in other European Union (EU) countries. In this world, where science and technology influence our lives and economies, is difficult to evade that problem. Our scientific limitations nowadays will be technologic dependence tomorrow, in other words, economic limitations in international trade or incapacity of leading the world.

We get surprised when we hear of the bad results that our young people get in Mathematics, but this is only the consequence of the bad reputation that it also has in our society. Have not you ever heard (or said): “oh my poor kid, he/she has to suffer this torture?”. It is too easy to assign responsibilities to others when we are in trouble, and of course there are more than enough reasons to criticize science teaching, a teaching system that permits science subjects to go down in an alarming way.

This is not only an isolated fact. Most of us still remember PISA 2000 report [2]. PISA is a three-yearly survey of the knowledge and skills of 15-year-olds in the principal industrialised countries. It assesses how far students near the end of compulsory education have acquired some of the knowledge and skills that are essential for full participation in society. It presents evidence on students performance in reading, mathematical and scientific literacy, reveals factors that influence the development of these skills at home and at school, and examines what the implications are for policy development. We focus on the part of the survey relative to Mathematics, where we can find discouraging results about the skill of the young people in our area.

For much of the last century, the content of school mathematics and science curricula has been dominated by the need to provide the foundation for the professional training of a small number of mathematicians, scientists and engineers. Today, however, literacy in mathematics and science is important for all of us to understand medical, economic, environmental and other issues that shape modern societies, which rely heavily on technological and scientific advances. PISA looked at mathematics in relation to its use in people's lives and assessed the capacity of students to recognize and interpret mathematical problems; translate these problems into mathematical context; use mathematical knowledge and procedures to solve problems; interpret the results in terms of the original problem; reflect on the methods applied; and formulate and communicate the outcomes.

PISA found wide variation between countries in overall performance in mathematics across these different aspects. Countries with the higher investment in education do not reach the best marks in the mathematic and scientific skill. Other than that OCDE countries [1] perform worse that Corea, China or Iceland. This implies that some countries have managed to mitigate the influence of social background and some have achieved this while reaching a high overall level of performance. These are noteworthy achievements. Will other countries take up the challenge?

All these fails in Mathematics are very bad for our countries' future, but it is even worse, much worse, that all these young people -and all these adults that share with them not their fails but their ignorance- do not learn or understand the knowledge of, for instance, Pythagoras, Euclides, Newton, Leibnitz, Euler, Cauchy or Hilbert. It is not only that Mathematics are everywhere but, because of a reason we do not understand, Nature obeys laws that are expressed in Mathematical terms. As Albert Einstein asked himself: how can it be possible that Mathematics, something coming from human reasoning and independent from experience, is adapted so well to reality? Wherever you look at, you find Mathematics.

Mathematics are a great instrument to test our intellectual skill. They also give rise to unforgettable experiences as: understanding that even numbers are as many as natural numbers; understanding that $x^2 + y^2 = z^2$ has infinity many integer solutions (Pythagoras Theorem) but $x^n + y^n = z^n$ does not have any integer solution if $n > 3$ (last Fermat Theorem); understanding that π cannot be written as the quotient of two integers; being able to check that in a group of 23 people the probability of finding two of them that have the same birthday is higher than the probability that all of them have different birthdays; or being able to model a real world situation.

Because of the fact that Mathematics rule the world we live in, they give us the possibility of developing our thought and our technology, and for an unlimited number of argumentations, Mathematics can not die out [6].

However, either we change the trend in our teaching system and in the opinion of our societies or we are leading a disaster in our new future. It is too easy to criticize the system but the reality is that perhaps the method followed to teach is not adequate. It is clear that the classic teaching methodology by passive listening must play a role. However, there are a number of other possibilities that should be used to motivate and to wake up scientific interest and curiosity.

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[1]OECD (Organisation for Economic Co-operation and Development) countries participating in PISA 2000: Australia, Austria, Belgium, Canada, Czech Republic, Denmark, Finland, France, Germany, Hungary, Iceland, Ireland, Italy, Japan, Korea, Luxembourg, Mexico, Netherlands, New Zealand, Norway, Poland, Portugal, Spain, Sweden, Switzerland, United Kingdom, United States.

In this communication we present some teaching experiences developed within the Comenius 2.1 project: MAnagement MAthematics for EUropean SCHools (MaMaEuSch: 94342-CP-1-2001-1-DE-Comenius-C21). The goal of these experiences is to provide Mathematical teachers, in the High School level, with actual case oriented units that use only basic concept in Mathematics. Our aim is that these cases will open the curiosity and encourage pupils to study better and harder. This work is organized in three sections. Section 1 is the introduction. In section 2 we present the project, the consortium and the several goals pursued by the project. Finally, Section 3 includes three teaching units of MaMaEuSch developed in Seville.

§2. The project, partners and activities.

In many countries of the EU the students in pre-university schools think of Mathematics as a boring and non-practical mental exercise, which is a necessity to pass, but which is otherwise disliked, if not hated. In contrast to this, Mathematics is the discipline with the largest influence on our society. Many of our most exciting progresses are due to mathematical modelling and the application of these models in industry and society. Mathematics is virtually everywhere, for instance in technology, economy, natural science or management science.

The fifth EU framework included a general program called Comenius 2.1, specifically devoted to innovative teaching and educational experiences oriented to High School level. Identifying the deficit in the Mathematical skills of the European Union students motivated a consortium of 3 partners with the common goal of developing alternative teaching material for improving mathematical education.

This consortium gave rise to the project MaMaEuSch (Management Mathematics for European Schools) that got funded by the European Union within Comenius 2.1 program.

The aim of this project is to show the applicability of school mathematics in its form as management mathematics. The approach is to develop case oriented teaching material and course presentations for student teachers and for the continuing education of active teachers. Both groups will learn how to model real-world problems taken from their daily lives (industry, society,...). All material is developed in a bilingual mode which includes pairs of the languages of the proposal partners. Additional languages of the EU can be added upon request. The presentation of the material is both in print and on the internet. The design of the internet material allows an expandability by adding on novel solution approaches.

The success of the approach is being measured by the feedback from the participating teachers and student teachers, but even more important, by the feedback of the pupils in the classes and their parents, which are confronted with the new material through their teachers. The feedback is evaluated systematically by corresponding questionnaires.

The consortium of the project is composed by three partners: University of Kaiserslautern, University of Klagenfurt and University of Seville. It has a duration of three years and full economic support to hire several research positions working full time.

The **University of Kaiserslautern**, Germany, is a modern university specialized in areas of technology and natural science. Student teachers in mathematics have to choose a second area of expertise, which can be a subject from natural science, management science or social science. The department of mathematics has a large population of non-German students. After the intermediate exam in the fourth semester approximately 50% of the students are coming from abroad. The teaching language is mostly English.

The department has close ties with the Fraunhofer Institute fuer Techno- und Wirtschafts-mathematik (ITWM) which is a mathematical research institute consulting companies and public institutions in applying mathematical solutions to real-world problems.

MaMaEuSch is headed in Kaiserslautern by Horst W. Hamacher, who is a professor in Wirtschaftsmathematik (management mathematics) and one of the directors of the ITWM. He has published some 70 articles in professional journals, 12 books and is editor and co-editor of several professional journals. He is a part-time teacher in schools, where he implements the project oriented approach to teaching mathematics in schools on a small scale.

Because of its experiences with real-world problems the University of Kaiserslautern group is - in cooperation with the ITWM - the main source for “good” real-word projects, which will be transferred to the school environment. The group also plays a leading role in developing the teaching aids, both in print and in electronic form. It has experiences in developing bilingual teaching material for university courses. It coordinates the activities for all partners, set up and update internet presentations and conferences and will organize the international workshop at the end of the funding period.

The **University of Klagenfurt**, Austria, has a long tradition in the training of teachers for mathematics. The mathematics department regularly hosts the congress on school mathematics and is well established in the international community on didactics for mathematics. In the course of the expansion of the university the mathematics department was extended to cover optimization and operations research. In addition to the teachers program in mathematics, the university offers also an engineering degree in applied mathematics. Both programs are set up to run 5 years.

Franz Rendl is leading the optimization group of the mathematics department and is in charge of MaMaEuSch in Klagenfurt. His research interests focus on discrete and nonlinear optimization. He is currently in the editorial board of several major journals on optimization and Operations Research. He also has a wide experience in scientific consulting in industry.

Within the MaMaEuSch consortium Klagenfurt is the leader in the mathematical optimization area. Optimization is based on various mathematical techniques which are part of the standard school curriculum. To see how these methods are used to tackle and solve real-world problems and to interrelate them with methods from stochastics and computer science is in the center of the Klagenfurt MaMaEuSch staff. Due to its geographical closeness to the eastern European EU acceding countries, Klagenfurt also leads the way to extending the MaMaEuSch programme to these countries after the funding period. Preparatory work can and is being done during the funding period.

The **University of Seville**, Spain, is one of the oldest universities in Spain with more than 400 years of history. It is a leading center of research and educational programs in many different areas. Student teachers in mathematics have to follow a 5 years program with a mathematical, methodological orientation plus a 1 year program which is pedagogically oriented. The Faculty of Mathematics has a large tradition in the education of mathematical teachers since more than 50% of their math students opt for such a carrier.

MaMaEuSch is headed in Seville by Justo Puerto-Albandoz, who is a professor in Statistics and Operations Research. He has published over 60 papers in professional journals, 2 books and is co-editor of several journals of Operations Research. He has collaborated in applied projects on optimization of health services, design of routes, development of decision support systems for administrations and companies, etc.

The expertise of the group in Seville is Mathematical Statistics and Operations Research. Therefore, the Sevillian group is leading the applications of Mathematical Statistics and Probability Theory (stochastic analysis and statistical estimation). For several years this area is part of the curriculum in most European countries. MaMaEuSch shows how methods of probability theory, statistical analysis, sampling and stochastic analysis can be applied in several fields of the real-world. In particular, the design of survey sampling, discrete simulation, parameter estimation and data analysis is essential in any market study, audience panel, experimental design in biology and agriculture, etc.

The project is being running for almost two years. In this period, the different partners have developed several cases among them we mention: coordination of an assembly line; location of flying ambulances; how find navigation systems the shortest way to the destination?; how can mathematics help to improve cancer therapy?; line planning in local traffic; how much should be the bus fare?; delay management in public transport; how to locate train stations?; introduction to the use of the calculator TI-92 and simple applications of matrix calculation. For the sake of brevity, we will only focus on three easy cases developed in Seville. Those who are interested in other cases or in a more detailed description are referred to the web page of the project [1].

§3. Cases developed in Sevilla

As mentioned above the group in Seville leads the cases oriented to Statistical Analysis and Operations Research. We have developed several cases dealing with: how to model competition among 'rational' players, experimental games, modelling recursion by mean of the popular TV quiz show "Who wants to be a millionaire©?", survey design, simple sampling schemes,...

We present only three of our simplest, but nevertheless more successful, cases. We restrict ourselves to a concise description that concentrates in the presentation of the actual situations, the model that is being applied and the goal pursued by the experience. Further details can be found in the complete description included in the homepage of the project [1].

The first of the cases presented is on a very popular game of cards in Spain, quite similar to Black Jack. The second one deals with the concept of fairness from a Game Theory point of view, but everything is explained by using the Mathematical tools taught in High School. Finally, we present several examples to introduce basic concepts on sampling and inference.

3.1. Seven and a Half

This case is designed to show how a game of cards can be played in a logical way by using basic tools of probability theory.

The game that we deal with is called *Seven and a half*. To play this game we need a Spanish card deck. How is that deck of cards? It consists of forty cards divided into four different groups called: gold, sword, cup and rough. So we will talk about golds, swords, cups and roughs.

Each group is made of ten cards and each of these cards has associated a number. Those numbers are: one, two, three, four, five, six, seven, ten, eleven and twelve. The cards that have associated a number between one and seven are called as the number they have, and the cards that have associated a ten, an eleven or a twelve are called jack, horse or king. That makes us talk about a three of cups, a seven of golds or a king of swords.

However, for our game each card has a value depending on the number it has associated. The way to find out the value of a card is as follows:

- If the number of the card is between one and seven then the value of that card is the same as its number.

- If the number of the card is ten, eleven or twelve (that is, the card is a jack, a horse or a king), the value of that card is half.

3.1.1. How to play?

The goal of the game is to get points until being the closest possible to seven and a half but without being over. The punctuation of each player is the sum of the values of her/his cards.

The game consists of the dealer giving a card to each player. After looking at his or her card, each player has to decide if he/she wants one more card or not depending on the value of the cards he/she has. For example, if you get a seven as your first card you had better not ask for more cards, because you have a very good punctuation and it is very easy to receive a card that will make your punctuation higher than seven and a half. But, if you receive a face card (a jack, a horse or a king) as your first card the logical thing is to ask for another card, because it is impossible to have a higher punctuation than seven and a half with only half point (the value of a face card).

If a player chooses to continue asking for cards, he/she will receive cards until he/she chooses stop or is eliminated. When one of those two things happens, the dealer has to ask the next player if he/she wants another card.

Each player can keep one of his/her cards down. This card could be different after receiving a new card. If you have one of your card down and you want another card, it will come up. If you have all your cards up and you ask for another card, it will come down.

A player who gets seven and a half has to show all his/her cards when receiving the last card. When every one has finished asking for cards, they have to show their cards and the one who has the highest punctuation, without being over seven and a half, is the winner of the game.

The player who deals the cards (the dealer) always wins in case of tie. If another player gets seven and a half and the dealer does not, this player will be the dealer in the following game. If no one gets seven and a half, the dealer will keep his/her position in the following game even when loosing. The dealer is always the last player to receive cards.

3.1.2. General idea of decision rules.

Basically, a decision rule is a description of the way to act according to the cards we have, that is to say, looking at the cards we have and the cards we can see of the others players we make a decision according to our rule. With a programmed decision rule an automaton (a computer or a robot) could play this game and practically any other game of cards.

A possible decision rule could be: "Asking cards until the probability of winning is higher than $0'8$". That is a decision rule because just making calculations about the probability of winning we know what to do in any situation: we stop asking cards if and only if our probability of winning is higher than $0'8$.

Unfortunately calculating the probability of winning is not always easy. Sometimes we need a computer if we want to calculate quickly that probability; this is one of the reasons why it is not allowed to use computers in casinos.

Our decision rule. The application of basic knowledge of probability theory allows us to propose to the attendants the following decision rule to play the game:

- Let p_1 be the probability of winning if the player does not ask for another card and p_2 the probability of being over seven and a half if the player asks for another card.
- We define f as follows:

$$f(p_1, p_2) = \begin{cases} S & \text{if } \{p_1 \geq 0'7\} \text{ or } \{p_1 \in [0'1, 0'7) \text{ and } p_2 \geq 0'55\} \\ C & \text{otherwise} \end{cases},$$

  where S means stopping asking for cards and C means continuing asking for cards.

That decision rule seems to be difficult but it is not. Basically it says that we stop asking for cards if our probability of winning is high enough ($p_1 \geq 0'7$) or if the probability of being over seven and a half is too high ($p_2 \geq 0'55$) and, although it is small, the probability of winning is not too small ($p_1 \in [0'1, 0'7)$).

In order to choose an appropriate decision rule we have to take into account several things, among them we mention:

- Measuring the risk we want to take.
- Trying to measure the risk that our rivals are inclined to take.

After estimating those risk levels we can choose a decision rule that complies with the information we have about our rivals and about ourselves.

3.1.3. Necessary tools to solve the game.

If we want to play "Seven and a half", using the decision rule we gave above, we have to calculate some probabilities. Those probabilities are p_1 and p_2 and we calculate them by using the basic tools of calculus of probabilities and the information that we have about:

- The face up cards our opponents have.
- The face down cards our opponents have.

When one plays a move of this game following the decision rule some techniques of probabilities appear. For instance it is necessary to use Conditional Probability, the Theorem of Total Probability and Bayes' rule [5]. For further details, the interested reader is referred to the web page of the project [1].

3.2. Cost allocation problem

In many day-to-day situations we find optimization problems. It normally happens that when several agents (people, companies, etc.) join their efforts to carry out an action in common which will help all of them, the total expense is lower than the expense generated by executing that action separately. Thus, it seems clear that cooperation is advantageous, because the agents will spend less money by acting in common and they will have the same service. But a new problem shows up: how the expenses generated by the common action should be allocated. This case brings up the discussion on the concept of fairness.

The case that we propose is the problem that results after building an elevator in a block, which has to be paid among all the neighbors.

3.2.1. The case

There still exist buildings which do not have any elevator. At a given point in time their neighbors think this is time to install it. In those cases the controversy of how to pay the elevator appears. The way of allocating the common costs motivate that some neighbors decide to act on their own without taking into account the remainder. In spite of the total expense is decreased, their own cost is higher than the expense they would have if acting by himself or forming smaller coalitions (groups of neighbors). Let us see the following case:

In a five storey block, with one apartment in each floor, an elevator is going to be installed. The company in charge of building the elevator has a fixed price depending on the number of floors the elevator has. Due to technical problems the cost of each floor is increased with its height, that is, it is more expensive to build the part of the elevator between the sixth and the seventh floor than the one between the first and the second. The prices of the company are shown in the following table:

Number of storeys	Cost of the elevator
1	10000
2	21000
3	33000
4	46000
5	60000

The draft of the elevator is shown in Figure 1.

In the resident's association was proposed the following cost allocation: each one pays proportionally, that is, 12000 Euros each. Is this allocation "fair"? How would you allocate the expenses among all the neighbors?

3.2.2. Sharing expenses

Let us see three different rules to share costs:

1. Proportional allocation says that each neighbor has to pay 12000 Euros. But let us thing a bit, if the neighbor of the first floor wanted to build an elevator only for his own use, he would have to pay less (10000 Euros), so he will not want to be part of the joint construction of the elevator.

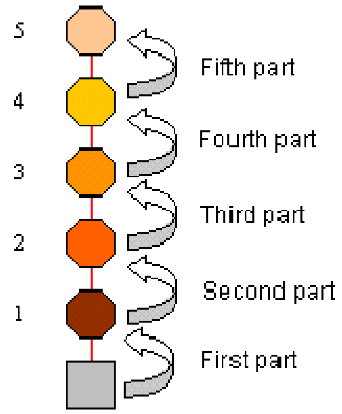

Figure 1: Elevator draft.

2. Bird's rule. When using that rule to share costs no subgroup of neighbors will be able to build an elevator for themselves at less cost. In this case, Bird's rule proposes the following cost allocation:

   The neighbor of the first floor pays 10000 Euros (the cost of building an elevator from the hall to the first floor). The neighbor of the second floor pays 11000 (the cost of building the elevator from the first floor to the second) and so on. With this allocation rule the neighbor of the third floor pays 12000 Euros, 13000 has to pay the one living in the fourth floor and the neighbor of the fifth floor pays 14000. This kind of allocations, so that there is no subgroup of agents that can spend less money by acting by themselves, are known in the Cooperative Game Theory as core allocations [4].

3. Another rule that can be proposed is the next: each neighbor pays for the number of floors that he has to pass through in order to get his to floor, that is, the neighbor of the third floor has to pay for the first part of the elevator, the second and the third. In this case each neighbor pays:

   - The first part costs 10000 Euros and it is paid among all the neighbors, because all of them use it ($\frac{10000}{5} = 2000$ Euros).
   - The part between the first and the second storey is paid by the neighbors from the second floor to the fifth ($\frac{11000}{4} = 2750$ Euros).
   - The third part costs 12000 and is paid by the neighbors of the third, fourth and fifth floor ($\frac{12000}{3} = 4000$ Euros).
   - The part between the third and the fourth storey is 13000 Euros and it has to be paid by those neighbors who use it, that is the ones living in the fourth and the fifth floor, ($\frac{13000}{2} = 6500$ Euros).
   - The last part, between the fourth and the fifth is paid only by the neighbor of the fifth floor, ($\frac{14000}{1} = 14000$ Euros).

   So the payoff vector is:

$$(2000, 4750, 8750, 15250, 29250),$$

where the i^{th} neighbor pays the i^{th} component of the vector above.

Although that is a stable allocation belonging the Core of the corresponding cooperative game, it seems to be exaggerated that the neighbor of the fifth floor pays 15 times what the neighbor of the first storey pays.

3.3. Sampling and inference

In this subsection we present examples which can be used to introduce basic concepts on sampling and inference.

3.3.1. Sampling

The goal which is to be achieved with this case is to introduce a real problem where one can use sampling as a way to get useful information from a population.

The data of this experience corresponds to an application in a particular high school. Our population consists of all the students of the high school Drago in Cádiz (Spain), and we want to have information about the number of left-handed students, number of students who have internet access at home, height of the students and the pocket money they receive per week.

All this information can be useful, for instance, to plan the number of special chairs for left-handed people, or to check whether it is a good decision to put some material for the students in the internet so that they can download it at home.

After these considerations, we proposed to the attendants to make sampling to get some information about the students of the high school but avoiding asking to all of them. What we know of the population is how they are distributed in levels and groups:

	A	B	C	D	E	Total
1ž ESO	33	20				53
2ž ESO	20	15	30			65
3ž ESO	20	15	26	14		75
4ž ESO	27	27	25			79
1ž Bach	33	28	30	31	23	145
2ž Bach	30	34	32	31		127

So we have a population of 558 students of a high school. A first decision to make is the sampling size. We decide that we want to have a maximum sample of 60 students which will be the maximum number we allow [3].

Now, we could start by calculating the sampling quotient and the elevation factor:

$$f = \frac{n}{N} = \frac{60}{558} = 0.1075 \qquad E = \frac{N}{n} = \frac{558}{60} = 9.3$$

and this means we are sampling around 10% of the population and that each student in the sample represents around 9 people in the population [5].

The are two different situations. The attendants were asked if the division we have into levels and groups affects the variables in some way. It is normal to consider that height and pocket money change depending on the levels, this is, we can consider that the age can influence them, while this will not happen for the internet access or for being left-handed. This fact makes us to propose two different situations.

Case 1:

We can think that the division in levels is for us a division in strata because levels are homogeneous inside them considering the age (and that means that they are homogeneous considering pocket money and height too), and so, as we have seen that age has an influence in our variable, we want to have a representation of every stratus in our sample. So, it is proposed to do a stratified random sampling.

We choose the size of the sample in the strata proportional to its size and the experience gets the following sample size for them:

$$\begin{array}{ll} n_1 = 6 & n_2 = 8 \\ n_3 = 8 & n_4 = 8 \\ n_5 = 16 & n_6 = 14 \end{array}$$

Once fixed the sampling sizes it is done the real random sampling in each stratus, and when we get the data, we can make the estimations.

We got an estimation of 168.83 for the average height and an error estimation of 0.7541, while we got an estimation of the average pocket money of 8.72 euros with an error estimation of 0.68.

Case 2:

Analyzing the variables being left-handed and having internet access at home, the division into strata is not useful. We propose again to choose a sample of 60 students and now the attendants consider that groups are as small populations for these variables. The meaning is that they assume our population divided into clusters, so that one should apply cluster sampling in this case.

We decide to include in our sample 2 or 3 clusters (they don't have all the same size) in order to keep 60 as the number of elements of the final sample.

Finally we proposed 3 clusters and the attendants got the information they needed from them. We got the following estimations: an estimated value for the proportion of left-handed students of 7.35$\%$ with an error measure of 1.2$\%$ and an estimated value for the proportion of students who have internet access at home of 69.11$\%$ with an error measure of 3.8$\%$.

3.3.2. Inference

We start by assuming the fact that we have a random sample of 25 students. We want to make a survey which has 3 goals:

1. We want to estimate the price of t-shirts to get some money for the students trip in the end of the year. We will use the data to find a confidence interval of the average of the pocket money, which can help us to decide how much the students can pay for the t-shirts.

2. Last surveys appeared in the media say that young people devote much time to be in the internet. So, with our information, can we say that the students of the high school devote more than one hour each day surfing in the internet?

3. We want to check if it is true, for our population, the usual belief that 10% of the population is left-handed.

Once we have the data from the sample, the attendants are asked to answer quantitatively these questions.

1. They found a confidence interval for the average. The first thing was to fix an appropriate confidence level, of 90% and then notice that they were in the case of unknown variance, so the interval is as follows:

$$\left(\overline{x} - t_{\frac{\alpha}{2}}(n-1)\frac{S_c}{\sqrt{n}}\sqrt{\frac{N-n}{N-1}}, \overline{x} + t_{\frac{\alpha}{2}}(n-1)\frac{S_c}{\sqrt{n}}\sqrt{\frac{N-n}{N-1}},\right)$$

   where a factor of $\sqrt{\frac{N-n}{N-1}}$ is applied because the sampling is made without replacement. Substituting the data the interval is

$$(5.2785, 11.2335).$$

   They proposed to choose a price between 5.27 and 11.23 euros for the t-shirts.

2. For the second situation, the proposal was to use a unilateral hypothesis test for the average of the daily time connected to the internet, considering the fact that the variance is unknown. The hypothesis would be:

   H_0 : the average of the daily time in the internet is higher or equal to 60 minutes.
   H_1 : the average of the daily time in the internet is lower than 60 minutes.

   The statistic used for the test was

$$T = \frac{\overline{x} - \mu_0}{S_c/\sqrt{n}}$$

   and for this case, its value is

$$T = -1.9888$$

   Thus, as the rejection region for this test is $T < -1.711$, they reject the null hypothesis and one can not assume that the students devote more than an hour a day to be connected to the internet.

3. To answer the third question, the proposed was a hypothesis test on the proportion of left-handed people of the population. The test is the following:

   H_0 : the proportion of lett-handed people is equal to 0.1
   H_1 : the proportion of lett-handed people is not equal to 0.1

The statistic used for this contrast was

$$T = \frac{\overline{p} - p_0}{\sqrt{\frac{p_0(1-p_0)}{n}}}$$

and it takes the value $T = 1.\widehat{6}$ being the rejection region $|T| \geq 1.96$. Therefore, the topic that there are a 10% of left-handed students in the high school can not be rejected.

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References

[1] http://www.mathematik.uni-kl.de/ mamaeusch/.

[2] http://www.pisa.oecd.org/.

[3] FERNÁNDEZ, F.R., MAYOR, J.A. *Muestreo en poblaciones finitas.* EUB. Ediciones Universitarias de Barcelona, Barcelona, 1995.

[4] OWEN, G. *Game Theory.* Academic Press, San Diego, 1995.

[5] ROHATGI, VIJAY K., EHSANES SALEH, A.K.MD. *An introduction to probability and statistics.* Wiley Series in Probability and Statistics, Chichester, 2001.

[6] SÁNCHEZ RON, J.M. ¡Vivan las Matemáticas! *El País*, 19^{th} September, 2003.

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