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Supply Management in Multiproduct Firms with Fixed Proportions Technology

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This paper studies the supply management of a primary input, where this input gives rise to multiple products in fixed proportions. My objective is twofold. First, I study fixed proportions technology under demand uncertainty in comparison with the flexible and dedicated technologies. I show that fixed proportions technology has a *cost-pooling* value over dedicated technology, which is larger than the *capacity-pooling* value of flexible technology over dedicated technology. I identify the critical role that demand correlation plays with the fixed proportions technology: in contrast to the capacity-pooling value, which *decreases* in demand correlation, the cost-pooling value *increases* in demand correlation. Second, focusing on the fixed proportions technology, I study supply management in the presence of contract and spot markets. I investigate how the optimal supply management strategy should respond to changing market uncertainties, and the differences in this response based on the contract type. I find that when the exercise price of the contract is high, a higher contract market dependence is the best response to the increasing demand correlation or spot price variability. However, a lower contract market dependence is the best response to the same when the exercise price is low. Managerially, these results are important because they imply that the supply management strategy adopted as a response to a change in the business environment should differ depending on the contract type. My results have implications about the new product strategy and the procurement contract choice of the processors in the agricultural industries.

Keywords: contracting; risk management; multiproduct newsvendor; flexibility; spot market; agriculture; coproduction

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1. Introduction

This paper develops a theoretical basis for understanding the trade-offs facing a processor in the supply management of a primary input, where this input gives rise to multiple products in fixed proportions. The problem considered here is relevant for several agricultural industries. For example, in the cocoa industry, cocoa beans are processed (by cleaning, roasting, and grinding) to produce cocoa liquor, which is further processed (by pressing and milling) to produce cocoa butter and cocoa powder. In the sugar industry, sugarcane is processed (by grinding) to produce sugarcane juice, which is then heated to extract the white table sugar. The remaining crystalized sugar particles are further processed to be sold as animal feed. In the wheat industry, wheat seeds are processed (by grounding and sieving) to produce wheat bran and coarse powder flour, which is further processed and sold as animal feed. In all of these industries, a primary input is processed into multiple products in fixed proportions.

In the operations management literature, as also highlighted by Chen et al. (2013), multiproduct firms with proportional production technology have received very limited attention. Motivated by the semiconductor industry, a stream of papers in this literature studies coproduction systems, where multiple products are produced in a single run with random yields. Because the focus is on yield uncertainty, the majority of these papers assume deterministic demands. In practice, given the escalating levels of uncertainty in the business environment today, including in the agricultural industries, one of the key determinants of supply management is demand uncertainty. This stream of papers remains silent on the impact of demand uncertainty. A vast amount of papers in the operations management literature study this impact in multiproduct firms with flexible production technology. These papers showcase the flexible technology as a hedge against demand uncertainty, and provide insights on how the profitability and the supply management strategy is affected by this uncertainty (Van Mieghem and Rudi 2002). With the flexible technology, paralleling the fixed proportions technology, a single input is capable of producing multiple products; but unlike the fixed proportions technology, each unit of input is used only in one product. Therefore, it is an open question whether the insights coming from the flexible technology analysis are applicable to the fixed proportions technology.

The first objective of this paper is to answer this question by studying the fixed proportions production technology in comparison with the flexible production technology. I analyze the impact of demand uncertainty on the optimal supply management strategy and the profitability with the fixed proportions technology, and investigate whether there exist any structural differences between the two technologies based on this impact.

A common feature of the processors in agricultural industries is that there are two markets of interest for supply management: the contract market and the spot market. Contract markets feature long-term arrangements between the processor and its suppliers. These contracts may take different forms in terms of pricing and delivery requirement. The most common contract form used in practice is the quantity flexibility contract (Kleindorfer and Wu 2003). A quantity flexibility contract specifies the capacity reserved in advance of the spot market. The actual delivery volume is decided within this reserved capacity on the day. This contract form also encompasses the quantity commitment contract where all the contracted volume is delivered. Spot markets are regional markets primarily used as a topping up of the contracts on the day. In agricultural industries, spot price shows considerable variability (Meyer 2013) and constitutes another source of uncertainty besides demand. Therefore, it is important for the processors to understand the impact of these uncertainties when choosing the right supply management strategy. As highlighted in Kleindorfer and Wu (2003), since the processors use different contract types (such as quantity flexibility or quantity commitment contracts), it is also important to understand how this impact changes based on the contract type.

The second objective of this paper is to develop this knowledge base. In the operations management literature, a vast amount of papers study supply management in the presence of contract and spot markets. Barring Boyabatlı et al. (2011), there is no work in this literature that focuses on the fixed proportions technology. Boyabatlı et al. (2011) study the optimal contracting decision of a multiproduct firm that uses a quantity commitment contract. I focus on a more general contract form, quantity flexibility contract, and investigate whether there exist any structural differences in the supply management strategy adopted as a response to changing spot price and demand uncertainties based on the contract type.

To analyze this problem, I propose a stylized model in which I consider a firm (processor) that operates under the fixed proportions technology. The firm procures a single input and produces and sells two outputs in fixed proportions of this input in a single period so as to maximize its expected profit. The output prices are fixed and the demands for the outputs are stochastic. The input can be sourced from a spot market, and from a contract market using a quantity flexibility contract, that is characterized by a unit reservation price and a unit exercise price. The firm chooses the contract volume under the spot price and the demand uncertainties. After these uncertainties are realized, the firm decides the exercise quantity from the contract, and the quantity for the spot market transactions (procurement and sales), which collectively determine the processing volume, and in turn, the production quantity of each output via the fixed proportions of this processing volume.

My consideration of the product market characteristics, i.e., the fixed product prices with uncertain demand, is consistent with practice in several agricultural markets (including cocoa, sugar, and wheat markets), and is motivated by my conversations with the procurement managers of a multinational consumer goods company.[1] This company procures a wide range of products from processors to be used as input for its food and beverage manufacturing operations. For the majority of its agricultural input (such as cocoa powder and cocoa butter) and chemical input originated from agricultural products (such as sweetener, enzymes, preservatives), a fixed-price contract with minimum and maximum delivery limits is used. The contract price is fixed for two to three months, which is consistent with the time frame considered in my single-period model.[2] The actual procurement volume is determined on the delivery date, but the minimum and the maximum delivery limits are specified when these contracts are signed. In summary, the upstream suppliers, i.e., the processors, face stochastic demand for their final products with fixed prices, consistent with my framework.

With this model, I first analyze the fixed proportions production technology in comparison with no-fixed proportions technology benchmarks. As benchmark cases, I consider a firm that operates under the flexible production technology, where a single input is capable of producing two products, and each unit of input yields an output only in one market; and a firm that operates under the dedicated production technology, where there are two inputs that can each produce a single product. To generate

[1] This company is one of the world's largest consumer goods company in revenues, and its main products involve foods, beverages, cleaning agents, and personal care products.

[2] The fixed-price contract has several implementation advantages for this company. On the upstream markets, it facilitates setting up invoice systems with its suppliers. On the downstream markets, it enables the company to keep a fixed market price for its own products, as preferred by its customers.

sharper managerial insights, I focus only on the contract market for supply management. With the flexible technology, as established in the extant literature, since the input can be allocated between the two products in response to demand realizations, there exists a *capacity-pooling* benefit over the dedicated technology. With the fixed proportions technology, since the input is used for both products at the same time, there exists a *cost-pooling* benefit over the dedicated technology. I show that the cost-pooling benefit of the fixed proportions technology is larger than the capacity-pooling benefit of the flexible technology. I identify the critical role that demand correlation plays with the fixed proportions technology: in contrast to the capacity-pooling value, which *decreases* in demand correlation, the cost-pooling value *increases* in demand correlation. A higher demand correlation is beneficial with the fixed proportions technology because it decreases the demand imbalance in the product markets facilitating the effective usage of the same input for both products. Whereas the optimal contract volume always increases in demand correlation with the fixed proportions technology, the opposite holds true with the flexible technology when this volume is smaller than the total expected demand.

These results have important implications for the new product strategy of processors in the agricultural industries. In particular, converting the biomass (the organic residue from processing) into a by-product, which benefits from cost pooling, is an alternative strategy to creating a differentiated product by customizing the input (through, e.g., different packaging or flavoring), which benefits from capacity pooling. My results underline the need for processors to take a holistic view of their supply management, and to manage it together with their new product strategy.

I next analyze the optimal supply management in the presence of spot market, focusing on the fixed proportions technology model. After the spot price and the demand uncertainties are realized, the firm can use the spot market to sell the contracted input or to source additional input. These spot market options are taken into account when the optimal contract volume is decided in the presence of uncertainties. I conduct sensitivity analysis to investigate how the optimal contract volume should respond to a change in demand correlation or spot price variability, and analyze whether there exist any structural differences in this response based on the contract type. The interplay between the exercise price of the contract and the impact of these uncertainty parameters provide the following insights: When the exercise price of the contract is high, a *higher* contract market dependence is the best response to the increasing demand correlation or spot price variability. However, a *lower* contract market dependence is the best response to the same when the exercise price is low. Managerially, these results are important because they imply that the optimal supply management strategy adopted as a response to a change in the business environment should differ depending on the contract type. Thus, indiscriminately employing the same response with different contracts, i.e., increasing or decreasing the contract market dependence, can be a detrimental strategy.

The remainder of this paper is organized as follows: Section 2 surveys the related literature and discusses the contribution of my work. Section 3 describes the basic fixed proportions technology model. Section 4 derives the optimal strategy. Section 5 studies the impact of demand correlation on the fixed proportions technology in comparison with the flexible and the dedicated technologies. Section 6 extends the fixed proportions technology model to incorporate the spot market, and analyzes the impact of spot price variability and demand correlation on the optimal supply management. Section 7 discusses the impact of relaxing two of my assumptions. Section 8 concludes with a discussion of main insights and future research directions.

2. Literature Review

Two streams of literature are relevant to my study: the first explores the supply management in multiproduct firms and the second studies the supply management in the presence of spot market. I now discuss my contribution to each literature.

In the literature on *supply management in multiproduct firms*, fixed proportions production technology has received very limited attention. Motivated by the semiconductor industry, a few papers study coproduction systems, where multiple products are produced in a single run with random yields. I refer the readers to Chen et al. (2013) for a review of the papers in this stream. The standard coproduction problem foresees different grades or quality levels of product, where the demand for a lower-quality product can be filled by converting a higher-quality product. Because of these product substitution possibilities, and the random yields (proportions) of the products, the supply management problem in a coproduction system is more complex than the one considered in my paper. Since the primary focus is on the yield uncertainty, the majority of papers in this stream assume deterministic demands to simplify the analysis. Among the papers assuming stochastic demands, Hsu and Bassok (1999), Rao et al. (2004), and Ng et al. (2012) propose different heuristic solutions to the optimal supply management in a price-taker newsvendor setting. Focusing on a price-setting newsvendor problem with a utility-maximizing customer demand model, Tomlin and Wang (2008) solve for the optimal production, pricing, and allocation decisions and analyze the value of different operational flexibilities. All these papers remain silent on the impact of demand correlation.

Another stream of papers in this literature investigates the impact of demand correlation albeit focusing on the flexible production technology. These papers study the supply management of a primary input, where the input may refer to inventory or capacity, with the flexible production technology in comparison with the dedicated production technology (where there are multiple inputs that can each produce a single product) in a variety of settings. I refer the readers to Boyabatlı et al. (2015) for a review of papers in this stream.

I contribute to the literature on supply management in multiproduct firms by (i) delineating the impact of the fixed proportions technology on supply management by making a comparison with the flexible and dedicated technologies, and (ii) studying the impact of demand correlation with the fixed proportions technology in comparison with the flexible technology. I show that the impact of demand correlation is fundamentally different for each technology. For example, whereas the flexible technology benefits from lower demand correlation, the fixed proportions technology benefits from higher demand correlation.

In the literature on *supply management in the presence of spot market*, the majority of papers focus on single-product firms where the fixed proportions technology is irrelevant. The papers in this stream provide conditions under which contract market is used as a part of the optimal supply management portfolio in the presence of spot market. For example, Mendelson and Tunca (2007) provide a rationale for the existence of forward contracts, based on strategic spot trading. Secomandi and Kekre (2014) demonstrate that forward contracts are beneficial when transaction costs of spot procurement are higher than that of contract procurement. I refer the readers to Kleindorfer and Wu (2003) for a review of the early literature and to Kouvelis et al. (2013) for a review of the recent papers in this area.

In this literature, only a few papers study multiproduct firms with the fixed proportions technology, and barring Boyabatlı et al. (2011), there is no work in this stream that studies contract and spot markets for supply management. Focusing on a petroleum refinery, Dong et al. (2014) analyze the optimal spot procurement volume of two crude oils with different quality levels that are processed into two products. They study the value of two operational flexibilities, range flexibility (the ability to process crude oil of diverse quality), and conversion flexibility (the ability to convert low-quality crude oil to high-quality crude oil). Focusing on an oilseed pressing mill, Boyabatlı et al. (2014) analyze the spot procurement volume of an input (oilseed) that is processed into a main product (crude vegetable oil) and a by-product (animal feed). They study the capacity investment decisions of the mill, the pressing capacity for the input, and the storage capacity for the main product. None of these papers consider contract market or demand uncertainty.

Boyabatlı et al. (2011) analyze the optimal supply management decision of a meatpacker in the beef industry, where the meatpacker processes fed-cattle to produce a high-quality product (boxed beef) and a low-quality product (ground beef) in fixed proportions. The meatpacker's supply management portfolio consists of spot procurement and a quantity commitment contract whose price is benchmarked on the prevailing spot price. Paralleling the practice in the beef industry, they assume a price-setting newsvendor model in each product market with downward substitution possibility. They numerically show that a lower contract volume is the best response to the increasing spot price variability and demand correlation. Motivated by other agricultural industries, I assume a price-taker newsvendor model in the absence of downward substitution. I focus on a more general contract form, quantity flexibility contract, which encompasses the quantity commitment contract as a special case, and investigate whether there exist any structural differences in the impact of uncertainties based on the contract type. In particular, I provide analytical conditions under which a lower contract volume is the best response to the increasing spot price variability and demand correlation, and prove that the opposite result may hold based on the contract parameters. I also delineate the impact of the fixed proportions production technology by studying this technology in comparison with the flexible and the dedicated technologies.

In summary, although a vast amount of papers study supply management in the presence of contract and spot markets in a single-product setting, there is no work (barring Boyabatlı et al. 2011) that studies multiproduct firms with the fixed proportions technology. I contribute to this literature by (i) characterizing the optimal supply management strategy with the fixed proportions technology in a new modeling setting that is relevant for agricultural industries, (ii) analyzing how this strategy should respond to changing spot price variability or demand correlation, and (iii) investigating whether there exist any structural differences in this response based on the contract type.

The following mathematical representation is used throughout the text: A realization of the random variable $\tilde{y}$ is denoted by y. Boldface letters represent column vectors of the required size, a prime ($'$) denotes the transpose operator, $\mathbb{E}$ denotes the expectation operator, $\Pr(\cdot)$ denotes probability, and $(x)^+ = \max(x, 0)$. The monotonic relations (increasing, decreasing) are used in the weak sense unless otherwise stated.

3. Model Description and Assumptions

I consider a firm that produces and sells two products in a single selling season so as to maximize its expected profit. The firm has fixed proportions production technology, which I call *proportional* (P) technology. With this technology, a single input is capable of producing two products, and each unit of processed input yields a_1 and a_2 units of products 1 and 2, respectively, where $a_1 + a_2 \leq 1$. I model the firm's decisions as a two-stage problem: the firm makes its procurement decision under demand uncertainty (stage 1); and the firm makes its processing decision after the resolution of this uncertainty (stage 2).

For procurement, I assume that the firm uses a quantity flexibility contract that is characterized by a unit reservation price $\beta > 0$ and a unit exercise price $b \geq 0$. This contract encompasses the quantity commitment contract where $b = 0$. The firm decides the input volume to reserve Q^P by incurring the unit cost β with respect to the demand uncertainty. After the resolution of this uncertainty, the firm decides the input volume to be delivered within the reserved capacity, i.e., the processing volume $z^P \leq Q^P$, by incurring the unit cost b. I assume that the firm incurs a unit processing cost of $\omega > 0$.

The firm faces a stochastic demand in each product market, represented by $\tilde{\mathbf{D}}' = (\tilde{D}_1, \tilde{D}_2)$. After the demands are realized, the processed input is converted into the final product in market j by incurring a unit production cost $c_j \geq 0$. The final product is salvaged from a unit price $s_j \geq 0$ if there is no unsatisfied demand; otherwise, it is sold from a unit price $p_j \geq \max(s_j, c_j)$. Since one unit of the processed input yields a_j units of product j, the firm faces a demand D_j/a_j for the processed input with a unit sales revenue $a_j(p_j - c_j)$ and a unit salvage revenue $a_j(s_j - c_j)^+$. I assume that $(\tilde{D}_1/a_1, \tilde{D}_2/a_2)$ follows a bivariate distribution with mean (μ_1, μ_2), and covariance matrix $\mathbf{\Sigma}$, where $\mathbf{\Sigma}_{jj} = \sigma_j^2$ for $j = 1, 2$ and $\mathbf{\Sigma}_{12} = \rho\sigma_1\sigma_2$ and ρ denotes the correlation coefficient. For comparative statics analysis, I assume $(\tilde{D}_1/a_1, \tilde{D}_2/a_2)$ to follow a bivariate normal distribution.

4. The Optimal Solution for the Proportional Technology

In this section, I describe the optimal solution for the firm's procurement and processing decisions. I solve the firm's problem using backward induction starting from stage 2. All the proofs are relegated to the appendix.

In stage 1, the firm reserved Q^P units of input. In stage 2, the firm observes the demand realizations (D_1, D_2), and, constrained by Q^P, decides the processing volume z^P to maximize the profit. Let $\Pi^P(z^P)$ denote the profit for a given z^P, and π^P denote the optimal profit, i.e., $\pi^P = \max_{0 \leq z^P \leq Q^P} \Pi^P(z^P)$. The stage 2 objective function is given by

$$\Pi^P(z^P) \doteq -(b+\omega)z^P + \sum_{j=1}^{2}\left[a_j(p_j - c_j)\min\left(z^P, \frac{D_j}{a_j}\right) + a_j(s_j - c_j)^+\left(z^P - \frac{D_j}{a_j}\right)^+\right].$$

The first term is the sum of the exercise price and the processing cost, and the second term denotes the total revenues from the product markets. A processed input generates revenue in each market, where the firm has a sales revenue if there is unsatisfied demand, and a salvage revenue otherwise.

The stage 2 objective function $\Pi^P(z^P)$ is piecewise linear and concave in z^P. Therefore, the optimal solution occurs at the breakpoints $\{0, D_1/a_1, D_2/a_2, Q^P\}$. The optimal processing volume z^{P*} is determined by comparing the procurement cost at this stage (b) with the unit processing margin, that is, the unit revenue from production minus the processing cost, leading to the various price breakpoints indicated:

$$z^{P*} = \begin{cases} 0 & \text{if } a_1(p_1-c_1)+a_2(p_2-c_2)-\omega \leq b, \\ \min\left(\dfrac{D_{[1]}}{a_{[1]}}, Q^P\right) & \text{if } a_1(p_1-c_1)+a_2(p_2-c_2)-\omega > b \\ & \quad \geq a_{[1]}(s_{[1]}-c_{[1]})^+ + a_{[2]}(p_{[2]}-c_{[2]})-\omega, \\ \min\left(\dfrac{D_{[2]}}{a_{[2]}}, Q^P\right) & \text{if } a_{[1]}(s_{[1]}-c_{[1]})^+ + a_{[2]}(p_{[2]}-c_{[2]}) \\ & \quad -\omega > b \geq a_1(s_1-c_1)^+ + a_2(s_2-c_2)^+ - \omega, \\ Q^P & \text{if } a_1(s_1-c_1)^+ + a_2(s_2-c_2)^+ - \omega > b, \end{cases} \tag{1}$$

where [1] denotes the index of the product with $\min(D_1/a_1, D_2/a_2)$ and [2] denotes the other.

In stage 1, the firm chooses the optimal contract volume (input volume to reserve) Q^{P*} with respect to the demand uncertainty $\tilde{\mathbf{D}}$ so as to maximize the expected profit $V^P(Q^P) = \mathbb{E}[\pi^P(Q^P; \tilde{\mathbf{D}})] - \beta^P Q^P$. The optimal contract volume is determined by comparing the reservation price β with the expected marginal revenue of an additional contracted unit. At stage 2, the marginal revenue depends on the exercise decision of the firm. When the difference between the unit processing margin and the exercise price is positive, the firm optimally exercises this additional contracted unit, and the marginal revenue is given by this difference. Otherwise, the unit is not exercised, and the marginal revenue is zero.

Proposition 1. *$Q^{P*}=0$ if $\beta \geq (a_1(p_1-c_1)+a_2(p_2-c_2)-\omega-b)^+$ and $Q^{P*}\to\infty$ if $\beta \leq (a_1(s_1-c_1)^+ + a_2(s_2-c_2)^+ - \omega-b)^+$. Otherwise, Q^{P*} solves $\partial V^P/\partial Q^P|_{Q^{P*}}=0$, where*

$$\begin{aligned}\frac{\partial V^P}{\partial Q^P} = -\beta &+ \Pr\left(\frac{\tilde{D}_1}{a_1} > Q^P, \frac{\tilde{D}_2}{a_2} > Q^P\right) \\ &\cdot (a_1(p_1-c_1)+a_2(p_2-c_2)-\omega-b)^+ \\ &+ \Pr\left(\frac{\tilde{D}_1}{a_1} > Q^P, \frac{\tilde{D}_2}{a_2} \leq Q^P\right) \\ &\cdot (a_1(p_1-c_1)+a_2(s_2-c_2)^+-\omega-b)^+ \\ &+ \Pr\left(\frac{\tilde{D}_1}{a_1} \leq Q^P, \frac{\tilde{D}_2}{a_2} > Q^P\right) \\ &\cdot (a_1(s_1-c_1)^+ + a_2(p_2-c_2)-\omega-b)^+ \\ &+ \Pr\left(\frac{\tilde{D}_1}{a_1} \leq Q^P, \frac{\tilde{D}_2}{a_2} \leq Q^P\right) \\ &\cdot (a_1(s_1-c_1)^+ + a_2(s_2-c_2)^+-\omega-b)^+. \end{aligned} \tag{2}$$

When there is unsatisfied demand in each market, the marginal revenue of an additional contracted unit is a function of the total sales revenues from both markets. When there is no unsatisfied demand in each market, the marginal revenue is a function of the total salvage revenues from both markets. Otherwise, it is a function of the sales revenue from the high demand market and the salvage revenue from the low demand market.

5. The Value of Fixed Proportions Technology: The Role of Demand Correlation

In this section, I conduct sensitivity analysis to study the impact of demand correlation on the *proportional* (P) technology. To increase the understanding, I make comparisons with two other production technology benchmarks that are commonly discussed in the literature. In particular, I consider *flexible* (F) and *dedicated* (D) technologies. With the flexible technology, a single input is capable of producing two products, and each unit of processed input is used only in one market, and yields either a_1 units of product 1 or a_2 units of product 2. With the dedicated technology, there are two inputs that can each produce a single product, and each unit of processed input in market j yields a_j units of product j. The production technologies are summarized in Figure 1. Let Q^P and Q^F denote the reserved input volume with the proportional and the flexible technology, respectively, whereas Q_j^D denotes the reserved input volume with the dedicated technology in market $j=1,2$. Let β^i denote the unit reservation price and V^{i*} denote the optimal expected profit with technology $i \in \{D, F, P\}$.[3]

I assume that the first product is the premium product such that the sales and the salvage revenues are higher in the first market, i.e., $a_1(p_1-c_1) > a_2(p_2-c_2)$ and $a_1(s_1-c_1)^+ \geq a_2(s_2-c_2)^+$. I also assume that the sales revenue in the second market is higher than the salvage revenue in the first market, i.e., $a_2(p_2-c_2) > a_1(s_1-c_1)^+$. These assumptions imply that, with the flexible technology, the firm has a sales revenue in the second market only if there is no unsatisfied demand in the first market, and the firm has a salvage revenue only in the first market, which is the case when there is no unsatisfied demand in each market.

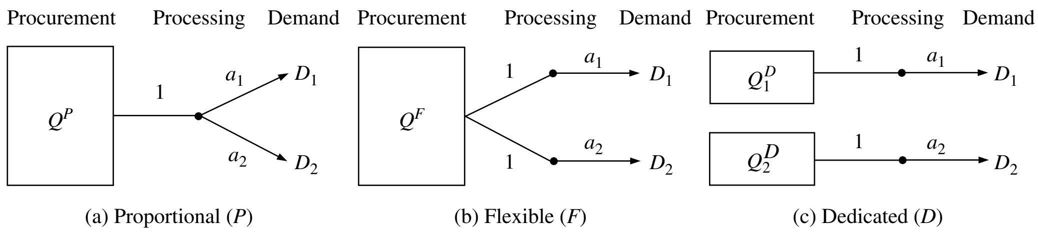
The extant literature studies the impact of demand correlation with the flexible technology in comparison with the dedicated technology by establishing a unique flexible cost threshold for a given dedicated cost such that the profits are identical with each technology, and analyzing how this threshold changes in the demand correlation (see, e.g., Boyabatlı and Toktay 2011, Goyal and Netessine 2011). I follow a similar approach. In particular, for a given β^D, I establish a unique unit reservation price threshold $\bar{\beta}^i(\beta^D)$ for technology $i \in \{F, P\}$ such that $V^{i*} \geq V^{D*}$ when $\beta^i \leq \bar{\beta}^i(\beta^D)$ and $V^{i*} < V^{D*}$ otherwise. I then conduct sensitivity analysis to study how $\bar{\beta}^i(\beta^D)$ changes in demand correlation. To avoid uninteresting cases, I assume β^D and b are such that the firm procures a positive and finite volume in each market with the dedicated technology, i.e., $b < a_2(p_2-c_2)-\omega$ and $(a_1(s_1-c_1)^+ - \omega - b)^+ < \beta^D < a_2(p_2-c_2)-\omega-b$.[4]

Paralleling the extant literature, I can show that $\bar{\beta}^F(\beta^D) \geq \beta^D$ in my setting, i.e., the firm can sustain a higher unit reservation price with the flexible technology than the dedicated technology. The reason is that the input can be allocated between the two products in response to demand realizations, and thus, the flexible technology has a capacity-pooling benefit over

[3] If Q^P, Q^F, and Q_j^D refer to the capacity investment level instead of the procurement volume, then β^i denotes the unit capacity investment cost with technology i, and b denotes the unit production cost.

[4] These conditions can be easily verified using Proposition 1: dedicated technology is a special case of proportional technology where each input is used only in market j, i.e., $a_{-j}=0$.

Figure 1 Production Technology Structures



Notes. With proportional (P) technology, the input volume Q^P serves both markets, and one unit of input yields a_j units of product in each market. With flexible (F) technology, Q^F serves both markets, and one unit of input yields a_j units of product in market j. With dedicated (D) technology, there is an input volume Q_j^D for market j, and one unit of input yields a_j units of product in this market.

the dedicated technology. Proposition 2 demonstrates that the threshold is the largest with the proportional technology.

Proposition 2. $\bar{\beta}^P(\beta^D) > \beta^D$ *and* $\bar{\beta}^P(\beta^D) > \bar{\beta}^F(\beta^D)$.

With the dedicated technology, to produce one unit of each product, $1/a_j$ units of input are required in market $j = 1, 2$. With the proportional technology, to produce one unit of each product, $\max(1/a_1, 1/a_2)$ units of input are required. Since the same input is used for producing both products, the proportional technology has a cost-pooling benefit over the dedicated technology, and thus, $\bar{\beta}^P(\beta^D) > \beta^D$. With the flexible technology, to produce one unit of each product, $1/a_1 + 1/a_2$ units of input are required. Since $\max(1/a_1, 1/a_2) < 1/a_1 + 1/a_2$, the proportional technology requires a smaller input volume than the flexible technology. Therefore, the cost-pooling benefit of the proportional technology is larger than the capacity-pooling benefit of the flexible technology, and $\bar{\beta}^P(\beta^D) > \bar{\beta}^F(\beta^D)$.

The cost-pooling feature of the proportional technology is different from the capacity-pooling feature of the flexible technology. With the flexible technology, paralleling the proportional technology, a single input is capable of producing two products. However, unlike the proportional technology, each unit of input is used in one product. A key distinction between two pooling features is the impact of demand correlation on their value:

Proposition 3. *Let* $(\tilde{D}_1/a_1, \tilde{D}_2/a_2)$ *follow a bivariate normal distribution*: $\partial\bar{\beta}^F(\beta^D)/\partial\rho \le 0$ *and* $\partial\bar{\beta}^P(\beta^D)/\partial\rho \ge 0$.

The threshold $\bar{\beta}^F(\beta^D)$ captures the capacity-pooling value of the flexible technology, whereas $\bar{\beta}^P(\beta^D)$ captures the cost-pooling value of the proportional technology. The common intuition prevalent in the academic literature argues that the capacity-pooling value of the flexible technology decreases in the demand correlation (see, e.g., Van Mieghem and Rudi 2002). Paralleling this intuition, $\bar{\beta}^F(\beta^D)$ decreases in the demand correlation in my setting. Interestingly, Proposition 3 demonstrates that the cost-pooling value of the proportional technology increases in the demand correlation. With low demand correlation, when the demand for one product is high, the demand for the other is low. Therefore, the firm can enjoy a sales revenue only in one of the markets. With high demand correlation, when the demand for one product is high, the demand for the other is also high. Therefore, the firm can enjoy a sales revenue in each market. In summary, a higher correlation decreases the demand imbalance in the product markets, and facilitates the effective usage of the proportional technology. Therefore, $\bar{\beta}^P(\beta^D)$ increases in the demand correlation.

I next analyze the impact of demand correlation on the optimal contract volume with each technology, focusing on the case where the firm has identical profit.

Proposition 4. *Let* $\beta^P = \bar{\beta}^P(\beta^D)$ *and* $\beta^F = \bar{\beta}^F(\beta^D)$, *i.e.*, $V^{P*} = V^{F*} = V^{D*}$; *and let* $(\tilde{D}_1/a_1, \tilde{D}_2/a_2)$ *follow a bivariate normal distribution. For these given* β^i *for* $i \in \{D, F, P\}$:

(i) *Dedicated* (D) *technology*: $\partial Q_j^{D*}/\partial\rho = 0$ *for* $j = 1, 2$.

(ii) *Flexible* (F) *technology*: $\partial Q^{F*}/\partial\rho > 0$ *if* $Q^{F*} > \mu_1 + \mu_2$ *and* $\partial Q^{F*}/\partial\rho < 0$ *if* $Q^{F*} < \mu_1 + \mu_2$.

(iii) *Proportional* (P) *technology*: $\partial Q^{P*}/\partial\rho > 0$ *if* $b > (a_1(s_1 - c_1)^+ + a_2(s_2 - c_2)^+ - \omega)^+$, *and* $\partial Q^{P*}/\partial\rho = 0$ *otherwise*.

With the dedicated technology, intuitively, the optimal contract volume in each market is independent of the demand correlation. With the flexible technology, since the input is allocated to the first market before the second market, the marginal revenue of an additional contracted unit at stage 2 is characterized by the realizations of the first product demand $\tilde{D}_1/a_1$, and the total demand $\tilde{D}_1/a_1 + \tilde{D}_2/a_2$. Because the total demand matters, the impact of demand correlation critically depends on the total expected demand. In particular, the optimal contract volume increases in the demand correlation only when this volume is greater than the total expected demand. With the proportional technology, as demonstrated in Proposition 1, joint demand from both markets matters. Interestingly, the optimal contract volume always (weakly) increases in the demand correlation. This is because the value of cost pooling increases, and thus, the firm takes more investment risk by increasing Q^{p*}.

In summary, the impact of demand correlation on the fixed proportions technology is structurally different from the impact on the flexible and the dedicated technologies. My results have important managerial implications about the new product strategy of processors in the agricultural industries, as I discuss in §8.

6. Supply Management in the Presence of Spot Market

In the previous sections, I studied the characterization of the optimal contract volume with the fixed proportions technology, and the impact of demand correlation on this volume in comparison with the flexible and the dedicated production technologies. In this section, focusing on the fixed proportions technology, I study the same in the presence of spot market, highlighting the impact of the spot market access on my results. This is because the processing firms in agricultural industries may also rely on the spot market for procurement of their primary input. Motivated by the empirical observations that document the high degree of uncertainty in the spot market prices in these industries, I conduct sensitivity analysis to study the impact of spot price variability on the optimal contract volume.

In practice, the processing firms use quantity flexibility contracts with different reservation and exercise prices. Consider two contracts, one with the lower reservation price and the other with the lower exercise price. In the literature, the differences between the two contracts in terms of their profitability is well understood. However, the literature remains silent on the differences in terms of their response to uncertainties. I attempt to fill this void by studying whether there exist any structural differences in my sensitivity analysis results based on the exercise price of the contract.

The remainder of this section is organized as follows: Section 6.1 discusses the additional modeling assumptions introduced beyond my model in §3. Section 6.2 derives the optimal supply management strategy in the presence of spot market. Section 6.3 investigates the impact of demand correlation and spot price variability on the optimal contract volume, emphasizing the differences among the contract types.

6.1. Additional Modeling Assumptions

For procurement, besides the quantity flexibility contract, I consider spot market. The input can be sourced from the spot market on the day at the prevailing spot price S. The firm can also resell the contracted input to the spot market on the day at a unit price $S(1-t)$, where $0 \leq t \leq 1$. Here, $t < 1$ may represent the transaction cost paid to the exchange market or the cost incurred for the transportation of the input sold to the buyer's premises. When $t = 1$, there is no value of spot sale in my model. In practice, this represents the case where the firm chooses not to participate in the spot resale market as a part of its procurement strategy. This case is relevant for the majority of the processors in the cocoa industry, as well as the processors in the beef industry (Boyabatlı et al. 2011).

To model the uncertainties, I assume that $(\tilde{S}, \tilde{D}_1/a_1, \tilde{D}_2/a_2)$ follows a trivariate distribution, where $\tilde{S}$ has a continuous marginal distribution with c.d.f. $F(\tilde{S})$, mean μ_S, and standard deviation σ_S; and $(\tilde{D}_1/a_1, \tilde{D}_2/a_2)$ follows a bivariate distribution as described in §3. Let $\bar{\rho}_{Sj}$ denote the correlation coefficient between $\tilde{S}$ and $\tilde{D}_j/a_j$ for $j = 1, 2$. In a short-term planning horizon, as considered in this paper, paralleling Boyabatlı et al. (2011), it is reasonable to assume $\bar{\rho}_{S1} = \bar{\rho}_{S2} = 0$. Over a longer time period, the information from the downstream demand is carried to the upstream spot market. Therefore, this correlation can be different from zero, and is determined by the relationship between the firm-specific demand $\tilde{D}_j/a_j$ and the (aggregate) industry demand for product j. A higher industry demand signals a higher dependence on the input in the future, and thus, increases the spot price for the input. Typically, the firm-specific demand $\tilde{D}_j/a_j$ follows the same pattern with the industry demand for product j, yielding $\bar{\rho}_{Sj} > 0$. In some cases, the firm-specific demand may follow the opposite pattern with the industry demand. For example, consider a wheat processor serving animal feed to feedlots. In anticipation of the future price increase in the wheat markets due to a high industry demand, some of these feedlots may decide to source animal feed from soybean or sugar processors. In this case, the firm-specific demand for the animal feed made of wheat is low, but the spot price of wheat is high due to a high industry demand, yielding $\bar{\rho}_{Sj} < 0$. I do not make any assumptions about $\bar{\rho}_{Sj}$ in my analysis, however, I focus on the more practical case of $\bar{\rho}_{S1} = \bar{\rho}_{S2} = 0$ when delineating my managerial insights in §8. For comparative statics analysis, I assume $(\tilde{S}, \tilde{D}_1/a_1, \tilde{D}_2/a_2)$ to follow a multivarite normal distribution.

For the quantity flexibility contract, I assume $\beta > \mathbb{E}[(\tilde{S}(1-t) - b)^+]$, i.e., the unit reservation price is higher than the expected revenue from (profitable) spot sales. Otherwise, the firm optimally reserves an infinite volume of input. I also assume $b < a_1(p_1 - c_1) + a_2(p_2 - c_2) - \omega$, i.e., the exercise price is lower than the unit processing margin when both products are sold.

Otherwise, the contract is never exercised on the day for processing.

For processing, I assume $\omega \geq a_1(s_1 - c_1)^+ + a_2(s_2 - c_2)^+$, i.e., the unit processing margin is nonpositive when both products are salvaged. Otherwise, when the spot price is sufficiently low, the firm optimally buys infinite volume of input from the spot market. Let

$$\begin{aligned} h_3 &\doteq a_1(p_1 - c_1) + a_2(p_2 - c_2) - \omega, \\ h_2 &\doteq a_1(p_1 - c_1) + a_2(s_2 - c_2)^+ - \omega, \\ h_1 &\doteq a_1(s_1 - c_1)^+ + a_2(p_2 - c_2) - \omega \end{aligned} \tag{3}$$

denote the unit processing margins, which correspond to the first three marginal revenue terms in (2) of Proposition 1. Without loss of generality, I assume $h_1 \leq h_2$. I also assume that ω is sufficiently low such that all three unit processing margins are positive.[5]

Paralleling §3, I model the firm's decisions as a two-stage problem. In stage 1, the firm chooses the contract volume under the spot price and the demand uncertainties. In stage 2, these uncertainties are realized and the firm decides the input volume to exercise from the contract, and the input volume for the spot market transactions (procurement and sales) that collectively determine the processing volume, and in turn, the production quantity of each output via the fixed proportions of this processing volume.

6.2. The Optimal Strategy

In this section, I describe the optimal solution for the firm's decisions. I make comparisons with the optimal solution in §4 to highlight the impact of spot market access.

In stage 2, the firm observes the demand (D_1, D_2) and the spot price S realizations. The firm decides the processing volume, how to source this volume from the reserved input Q^P and the spot procurement, and the spot resale volume of the contracted input. The firm's decision problem can be defined as a single-variable optimization problem over the processing volume z^P where the stage 2 objective function is given by

$$\begin{aligned} \Pi^P(z^P) \doteq & -\min(z^P, Q^P)\max(\min(S, b), S(1-t)) \\ & - (z^P - Q^P)^+ S + Q^P(S(1-t) - b)^+ \\ & - \omega z^P + \sum_{j=1}^{2}\left[a_j(p_j - c_j)\min\left(z^P, \frac{D_j}{a_j}\right)\right. \\ & \left. + a_j(s_j - c_j)^+\left(z^P - \frac{D_j}{a_j}\right)^+\right]. \end{aligned} \tag{4}$$

[5] My model requires $h_3 > 0$; otherwise, the firm cannot generate any revenues. When h_1 or h_2 takes negative values, the characterization of the optimal solution continues to hold. However, the discussion of my comparative statics results are more involved. To avoid unnecessary complications, I assume $h_1 > 0$.

The first line denotes the total procurement cost, whereas the second line denotes the total revenue from the product markets minus the processing cost. The total procurement cost is given by the sum of the procurement cost for the first $\min(z^P, Q^P)$ units and the same for the remaining $(z^P - Q^P)^+$ units minus the spot sale revenue when the input volume Q^P is profitably sold. For $0 \leq z^P \leq Q^P$, when the spot sale is not profitable, i.e., $S(1-t) < b$, the unit procurement cost is given by the cost of the cheapest source (sourced either from the contract at a cost of b, or from the spot market at a cost of S). When the spot sale is profitable, i.e., $S(1-t) \geq b$, it is also cheaper to source from the contract than the spot market. Therefore, the unit procurement cost is the opportunity cost of not selling this input to the spot market, i.e., $S(1-t)$. Combining these two scenarios, the unit procurement cost is given by $\max(\min(S, b), S(1-t))$. For the processing volume exceeding Q^P, the input can only be sourced from the spot market, and the unit procurement cost is S.

For $D_2/a_2 \leq D_1/a_1$, the optimal processing volume z^{P*} is given by

$$z^{P*} = \begin{cases} 0 & \text{if } h_3 \leq \max(\min(S,b), S(1-t)), \\ \min\left(\dfrac{D_2}{a_2}, Q^P\right) & \text{if } h_2 \leq \max(\min(S,b), S(1-t)) \leq h_3 \leq S, \\ \min\left(\dfrac{D_1}{a_1}, Q^P\right) & \text{if } \max(\min(S,b), S(1-t)) \leq h_2 \leq h_3 \leq S, \\ \dfrac{D_2}{a_2} & \text{if } h_2 \leq \max(\min(S,b), S(1-t)) \leq S \leq h_3, \\ \max\left(\min\left(\dfrac{D_1}{a_1}, Q^P\right), \dfrac{D_2}{a_2}\right) & \text{if } \max(\min(S,b), S(1-t)) \leq h_2 \leq S \leq h_3, \\ \dfrac{D_1}{a_1} & \text{if } S \leq h_2, \end{cases} \tag{5}$$

where h_2 and h_3 are the unit processing margins as defined in (3). For $D_1/a_1 \leq D_2/a_2$, z^{P*} is obtained by interchanging D_1/a_1 with D_2/a_2, and substituting h_1 with h_2 in (5). In comparison with z^{P*} in the absence of spot market, as given in (1), the unit procurement cost takes two different forms, $\max(\min(S, b), S(1-t))$ or S, incorporating the spot procurement and resale options. When $t = 1$ such that there is no spot resale, and $S > h_3$ such that spot procurement is never optimal, (5) is identical to the characterization in (1) except for one modification: since I assume $\omega \geq a_1(s_1 - c_1)^+ + a_2(s_2 - c_2)^+$, the firm never processes up to Q^P when there is no unsatisfied demand in either market.

In stage 1, the firm determines the optimal volume of input to reserve Q^{P*} with respect to the demand $\tilde{\mathbf{D}}$

and spot price $\tilde{S}$ uncertainties so as to maximize the expected profit $V^P(Q^P) = \mathbb{E}[\pi^P(Q^P; \tilde{S}, \tilde{\mathbf{D}})] - \beta Q^P$.

Proposition 5. *$Q^{P*} = 0$ if $\beta \geq -b + \mu_S(1-t) + \mathbb{E}[\max(\min(\tilde{S}, h_3), b) - \tilde{S}(1-t) \mid \tilde{S} \leq h_3/(1-t)]$. Otherwise, Q^{P*} solves $(\partial V^P/\partial Q^P)\,|_{Q^{P*}} = 0$, where*

$$\begin{aligned}\frac{\partial V^P}{\partial Q^P} = -\beta &+ \int_b^{h_3} (\tilde{S}-b) H_1(\tilde{S})\, dF(\tilde{S}) \\ &+ \int_{h_3}^{h_3/(1-t)} (h_3 - b) H_1(\tilde{S})\, dF(\tilde{S}) \\ &+ \int_{h_3/(1-t)}^{\infty} \big(\tilde{S}(1-t) - b\big) H_1(\tilde{S})\, dF(\tilde{S}) \\ &+ \int_b^{\max(b,\,h_2)} (\tilde{S}-b) H_2(\tilde{S})\, dF(\tilde{S}) \\ &+ \int_{\max(b,\,h_2)}^{\max(b,\,h_2)/(1-t)} \big(\max(b, h_2) - b\big) H_2(\tilde{S})\, dF(\tilde{S}) \\ &+ \int_{\max(b,\,h_2)/(1-t)}^{\infty} \big(\tilde{S}(1-t) - b\big) H_2(\tilde{S})\, dF(\tilde{S}) \\ &+ \int_b^{\max(b,\,h_1)} (\tilde{S}-b) H_3(\tilde{S})\, dF(\tilde{S}) \\ &+ \int_{\max(b,\,h_1)}^{\max(b,\,h_1)/(1-t)} \big(\max(b, h_1) - b\big) H_3(\tilde{S})\, dF(\tilde{S}) \\ &+ \int_{\max(b,\,h_1)/(1-t)}^{\infty} \big(\tilde{S}(1-t) - b\big) H_3(\tilde{S})\, dF(\tilde{S}) \\ &+ \int_{b/(1-t)}^{\infty} \big(\tilde{S}(1-t) - b\big)\big[1 - H_1(\tilde{S}) - H_2(\tilde{S}) \\ &\qquad\qquad - H_3(\tilde{S})\big]\, dF(\tilde{S}),\end{aligned}$$

$$H_1(\tilde{S}) \doteq \Pr\left(\frac{\tilde{D}_1}{a_1} > Q^P, \frac{\tilde{D}_2}{a_2} > Q^P \,\middle|\, \tilde{S}\right),$$

$$H_2(\tilde{S}) \doteq \Pr\left(\frac{\tilde{D}_1}{a_1} > Q^P, \frac{\tilde{D}_2}{a_2} \leq Q^P \,\middle|\, \tilde{S}\right),$$

$$H_3(\tilde{S}) \doteq \Pr\left(\frac{\tilde{D}_1}{a_1} \leq Q^P, \frac{\tilde{D}_2}{a_2} > Q^P \,\middle|\, \tilde{S}\right).$$

The optimal contract volume is characterized by comparing the unit reservation price β with the expected marginal revenue of an additional unit of contracted input. At stage 2, the marginal revenue takes different forms as it depends on the spot price and the demand realizations. To provide the intuition, I focus on the case where $D_1/a_1 > Q^P$ and $D_2/a_2 > Q^P$ for a given S. In this case, the unit processing margin is given by h_3. For $S \leq b$, the contract is not exercised and the marginal revenue is zero. For $b \leq S \leq h_3$, it is profitable to source from the spot market for processing. Therefore, the marginal revenue is given by the opportunity gain of not buying from the spot market minus the exercise price b. For $h_3 \leq S \leq h_3/(1-t)$, it is not profitable to source from the spot market for processing, and thus, the marginal revenue is given by the unit processing margin h_3 minus the exercise price b. For $S \geq h_3/(1-t)$, spot resale is more profitable than processing, and thus, the marginal revenue is given by the spot sale revenue $S(1-t)$ minus the exercise price b.

The rest of the marginal revenue expression in Proposition 5 follows a similar structure. At stage 2, for a given spot price realization S, the marginal revenue is a function of three different identities when it is profitable to exercise: the spot procurement cost, the unit processing margin, or the spot sale revenue. The unit processing margin, h_1, h_2, or h_3, is determined by the demand realization in each market. When there is no unsatisfied demand in either market, processing is not profitable, and thus, only spot sale revenue is relevant.

In summary, the firm incorporates the spot procurement and resale options when deciding the optimal contract volume in stage 1. To make comparison with the characterization of the optimal contract volume in the absence of the spot market, I focus on the special case where the spot price and the demand have independent distributions.

Corollary 1. *When the spot price is independent of demand, i.e., $\bar{\rho}_{S1} = \bar{\rho}_{S2} = 0$, the first-order condition in Proposition 5 is given by*

$$\begin{aligned}\frac{\partial V^P}{\partial Q^P} = -\beta - b + \mu_S(1-t) &+ \Pr\left(\frac{\tilde{D}_1}{a_1} > Q^P, \frac{\tilde{D}_2}{a_2} > Q^P\right) G(b, h_3) \\ &+ \Pr\left(\frac{\tilde{D}_1}{a_1} > Q^P, \frac{\tilde{D}_2}{a_2} \leq Q^P\right) G(b, \max(b, h_2)) \\ &+ \Pr\left(\frac{\tilde{D}_1}{a_1} \leq Q^P, \frac{\tilde{D}_2}{a_2} > Q^P\right) G(b, \max(b, h_1)) \\ &+ \Pr\left(\frac{\tilde{D}_1}{a_1} \leq Q^P, \frac{\tilde{D}_2}{a_2} \leq Q^P\right) G(b, b),\end{aligned}$$

where $G(b, \lambda_1) \doteq \mathbb{E}[\max(\min(\tilde{S}, \lambda_1), b) - \tilde{S}(1-t) \mid \tilde{S} \leq \lambda_1/(1-t)]$.

In this case, the marginal cost of an additional unit of contract is given by the total procurement cost $\beta + b$. The expected marginal revenue is given by the sum of the expected spot sale revenue $\mu_S(1-t)$ and the expected processing revenue in excess of the spot sale, which also includes the optimal no exercise decision of the contract. In particular,

$$\begin{aligned}G(b, \lambda_1) \doteq \int_0^b \Big(b - \tilde{S}(1-t)\Big)\, dF(\tilde{S}) &+ \int_b^{\lambda_1} t\tilde{S}\, dF(\tilde{S}) \\ &+ \int_{\lambda_1}^{\lambda_1/(1-t)} \Big(\lambda_1 - \tilde{S}(1-t)\Big)\, dF(\tilde{S})\end{aligned}$$

denotes the expected processing revenue in excess of the spot sale with the unit processing margin λ_1 and the exercise price b. When $S \leq b$, the contract is optimally not exercised, and thus, the firm receives back b, which is deducted as a part of the marginal cost. The remaining terms are in parallel with my earlier discussion. The unit processing margin λ_1 is determined by the demand realization in each market, which explains the probability terms in Corollary 1. When $t = 1$ such that there is no spot resale, and $S > h_3$ such that spot procurement is never optimal, $G(b, \lambda_1) = \lambda_1$, and the first-order condition in Corollary 1 is identical to that of Proposition 1.

6.3. The Impact of Demand Correlation and Spot Price Variability on Contract Procurement

In this section, I conduct sensitivity analysis to study how firms should adjust their contract volume in the presence of spot market as a response to changing demand correlation and spot price variability. I investigate whether there exist any structural differences in this response based on the contract parameters. To this end, I analyze how my comparative static results are impacted as the exercise price of the contract changes.

Proposition 6. *Let $(\tilde{S}, \tilde{D}_1/a_1, \tilde{D}_2/a_2)$ follow a multivarite normal distribution. For $b \geq h_1$, $\partial Q^{P*}/\partial \rho \geq 0$. For $b < h_1$, when the spot price is independent of demand, i.e., $\bar{\rho}_{S1} = \bar{\rho}_{S2} = 0$, there exists a unique $\hat{b} \in [0, h_1)$ such that $\partial Q^{P*}/\partial \rho \leq 0$ if $b \leq \hat{b}$ and $\partial Q^{P*}/\partial \rho \geq 0$ if $b \geq \hat{b}$. In this case, $\hat{b} > 0$ if $G(0, h_3) < G(0, h_2) + G(0, h_1)$, and $\hat{b} = 0$ otherwise, where $G(b, \lambda_1)$ is as defined in Corollary* 1.

Proposition 6 demonstrates that the impact of the demand correlation on the firm's optimal supply management strategy crucially depends on the contract parameter: When the exercise price is sufficiently high, a higher contract market dependence is the best response to the increasing demand correlation. However, a lower contract market dependence may be the best response to the same when the exercise price is sufficiently low. I now explain the intuition behind this result. A higher demand correlation ρ increases the probability of unsatisfied demand in each market (with a processing margin h_3). It also decreases the probability of unsatisfied demand in the low demand market and no unsatisfied demand in the high demand market (with a processing margin h_1 or h_2). Therefore, the impact on Q^{P*} is determined by comparing the increase in the expected marginal revenue of contracting based on the h_3 case with the decrease in the same based on the h_1 and h_2 cases. When $b \geq h_1$, the contract is not exercised when the processing margin is h_1, and the former effect outweighs the latter, increasing Q^{P*}. When $b < h_1$, the net impact cannot be proven analytically except for the $\bar{\rho}_{S1} = \bar{\rho}_{S2} = 0$ case. In this case, under the condition given in Proposition 6, Q^{P*} decreases in ρ when the exercise price b is sufficiently small. This condition is satisfied, for example, when there is no spot sale $(t = 1)$ and $\omega = a_1(s_1 - c_1)^+ + a_2(s_2 - c_2)^+$.

In the absence of spot market, $G(b, \lambda_1) = \lambda_1$, and because $h_3 > h_2 + h_1$ by definition, the condition in Proposition 6 is never satisfied. Therefore, Q^{P*} increases in ρ for any b, as shown in Proposition 4. In the presence of spot market, the expected processing revenue in excess of the spot sale is lower than the processing margin due to the spot procurement option. As a result, the condition in Proposition 6 can be satisfied.

To understand the impact of demand correlation for the $b < h_1$ case without the $\bar{\rho}_{S1} = \bar{\rho}_{S2} = 0$ assumption, I conduct numerical experiments. The parameter levels for these experiments are chosen based on my interactions with the procurement managers of a cocoa processor and a multinational consumer goods company that procures cocoa butter and cocoa powder. I assume that $(\tilde{S}, \tilde{D}_1/a_1, \tilde{D}_2/a_2)$ follows a trivariate normal distribution, where $\tilde{S}$ has a marginal distribution with mean $\mu_S \in \{1{,}300, 1{,}500, 1{,}700\}$ and standard deviation $\sigma_S \in \{4\%, 8\%, 12\%\}$ of μ_S; and $(\tilde{D}_1/a_1, \tilde{D}_2/a_2))$ follows a symmetric bivariate distribution with mean $\mu_1 = \mu_2 \in \{12{,}000, 14{,}000, 16{,}000\}$, standard deviation $\sigma_1 = \sigma_2 \in \{4\%, 8\%, 12\%\}$ of μ_j, and correlation coefficient $\rho \in \{0, 0.2, 0.4, 0.6, 0.8\}$. For the quantity commitment contract, I assume a reservation price $\beta \in \{5\%, 20\%, 50\%\}$ of μ_S.[6] I use $a_1 = a_2 = 0.5$, $h_3 = 1{,}650$, $h_2 = 1{,}225$, $h_1 = 425$, and $t = 1$.[7] For the correlation between the spot price and the demand, I assume $\bar{\rho}_{S1} = \bar{\rho}_{S2} \in \{-0.25, 0.25, 0.5\}$. For each $\bar{\rho}_{Sj}$, I focus on 243 numerical instances to investigate the impact of ρ on the optimal contract volume. To study how the sign of this impact changes with the exercise price, I consider 10 $b \in [0, h_1)$ values. Let $Q^{P*}(\rho^k, b)$ denote the optimal contract volume with the kth ρ level and the exercise price b. I calculate the sign of $Q^{P*}(\rho^{k+1}, b) - Q^{P*}(\rho^k, b)$, and investigate how this sign changes as b increases. In all my numerical experiments, this sign changes once from negative to positive, i.e., Q^{P*} first decreases then increases in ρ, as b increases except for nine instances in the $\bar{\rho}_{Sj} = 0.5$

[6] These reservation prices satisfy $\beta > (1 - t)\mu_S$ such that the firm does not reserve an infinite volume of input and $\beta < \mu_S(1 - t) + G(0, h_3)$ such that the firm reserves a positive input volume for $b \geq 0$.

[7] Cocoa beans are first processed into cocoa liquor, which is further processed into cocoa butter and powder. These proportions reflect the yields out of the cocoa liquor. There is a 20% yield loss in the processing of cocoa beans into cocoa liquor. To capture this yield loss, I adjust the cocoa bean mean spot price representing the period between 2005 and 2007 by 1/0.8 to determine μ_S in my experiments.

case where the sign is positive for all b values considered.[8] These observations are consistent with the existence of $\hat{b} \in [0, h_1)$, as shown in Proposition 6.

As in Proposition 6, the impact of spot price variability on the optimal contract volume crucially depends on the exercise price of the contract:

PROPOSITION 7. *Assume that there is no spot sale possibility, i.e., $t = 1$; and let $\tilde{S}$ follow a normal distribution that is independent of demand, i.e., $\bar{\rho}_{S1} = \bar{\rho}_{S2} = 0$:*

(i) *When $\mu_S \geq (h_2 + h_3)/2$, $\partial Q^{P*}/\partial \sigma_S \geq 0$ if $b \geq \min(2\mu_S - h_3, h_3)$ and $\partial Q^{P*}/\partial \sigma_S \leq 0$ otherwise.*

(ii) *When $(h_2 + h_3)/2 > \mu_S \geq (h_1 + h_3)/2$, $\partial Q^{P*}/\partial \sigma_S \geq 0$ if $b \geq \min(\mu_S, h_2)$ and $\partial Q^{P*}/\partial \sigma_S \leq 0$ if $b \leq h_1$.*

(iii) *When $(h_1 + h_3)/2 > \mu_S \geq (h_1 + h_2)/2$, $\partial Q^{P*}/\partial \sigma_S \geq 0$ if $b \geq \min(\mu_S, h_2)$.*

(iv) *When $(h_1 + h_2)/2 > \mu_S \geq h_1/2$, $\partial Q^{P*}/\partial \sigma_S \geq 0$ if $b \geq \min(\mu_S, h_1)$.*

(v) *When $\mu_S < h_1/2$, $\partial Q^{P*}/\partial \sigma_S \geq 0$.*

As follows from Corollary 1, the impact of σ_S on Q^{P*} is determined by its effect on the expected marginal revenue from processing terms $G(b, h_3)$, $G(b, \min(b, h_2))$ and $G(b, \min(b, h_1))$. The overall effect can be characterized in five cases based on the mean spot price μ_S. When $\mu_S \geq h_2 + h_3/2$ (case (i)), Proposition 7 establishes a unique exercise price threshold $\bar{b}$, which is critical for the impact of σ_S on Q^{P*}. In particular, when the exercise price is larger than $\bar{b}$, Q^{P*} increases in σ_S. The reason is that, with an increase in σ_S, the contract benefits from high S realizations (when the marginal revenue at stage 2 is characterized by the opportunity gain of not using spot procurement), whereas it is not negatively affected from low S realizations due to the downside protection b (when the contract is not exercised at stage 2). When the exercise price is smaller than $\bar{b}$, Q^{P*} decreases in σ_S. This is because, with an increase in σ_S, the contract is negatively affected from low S realizations, whereas it does not benefit from high S realizations (when the marginal revenue is characterized by the unit processing margin). When $\mu_S < h_1/2$ (case (v)), $\bar{b} = 0$, and thus, Q^{P*} increases in σ_S for all the b values considered.

In the remaining μ_S range (cases (ii)–(iv)), Proposition 7 provides a partial characterization based on the exercise price b. Therefore, I resort to numerical experiments. In these experiments, I use the same parameter set as the demand correlation analysis except for $\rho \in \{0, 0.4, 0.8\}$, and $\sigma_S \in \{4\%, 6\%, 8\%, 10\%, 12\%\}$ of μ_S. For $\bar{\rho}_{S1} = \bar{\rho}_{S2} = 0$, I focus on 243 numerical instances to investigate the impact of σ_S on the optimal contract volume. To study how the sign of this impact changes with the exercise price, I consider 15 $b \in [0, h_1)$, 30 $b \in [h_1, h_2)$, and 15 $b \in [h_2, h_3)$ values. Let $Q^{P*}(\sigma_S^k, b)$ denote the optimal contract volume with the kth σ_S level and the exercise price b. Focusing only on the interior optimal solutions (with strictly positive contract volume), I calculate the sign of $Q^{P*}(\sigma_S^{k+1}, b) - Q^{P*}(\sigma_S^k, b)$, and investigate how this sign changes as b increases. In all my numerical experiments, either this sign changes once from negative to positive as b increases or this sign is always negative. In the latter case, the positive values are not observed because high b values lead to $Q^{P*} = 0$.[9]

In summary, Proposition 7 and my numerical experiments demonstrate that the impact of spot price variability on the firm's optimal supply management strategy crucially depends on the contract parameter: when the exercise price is high, a higher contract market dependence is the best response to the increasing spot price variability, whereas a lower contract market dependence is the best response to the same when the exercise price is low.

To understand the impact of spot price variability without the $\bar{\rho}_{S1} = \bar{\rho}_{S2} = 0$ assumption, I replicate my numerical experiments with $\bar{\rho}_{S1} = \bar{\rho}_{S2} \in \{-0.25, 0.25, 0.5\}$. For the $\bar{\rho}_{Sj} = -0.25$ case, paralleling the $\bar{\rho}_{Sj} = 0$ case, I observe that the sign of $Q^{P*}(\sigma_S^{k+1}, b) - Q^{P*}(\sigma_S^k, b)$ either changes once from negative to positive as b increases, or is always negative. For the other $\bar{\rho}_{Sj}$ cases, the result is inconclusive as I observe additional patterns, including the sign changing once from positive to negative, and the sign changing multiple times. The most significant consequence of this analysis is that, in a long-term planning horizon where the correlation between the spot price and the demand is typically different from zero, this correlation is instrumental in deciding whether to increase or decrease the contract procurement as a response to increasing spot price variability.

7. Discussion of Assumptions and Extensions

In this section, I discuss relaxing two of my assumptions, the fixed-price sales contract and the absence of output spot procurement. I only provide a qualitative summary of my results. The details of the analysis are available upon request.

[8] The condition in Proposition 6 is satisfied in my numerical setting. To validate my numerical code, I carry out additional experiments for the $\bar{\rho}_{S1} = \bar{\rho}_{S2} = 0$ case. In all these experiments, paralleling Proposition 6, the sign of $Q^{P*}(\rho^{k+1}, b) - Q^{P*}(\rho^k, b)$ changes once from negative to positive.

[9] The μ_S values considered (which are representative of the cocoa industry) correspond to cases (i) and (ii) of Proposition 7. To analyze cases (iii) and (iv), I replicate my experiments with $\mu_S \in \{700, 900, 1{,}100\}$. Besides the two patterns observed in the original experiments, I also observe instances where the optimal contract volume increases in σ_S for all b values considered, paralleling case (v) of Proposition 7.

7.1. Pass-Through Sales Contract

Throughout the paper, I assume that the firm uses a fixed-price sales contract. Another common form of sales contract used in the agricultural industries is the pass-through contract, where the sales price of the product includes a fixed processing margin and a variable component that is indexed on the spot price of the input.[10] In this section, I study the optimal supply management strategy with the pass-through sales contract, and investigate the impact of demand correlation and spot price variability on this strategy.

To analyze this problem, I assume that the processor uses a pass-through sales contract in the first market, and the sales price is given by $p_1 + \alpha S$, where $\alpha \geq 0$ denotes the degree of pass-through. When $\alpha \geq 1/a_1$, the spot procurement cost of the input volume required to produce one unit of the first product (S/a_1) is fully charged to the consumer. Therefore, I denote $\alpha \geq 1/a_1$ as the *full pass-through* sales contract. Similarly, I denote $\alpha < 1/a_1$ as the *partial pass-through* sales contract, where $\alpha = 0$ corresponds to the fixed-price sales contract. When $\alpha > 0$, the sales price depends on the spot price, and thus, it is uncertain at the time of the input contract procurement (stage 1). Throughout the analysis, I focus on the case where there is no spot sale possibility, i.e., $t = 1$.

In stage 2, the characterization of the optimal processing volume is identical to (5) except for the unit processing margins h_3 and h_2 are substituted by $\hat{h}_3(S) \doteq h_3 + a_1\alpha S$ and $\hat{h}_2(S) \doteq h_2 + a_1\alpha S$, respectively. In stage 1, the characterization of the optimal contract volume critically depends on the pass-through level α. To provide the intuition, I focus on the special case where the spot price and the demand have independent distributions. In this case, the first-order condition in Corollary 1 is given by

$$\begin{aligned}\frac{\partial V^P}{\partial Q^P} = -\beta - b + H_1\bigg[G\bigg(b, \max\bigg(b, \frac{h_3}{(1-a_1\alpha)^+}\bigg)\bigg) + a_1\alpha \\ \mathbb{E}\bigg[\bigg(\tilde{S} - \max\bigg(\frac{b-h_3}{a_1\alpha}, \frac{h_3}{(1-a_1\alpha)^+}\bigg)\bigg)^+\bigg]\bigg] \\ + H_2\bigg[G\bigg(b, \max\bigg(b, \frac{h_2}{(1-a_1\alpha)^+}\bigg)\bigg) + a_1\alpha \\ \mathbb{E}\bigg[\bigg(\tilde{S} - \max\bigg(\frac{b-h_2}{a_1\alpha}, \frac{h_2}{(1-a_1\alpha)^+}\bigg)\bigg)^+\bigg]\bigg] \\ + H_3 G(b, \max(b, h_1)) + b(1 - H_1 - H_2 - H_3),\end{aligned}$$

where $H_1 \doteq \Pr(\tilde{D}_1/a_1 > Q^P,\ \tilde{D}_2/a_2 > Q^P)$, $H_2 \doteq \Pr(\tilde{D}_1/a_1 > Q^P,\ \tilde{D}_2/a_2 \leq Q^P)$, and $H_3 \doteq \Pr(\tilde{D}_1/a_1 \leq Q^P,\ \tilde{D}_2/a_2 > Q^P)$. With the full pass-through sales contract ($\alpha \geq 1/a_1$), when $D_1/a_1 > Q^P$, the expected processing revenue is given by $G(b, \infty) = \mathbb{E}[\max(\tilde{S}, b)]$: In stage 2, it is always profitable to source from the spot market for processing, i.e., $S \leq \hat{h}_j(S) = h_j + a_1\alpha S$ for $j = 2, 3$, and thus, the processing revenue is the opportunity gain of not procuring from the spot market. With the partial pass-through sales contract ($\alpha < 1/a_1$), when $D_1/a_1 > Q^P$, the expected processing revenue is given by the sum of two terms, the revenue with the fixed processing margin $h_j/(1 - a_1\alpha)$, and the additional revenue over this fixed margin due to the pass-through of the input spot price. With the fixed-price sales contract ($\alpha = 0$), only the first term is relevant, and the expected processing revenue is $G(b, \max(b, h_j))$, as given in Corollary 1.

The impact of demand correlation on the optimal contract volume. With the partial pass-through sales contract, paralleling the fixed-price sales contract, when the exercise price is high (low), a higher (lower) contract market dependence is the best response to the increasing demand correlation. With the full pass-through sales contract, unlike the fixed-price sales contract, a lower contract market dependence is the best response to the same regardless of the exercise price. This is because when there is unsatisfied demand in the first market, it is always profitable to source from the spot market for processing.

The impact of spot price variability on the optimal contract volume. With the partial pass-through sales contract, paralleling the fixed-price sales contract, there exists an exercise price threshold such that when the exercise price is higher (lower) than this threshold, a higher (lower) contract market dependence is the best response to the increasing spot price variability. Unlike the fixed-price sales contract, this threshold can be equal to zero such that a higher contract market dependence may be the best response to the increasing spot price variability regardless of the exercise price. The reason is that the expected value of processing increases due to the pass-through of the spot price. With the full pass-through sales contract, a higher contract market dependence is always the best response to the increasing spot price variability regardless of the exercise price of the procurement contract. The reason is that the expected value of the opportunity gain of not procuring from the spot market increases in the spot price variability.

7.2. Access to Output Spot Procurement

Throughout the paper, I assume that the demand is satisfied from the processed input. If there exists a spot market for the output, then the firm may also satisfy the demand by sourcing from this spot market. In practice, this observation is relevant, for example,

[10] For example, in the cocoa industry, the sales price of the cocoa butter can be linked to the spot price of the cocoa beans based on the extraction rate of the butter from the beans, a practice called ratio pricing.

for the oilseed processors.[11] In this section, I study the optimal supply management strategy, and the impact of demand correlation and (input) spot price variability on this strategy in the presence of the input and the output spot markets in comparison with the benchmark case (in the absence of the output spot market), as considered in §6.

To analyze this problem, I assume that the first product can be sourced from the output spot market at the prevailing price M on the day (stage 2), which is uncertain at the time of the contract procurement (stage 1), where $(\tilde{M}, \tilde{S}, \tilde{D}_1/a_1, \tilde{D}_2/a_2)$ follows a multivariate distribution. I normalize the salvage values and the product-specific processing costs to zero, i.e., $s_1 = s_2 = c_1 = c_2 = 0$. With this model, the firm's stage-2 objective function is identical to (4) except for one modification: $p_1 \min(a_1 z^P, D_1)$ is substituted by $D_1(p_1 - M)^+ + \min(a_1 z^P, D_1)\min(p_1, M)$. In other words, the revenue from the first product market is given by the sum of the revenue when the demand is (profitably) satisfied from the output spot market and the additional revenue from processing. The unit revenue from processing is given by the opportunity gain of not procuring that unit from the (output) spot market when it is profitable to satisfy the demand from the spot market; and it is given by the sales price otherwise. In stage 1, the characterization of the optimal contract volume is identical to Proposition 5, except for two minor modifications: h_3 and h_2 are substituted by the unit processing margins $\hat{h}_3(M) \doteq a_1 \min(p_1, M) + a_2 p_2 - \omega$ and $\hat{h}_2(M) \doteq a_1 \min(p_1, M) - \omega$, respectively; and an expectation is taken with respect to the $\tilde{M}$ distribution. In comparison with the benchmark case, because the processing revenue is smaller in the first market, the optimal contract volume is lower. In contrast, the optimal expected profit is higher because of the output spot procurement option.

To conduct sensitivity analysis to investigate the impact of demand correlation and (input) spot price variability on the optimal contract volume, I introduce further assumptions. In particular, I normalize the processing cost to zero, i.e., $\omega = 0$, and focus on the case where there is no input spot sale possibility, i.e., $t = 1$. I assume that the input and the output spot prices are perfectly correlated, $\tilde{M} \doteq \gamma \tilde{S}$, and leave the analysis with a more general correlation structure for future research. To focus on the practically relevant cases, because one unit of the processed input leads to a_1 units of the first product, I assume that γ is in the vicinity of $1/a_1$.[12] I also assume $(\tilde{S}, \tilde{D}_1/a_1, \tilde{D}_2/a_2)$, which is sufficient to characterize the uncertainties, follows a trivariate normal distribution. With these assumptions, the impact of demand correlation is structurally the same with the benchmark case. In particular, Proposition 6 continues to hold except for one modification: h_3 and h_2 are substituted by $\hat{h}_3(S) = h_3 - (h_2 - a_1\gamma S)^+$ and $\hat{h}_2(S) = h_2 - (h_2 - a_1 \gamma S)^+$. For the impact of input spot price variability, an analogue of Proposition 7 and additional numerical experiments demonstrate that this impact is structurally the same with the benchmark case: when the exercise price is high (low), a higher (lower) contract market dependence is the best response to the increasing spot price variability.

8. Conclusion

The first contribution of this paper is to the literature on supply management in multiproduct firms. The majority of papers in this stream study flexible production technology, where a single input is capable of producing multiple outputs, and gives rise to one output in each production run. In practice, a common feature of the processors in agricultural industries is that they operate under the fixed proportions production technology, where a single input gives rise to multiple outputs in fixed proportions in each production run. In this literature, a few papers study coproduction systems, where multiple products are produced in a single production run with random yields. These papers remain silent on the impact of demand uncertainty. I contribute to this literature by studying the fixed proportions technology under demand uncertainty and the impact of demand correlation on this technology in comparison with the flexible technology.

To analyze this problem, I consider a firm (processor) that procures a single input and sells two outputs that are produced in fixed proportions of this input. The firm chooses its procurement volume under demand uncertainty, and the processing volume after this uncertainty is resolved. As a benchmark case, I consider a firm that operates under the flexible technology. With the flexible technology, as established in the extant literature, since the input can be allocated between the two outputs in response to demand realizations, there exists a capacity-pooling benefit. With the fixed proportions technology, since the same input is used for producing both outputs, there exists a cost-pooling benefit. I show that when the two production systems have the same profitability, the cost-pooling benefit of the fixed proportions technology is larger than the capacity-pooling benefit

[11] An oilseed pressing plant processes the oilseed (rapeseed, sunflower seed, coconut seed, palm fruit, and soybean) to produce crude vegetable oil (rapeseed oil, sunflower oil, coconut oil, palm oil, and soybean oil) and a by-product (meal, cake, kernel) in fixed proportions. Because the oilseed and the crude vegetable oil are commodities, there exist spot markets for the input and the output.

[12] For the results presented in this section, I assume $\gamma \geq h_2/(a_1 h_3)$.

Table 1 Impact of a Higher Demand Correlation with Each Production Technology

	Pooling value	Contract volume
Fixed proportions technology	Cost-pooling value increases	Increase the contract volume
Flexible technology	Capacity-pooling value decreases	Increase the contract volume when total expected demand is smaller than this volume; and decrease it otherwise

of the flexible technology. My analysis identifies the critical role that demand correlation plays with each technology, as summarized in Table 1.

These results have important implications for the new product strategy of the processors in the agricultural industries. One such strategy, that has been suggested in the extant literature, is to create a differentiated product by mixing the input with different ingredients (such as flavors) or by changing the packaging of the product. Because the differentiated product uses the same input with the existing product, this strategy banks on the capacity-pooling value of the flexible production technology. My results suggest that converting the biomass (the organic residue from processing) into a by-product is another potent strategy that banks on the cost-pooling value of the fixed proportions technology. In practice, this strategy has already been implemented by the wood processors (White 2011), where the biomass is marketed as fuel, and by the sugarcane processors (Bose 2003), where the biomass is marketed as fuel or as a wood-substitute raw material for paper manufacturing. My results in Table 1 point out that there are fundamental differences between the two strategies, suggesting distinct managerial actions. For example, when the demand correlation between two products increases in the number of overlapping customers for each product, my results suggest that a differentiated product should be targeted to a new customer base to decrease demand correlation with the existing product, whereas a by-product should be targeted to the existing customer base to increase this correlation.

The second contribution of this paper is to the literature on supply management in the presence of contract and spot markets. Barring Boyabatlı et al. (2011), there is no work in this literature that focuses on the fixed proportions technology. I contribute to this literature by studying how the supply management strategy should respond to changing spot price variability and demand correlation in a new modeling setting that is relevant for agricultural industries, and investigating whether there exist any structural differences in this response based on the contract type. To analyze this problem, I extend my fixed proportions technology model to incorporate for the spot market access of the firm. My analysis provides the results summarized in Table 2.

Table 2 The Optimal Response to an Increase in the Spot Price Variability or the Demand Correlation Based on the Contract Type in a Short-Term Planning Horizon

	Low reservation price High exercise price	High reservation price Low exercise price
Spot price variability	Increase the contract volume	Decrease the contract volume
Demand correlation	Increase the contract volume	Decrease the contract volume

These results have important implications for the procurement contract choice of the processors in the agricultural industries. As established in the literature, two contracts, one with the lower reservation price and the other with the lower exercise price, can generate the same expected profit when the reservation and exercise prices are appropriately chosen.[13] In this case, a processor that is only concerned with the profit impact of its contract choice is indifferent between the two contracts. However, the processor may also be concerned with the volume impact of the contract choice on its supplier: the processor typically has a long-term relationship with its supplier, and in order not to harm this relationship, the processor prefers not to decrease its procurement volume as a response to a change in the business environment. In agricultural industries, two such changes have been documented in the last two decades. First, spot prices have shown increasing variability (Meyer 2013). Second, the food manufacturers, retailers, and consumer goods companies (the customers of the processors) have consolidated their supplier base because of food safety concerns (Maitland 1997), and thus, multiple products are sourced from the same processor increasing the demand correlation between these products. A key implication of my results in Table 2 is that the contract with the lower reservation price should be preferred over the contract with the lower exercise price. This is because the former does not require decreasing of the contract volume with an increase in the demand correlation or the spot price variability.

My work comes with several limitations. For brevity, I focus on a firm with two outputs. Although the analysis would be more complex, I expect my main results to continue to hold with n outputs. In my model, I assume specific forms of procurement and sales contracts that are commonly used in agricultural industries. Analyzing other contract forms should prove to be an interesting problem for future research.

[13] This result can be proven in my setting using the fact that when the firm optimally uses a contract market, the optimal expected profit strictly decreases in the reservation or the exercise price of the contract.

On the procurement side, the exercise price can be a function of the contract volume due to volume discounts (Pei et al. 2011), or can be a function of the input spot price (Boyabatlı et al. 2011). On the sales side, the output price can be a function of the production volume (Kazaz and Webster 2011). My model implicitly assumes that the firm has perfect information about the supplier when sourcing from the contract market. In the literature, a stream of papers studies the optimal supply management in the presence of imperfect information about the supplier's operational characteristics including the reliability (Yang et al. 2009), the production cost (Zhang 2010), and the product quality (Babich and Tang 2012), all in a single-product firm. It would be interesting to analyze whether the insights coming from these papers continue to hold with the fixed proportions technology.

Other interesting future research directions remain. Throughout the paper, I assume that the contract parameters are exogenous. In practice, suppliers may adjust the contract parameters as a response to changes in the demand and the spot price uncertainties. It would be interesting to analyze the impact of these uncertainties on the optimal supply management strategy in an equilibrium setting, following the examples of Wu and Kleindorfer (2005) and Pei et al. (2011), who provide some results in a single-product firm in the absence of the fixed proportions technology. I also assume that all the reserved input volume is available on the spot day. As highlighted in Tomlin (2006), the contract procurement can be unreliable and the reserved volume is only partially available (or completely unavailable) on the day. In agricultural industries, this unreliability can be driven by farm-yield uncertainty due to weather conditions. Modeling the farm-yield uncertainty requires additional features such as the yield-dependent spot price (Kazaz and Webster 2011), the yield-dependent processing cost (Kazaz 2004), and the yield-dependent proportions (Boyabatlı and Wee 2013). This is clearly a different problem and merits a separate study. Finally, I assume that the firm sources from a single contract. In the literature, a stream of papers studies sourcing from multiple contracts and investigates the conditions under which different sourcing policies (such as single versus dual sourcing) are optimal in a variety of models. These papers focus on contracts that have heterogeneous characteristics such as procurement cost (Kleindorfer and Wu 2003, Martínez-de-Albéniz and Simchi-Levi 2005), unreliability (Tomlin and Wang 2005, Babich et al. 2007), procurement capacity (Wang et al. 2010), and delivery lead time (Wu and Zhang 2014). There is no work in this literature that analyzes a multiproduct firm. It would be interesting to study how the optimal sourcing policy is affected by the fixed proportions technology.

Appendix

Let $\phi(\cdot)$ and $\Phi(\cdot)$ denote the standard normal p.d.f. and c.d.f., respectively; $\phi'(z) = -z\phi(z)$, $\int_{-\infty}^{v} z\phi(z)\,dz = -\phi(v)$. For a bivariate normal $(\tilde{X}_1, \tilde{X}_2)$ with correlation coefficient ρ, $\Pr(\tilde{X}_1 \le x_1, \tilde{X}_2 \le x_2)$ is increasing in ρ for a fixed (x_1, x_2) as follows from the Slepian's inequality (Tong 1990, p. 21). I use the following result from Müller (2001):

Lemma A.1. *Let $\tilde{\mathbf{X}}$ ($\underline{\tilde{\mathbf{X}}}$) have a bivariate normal distribution with mean $\boldsymbol{\mu}$ ($\underline{\boldsymbol{\mu}}$) and covariance matrix Σ ($\underline{\Sigma}$). If $\boldsymbol{\mu} = \underline{\boldsymbol{\mu}}$, $\tilde{\mathbf{X}}$ and $\underline{\tilde{\mathbf{X}}}$ have the same marginal distributions, $\Sigma_{ij} \le \underline{\Sigma}_{ij}$, then $\tilde{\mathbf{X}} \le \underline{\tilde{\mathbf{X}}}$ in the supermodular order, i.e. $\mathbb{E}[f(\tilde{\mathbf{X}})] \le \mathbb{E}[f(\underline{\tilde{\mathbf{X}}})]$ for any supermodular function f.*

Proof of Proposition 1. The proof is omitted. □

Proof of Proposition 2. It is sufficient to prove $\overline{\beta}^P(\beta^D) > \overline{\beta}^F(\beta^D)$. Because $\partial V^{P*}/\partial\beta^P < 0$, it is sufficient to show that $V^{P*} > V^{F*}$ when $\beta^P = \beta^F = \beta$. From the optimality of Q^{P*}, it is sufficient to prove that $V^P(Q^P) > V^F(Q^F)$ for $Q^P = Q^F = Q$. $V^i(Q) = \mathbb{E}[\pi^i(Q; \tilde{\mathbf{D}})] - \beta Q$ for $i \in \{F, P\}$, where $\pi^i(Q; \mathbf{D}) = \max_{0 \le z^i \le Q} \Pi^i(z^i)$, and $\Pi^i(z^i) = -(b+\omega)z^i + \Psi^i(z^i)$,

$$\begin{aligned}
\Psi^P(z^P) &\doteq \sum_{j=1}^{2}\Bigg[a_j(p_j - c_j)\min\left(z^P, \frac{D_j}{a_j}\right) \\
&\qquad + a_j(s_j - c_j)^+\left(z^P - \frac{D_j}{a_j}\right)^+\Bigg], \\
\Psi^F(z^F) &\doteq a_1(p_1 - c_1)\min\left(z^F, \frac{D_1}{a_1}\right) \\
&\qquad + a_2(p_2 - c_2)\min\left(\left(z^F - \frac{D_1}{a_1}\right)^+, \frac{D_2}{a_2}\right) \\
&\qquad + a_1(s_1 - c_1)^+\left(z^F - \frac{D_1}{a_1} - \frac{D_2}{a_2}\right)^+.
\end{aligned}$$

For $z^F = z^P = z$, $\Pi^P(z) \ge \Pi^F(z)$ because proportional technology has a higher salvage value in the first market due to $(z - D_1/a_1)^+ \ge (z - D_1/a_1 - D_2/a_2)^+$; a higher sales revenue in the second market due to $\min(z, D_2/a_2) \ge \min((z - D_1/a_1)^+, D_2/a_2)$; and a higher salvage revenue in the second market (which does not exist with the flexible technology). Since $\Pi^P(z) \ge \Pi^F(z)$, where the equality is strict for some demand realizations, $\Pi^P(z^{P*}) \ge \Pi^F(z^{F*})$ from the optimality of z^{P*}, and thus, $\pi^P(Q; \mathbf{D}) \ge \pi^F(Q; \mathbf{D})$, where the equality is strict for some demand realizations. Therefore, $\mathbb{E}[\pi^P(Q; \tilde{\mathbf{D}})] > \mathbb{E}[\pi^F(Q; \tilde{\mathbf{D}})]$. □

Proof of Proposition 3. I only provide the proof for the proportional technology. Because $\partial V^{D*}/\partial\rho = 0$ and $\partial V^{P*}/\partial\beta^P < 0$, it is sufficient to show that, for a given β^P, $\partial V^{P*}/\partial\rho \ge 0$, which holds true when $\partial V^P(Q)/\partial\rho \ge 0$ for a given $Q^P = Q$. It follows from Lemma A.1 that increasing ρ leads to another bivariate normal distribution that is preferred over $\tilde{\mathbf{D}}/a$ in the supermodular order. Because $V^P(Q) = \mathbb{E}[\pi^i(Q; \mathbf{D}/a)] - \beta Q$, it is sufficient to prove that $\pi^P(Q; \mathbf{D}/a)$ is supermodular in $\tilde{\mathbf{D}}/a$. I now provide that proof:

It follows from z^{P*} in (1) that, the optimal stage 2 profit function $\pi^P(Q; \mathbf{D}/a)$ is a function of the ordering between b and $a_1(s_1 - c_1)^+ + a_2(s_2 - c_2)^+ - \omega \le a_1(s_1 - c_1)^+ +$

$a_2(p_2-c_2)-\omega \le a_1(p_1-c_1)+a_2(s_2-c_2)^+-\omega \le a_1(p_1-c_1)+a_2(p_2-c_2)-\omega$. Therefore, I have four cases to consider. I only analyze the case $a_1(s_1-c_1)^+ + a_2(p_2-c_2)-\omega \le b \le a_1(p_1-c_1)+a_2(s_2-c_2)^+-\omega$. The other cases can be analyzed in a similar fashion:

$$\pi^P(Q;\mathbf{D}/a) = \begin{cases} \left[a_1(p_1-c_1)+a_2(s_2-c_2)^+-\omega-b\right]\dfrac{D_1}{a_1} \\ \quad +\left[a_2(p_2-c_2)-a_2(s_2-c_2)^+\right]\dfrac{D_2}{a_2} \\ \qquad \text{if } \dfrac{D_2}{a_2} \le \dfrac{D_1}{a_1} \le Q, \\ \left[a_1(p_1-c_1)+a_2(p_2-c_2)-\omega-b\right]\dfrac{D_1}{a_1} \\ \qquad \text{if } \dfrac{D_1}{a_1} < \dfrac{D_2}{a_2} \le Q, \\ \left[a_1(p_1-c_1)+a_2(p_2-c_2)-\omega-b\right]\dfrac{D_1}{a_1} \\ \qquad \text{if } \dfrac{D_1}{a_1} \le Q < \dfrac{D_2}{a_2}, \\ \left[a_1(p_1-c_1)+a_2(s_2-c_2)^+-\omega-b\right]Q \\ \quad +\left[a_2(p_2-c_2)-a_2(s_2-c_2)^+\right]\dfrac{D_2}{a_2} \\ \qquad \text{if } \dfrac{D_2}{a_2} \le Q < \dfrac{D_1}{a_1}, \\ \left[a_1(p_1-c_1)+a_2(p_2-c_2)-\omega-b\right]Q \\ \qquad \text{if } Q < \dfrac{D_2}{a_2} \le \dfrac{D_1}{a_1}, \\ \left[a_1(p_1-c_1)+a_2(p_2-c_2)-\omega-b\right]Q \\ \qquad \text{if } Q < \dfrac{D_1}{a_1} < \dfrac{D_2}{a_2}. \end{cases}$$

To prove the supermodularity of $\pi^P(Q;\mathbf{D}/a)$ in $\tilde{\mathbf{D}}/a$, it is sufficient to show $\partial^2\pi^P/\partial(\mathbf{D}_1/a_1)\partial(\mathbf{D}_2/a_2) \ge 0$.

$$\frac{\partial \pi^P(Q;\mathbf{D}/a)}{\partial(D_1/a_1)} = \begin{cases} a_1(p_1-c_1)+a_2(s_2-c_2)^+-\omega-b \\ \qquad \text{if } \dfrac{D_2}{a_2} \le \dfrac{D_1}{a_1} \le Q, \\ a_1(p_1-c_1)+a_2(p_2-c_2)-\omega-b \\ \qquad \text{if } \dfrac{D_1}{a_1} < \dfrac{D_2}{a_2} \le Q, \\ a_1(p_1-c_1)+a_2(p_2-c_2)-\omega-b \\ \qquad \text{if } \dfrac{D_1}{a_1} \le Q < \dfrac{D_2}{a_2}, \\ 0 \qquad \text{if } \dfrac{D_2}{a_2} \le Q < \dfrac{D_1}{a_1}, \\ 0 \qquad \text{if } Q < \dfrac{D_2}{a_2} \le \dfrac{D_1}{a_1}, \\ 0 \qquad \text{if } Q < \dfrac{D_1}{a_1} < \dfrac{D_2}{a_2}. \end{cases}$$

It is easy to show that for a given D_1/a_1, $\partial\pi^P(Q;\mathbf{D}/a)/\partial(D_1/a_1)$ increases as D_2/a_2 increases. For example, for a given $D_1/a_1 < Q$, as D_2/a_2 increases, $\partial\pi^P(Q;\mathbf{D}/a)/\partial(D_1/a_1)$ traces the first three regions in (6) in this order. Since $a_1(p_1-c_1)+a_2(p_2-c_2)-\omega > a_1(p_1-c_1)+a_2(s_2-c_2)^+-\omega$, $\partial^2\pi^P(Q;\mathbf{D}/a)/\partial(D_1/a_1)\partial(D_2/a_2) \ge 0$. □

Proof of Proposition 4. With the dedicated technology, the first order condition for market j is obtained by substituting $a_{-j}=0$ in (2) of Proposition 1. Because Q_j^{D*} is a function of $\tilde{D}_j/a_j$, it is independent of ρ. For the other technologies, let $J^i \doteq \partial V^i/\partial Q^i$ for $i \in \{F, P\}$. From the implicit function theorem, $\operatorname{sgn}(\partial Q^{i*}/\partial\rho) = \operatorname{sgn}(\partial J^i/\partial\rho|_{Q^{i*}})$.

With the flexible technology, since $b < a_2(p_2-c_2)-\omega$ by assumption, I obtain

$$\begin{aligned} J^F &= -\beta^F + a_2(p_2-c_2)-\omega-b \\ &\quad + \Pr\left(\frac{\tilde{D}_1}{a_1} > Q^F\right)\left(a_1(p_1-c_1)-a_2(p_2-c_2)\right) \\ &\quad + \Pr\left(\frac{\tilde{D}_1}{a_1}+\frac{\tilde{D}_2}{a_2} \le Q^F\right) \\ &\quad \cdot\left((a_1(s_1-c_1)^+-\omega-b)^+ - (a_2(p_2-c_2)-\omega-b)\right), \end{aligned}$$

$$\left.\frac{\partial J^F}{\partial\rho}\right|_{Q^{F*}} = \frac{\partial \Pr(\tilde{D}_1/a_1+\tilde{D}_2/a_2 \le Q^{F*})}{\partial\rho} \cdot\left((a_1(s_1-c_1)^+-\omega-b)^+ - (a_2(p_2-c_2)-\omega-b)\right),$$

where the second term is negative from $b < a_2(p_2-c_2)-\omega$. $\tilde{D}_1/a_1+\tilde{D}_2/a_2$ follows a normal distribution with mean $\hat{\mu} = \mu_1+\mu_2$ and standard deviation $\hat{\sigma} = \sqrt{\sigma_1^2+\sigma_2^2+2\rho\sigma_1\sigma_2}$.

$$\frac{\partial \Pr(\tilde{D}_1/a_1+\tilde{D}_2/a_2 \le Q^{F*})}{\partial\rho} = -\frac{\phi(Q^{F*}-\hat{\mu})}{\hat{\sigma}}\,\frac{Q^{F*}-\hat{\mu}/\hat{\sigma}}{\partial\hat{\sigma}/\partial\rho},$$

which is negative for $Q^{F*} > \hat{\mu}$, and positive otherwise.

With the proportional technology, J^P is as given in (2). Let $A \doteq \{\tilde{D}_1/a_1 \le Q^{P*}\}$ and $B \doteq \{\tilde{D}_2/a_2 \le Q^{P*}\}$. Using the identities, $\Pr(\bar{A}) = 1-\Pr(A)$, $\Pr(\bar{B}) = 1-\Pr(B)$, $\Pr(A\bar{B}) = \Pr(A)-\Pr(AB)$, $\Pr(\bar{A}B) = \Pr(B)-\Pr(AB)$, $\Pr(\bar{A}\,\bar{B}) = 1-\Pr(A)-\Pr(B)+\Pr(AB)$, I obtain $\partial J^P/\partial\rho|_{Q^{P*}} = (\partial\Pr(AB)/\partial\rho)H$, where $H = (a_1(s_1-c_1)^+ + a_2(s_2-c_2)^+ - \omega - b)^+ - (a_1(s_1-c_1)^+ + a_2(s_2-c_2)^+ - \omega - b)$. It follows from the Slepian inequality that $\partial\Pr(AB)/\partial\rho \ge 0$. Moreover, $H > 0$ for $b > (a_1(s_1-c_1)^+ + a_2(s_2-c_2)^+ - \omega)^+$, and $H = 0$ otherwise. □

Proof of Proposition 5. The proof is omitted. □

Proof of Corollary 1. The result follows from Proposition 5 by using $\bar{\rho}_{S1} = \bar{\rho}_{S2} = 0$. □

Proof of Proposition 6. For a given S, $(\tilde{D}_1/a_1, \tilde{D}_2/a_2|S)$ follow a bivariate normal distribution with mean vector $\hat{\boldsymbol{\mu}}' = (\mu_1+\bar{\rho}_{S1}(\sigma_1/\sigma_S)(S-\mu_S),\ \mu_2+\bar{\rho}_{S2}(\sigma_2/\sigma_S)(S-\mu_S))$, standard deviation vector $\hat{\boldsymbol{\sigma}}' = (\sigma_1\sqrt{1-\bar{\rho}_{S1}^2},\ \sigma_2\sqrt{1-\bar{\rho}_{S2}^2})$, and correlation coefficient $\hat{\rho} = (\rho-\bar{\rho}_{S1}\bar{\rho}_{S2})/(\sqrt{1-\bar{\rho}_{S1}^2}\sqrt{1-\bar{\rho}_{S2}^2})$. It follows that increasing ρ increases $\hat{\rho}$, and does not change the mean $\hat{\boldsymbol{\mu}}$ or the standard deviation $\hat{\boldsymbol{\sigma}}$. Let $J^P \doteq \partial V^P/\partial Q^P$ as defined in Proposition 5. From the implicit function theorem, $\operatorname{sgn}(\partial Q^{P*}/\partial\rho) = \operatorname{sgn}(\partial J^P/\partial\rho|_{Q^{P*}})$. Let $A \doteq \{(\tilde{D}_1/a_1|S) \le Q^{P*}\}$ and $B \doteq \{(\tilde{D}_2/a_2|S) \le Q^{P*}\}$. For $b \ge h_1$, using the identities, $\Pr(\bar{A}) = 1 - \Pr(A)$, $\Pr(\bar{B}) = 1 - \Pr(B)$, $\Pr(\bar{A}B) = \Pr(B) - \Pr(AB)$, $\Pr(\bar{A}\bar{B}) = 1 - \Pr(A) - \Pr(B) + \Pr(AB)$, J^P can be rewritten in terms of $\Pr(A)$, $\Pr(B)$, and $\Pr(AB)$. Let $X(\tilde{S}) = \partial \Pr(AB)/\partial \rho$. Since $\hat{\rho}$ is increasing in ρ, $X(\tilde{S}) \geq 0$ from the Slepian's inequality. I obtain

$$\begin{aligned}\left.\frac{\partial J^P}{\partial \rho}\right|_{Q^{P*}} &= \int_b^{h_3} (\tilde{S}-b)X(\tilde{S})\,dF(\tilde{S}) + \int_{h_3}^{h_3/(1-t)} (h_3-b)X(\tilde{S})\,dF(\tilde{S})\\ &\quad+\int_{h_3/(1-t)}^{\infty} (\tilde{S}(1-t)-b)X(\tilde{S})\,dF(\tilde{S})\\ &\quad-\int_b^{h_2} (\tilde{S}-b)X(\tilde{S})\,dF(\tilde{S})\\ &\quad-\int_{h_2}^{h_2/(1-t)} (h_2-b)X(\tilde{S})\,dF(\tilde{S})\\ &\quad-\int_{h_2/(1-t)}^{\infty} (\tilde{S}(1-t)-b)X(\tilde{S})\,dF(\tilde{S}).\end{aligned}$$

I have two cases to consider. For $h_2/(1-t) \leq h_3$,

$$\begin{aligned}\left.\frac{\partial J^P}{\partial \rho}\right|_{Q^{P*}} &= \int_{h_2}^{h_2/(1-t)} (\tilde{S}-h_2)X(\tilde{S})\,dF(\tilde{S})\\ &\quad+\int_{h_2/(1-t)}^{h_3} (t\tilde{S})X(\tilde{S})\,dF(\tilde{S})\\ &\quad+\int_{h_3}^{h_3/(1-t)} (h_3-\tilde{S}(1-t))X(\tilde{S})\,dF(\tilde{S}) \geq 0;\end{aligned}$$

and for $h_3 \leq h_2/(1-t)$,

$$\begin{aligned}\left.\frac{\partial J^P}{\partial \rho}\right|_{Q^{P*}} &= \int_{h_2}^{h_3} (\tilde{S}-h_2)X(\tilde{S})\,dF(\tilde{S})\\ &\quad+\int_{h_3}^{h_2/(1-t)} (h_3-h_2)X(\tilde{S})\,dF(\tilde{S})\\ &\quad+\int_{h_2/(1-t)}^{h_3/(1-t)} (h_3-\tilde{S}(1-t))X(\tilde{S})\,dF(\tilde{S}) \geq 0.\end{aligned}$$

When $\bar{\rho}_{S1} = \bar{\rho}_{S2} = 0$, J^P is given by (6) in Corollary 1. Defining $A \doteq \{\tilde{D}_1/a_1 \leq Q^{P*}\}$ and $B \doteq \{\tilde{D}_2/a_2 \leq Q^{P*}\}$, for $b \leq h_1$, $\partial J^P/\partial \rho|_{Q^{P*}} = (\partial \Pr(AB)/\partial \rho)[G(b,b) + G(b,h_3) - G(b,h_1) - G(b,h_2)]$, where the first term is positive from the Slepian's inequality. Therefore, for a given b, $\operatorname{sgn}(\partial Q^{P*}/\partial \rho)$ is given by the sign of the second term. Let $H(b)$ denote this term. From $G(b,\lambda) = \int_0^b (b - \tilde{S}(1-t))\,dF(\tilde{S}) + \int_b^\lambda t\tilde{S}\,dF(\tilde{S}) + \int_\lambda^{\lambda/(1-t)} (\lambda - \tilde{S}(1-t))\,dF(\tilde{S})$, $\partial H(b)/\partial b = 1 - F(b) > 0$. Since $H(h_1) > 0$, if $H(0) \geq 0$, then $\partial Q^{P*}/\partial \rho \geq 0$ for $b \geq 0$. If $H(0) < 0$, then there exists a unique $\hat{b} \in (0, h_1)$ such that $\partial Q^{P*}/\partial \rho < 0$ for $b < \hat{b}$, and $\partial Q^{P*}/\partial \rho \geq 0$ otherwise. $\square$

Proof of Proposition 7. Let $J^P \doteq \partial V^P/\partial Q^P$ as given by (6) in Corollary 1. When $t = 1$, $G(b,\lambda) = \mathbb{E}[\min(\max(\tilde{S}, b), \lambda)]$ for $b \leq \lambda$. If $\tilde{S}$ follows a normal distribution with mean μ_S and standard deviation σ_S, $G(b,\lambda) = \lambda + \sigma_S(L((b-\mu_S)/\sigma_S) - L((\lambda-\mu_S)/\sigma_S))$ where $L(\eta) = \int_{-\infty}^{\eta} (\eta - z)\phi(z)\,dz$ is the standard-normal loss function. $\partial G(b,\lambda)/\partial \sigma_S = \phi((b-\mu_S)/\sigma_S) - \phi((\lambda-\mu_S)/\sigma_S)$, which is positive if $\mu_S \leq (b+\lambda)/2$, and negative otherwise.

From the implicit function theorem, $\operatorname{sgn}(\partial Q^{P*}/\partial \sigma_S) = \operatorname{sgn}(\partial J^P/\partial \sigma_S|_{Q^{P*}})$. Let $A \doteq \{\tilde{D}_1/a_1 \leq Q^{P*}\}$ and $B \doteq \{\tilde{D}_2/a_2 \leq Q^{P*}\}$. I obtain the following:

$$\begin{aligned}&P\left.\frac{\partial J^P}{\partial \sigma_S}\right|_{Q^{P*}}\\ &\quad= \Pr(\bar{A}\bar{B})\left[\phi\left(\frac{b-\mu_S}{\sigma_S}\right) - \phi\left(\frac{h_3-\mu_S}{\sigma_S}\right)\right]\\ &\qquad+\Pr(\bar{A}B)\left[\phi\left(\frac{b-\mu_S}{\sigma_S}\right) - \phi\left(\frac{\max(b,h_2)-\mu_S}{\sigma_S}\right)\right]\\ &\qquad+\Pr(A\bar{B})\left[\phi\left(\frac{b-\mu_S}{\sigma_S}\right) - \phi\left(\frac{\max(b,h_1)-\mu_S}{\sigma_S}\right)\right]. \qquad (6)\end{aligned}$$

Let $H(b)$ denote the right-hand side of this equation for a given Q^{P*}. Because $\phi(z)$ is decreasing in z for $z > 0$, $H(b) > 0$ for $b > \mu_S$. The sign of $H(b)$ can be analyzed in three cases based on b.

Case I ($h_2 \leq b < h_3$). Only the first term in $H(b)$ is nonzero. Within this b range, $H(b) > 0$ for $\mu_S < (h_2 + h_3)/2$. For $\mu_S \geq (h_2 + h_3)/2$, $H(b) < 0$ if $b < \min(2\mu_S - h_3, h_3)$, and $H(b) > 0$ otherwise.

Case II ($h_1 \leq b < h_2$). The first two terms in $H(b)$ are nonzero. Within this b range, $H(b) > 0$ for $\mu_S < (h_1 + h_2)/2$, and $H(b) < 0$ for $\mu_S \geq (h_2 + h_3)/2$.

Case III ($0 \leq b < h_1$). All three terms in $H(b)$ are nonzero. Within this b range, $H(b) > 0$ for $\mu_S < h_1/2$, and $H(b) < 0$ for $\mu_S \geq (h_1 + h_3)/2$.

Rearranging these cases, and using $H(b) > 0$ for $b > \mu_S$ give the desired result. $\square$

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