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Technical Appendix to "Integrating Long-term and Short-term Contracting in Beef Supply Chains"

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TECHNICAL APPENDIX TO INTEGRATING LONG-TERM AND SHORT-TERM CONTRACTING IN BEEF SUPPLY CHAINS

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The proofs for Propositions 1 and 2 are omitted. §A provides the equivalent formulation to the stage 1 optimization problem. The optimal processing decision z^* is relegated to §B. §C illustrates the proof for Proposition 3. The proofs for technical statements in the general "window contracts" model and the beef market model (as summarized in Table 3) are provided in §D and §E, respectively.

A Characterization of Stage 1 Optimization Problem

Proposition A.1 *The stage 1 optimization problem in (1) can be restated as* $\Pi(Q^C; P^S, \boldsymbol{\xi}) = \max_{0\leq z\leq K} \Lambda(z)$ *where* $\Lambda(.)$ *is continuous and strictly concave in* z*. We have*

$$\Lambda(z) = \begin{cases} \Lambda^{1,C}(z) & \textit{for} \quad 0 \leq z \leq \min(I(M), Q^C, K) \\ \Lambda^{2,C}(z) & \textit{for} \quad \min(I(M), Q^C, K) < z \leq \min(Q^C, K) \\ \Lambda^{1,S}(z) & \textit{for} \quad \min(Q^C, K) < z \leq \min\left(\max(I(S), Q^C), K\right) \\ \Lambda^{2,S}(z) & \textit{for} \quad \min\left(\max(I(S), Q^C), K\right) < z \leq K, \end{cases} \tag{9}$$

$$\Lambda(z) = \begin{cases} \Lambda^{3,C}(z) & \textit{for} \quad 0 \leq z \leq \min(II, Q^C, K) \\ \Lambda^{2,C}(z) & \textit{for} \quad \min(II, Q^C, K) < z \leq \min(Q^C, K) \\ \Lambda^{3,S}(z) & \textit{for} \quad \min(Q^C, K) < z \leq \min\left(\max(II, Q^C), K\right) \\ \Lambda^{2,S}(z) & \textit{for} \quad \min\left(\max(II, Q^C), K\right) < z \leq K, \end{cases} \tag{10}$$

for $\xi_1 \geq \xi_2$ *and for* $\xi_1 < \xi_2$*, respectively, where*

$$\begin{aligned}
\Lambda^{k,C}(z) &= -Q^C\left[\max\left(\min(u, P^S+\nu), u\right)\right] + (1-\omega)P^S[Q^C - z] - c_0 z - c_1(K-z)^2 + \pi^k(a_1^C z, a_2^C z, \boldsymbol{\xi}),\\
\Lambda^{k,S}(z) &= -Q^C\left[\max\left(\min(u, P^S+\nu), u\right)\right] - (z-Q^C)(P^S+t) - c_0 z - \delta(z-Q^C) - c_1(K-z)^2\\
&\quad +\pi^k\left((a_1^C - a_1^S)Q^C + a_1^S z, (a_2^C - a_2^S)Q^C + a_2^S z, \boldsymbol{\xi}\right),
\end{aligned}$$

for $k \in \{1,2,3\}$ *and* $I(j) \doteq \frac{\frac{\xi_1-\xi_2}{2} - Q^C\left[(b_1-e)(a_1^C - a_1^j) + (b_2-e)(a_2^j - a_2^C)\right]}{(b_1-e)a_1^j - (b_2-e)a_2^j}$ *for* $j \in \{C, S\}$ *and* $II \doteq \frac{\xi_2-\xi_1}{2(b_2-e)s}$.

B Characterization of The Optimal Processing Decision z^*

Proposition B.1 *For* $\xi_1 \geq \xi_2$ $(\xi_1 < \xi_2)$*, there exist 8 spot price thresholds* $\overline{P}^{(\cdot)}$ $(\underline{P}^{(\cdot)})$ *that characterizes the optimal processing decision* z^**. These spot price thresholds are given by*

$$\begin{aligned}
\overline{P}^0 &\doteq \frac{\xi_1 a_1^C + \xi_2 a_2^C + 2c_1 K - c_0 - \alpha s \mu_S}{1-\omega-\alpha s},\\
\overline{P}^1(\min(I(M), Q^C, K)) &\doteq \frac{\xi_1 a_1^C + \xi_2 a_2^C + 2c_1 K - c_0 - \alpha s \mu_S}{1-\omega-\alpha s}\\
&\quad - \frac{2\left[b_1(a_1^C)^2 + b_2(a_2^C)^2 + 2e a_1^C a_2^C + c_1\right]\min(I(M), Q^C, K)}{1-\omega-\alpha s},\\
\overline{P}^2(\min(I(M), Q^C, K)) &\doteq \frac{\frac{[\xi_1(b_2-e)+\xi_2(b_1-e)]s}{b_1+b_2-2e} + 2c_1 K - c_0 - \alpha s\mu_S - 2\left[\frac{(b_1 b_2 - e^2)s^2}{b_1+b_2-2e} + c_1\right]\min(I(M), Q^C, K)}{1-\omega-\alpha s},\\
\overline{P}^3(\min(Q^C, K)) &\doteq \frac{\frac{[\xi_1(b_2-e)+\xi_2(b_1-e)]s}{b_1+b_2-2e} + 2c_1 K - c_0 - \alpha s\mu_S - 2\left[\frac{(b_1 b_2 - e^2)s^2}{b_1+b_2-2e} + c_1\right]\min(Q^C, K)}{1-\omega-\alpha s},\\
\overline{P}^4(\min(Q^C, K)) &\doteq (1-\alpha s)^{-1}\left[\xi_1 a_1^S + \xi_2 a_2^S + 2c_1 K - c_0 - t - \delta - \alpha s \mu_S\right.\\
&\quad -2Q^C\left[(a_1^C - a_1^S)(b_1 a_1^S + e a_2^S) + (a_2^C - a_2^S)(b_2 a_2^S + e a_1^S)\right]\\
&\quad \left.-2\left[b_1(a_1^S)^2 + b_2(a_2^S)^2 + 2e a_1^S a_2^S + c_1\right]\min(Q^C, K)\right],\\
\overline{P}^5\left(\min\left[\max(Q^C, I(S)), K\right]\right) &\doteq (1-\alpha s)^{-1}\left[\xi_1 a_1^S + \xi_2 a_2^S + 2c_1 K - c_0 - t - \delta - \alpha s \mu_S\right.\\
&\quad -2Q^C\left[(a_1^C - a_1^S)(b_1 a_1^S + e a_2^S) + (a_2^C - a_2^S)(b_2 a_2^S + e a_1^S)\right]\\
&\quad \left.-2\left[b_1(a_1^S)^2 + b_2(a_2^S)^2 + 2e a_1^S a_2^S + c_1\right]\min\left[\max(Q^C, I(S)), K\right]\right],\\
\overline{P}^6\left(\min\left[\max(Q^C, I(S)), K\right]\right) &\doteq (1-\alpha s)^{-1}\left[\frac{[\xi_1(b_2-e)+\xi_2(b_1-e)]\,s}{b_1+b_2-2e} + 2c_1 K - c_0 - t - \delta - \alpha s\mu_S\right.\\
&\quad \left.-2\left[\frac{(b_1 b_2 - e^2)s^2}{b_1+b_2-2e} + c_1\right]\min\left[\max(Q^C, I(S)), K\right]\right],\\
\overline{P}^7 &\doteq (1-\alpha s)^{-1}\left[\frac{[\xi_1(b_2-e)+\xi_2(b_1-e)]\,s}{b_1+b_2-2e} - c_0 - t - \delta - \alpha s\mu_S - 2\left[\frac{(b_1 b_2 - e^2)s^2}{b_1+b_2-2e}\right]K\right]
\end{aligned}$$

$$
\begin{aligned}
\underline{P}^0 &\doteq \frac{\xi_2 s + 2c_1 K - c_0 - \alpha s \mu_S}{1-\omega-\alpha s},\\
\underline{P}^1(\min(II, Q^C, K)) &\doteq \frac{\xi_2 s + 2c_1 K - c_0 - \alpha s \mu_S - 2\left[b_2 s^2 + c_1\right]\min(II, Q^C, K)}{1-\omega-\alpha s},\\
\underline{P}^2(\min(II, Q^C, K)) &\doteq \frac{\frac{[\xi_1(b_2-e)+\xi_2(b_1-e)]s}{b_1+b_2-2e} + 2c_1 K - c_0 - \alpha s \mu_S - 2\left[\frac{(b_1 b_2 - e^2)s^2}{b_1+b_2-2e} + c_1\right]\min(II, Q^C, K)}{1-\omega-\alpha s},\\
\underline{P}^3(\min(Q^C, K)) &\doteq \overline{P}^3(\min(Q^C, K)),\\
\underline{P}^4(\min(Q^C, K)) &\doteq (1-\alpha s)^{-1}\left[\xi_2 s + 2c_1 K - c_0 - t - \delta - \alpha s \mu_S - 2\left[b_2 s^2 + c_1\right]\min(Q^C, K)\right],\\
\underline{P}^5(\min\left[\max(Q^C, II), K\right]) &\doteq (1-\alpha s)^{-1}\left[\xi_2 s + 2c_1 K - c_0 - t - \delta - \alpha s \mu_S - 2\left[b_2 s^2 + c_1\right]\min\left[\max(Q^C, II), K\right]\right],\\
\underline{P}^6(\min\left[\max(Q^C, II), K\right]) &\doteq (1-\alpha s)^{-1}\left[\frac{\left[\xi_1(b_2-e)+\xi_2(b_1-e)\right]s}{b_1+b_2-2e} + 2c_1 K - c_0 - t - \delta - \alpha s \mu_S\right.\\
&\qquad \left. -2\left[\frac{(b_1 b_2 - e^2)s^2}{b_1+b_2-2e} + c_1\right]\min\left[\max(Q^C, II), K\right]\right],\\
\underline{P}^7 &\doteq \overline{P}^7.
\end{aligned}
$$

where in $\overline{P}^k(y)$ ($\underline{P}^k(y)$), *for* $k \in \{1,2,3,4,5,6\}$, *the argument* y *refers to the last term in the definition of the thresholds on the right-hand side.*

For $\boldsymbol{\xi} \in \Omega^1$, *the unique optimal processing decision* z^* *is characterized by*

$$
z^* = \begin{cases}
0 & \text{if } P^S \geq \overline{P}^0\\
z^*_{1,C} = \frac{(1-\omega-\alpha s)(\overline{P}^0 - P^S)}{2\left[b_1(a_1^C)^2 + b_2(a_2^C)^2 + 2e a_1^C a_2^C + c_1\right]} & \text{if } \overline{P}^0 > P^S \geq \overline{P}^1(\min(Q^C, K))\\
\min(Q^C, K) & \text{if } \overline{P}^1(\min(Q^C, K)) > P^S \geq \overline{P}^4(\min(Q^C, K))\\
z^*_{1,S} = \min(Q^C, K) + \frac{(\overline{P}^4(\min(Q^C, K)) - P^S)(1-\alpha s)}{2\left[b_1(a_1^S)^2 + b_2(a_2^S)^2 + 2e a_1^S a_2^S + c_1\right]} & \text{if } \overline{P}^4(\min(Q^C, K)) > P^S \geq \overline{P}^5(K)\\
K & \text{if } \overline{P}^5(K) > P^S.
\end{cases}
$$

For $\boldsymbol{\xi} \in \Omega^2$, *the unique optimal processing decision* z^* *is characterized by*

$$
z^* = \begin{cases}
0 & \text{if } P^S \geq \overline{P}^0\\
z^*_{1,C} = \frac{(1-\omega-\alpha s)(\overline{P}^0 - P^S)}{2\left[b_1(a_1^C)^2 + b_2(a_2^C)^2 + 2e a_1^C a_2^C + c_1\right]} & \text{if } \overline{P}^0 > P^S \geq \overline{P}^1(Q^C)\\
Q^C & \text{if } \overline{P}^1(Q^C) > P^S \geq \overline{P}^4(Q^C)\\
z^*_{1,S} = Q^C + \frac{(\overline{P}^4(Q^C) - P^S)(1-\alpha s)}{2\left[b_1(a_1^S)^2 + b_2(a_2^S)^2 + 2e a_1^S a_2^S + c_1\right]} & \text{if } \overline{P}^4(Q^C) > P^S \geq \overline{P}^5(I(S))\\
z^*_{2,S} = I(S) + \frac{(\overline{P}^6(I(S)) - P^S)(1-\alpha s)}{2\left[\frac{(b_1 b_2 - e^2)s^2}{b_1+b_2-2e} + c_1\right]} & \text{if } \overline{P}^5(I(S)) = \overline{P}^6(I(S)) > P^S \geq \overline{P}^7\\
K & \text{if } \overline{P}^7 > P^S.
\end{cases}
$$

For $\boldsymbol{\xi} \in \Omega^3$, the unique optimal processing decision z^* is characterized by

$$z^* = \begin{cases} 0 & \text{if } P^S \geq \overline{P}^0 \\ z^*_{1,C} = \frac{(1-\omega-\alpha s)(\overline{P}^0 - P^S)}{2\left[b_1(a_1^C)^2 + b_2(a_2^C)^2 + 2ea_1^C a_2^C + c_1\right]} & \text{if } \overline{P}^0 > P^S \geq \overline{P}^1(I(M)) \\ z^*_{2,C} = I(M) + \frac{(1-\omega-\alpha s)(\overline{P}^2(I(M)-P^S)}{2\left[\frac{(b_1 b_2 - e^2)s^2}{b_1+b_2-2e} + c_1\right]} & \text{if } \overline{P}^1(I(M)) = \overline{P}^2(I(M)) > P^S \geq \overline{P}^3(\min(Q^C, K)) \\ \min(Q^C, K) & \text{if } \overline{P}^3(\min(Q^C, K)) > P^S \geq \overline{P}^6(\min(Q^C, K)) \\ z^*_{2,S} = \min(Q^C, K) + \frac{(\overline{P}^6(\min(Q^C,K)) - P^S)(1-\alpha s)}{2\left[\frac{(b_1 b_2 - e^2)s^2}{b_1+b_2-2e} + c_1\right]} & \text{if } \overline{P}^6(\min(Q^C, K)) > P^S \geq \overline{P}^7 \\ K & \text{if } \overline{P}^7 > P^S. \end{cases}$$

For $\boldsymbol{\xi} \in \Omega^4$, the unique optimal processing decision z^* is characterized by

$$z^* = \begin{cases} 0 & \text{if } P^S \geq \underline{P}^0 \\ z^*_{3,C} = \frac{(1-\omega-\alpha s)(\underline{P}^0 - P^S)}{2[b_2 s^2 + c_1]} & \text{if } \underline{P}^0 > P^S \geq \underline{P}^1(II) \\ z^*_{2,C} = II + \frac{(1-\omega-\alpha s)(\underline{P}^2(II) - P^S)}{2\left[\frac{(b_1 b_2 - e^2)s^2}{b_1+b_2-2e} + c_1\right]} & \text{if } \underline{P}^1(II) = \underline{P}^2(II) > P^S \geq \underline{P}^3(\min(Q^C, K)) \\ \min(Q^C, K) & \text{if } \underline{P}^3(\min(Q^C, K)) > P^S \geq \underline{P}^6(\min(Q^C, K)) \\ z^*_{2,S} = \min(Q^C, K) + \frac{(\underline{P}^6(\min(Q^C,K)) - P^S)(1-\alpha s)}{2\left[\frac{(b_1 b_2 - e^2)s^2}{b_1+b_2-2e} + c_1\right]} & \text{if } \underline{P}^6(\min(Q^C, K)) > P^S \geq \underline{P}^7 \\ K & \text{if } \underline{P}^7 > P^S. \end{cases}$$

For $\boldsymbol{\xi} \in \Omega^5$, the unique optimal processing decision z^* is characterized by

$$z^* = \begin{cases} 0 & \text{if } P^S \geq \underline{P}^0 \\ z^*_{3,C} = \frac{(1-\omega-\alpha s)(\underline{P}^0 - P^S)}{2[b_2 s^2 + c_1]} & \text{if } \underline{P}^0 > P^S \geq \underline{P}^1(Q^C) \\ Q^C & \text{if } \underline{P}^1(Q^C) > P^S \geq \underline{P}^4(Q^C) \\ z^*_{3,S} = Q^C + \frac{(\underline{P}^4(Q^C) - P^S)(1-\alpha s)}{2[b_2 s^2 + c_1]} & \text{if } \underline{P}^4(Q^C) > P^S \geq \underline{P}^5(II) \\ z^*_{2,S} = II + \frac{(\underline{P}^6(II) - P^S)(1-\alpha s)}{2\left[\frac{(b_1 b_2 - e^2)s^2}{b_1+b_2-2e} + c_1\right]} & \text{if } \underline{P}^5(II) = \underline{P}^6(II) > P^S \geq \underline{P}^7 \\ K & \text{if } \underline{P}^7 > P^S. \end{cases}$$

For $\boldsymbol{\xi} \in \Omega^6$, the unique optimal processing decision z^* is characterized by

$$z^* = \begin{cases} 0 & \text{if } P^S \geq \underline{P}^0 \\ z^*_{3,C} = \frac{(1-\omega-\alpha s)(\underline{P}^0 - P^S)}{2[b_2 s^2 + c_1]} & \text{if } \underline{P}^0 > P^S \geq \underline{P}^1(\min(Q^C, K)) \\ \min(Q^C, K) & \text{if } \underline{P}^1(\min(Q^C, K)) > P^S \geq \underline{P}^4(\min(Q^C, K)) \\ z^*_{3,S} = \min(Q^C, K) + \frac{(\underline{P}^4(\min(Q^C,K)) - P^S)(1-\alpha s)}{2[b_2 s^2 + c_1]} & \text{if } \underline{P}^4(\min(Q^C, K)) > P^S \geq \underline{P}^5(K) \\ K & \text{if } \underline{P}^5(K) > P^S. \end{cases}$$

C Characterization of the First-Order Condition at Stage 0

Proof of Proposition 3: Using Proposition B.1, we can characterize the expected profit $\mathbb{E}[\Pi(Q^C)]$ for $Q^C \leq K$ and $Q^C > K$. Let $f(\tilde{\xi}_1, \tilde{\xi}_2)$ denote the density function of $\tilde{\boldsymbol{\xi}}' = (\tilde{\xi}_1, \tilde{\xi}_2)$. We define $\Pi^k(Q^C, \tilde{\boldsymbol{\xi}})$ for $k = 1, .., 6$ such that $\mathbb{E}[\Pi(Q^C)] = \sum_{k=1}^{6} \mathbb{E}[\Pi^k(Q^C, \tilde{\boldsymbol{\xi}}) | \tilde{\boldsymbol{\xi}} \in \Omega^k] Pr\{\tilde{\boldsymbol{\xi}} \in \Omega^k\}$. For example, for $Q^C \leq K$, we have $\Pi^1(Q^C, \tilde{\boldsymbol{\xi}}) = \int_{\overline{P}^0}^{\infty} \Lambda^{1,C}(0)\ dF(\tilde{P}^S) + \int_{\overline{P}^1(Q^C)}^{\overline{P}^0} \Lambda^{1,C}(z^*_{1,C})\ dF(\tilde{P}^S) + \int_{\overline{P}^4(Q^C)}^{\overline{P}^1(Q^C)} \Lambda^{1,C}(Q^C)\ dF(\tilde{P}^S) + \int_{\overline{P}^5(K)}^{\overline{P}^4(Q^C)} \Lambda^{1,S}(z^*_{1,S})\ dF(\tilde{P}^S) + \int_0^{\overline{P}^5(K)} \Lambda^{1,S}(K)\ dF(\tilde{P}^S)$. $\Pi^k(Q^C, \tilde{\boldsymbol{\xi}})$ for the other regions can be established in the same manner, and is omitted. For $Q^C > K$, we have $\Omega^2 = \Omega^5 = \emptyset$, and we obtain

$$\frac{\partial \mathbb{E}[\Pi(Q^C)]}{\partial Q^C} = -\mathbb{E}\left[\max\left(\min(u, \tilde{P}^S + \nu), l\right)\right] + \mathbb{E}[\tilde{P}^S(1-\omega)] < 0 \tag{11}$$

by assumption. For $Q^C \leq K$, we analyze each $\frac{\partial \Pi^k}{\partial Q^C}$ separately. We only provide the characterization for $\tilde{\boldsymbol{\xi}} \in \Omega^1$, the rest can be established similarly. We obtain $\frac{\partial \Pi^1}{\partial Q^C} =$

$$\begin{aligned}
- &\quad \mathbb{E}\left[\max\left(\min(u, \tilde{P}^S + \nu), l\right)\right] + \int_{\overline{P}^1(Q^C)}^{\infty} \left[\tilde{P}^S(1-\omega)\right] dF(\tilde{P}^S) \\
+ &\quad \int_{\overline{P}^4(Q^C)}^{\overline{P}^1(Q^C)} \Big[\tilde{\xi}_1 a_1^S + \tilde{\xi}_2 a_2^S + \Delta(\tilde{\xi}_1 - \tilde{\xi}_2) + 2c_1 K - c_0 + \alpha s(\tilde{P}^S - \mu_S) \\
&\quad -2(Q^C)[b_1(a_1^S)^2 + b_2(a_2^S)^2 + 2e a_1^S a_2^S + c_1 + (\Delta)^2(b_1 + b_2 - 2e) + 2\Delta[(b_1 - e)a_1^S - (b_2 - e)a_2^S]]\Big]\, dF(\tilde{P}^S) \\
+ &\quad \int_{\overline{P}^5(K)}^{\overline{P}^4(Q^C)} \Bigg[\tilde{P}^S + t + \delta + \Delta[\tilde{\xi}_1 - \tilde{\xi}_2] - 2Q^C(\Delta)^2[b_1 + b_2 - 2e] - \frac{\Delta[(b_1 - e)a_1^S - (b_2 - e)a_2^S]}{[b_1(a_1^S)^2 + b_2(a_2^S)^2 + 2e a_1^S a_2^S + c_1]} \\
&\quad \left[\tilde{\xi}_1 a_1^S + \tilde{\xi}_2 a_2^S + 2c_1 K - c_0 - \alpha s \mu_S - \tilde{P}^S(1 - \alpha s) - t - \delta - 2\Delta Q^C[(b_1 - e)a_1^S - (b_2 - e)a_2^S]\right]\Bigg]\, dF(\tilde{P}^S) \\
+ &\quad \int_0^{\overline{P}^5(K)} \left[\tilde{P}^S + t + \delta + \Delta[\tilde{\xi}_1 - \tilde{\xi}_2] - 2Q^C(\Delta)^2[b_1 + b_2 - 2e] - 2\Delta K[(b_1 - e)a_1^S - (b_2 - e)a_2^S]\right] dF(\tilde{P}^S)
\end{aligned}$$

To establish the concavity of $\mathbb{E}[\Pi(Q^C)]$, we obtain $\frac{\partial^2 \mathbb{E}[\Pi(Q^C)]}{\partial (Q^C)^2} = \sum_{k=1}^{6} \int\int_{\Omega^k} \frac{\partial^2 \Pi^k(Q^C, \tilde{\boldsymbol{\xi}})}{\partial {Q^C}^2} f(\tilde{\xi}_1, \tilde{\xi}_2) d\tilde{\xi}_1 d\tilde{\xi}_2$. From (11), we have $\frac{\partial^2 \mathbb{E}[\Pi(Q^C)]}{\partial (Q^C)^2} = 0$; hence $\mathbb{E}[\Pi(Q^C)]$ is concave for $Q^C > K$. For $Q^C < K$, for concavity, it is sufficient to prove that $\frac{\partial^2 \Pi^k(Q^C)}{\partial (Q^C)^2} < 0$ for $k = 1, .., 6$. For $\tilde{\boldsymbol{\xi}} \in \Omega^1$, we obtain $\frac{\partial^2 \Pi^1}{\partial (Q^C)^2} =$

$$\int_{\overline{P}^4(Q^C)}^{\overline{P}^1(Q^C)} -2\left[b_1(a_1^S)^2 + b_2(a_2^S)^2 + 2e a_1^S a_2^S + c_1 + (\Delta)^2(b_1 + b_2 - 2e) + 2\Delta[(b_1 - e)a_1^S - (b_2 - e)a_2^S]\right] dF(\tilde{P}^S)$$

$$+ \int_{\overline{P}^5(K)}^{\overline{P}^4(Q^C)} -\left[2(\Delta)^2 \frac{(b_1 b_2 - e^2)(a_1^S + a_2^S)^2 + (b_1 + b_2 - 2e)c_1}{b_1(a_1^S)^2 + b_2(a_2^S)^2 + 2e a_1^S a_2^S + c_1}\right] dF(\tilde{P}^S) + \int_0^{\overline{P}^5(K)} -2(\Delta)^2(b_1 + b_2 - 2e) dF(\tilde{P}^S) < 0.$$

The other regions can be established in the same manner, and the proof is omitted. Combining all Ω^k, we have $\frac{\partial^2 \mathbb{E}[\Pi(Q^C)]}{\partial (Q^C)^2} < 0$ for $Q^C < K$; hence $\mathbb{E}[\Pi(Q^C)]$ is also concave for $Q^C < K$. It is easy to establish that $\mathbb{E}[\Pi(Q^C)]$ is kinked at $Q^C = K$. Therefore it is not differentiable at $Q^C = K$. It is easy to establish that $\left.\frac{\partial \mathbb{E}[\Pi(Q^C)]}{\partial Q^C}\right|_{K^-} > \left.\frac{\partial \mathbb{E}[\Pi(Q^C)]}{\partial Q^C}\right|_{K^+}$. Therefore $\mathbb{E}[\Pi(Q^C)]$ is globally concave.

By using the definitions of $\overline{P}^{(.)}$, $\overline{P}^{(.)}$ and $z^*_{(.)}$, for $Q^C < K$, we obtain

$$
\begin{aligned}
\frac{\partial \mathbb{E}[\Pi(Q^C)]}{\partial Q^C} &= -\mathbb{E}\left[\max\left(\min(u, \tilde{P}^S+\nu), l\right)\right] \qquad (12)\\
&+ \mathbb{E}[(1-\omega)\tilde{P}^S + (1-\omega-\alpha s)(\overline{P}^1(Q^C)-\tilde{P}^S)^+ | \tilde{\boldsymbol{\xi}} \in \Omega_{12}] Pr\left\{\tilde{\boldsymbol{\xi}} \in \Omega_{12}\right\}\\
&+ \mathbb{E}[(1-\omega)\tilde{P}^S + (1-\omega-\alpha s)(P^3(Q^C)-\tilde{P}^S)^+ | \tilde{\boldsymbol{\xi}} \in \Omega_{34}] Pr\left\{\tilde{\boldsymbol{\xi}} \in \Omega_{34}\right\}\\
&+ \mathbb{E}[(1-\omega)\tilde{P}^S + (1-\omega-\alpha s)(\underline{P}^1(Q^C)-\tilde{P}^S)^+ | \tilde{\boldsymbol{\xi}} \in \Omega_{56}] Pr\left\{\tilde{\boldsymbol{\xi}} \in \Omega_{56}\right\}\\
&- \mathbb{E}\left[\int_0^{\overline{P}^4(Q^C)} \left[(\overline{P}^4(Q^C)-\tilde{P}^S)(1-\alpha s)\right] dF(\tilde{P}^S)\right.\\
&\quad \left. + \int_{\overline{P}^5(K)}^{\overline{P}^4(Q^C)} 2\Delta h(z^*_{1,S}-Q^C) dF(\tilde{P}^S) + \int_0^{\overline{P}^5(K)} 2\Delta h(K-Q^C) dF(\tilde{P}^S) \right| \tilde{\boldsymbol{\xi}} \in \Omega_1 \Bigg] Pr\left\{\tilde{\boldsymbol{\xi}} \in \Omega_1\right\}\\
&- \mathbb{E}\left[\int_{\overline{P}^5(I(S))}^{\overline{P}^4(Q^C)} \left[(\overline{P}^4(Q^C)-\tilde{P}^S)(1-\alpha s)\right] dF(\tilde{P}^S)\right.\\
&\quad \left. + \int_{\overline{P}^5(K)}^{\overline{P}^4(Q^C)} 2\Delta h(z^*_{1,S}-Q^C) dF(\tilde{P}^S) + \int_0^{\overline{P}^6(I(S))} 2\Delta h(I(S)-Q^C) dF(\tilde{P}^S) \right| \tilde{\boldsymbol{\xi}} \in \Omega_2 \Bigg] Pr\left\{\tilde{\boldsymbol{\xi}} \in \Omega_2\right\}\\
&- \mathbb{E}\left[\left.\int_0^{P^6(Q^C)} \left[(P^6(Q^C)-\tilde{P}^S)(1-\alpha s)\right] \; dF(\tilde{P}^S)\right| \tilde{\boldsymbol{\xi}} \in \Omega_{34}\right] Pr\left\{\tilde{\boldsymbol{\xi}} \in \Omega_{34}\right\}\\
&- \mathbb{E}\left[\left.\int_0^{\underline{P}^4(Q^C)} \left[(\underline{P}^4(Q^C)-\tilde{P}^S)(1-\alpha s)\right] dF(\tilde{P}^S)\right| \tilde{\boldsymbol{\xi}} \in \Omega_{56}\right] Pr\left\{\tilde{\boldsymbol{\xi}} \in \Omega_{56}\right\}.
\end{aligned}
$$

where $h = (b_1-e)a_1^S - (b_2-e)a_2^S$. From (11), we have $\frac{\partial \mathbb{E}[\Pi(Q^C)]}{\partial Q^C} < 0$ for $Q^C > K$; hence $Q^{C^*} \le K$. Since $\mathbb{E}[\Pi(Q^C)]$ is concave function, $Q^{C^*} = 0$ if $\frac{\partial \mathbb{E}[\Pi(Q^C)]}{\partial Q^C} |_{0^+} \le 0$. $Q^{C^*} = K$ if $\frac{\partial \mathbb{E}[\Pi(Q^C)]}{\partial Q^C} |_{K^-} > 0$. Otherwise Q^{C^*} is the solution to the first order condition as depicted in (12).The equivalence between (12) and the optimality condition in (4) can be obtained after standardizing $\tilde{P}^S$ as $\mu_S + z\sigma_S$, and using the identities of the standard normal distribution. ■

D Proofs for the "Window Contracts" Model

Proof of Proposition 4: We have $V(Q^C) = \sum_{l=1}^6 \mathbb{E}_{\tilde{\boldsymbol{\xi}}}\left[\mathbb{E}_{\tilde{P}^S}\left[\Pi^l(Q^C, \tilde{\boldsymbol{\xi}}, \tilde{P}^S)\right]\Big| \tilde{\boldsymbol{\xi}} \in \Omega^l\right] Pr\{\tilde{\boldsymbol{\xi}} \in \Omega^l\}$. We define $G(l,u) \doteq \mathbb{E}\left[\max\left(\min(u, \tilde{P}^S+\nu), l\right)\right]$. For a given Q^C, we can separate $V(Q^C)$ as follows:

$$V(Q^C) = -G(l,u)Q^C + \mu_S(1-\omega)Q^C + \sum_{l=1}^{6} \mathbb{E}_{\tilde{\boldsymbol{\xi}}}\left[\mathbb{E}_{\tilde{P}^S}\left[\Pi^l_\Theta(Q^C, \tilde{\boldsymbol{\xi}}, \tilde{P}^S)\right]\Big| \tilde{\boldsymbol{\xi}} \in \Omega^l\right] Pr\{\tilde{\boldsymbol{\xi}} \in \Omega^l\} \quad (13)$$

where the first term is the expected contract procurement cost, the second term is the expected revenues from spot sales, and the remaining terms denote the additional expected profit from processing over spot sale. For $Q^C < K$, we have in Ω^1 region, $\mathbb{E}_{\tilde{P}^S}[\Pi^1_\Theta] = \int_{\overline{P}^0}^{\infty} \left[-c_1K^2\right] dF(\tilde{P}^S)$

$$
\begin{aligned}
&+ \int_{\overline{P}^1(Q^C)}^{\overline{P}^0} \Big[-c_1K^2 + \\
&\qquad \frac{[\xi_1 a_1^S + \xi_2 a_2^S + \Delta(\xi_1-\xi_2) + 2c_1K - c_0 - \alpha s \mu_S - \tilde{P}^S(1-\phi-\alpha s)]^2}{4[b_1(a_1^S)^2 + b_2(a_2^S)^2 + 2e a_1^S a_2^S + c_1 + (\Delta)^2(b_1+b_2-2e) + 2\Delta[(b_1-e)a_1^S - (b_2-e)a_2^S]]}\Bigg] dF(\tilde{P}^S) \\
&+ \int_{\overline{P}^4(Q^C)}^{\overline{P}^1(Q^C)} \Big[-\tilde{P}^S(1-\omega)Q^C - c_1K^2 + Q^C[\xi_1 a_1^S + \xi_2 a_2^S + \Delta(\xi_1-\xi_2) + 2c_1K - c_0 + \alpha s(\tilde{P}^S - \mu_S)] \\
&\qquad -(Q^C)^2[b_1(a_1^S)^2 + b_2(a_2^S)^2 + 2e a_1^S a_2^S + c_1 + (\Delta)^2(b_1+b_2-2e) + 2\Delta[(b_1-e)a_1^S - (b_2-e)a_2^S]]\Big] dF(\tilde{P}^S) \\
&+ \int_{\overline{P}^5(K)}^{\overline{P}^4(Q^C)} \Big[-\tilde{P}^S(1-\omega)Q^C + Q^C(\tilde{P}^S + t + \delta) - c_1K^2 + Q^C\Delta[\xi_1-\xi_2] - (Q^C)^2(\Delta)^2[b_1+b_2-2e] \\
&\quad + \frac{[\xi_1 a_1^S + \xi_2 a_2^S + 2c_1K - c_0 - \alpha s\mu_S - \tilde{P}^S(1-\alpha s) - t - \delta - 2\Delta Q^C[(b_1-e)a_1^S - (b_2-e)a_2^S]]^2}{4[b_1(a_1^S)^2 + b_2(a_2^S)^2 + 2e a_1^S a_2^S + c_1]}\Bigg] dF(\tilde{P}^S) \\
&+ \int_0^{\overline{P}^5(K)} \Big[-\tilde{P}^S(1-\omega)Q^C + Q^C(\tilde{P}^S + t + \delta) + Q^C\Delta[\xi_1-\xi_2] - (Q^C)^2(\Delta)^2[b_1+b_2-2e] \\
&\quad +K[\xi_1 a_1^S + \xi_2 a_2^S - c_0 - \alpha s\mu_S - \tilde{P}^S(1-\alpha s) - t - \delta - 2\Delta Q^C[(b_1-e)a_1^S - (b_2-e)a_2^S]] \\
&\quad -K^2[b_1(a_1^S)^2 + b_2(a_2^S)^2 + 2e a_1^S a_2^S]\Big] dF(\tilde{P}^S).
\end{aligned}
$$

$\mathbb{E}_{\tilde{P}^S}[\Pi^1_\Theta]$ for the other Ω^l regions can be characterized in a similar fashion. By using the normality assumption of $\tilde{P}^S$, we obtain $G(l,u) = \left[u + \sigma_S\left(L\left(\frac{l-\nu-\mu_S}{\sigma_S}\right) - L\left(\frac{u-\nu-\mu_S}{\sigma_S}\right)\right)\right]$ where $L(z) = z\Phi(z) + \phi(z)$ is the standard normal loss function, and $\Phi(.)$ and $\phi(.)$ is cdf and pdf of standard normal random variable, respectively. Using the identity $\phi'(z) = -z\phi(z)$, we obtain $\frac{\partial G(l,u)}{\partial \sigma_S} = \phi\left(\frac{l-\nu-\mu_S}{\sigma_S}\right) - \phi\left(\frac{u-\nu-\mu_S}{\sigma_S}\right)$. It follows that $\frac{\partial G(l,u)}{\partial \sigma_S} > (<) 0$ if $\mu_S + \nu < \frac{l+u}{2}$ ($\mu_S + \nu > \frac{l+u}{2}$); and $\frac{\partial G(l,u)}{\partial \sigma_S} = 0$ if $\mu_S + \nu = \frac{l+u}{2}$ or $l = u$ or $l \to -\infty, u \to \infty$.

We now analyze the effect of σ_S on the expected value from processing over spot sale. We have

$$
\sum_{l=1}^{6} \mathbb{E}_{\tilde{\boldsymbol{\xi}}}\left[\mathbb{E}_{\tilde{P}^S}\left[\Pi^l_\Theta(Q^C, \tilde{\boldsymbol{\xi}}, \tilde{P}^S)\right]\Big| \tilde{\boldsymbol{\xi}} \in \Omega^l\right] Pr\{\tilde{\boldsymbol{\xi}} \in \Omega^l\} = \mathbb{E}_{\tilde{P}^S}\left[\sum_{l=1}^{6} \mathbb{E}_{\tilde{\boldsymbol{\xi}}}\left[\Pi^l_\Theta(Q^C, \tilde{\boldsymbol{\xi}}, \tilde{P}^S)\Big| \tilde{\boldsymbol{\xi}} \in \Omega^l\right] Pr\{\tilde{\boldsymbol{\xi}} \in \Omega^l\}\right]
$$

Let $\mathbb{E}_{\tilde{P}^S}[\Psi(\tilde{P}^S)]$ denote the right-hand side term. We use the following result from Müller (2001):

Lemma D.1 *Let $\tilde{P}^S$ ($\underline{\tilde{P}}^S$) to have a normal distribution with mean μ_S ($\underline{\mu}_S$) and standard deviation σ_S ($\underline{\sigma}_S$). If $\mu_S = \underline{\mu}_S$ and $\sigma_S \le \underline{\sigma}_S$, then, $\tilde{P}^S \le \underline{\tilde{P}}^S$ in the convex order, i.e. $\mathbb{E}[f(\tilde{P}^S)] \le \mathbb{E}[f(\underline{\tilde{P}}^S)]$ for any convex function f.*

For convexity of $\Psi(P^S)$ in P^S, it is sufficient to show that each Π^l_Θ is a convex function of P^S. We will only provide the proof for Ω^1 region, i.e. Π^1_Θ. The same result for the other regions can be proven in a similar fashion. We obtain

$$\frac{\partial \Pi^1_\Theta}{\partial P^S} = \begin{cases} 0 & \text{if } P^S \in [\overline{P}^0, \infty) \\ (\alpha s + \omega - 1)\frac{f_1(P^S)}{2h_1} & \text{if } P^S \in [\overline{P}^1(Q^C), \overline{P}^0) \\ (\alpha s + \omega - 1)Q^C & \text{if } P^S \in [\overline{P}^4(Q^C), \overline{P}^1(Q^C)) \\ \omega Q^C - (1-\alpha s)\frac{f_2(P^S)}{2h_2} & \text{if } P^S \in [\overline{P}^5(K), \overline{P}^4(Q^C)) \\ \omega Q^C - (1-\alpha s)K & \text{if } P^S \in [0, \overline{P}^5(K)) \end{cases} \tag{14}$$

where f_1, h_1, f_2, h_2 are given by

$$\begin{aligned} f_1(P^S) &= \xi_1 a_1^S + \xi_2 a_2^S + \Delta(\xi_1 - \xi_2) + 2c_1 K - c_0 - \alpha s \mu_S - P^S(1-\omega-\alpha s) \\ h_1 &= b_1(a_1^S)^2 + b_2(a_2^S)^2 + 2e a_1^S a_2^S + c_1 + \Delta^2(b_1 + b_2 - 2e) + 2\Delta[(b_1 - e)a_1^S - (b_2 - e)a_2^S] \\ f_2(P^S) &= \xi_1 a_1^S + \xi_2 a_2^S + 2c_1 K - c_0 - \alpha s \mu_S - P^S(1-\alpha s) - t - \delta - 2\Delta Q^C[(b_1 - e)a_1^S - (b_2 - e)a_2^S] \\ h_2 &= b_1(a_1^S)^2 + b_2(a_2^S)^2 + 2e a_1^S a_2^S + c_1. \end{aligned} \tag{15}$$

From (14), it can be easily established that Π^1_Θ is convexly decreasing in P^S by using $\frac{\partial f_1}{\partial P^S} = \frac{(1-\omega-\alpha s)^2}{2h_1} > 0$, $\frac{\partial f_2}{\partial P^S} = \frac{(1-\alpha s)^2}{2h_2} > 0$ and the fact that Π^1_Θ is a smooth function of P^S, i.e. left-hand side and right-hand side derivative at boundaries in (14) are equal. This concludes the proof. ■

Proof of Proposition 5: The correlation parameter α only affects the expected value of processing over spot sale in (13). For Ω^1 region, we obtain $\frac{\partial \mathbb{E}_{\tilde{P}^S}[\Pi^1_\Theta]}{\partial \alpha} =$

$$\begin{aligned} &\int_{\overline{P}^1(Q^C)}^{\overline{P}^0} [s(\tilde{P}^s - \mu_S)\frac{f_1(\tilde{P}^S)}{2h_1}]\, dF(\tilde{P}^S) + \int_{\overline{P}^4(Q^C)}^{\overline{P}^1(Q^C)} [s(\tilde{P}^s - \mu_S)Q^C]\, dF(\tilde{P}^S) \\ &+ \int_{\overline{P}^5(K)}^{\overline{P}^4(Q^C)} [s(\tilde{P}^s - \mu_S)\frac{f_2(\tilde{P}^S)}{2h_2}]\, dF(\tilde{P}^S) + \int_0^{\overline{P}^5(K)} [s(\tilde{P}^s - \mu_S)K]\, dF(\tilde{P}^S) \end{aligned} \tag{16}$$

where f_1, h_1, f_2, h_2 are given in (15). Observe that $\frac{f_1(P^S)}{2h_1} = z^*_{1,C}$, $\frac{f_2(P^S)}{2h_2} = z^*_{1,S}$. Thus, using Proposition B.1, (16) can be written as $\mathbb{E}_{\tilde{P}^S}\left[Z^*(\tilde{P}^S)s(\tilde{P}^s - \mu_S)\right]$ where Z^* is the random variable that denotes the optimal processing decision. Since $\tilde{P}^S$ is normally distributed, we have $\mathbb{E}_{\tilde{P}^S}\left[Z^*(\tilde{P}^S)s(\tilde{P}^s - \mu_S)\right] = s\sigma_S \mathbb{E}\left[Z^*(\mu_S + z\sigma_S)z\right]$ where the second expectation is taken over the standard normal random variable. As follows from Stein's Lemma, for a differentiable function g and a standard normal random variable z, we have $\mathbb{E}[g(z)z] = \mathbb{E}[g'(z)]$ (see for example, Rubinstein (1976)). By using this identity, we obtain

$$\mathbb{E}\left[Z^*(\mu_S + z\sigma_S)z\right] = \int_{\overline{P}^1(Q^C)}^{\overline{P}^0} \frac{-(1-\omega-\alpha s)}{2h_1}\, dF(\tilde{P}^S) + \int_{\overline{P}^5(K)}^{\overline{P}^4(Q^C)} \frac{-(1-\alpha s)}{2h_2}\, dF(\tilde{P}^S) < 0$$

as $\alpha < \frac{1-\omega}{s}$. The desired result follows as this argument also holds for the other $\Omega^{(\cdot)}$ regions. ■

Proof of Proposition 6: As can be observed from (13), the comparison of $V(Q^C)$ with window contract and forward contract reduces to the comparison of the expected contract procurement cost $G(l, u)$. We define $H(\overline{F}) \doteq G(\overline{F} - \tau, \overline{F} + \tau) - \overline{F}$ as the cost differential between the window and forward contract for $\tau < \overline{F}$. We obtain $\frac{\partial H}{\partial \overline{F}} = \Phi\left(\frac{\overline{F}-\tau-\nu-\mu_S}{\sigma_S}\right) - \phi\left(\frac{\overline{F}+\tau-\nu-\mu_S}{\sigma_S}\right) < 0$. By using $\phi(z) = \phi(-z)$ and $\Phi(z) = 1 - \Phi(-z)$ for the standard normal distribution, it is easy to establish that $H(\mu_S + \nu) = 0$. Therefore, if $\overline{F} > \mu_S + \nu$ ($\overline{F} < \mu_S + \nu$), the expected cost of window contract is higher (lower) than the forward contract. As follows from (12), the type of the contract only affects the expected marginal procurement cost $G(l, u)$ of C-input in the optimality condition. Since $V(Q^C)$ is a concave function of Q^C, it follows that Q^{C^*} is lower (higher) with the window contract if $\overline{F} > \mu_S + \nu$ ($\overline{F} < \mu_S + \nu$). It is easy to establish that the expected spot procurement at the optimal solution depends on the contract type only through the optimal volume of C-input, and is decreasing in Q^{C^*}. This concludes the proof. ■

E Proofs for the Analytical Statements in Table 3

We only provide the proof for the impact of ρ_ξ and σ_ξ on the expected profit by using the assumption that all the probability mass of $\tilde{\boldsymbol{\xi}}$ is located in Ω^1 region. The proof for the impact of σ_S follows from Proposition 4, and the proof for μ_S and μ_i can be obtained using a similar technique. In each of the proofs, we will demonstrate the impact on $V(Q^C)$ for $Q^C < K$. This also implies the same effect on the expected optimal profit $V^*(Q^{C^*})$. For notational convenience, we define $\Upsilon(\boldsymbol{\xi}) \doteq \mathbb{E}_{\tilde{P}^S}\left[\Pi^l_\Theta(Q^C, \boldsymbol{\xi}, \tilde{P}^S)\right]$ so that $V(Q^C) \doteq \mathbb{E}_{\tilde{\boldsymbol{\xi}}}\left[\Upsilon(\tilde{\boldsymbol{\xi}})\right]$.

Proof of ρ_ξ effect on $V(Q^C)$: We use the following result result from Müller (2001):

Lemma E.1 *Let $\tilde{\boldsymbol{\xi}}$ ($\tilde{\underline{\boldsymbol{\xi}}}$) to have a bivariate normal distribution with mean $\boldsymbol{\mu}$ ($\underline{\boldsymbol{\mu}}$) and covariance matrix Σ ($\underline{\Sigma}$). If $\boldsymbol{\mu} = \underline{\boldsymbol{\mu}}$, $\tilde{\boldsymbol{\xi}}$ and $\tilde{\underline{\boldsymbol{\xi}}}$ have the same marginal distributions, $\Sigma_{ij} \le \underline{\Sigma}_{ij}$, then $\tilde{\boldsymbol{\xi}} \le \tilde{\underline{\boldsymbol{\xi}}}$ in the supermodular order, i.e. $\mathbb{E}[f(\tilde{\boldsymbol{\xi}})] \le \mathbb{E}[f(\tilde{\underline{\boldsymbol{\xi}}})]$ for any supermodular function f.*

Since we have symmetric σ_ξ, it follows from Lemma E.1 that increasing ρ_ξ leads to another bivariate normal distribution that is preferred over $\tilde{\boldsymbol{\xi}}$ in the supermodular order. It is sufficient to show that $\Upsilon(\boldsymbol{\xi})$ is supermodular in $\boldsymbol{\xi}$. To prove supermodularity, it is sufficient to show $\frac{\partial^2 \Upsilon(\boldsymbol{\xi})}{\partial \xi_1 \partial \xi_2} \ge 0$. We obtain

$$\frac{\partial^2 \Upsilon(\boldsymbol{\xi})}{\partial \xi_1 \partial \xi_2} = \int_{\overline{P}^1(Q^C)}^{\overline{P}^0} \frac{a_1^C a_2^C}{2h_1}\, dF(\tilde{P}^S) + \int_{\overline{P}^5(K)}^{\overline{P}^4(Q^C)} \frac{a_1^S a_2^S}{2h_2}\, dF(\tilde{P}^S) > 0$$

where h_1 and h_2 are as defined in (15). This concludes the proof. ■

Proof of σ_ξ effect on $V(Q^C)$: We use the following result result from Müller (2001):

Lemma E.2 *Let $\tilde{\boldsymbol{\xi}}$ ($\tilde{\underline{\boldsymbol{\xi}}}$) to have a bivariate normal distribution with mean $\boldsymbol{\mu}$ ($\underline{\boldsymbol{\mu}}$) and covariance matrix Σ ($\underline{\Sigma}$) with $\sigma_{\xi_1} = \sigma_{\xi_2} = \sigma_\xi$ ($\sigma_{\underline{\xi}_1} = \sigma_{\underline{\xi}_2} = \sigma_{\underline{\xi}}$). If $\boldsymbol{\mu} = \underline{\boldsymbol{\mu}}$, and $\sigma_\xi \le \sigma_{\underline{\xi}}$ then $\tilde{\boldsymbol{\xi}} \le \tilde{\underline{\boldsymbol{\xi}}}$ in the convex order, i.e. $\mathbb{E}[f(\tilde{\boldsymbol{\xi}})] \le \mathbb{E}[f(\tilde{\underline{\boldsymbol{\xi}}})]$ for any convex function f.*

To prove the result, as defined in $V(Q^C) = \mathbb{E}_{\tilde{\boldsymbol{\xi}}}\left[\Upsilon(\tilde{\boldsymbol{\xi}})\right]$, it is sufficiently show that $\Upsilon(\boldsymbol{\xi})$ is jointly convex in $\boldsymbol{\xi}$. We obtain

$$\begin{aligned}\frac{\partial^2 \Upsilon(\boldsymbol{\xi})}{\partial \xi_1^2} &= \int_{\overline{P}^1(Q^C)}^{\overline{P}^0} \frac{a_1^C a_1^C}{2h_1}\, dF(\tilde{P}^S) + \int_{\overline{P}^5(K)}^{\overline{P}^4(Q^C)} \frac{a_1^S a_1^S}{2h_2}\, dF(\tilde{P}^S) > 0,\\ \frac{\partial^2 \Upsilon(\boldsymbol{\xi})}{\partial \xi_2^2} &= \int_{\overline{P}^1(Q^C)}^{\overline{P}^0} \frac{a_2^C a_2^C}{2h_1}\, dF(\tilde{P}^S) + \int_{\overline{P}^5(K)}^{\overline{P}^4(Q^C)} \frac{a_2^S a_2^S}{2h_2}\, dF(\tilde{P}^S) > 0\end{aligned}$$

where h_1 and h_2 are as defined in (15) and

$$\frac{\partial^2 \Upsilon(\boldsymbol{\xi})}{\partial \xi_1^2}\frac{\partial^2 \Upsilon(\boldsymbol{\xi})}{\partial \xi_2^2} - \left(\frac{\partial^2 \Upsilon(\boldsymbol{\xi})}{\partial \xi_i \xi_j}\right)^2 = (s\Delta)^2 \left(\int_{\overline{P}^1(Q^C)}^{\overline{P}^0} dF(\tilde{P}^S)\right)\left(\int_{\overline{P}^5(K)}^{\overline{P}^4(Q^C)} dF(\tilde{P}^S)\right) \geq 0.$$

Hence, $\Upsilon(\boldsymbol{\xi})$ is jointly convex in $\boldsymbol{\xi}$. This concludes the proof. ■

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