5-2010

Optimal Monetary Policy in a Model with Recursive Preferences

Sungbae An
Singapore Management University, sungbae@smu.edu.sg

Optimal Monetary Policy in a Model with Recursive Preferences

Sungbae An*

Abstract

This paper provides a simple and elegance approach for an empirical investigation of a model with Epstein-Zin (1989) preferences. The perturbation method implemented in Dynare is readily applicable for computation of equilibrium and welfare. A stylized new Keynesian economy with sticky prices is analyzed and optimal simple rules are accessed across various types of monetary policy rules.

Keywords: Recursive preference, perturbation method, Dynare, Ramsey steady state, optimal simple rule,

*Assistant Professor, School of Economics, Singapore Management University, 90 Stamford Road, Singapore 178903 (email: sungbae@smu.edu.sg)

1 Introduction

The recursive preferences have long been considered to provide a possible linkage between the aggregate macro data and the financial market data. With an extra degree of freedom that comes from the disentanglement between the intertemporal elasticity of substitution (EIS) and relative risk aversion, we can tackle various puzzles regarding financial data such as risk premium and risk-free rate puzzles, at least in principle. The main reason that the recursive utility specification has not been popular in the literature is the computational cost. The linearization method generates exactly the same results with the time and event separable expected utility specification, à la von-Neumann and Morgenstern. Recently, many solution methods for empirical investigation of the recursive type utility are suggested and applied. Tallarini (2000) is among the first authors who applies the risk-sensitivity theory to compute an equilibrium asset pricing model under the unit EIS. Croce (2008) also uses the same approach in a model with investment adjustment cost based on Jermann (1998). Hansen, Heaton, and Li (2008) uses the Taylor approximation on the parameter around the unit EIS to investigate the effect of the longrun risk in the consumption growth. Amisano and Tristani (2009) uses perturbation method in studying the term structure model with this utility specification. Papers by Caldara, Fernández-villaverde, Rubio-Ramírez, and Yao (2009) and Aldrich and Kung (2009) compares the performance of the approximation methods including the projection and the perturbation approach. Even though the high volatility of the shock can generate sizeable approximation error in a perturbation approach, they show that the second order perturbation works well especially the EIS is bigger than the unity as in Bansal and Yaron (2004).

In this paper we provide a simple way to use the perturbation method for models with recursive preferences. This approach is particularly interesting because it is readily *code-able* in Dynare without any complications. The main idea for this implementation is just to map the continuation value in the recursive utility as another time-information consistent variable. Since the utility is already specified in a recursive form, the dynamic programming, the solution via the Bellman equation so to speak, is directly applicable for agent's optimization problem, and moreover, the welfare calculation is intuitively simple.

For an empirical application, we consider a new Keynesian economy that features the sticky prices via Calvo (1983) and Yun (1996) pricing apparatus. In the literature, this model is usually linearized around the zero-inflation steady state when the model is not equipped with the price indexation. Following Schmitt-

Grohé and Uribe (2007), we use the second-order approximation via perturbation method to analyze the welfare implication of various monetary policy rules around the nonstochastic Ramsey steady state.

The main findings of the paper are two-fold. First, the computational cost regarding the model with recursive preferences turns out to be small when we use the perturbation method to approximate the policy function. We can even introduce the recursive preference specifications to the medium scale workhorse models such as Christiano, Eichenbaum, and Evans (2005) and Smets and Wouters (2007). It would be particularly interesting because larger models can contain the financial sector where the recursive preferences can play a great role.

Secondly, the optimal simple rule in an economy with recursive preferences almost attains the welfare level that the Ramsey steady state provides. Moreover, the welfare implications are quite different whether the optimal simple rule or the Taylor rule are exercised.

The remainder of the paper proceeds as follows. Section 2 describes the model economy that consists of the representative household with recursive preference and the firms who are subject to Calvo-Yun type pricing reoptimization chances. Section 3 explains the solution method to implement the model equilibrium in the Dynare and the welfare evaluation with recursive preferences. Section 4 describes the calibration and the results, and Section 5 concludes.

2 Model

The model is a version of Schmitt-Grohé and Uribe (2007) with augmentation with recursive preferences. The model economy consists of the representative household, the final good producing firm, the monopolistically competitive intermediate good producing firms, and the consolidated government who exercises the fiscal and the monetary policy.

2.1 Household

The representative household has the Epstein-Zin type recursive preference, that is, the household maximized the following utility by choosing consumption c_t and provides h_t hours of work in a competitive labor market.

$$v_t = \max \left\{ (1-\beta)\left(c_t(1-n_t)^{\nu}\right)^{\frac{1-\gamma}{1+\nu}} + \beta\left(\mathbf{E}_t\left[v_{t+1}^{1-\chi}\right]\right)^{\frac{1-\gamma}{1-\chi}} \right\}^{\frac{1}{1-\gamma}}$$

where γ denotes the inverse of the EIS, χ governs the risk aversion, and ν controls the labor supply. For the notational clarification, we rewrite the above preference specification as

$$v_t = \max \left\{ (1-\beta) u_t^{1-\gamma} + \beta \mathcal{W}_t^{1-\gamma} \right\}^{\frac{1}{1-\gamma}} \quad (1)$$

with the instantaneous utility u_t and the continuation value $\mathcal{W}_t$:

$$u_t = c_t^{\frac{1}{1+\nu}} (1-h_t)^{\frac{\nu}{1+\nu}}, \quad (2)$$

$$\mathcal{W}_t = \left(\mathbf{E}_t \left[v_{t+1}^{1-\chi} \right] \right)^{\frac{1}{1-\chi}} \quad (3)$$

Hence, the representative household maximizes her lifetime utility that is characterized as a CES aggregate of the current utility and the continuation value weighted by $(1-\beta)$ and β, respectively. We note that the continuation value $\mathcal{W}_t$ is again a CES aggregate of the future welfare across the state of the world that is governed by a stochastic process. As a special case when $\gamma = \chi$, the above preference specification boils down to the von Neuman-Morgenstern expected utility that is additively separable both in time and state.

Assuming that households have access to a complete asset market and also own capital for renting to firms, the utility maximization is subject to household's budget constraint and the capital evolution. The budget constraint is given as

$$\begin{aligned} c_t + i_t + \mathbf{E}_t \left[Q_{t,t+1} \frac{D_{t+1}}{P_t} \right] + \frac{M_t + B_t}{P_t} \\ \leq w_t h_t + r_t^K k_{t-1} + \frac{D_t}{P_t} + \frac{M_{t-1} + R_{t-1} B_{t-1}}{P_t} + \frac{\Pi_t}{P_t} - \frac{T_t}{P_t} \end{aligned} \quad (4)$$

where i_t denotes the investment, P_t denotes the price of consumption and investment goods, D_t denotes the random nominal payment from holding risky asset, $Q_{t,t+1}$ denotes the stochastic discount factor for nominal payoffs, M_t denotes nominal money holding, B_t denotes the riskless government issued nominal bond that pays the gross return R_t, w_t denotes the real wage, r_t^K denotes the real rental rate of capital, Π_t denotes profits received from the ownership of the (intermediate goods producing) firms, and T_t denotes the lump-sum tax paid to the government. The household-owned capital is depreciated at a constant rate δ:

$$k_t = (1-\delta) k_{t-1} + i_t. \quad (5)$$

The household's decision problem is (1) subject to (4) and (5). Let λ_t be the Lagrange multiplier for (4) with substitution of (5). Since the lifetime utility is already given in a recursive form, the programming problem is a straight application of Bellman equation solution. From the first order conditions we can write the stochastic discount factor as

$$Q_{t,t+1} = \beta \left(\frac{v_{t+1}}{v_t} \right)^{-\gamma} \frac{\lambda_{t+1}}{\lambda_t} \frac{P_t}{P_{t+1}} \left(\frac{v_{t+1}}{\mathcal{W}_t} \right)^{\gamma - \chi}.$$

The most distinctive feature of the above stochastic discount factor is in the last term, the ratio between the future value and the CES aggregate of the future value across the state. In a standard expected utility specification, the last term drops out since the inverse of the EIS and the risk aversion are assumed to be identical, that is, $\gamma = \chi$. The FOCs are summarized as

$$\nu \frac{c_t}{1 - h_t} = w_t \tag{6}$$

$$Q_{t-1,t} = \beta \left(\frac{c_t}{c_{t-1}} \right)^{-\frac{\gamma+\nu}{1+\nu}} \left(\frac{1 - h_t}{1 - h_{t-1}} \right)^{\frac{\nu(1-\gamma)}{1+\nu}} \pi_t^{-1} \left(\frac{v_t}{\mathcal{W}_{t-1}} \right)^{\gamma - \chi} \tag{7}$$

$$\mathbf{E}_t \left[Q_{t,t+1} R_t \right] = 1 \tag{8}$$

$$\mathbf{E}_t \left[Q_{t,t+1} \pi_{t+1} \left(r_{t+1}^K + (1 - \delta) \right) \right] = 1 \tag{9}$$

where $w_t = W_t / P_t$ denotes the real wage, $\pi_t = P_t / P_{t-1}$ denotes the gross inflation, and $r_t^K = R_t^K / P_t$ denotes the real rental rate of capital. The first condition (6) is usual labor supply equation and (8) and (9) are the asset pricing equations for returns on the bond purchase and the physical investment, respectively. The only different consequence from the standard expected utility maximizing household is the definition of the stochastic factor (7).

2.2 Final Good Producing Firm

The perfectly competitive final good producing firm combines a continuum variety of intermediate goods indexed by $j \in [0, 1]$ using the CES aggregation technology

$$y_t = \left(\int_0^1 y_t(j)^{1 - \frac{1}{\eta}} \, dj \right)^{\frac{\eta}{\eta - 1}}$$

Here $\eta > 1$ represents the elasticity of substitution for each intermediate good. The firm takes input prices $P_t(j)$ and output prices P_t as given. Hence, the demand for the intermediate good j is determined by

$$y_t(j) = \left(\frac{P_t(j)}{P_t} \right)^{-\eta} y_t$$

The relationship between intermediate goods prices and the price of the final good can be written as

$$P_t = \left(\int_0^1 P_t(j)^{1-\eta} \, dj \right)^{\frac{1}{1-\eta}} \tag{10}$$

2.3 Intermediate good producing firms

Intermediate good j is produced by a monopolist who has access to an identical technology, represented by the production function

$$y_t(j) \; = \; z_t \left(k_t^d(j)\right)^{\alpha} \Big(n_t(j)\Big)^{1-\alpha} - \kappa$$

where z_t is an exogenous productivity process that is common to all firms, $n_t(j)$ is the labor input of firm j, and $k_t^d(j)$ is capital used by firm j. Labor and capital are hired in perfectly competitive factor markets at W_t and R_t^K, respectively. The parameter α denotes the cost share of capital and κ denotes the fixed cost of production which ensures the zero profit at the steady state. Firms minimize the total cost of production

$$W_t n_t(j) + R_t^K k_t^d(j) + \kappa$$

of which the first-order condition implies

$$\frac{k_t^d(j)}{n_t(j)} = \frac{\alpha}{1-\alpha} \frac{W_t}{R_t^K}$$

Noting that firms face the same factor prices, the input capital-labor ratio should be equal across all the intermediate good producing firms. The total cost function for firm j can be written as

$$\mathcal{C}_t(j) = \left(\frac{W_t}{1-\alpha}\right)^{1-\alpha} \left(\frac{R_t^K}{\alpha}\right)^{\alpha} \left(\frac{y_t(j)+\kappa}{z_t}\right) + \kappa \tag{11}$$

While each firm can set the price of its own output $P_t(j)$, utilizing its monopolistic power, only a fraction $(1-\theta)$ of firms are given chances for full adjustment at any given period, independent of the time elapsed since the last adjustment. Thus, each period a measure $(1-\theta)$ of producers reset their output prices, while a fraction θ keep their prices unchanged. As a result, the average duration of a price is given by $(1-\theta)^{-1}$. In this context, θ becomes a natural index of price stickiness. To put it differently, each period t firm j's output price is set as

$$P_t(j) = \begin{cases} P_{t-1}(j) & \text{with prob. } \theta \\ P_t^*(j) & \text{with prob. } 1-\theta \end{cases} \tag{12}$$

where $P_t^*(j)$ denotes the fully adjusted price reset at period t. In this model we do not introduce the price indexation which is often assumed in the literature to introduce the positive steady state inflation. Eventually with the higher order approximation, we can approximate the equilibrium around the nonstochastic steady state where the inflation is strictly positive.

Now firm j who has a chance to reoptimize chooses the output price $P_t^*(j)$ to maximize the present value of future profits

$$\max_{P_t^*(j)} \quad \mathbf{E}_t \left[\sum_{k=0}^{\infty} \theta^k Q_{t,t+k} \left(P_t^*(j) y_{t+k}^*(j) - \mathcal{C}_{t+k}^*(j) \right) \right]$$

subject to the sequence of demand constraints

$$y_{t+k}^*(j) = \left(\frac{P_t^*(j)}{P_{t+k}} \right)^{-\eta} y_{t+k}$$

for $k = 0, 1, \ldots$ and $\mathcal{C}_t^*(j)$ denotes the total cost of production at period t when the output is $y_t^*(j)$. The stochastic discount factor between t and $t+k$ is obtained by applying one period stochastic discount factor successively

$$Q_{t,t+k} = Q_{t,t+1} Q_{t+1,t+2} \cdots Q_{t+k-1,t+k}$$

Hence, the first-order condition for price adjustment is given by

$$\mathbf{E}_t \left[\sum_{k=0}^{\infty} \theta^k Q_{t,t+k} y_{t+k}^*(j) \left\{ P_t^*(j) - \frac{\eta}{\eta - 1} \mathcal{MC}_{t+k}^*(j) \right\} \right] = 0 \tag{13}$$

where $\mathcal{MC}_t^*(j) = \frac{\partial \mathcal{C}_t^*(j)}{\partial y_t^*(j)}$, the nominal marginal cost of production of firm j.

2.4 Government

The consolidated government prints money M_t, issues one period riskless nominal bond B_t that pays the nominal gross return R_t, collects the lump-sum tax T_t from households, and purchases the final good amount of g_t that is subject to an exogenous process. We further assume that the government runs a balanced budget every period:

$$M_{t-1} + R_{t-1} B_{t-1} + P_t g_t \quad = \quad M_t + B_t + T_t$$

In a cashless economy, the government budget constraint can be written as

$$\frac{R_{t-1}}{\pi_t} b_{t-1} + (g_t - \tau_t) = b_t \tag{14}$$

where $b_t = B_t/P_t$ denotes the real bond outstanding and $\tau_t = T_t/P_t$ denotes the real lump-sup tax.

Monetary policy is described by an interest rate feedback rule of the form

$$\log\left(\frac{R_t}{R^*}\right) = \rho_R \log\left(\frac{R_{t-1}}{R^*}\right) + (1-\rho_R)\mathbf{E}_t\left[\phi_\pi \log\left(\frac{\pi_{t-s}}{\pi^*}\right) + \phi_y \log\left(\frac{y_{t-s}}{y^*}\right)\right] \quad (15)$$

for $s = -1, 0, 1$ where R^* and π^* denote the target interest rate and inflation when the monetary authority follows Ramsey policy and y^* denotes the nonstochastic Ramsey steady-state level of aggregate output. That is, the monetary authority responds to the inflation gap and the output gap from the nonstochastic Ramsey steady state level. The interest rate feedback rule (15) can be either contemporaneous, forward-, or backward-looking.

2.5 Aggregation and Equilibrium

The price level of the economy is determined by (10) where intermediate good price is set according to (12) and (13). Noting that each intermediate good producing firm is *ex ante* identical, its pricing scheme is identical. That is, $P_t^* = P_t^*(j)$ when firm j has a chance to reoptimize the output price. Hence, the pricing behavior of each firm can be aggregated as

$$1 = (1-\theta)(p_t^*)^{1-\eta} + \theta\pi_t^{\eta-1} \quad (16)$$

where $p_t^* = P_t^*/P_t$ is the relative price of intermediate good to the final good and $\pi_t = P_t/P_{t-1}$ denotes the gross inflation of the final good.

With aggregate demand for labor and capital, $n_t = \int_0^1 n_t(j)\,dj$ and $k_t^d = \int_0^1 k_t^d(j)\,dj$, firm's cost minimization is aggregated as

$$\frac{k_t^d}{n_t} = \frac{\alpha}{1-\alpha}\frac{w_t}{r_t^K}$$

From (11) the aggregate total cost of production is

$$\mathcal{C}_t = \int_0^1 \mathcal{C}_t(j)\,dj = \kappa\Big(1+\Omega_t\Big) + \Omega_t y_t s_t$$

where $\Omega_t = \frac{1}{z_t}\left(\frac{W_t}{1-\alpha}\right)^{1-\alpha}\left(\frac{R_t^K}{\alpha}\right)^{\alpha}$ and the price dispersion is defined as

$$s_t = \int_0^1 \left(\frac{P_t(j)}{P_t}\right)^{-\eta} dj$$

Noting that the marginal cost of production for firm j is $\mathcal{MC}_t(j) = \Omega_t$, the real marginal cost of aggregate production is

$$mc_t \quad = \quad = s_t \frac{\mathcal{MC}_t(j)}{P_t}$$

With the aggregate capital-labor ratio, we write the marginal cost as the ratio of the factor cost to its marginal product:

$$\frac{mc_t}{s_t} \alpha z_t \left(\frac{k_{t-1}^d}{n_t} \right)^{\alpha - 1} = r_t^K \tag{17}$$

$$\frac{mc_t}{s_t} (1-\alpha) z_t \left(\frac{k_{t-1}^d}{n_t} \right)^{\alpha} = w_t \tag{18}$$

Hence, the price adjustment decision (13) can be written more compactly as

$$\mathbf{E}_t \left[\sum_{k=0}^{\infty} \theta^k Q_{t,t+k} \left(\frac{P_t^*}{P_{t+k}} \right)^{-1-\eta} y_{t+k} \left\{ \frac{P_t^*}{P_{t+k}} - \frac{\eta}{\eta - 1} \frac{mc_{t+k}}{s_{t+k}} \right\} \right] = 0 \tag{19}$$

To facilitate the recursive representation, let

$$\xi_t \quad = \quad \mathbf{E}_t \left[\sum_{k=0}^{\infty} \theta^k Q_{t,t+k} \left(\frac{P_t^*}{P_{t+k}} \right)^{-\eta} y_{t+k} \right]$$

$$\zeta_t \quad = \quad \mathbf{E}_t \left[\sum_{k=0}^{\infty} \theta^k Q_{t,t+k} \left(\frac{P_t^*}{P_{t+k}} \right)^{-1-\eta} \frac{y_{t+k} \cdot mc_{t+k}}{s_{t+k}} \right]$$

Then we can write (19)

$$(p_t^*)^{\eta} \xi_t \quad = \quad y_t + \theta \mathbf{E}_t \left[Q_{t,t+1} \left(\pi_{t+1} p_{t+1}^* \right)^{\eta} \xi_{t+1} \right] \tag{20}$$

$$(p_t^*)^{1+\eta} \zeta_t \quad = \quad \frac{y_t \cdot mc_t}{s_t} + \theta \mathbf{E}_t \left[Q_{t,t+1} \left(\pi_{t+1} p_{t+1}^* \right)^{1+\eta} \zeta_{t+1} \right] \tag{21}$$

and

$$\xi_t = \frac{\eta}{\eta - 1} \zeta_t \tag{22}$$

The price dispersion can be shown to evolve as

$$s_t \quad = \quad (1-\theta)(p_t^*)^{-\eta} + \theta \pi_t^{\eta} s_{t-1} \tag{23}$$

In an equilibrium all the goods and factor markets should clear at each period. For labor and capital markets we have

$$h_t \quad = \quad n_t \tag{24}$$

$$k_{t-1} = k_t^d \tag{25}$$

Intermediate goods market should also clear for each firm's product j. That is,

$$y_t(j) = \left(\frac{P_t(j)}{P_t}\right)^{-\eta} y_t$$

and the aggregation gives us

$$z_t \left(k_t^d\right)^\alpha n_t^{1-\alpha} - \kappa = s_t y_t \tag{26}$$

because the capital-labor ratio for each firm is identical. The final goods market clearing condition is given by

$$y_t = c_t + i_t + g_t \tag{27}$$

Now we can define the competitive equilibrium. The competitive equilibrium consists of a set of processes c_t, h_t, i_t, k_t, $Q_{t-1,t}$, k_t^d, n_t, y_t, mc_t, p_t^*, w_t, r_t^K, R_t, π_t, ξ_t, ζ_t, s_t, v_t, u_t, and $\mathcal{W}_t$ for $t = 0, 1, \ldots$ that satisfy equations (1)–(3), (5)–(9), (14)–(18), and (20)–(27), given initial values for k_{-1} and s_{-1}, and exogenous stochastic processes g_t and z_t.

Moreover, the two exogenous processes, the common technological progress shock that enters in the intermediate good production firms' production functions and the government spending shock, are specified as AR(1) processes.

$$\log(z_t) = \rho_z \log(z_{t-1}) + \epsilon_{z,t}$$
$$\log(g_t) = (1 - \rho_g) \log(g) + \rho_g log(g_{t-1}) + \epsilon_{g,t}$$

A partial set of equilibrium conditions are listed in Appendix A.

2.6 Ramsey-Optimal Policy

In terms of setting the policy, the social planner may want to maximize the welfare given the decision of the private sector without relying on a particular form of policy rule such as (15). The welfare of the economy which is the planner's objective is given as the same form as the lifetime utility of the representative agent, that is, a recursive form utility of Epstein-Zin type

$$\widetilde{v}_t = \max \left\{ (1 - \tilde{\beta}) \left(c_t (1 - n_t)^\nu \right)^{\frac{1-\gamma}{1+\nu}} + \tilde{\beta} \left(\mathbf{E}_t \left[\widetilde{v}_{t+1}^{1-\chi} \right] \right)^{\frac{1-\gamma}{1-\chi}} \right\}^{\frac{1}{1-\gamma}}$$

where $\tilde{\beta}$ denotes the planner's subjective discount factor, $\tilde{v}_t$ denotes the social welfare arises from the planner's choice of policy. The constrained optimization is subject to the equilibrium characterization for endogenous variables, given in Appendix A, (A1)–(A16). We can see that there 17 endogenous variables are included in 16 constraints. Hence the planner's optimization procedure would pin down the variable with extra degree of freedom, namely the nominal interest rate schedule. Since this is again a dynamic programming problem, we can utilize the recursive structure of the planner's objective. Let μ_t^i denote the Lagrange multiplier for the equilibrium condition (Ai). Then the first order conditions for the Ramsey problem can be listed as (C1)–(C18) as well as the equilibrium behavior of the private sector, (A1)–(A16), and this will characterize the equilibrium under the Ramsey-optimal policy.

3 Solution Method

3.1 Perturbation and Recursive Form

The equilibrium characterization of the model is analyzed using the second-order perturbation method. As shown in Caldara, Fernández-villaverde, Rubio-Ramírez, and Yao (2009) the approximation error from the second-order approximation is comparable to those from other global approximation methods such as the projection with Chebyshev polynomial and the value function iteration. Hence, we will stick to the second-order approximation in calculating equilibrium behavior of our model. The details of the second order approximation can be found in Judd (1988), Schmidtt-Grohé and Uribe (2004), Kim, Kim, Shaumburg, and Sims (2005) among others.

The equilibrium condition of the model can generally be expressed as the following expectational equation vector

$$\mathbf{E}_t \left[f(x_t, x_{t+1}, y_t, y_{t+1}) \right] = 0 \tag{28}$$

where x_t denotes the predetermined state variable and y_t is non-predetermined endogenous variable. Once we write down the equilibrium conditions for a particular model in this form, the policy function can be easily approximated by the perturbation method. Dynare is a powerful computational tool that provides an easy implementation of this numerical approximation on a standard desktop computer. Actually most of the equilibrium conditions are fit into (28). Static conditions such as (A3) and evolutionary equation such as (A1) are not even subject to the expectation operator. Expectational equations such as (A4) also fits (28). The problem arises when we need to deal with such forms as (19). Noting that the infinite sums can be cast as a recursive representation, (19) can be included in (28). In this respect, the Epstein-Zin preference specification should easily be written as a part of (28). The only complication is related to the expression of the continuation value. As we have seen before, the continuation value in the recursive preference is the CES aggregator of the future welfare across the possible states of the world. When it is written in a stochastic discount factor, as is often written in a formidable form such as

$$Q_{t,t+1} = \beta \left(\frac{c_{t+1}}{c_t} \right)^{-\gamma} \left(\frac{v_{t+1}}{\left(\mathbf{E}_t \left[v_{t+1}^{1-\chi} \right] \right)^{\frac{1}{1-\chi}}} \right)^{\gamma-\chi}$$

it is not so clear how the perturbation approach can deal with this expression. One immediate and simple solution for this problem is to introduce an additional variable as we did in Section 2. Let $\mathcal{W}_t$ denote the continuation value and define as (3) then we can simply write

$$\mathcal{W}_t^{1-\gamma} = \mathbf{E}_t \left[v_{t+1}^{1-\chi} \right]$$

and include it in (28). By introducing $\mathcal{W}_t$, now we can code all the equilibrium conditions in Dynare language.

3.2 Ramsey Steady State

As previously discussed, it is necessary to linearize the model around the zero inflation steady state when the model abstracts from the price indexation. However, the approximation can be performed around any nonstochastic steady state if the higher order perturbation is used. We assumed that the monetary authority in our model exercises various versions of interest rate feedback rules and its target interest rate and the inflation rate are set to the nonstochastic Ramsey steady state. For an empirical investigation we need to calculate the Ramsey steady state first.

The Ramsey equilibrium solves the system of equations made up of the private sector equilibrium condition (A1)–(A16) and the additional Ramsey first order conditions (C1)–(C18). The nonstochastic steady state of the Ramsey equilibrium can be obtained by the following procedure. First, guess a steady state interest rate and calculates the nonstochastic steady state of the private sector equilibrium conditions. Given the steady state values, the remaining Ramsey first order conditions are now a system of linear equations where all the unknowns are the Lagrange multipliers μ_t's. That is, the steady state values of the variables given the interest rate solves the private sector equilibrium and hence the steady state values can be plugged into the additional Ramsey first order conditions. The number of equations in this linear system is 17 while the number of unknown Lagrange multipliers are 16. Hence the least squares are used to evaluate the fit and the guess on the interest rate will be updated to minimize the residual.

3.3 Calibration

For further analysis we calibrate the model at the nonstochastic steady state of the competitive equilibrium. Table 1 reports the calibration of the structural parameters. The model is calibrated to the postwar U.S. economy. The steady state inflation is calibrated to 4.5 percent per annum and the government spending is set to 17 percent of the output at the steady state which is the long run average of the U.S. postwar observation. The capital share parameter α is 0.3 that implies that the ratio of the labor cost in the production is 70 percent with the Cobb-Douglas production function. The time discount factor is set to 0.99 that matches the annual real interest rate around 4 percent. Noting that the annual depreciation is around 10 percent so the depreciation rate is set at 0.25. The elasticity of substitution between intermediate goods is chosen as 5 so that the price markup over the marginal cost is 25 percent at the steady state. The Calvo parameter θ which governs the price stickiness is set at 0.8, which implies that the average duration of price renewal is 5 quarters.

The disentanglement of the EIS $1/\gamma$ and the risk aversion χ is a distinctive feature with the recursive preference. To facilitate the extra degree of freedom, we set the risk aversion on a high side, 10. Since the model used in this paper does not target to match the financial data anomalies such as the risk premium puzzle, it is still the reasonable values. However, this high risk aversion by itself cannot match the empirical anomalies in the financial data. Introducing the real adjustment cost on the investment in line with Jermann (1998) helps a lot in explaining these puzzles. One direct implication of high risk aversion in an equilibrium model is that it breaks the comovement between the output and the consumption. With the standard value for γ in the literature 2, our model demonstrates the correlation to be too low, around 0.2. One possible solution to deal with this problem is to adjust the EIS. With higher EIS the representative agent is more willing to move the consumption across time horizon, that is, she becomes more sensitive to the changes in income. We follow Bansal and Yaron (2004) that the EIS is even bigger than the unity and set it to 1.25, equivalently $\gamma = 0.8$. With this adjustment, the correlation between the output and the consumption is around 0.8. The parameter

ν governs the elasticity of substitution between consumption and leisure and hence the labor supply is calibrated to 4.4543 so that the steady state hours of work is around 20 percent of the endowment.

Finally, we set κ that governs the fixed cost of the production to 0.897 so that firms entertains the zero profit in the steady state. For the two shock processes, the technology shock and the government spending shock, we follow Schmitt-Grohé and Uribe (2007) for the calibration of the persistence of shocks and the standard deviations of innovation terms, which is not quite different from the values frequently used in the business cycles literature.

4 Empirical Results

Before we explain the empirical results, we should note the difficulty in calculating the welfare cost. The welfare level at the aquarium, either conditional or unconditional, is easy to calculate. It is because the preference is already given in a recursive formulation, and it is one of the variables that characterize the equilibrium; hence, Dynare will automatically these values when we solve the model economy on a computer. However, the welfare cost, as is often measured in the loss of the consumption stream from the reference level of consumption, the Ramsey equilibrium in the context of our paper, is not as intuitive as in the expected utility case. So we reports the level of the welfare directly rather than using a relevant measure.

We first calculate the Ramsey steady state so that it can be referenced to the monetary authority in choosing the monetary policy parameters. The nonstochastic Ramsey steady state interest rate is 5.4 percent per annum and the inflation is 1.25 percent annum. The welfare level at the Ramsey steady state is 0.636889.

Table 2 is a reproduction of Table 2 Panel A in Schmitt-Grohé and Uribe (2007). Various monetary policy rule in a class of the interest rate feedback rule is evaluated at the cashless economy with recursive preferences. To find optimal parameter values, we search for the maximum attainable conditional welfare level on a two-dimensional grid of ψ_π and ψ_y for each type of policy, which includes the one that responds only to contemporaneous output gap and inflation gap, the backward looking one ($i = 1$), and the forward looking one ($i = -1$). The optimized rules are also compared to non-optimizing rules such as the Taylor rule ($\psi_\pi = 1.5$ and $\psi_y = 0.5$) and the simple Taylor rule ($\psi_\pi = 1.5$ and $\psi_y = 0$). The independent search on a grid of ρ_R has reached the boundary of the grid in every optimized policy rule.

First, we note that the optimal response on the output gap is almost zero. For all the optimizing policy rule scenario, not responding to the output gap increases the welfare level. Result from the non-optimizing rules also confirms this finding: The simple Taylor rule attains a higher welfare level than the standard Taylor rule.

Secondly, the welfare level attained by contemporaneous smoothing rule is very close to the welfare level at the Ramsey steady state. Also for all the optimized rule the conditional welfare differ only at the fifth decimal level. However, we do not have an appropriate welfare cost measure with recursive preferences, so it is hard to tell if this tiny difference refers to relatively large in terms of the consumption stream. As an alternative measure of the welfare level, we refer to the volatility of the inflation and the interest rate. As shown in Woodford (2003), the welfare level can be accurately approximated by linear combination of the variances of the inflation, the output gap, and the interest rate up to second order, at least in a simple new Keynesian model. The last two columns of Table 2 reports the standard deviations of the inflation and the interest rate. Again the changes in the standard deviation across the optimized simple rule are quite small. This finding is well fit to the finding of Schmitt-Grohé and Uribe (2007).

Thirdly, the non-optimizing type monetary policy rules show significant differences in welfare levels. These big differences stem from restricting the inflation gap responsiveness ψ_π to 1.5, both in the standard and the simple Taylor rules.

Figure 1 depicts the impulse responses to a technology shock. The monetary authority plays either the contemporaneous smoothing optimal policy (solid line) or the standard Taylor rule (dashed line). Basically, the optimal simple rule mimics the Ramsey equilibrium where nominal interest rate, price dispersion, and marginal costs are instantly adjusted to the steady state level. With Taylor rule, however, responses of the aforementioned variables show prolonged effects. With the high EIS in recursive preferences, the consumption responds quite quickly to the technology shock. Impacts on the capital are different in an order of magnitude. The initial responses to the hours worked even have different signs. Hence, the welfare implications of the technology shock are very different when the monetary authority adopts different policy rule. With the optimal simple rule, the welfare shows much more prolonged responses.

5 Conclusion

This paper provides a simple way to analyze an economy where the representative household's utility is Epstein and Zin type. The welfare implication of the optimal monetary policy is analyzed within this context.

References

Aldrich, Eric M., and Howard Kung (2009): "Computational Methods for Production-Based Asset Pricing Models with Recursive Utility," *Working Paper,* Duke University.

Bansal, R., and A. Yaron (2004): "Risks for the Long Run: A Potential Resolution of Asset Pricing Puzzles," *Journal of Finance*, 59, 1481–1509.

Caldara, Dario, Jesús Fernández-Villaverde, Juan F. Rubio-Ramírez, and Wen Yao (2009): "Computing DSGE Models with Recursive Preferences," *Working Paper,* University of Pennsylvania.

Calvo, Guillermo A. (1983): "Staggered Prices in a Utility-maximizing Framework," *Journal of Monetary Economics*, 12(3), 383–398.

Christiano, Lawrence J., Martin Eichenbaum, and Charles L. Evans (2005): "Nominal Rigidities and the Dynamic Effects of a Shock to Monetary Policy," *Journal of Political Economy*, 113(1), 1–45.

Croce, Mariano M. (2008): "Long-Run Productivity Risk: A New Hope for Production-Based Asset Pricing," Working paper, University of North Carolina at Chapel Hill.

Epstein, Larry G., and Stanley E. Zin (1989): "Substitution, Risk Aversion, and the Temporal Behavior of Consumption and Asset Returns: A Theoretical Framework," *Econometrica*, 57(4), 937–969.

Hansen, L. P., J. Heaton, and Nan Li (2008): "Consumption Strikes Back? Measuring Long-Run Risk," *Journal of Political Economy*, 116, 260–302.

Jermann, Urban J. (1998): "Asset Pricing in Production Economies," *Journal of Monetary Economics*, 41(2), 257–275.

Judd, Kenneth L. (1998): *Numerical Methods in Economics*. Cambridge, MA: MIT Press.

Kim, Jinill, Sunghyun Henry Kim, Ernst Schaumburg, and Christopher A. Sims (2005): "Calculating and Using Second Order Accurate Solutions of discrete Time Dynamic Equilibirum Models," *Manuscript*, Princeton University.

Levin, Andrew T., and J. David López-Salido (2004): "Optimal Monetary Policy with Endogenous Capital Accumulation," *Manuscript*, Federal Reserve Board of Governors.

Levin, Andrew T., Alexei Onatski, John C. Williams, and Noah Williams (2006): "Monetary Policy Under Uncertainty in Micro-Founded Macroeconometric Models," in *NBER Macroeconomics Annual 2005*, ed. by Mark Gertler, and Kenneth Rogoff. Cambridge, MA: MIT Press, 229–287.

Schmitt-Grohé, Stephanie, and Martín Uribe (2004): "Solving Dynamic General Equilibrium Models Using a Second-order Approximation to the Policy Function," *Journal of Economic Dynamics and Control*, 28(4), 755–775.

Schmitt-Grohé, Stephanie, and Martín Uribe (2007): "Optimal Simple and Implementable Monetary and Fiscal Rules," *Journal of Monetary Economics*, 54(6), 1702–1725.

Smets, Frank, and Rafael Wouters (2007): "Shocks and Frictions in US Business Cycles: A Bayesian DSGE Approach," *American Economic Review*, 97(3), 586–606.

Tallarini, Jr., Thomas D. (2000): "Risk-Sensitive Real Business Cycles," *Journal of Monetary Economics*, 45(3), 507–532.

Yun, Tack (1996): "Nominal price rigidity, money supply endogeneity, and business cycles," *Journal of Monetary Economics*, 37(2), 345–370.

Table 1: Calibration of Model Parameters

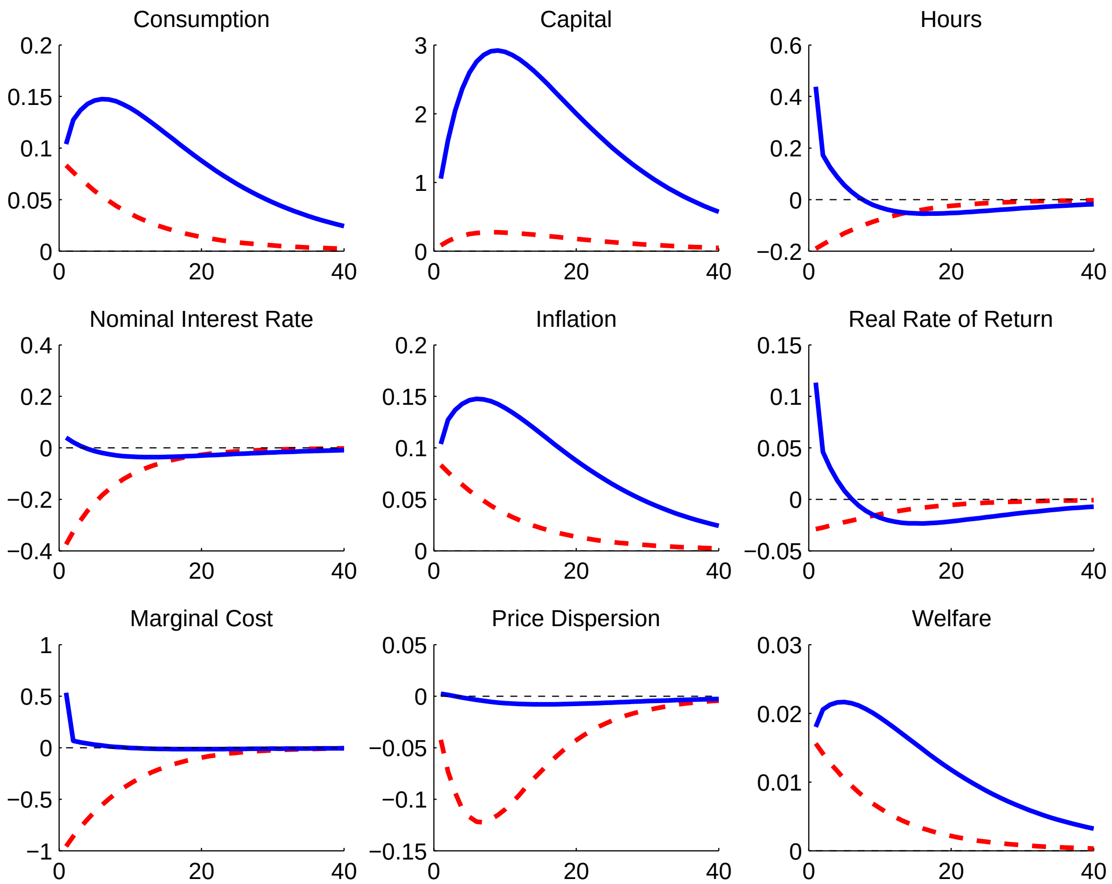
Parameter	Value	Description
α	0.3	Capital share
β	0.99	Discount factor
γ	0.8	Inverse of EIS
δ	0.25	Depreciation rate
ν	4.543	Elasticity of substitution b/w consumption and labor
χ	10	Risk aversion
η	5	Price elasticity of demand for intermediate goods
θ	0.8	Price stickiness: Calvo parameter
κ	0.0897	Fixed cost of production
$\bar{g}$	0.616	Steady state level of government purchase
ρ_z	0.8556	Technology shock persistence
ρ_g	0.87	Government spending shock persistence
σ_z	0.0064	Standard deviation of technology shock
σ_g	0.016	Standard deviation of government purchasing shock

Table 2: Optimal Monetary Policy

$$\hat{R}_t = \rho_R \hat{R}_{t-1} + (1-\rho_R)\left(\psi_\pi \mathbf{E}_t \hat{\pi}_{t-i} + \psi_y \mathbf{E}_t \hat{y}_{t-i}\right)$$

	ψ_π	ψ_y	ψ_R	v_c	v_u	σ_π	σ_R
Optimized Rules							
Contemporaneous (i=0)							
Smoothing	3	0.01	0.56	0.636887	0.636892	0.04	0.10
No Smoothing	3	0.01		0.636876	0.636885	0.04	0.11
Backward (i=1)	3	0.05	0.74	0.636878	0.636886	0.07	0.09
Forward (i=-1)	3	0.03	0.93	0.636839	0.636865	0.09	0.07
Non-Optimized Rules							
Taylor Rule (i=0)	1.5	0.5		0.636068	0.636457	0.50	0.47
Simple Taylor Rule (i=0)	1.5			0.636816		0.30	0.30

Figure 1: Impulse Response Function: Technology Shock



Notes: Impulse responses to technology shock with optimal simple rule (solid) and Taylor rule (dashed)

A Equilibrium Characterization

Let us denote $Q_{t-1,t}$ by Q_t. Given R_t, the equilibrium can be chracterized by the following system of equations:

$$k_t = (1-\delta)k_{t-1} + i_t \tag{A1}$$

$$Q_t = \beta \left(\frac{c_t}{c_{t-1}}\right)^{-\frac{\gamma+\nu}{1+\nu}} \left(\frac{1-n_t}{1-n_{t-1}}\right)^{\frac{\nu(1-\gamma)}{1+\nu}} \pi_t^{-1} \left(\frac{v_t}{\mathcal{W}_{t-1}}\right)^{\gamma-\chi} \tag{A2}$$

$$\nu \frac{c_t}{1-n_t} = w_t \tag{A3}$$

$$\mathbf{E}_t \left[Q_{t+1}\pi_{t+1}\left(r^K_{t+1} + 1 - \delta\right)\right] = 1 \tag{A4}$$

$$\mathbf{E}_t \left[Q_{t+1}R_t\right] = 1 \tag{A5}$$

$$\frac{mc_t}{s_t}(1-\alpha)z_t \left(\frac{k_{t-1}}{n_t}\right)^{\alpha} = w_t \tag{A6}$$

$$\frac{mc_t}{s_t}\alpha z_t \left(\frac{k_{t-1}}{n_t}\right)^{\alpha-1} = r^K_t \tag{A7}$$

$$1 = (1-\theta)(p^*_t)^{1-\eta} + \theta\pi_t^{\eta-1} \tag{A8}$$

$$(p^*_t)^{1+\eta}\zeta_t = \frac{y_t \cdot mc_t}{s_t} + \theta\mathbf{E}_t\left[Q_{t+1}\left(\pi_{t+1}p^*_{t+1}\right)^{1+\eta}\zeta_{t+1}\right] \tag{A9}$$

$$(p^*_t)^{\eta}\xi_t = y_t + \theta\mathbf{E}_t\left[Q_{t+1}\left(\pi_{t+1}p^*_{t+1}\right)^{\eta}\xi_{t+1}\right] \tag{A10}$$

$$\xi_t = \frac{\eta}{\eta-1}\zeta_t \tag{A11}$$

$$z_t {k_{t-1}}^{\alpha} n_t^{1-\alpha} - \kappa \;=\; s_t y_t \tag{A12}$$

$$y_t = c_t + i_t + g_t \tag{A13}$$

$$s_t = (1-\theta)(p^*_t)^{-\eta} + \theta\pi_t^{\eta}s_{t-1} \tag{A14}$$

$$v_t^{1-\gamma} = (1-\beta)\left(c_t(1-n_t)^{\nu}\right)^{\frac{1-\gamma}{1+\nu}} + \beta\mathcal{W}_t^{1-\gamma} \tag{A15}$$

$$\mathcal{W}_t^{1-\chi} = \mathbf{E}_t\left[v_{t+1}^{1-\chi}\right] \tag{A16}$$

$$\log(z_t) = \rho_z \log(z_{t-1}) + \epsilon_{z,t}$$

$$\log(g_t) = (1-\rho_g)\log(\bar{g}) + \rho_g \log(g_{t-1}) + \epsilon_{g,t}$$

B Steady state

The nonstochastic steady state given R is obatained as follows.

B.1 Steady State: Step 1

$$\frac{i}{k} = \delta$$

$$\pi = \beta Q^{-1} = \beta R \qquad \text{(4) from (*)}$$

$$\frac{c}{k} = \frac{w}{\nu}\left(\frac{1}{k} - \frac{n}{k}\right)$$

$$r^K = \frac{1}{\beta} - 1 + \delta \qquad (5)$$

$$Q = \frac{1}{R} \qquad (3)$$

$$w = \frac{mc}{s}(1-\alpha)\left(\frac{k}{n}\right)^{\alpha}$$

$$\frac{k}{n} = \left(\frac{mc}{s}\frac{\alpha}{r^K}\right)^{\frac{1}{1-\alpha}}$$

$$p^* = \left[\frac{1-\theta\pi^{\eta-1}}{1-\theta}\right]^{\frac{1}{1-\eta}} \qquad (6)$$

$$\zeta = \frac{y \cdot mc}{s\left(1-\theta\beta\pi^{\eta}\right)\left(p^*\right)^{1+\eta}}$$

$$\xi = \frac{y}{\left(1-\theta\beta\pi^{\eta-1}\right)\left(p^*\right)^{\eta}}$$

$$\xi = \frac{\eta}{\eta-1}\zeta$$

$$\frac{y}{k} = \frac{1}{s}\left[\left(\frac{k}{n}\right)^{\alpha-1} - \frac{\kappa}{k}\right]$$

$$\frac{y}{k} = \frac{c}{k} + \frac{i}{k} + \frac{g}{k}$$

$$s = \frac{(1-\theta)(p^*)^{-\eta}}{1-\theta\pi^{\eta}} \qquad (7)$$

$$v = \left(c(1-n)^{\nu}\right)^{\frac{1}{1+\nu}}$$

$$\mathcal{W} = v \qquad (*)$$

$$z = 1 \qquad (1)$$

$$g = \bar{g} \qquad (2)$$

B.2 Steady State: Step 2

$$i = \delta k \qquad (13)$$

$$c = \frac{w}{\nu}\,(1-n) \qquad (12)$$

$$n = \left(\frac{mc}{s}\frac{\alpha}{r^K}\right)^{-\frac{1}{1-\alpha}} k \qquad (11)$$

$$w = \frac{mc}{s}(1-\alpha)\left(\frac{k}{n}\right)^{\alpha} \qquad (9)$$

$$\zeta = \left(1-\frac{1}{\eta}\right)\xi \qquad (18)$$

$$\xi = \frac{y}{\left(1-\theta\beta\pi^{\eta-1}\right)(p^*)^{\eta}} \qquad (17)$$

$$mc = \left(1-\frac{1}{\eta}\right)\frac{1-\theta\beta\pi^{\eta}}{1-\theta\beta\pi^{\eta-1}}\,p^*\,s \qquad (8)$$

$$y = \frac{1}{s}\left(k^{\alpha}n^{1-\alpha}-\kappa\right) \qquad (16)$$

$$k = \frac{\frac{\kappa}{s}+\frac{w}{\nu}+g}{\frac{1}{s}\left(\frac{k}{n}\right)^{\alpha-1}+\frac{w}{\nu}\left(\frac{k}{n}\right)^{-1}-\delta} \qquad (10)$$

$$v = \left(c(1-n)^{\nu}\right)^{\frac{1}{1+\nu}} \qquad (14)$$

$$\mathcal{W} = v \qquad (15)$$

C First-order Conditions for the Ramsey Problem

$$k_t: \quad -\mu_t^1 + \mathbf{E}_t\left[\frac{\partial \tilde{v}_t}{\partial \tilde{v}_{t+1}}\frac{\partial \tilde{v}_{t+1}}{\partial k_t}\right] = 0 \tag{C1}$$

$$\frac{\partial \tilde{v}_t}{\partial \tilde{v}_{t+1}} = \tilde{\beta}\left(\frac{\tilde{v}_{t+1}}{\tilde{v}_t}\right)^{-\gamma}\left(\frac{\tilde{v}_{t+1}}{\left(\mathbf{E}_t\left[\tilde{v}_{t+1}^{1-\chi}\right]\right)^{\frac{1}{1-\chi}}}\right)^{\gamma-\chi}$$

$$\frac{\partial \tilde{v}_t}{\partial k_{t-1}} = \mu_t^1(1-\delta) - \mu_t^6\frac{\alpha w_t}{k_{t-1}} - \mu_t^7\frac{(\alpha-1)r_t^K}{k_{t-1}} - \mu_t^{12}\alpha z_t k_{t-1}^{\alpha-1}n_t^{1-\alpha}$$

$$i_t: \quad \mu_t^1 + \mu_t^{13} = 0 \tag{C2}$$

$$c_t: \quad (1-\tilde{\beta})\tilde{v}_t^{\gamma}\frac{\left(c_t(1-n_t)^{\nu}\right)^{\frac{1-\gamma}{1+\nu}}}{(1+\nu)c_t} - \mu_t^2\frac{\gamma+\nu}{1+\nu}\frac{Q_t}{c_t} - \mu_t^3\frac{\nu}{1-n_t} + \mu_t^{13}$$

$$+\mu_t^{15}(1-\beta)(1-\gamma)\frac{\left(c_t(1-n_t)^{\nu}\right)^{\frac{1-\gamma}{1+\nu}}}{(1+\nu)c_t} + \mathbf{E}_t\left[\frac{\partial \tilde{v}_t}{\partial \tilde{v}_{t+1}}\frac{\partial \tilde{v}_{t+1}}{\partial c_t}\right] = 0 \tag{C3}$$

$$\frac{\partial \tilde{v}_t}{\partial c_{t-1}} = \mu_t^2\frac{\gamma+\nu}{1+\nu}\frac{Q_t}{c_{t-1}}$$

$$n_t: \quad -(1-\tilde{\beta})\tilde{v}_t^{\gamma}\frac{\nu}{1+\nu}\frac{\left(c_t(1-n_t)^{\nu}\right)^{\frac{1-\gamma}{1+\nu}}}{1-n_t} - \mu_t^2\frac{\nu(1-\gamma)}{1+\nu}\frac{Q_t}{1-n_t} - \mu_t^3\frac{\nu c_t}{(1-n_t)^2}$$

$$+\mu_t^6\frac{\alpha w_t}{n_t} + \mu_t^7\frac{(\alpha-1)r_t^K}{n_t} - \mu_t^{12}(1-\alpha)z_t k_{t-1}^{\alpha}n_t^{-\alpha}$$

$$-\mu_t^{15}(1-\beta)\frac{\nu(1-\gamma)}{1+\nu}\frac{\left(c_t(1-n_t)^{\nu}\right)^{\frac{1-\gamma}{1+\nu}}}{1-n_t} + \mathbf{E}_t\left[\frac{\partial \tilde{v}_t}{\partial \tilde{v}_{t+1}}\frac{\partial \tilde{v}_{t+1}}{\partial n_t}\right] = 0 \tag{C4}$$

$$\frac{\partial \tilde{v}_t}{\partial n_{t-1}} = \mu_t^2\frac{\nu(1-\gamma)}{1+\nu}\frac{Q_t}{1-n_{t-1}}$$

$$Q_{t+1}: \quad -\mu_t^4\pi_{t+1}\left(r_{t+1}^K + 1 - \delta\right) - \mu_t^5 R_t + \mu_t^9\theta\left(\pi_{t+1}p_{t+1}^*\right)^{1+\eta}\zeta_{t+1}$$

$$+\mu_t^{10}\theta\left(\pi_{t+1}p_{t+1}^*\right)^{\eta}\xi_{t+1} + \frac{\partial \tilde{v}_t}{\partial \tilde{v}_{t+1}}\frac{\partial \tilde{v}_{t+1}}{\partial Q_{t+1}} = 0 \tag{C5}$$

$$\frac{\partial \tilde{v}_t}{\partial Q_t} = -\mu_t^2$$

$$w_t: \quad \mu_t^3 + \mu_t^6 = 0 \tag{C6}$$

$$r_{t+1}^K: \quad -\mu_t^4 Q_{t+1}\pi_{t+1} + \frac{\partial \tilde{v}_t}{\partial \tilde{v}_{t+1}}\frac{\partial \tilde{v}_{t+1}}{\partial r_{t+1}^K} = 0 \tag{C7}$$

$$\frac{\partial \tilde{v}_t}{\partial r_t^K} = \mu_t^7$$

$$\pi_{t+1}: \quad -\mu_t^4 Q_{t+1}\left(r_{t+1}^K + 1 - \delta\right) + \mu_t^9\theta(1+\eta)Q_{t+1}\left(p_{t+1}^*\right)^{1+\eta}\pi_{t+1}^{\eta}\zeta_{t+1}$$

$$+\mu_t^{10}\theta\eta Q_{t+1}\left(p_{t+1}^*\right)^{\eta}\pi_{t+1}^{\eta-1}\xi_{t+1}+\frac{\partial\tilde{v}_t}{\partial\tilde{v}_{t+1}}\frac{\partial\tilde{v}_{t+1}}{\partial\pi_{t+1}}=0 \tag{C8}$$

$$\frac{\partial\tilde{v}_t}{\partial\pi_t}=-\mu_t^2\frac{Q_t}{\pi_t}+\mu_t^8\theta(\eta-1)\pi_t^{\eta-2}+\mu_t^{14}\theta\eta\pi_t^{\eta-1}s_{t-1}$$

$$mc_t:\quad -\mu_t^6\frac{w_t}{mc_t}-\mu_t^7\frac{r_t^K}{mc_t}+\mu_t^9\frac{y_t}{s_t}=0 \tag{C9}$$

$$p_{t+1}^*:\quad \mu_t^9\theta(1+\eta)Q_{t+1}\pi_{t+1}^{1+\eta}\left(p_{t+1}^*\right)^{\eta}\zeta_{t+1}+\mu_t^{10}\theta\eta Q_{t+1}\pi_{t+1}^{\eta}\left(p_{t+1}^*\right)^{\eta-1}\xi_{t+1}$$

$$+\frac{\partial\tilde{v}_t}{\partial\tilde{v}_{t+1}}\frac{\partial\tilde{v}_{t+1}}{\partial p_{t+1}^*}=0 \tag{C10}$$

$$\frac{\partial\tilde{v}_t}{\partial p_t^*}=\mu_t^8(1-\theta)(1-\eta)(p_t^*)^{-\eta}-\mu_t^9(1+\eta)\left(p_t^*\right)^{\eta}\zeta_t-\mu_t^{10}\eta\left(p_t^*\right)^{\eta-1}\xi_t$$

$$-\mu_t^{14}(1-\theta)\eta\left(p_t^*\right)^{-\eta-1}$$

$$\zeta_{t+1}:\quad \mu_t^9\theta Q_{t+1}\left(\pi_{t+1}p_{t+1}^*\right)^{1+\eta}+\frac{\partial\tilde{v}_t}{\partial\tilde{v}_{t+1}}\frac{\partial\tilde{v}_{t+1}}{\partial\zeta_{t+1}}=0 \tag{C11}$$

$$\frac{\partial\tilde{v}_t}{\partial\xi_t}=-\mu_t^9\left(p_t^*\right)^{1+\eta}+\mu_t^{11}\frac{\eta}{\eta-1}$$

$$\xi_{t+1}:\quad \mu_t^{10}\theta Q_{t+1}\left(\pi_{t+1}p_{t+1}^*\right)^{\eta}+\frac{\partial\tilde{v}_t}{\partial\tilde{v}_{t+1}}\frac{\partial\tilde{v}_{t+1}}{\partial\xi_{t+1}}=0 \tag{C12}$$

$$\frac{\partial\tilde{v}_t}{\partial\xi_t}=-\mu_t^{10}\left(p_t^*\right)^{\eta}-\mu_t^{11}$$

$$y_t:\quad \mu_t^9\frac{mc_t}{s_t}+\mu_t^{10}+\mu_t^{12}s_t-\mu_t^{13}=0 \tag{C13}$$

$$s_t:\quad \left[\mu_t^6\frac{w_t}{s_t}+\mu_t^7\frac{r_t^K}{s_t}-\mu_t^9\frac{y_t\cdot mc_t}{s_t^2}\right]+\mu_t^{12}y_t-\mu_t^{14}+\mathbf{E}_t\left[\frac{\partial\tilde{v}_t}{\partial\tilde{v}_{t+1}}\frac{\partial\tilde{v}_{t+1}}{\partial s_t}\right]=0 \tag{C14}$$

$$\frac{\partial\tilde{v}_t}{\partial s_{t-1}}=\mu_t^{14}\theta\pi_t^{\eta}$$

$$R_t:\quad -\mu_t^5\mathbf{E}_t\left[Q_{t+1}\right]+\mathbf{E}_t\left[\frac{\partial\tilde{v}_t}{\partial\tilde{v}_{t+1}}\frac{\partial\tilde{v}_{t+1}}{\partial R_t}\right]=0 \tag{C15}$$

$$\frac{\partial\tilde{v}_t}{\partial R_{t-1}}=-\mu_t^{16}\frac{b_{t-1}}{\pi_t}$$

$$v_{t+1}:\quad \mu_t^{16}(1-\chi)v_{t+1}^{-\chi}+\frac{\partial\tilde{v}_t}{\partial\tilde{v}_{t+1}}\frac{\partial\tilde{v}_{t+1}}{\partial v_{t+1}}=0 \tag{C16}$$

$$\frac{\partial\tilde{v}_t}{\partial v_t}=\mu_t^2(\gamma-\chi)\frac{Q_t}{v_t}-\mu_t^{15}(1-\gamma)v_t^{-\gamma}$$

$$\mathcal{W}_t:\quad \mu_t^{15}\beta(1-\gamma)\mathcal{W}_t^{-\gamma}-\mu_t^{16}(1-\chi)\mathcal{W}_t^{-\chi}+\mathbf{E}_t\left[\frac{\partial\tilde{v}_t}{\partial\tilde{v}_{t+1}}\frac{\partial\tilde{v}_{t+1}}{\partial\mathcal{W}_t}\right]=0 \tag{C17}$$

$$\frac{\partial\tilde{v}_t}{\partial\mathcal{W}_{t-1}}=-\mu_t^2(\gamma-\chi)\frac{Q_t}{\mathcal{W}_{t-1}} \tag{C18}$$