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Education, Technological Progress and Economic Growth

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Abstract

An important role of education – and the resultant accumulation of human capital – for a less-developed economy is to facilitate technology diffusion in order for it to catch up with developed economies. This paper presents a model linking education, the accumulation of physical capital and technological progress. In the model, investment in education and the accumulation of physical capital are complementary, and intertwine with the technology progress through related effects on technology diffusion and the expansion of the technology frontier. The allocation of effort to education, the optimal savings rate and the technology gap are endogenously determined in the steady-state balanced growth equilibrium.

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1. Introduction

A major theme in the research on economic growth is the explanation of the divergent growth experiences of nations. Specifically, the recent growth literature (as surveyed in Grossman and Helpman (1991), Barro and Sala-i-Martin (1995), Aghion and Howitt (1998) and Temple (1999b)) has endeavored to uncover the economic, social and political factors that aided or hindered the convergence of economic growth rates. Empirical studies on the growth performance of countries over the second half of the twentieth century have not shown conclusive evidence of convergence in the fortunes between the rich and the poor nations (see Pritchett (1997)). In fact, these studies have found that different countries appeared to have remained on disparate growth paths for long periods of time. A review of the data from the 2002 World Bank Development Indicators shows that high-income countries recorded an average annual GDP per capita growth rate of 2.77 percent over the four decades from 1960 to 2000. By contrast, the corresponding statistic for middle-income countries and low-income countries are, 2.70 percent and 1.60 percent, respectively.[1] There were only brief periods in the 1970s and the 1990s that middle-income countries grew faster than the high-income ones.

The theoretical literature that has been developed to explain these disparities in growth experiences have taken several directions. One line of research (as exemplified in the work of Jones and Manuelli (1990), King and Rebelo (1990) and Rebelo (1991)) emphasizes the accumulation of physical capital and human capital as the driving force of continual growth. As Lucas (1993) pointed out, the accumulation of human capital – specifically, knowledge – is a key factor in explaining the growth experiences of countries. Another line of theory (following Arrow (1962), Romer (1986) and Lucas (1988)) stresses the externalities in the capital accumulation process, so that economic growth can be sustained by continuing accumulation of the inputs that produces positive externalities. In an influential paper,

[1] The definitions of high-income, middle-income and low-income country groups follow the definitions set by the World Bank.

Mankiw, Romer and Weil (1992) argues that the evidence on the international disparity in levels of per capita income and rates of growth is consistent with the neoclassical Solow model, once it has been augmented to include human capital as an accumulable factor and to allow for cross-country differences in savings rates. More recently, authors such as Romer (1990a), Aghion and Howitt (1992), Grossman and Helpman (1994) and Basu and Weil (1998), among others, have focused on the role of innovation and technological progress and cast industrial innovation as the engine of long-run sustained growth. A common thread to all these lines of theoretical research on economic growth is the emphasis on the role played by education in facilitating the accumulation of physical capital (by strengthening learning-by-doing externality effects) and human capital (by increasing the level of technical skills), and in innovation (by increasing the productivity of R&D efforts).

The impact of education on human capital accumulation is well-established in the theoretical growth literature. In this paper, we examine the role that education plays in fostering technological diffusion and economic growth. We present a theoretical model – inspired by Arrow (1962), Nelson and Phleps (1966) and Lucas (1988) – that examines the endogenous allocation of effort to production and education activities, and the resultant impact on physical capital accumulation and technological progress. In the model, a greater proportion of effort allocated to production allows for the accumulation of a larger stock of physical capital, which, through learning-by-doing externality effects, advances the economy's appropriate technology frontier. On the other hand, a greater proportion of effort allocated to education allows for a faster pace of technology diffusion, and, consequently, a more rapid expansion of the technology in practice. Our analysis shows that, in the steady-state growth equilibrium, the accumulation of human capital and physical capital are complementary in facilitating technology progress – in the sense that a larger (optimal) level of effort devoted to education is associated with a higher rate of net savings. In the model, the steady-state growth rate is determined by the rate of expansion of the global technology frontier, the share of capital in production and the strength of localized learning-by-doing externality effects. Even if different economies possess the same steady-state growth rates, they may still differ in the optimal savings rate, the optimal allocation of effort to education (equivalent to the rate of accumulation of human capital), as well as the optimal technology gaps.

The rest of the paper is organized as follows. In Section 1.1, we briefly discuss the role that education plays in the economic growth process. Section 2 introduces the theoretical model and solves for the optimal growth paths. Section 3 characterizes the balanced growth steady-state equilibrium, in terms of the endogenously chosen optimal allocation of effort, optimal savings rates and optimal technology gap. Section 4 presents the comparative statics results. The last section concludes the paper.

1.1 How important is Education to Economic Growth?

A key contribution of education to the economic growth of a less developed country is to facilitate a more rapid pace of technology diffusion in order for it to catch up with the more developed countries.[2] Clearly, a direct effect of education is to raise the level of skills of workers. Heckman (2002) noted that an individual who has undergone training in a discipline (say, accountancy) would improve his performance in that discipline. An equally important indirect effect of education is to increase the flexibility of the labor force and its capacity to learn new ideas, adapt to new technologies, improve local technologies on the job, as well as to better-equip workers to undertake scientific research and innovation. In the endogenous growth literature, human capital is a central input for innovation and R&D activities. An increase in investment in education accelerates technological progress through the creative destruction of old ideas and processes.

In a rapidly growing economy, the benefits of education is evident, not only through the training of workers to work with more sophisticated technology, but also in developing a group of specialized labor that may be devoted to R&D and innovation activities. Nelson and

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[2] Mincer (1984) and Romer (1990b) provides discussions on the different channels through which education plays in fostering technology diffusion and economic growth. See also Psacharopoulos (1994) and Islam (1995) for discussion of the education in the growth process.

Phelps (1966) noted that "education has a positive payoff only if the technology is always improving". The key idea is that by facilitating the diffusion of knowledge and technology to offset the diminishing returns to physical capital that otherwise occurs, the return to education is higher the more rapid the improvement in technology. In turn, the higher the overall level of educational investment and attainment, the higher is the level of technology that could potentially be achieved, thus reinforcing the benefits to education, through a virtuous cycle. Therefore, physical capital and human capital are complementary, as each factor raises the productivity of the other factor.

Equally important is the role of education in fostering social cohesion. This aspect of education in the growth process has been emphasized recently by Gradstein and Justman (2002). By instilling civic virtues from an early age, through education, this can potentially reduce the cost of enforcing desirable social norms, which in turn facilitates a more rapid economic growth process.

Table 1: Annual Expenditure on Education and Health as a Percentage of Expenditure on Gross Capital Formation: 1960-2000

Country Groups	On education	On health	Total
High-income	41.92	40.95	82.87
Middle-income	29.66	21.29	50.93
Low-income	28.37	19.39	47.76

Source: World Development Indicators, 2002.

While the importance of education in economic developed is well-accepted by policy makers, historical experience (as shown in Table 1) suggests that unlike high-income countries, middle- and lower-income countries have placed heavier emphasis on the accumulation in physical capital than on education and investment in human capital. This may be due to several factors. For instance, market failures in the provision of private education may be more prevalent in a developing country. Imperfect financial markets may also make it difficult to borrow to finance one's education since the return is long-term and may be uncertain. Government policies may also be myopic, favoring physical capital investment that brings more immediate and certain returns. International aids, especially those tied to the purchase of the donor country's products, also tend to encourage physical capital accumulation. In a study on China, Heckman (2002) found significant market distortions in the private returns to education. Furthermore, the existence of positive externalities meant that there had been an underinvestment in education in the Chinese economy.

Although education and human capital accumulation is central to the growth process, empirical evidence on the contribution of education to growth has been mixed. An influential and much-cited cross-national econometric study by Pritchett (1996) suggests that increases in education capital resulting from improvements in the educational attainment of the labor force have had little impact on the growth rate of output per worker. In fact, the study found that the estimated impact of the growth of human capital on total factor productivity is large, strongly significant, and negative. Pritchett's findings appear to contradict the conventional wisdom about the importance of education, as well as empirical studies on the subject.

However, in a study on Asian economies, Mchahon (1998) found that an increase in investment in secondary education was significant in achieving high rates of investment and high per-capita GDP growth. Earlier work by researchers such as Barro and Sala-i-Martin (1991, 1995) and Benhabib and Spiegel (1994) have also found a weakly positive relationship between schooling and per capita GDP growth rate across countries.[3] More recently, Hanushek and Kimko (2000) found direct causal links between international mathematics and

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[3] These studies have found that the specific measures of school attainment that are significant explanatory variables for the growth process are average years of male secondary and higher schooling and average years of female secondary and higher schooling. The explanatory power for growth rates is greater in this form – which distinguishes years of attainment at the secondary and higher levels – than with an alternative nonlinear specification in terms of total years of schooling. Attainment at the primary level turns out not to be significantly related to growth rates. See also Barro and Lee (1996) for a discussion of the international measures of schooling quality.

science test scores, the quality of the labor-force, and economic growth. Bils and Klenow (2000) also found that anticipated economic growth induces more people to take up training or to stay longer in schools, as an increase in private returns on education makes the investment worthwhile.

In a recent study, Temple (1999a) examined the dataset on education and economic growth used by Benhabib and Spiegel (1994) and concluded that, when unrepresentative observations are excluded from the sample, "there is clear evidence that output growth is positively correlated with the change in educational attainment, even when one conditions on physical capital accumulation." Adopting a similar approach, Temple (2001) examined Pritchett's findings using alternative specifications of the production function to account for the accumulation of human capital, and concluded that by choosing specifications judiciously in econometric analyses, the importance of educational attainment in growth is substantially increased. The level of education also remains an important determinant in explaining subsequent rate of economic growth.

The conclusion that one may draw from these studies is that as the contribution of education to economic growth is multi-faceted, looking at only tangible measures of its contribution to the accumulation of human capital will often underestimate its contribution. Most empirical studies have shown that broad measures of educational attainment – in terms of the number of years of schooling, the school enrolment rate of the proportion of the workforce with secondary education – do not adequately capture the linkage between education and growth. The quality of schooling differs across countries, and there may be little incentive to ensure that public expenditure on education is wisely spent to maximize the quantity and quality of educational output. Furthermore, there are other elements of human capital besides general educational attainment captured by formal schooling enrolment statistics. Improvements in productivity also occur with better physical and mental health, formal and informal occupational training programmes as well as on-the-job training for employees.

2. A Model of Education and Technological Progress

In the context of economic development, an important role of education is to increase an individual's capacity, first, to *innovate* (i.e. to create new products and new technologies) and, second, to adapt to new technologies, thereby accelerating technological diffusion in the economy. Our modeling of education in the growth process follows the approach of Nelson and Phleps (1966).[4] In this approach, the accumulation of human capital is inseparable from technological progress, and may occur in different ways – through learning in schools, research and innovations in laboratories and in the course of production and commerce.

We consider a closed economy with a constant population of identical infinitely-lived agents that produces and consumes a homogenous good using the following production function $Y(t) = Y\left(A(t), K(t), L(t)\right)$ where A is an index of technology, K is the stock of physical capital, and L is the flow of labor effort devoted to production. By normalizing the flow of labor effort to 1, we may write

$$Y(t) = Y\left(A(t), K(t), u(t)\right) \qquad (1)^{5}$$

where $0 \leq u(t) \leq 1$ is the fraction of time each agent spends in production instead of engaging in educational activities. As Lucas (1993) noted, actual schooling decisions take place in a life-cycle context, with the main phase of education preceding work and each individual deciding on the length of these two careers. Since agents are infinitely-lived in

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[4] The Nelson-Phleps framework predicted that productivity growth and the rate of innovations should increase with the level of educational attainment, particularly with the enrolment in secondary and higher education. Also, marginal productivity of education is an increasing function of the rate of technological progress. These predictions have empirical support in the studies of Barro and Sala-i-Martin (1995), Benhabib and Spiegel (1994) and Mchahon (1998).

[5] Our model emphasizes the effect of education on technological change is through the expansion of the current technology in practice $A(t)$. This formulation is similar to the one in Lucas (1988), where $Y(t) = Y\left(A(t), K(t), h(t)u(t)\right)$, and $h(t)$ represents embodied human capital. By implicitly assuming $h(t) = 1$, our formulation abstracts from the effect of productive effort $u(t)$ on embodied human capital studied in Lucas (1988). This effect can be incorporated in an expansion of the model.

our model, it is perhaps more appropriate in our model to interpret educational investment as technical training for the purpose of adopting increasingly more sophisticated technology.[6]

We further make a further simplifying assumption that Y is linearly homogeneous in K and u, and follows the Cobb-Douglas specification:

$$Y(t) = A(t)K(t)^{\beta}u(t)^{1-\beta}\,. \tag{2}$$

where $0 < \beta < 1$. Assuming no depreciation, the capital accumulation function is given by

$$\dot{K}(t) = Y(t) - c(t) \tag{3}$$

where $c(t)$ is the consumption at time t. There is no disutility from work and each agent maximizes a stream of discounted utilities, given by an intertemporal utility function

$$\int_0^{\infty} e^{-\rho t}\frac{1}{1-\sigma}\left[c(t)^{1-\sigma}-1\right]dt\,. \tag{4}$$

where ρ is the rate of intertemporal rate of discount and σ is the coefficient of relative risk aversion (equivalently, σ^{-1} is the intertemporal elasticity of substitution).

2.1 Appropriate Technology Frontier

Countries differ not only in their factor endowments, but also in terms of the menu of technologies that they have access to and are able to deploy at any point in time. To capture this feature in our model, we follow Nelson and Phelps (1966), and more recently, Basu and Weil (1998) to distinguish between the actual technology currently in practice $A(t)$ − which represents that average level of the best-practice technology embodied in the production − from the frontier technology $\tilde{A}(t)$ − which represents the a body of state-of-the-art scientific knowledge and innovations that the economy acquire over time.

Technological progress and physical capital accumulation often go hand in hand, as a significant amount of technological advancement is embodied in physical capital.[7] We may

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[6] We may extend the model to include population growth, but the qualitative aspects of our results regarding the complementary nature of education and physical capital accumulation remain the same.

distinguish between two kinds of technological progress: (i) a general improvement in production techniques, brought on by fundamental discoveries in science resulting from investment in R&D, and which is reflected in a *general* outward shift of an economy's aggregate production function; (ii) a '*localized*' effect that involves learning by doing externalities, and is dependent on factors such as the current stock of physical capital, the capital intensity of production or the distribution of skills in the labor force. In this sense, there is a positive externality associated the accumulation of physical capital, as it also opens up further technology possibilities for adoption later on. The concept of localized technological progress was first discussed by Atkinson and Stiglitz (1969) and analyzed recently by Basu and Weil (1998) in the context of appropriate technologies, which are specific to particular capital-labor combinations.[8]

The possibility of localized technological progress implies that the *appropriate* technology frontier that an economy faces is a function of both the global technology frontier and the economy's current stock of physical capital. Accordingly, in our model, we suppose the rate of expansion of the appropriate technology frontier is a function of both the rate of expansion of the global technology frontier and the rate of accumulation of physical capital. Formally, let $\tilde{A}(t)$ denote the *appropriate* technology frontier where

$$\tilde{A}(t) = \tilde{A}\left(K(t), \overline{A}(t)\right) \tag{5}$$

where $\overline{A}(t)$ denotes the global technology frontier at time *t*. Individually, most countries are small relative to the world economy in terms of their contributions to this dynamic knowledge

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[7] This linkage between technological progress and physical capital accumulation is discussed in Romer (1986), Rebelo (1991), De Long and Summers (1991, 1993), among others.

[8] Atkinson and Stiglitz (1969) pointed out that the technology relevant to a less developed country with abundant labor is different from that relevant to a developed one with abundant capital. The former may be more interested in improving her productivity in handcarts while the latter forklift trucks. Basu and Weil (1998) noted that "an advance in transportation technology in Japan may take the form of a refinement of the newest maglev train. Such an advance may have very few spillovers to the technology of the transportation sector in Bangladesh, which relies in large part on bicycles and bullock carts."

base[9]; thus, most countries would treat the global frontier technology $\bar{A}(t)$, and its associated rate of change $g_{\bar{A}} \equiv \dot{\bar{A}}/\bar{A}$, as exogenously given.[10] We shall adopt this assumption, and further assume that $g_{\bar{A}}$ is constant, so that $\bar{A}(t) = \bar{A}(0)e^{g_{\bar{A}}t}$; $g_{\bar{A}} > 0$. For sharp analytical results, we consider the following specific functional form of $\tilde{A}(t)$:

$$\tilde{A}(t) = \eta K(t)^{\alpha} \bar{A}(t) \tag{6}$$

where $\alpha \in (0,\ 1)$ is a measure of the strength of the learning-by doing effects associated with the accumulation of physical capital, while $\eta > 0$ is a scaling constant. Equation (6) captures the notion that the expansion of the appropriate technology frontier is a function of both the pace of capital accumulation and the global advancement in science and technology. Taking logs on both sides of Equation (6) and differentiate with respect to *t*, we obtain

$$g_{\tilde{A}}(t) = \alpha g_K(t) + g_{\bar{A}} \tag{7}$$

There are various means through which education can accelerate technology diffusion, enhance the technology in practice and narrow the technology gap $\left[\bar{A}(t) - A(t)\right]$. For instance, raising the level of educational attainment promotes more rapid absorption of new ideas and production methods. In turn, this enables more rapid technology diffusion and transfer and facilitates the adoption of more sophisticated technology further up the technology ladder. Therefore, the impact of education on human capital accumulation is intertwined and inextricable with the process of technological progress. We model the technology diffusion process as follows:

$$\dot{A}(t) = f\left(1-u(t)\right)\cdot\left[\tilde{A}(t) - A(t)\right] \;; \qquad f(\ \cdot\ ) > 0 \tag{8}$$

In the formulation in (8), the rate of technological diffusion is increasing in educational effort (and attainment) and proportional to the technological gap. For sharper analytical results, we adopt a linear specification of $f(\ \cdot\ )$:

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9 The exceptions are perhaps the United States and the leading industrial nations.

10 Throughout this paper, g_x denotes the growth rate of the variable *x*, and $\dot{x}$ for its time derivative.

$$\dot{A}(t) = \delta\left[1-u(t)\right]\cdot\left[\tilde{A}(t)-A(t)\right]; \qquad \delta > 0 \tag{9}$$

where δ is a measure of the effectiveness of educational effort in effecting technological change. Equation (9) is motivated by Nelson and Phelps (1966, pp 73, Equation 8).[11] Note that the impact of educational effort on technological change $\dot{A}(t)$ is directly related to the size of the technology gap $\left[\tilde{A}(t)-A(t)\right]$. Starting with an initially wide technology gap, the returns to educational investment are high as technology transfer, learning and imitation strategies are relatively straightforward given an established body of knowledge to draw from. As the technology gap narrows, the returns from learning and technology transfer becomes progressively more difficult, as the body of scientific knowledge at the frontier may be undergoing further research, experimentation and refinement. Therefore, as the country gets closer to the technology frontier, educational efforts produce progressively lower returns.

2.2 Optimal Growth Paths

The optimal growth paths can be solved from the perspective of a representative agent in the economy. The equations of motions are given in (3) and (9). The current-value Hamiltonian H for the optimization problem contains two state variables $K(t)$ and $A(t)$, two control (i.e. choice) variables $c(t)$ and $u(t)$, and two co-state variables $\theta_1(t)$ and $\theta_2(t)$:

$$H(K,\, A,\, \theta_1,\, \theta_2,\, c,\, u,\, t) = \frac{c^{1-\sigma}-1}{1-\sigma} \;+\; \theta_1\left(AK^{\beta}u^{1-\beta}-c\right) \;+\; \theta_2\delta\left[1-u\right]\left(\eta K^{\alpha}\bar{A}-A\right) \tag{10}$$

To characterize the optimal growth paths $\left\{c(t),\, u(t)\right\}_{t=0}^{t=\infty}$, an application of Pontryagin's Maximum Principle allows us to solve for a the pair of first order conditions related to the pair of control variables; they are

$$\theta_1(t) = c(t)^{-\sigma} \tag{11}$$

$$\theta_1(t)(1-\beta)\left[\frac{K(t)}{u(t)}\right]^{\beta} = \theta_2(t)\delta\left[\frac{\eta K(t)^{\alpha}\bar{A}(t)}{A(t)}-1\right] \tag{12}$$

[11] We may also consider $\delta\left[1-u(t)\right]$ as a measure of the intensity of educational human capital, along the lines of Nelson and Phleps (1966, pp 72).

Condition (11) says that the optimal allocation of the economy's output is such that, at the margin, they are equally valuable for use as consumption or as investment, while Condition (12) says that at the margin, time spent on production and on education must be equally valuable. Next, the pair of time-paths for the co-state variables is

$$\frac{\dot{\theta}_1(t)}{\theta_1(t)} = \rho - \beta A(t)\left[\frac{u(t)}{K(t)}\right]^{1-\beta} - \frac{\theta_2(t)}{\theta_1(t)}\alpha\delta\left[1-u(t)\right]\frac{\left[\eta K(t)^{\alpha}\bar{A}(t)\right]}{K(t)} \tag{13}$$

$$\frac{\dot{\theta}_2(t)}{\theta_2(t)} = \rho + \delta\left[1-u(t)\right] - \frac{\theta_1(t)}{\theta_2(t)}K(t)^{\beta}u(t)^{1-\beta} \tag{14}$$

Lastly, the relevant pair of transversality conditions for the optimization problems are $\lim_{t\to\infty} e^{-\rho t}\theta_1(t)K(t) = 0$ and $\lim_{t\to\infty} e^{-\rho t}\theta_2(t)A(t) = 0$. The first of these two conditions implies that the shadow value of the accumulated capital stock must vanish eventually, is familiar. In the second, this requires that the shadow value of $A(t)$ must fall faster than $g_{\bar{A}}$, the rate of growth of the frontier technology. Together, the conditions in (11), (12), (13), (14) and the pair of transversality conditions allow us to characterize the optimal growth paths in Proposition 1. The proofs of the Propositions are given in Appendix A.

Proposition 1: The equilibrium conditions for the optimal growth paths are:

$$g_c(t) = \frac{1}{\sigma}\left[-\rho + A(t)\left[\frac{u(t)}{K(t)}\right]^{1-\beta}\left\{\beta + \alpha(1-\beta)\left[\frac{1-u(t)}{u(t)}\right]\left[\frac{\eta K(t)^{\alpha}\bar{A}(t)}{\eta K(t)^{\alpha}\bar{A}(t) - A(t)}\right]\right\}\right] \tag{15}$$

$$g_K(t) = A(t)\left[\frac{u(t)}{K(t)}\right]^{1-\beta} - \frac{c(t)}{K(t)} \tag{16}$$

$$A(t) = \bar{A}(0)\int_0^t \left\{\delta\left[1-u(y)\right]\eta K(y)^{\alpha}\exp\left[g_{\bar{A}}y - \delta\int_y^t\left[1-u(z)\right]dz\right]\right\}dy$$
$$+ A(0)\exp\left[-\delta\int_0^t\left[1-u(z)\right]dz\right] \tag{17}$$

3. The Balanced Growth Steady State Equilibrium

While there are potentially multiple equilibrium growth paths, we focus our analysis on the steady-state balanced growth path along which consumption and the accumulation of physical capital proceed at constant percentage rates and the optimal effort allocation $u(t)$ is a constant amount. Let u^* denote the optimal effort allocated to production in the steady-state equilibrium.

Proposition 2: As $t \to \infty$, the equilibrium growth path of the technology in practice $A(t)$ converges to

$$A(t) = \frac{\delta\left(1-u^*\right)}{g_{\bar{A}} + \alpha g_K + \delta\left(1-u^*\right)} \eta K(t)^{\alpha} \bar{A}(0) e^{g_{\bar{A}} t} \tag{18}$$

The steady-state equilibrium expansion path of the technology in practice is independent of the initial capital stock of the economy. Since the rate of expansion of the appropriate technology frontier $\tilde{A}(t)$ is $g_{\tilde{A}} = g_{\bar{A}} + \alpha g_K$ along the balanced growth path, this implies that $A(t)$ grows at the same rate as the appropriate technology frontier $\tilde{A}(t)$.

Proposition 3: If $1-\beta-\alpha > 0$, a steady-state equilibrium exists, characterized by[12]

$$g_Y = g_c = g_K = \frac{g_{\bar{A}}}{1-\beta-\alpha} \quad \text{and} \quad g_{\tilde{A}} = \frac{1-\beta}{1-\beta-\alpha} g_{\bar{A}} \tag{19}$$

The steady-state growth rates in Proposition 3 are familiar results of the neo-classical Solow model, where the intertemporal rate of discount ρ, the degree of risk aversion σ, or the effectiveness of education δ, have no bearing on the steady-state growth rate of the economy, denoted g_Y, which equals $g_{\bar{A}}$. However, as we show in Proposition 4, these parameters influence the choice of the optimal savings rate s^*, the optimal production effort u^*, and the optimal technology gap G^*. The implication is that although different economies may possess same steady-state growth rates, they may still differ in terms of the optimal savings rate, their level of wealth and their distance to the global technology frontier.

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12 Since the share of capital β in output is empirically estimated to be around 0.3, this restriction effectively implies that α must less than 0.7.

Proposition 4: In the steady-state equilibrium, the optimal technology gap G^* , the optimal savings rate s^*, and the optimal amount effort allocated to production u^*, are

$$G^* = \frac{(1-\beta)g_{\bar{A}}}{\delta\left(1-u^*\right)(1-\beta-\alpha)} \tag{20}$$

$$s^* = \frac{\left[\beta u^* + \alpha(1-\beta)\left(1-u^*\right)\right]g_{\bar{A}} + \alpha\delta\left(1-u^*\right)^2}{\left[\rho(1-\beta-\alpha)+\sigma g_{\bar{A}}\right]u^*} \tag{21}$$

$$u^* = \frac{(1-\beta+\sigma)g_{\bar{A}} + (1-\beta-\alpha)(2\delta+\rho) - \sqrt{\left[(1-\beta+\sigma)g_{\bar{A}} + \rho(1-\beta-\alpha)\right]^2 + 4\delta(1-\beta-\alpha)}}{2\delta(1-\beta-\alpha)} \tag{22}$$

In the steady-state equilibrium, the optimal allocation of effort to production and education involves a tradeoff between the generation of output for current consumption and capital accumulation versus investing time on education, which, by enhancing the technology in practice, leads to a higher level of output productivity in future production. In turn, the optimal effort allocation has a direct bearing on the optimal size of the technology gap.

In order that u^* and s^* exist as interior solutions, certain regularity conditions must be satisfied. Firstly, it is straightforward to verify that u^* is less than 1, and that u^* is greater than zero if the following condition is satisfied:

$$(\rho+\delta)(1-\beta-\alpha)+(1-\beta+\sigma)g_{\bar{A}} > 1 \tag{23}$$

Next, in order that the transversality condition $\lim_{t\to\infty} e^{-\rho t}\theta_1(t)K(t) = 0$ is satisfied, we require s^* to be less than 1. This requires that $\left(1-u^*\right)$ satisfies a regularity condition which we derive in Appendix C. The regularity condition is easily understood in the special case when $\alpha = 0$, i.e. when the effects of learning-by-doing externality effects in expanding the technology frontier are absent, so that the economy's appropriate technology frontier is simply the global technology frontier. The optimal savings rate in (21) simplifies to

$$s^* = \frac{\beta g_{\bar{A}}}{\rho(1-\beta)+\sigma g_{\bar{A}}} \tag{24}$$

so that the regularity condition that must be satisfied in order for s^* to be less than 1 is $\rho(1-\beta)+(\sigma-\beta)g_{\bar{A}}>0$. Essentially, this condition places a restriction on the minimum value of σ, so that if the coefficient of intertemporal substitutability σ^{-1} is too high, there is no steady-state equilibrium in our model.

4. Comparative Statics Analysis

In this section, we examine the variation of the optimal allocation of production effort u^*, the optimal technology gap G^* and the optimal savings rate s^* with respect to the system parameters in the economy. The details of the comparative statics results are presented in Appendix B and summarized in Table 2 below.

Table 2: Partial Derivatives of Optimal Effort u^*, Optimal Technology Gap G^* and Optimal Savings Rate s^*

	Optimal Work Effort u^*		Optimal Technology Gap G^*		Optimal Savings Rate s^*	
X	$\frac{\partial u^*}{\partial X}$	$\frac{\partial^2 u^*}{\partial X^2}$	$\frac{\partial G^*}{\partial X}$	$\frac{\partial^2 G^*}{\partial X^2}$	$\frac{\partial s^*}{\partial X}$	$\left.\frac{\partial^2 s^*}{\partial X^2}\right\|_{\alpha=0}$
Intertemporal rate of discount ρ	+	–	+	+	–	+
Coefficient of risk aversion σ	+	–	+	+	–	+
Share of capital in output β	–	–	–[1]	–[1]	+[2]	+
Growth in frontier technology $g_{\bar{A}}$	–	+	+	–	+[1]	–
Effectiveness of education δ	+	–	–	+	0[1]	0
Learning-by-doing effects α	–	+	–	+	+	+

Notes: 1. Sufficient condition: $\alpha = 0$ 2. Sufficient condition: $u^* > \frac{\alpha}{1+\alpha}$

First, we note that the partial derivatives of u^* and G^* (except in the case of β) are monotonic with respect to each of the parameters in the economic system. However, the signs of most of the partial derivatives of s^* depend on the specific configuration of the economic parameters[13]. It is only when $\alpha = 0$ (and by continuity, for sufficiently small α) that all the partial derivatives of s^* (and G^* with respect to β) are monotonic in their arguments. The case where $\alpha = 0$ corresponds to the situation where there are no learning-by-doing externality effects, through $K(t)^{\alpha}$, that helps to expand the appropriate technology frontier. The result implies that when learning-by-doing externality effects are important (i.e. $\alpha > 0$) in determining the appropriate technology frontier, different optimal technology gaps and savings behaviors are consistent with the same steady-state growth rates.

To understand the comparative statics results, we note the complementary nature between the accumulation of *physical* capital and the accumulation of *human* capital, as reflected in the effect of educational effort in closing the technology gap. Since educational effort is $\left(1-u^*\right)$ in the balanced growth equilibrium, the first-order partial derivatives of $\left(1-u^*\right)$ with respect to ρ, σ and α have the same sign as the first-order partial derivatives for s^*. When sufficient conditions hold, this is also true in the case of β and $g_{\bar{A}}$. The optimal saving rate s^* is invariant with respect to δ. Therefore, a higher optimal rate of savings is generally associated with a higher level of educational effort. In turn, this is reflected in a smaller technology gap, which is brought about by a lower rate of intertemporal discount, a lower degree of risk aversion or a larger share of physical capital in production. We consider these effects in turn.

When the discount rate ρ is larger, current consumption is more valuable than future consumption, so less effort is devoted to education, and more of the output generated is

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[13] As shown in Appendix B, sufficient conditions can be found so that the first-order derivatives of s^* are monotonic. We derive the second-order partial derivatives of s^* when $\alpha = 0$.

consumed rather than accumulated to aid future production. Similarly, a larger σ reflects a lower degree of intertemporal substitutability of consumption (in other words, a weaker desire to smooth consumption). Therefore, current consumption is again more valuable, and a smaller amount of goods produced is saved and less emphasis is placed on technological progress. Lastly, if physical capital commands a larger share β in the production function, a larger proportion of goods produced will be accumulated for future production (i.e. s^* is higher) and more effort will be devoted to speed up the diffusion of technology and expand the technology in practice.

Next, if the effectiveness of education δ in facilitating technology diffusion improves, the optimal technology gap is smaller even though a smaller amount of effort is devoted to education in the steady state equilibrium. Finally, when the exogenous rate of expansion of the global technology frontier $g_{\bar{A}}$ increases, the optimal technology gap is wider even though a greater proportion of effort is exerted to accelerate the expansion of the technology in use. Furthermore, by enhancing the marginal product of physical capital, this also induces a higher steady-state equilibrium savings rates, when α is sufficiently small. This result echoes the point made in Nelson and Phleps (1966) that the "Golden Rule growth requires more education the more technologically progressive is the economy" (pp 74, Footnote 4).

The comparative statics results suggest that an optimal growth policy is a balanced approach to the accumulation of physical and human capital. In our model, a more rapid pace of physical capital accumulation generates stronger learning-by-doing externality effects. By accelerating the expansion of the appropriate technology frontier, a faster rate of physical capital accumulation also increases the effectiveness of educational effort in advancing the technology in practice, since this impact is directly proportional to the size of the technology gap.

5. Conclusion

In this paper, we present a model that links education and the accumulation of physical capital with technological progress. Our aim is to understand the endogenous determination of the allocation of effort to education and production, and the consequent impact on optimal savings rate and the optimal technology gap. Our analysis indicates that education and accumulation of human capital are complementary to the accumulation of physical capital. Moreover, they are intertwined with technological progress through separate impacts on technology diffusion and the expansion of the appropriate technology frontier.

If the accumulations of physical and human capital are complementary in facilitating technological progress, a balanced investment approach in both types of capital would act to maintain the marginal product of both capital and labor as the technology in use expands at an optimal rate. The recent debate on the total productivity performance of many Asian economies (as discussed in Young (1992, 1995), Krugman (1994) and Hsieh (2002)) highlights the emphasis placed on the accumulation of physical capital in emerging Asian economies. A possible linkage of the present paper to this discussion is that if a rapid pace of physical capital accumulation generates an externality effect in pushing out the appropriate technology frontier, then the contribution of physical capital accumulation to the growth process may possibly be understated. Furthermore, a normative implication is that these Asian countries could have performed even better if there had been greater investment in education.

Although the debate continues over the relative contribution of technological progress and factor accumulation to economic growth (see Temple (1999b) and Easterly and Levine (2001)), we hope that our analysis has contributed to a better understanding of the complementary roles of investment in human capital (through education) and accumulation of physical capital in facilitating technological progress.

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Appendix A

Proof of Proposition 1: Differentiating (11) with respect to *t*, we obtain $\frac{\dot{\theta}_1(t)}{\theta_1(t)} = -\sigma \frac{\dot{c}(t)}{c(t)}$.

Next, using (13) and (14), we obtain

$$\theta_2(t) = \frac{1}{\delta}(1-\beta)c(t)^{-\sigma}\left[\frac{K(t)}{u(t)}\right]^{\beta}\left[\frac{A(t)}{\eta K(t)^{\alpha}\bar{A}(t) - A(t)}\right] \tag{A.1}$$

which upon substitution into (13), leads to $g_c(t)$ in (15). Equation (16) follows from the definition of $g_K(t) = \left[Y(t) - c(t)\right]K(t)^{-1}$. Lastly, re-arranging equation (9), and utilizing the definition of $\tilde{A}(t)$ in (6), $\left[\dot{A}(t) + \delta\left[1-u(t)\right]A(t)\right] = \delta\left[1-u(t)\right]\eta K(t)^{\alpha}\bar{A}(t)$, so that multiplying both sides by $\exp\left[\delta\int_0^t\left[1-u(z)\right]dz\right]$, the expansion path of $A(t)$ is solved, as in (17). Q.E.D.

Proof of Proposition 2: With optimal constant at u^*, $A(t)$ in (17) simplifies to

$$\begin{aligned} A(t) = &\left[A(0) - \frac{\delta\left(1-u^*\right)}{g_{\bar{A}} + \delta\left(1-u^*\right)}\eta K(0)^{\alpha}\bar{A}(0)\right]e^{-\delta\left(1-u^*\right)t} + \frac{\delta\left(1-u^*\right)}{g_{\bar{A}} + \delta\left(1-u^*\right)}\eta K(t)^{\alpha}\bar{A}(0)e^{g_{\bar{A}}t} \\ &- \frac{\delta\left(1-u^*\right)}{g_{\bar{A}} + \delta\left(1-u^*\right)}\eta\alpha\bar{A}(0)\int_0^t\left\{g_K(y)K(y)^{\alpha}\exp\left[g_{\bar{A}}y - \delta\left(1-u^*\right)(t-y)\right]\right\}dy \end{aligned} \tag{A.2}$$

Since the rate of capital accumulation, g_K, is constant in the steady-state equilibrium, we utilize (17) to simplify the third term on the right-hand side of (A.2). Rewriting, we have

$$\begin{aligned} A(t) = &\left[A(0) - \frac{\delta\left(1-u^*\right)}{g_{\bar{A}} + \delta\left(1-u^*\right)}\eta K(0)^{\alpha}\bar{A}(0)\right]e^{-\delta\left(1-u^*\right)t} + \frac{\delta\left(1-u^*\right)}{g_{\bar{A}} + \delta\left(1-u^*\right)}\eta K(t)^{\alpha}\bar{A}(0)e^{g_{\bar{A}}t} \\ &-\alpha g_K\frac{A(t) - A(0)e^{-\delta\left(1-u^*\right)t}}{g_{\bar{A}} + \delta\left(1-u^*\right)} \end{aligned} \tag{A.3}$$

As $t \to \infty$, the term $e^{-\delta\left(1-u^*\right)t}$ tends to zero, leading to the result in (18) Q.E.D.

Proof of Proposition 3: By definition, the steady-state rate of capital accumulation g_K is given in (16). Substituting this and (18) into (15), we derive the steady-state growth rate of consumption as

$$g_c = \frac{1}{\sigma}\left[-\rho + \left[g_K + \frac{c(t)}{K(t)}\right]\left\{\beta + \alpha(1-\beta)\left[\frac{1-u^*}{u^*}\right]\left[1 + \frac{\delta\left(1-u^*\right)}{g_{\bar{A}} + \alpha g_K}\right]\right\}\right] \tag{A.4}$$

Since g_c is a constant, inspection of the right-hand side of (A.4) implies that the ratio $\frac{c(t)}{K(t)}$ must also be constant along the balanced growth path; hence, g_c equals g_K. Since $g_K + \frac{c(t)}{K(t)}$ is constant in (A.4), it follows that $A(t)\left[\frac{u^*}{K(t)}\right]^{1-\beta}$ in (16) must be constant too. Upon differentiation, we obtain $g_c = g_K = g_{\tilde{A}}/(1-\beta) = (g_{\bar{A}} + \alpha g_K)/(1-\beta)$, This leads to the results in Proposition 3. Q.E.D.

Proof of Proposition 4: Using the results in Proposition 2 and Proposition 3, it is straightforward to derive the optimal technology gap, $G^* \equiv \frac{\tilde{A}(t)}{A(t)} - 1$. Next, substituting the results in Proposition 3 into Equation (A.4), we obtain

$$c(t) = \frac{g_{\bar{A}}}{1-\beta-\alpha}\left\{\frac{\left[\rho(1-\beta-\alpha) + \sigma g_{\bar{A}}\right]u^*}{\left[\beta u^* + \alpha(1-\beta)\left(1-u^*\right)\right]g_{\bar{A}} + \alpha\delta\left(1-u^*\right)^2} - 1\right\}K(t) \tag{A.5}$$

From Equation (16), we obtain $Y(t) = g_K K(t) + c(t)$, so that we have

$$Y(t) = \frac{g_{\bar{A}}}{1-\beta-\alpha}\left\{\frac{\left[\rho(1-\beta-\alpha) + \sigma g_{\bar{A}}\right]u^*}{\left[\beta u^* + \alpha(1-\beta)\left(1-u^*\right)\right]g_{\bar{A}} + \alpha\delta\left(1-u^*\right)^2}\right\}K(t) \tag{A.6}$$

so that we obtain $s^* \equiv 1 - c(t)/Y(t)$ in (21). We can simplify Equation (12) to yield $(1-\beta)c(t)^{-\sigma}\left[\frac{K(t)}{u^*}\right]^{\beta} = \theta_2(t)G^*$. Taking log on both sides, and differentiating with respect to t, this yields $\dot{\theta}_2(t)/\theta_2(t) = (\beta-\sigma)g_{\bar{A}}/(1-\beta-\alpha)$, which, upon substituting into (14) and utilizing (11) and (A.1), leads to a characterization of the optimal effort allocation :

$$\delta(1-\beta-\alpha)\left(1-u^*\right)^2 + \left[\rho(1-\beta-\alpha) + (1-\beta+\sigma)g_{\bar{A}}\right]\left(1-u^*\right) - g_{\bar{A}} = 0 \tag{A.7}$$

The positive root of the above equation is the solution for $\left(1-u^*\right)$, which then leads to the result in (22). Q.E.D.

Appendix B

The partial derivatives of u^* are

Intertemporal rate of discount:

$$\frac{\partial u^*}{\partial \rho} = \frac{(1-\beta-\alpha)\left(1-u^*\right)^2}{g_{\bar{A}} + \delta(1-\beta-\alpha)\left(1-u^*\right)^2} > 0 \tag{B.1}$$

$$\frac{\partial^2 u^*}{\partial \rho^2} = \frac{-2(1-\beta-\alpha)\left(1-u^*\right)g_{\bar{A}}}{\left[g_{\bar{A}} + \delta(1-\beta-\alpha)\left(1-u^*\right)^2\right]^2}\frac{\partial u^*}{\partial \rho} < 0$$

Coefficient of risk aversion:

$$\frac{\partial u^*}{\partial \sigma} = \frac{g_{\bar{A}}\left(1-u^*\right)^2}{g_{\bar{A}} + \delta(1-\beta-\alpha)\left(1-u^*\right)^2} > 0 \tag{B.2}$$

$$\frac{\partial^2 u^*}{\partial \sigma^2} = \frac{-2\left(1-u^*\right)\left(g_{\bar{A}}\right)^2}{\left[g_{\bar{A}} + \delta(1-\beta-\alpha)\left(1-u^*\right)^2\right]^2}\frac{\partial u^*}{\partial \sigma} < 0$$

Share of capital in output:

$$\frac{\partial u^*}{\partial \beta} = \frac{-\left(1-u^*\right)^2\left[\delta\left(1-u^*\right)+\rho+g_{\bar{A}}\right]}{g_{\bar{A}} + \delta(1-\beta-\alpha)\left(1-u^*\right)^2} < 0 \tag{B.3}$$

$$\frac{\partial^2 u^*}{\partial \beta^2} = \left\{\frac{2\left(1-u^*\right)\left[\delta^2(1-\beta-\alpha)\left(1-u^*\right)^3 + g_{\bar{A}}\left[2\delta\left(1-u^*\right)+\rho+g_{\bar{A}}\right]\right]}{\left[g_{\bar{A}} + \delta(1-\beta-\alpha)\left(1-u^*\right)^2\right]^2}\right\}\frac{\partial u^*}{\partial \beta} < 0$$

Growth in global frontier technology:

$$\frac{\partial u^*}{\partial g_{\bar{A}}} = \frac{-(1-\beta-\alpha)\left(1-u^*\right)^2\left[\rho+\delta\left(1-u^*\right)\right]}{g_{\bar{A}}\left[g_{\bar{A}} + \delta(1-\beta-\alpha)\left(1-u^*\right)^2\right]} < 0 \tag{B.4}$$

$$\frac{\partial^2 u^*}{\partial g_{\bar{A}}^2} = \frac{-2\left(1-u^*\right)\left[(1+\sigma-\beta)g_{\bar{A}} + \delta(1-\beta-\alpha)\left(1-u^*\right)\right]}{\left[g_{\bar{A}} + \delta(1-\beta-\alpha)\left(1-u^*\right)^2\right]^2}\frac{\partial u^*}{\partial g_{\bar{A}}} > 0$$

Effectiveness of education:

$$\frac{\partial u^*}{\partial \delta} = \frac{(1-\beta-\alpha)\left(1-u^*\right)^3}{g_{\bar{A}} + \delta(1-\beta-\alpha)\left(1-u^*\right)^2} > 0 \tag{B.5}$$

$$\frac{\partial^2 u^*}{\partial \delta^2} = \frac{-2(1-\beta-\alpha)\left(1-u^*\right)^2\left[2g_{\bar{A}} + \delta(1-\beta-\alpha)\left(1-u^*\right)^2\right]}{\left[g_{\bar{A}} + \delta(1-\beta-\alpha)\left(1-u^*\right)^2\right]^2}\frac{\partial u^*}{\partial \delta} < 0$$

Learning-by-doing externality effects:

$$\frac{\partial u^*}{\partial \alpha} = \frac{-\rho\left(1-u^*\right)^2}{g_{\bar{A}} + \delta(1-\beta-\alpha)\left(1-u^*\right)^2} < 0 \tag{B.6}$$

$$\frac{\partial^2 u^*}{\partial \alpha^2} = \frac{-2\rho g_{\bar{A}}\left(1-u^*\right)}{\left[g_{\bar{A}} + \delta(1-\beta-\alpha)\left(1-u^*\right)^2\right]^2}\frac{\partial u^*}{\partial \alpha} > 0$$

The partial derivatives of G^* are

Intertemporal rate of discount:

$$\frac{\partial G^*}{\partial \rho} = \frac{g_{\bar{A}}(1-\beta)}{\delta(1-\beta-\alpha)\left(1-u^*\right)^2}\frac{\partial u^*}{\partial \rho} > 0 \tag{B.7}$$

$$\frac{\partial^2 G^*}{\partial \rho^2} = \frac{2g_{\bar{A}}(1-\beta)(1-\beta-\alpha)\left(1-u^*\right)}{\left[g_{\bar{A}} + \delta(1-\beta-\alpha)\left(1-u^*\right)^2\right]^2}\frac{\partial u^*}{\partial \rho} > 0$$

Coefficient of risk aversion:

$$\frac{\partial G^*}{\partial \sigma} = \frac{g_{\bar{A}}(1-\beta)}{\delta(1-\beta-\alpha)\left(1-u^*\right)^2}\frac{\partial u^*}{\partial \sigma} > 0 \tag{B.8}$$

$$\frac{\partial^2 G^*}{\partial \sigma^2} = \frac{2(1-\beta)\left(1-u^*\right)\left(g_{\bar{A}}\right)^2}{\left[g_{\bar{A}} + \delta(1-\beta-\alpha)\left(1-u^*\right)^2\right]^2}\frac{\partial u^*}{\partial \sigma} > 0$$

Share of capital in output:

$$\frac{\partial G^*}{\partial \beta} = \frac{g_{\bar{A}}\left[\alpha g_{\bar{A}} - (1-\beta-\alpha)\left(1-u^*\right)\left[\delta(1-\beta-\alpha)\left(1-u^*\right) + (1-\beta)(\rho+g_{\bar{A}})\right]\right]}{\delta(1-\beta-\alpha)^2\left(1-u^*\right)\left[g_{\bar{A}} + \delta(1-\beta-\alpha)\left(1-u^*\right)^2\right]} \tag{B.9}$$

$$\frac{\partial^2 G^*}{\partial \beta^2} = G^*\left\{\frac{1}{G^{*2}}\left[\frac{\partial G^*}{\partial \beta}\right]^2 + \frac{1}{\left(1-u^*\right)^2}\left[\frac{\partial u^*}{\partial \beta}\right]^2 + \frac{1}{\left(1-u^*\right)}\frac{\partial^2 u^*}{\partial \beta^2} + \frac{\left[2(1-\beta)-\alpha\right]\alpha}{(1-\beta)^2(1-\beta-\alpha)^2}\right\}$$

Sufficient condition for $\frac{\partial G^*}{\partial \beta} < 0$ and $\frac{\partial^2 G^*}{\partial \beta^2}$ <0 is that α is sufficiently close to zero.

Growth in global frontier technology:

$$\frac{\partial G^*}{\partial g_{\bar{A}}} = \frac{(1-\beta)\left[(1-\beta+\sigma)g_{\bar{A}} + \delta(1-\beta-\alpha)\left(1-u^*\right)\right]}{\delta(1-\beta-\alpha)\left[g_{\bar{A}} + \delta(1-\beta-\alpha)\left(1-u^*\right)^2\right]} > 0 \tag{B.10}$$

$$\frac{\partial^2 G^*}{\partial g_{\bar{A}}^2} = \frac{2(1-\beta)\left(1-u^*\right)\left[\delta(1-\beta-\alpha)\left(1-u^*\right) + (1+\sigma-\beta)g_{\bar{A}}\right]}{\left[g_{\bar{A}} + \delta(1-\beta-\alpha)\left(1-u^*\right)^2\right]^2}\frac{\partial u^*}{\partial g_{\bar{A}}} < 0$$

Effectiveness of education:

$$\frac{\partial G^*}{\partial \delta}=\frac{-(1-\beta)\left(g_{\bar{A}}\right)^2}{\delta^2(1-\beta-\alpha)\left(1-u^*\right)\left[g_{\bar{A}}+\delta(1-\beta-\alpha)\left(1-u^*\right)^2\right]}<0 \tag{B.11}$$

$$\frac{\partial^2 G^*}{\partial \delta^2}=\frac{\left(g_{\bar{A}}\right)^2(1-\beta)}{\delta^3(1-\beta-\alpha)^2\left(1-u^*\right)^5}\left\{\left[2\delta+\left(1-u^*\right)\right]\frac{\partial u^*}{\partial \delta}-\delta\left(1-u^*\right)\frac{\partial^2 u^*}{\partial \delta^2}\right\}>0$$

Learning-by-doing externality effects:

$$\frac{\partial G^*}{\partial \alpha}=\frac{G^*}{\left(1-u^*\right)}\frac{\partial u^*}{\partial \alpha}<0 \tag{B.12}$$

$$\frac{\partial^2 G^*}{\partial \alpha^2}=\frac{G^*}{\left(1-u^*\right)}\left\{\frac{\partial^2 u^*}{\partial \alpha^2}+\frac{2}{\left(1-u^*\right)}\left[\frac{\partial u^*}{\partial \alpha}\right]^2\right\}>0$$

The partial derivatives of s^* are

Intertemporal rate of discount:

$$\begin{aligned}\frac{\partial s^*}{\partial \rho}=&-\frac{\alpha\left[(1-\beta)g_{\bar{A}}+\delta\left(1-u^{*2}\right)\right]}{\left[\rho(1-\beta-\alpha)+\sigma g_{\bar{A}}\right]u^{*2}}\frac{\partial u^*}{\partial \rho}\\&-\frac{(1-\beta-\alpha)\left\{\left[\beta u^*+\alpha(1-\beta)\left(1-u^*\right)\right]g_{\bar{A}}+\alpha\delta\left(1-u^*\right)^2\right\}}{\left[\rho(1-\beta-\alpha)+\sigma g_{\bar{A}}\right]^2u^*}<0\end{aligned} \tag{B.13}$$

$$\left.\frac{\partial^2 s^*}{\partial \rho^2}\right|_{\alpha=0}=\frac{2(1-\beta)^2\beta g_{\bar{A}}}{\left[\rho(1-\beta)+\sigma g_{\bar{A}}\right]^3}>0$$

Coefficient of risk aversion:

$$\begin{aligned}\frac{\partial s^*}{\partial \sigma}=&\frac{-\alpha\left[(1-\beta)g_{\bar{A}}+\delta\left(1-u^{*2}\right)\right]}{\left[\rho(1-\beta-\alpha)+\sigma g_{\bar{A}}\right]u^{*2}}\frac{\partial u^*}{\partial \sigma}\\&-\frac{g_{\bar{A}}\left\{\left[\beta u^*+\alpha(1-\beta)\left(1-u^*\right)\right]g_{\bar{A}}+\alpha\delta\left(1-u^*\right)^2\right\}}{\left[\rho(1-\beta-\alpha)+\sigma g_{\bar{A}}\right]^2u^*}<0\end{aligned} \tag{B.14}$$

$$\left.\frac{\partial^2 s^*}{\partial \sigma^2}\right|_{\alpha=0}=\frac{2\beta\left(g_{\bar{A}}\right)^3}{\left[\rho(1-\beta)+\sigma g_{\bar{A}}\right]^3}>0$$

Share of capital in output:

$$\begin{aligned}\frac{\partial s^*}{\partial \beta}=&-\frac{\alpha\left[(1-\beta)g_{\bar{A}}+\delta\left(1-u^{*2}\right)\right]}{\left[\rho(1-\beta-\alpha)+\sigma g_{\bar{A}}\right]u^{*2}}\frac{\partial u^*}{\partial \beta}\\&+\frac{\left[\rho(1-\beta-\alpha)+\sigma g_{\bar{A}}\right]\left[u^*(1+\alpha)-\alpha\right]+\rho\left\{g_{\bar{A}}\left[\beta u^*+\alpha(1-\beta)\left(1-u^*\right)\right]+\alpha\delta\left(1-u^*\right)^2\right\}}{\left[\rho(1-\beta-\alpha)+\sigma g_{\bar{A}}\right]^2u^*}\end{aligned} \tag{B.15}$$

Sufficient condition for $\frac{\partial s^*}{\partial \beta}>0$ is $u^*>\frac{\alpha}{1+\alpha}$

$$\left.\frac{\partial^2 s^*}{\partial \beta^2}\right|_{\alpha=0} = \frac{2\left(\rho + \sigma g_{\bar{A}}\right)\rho g_{\bar{A}}}{\left[\rho(1-\beta) + \sigma g_{\bar{A}}\right]^3} > 0$$

Growth in frontier technology:

$$\frac{\partial s^*}{\partial g_{\bar{A}}} = -\frac{\alpha\left[(1-\beta)g_{\bar{A}} + \alpha\delta\left(1-u^{*2}\right)\right]}{\left[\rho(1-\beta-\alpha)+\sigma g_{\bar{A}}\right]u^{*2}}\frac{\partial u^*}{\partial g_{\bar{A}}} + \frac{\left\{\rho(1-\beta-\alpha)\left[\beta u^* + \alpha(1-\beta)\left(1-u^*\right)\right] - \alpha\sigma\delta\left(1-u^*\right)^2\right\}}{\left[\rho(1-\beta-\alpha)+\sigma g_{\bar{A}}\right]^2 u^*} \tag{B.16}$$

Sufficient condition for $\frac{\partial s^*}{\partial g_{\bar{A}}} > 0$ is that α is sufficiently close to zero.

$$\left.\frac{\partial^2 s^*}{\partial g_{\bar{A}}^{\ 2}}\right|_{\alpha=0} = \frac{-2\sigma\rho\beta(1-\beta)}{\left[\rho(1-\beta) + \sigma g_{\bar{A}}\right]^3} < 0$$

Effectiveness of education:

$$\frac{\partial s^*}{\partial \delta} = -\frac{\alpha\left[(1-\beta)g_{\bar{A}} + \delta\left(1-u^{*2}\right)\right]}{\left[\rho(1-\beta-\alpha)+\sigma g_{\bar{A}}\right]u^{*2}}\frac{\partial u^*}{\partial \delta} + \frac{\alpha\left(1-u^*\right)^2}{\left[\rho(1-\beta-\alpha)+\sigma g_{\bar{A}}\right]u^*} \tag{B.17}$$

$$\left.\frac{\partial^2 s^*}{\partial \delta^2}\right|_{\alpha=0} = 0$$

Learning-by-doing externality effects:

$$\frac{\partial s^*}{\partial \alpha} = -\frac{\alpha\left[(1-\beta)g_{\bar{A}} + \delta\left(1-u^{*2}\right)\right]}{\left[\rho(1-\beta-\alpha)+\sigma g_{\bar{A}}\right]u^{*2}}\frac{\partial u^*}{\partial \alpha} + \frac{\left(1-u^*\right)\left[(1-\beta)g_{\bar{A}} + \delta\left(1-u^*\right)\right]}{\left[\rho(1-\beta-\alpha)+\sigma g_{\bar{A}}\right]u^*} + \frac{\rho\left\{g_{\bar{A}}\left[\beta u^* + \alpha(1-\beta)\left(1-u^*\right)\right] + \alpha\delta\left(1-u^*\right)^2\right\}}{\left[\rho(1-\beta-\alpha)+\sigma g_{\bar{A}}\right]^2 u^*} > 0 \tag{B.18}$$

$$\left.\frac{\partial^2 s^*}{\partial \alpha^2}\right|_{\alpha=0} = -2\frac{(1-\beta)g_{\bar{A}} + \delta\left(1-u^{*2}\right)}{\left[\rho(1-\beta)+\sigma g_{\bar{A}}\right]u^{*2}}\left.\frac{\partial u^*}{\partial \alpha}\right|_{\alpha=0} + \frac{\rho}{\rho(1-\beta)+\sigma g_{\bar{A}}}\left.\frac{\partial s^*}{\partial \alpha}\right|_{\alpha=0} > 0$$

Appendix C

In order that the steady-state savings rate in (21) is less than 1, we require that $\left(1-u^{*}\right)$ satisfy the condition

$$\left[\beta u^{*}+\alpha(1-\beta)\left(1-u^{*}\right)\right]g_{\bar{A}}+\alpha\delta\left(1-u^{*}\right)^{2}<\left[\rho(1-\beta-\alpha)+\sigma g_{\bar{A}}\right]u^{*} \tag{C.1}$$

Utilizing Equation (A.7) which defines the optimal effort allocated to education $\left(1-u^{*}\right)$, we can eliminate the term $\left(1-u^{*}\right)^{2}$ in (C.1) to obtain the equivalent condition:

$$\Omega\left(1-u^{*}\right)<\Phi \tag{C.2}$$

where $\Omega\equiv\rho(1-\beta-2\alpha)(1-\beta-\alpha)-\left[\alpha(1-\beta)(\alpha+\beta)-(1-\beta-\alpha)(\sigma-\beta)+\alpha\sigma\right]g_{\bar{A}}$

$\Phi\equiv\rho(1-\beta-\alpha)^{2}+\left[(1-\beta-\alpha)(\sigma-\beta)+\alpha\right]g_{\bar{A}}$.

Since

$$\Phi-\Omega=\alpha\left[\rho(1-\beta-\alpha)+\left[(1-\beta)(\alpha+\beta)+(1+\sigma)\right]g_{\bar{A}}\right]>0$$

we can rewrite (C.2) to obtain $-\Omega\left[u^{*}\right]<\Phi-\Omega$, so that if $\Omega\geq 0$, the condition in (C.2) would be satisfied. On the other hand, if $\Omega<0$, then we require that $u^{*}<1-\dfrac{\Phi}{\Omega}$, which is satisfied if $\Phi\geq 0$. Therefore, the condition in (C.2) is non-binding if (i) $\Phi>\Omega\geq 0$ or (ii) $\Phi\geq 0>\Omega$. A necessary and sufficient condition for either (i) or (ii) to hold is that $\sigma\geq\beta$. It is only when (iii) $0>\Phi>\Omega$ that the condition in (C.2) is binding. A necessary (but not sufficient) condition for (iii) to occur is that $\sigma<\beta$. The other necessary conditions for (iii) to occur are $\sigma<\beta-\dfrac{\alpha}{1-\beta-\alpha}$ and $g_{\bar{A}}<\dfrac{\rho(1-\beta-\alpha)^{2}}{(1-\beta-\alpha)(\beta-\sigma)-\alpha}$.