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Rational and boundedly rational behavior in a binary choice sender-receiver game

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Abstract

We consider a signalling game in which a population of receivers decide on the outcome by majority rule, sender and receivers have conflicting interests, and there is uncertainty about both players' types. We model players rationality along the lines of recent findings in behavioral game theory. We characterize the structure of the equilibria in the reduced game so obtained. We find that all pure strategy equilibria are consistent with successful attempts to mislead the receivers, and relate them to the message bin Laden sent on the eve of the 2004 US Presidential elections. The same result holds if we allow for some uncertainty about the sign of the correlation between the sender's and the receivers' payoffs.

1 Introduction

The weekend before 2004 US presidential elections the Al-Jazeera TV network aired a videotaped address from Osama bin Laden (OBL henceforth) whose content reached also American people. Apparently the message was not about the impending elections: [1]

> People of America this talk of mine is for you and concerns the ideal way to prevent another Manhattan, and deals with the war and its causes and results.

[1] A full transcript of the speech can be found on the following website http://english.aljazeera.net/NR/exeres/79C6AF22-98FB-4A1C-B21F-2BC36E87F61F.htm

The timing of the message and its very ending though, leave little doubts about the fact that it was indeed meant to influence the American voters. Quoting again from the message:

> [...] The wise man doesn't squander his security, wealth and children for the sake of the liar in the White House. In conclusion, I tell you in truth, that your security is not in the hands of Kerry, nor Bush, nor al-Qaida. No. Your security is in your own hands.

It is clearly not an easy task to ascertain whether or not the message had a significant impact on the vote. However, several commentators agreed on the conclusion that the message actually helped re-electing George W. Bush. [2] For example, quoting from Krauthammer [8]:

> With the election hanging in the balance, the campaign awaited some improbable development to tip the scale. Re-enter OBL. By reminding us 9/11 and the war on terrorism, OBL invoked the only thing that could trump Iraq - and save the President. His chilling reappearance reminded us of our peril, put Iraq in perspective and played precisely to the President's success and strength - success and strength that he so squandered in Baghdad. Bin Laden was never one to remotely understand the American mind. He spectacularly misjudged 9/11 - and he pulled his nemesis over the finish line.

In an experimental setting (conducted in late September 2004), Cohen et al. [4] show that shifting the public's attention onto themes such as terrorism and the recollection of the facts of 9/11 would tend to substantially shift votes from Kerry to Bush. We take this result as evidence that OBL's sortie in the closing days of the campaign might have had a significant impact on vote, helping President Bush's re-election, and formulate the following working hypothesis: the message was indeed targeting the U.S. voters and had an effect in the outcome of the presidential elections. In other words, we stick to the interpretation that the face value of OBL's words boils down to opposing George W. Bush and hence "endorsing" John F. Kerry, and that voters definitely responded to those words. We then propose a model

[2] The defeated candidate himself later ascribed a decisive importance to the videotape for the presidential race. Senator Kerry indeed said that (See Nagourney [11]) "... the attacks of Sept. 11 were the "central deciding thing" in his contest with President Bush and that the release of an OBL videotape the weekend before Election Day had effectively erased any hope he had of victory."

that deals with the strategic rationale of this message and its ultimate effect on the polls. We want to understand the following issues: did OBL really misunderstand the American voters? Was his implied stand on U.S. election (opposing Bush / endorsing Kerry) a lie or a truthful revelation of his view? Did the American voters believe him? OBL's message is an example of an attempt to influence the result of an election by a third-party using a costless message. The number of cases in which third parties endorse (more or less explicitly) one candidate is surely large. An example is the Economist's strong position against Mr. Berlusconi's fitness to lead Italy before both the 2001 and the 2006 Italian general elections. Another is sub-commander Marcos' "endorsement", on national television in May 2006, of one of the candidates for the incoming Presidential elections in Mexico. Nevertheless, OBL's role and his message's timing, content and echo in the media, make it a rather novel event in the political scenery. We think it is interesting to understand why such an announcement was publicly made, and how the American voters reacted to it through the vote.

To this end, we consider a game *à la* Crawford and Sobel [6], played between one sender and a population of receivers. The sender has private information about a state variable while the receivers take a binary action upon observing a public message from the sender. Majority rule determines the choice implemented, which, together with the prevailing state, determines the payoffs. Players have opposite interests. Under this setup in any equilibrium the sender uses only uninformative messages, and the receivers choose the action that, ex ante, maximizes their expected payoff. This being the case, OBL's message could not have shifted votes in favor of Bush. To account for this possibility, we introduce a structural model of interaction between fully and behaviorally rational players that resembles Crawford [5] analysis of Allies' successful attempt to mislead the Germans about where they would land on D-Day in 1944. In this setup, both receivers and sender are uncertain about the degree of sophistication of their opponent. The equilibria of this incomplete information game are studied as a function of the parameters governing this uncertainty.

The strategic interaction between OBL and the U.S. voters we have in mind can be explained by the following example: the state of the world represents the exogenous seriousness of OBL's imminent threat to U.S. national security. In particular, we may think of it as OBL's "health" condition in relation to his (and al-Qaida's) ability to plan and carry out another strike relatively quickly. [3] In a "low" state, OBL is capable of organizing another

[3] Ample speculation about the state of OBL's kidney disorder has been publicly made.

attack within few months after the election, and the U.S. voters prefer an aggressive foreign policy. Therefore they see Bush as the better option. In a "high" state, OBL's health represents a serious threat to his ability to organize a new strike, and the U.S. citizens prefer a foreign policy more inclined towards diplomacy, that would probably cost them less. In this case they see Kerry as a better alternative.

As for OBL, we make the assumption that his payoffs are in sharp contrast with the U.S. citizens. In particular, in a "high" state he prefers Bush, because this option is more likely to turn him into a martyr. Conversely, in a "low" state, OBL prefers Kerry because the time of diplomacy may be long enough to allow him (and al-Qaida) to prepare for future terroristic activities.

We want to stress that at the time of the message, candidates positions were spelled out clearly and no time for changes was available. In other words, positions on foreign policy issues were given to the voters. In this respect, we believe it is a sensible assumption to focus on the game between OBL and the U.S. voters.

The type of uncertainty is modeled along the lines of Crawford [5], as this approach seems quite natural given the idiosyncratic nature of the events. In fact, unlike similar applications of strategic information transmission, such as Sobel [13] and Benabou and Laroque [1], the novelty of the scenario does not leave room for learning or reputation considerations. The same lack of repetition cannot even allow for a test of equilibrium play in mixed strategies. [4] Therefore we assume that players are characterized by their expectations about how the others will play, and choose their best plan of action accordingly. The class of expectations we consider has been shown to be a well-grounded starting point for interpreting how people react to novel situations. Examples include first-round behavior in experimental guessing games, as in Nagel [10] and estimated types in a series of two-person normal-form games in Costa-Gomes et al. [9]. The related model of cognitive hierarchy developed by Camerer et al. [2] also shows good fit to empirical data from various experimental games.

Finally, our model relates also to Farrell and Gibbons [7], where the sender faces two different receivers. We differenciate from them since no receiver can be excluded from observing the message, and the final outcome is determined by majority rule, so that, in particular, the sender's payoff is not additively separable in the actions of different groups within the population

[4] This approach can be used for sports, were data can be (and in fact are) gathered. See for example Chiappori et al. [3], Palacios-Huerta [12] and Walker and Wooders [14].

$$
\begin{array}{cc}
 & P1 \\
P2 \begin{array}{c} I(U)=1 \\ I(U)=0 \end{array} & \begin{array}{|c|}\hline b,-b \\ \hline 0,\quad 0 \\ \hline \end{array} \\
 & S_1
\end{array}
\qquad\qquad
\begin{array}{cc}
 & P1 \\
P2 \begin{array}{c} I(U)=1 \\ I(U)=0 \end{array} & \begin{array}{|c|}\hline 0,\quad 0 \\ \hline 1,-1 \\ \hline \end{array} \\
 & S_2
\end{array}
$$

Figure 1: Payoffs given $P2$'s action and the state of nature, with $b > 0$.

of receivers. Majority rule per se does not change the outcome with respect to the one receiver's case. It's the addition of uncertainty about players' degree of sophistication that makes our model, we believe, interesting.

The remainder of the paper is organized as follows: Section 2, discusses the model in its general form. Section 3 extends the model to account for uncertainty about the payoffs correlation between sender and receiver. Section 4 concludes.

2 The model

Suppose there are two states of the world, $\{S_1, S_2\} = S$, whose probabilities are, respectively, p and $1-p$. There are two sets of players, a sender, $P1$, who knows the state of the world, and a population (continuum) of receivers, $P2$, who do not know it. $P1$ sends a message $m \in M = \{m_1, m_2\}$ to $P2$, which in turn chooses an action $a \in A = \{U, D\}$. Let $I(U)$ be the indicator function that takes value 1 (resp. 0) if the fraction of receivers that choose action U is larger (resp. smaller) than 1/2. The payoffs for both players depend on the state of nature and on the action of the receivers and are reported in figure 1, where the parameter $b > 0$.

A (pure) strategy for the sender is a function from the state of the world space (S) to the message space (M). A (pure) strategy for the receivers is a function from the message space (M) to the action space (A). (m_1, m_2) denotes the pure strategy that corresponds to sending message m_1 if the state is S_1 and sending the message m_2 if the state is S_2. (U, D) denotes the strategy that corresponds to choosing action U if message is m_1 and action D if message is m_2.

In the remainder of this section we describe first the equilibria of the game with rational players (which represents the benchmark case), as a function of the parameters p and b, and then focus on the equilibria of the reduced game where some uncertainty about types of players is introduced along the lines of Crawford [5]. Attention will be restricted to the cases when $pb = 1 - p$ which resembles a close election, one that, probably, best resembles the 2004 Presidential election. [5]

One final remark before we proceed with the analysis: in the sub-game of the vote between receivers, we restrict attention to symmetric un-dominated strategies, which basically means that receivers choose the action that maximizes their expected payoff were they pivotal to the election and that they mix with the same probability when indifferent.

If the game is played by standard players, then we know that in equilibrium no message can reveal any extra information. As a result, receivers choose an action so as to maximize expected payoffs given the prior probabilities on the states of nature.

Lemma 1 *Suppose the players are rational and $pb = 1 - p$: then the Nash equilibria of the game are described by*

$$\begin{aligned} &[\alpha_1(m_1, m_1) + \alpha_2(m_1, m_2) + \alpha_2(m_2, m_1) + \alpha_4(m_2, m_2)] \\ &[\beta_1(U, U) + \beta_2(U, D) + \beta_2(D, U) + \beta_4(D, D)] \\ &\textit{with } \alpha_i \geq 0,\ \alpha_1 + 2\alpha_2 + \alpha_4 = 1, \textit{ and } \beta_i \geq 0,\ \beta_1 + 2\beta_2 + \beta_4 = 1 \end{aligned}$$

where the first row reads: $P1$ mixes among (U, U), (U, D), (D, U), and (D, D) with probability α_1, α_2, α_2, and α_4. The second row reads in a similar fashion. [6]

If the game is played by standard players, then in equilibrium messages will reveal no information, which otherwise can be exploited by $P2$. No prediction regarding the outcome can then be based on the model when $pb = 1 - p$ and the players are fully rational. If one takes the claim that the message had effects seriously, as we do, then an enlarged model is needed. Therefore we introduce a structural model of uncertainty about types of players that draws from behavioral economics. Players are unsure about the degree of rationality of their opponents. They only know that they can belong to the family of either mortal or sophisticated players. Mortal players'

[5] We believe this to be the more interesting case since it is the one where information to the receiver may have an impact on her decision. In a previous draft, we also considered the case where $pb > 1 - p$.

[6] For the class of equilibria where $P1$ sends only one message, a specification of $P2$'s beliefs for off equilibrium messages would complete the description of the equilibria. We just skip this part as those equilibria do not arise in the reduced game we are about to analyze.

strategies are determined by iteration of best responses, starting from a naive definition of types of senders. In particular, we assume that some senders always choose (m_1, m_2). We call them *truth-tellers*. Some receivers believe that senders are only truth-tellers and therefore optimally choose (U, D). These receivers are called *believers*. Some other senders (which we call *liars*) think the population of receivers is made only of *believers*, and therefore optimally choose (m_2, m_1). Last, some receivers (which we call *inverters*) believe the senders are all liars, and therefore optimally choose (D, U). Note that we could increase the level of sophistication of the players arbitrarily, but after the last iteration we would end up with types of mortal players whose behavior is identical to those just introduced. Therefore there is no loss in generality in considering only the two pairs of mortal types: truth-tellers believers, and liars inverters.

Let x_t, x_l, and x_s denote the probability a sender is, respectively, a truth-teller, a liar, or a sophisticated type. Similarly, y_b, y_i, and y_s denote, the population's share of believers, inverters and sophisticated types. Sophisticated players are the standard rational players, and their behavior comes from equilibrium considerations, given that the structure of the game (payoffs, strategy and uncertainty about types) is common knowledge.

Lemma 2 *The sophisticated receiver's best response is given by*

$$\begin{array}{ll} (U, D) & \text{if } x_s(1 + w_1 - w_2) > 1 - 2x_t \\ (D, U) & \text{if } x_s(1 + w_1 - w_2) < 1 - 2x_t \end{array}$$

Proof. A (pivotal) sophisticated receiver will choose the action maximizing expected payoff. Upon observing message m_1, the probability of being in state S_1 and S_2 is proportional to, respectively, $p(x_t + w_1 x_s)$ and $(1 - p)(x_l + w_2 x_s)$. Therefore, since $pb = 1 - p$ U is optimal under m_1 if

$$x_t + w_1 x_s > x_l + w_2 x_s,$$

i.e. if

$$x_s(1 + w_1 - w_2) > 1 - 2x_t.$$

Similarly, upon observing message m_2, the probability of being in state S_1 and S_2 is proportional to, respectively, $x_l + (1 - w_1)x_s$ and $x_t + (1 - w_2)x_s$. Therefore U is optimal under m_2 if

$$x_l + (1 - w_1)x_s > x_t + (1 - w_2)x_s,$$

i.e. if

$$1 - 2x_t > (1 + w_1 - w_2)x_s.$$

■

Therefore the only pure strategy best responses for the sophisticated receiver are (U, D) or (D, U). Let β denote the probability of the sophisticated receiver playing (U, D) and $1 - \beta$ the probability of playing (D, U).

Lemma 3 *the sophisticated sender's best response to* $\beta(U, D)+(1-\beta)(D, U)$ *is given by*

$$\begin{array}{ll} (m_1, m_2) & \text{if } \beta y_s < \dfrac{1}{2} - y_b \\ (m_2, m_1) & \text{if } \beta y_s > \dfrac{1}{2} - y_b \end{array}$$

Proof. The sophisticated sender is facing a distribution of strategies given by

$$(y_b + \beta y_s)(U, D) + (y_i + (1 - \beta)y_s)(D, U). \tag{1}$$

Thus, in state S_1, message m_1 is best if

$$y_b + \beta y_s < \tfrac{1}{2} < y_i + (1 - \beta)y_s, \tag{2}$$

while m_2 is best if

$$y_i + (1 - \beta)y_s < \tfrac{1}{2} < y_b + \beta y_s. \tag{3}$$

Last, indifference between m_1 and m_2 is obtained whenever neither (2) nor (3) hold. Now observe that

- $y_b + \beta y_s + y_i + (1 - \beta)y_s = 1$;
- In state S_2 message m_i is best whenever in state S_1 m_{3-i} is best.

Therefore the only possible configurations are (m_1, m_2) and (m_2, m_1), and the former holds whenever

$$y_b + \beta y_s < \frac{1}{2},$$

while the latter holds whenever the opposite (strict) inequality holds. ■

We are now in the position to characterize the pure strategy (sequential) Nash equilibria of the game, under the assumption that the sender is a sophisticated player.

Proposition 1 *The structure of the pure strategies (sequential) Nash equilibria in weekly dominant strategies is described in the following:*

$$
\begin{array}{lll}
EQ1: & [(m_1, m_2), (U, D)] & \text{if and only if } y_i > \frac{1}{2} \text{ and } x_l < \frac{1}{2} \\
EQ2: & [(m_2, m_1), (U, D)] & \text{if and only if } y_i < \frac{1}{2} \text{ and } x_t > \frac{1}{2} \\
EQ3: & [(m_1, m_2), (D, U)] & \text{if and only if } y_b < \frac{1}{2} \text{ and } x_l > \frac{1}{2} \\
EQ4: & [(m_2, m_1), (D, U)] & \text{if and only if } y_b > \frac{1}{2} \text{ and } x_t < \frac{1}{2}
\end{array}
$$

Further, for the sophisticated sender, all the above equilibria imply that, with majority rule, the outcome favorable to the sender will result, whatever the state.

Proposition 1 shows that the (sophisticated) sender can always take advantage of the uncertainty, whether it is about the receivers or about himself. In fact, whenever the population share of one mortal player is larger than 1/2 the sophisticated sender can just induce the most preferred outcome by fooling the majority of mortal players. On the other hand, when the probability that the sender is either mortal player is larger than 1/2, the sophisticated sender can just mimic the strategy of the least likely mortal and then fool even the sophisticated receivers, who nevertheless are acting optimally. Observe that if instead the sender is mortal then the outcome will favor a liar in EQ2 and EQ3 and it will favor the receivers in EQ2 and EQ4. The situation is reversed if the sender is a truth-teller.

Thus our framework offers a simple way to assess, for example, whether the final comment in Krauthammer [8] that OBL actually played against himself through his message is actually realistic. The argument we can give is based on the plausibility of the implied restrictions on the parameters that describe the ex-ante probabilities regarding the types. Indeed, without attaching any specific interpretation to states and actions, the pure-strategy equilibria we obtain all entail that voters are fooled by a sophisticated sender whatever the state. Whether such conclusion has a sort of "conspiracy theory" flavor we leave it the reader to decide.

It remains to consider the case where $\frac{1}{2} > \max\{x_t, x_l, y_b, y_i\}$. The equilibrium is in mixed strategy, as a quick glance at the best responses' functions suggests. (Figures 2 and 3 report the best responses for, respectively, $P1$ and $P2$ as a function of the parameters governing the uncertainty about players' types.) Let α_1, α_2, α_3, and α_4 denote the probability of, respectively, (m_1, m_1), (m_1, m_2), (m_2, m_1), and (m_2, m_2). $P2$ is indifferent

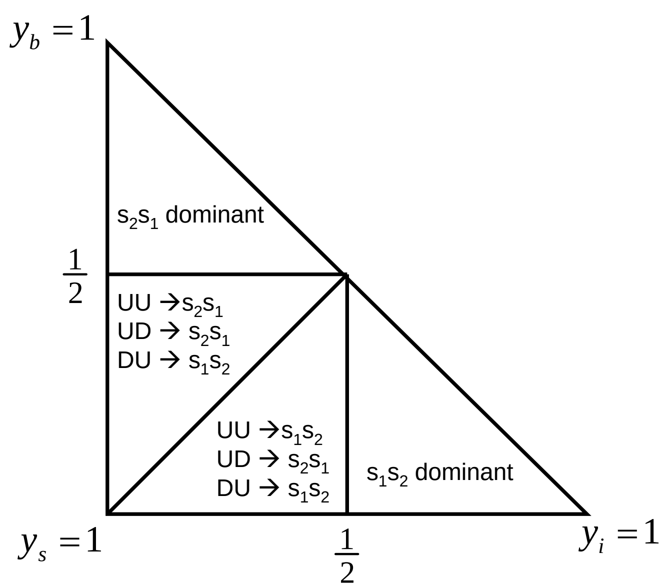

Figure 2: Best responses to pure strategies for sophisticated sender in the reduced game.

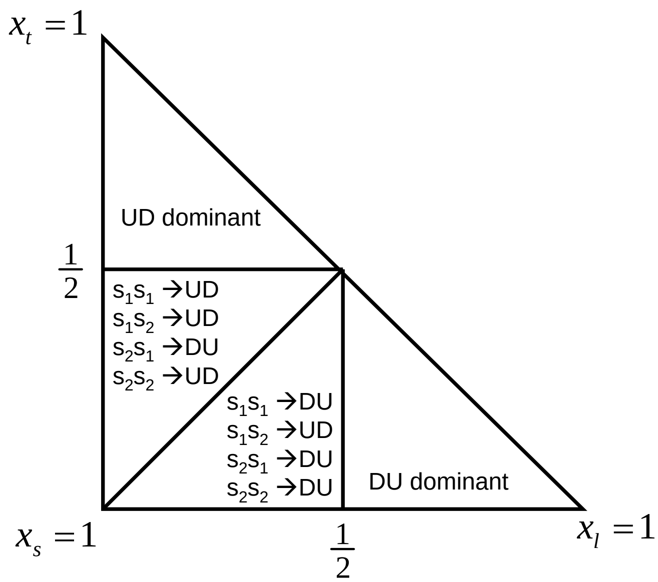

Figure 3: Best responses to pure strategies for sophisticated receiver in the reduced game.

between (U, D) and (D, U) if and only if

$$2x_t + x_s(\alpha_1 + 2\alpha_2 + \alpha_4) = 2x_l + x_s(\alpha_1 + 2\alpha_3 + \alpha_4),$$

which requires

$$\alpha_2 = \frac{x_l - x_t}{x_s} + \alpha_3.$$

The associated expected payoff will be

$$\begin{aligned}
2x_l + x_s(\alpha_1 + 2\alpha_3 + \alpha_4) &= \\
2x_l + x_s(\alpha_1 + \alpha_3 + \alpha_4 + \alpha_3) &= \\
2x_l + x_s(1 - \alpha_2 + \alpha_3) &= \\
2x_l + x_s(1 - \frac{x_l - x_t}{x_s} - \alpha_3 + \alpha_3) &= \\
2x_l + x_s(1 - \frac{x_l - x_t}{x_s}) &= \\
2x_l + x_s - x_l + x_t &= 1,
\end{aligned}$$

which corresponds to the expected payoff of choosing either (U, U) or (D, D). Thus when $P2$ is indifferent between (U, D) and (D, U), $P2$ is indifferent between any of the strategies available.

Similarly, let β_1, β_2, β_3, and β_4 denote, respectively, the probability of playing (U, U), (U, D), (D, U), and (D, D). $P2$ is indifferent between (m_1, m_2) and (m_2, m_1) if and only if

$$2y_b + y_s(\beta_1 + 2\beta_2 + \beta_4) = 2y_i + y_s(\beta_1 + 2\beta_3 + \beta_4)$$

which requires

$$\beta_2 = \frac{y_i - y_b}{y_s} + \beta_3$$

The associated expected payoff will be

$$\begin{aligned}
2y_i + y_s(\beta_1 + 2\beta_3 + \beta_4) &= \\
2y_i + y_s(\beta_1 + \beta_3 + \beta_4 + \beta_3) &= \\
2y_i + y_s(1 - \beta_2 + \beta_3) &= \\
2y_i + y_s(1 - \frac{y_i - y_b}{y_s} - \beta_3 + \beta_3) &= \\
2y_i + y_s(1 - \frac{y_i - y_b}{y_s}) &= \\
2y_i + y_s - y_i + y_b &= 1,
\end{aligned}$$

which corresponds to the expected payoff of choosing either (m_1, m_1) or (m_2, m_2). Thus when $P1$ is indifferent between (m_1, m_2) and (m_2, m_1), $P1$ is indifferent between any of the strategies available.

Proposition 2 *Assume* $\frac{1}{2} \geq \max\{x_t, x_l, y_i, y_b\}$. *The mixed strategies (sequential) equilibria are given by*

$$
\begin{aligned}
&[\alpha_1(m_1, m_1) + \alpha_2(m_1, m_2) + \alpha_3(m_2, m_1) + \alpha_4(m_2, m_2)] \\
&\qquad [\beta_1(U, U) + \beta_2(U, D) + \beta_3(D, U) + \beta_4(D, D)] \\
&\textit{with} \quad \alpha_2 = \frac{x_l - x_t}{x_s} + \alpha_3, \ \ \alpha_i \geq 0 \ \textit{ for all } i, \ \sum_i \alpha_i = 1 \\
&\textit{with} \quad \beta_2 = \frac{y_i - y_b}{y_s} + \beta_3, \ \ \beta_j \geq 0 \ \textit{ for all } j, \ \sum_j \beta_j = 1
\end{aligned}
$$

Observe that $\alpha_2 \gtreqless \alpha_3$ if and only if $x_l \gtreqless x_t$. That is, whenever there are more liars than truth-tellers, the sophisticated player needs to put higher weight on (m_1, m_2) (truth-telling) in the mixed strategies equilibrium. Moreover, this weight increases with the difference between the measure of these two types of mortal players. Similarly, $\beta_2 \gtreqless \beta_3$ if and only if $y_i \gtreqless y_b$.

Notice also that in all cases there is an ex-ante exact fifty-fifty expected split of the population between those that choose U and those that choose D, as a result of the use of the mixed strategy by the sophisticated receivers. So the outcome of the underlying election is undetermined: it will be the result of the discrepancy between β and the actual fractions of sophisticated receivers choosing either alternative (the actual realization of the mixed strategy over the population). This case resembles somehow the equilibrium in the full rationality case. In fact, as the probabilities of mortal players go to zero, this equilibrium converges to the mixed strategy equilibrium under full rationality. (This is not coincidental, as the Nash equilibria in the reduced game are continuous in the players' types' parameter space.)

To recap, we find that the sophisticated sender can achieve the most preferred outcome unless the probability of sophisticated players is sufficiently large. We find this result very interesting especially in relation to the tendency of mass media in shaping the public opinion on the (ir)rationality of some actors.

In the next section we extend the model to allow for uncertainty about the correlation between sender's and receivers' payoffs.

		$P1$	$P1$	$P1$	$P1$
$P2$	$I(U)=1$	$1,-1$	$1,1$	$0,0$	$0,\ \ 0$
	$I(U)=0$	$0,\ \ 0$	$0,0$	$1,1$	$1,-1$
		S_1	S_2	S_3	S_4

Figure 4: Player's payoffs under each state of the world.

3 What if $P1$ may have common interests with $P2$?

In this section we investigate the case in which $P1$ may have interests that, under some states of the world, coincide with those of $P2$. Assume therefore that the states of the world are $S = \{S_1, S_2, S_3, S_4\}$. Let p_i be the probability of state i. $P1$ can still send two different messages, which now we label as $M = \{m_1, m_2\}$, and a strategy for $P1$ is a function from S to M. $P2$ choose an action, $a \in A = \{U, D\}$ and a strategy for $P2$ is a function from the set of M to A. Let $I(U)$ be the indicator function that takes value 1 (resp. 0) if the fraction of receivers that choose action U is larger (resp. smaller) than 1/2. Players' payoffs as a function of the states and of $P2$'s actions are reported in figure 4. Compared to the previous model, we have added uncertainty about the payoffs structure but we have also restricted attention to the case where both alternatives give the same payoff to both players. In what follows we will consider the main case where

$$p_1 + p_2 = p_3 + p_4, \tag{4}$$

which, again, corresponds to a close election. We shall suppose further that either one of these two inequalities hold

$$p_2 + p_3 > p_1 + p_4 \tag{5}$$

$$p_2 + p_3 < p_1 + p_4 \tag{6}$$

Inequality (5) means that $P1$ is more likely to have positively correlated payoffs with $P2$ (we call him a *friend*), while inequality (6) means $P1$ is more likely to have negatively correlated payoffs with $P2$ (we call him an *enemy*). This model is similar to Sobel [13], for there is uncertainty on whether the sender is a friend or enemy, as well as on which action is best for the receiver.

Given the nature of M and S, a strategy for $P1$ can induce posterior probabilities on S that generate on of the following partitions of S:

$$S^0 = \{S\}$$
$$S^1 = \{\{S_i\}, \{S_{-i}\}\}$$
$$S^2 = \{\{S_i, S_h\}, \{S_j, S_k\}\}$$

S^0 corresponds to totally uninformative messages. S^1 corresponds to messages where one state, S_i, is revealed with probability 1. S^2 corresponds to messages where where two states, S_i and S_h, are separated from the other two, S_j and S_k.

Definition 1 *A partially revealing equilibrium is an equilibrium that induces either S^1 or S^2. A pooling equilibrium is an equilibrium that induces S^0.*

Observe that there cannot be partially revealing equilibria that induce S^1, since $P1$ has an incentive to deviate from the proposed strategy when the states are either S_1 or S_4.

Proposition 3 *Suppose condition (5) holds, then there exists an equilibrium in which $P1$ credibly informs $P2$ of his preferred states.*

The above is the analogous to Theorem 1 in Sobel [13], and it says that if the probability of $P1$ being a friend is larger than $1/2$, then there is an equilibrium in which the most preferred action for $P1$ is revealed with probability 1.

The next sections consider the equilibria of the reduced game among sophisticated players once types uncertainty is introduced.

3.1 $P1$ is an enemy

Consider first the case where $P1$ is an enemy. Conditions (4) and (6) imply that $p_4 > p_2$ and $p_1 > p_3$. Consider a baseline type of sender choosing (m_1, m_2, m_1, m_2), which means this type sends the message m_1 if $s \in \{S_1, S_3\}$, and send the message m_2 if $s \in \{S_2, S_4\}$. [7] This corresponds again to a *truth-telling* type, in the sense that the strategy separates the states where the sender prefers U to the states where the sender prefers D. By iterating best responses we have the following types' pairs:

[7] Adopting alternative definitions of senders' mortal behaviors at the first iteration of the best response results in more calculations without altering the main point of the paper.

truth-teller	(m_1, m_2, m_1, m_2)	*believer*	(U, D)
liar	(m_2, m_1, m_2, m_1)	*inverter*	(D, U)

where, again, (U, D) means that the receiver chooses action U if message is m_1, and action D if message is m_2. Observe that further iterating best responses would not return different types from those above. Therefore there is no loss in generality in stopping with the above characterization of types. Let x_t, x_l and x_s denote the probability of $P1$'s types. Similarly, y_b, y_l and y_s denote the population share of $P2$'s types.

It remains to define the behavior of the sophisticated players. Let w_i denote the probability that the sophisticated $P1$ sends message m_1 in state S_i.

Lemma 4 *The sophisticated receiver's best response is given by*

$$\begin{array}{ll} (U,D) & \text{if } (x_t + w_1 x_s)p_1 + (x_l + w_2 x_s)p_2 > (x_t + w_3 x_s)p_3 + (x_l + w_4 x_s)p_4 \\ (D,U) & \text{if } (x_t + w_1 x_s)p_1 + (x_l + w_2 x_s)p_2 < (x_t + w_3 x_s)p_3 + (x_l + w_4 x_s)p_4 \end{array}$$

Proof. A (pivotal) sophisticated receiver will choose the action maximizing expected payoff. Upon observing message m_1, the probability of being in state S_i is proportional to $(x_t + w_i x_s)p_i$ if $i \in 1, 3$ and to $(x_l + w_i x_s)p_i$ if $i \in 2, 4$; similarly, the probability of being in state S_i conditional to observing message m_2 is proportional to $(x_l + (1 - w_i)x_s)p_i$ if $i \in 1, 3$ and to $(x_t + (1 - w_i)x_s)p_i$ if $i \in 2, 4$. Using these probabilities, it is straightforward to verify that (U, D) is optimal if and only if $(x_t + w_1 x_s)p_1 + (x_l + w_2 x_s)p_2 > (x_t + w_3 x_s)p_3 + (x_l + w_4 x_s)p_4$ while (D, U) is optimal if and only if the opposite inequality holds. ■

Let β and $1 - \beta$ denote, respectively, the probability a sophisticated receiver chooses (U, D) and (D, U).

Lemma 5 *The best response for a sophisticated $P1$ is given by:*

$$\begin{array}{l} (m_1, m_2, m_1, m_2) \text{ if } \beta y_s < \dfrac{1}{2} - y_b \\ (m_2, m_1, m_2, m_1) \text{ if } \beta y_s > \dfrac{1}{2} - y_b \end{array}$$

Proof. The proof is virtually identical to the proof of lemma 5. Let $\mu(A|m)$ the share of receivers choosing action A after observing message m. We have that

$$
\begin{aligned}
\mu(D|m_1) =& y_i + (1-\beta)y_s \\
\mu(U|m_1) =& 1 - \mu(D|m_1) \\
\mu(D|m_2) =& y_b + \beta y_s \\
\mu(U|m_2) =& 1 - \mu(D|m_2)
\end{aligned}
$$

Consider first the states $\{S_1, S_3\}$, in which the sender's preferred outcome is D. Message m_1 is preferred if

$$\mu(D|m_1) > \frac{1}{2} > \mu(D|m_2), \tag{7}$$

whereas m_2 is preferred if

$$\mu(D|m_2) > \frac{1}{2} > \mu(D|m_1). \tag{8}$$

Last, the sophisticated is indifferent between m_1 and m_2 if neither (7) nor (8) hold. Observe that $\mu(D|m_1)+\mu(D|m_2) = 1$, which means that conditions (7) and (8) can be simplified to, respectively, $\mu(D|m_1) > \frac{1}{2}$ and $\mu(D|m_1) < \frac{1}{2}$.

Now consider the states $\{S_2, S_4\}$, in which the sender's preferred outcome is U. Message m_1 is preferred if

$$\mu(U|m_1) > \frac{1}{2} > \mu(U|m_2), \tag{9}$$

whereas m_2 is preferred if

$$\mu(U|m_2) > \frac{1}{2} > \mu(U|m_1). \tag{10}$$

Last, the sophisticated is indifferent between m_1 and m_2 if neither (9) nor (10) hold. Observe that $\mu(U|m_1) + \mu(U|m_2) = 1$, which means that conditions (9) and (10) can be simplified to, respectively, $\mu(U|m_1) > \frac{1}{2}$ and $\mu(U|m_1) < \frac{1}{2}$.

By inspecting these conditions, it can be seen that m_1 is optimal in $\{S_1, S_3\}$ if and only if m_2 is optimal in $\{S_2, S_4\}$. Thus the only possible configurations in pure strategies for the sophisticated $P1$ are either (m_1, m_2, m_1, m_2) or (m_2, m_1, m_2, m_1). The former works when $\mu(D|m_1)\frac{1}{2}$, i.e. when

$$y_i + (1-\beta)y_s > \frac{1}{2},$$

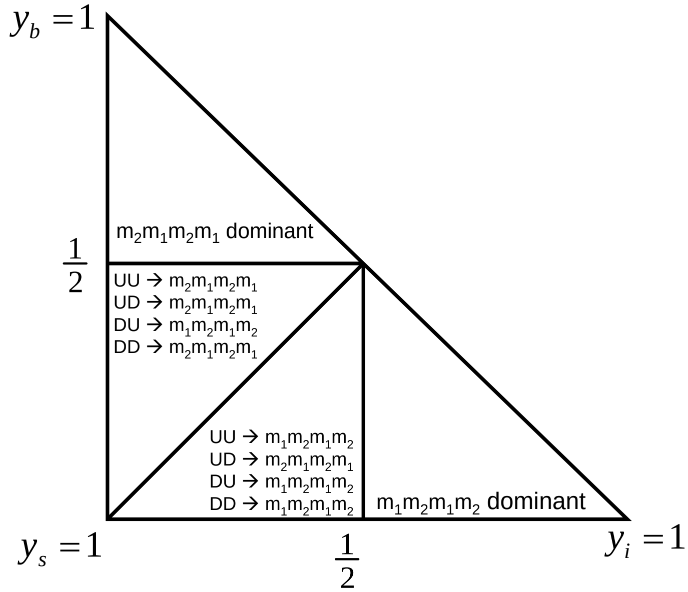

Figure 5: Best responses to pure strategies for sophisticated sender in the reduced game.

which holds if and only if

$$\beta y_s < \frac{1}{2} - y_b.$$

■

This allows us to characterize $P1$'s best response to any pure strategy from $P2$. Results similar to those in the base game arise. (See figure (5).)

In particular, for (m_1, m_2, m_1, m_2) (resp. (m_2, m_1, m_2, m_1)) to be a dominant strategy $y_i > \frac{1}{2}$ (resp. $y_b > \frac{1}{2}$) is needed. Therefore, no dominant strategy exists when $\frac{1}{2} > \max\{y_i, y_b\}$, but the best response is either (m_1, m_2, m_1, m_2) or (m_2, m_1, m_2, m_1).

Consider now $P2$. Let α, $1 - \alpha$ denote the probability that sophisticated $P1$ chooses, respectively, (m_1, m_2, m_1, m_2) and (m_2, m_1, m_2, m_1).

We are now in the position to characterize the pure strategy (sequential) Nash equilibria of the game.

Proposition 4 *Suppose conditions (4) and (6) hold. Then the pure strategy* *(sequential) Nash equilibria of the reduced game between sophisticated sender and receivers are given by*

$$
\begin{array}{lll}
EQ1: & [(m_1, m_2, m_1, m_2), (U, D)] & \textit{if and only if } y_i > \frac{1}{2} \textit{ and } x_l < \frac{1}{2} \\
EQ2: & [(m_1, m_2, m_1, m_2), (D, U)] & \textit{if and only if } y_b < \frac{1}{2} \textit{ and } x_l > \frac{1}{2} \\
EQ3: & [(m_2, m_1, m_2, m_1), (U, D)] & \textit{if and only if } y_i < \frac{1}{2} \textit{ and } x_t > \frac{1}{2} \\
EQ4: & [(m_2, m_1, m_2, m_1), (D, U)] & \textit{if and only if } y_b > \frac{1}{2} \textit{ and } x_t < \frac{1}{2}
\end{array}
$$

Further, for the sophisticated sender, all the above equilibria imply that, with majority rule, the outcome favorable to the sender will result, whatever the state.

Note that the equilibria correspond exactly to those in the initial case. Consider first the cases where one mortal receiver represents the majority of the population. In this case the sophisticated sender can take advantage of them and fool the majority of voters. (Equilibria 1 and 4). Now consider the case when the sophisticated receiver is pivotal. Since there is a probability larger than 1/2 of a mortal receiver, the sophisticated receiver will mimic the behavior of the other mortal and then fool the sophisticated receivers. Thus the logic is exactly the same as in the previous case. The only difference is that it is not true anymore that the receivers loose anytime the most preferred outcome of the sender is implemented. In particular, in states S_2 and S_3, letting the sophisticated sender determine the outcome helps the receivers as well.

It remains to find the equilibria in mixed strategies. Remember that these equilibria arise whenever $\frac{1}{2} > \max\{x_t, x_l, y_b, y_i\}$. Let α_i denote the probability the sophisticated sender chooses message m_1 in state S_i. Let β_i denote the probability the sophisticated receiver chooses action U if message m_i is received. Therefore the conditions for indifference among all the possible strategies for $P1$ and $P2$ are given by, respectively

$$
\begin{aligned}
x_s(p_3\alpha_3 + p_4\alpha_4 - p_1\alpha_1 - p_2\alpha_2) &= (p_3 - p_1)(x_t - x_l) \\
y_s(\beta_2 - \beta_1) &= y_b - y_i
\end{aligned}
$$

The role of the sophisticated players in the mixed strategies equilibrium is the same as in the standard model: they need to put more probability on choosing the action that is consistent with the mortal type whose probability is smaller. Moreover, we can see that the mixed strategy for the sophisticated receiver is the same as in the standard model, since the receiver only needs to condition the action to the message received. For the sophisticated sender there are more options available, but the nature of the mixed strategy equilibrium is the same as in the standard model.

An example of a possible mixed strategy equilibrium is given by

Proposition 5 *Let conditions (4) and (6) hold. Assume* $\frac{1}{2} > \max\{x_t, x_l, y_b, y_i\}$. *The following pair of strategies is a mixed strategy equilibrium*

$$[\alpha(m_1, m_2, m_1, m_2) + (1-\alpha)(m_2, m_1, m_2, m_1)] \qquad \alpha = \frac{1}{2} + \frac{x_l - x_t}{2x_s}$$
$$[\beta(U, D) + (1-\beta)(D, U)] \qquad \beta = \frac{1}{2} + \frac{y_b - y_i}{2y_s}$$

where β *denotes the probability of* (U, D), *and* α *denotes the probability of* (m_1, m_2, m_1, m_2).

Proof. $P1$ is indifferent between the two strategies if and only if

$$(p_2+p_3)[y_i+(1-\beta)y_s]-(p_1+p_4)[y_b+uy_s] = (p_2+p_3)[y_b+uy_s]-(p_1+p_4)[y_i+(1-\beta)y_s]$$
$$y_i + (1-\beta)y_s = y_b + uy_s$$
$$y_i + y_s - y_b = 2uy_s$$
$$\beta = \frac{1}{2} + \frac{y_i - y_b}{2y_s}$$

Observe first that β is well defined, since $\frac{1}{2} > \max\{y_b, y_i\}$. Moreover, that value of β makes the sender indifferent between any of the strategies available. In fact, by sending message m_1 the probability of $P2$ playing U is $y_b + uy_s = \frac{1}{2}$. Similarly, by sending message m_2, the probability of $P2$ playing U is $y_i + (1-\beta)y_s = \frac{1}{2}$. Therefore, none of $P1$'s pure strategies can make him better off than mixing between (m_1, m_2, m_1, m_2) and (m_2, m_1, m_2, m_1).

Similarly, $P2$ is indifferent between (U, D) and (D, U) if and only if

$$(p_1+p_4)[(1-\alpha)x_s+x_l]+(p_2+p_3)[wx_s+x_t] = (p_1+p_4)[wx_s+x_t]+(p_2+p_3)[(1-\alpha)x_s+x_l]$$
$$(p_1 + p_4 - p_2 - p_3)[(1-\alpha)x_s + x_l - wx_s - x_t] = 0$$
$$(1-\alpha)x_s + x_l - wx_s - x_t = 0$$
$$x_s + x_l - x_t = 2wx_s$$
$$\alpha = \frac{1}{2} + \frac{x_l - x_t}{2x_s}$$

Observe first that α is well defined since $\frac{1}{2} > \max\{x_t, x_l\}$. Moreover, that value of α makes the receiver indifferent between any of the strategies available. In fact, $(1-\alpha)x_s + x_l = \frac{1}{2} = wx_s + x_t$, so that the expected payoff of either (U, D) or (D, U) is $\frac{1}{2}$ which is also what $P2$ can guarantee himself by choosing either (D, D) or (U, U). ■

Therefore we can conclude that the introduction of uncertainty about correlation in payoffs between sender and receiver does not change the main result if there is at least a 50 % chance that the sender is indeed an enemy. We now turn to the opposite case, that is when the sender is more likely to be a friend, which we analyze in the next section.

3.2 P1 is a friend

Consider now the case where $P1$ is a friend (i.e. when (5) holds.) This means that $p_2 > p_4$, and $p_3 > p_1$. If we assume again that the most naive sender adopts (m_1, m_2, m_1, m_2), iteration of best responses generates the following set of distinct pairs of players' types:

truth-teller : (m_1, m_2, m_1, m_2)	*believer* (D, U)

As before, let w_i denote the probability the sophisticated sender chooses m_1 in state S_i.

Lemma 6 *Let conditions (4) and (5) hold. The sophisticated sender's best response is given by*

$$(U, D) \text{ if } (p_3 - p_1)x_t < x_s(p_1w_1 + p_2w_2 - p_3w_3 - p_4w_4)$$
$$(D, U) \text{ if } (p_3 - p_1)x_t > x_s(p_1w_1 + p_2w_2 - p_3w_3 - p_4w_4)$$

The proof is identical to the one for lemma 4 and therefore is omitted.

Similarly, let β, $1-\beta$ denote, respectively, the probability of choosing (U, D), and (D, U).

Lemma 7 *Let conditions (4) and (5) hold. $P1$ best responses are given by*

$$(m_1, m_2, m_1, m_2) \text{ if } \beta y_s < \frac{1}{2}$$
$$(m_2, m_1, m_2, m_1) \text{ if } \beta y_s > \frac{1}{2}$$

The proof is identical to the one for lemma 5 and therefore is omitted.

We are now ready to characterize the (pure strategy) Nash equilibria of this game.

Proposition 6 *Let conditions (4) and (5) hold. The pure strategies Nash equilibria of the game are given by*

$$\begin{array}{lll} EQ1: & [(m_2, m_1, m_2, m_1), (U, D)] & \textit{if } \frac{1}{2} > \max\{x_t, y_b\} \\ EQ2: & [(m_1, m_2, m_1, m_2), (D, U)] & \textit{for any parameters' value.} \end{array}$$

Further, for the sophisticated sender, all the above equilibria imply that, with majority rule, the outcome favorable to the sender will result, whatever the state.

This results shows that when payoffs the probability that the sender is a friend is higher than 1/2, the coordination on the type of messages received depends upon the distribution of mortal types only when the sophisticated sender chooses to behave differently from the mortal sender. Notice that, again, the outcome using the majority rule implies the sender wins in every state in any pure strategy equilibria.

4 Conclusion

We have analyzed a model with a sender interacting with a population of receivers. The sender has information that is relevant for the payoffs of both players. Receivers take a binary choice and the final outcome is determined by majority rule. In addition, there is uncertainty (similar to Crawford [5]) about the sophistication of both sender and receivers. Our main result is that pure strategy equilibria of the game share the property that if the sender is sophisticated the outcome will be favorable for him whatever the prevailing state. This result carries through also in the case when there are states in which the interests of the sender are not at odds with those of the receiver. Using this framework to analyze Osama bin Laden's videotape message to the American public on the eve of the 2004 U.S. Presidential elections, we can raise some skepticism about the view according to which "he pulled his nemesis over the finish line" [8].

We think that the game we propose is a parsimonious representation of the conflicting interests between U.S. voters and OBL alongside whit a hidden true motivation for the message on OBL's part. Nevertheless, a possible

[8] See Krauthammer [8]

alternative (and equally interesting) description of what happened can be devised if one believes that bin Laden's message's ultimate intended audience was not (or not only) the American public but, for example, (radical) groups within the islamic communities worldwide. In that case, Farrell and Gibbons [7] model of cheap talk with two audiences would be the natural starting point for the analysis, which can be the object of further research on the matter.

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