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# The Implications of Technology Networks on Diffusion and Economic Growth

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## Citation

LEUNG, Hing-Man. The Implications of Technology Networks on Diffusion and Economic Growth. (2002). 08-2002, 1-24. Research Collection School Of Economics.

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# **The Implications of Technology Networks on Diffusion and Economic Growth**

**George H M Leung**

July 2002

Paper No. 8-2002

# THE IMPLICATIONS OF TECHNOLOGY NETWORKS ON DIFFUSION AND ECONOMIC GROWTH

By

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## ABSTRACT

After the record-breaking run of high-speed growth in the United States during the late 1990s, a pressing question is “Has anything fundamental changed in our growth engine?” This paper examines an IT-led endogenous growth model driven by technology diffusion. Diffusion is in turn driven by network effect embodied in new technologies. The equilibrium long-term growth rate is however found to be independent of such technology networks. A novelty in our model is that innovation is discontinuous and it is separated by periods of diffusion. This (IT) network-diffusion is shown to be Sigmoid, and diffusion speed is slower than socially optimal.

Key words: network, diffusion, growth, IT, new economy

JEL Classification number: O40

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## **ABSTRACT**

After the record-breaking run of high-speed growth in the United States during the late 1990s, a pressing question is “Has anything fundamental changed in our growth engine?” This paper examines an IT-led endogenous growth model driven by technology diffusion. Diffusion is in turn driven by network effect embodied in new technologies. The equilibrium long-term growth rate is however found to be independent of such technology networks. A novelty in our model is that innovation is discontinuous and it is separated by periods of diffusion. This (IT) network-diffusion is shown to be Sigmoid, and diffusion speed is slower than socially optimal.

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A pressing question we face as we enter the twenty-first century is “Has anything fundamentally changed to our growth engine?” The record-breaking run of high-speed growth in the United States during the late 1990s, and the impact of the computer-led information technology revolution (IT) have raised the question: “Can a higher rate of growth than in previous decades be sustained?” Jorgenson and Stiroh (2000) are optimistic when they wrote, “A consensus is now emerging that something fundamental has changed, with ‘new economy’ proponents pointing to information technology (IT) as the causal factor behind the strong performance.” (p.125). Gordon (2002) is however more cautious, pointing out that “The 1995-2000 productivity growth revival was fragile, both because a portion rested on unsustainably rapid output growth in 1999-2000, and because much of the rest was the result of a doubling in the growth rate of computer investment after 1995 that could not continue forever.” (p.1) Events since 2000 have produced mixed clues as to where the U.S. economy is heading. The technology stock bubble burst, followed by the terrorist attacks of 9/11, and the Enron/WorldCom scandals did not make it easier to see whether long-term growth rates would be faster, slower, or indeed different in this *New Economy* than previously.

The purpose of this paper is to attempt a theoretical re-examination of the engine of growth. Our prevailing understanding of the growth mechanism can be improved on two counts. First, the three main mechanisms of endogenous growth – Lucas’ (1988) human capital accumulation, Romer’s (1990) technological change, and Aghion and Hewitt’s (1992) vertical innovation – are very general and do not address the issues of the IT directly. Second, the great majority of existing growth models has ignored the process of technological *diffusion*, which is an essential part of capitalist accumulation. With regards to diffusion, the difficulty seems to be that the search for a long-term steady state is easier when human capital or technological

accumulation is *continuous*. This continuity cannot generally be maintained when innovations are separated in time by periods of diffusion.<sup>1</sup> It turns out, as we will show in what follows, that a fruitful way to study the mechanics of IT-led growth is precisely to focus on its impacts on diffusion.<sup>2</sup>

One of the outstanding characteristics of IT is network effect that is often embodied in each generation of technology. It is now well documented that the usefulness of a fax machine increases when the size of the network – the number of fax machines owned – increases. The sizable industrial organization literature on network focuses mainly on the compatibility between rivalry networks, and on network externality.<sup>3</sup> *Moreover, old vintage network often needs to be abandoned for a new technology to take its place.* Old gramophone record collections were hard to get rid of and it delayed the decision by many to adopt the magnetic tape recording cassettes. Any revolutionary keyboard layout, however superior it might be, will be unlikely to replace the QWERTY network. In short, a strong old network tends to delay adoption of new technology. Yet a new technology with strong network may not be adopted quickly since the network benefit will not be realized – until it finally is.

It transpires that the interactions between technology networks have important implications on the diffusion process. The literature on diffusion has almost exclusively been concerned with explaining the Sigmoid diffusion curve.<sup>4</sup> None of the three traditional explanations – the ‘epidemic’, the ‘probit’, and the ‘information cascades’ models – has studied technology network as the driving force for diffusion. A contribution of the present paper is to show that networks can indeed drive diffusion, and via this network-diffusion interaction new lights can be shed on the mechanics of IT-led growth. Our two main conclusions are (a) networks hasten diffusion, but (b) this leaves the rate of innovation unchanged. We find no theoretical

evidence, at least not via the network - diffusion channel, for a fundamental change in the growth engine and the growth rate.

A new and emerging literature has recently developed under the heading of General Purpose Technology (GPT). The IT revolution that motivates the present study would probably qualify as a GPT.<sup>5</sup> We are however not interested in comparing between GPTs. It seems that one GPT, such as steam, is so drastically different from another, such as electricity, that comparing growth rate under one regime with another is almost akin to comparing apples with bananas. Instead we are interested in the diffusion of a single GPT, namely the IT-related technologies, and ask whether income growth will be faster with network-driven diffusions of IT technology. The stylized fact we aim to understand is that technology generations replace each other in a gradual process of diffusion during which both are used. In the existing GPT literature, the model developed by Helpman and Trajtenberg (1998a) did not allow diffusion, and in Helpman and Trajtenberg (1998b) diffusion from one GPT to the next is determined by R&D sequences in the adopting sectors, rather than by diffusion of the GPT itself. In Aghion and Howitt (1998) diffusion relies on a mechanical epidemic procedure. It is hope that the model developed below contributes to this literature by pointing out the IT-network effects and its impacts on the diffusion process of GPT.

The plan of this paper is as follows. In Section one we present the model of network-driven diffusion. Section two studies diffusion in the steady state. Section three examines the paths of prices and wages. Endogenous innovation and growth is examined in Section four. Section five is a brief summary and conclusion.

## 1. The Model of Diffusion and Welfare

The model to be presented describes a simple diffusion process in which an old technology is replaced by a new one. The unique feature of this process is that the creation of the new network necessitates the destruction of the old network; and the destruction of the old network first delays and then accelerates the installation of the new one. We will also establish in this section that this network diffusion process is typically slower than optimal.

The economy produces a single homogenous output  $x$  using labour  $L$  and capital (machine) as inputs. To keep the analysis simple we shall assume a fixed-proportion production method where one unit of labour is combined with one unit of machine to form a production unit.

Denote technology by a number  $G$  (generation), with a larger  $G$  indicating a newer generation. The output function of any  $G$  consists of two parts. The first part is  $a(G)$ , and  $a' > 0$  which reflects the simple impact of technological advance. The second part is network benefits which is the focus of this paper. Network benefit increases when more are using the same generation  $G$ . Define  $0 \leq s \leq 1$  as the share of total labour engaging in  $x$ -production using the same  $G$ . Network effect is said to assist production if  $s > 0$  contributes to output. Let this contribution be given by  $\alpha \cdot z(G) \cdot s(G)$ , where  $\alpha \geq 0$  is a parameter gauging the strength of network effects,  $z(G)$  is a constant just like  $a(G)$  and without loss of generality we shall replace  $z(G)$  by  $a(G)$  in what follows. The total output from a fixed-proportion input unit is therefore  $a + \alpha a s$ . When only  $G$  is used throughout the economy ( $s = 1$ ) the output function is  $a + \alpha a$ . A new generation  $G'$  yet to be adopted will only have (potential) output  $a(G')$  since  $s(G') \cong 0$ .



Network effects and the aging of machines jointly drive the diffusion process.<sup>6</sup> As machine ages, an increasing fraction of the labour is spent maintaining it, output per labour falls. Let  $g$  denote the age of a machine. Using the output function described in the last paragraph, let the output flow at time  $t$  per labour operating a machine with age  $g$  be

$$\frac{a + \alpha a s(t)}{\beta b^{g(t)}} \quad (1)$$

where  $\beta > 0$ ,  $b > 1$ , and both  $t$  and  $g$  are continuous. There is no other operating cost. Neither is there disutility of work. The machine once purchased is sunk.  $x$  is the numeraire, hence the expression in (1) is also the instantaneous revenue to the labour-firm. Each individual firm is assumed to be sufficiently small to take  $s$  as given. The attractiveness of staying with the old technology will fall as  $g$  rises in tandem with  $t$ . One might be reluctant to adopt a new technology because existing network effect with the old machine, as measured by  $\alpha$  and  $s$ , is large.

New technologies arrive at discrete time intervals. Innovation dates and thus the length of such intervals will be taken to be exogenous until Section four below. At the moment we focus on just two consecutive generations of machines. The new machine is attractive to potential adopters for two reasons. First, the old (current) generation offers  $a$  units of output per input, the new machine offers  $\lambda a$ ,  $\lambda > 1$  and is exogenous. Second, the newer machine needs less maintenance and offers greater output per ‘man’. Adoption is costly also for two reasons. First, one has to pay the inventor the price of the new machine. Second, the ‘cost’ of leaving an established network and joining (worse if starting) a new network could be significant if  $\alpha$  is large, and if  $s$  is small, until the diffusion is well underway.

Suppose at time  $t = 0$  a new generation technology  $G'$  is already some way along the process of replacing  $G$ . As already assumed just two generations  $G$  and

$G'$  are in use. Suppose  $s(\geq 0)$  of  $L$  has made the switch from  $G$  to  $G'$ .<sup>7</sup> Firms are familiar with the aging rate of their own machines. For simplicity assume the prospect of a next generation technology is sufficiently remote to be ignored. Assume further a constant discount rate  $r$ , and a constant price  $p$  payable upon adoption date  $t_1$ , say.<sup>8</sup> A firm with generation  $G$  machine of age  $g$  chooses  $t_1$  to maximize the following discounted income stream

$$\int_0^{t_1} \frac{a + \alpha a(1-s)}{\beta b^{g+t}} e^{-rt} dt + \int_{t_1}^{\infty} \frac{\lambda(a + \alpha as)}{\beta b^{t-t_1}} e^{-rt} dt - p e^{-rt_1}. \quad (2)$$

The first denominator has the power  $g + t$  since by assumption the firm's machine is already  $g$  periods old at  $t = 0$ . The second denominator has the power  $t - t_1$  since, upon adoption, the machine is brand new ( $g = 0$  at  $t = t_1$ ).

The two integrals when evaluated become  $\frac{(b^{t_1} e^{rt_1} - 1)[a + a(1-s)\alpha] b^{-g-t_1} e^{-rt_1}}{\beta(r + \ln b)}$

and  $\frac{\lambda a(1+s\alpha) e^{-rt_1}}{\beta(r + \ln b)}$ , respectively. Substituting into (2) and differentiating with

respect to  $t_1$ , the first order condition for the adoption date,  $t_1^*$  say, is

$$t_1^* = \frac{\ln \left\{ \frac{a[1 + (1-s)\alpha](r + \ln b)}{r[a\lambda(1+s\alpha) - p(r + \ln b)]} \right\}}{\ln b} - g. \quad (3)$$

While it might appear somewhat daunting, solution (3) in fact has many intuitive characteristics. Other things being equal, a firm with older machines (larger  $g$ ) will adopt earlier. We can turn it around and ask, "Given reasonable values of parameters appearing in the first term on the right-hand side, which firm of what machine-age would adopt immediately at  $t_1^* = 0$ ?" The answer is

$$g = \frac{\ln \left\{ \frac{a[1 + (1-s)\alpha](r + \ln b)}{r[a\lambda(1+s\alpha) - p(r + \ln b)]} \right\}}{\ln b}. \quad (4)$$

We can solve this to write

$$s = \frac{r[b^g pr + a(1 + \alpha - b^g \lambda)] + (a + b^g pr + a\alpha) \ln b}{a\alpha(r + b^g r \lambda + \ln b)}. \quad (5)$$

The Appendix shows that if  $p$  is not too high, which will in any case have to be true if the new technology is to be adopted at all, *equation (5) is a Sigmoid-shaped curve resembling the usual process of technological diffusion.*

The exact shape of this Sigmoid curve is not critical to the main results of this paper. To get a feel of what it looks like Figure one plots this curve assuming the following hypothetical parameter values:  $(a, \alpha, r, b, p, \lambda) = (1, 2, 0.05, 1.2, 2, 1.4)$ . The resemblance with the familiar diffusion curve is less than exact but the difference is trivial. Instead of having  $t$  on the horizontal axis, we have the age of machines that triggers off the adoption of new machines. The diffusion curve should therefore be read ‘backwards’. The oldest machine with  $g = 14.82$  (substituting  $s = 0$  into equation (5)) is the first to adopt. As  $t$  rises and  $s$  increases, younger and younger machines are replaced owing to the growing network of  $G'$  and the shrinking network of  $G$ . The last machine to be given up in favour of  $G'$  would only have  $g = 1.2$  (substituting  $s = 1$  into equation (5)).<sup>9</sup>

**Put Figure one about here**

*Market failure arising from network diffusion could be substantial.* To see this assume  $p = 0$ . Since  $\lambda > 1$  and  $b > 1$  the new technology is truly manna from heaven as far as the adopters are concerned. A social planner would have it adopted

immediately and by all firms since  $\frac{a + \alpha a}{\beta b^g} < \frac{\lambda(a + \alpha a)}{\beta}$ . Substituting  $p = 0$  into

equation (4) we get  $g|_{p=0, s=0} = \frac{\ln \left[ \frac{(1+\alpha)(r+\ln b)}{r\lambda} \right]}{\ln b}$ , which will be positive if

$\frac{(1+\alpha)(r+\ln b)}{r\lambda} > 1$  holds, i.e. if network effect is substantial ( $\alpha$  is ‘large’) and if

technical progress is not ( $\lambda$  is ‘small’). As an example it equals 12.61 if the parameters take the values used earlier:  $(a, \alpha, r, b, p, \lambda) = (1, 2, 0.05, 1.2, 2, 1.4)$ .

Adoption by private firms is held back for a substantial period by the old network and discouraged by the absence of a new network, and it would not commence until existing machines have sufficiently old. Market failure worsens if  $\alpha$  rises and if  $\lambda$  falls.

We pursue the issue of social optimality a little further by looking at the case of costly innovation instead of manna from heaven. Would private adoption incentives in the presence of old and new networks produce the socially optimal pattern of diffusion? An intuitive guess would seem to point to a negative answer since the old network continues to hold back adoption and the absence of a new network still discourages it. This is indeed the case and it can be verified as follows.

Assume the new machine costs the adopter  $p$  dollars at the time of adoption. We focus here on the optimality of the diffusion process and ignore the determination of  $p$ . Private profit maximization characterized by equation (2) yields results (3) and (4). A social planner would, in contrast, recognize the contribution each individual adoption has on creating the new network and on destroying the old one. She would in other words add a term,  $\Omega(\alpha, s)$  say, to the right-hand side of equation (2). Clearly  $\partial\Omega/\partial\alpha > 0$ , since the creation of the new network and the destruction of the old network are faster the greater is  $\alpha$ . In any case  $\Omega(t) > 0$  for as long as  $s(t) < 1$  since the social benefit will continue to accrue as long as diffusion is not yet complete. *By*

*ignoring this social benefit, private adoption decision would be too slow from a social welfare point of view.*

In short, private adoption and diffusion of new technology are in general slower than socially optimal when there are networks in the technologies. From this we return to the positive analysis and look for a steady state pattern of diffusion.

## **2. Diffusion in a steady state**

Not all technologies embody network, and if they do one network may be stronger than another. Nor is network exclusively a modern day IT phenomenon.<sup>10</sup> It seems true though that IT-related networks are particularly strong. We saw in the last section that networks can drive a diffusion process and influence the timing of adoption. In this section we are interested in two specific questions. Resolving these questions is necessary to establish long-term growth rate in Sections three and four. First, what are the long-term impacts of a network that emerges only once? That is, only one generation technology  $\tilde{G}$  embodies network effect, none of the other generations do before or after  $\tilde{G}$ . Second, what are the long-term impacts of network effect that is repeated in every generation? We continue to focus on the diffusion pattern.

Continue to assume that each generation of new technology arrives at regular time intervals of length  $T$ , say, so that we can focus exclusively on diffusion. The dependence of this process on networks is described by equation (4) above. If network were absence, we substitute  $\alpha = 0$  to get

$$\bar{g} = \frac{\ln \left\{ \frac{a(r + \ln b)}{r[a\lambda - p(r + \ln b)]} \right\}}{\ln b}, \quad (6)$$

where the upper-bar on  $g$  signifies its being a constant independent of  $s$ . Would steady state equilibrium exist and if ‘yes’ what does it look like? The easiest way to

think about this is to assume, to begin with, that every existing machine has exactly the same age. Adoption will be immediate and by all firms at time  $t_1$  which satisfies  $g(t_1) = \bar{g}$ , provided the new generation has arrived by  $t_1$ . For this to qualify as a steady state, i.e., the said innovation and adoption can be repeated an indefinite number of times, it is necessary that  $\bar{g} = T$  holds.<sup>11</sup> Figure two shows the diffusion curve being a vertical straight-line in  $s - g$  space.

**Put Figure two about here**

A more interesting situation is one where existing machines do not have the same age. Assume instead an age-profile of existing machines denoted by  $\tilde{g} \in [g(\max), g(\min)]$ . The diffusion path will proceed along  $\tilde{g} = \bar{g}$  defined by equation (6). It follows from the argument in the last footnote that a steady state exists if  $g(\max) - g(\min) = T$ . *This implies that any age-profile of existing machines satisfying this description of steady state will be preserved in subsequent innovations and diffusion.* The same diffusion curve re-emerges every time a new generation of technology arrives. Recall however this conclusion is reached assuming the absence of network in any generation of technology.

Consider an economy operating at such a steady state, absent any network, until a new generation arrives with positive network effect  $\alpha > 0$ . Using this in equation (4), it is immediate that initial adoption of the new generation machine will be delayed since  $g[s = 0, \alpha > 0] > g[s = 0, \alpha = 0]$ . By contrast, those at the tail of the diffusion process will adopt *earlier* since  $g[s = 1, \alpha > 0] < g[s = 1, \alpha = 0]$ . This is hardly surprising. As discussed in the last section network effect is partly responsible for the diffusion curve's Sigmoid shape. Compared to the case of no network, the

early adopters now wait longer until more are ready to adopt in order to benefit from the network effect. The late adopters wait less long being pulled along by the network benefits that continue to accumulate. *The conclusion must be that the existing age profile of machines is thereby compressed, and  $g(\max) - g(\min)$  falls if network effect  $\alpha > 0$  for just one generation of new machines.*

If network effect  $\alpha > 0$  emerges for one generation only, and  $\alpha = 0$  prevails thereafter, by the argument two paragraphs earlier this once-compressed age-profile will be preserved thereafter. What if network  $\alpha > 0$  persists? The answer must be that the age-profile is compressed in *every* round of innovation-diffusion. *The diffusion curve becomes forever more Sigmoid until, asymptotically,  $g(\max) = g(\min)$ , everyone adopts at the same time and the diffusion curve is a vertical straight-line shown in Figure two.* Notice that this conclusion does not assume  $\alpha$  to be a constant in every generation.

### **3. Prices and wages**

With the pattern of diffusion now established, the movement of the price of new machines and wages can readily be traced. Doing so also facilitates our discussion of endogenous innovation and growth in Section four below. Again we begin with the case without network:  $\alpha = 0$ . Assume innovative activity employs research workers  $L_r$ , who are drawn from the same pool of homogeneous labour for the production of final good.

New generations of technology continue to arrive at regular intervals of  $T$ . Each generation of technology is a measure  $\lambda$  superior to the preceding generation as described in equation (2). With  $\alpha = 0$ , adoption progresses along the diffusion curve according to the machine age profile at  $t = 0$ . The worker employed by the first

adopter earns  $\int_0^T \frac{\lambda a}{\beta b^g} e^{-rg} dg - p_0$  for the entire period  $[0, T]$  where  $p_0$  is the spot price of the new machine at  $t = 0$ . Since workers are homogenous and mobile between employments, this must be equal to  $\int_0^T \frac{a}{\beta b^g} e^{-rg} dg - p_0 e^{-rT}$ , earned by the last adopter of this technology for the period  $[0, T]$ . Equating the two expressions we have

$$p_0(1 - e^{-rT}) = (\lambda - 1) \int_0^T \frac{a}{\beta b^g} e^{-rg} dg. \quad (7)$$

Repeat the same argument for the next generation of technology for the period  $t \in [T, 2T]$ . The first adopter earns  $\int_0^T \frac{\lambda^2 a}{\beta b^g} e^{-rg} dg - p_1$ , which must be equal to that earned by the last adopter  $\int_0^T \frac{\lambda a}{\beta b^g} e^{-rg} dg - p_1 e^{-rT}$ . Equating the two we have

$$p_1(1 - e^{-rT}) = \lambda(\lambda - 1) \int_0^T \frac{a}{\beta b^g} e^{-rg} dg. \quad (8)$$

Using (7) and (8), extending to subsequent and all future generations of technology we have

$$\frac{p_1}{p_0} = \frac{p_2}{p_1} = \dots = \frac{p_G}{p_{G-1}} = \lambda. \quad (9)$$

*The spot price of new machine rises discontinuously by a factor  $\lambda$  over each time period  $T$ .*

Next we turn to labour wages. Recall that network is absent. The movement of labour earnings across generations of technology can be traced from the first adopter in each case. Denote this earning by  $w$  - the discounted present value 'wage' for the



entire period  $[0, T]$ . Constant returns and zero profits in the final output imply

$$w_0 = \int_0^T \frac{\lambda a}{\beta b^g} e^{-rg} dg - p_0 \text{ and } w_1 = \int_0^T \frac{\lambda^2 a}{\beta b^g} e^{-rg} dg - p_1. \text{ Using (9) we have}$$

$$\frac{w_1}{w_0} = \frac{w_2}{w_1} = \dots = \frac{w_G}{w_{G-1}} = \lambda. \quad (10)$$

In the absence of network, both  $p$  and  $w$  rise discontinuously and stepwise at a rate of  $(\lambda - 1)$  over the time period  $T$ . This closely resembles the concept of long-term steady state per capita income growth despite the discontinuity inherent in the model.

Next we introduce network in this innovation-diffusion process. Section two above establishes that asymptotically every innovation is adopted simultaneously by all users. Equilibrium in the final goods market during  $[0, T]$  therefore requires

$$Lw_0 = (L - L_r) \int_0^T \frac{\lambda(1 + \alpha)a}{\beta b^g} e^{-rg} dg, \quad (11)$$

where  $Lw_0$  is total expenditure on the final good, and the right-hand side is its supply.

Constant returns and zero profits in the final output yield

$$w_0 = \int_0^T \frac{\lambda(1 + \alpha)a}{\beta b^g} e^{-rg} dg - p_0. \quad (12)$$

Again zero profit in the final good implies

$$p_0 = \frac{L_r}{L} \int_0^T \frac{\lambda(1 + \alpha)a}{\beta b^g} e^{-rg} dg. \quad (13)$$

Now consider the next generation technology between  $t \in [T, 2T]$ . Using the same reasoning as in (11) to (13), the spot price  $p_1$  for the new technology at  $t = T$  is

$$p_1 = \frac{L_r}{L} \int_0^T \frac{\lambda^2(1 + \alpha)a}{\beta b^g} e^{-rg} dg. \quad (14)$$

It follows immediately and by extending to all future generations that

$$\frac{p_1}{p_0} = \frac{p_2}{p_1} = \dots = \frac{p_G}{p_{G-1}} = \lambda . \quad (15)$$

Using this in (12) we have

$$\frac{w_1}{w_0} = \frac{w_2}{w_1} = \dots = \frac{w_G}{w_{G-1}} = \lambda . \quad (16)$$

Since  $L$  is constant, and the final good production uses one unit of labour, the earning  $w$  is real per-capita income grows at the rate  $(\lambda - 1)$  over each time period of length  $T$ . Since  $\lambda$  is assumed fixed, the real growth rate of the economy in both aggregate and per capita terms depends entirely and negatively on the length of  $T$  - the equilibrium rate of replacement of technology generations.

#### 4. Innovation and growth

Finally we come to innovation, and to relax the assumption that  $T$  - the time interval between innovations is exogenous. The main question is whether network effect would lead to faster or slower innovation, i.e. a different  $T$ . Since real earning per capita increases by a factor  $\lambda$  every  $T$  (Section three above), the growth rate of real per capita income is inversely related to  $T$ .

Imagine a large number of potential innovative agents at  $t = 0$  each chooses to deploy resources to maximize profits from inventing the next generation technology at some  $t = T$ . As assumed earlier research activity uses only labour, denoted  $L_r$  drawn from the same homogeneous pool of  $L$  that also produces the final good. Once invented the marginal cost of providing the new technology to final good producers is zero. A single patent will be awarded to the earliest inventor of the next generation technology. The technology to research is itself well diffused so that competitive innovation drives innovative profit to zero, this closes the model and defines the equilibrium.

We continue to assume each new generation technology multiplies labour productivity by an exogenous factor  $\lambda > 1$ . There is well-documented evidence pointing to an inverse, concave relationship between the resource spent on an innovative project and the timing of its completion (see for example Mansfield (1961)). For simplicity assume  $T$  and  $L_r$  to have the unit elasticity relation

$$T = c/L_r, \quad c > 0. \quad (17)$$

Let  $R$  denote the revenue as discounted value accruing to a successful inventor at  $t = T$ .  $R$  depends on the price charged for the new generation of machine, as well as on the pattern of diffusion. We saw earlier both of these are influenced by networks. We assume however  $R$ , as far as the choice problem of the inventor is concerned, is *not* a function of  $L_r$ .<sup>12</sup> The cost of research is  $w_0 L_r$ , where  $w_0$  as specified earlier is the wage which the inventor takes as given. He then chooses  $L_r$  to maximize

$$R e^{-cr/L_r} - w_0 L_r. \quad (18)$$

The first order condition is  $\frac{Rcr}{L_r^2} e^{-cr/L_r} = w_0$ . Substituting this into the equilibrium zero profit condition we have

$$\frac{R(L_r - cr)}{L_r} e^{-cr/L_r} = 0. \quad (19)$$

At the equilibrium  $L_r - cr = 0$  must hold and from this the second order condition

$\frac{Rr(cr - 2L_r)}{L_r^4} e^{-cr/L_r} < 0$  is also satisfied. It follows from (19) that the equilibrium

research employment,  $L_r^*$  say, is entirely determined by the exogenous research parameter  $c$  and the interest rate  $r$ . The equilibrium time interval between inventions

is  $T = 1/r$ . The growth rate of per capita income may be written either as  $(\lambda - 1)/T$  or  $(\lambda - 1)r$ , and it is independent of network effect  $\alpha$ .

Two points should be made to appreciate the meaning of this result. First, the discount rate  $r$  plays such a prominent role in long-term growth because the primary incentive to innovate is modelled here as bringing forward the innovation revenue  $R$ . Should  $r \rightarrow 0$  this incentive disappears, so would innovation and growth. This is a different role played by the interest rate compared to the traditional one as inducement to save. Second, the equilibrium is independent of  $\alpha$  because of competitive innovation. The individual inventor's choice of  $L_r$  is influenced by  $\alpha$  as it is evident from the first order condition  $\frac{Rcr}{L_r^2} e^{-cr/L_r} = w_0$ , as we argued earlier both  $R$  and  $w_0$  are influenced by  $\alpha$ . Such influences however exactly offset each other in the process of competitive innovation, as  $R$  and  $w_0$  must together satisfy the zero profit condition (18).

## 5. Conclusions

We began our enquiry by posing the questions, "Has anything fundamentally changed in our growth engine?" and, "Can faster growth in the *New Economy* be sustained?" If the *New Economy* as Jorgenson and Stiroh (2000) put it is propelled by IT, and if networks as Katz and Shapiro (1994) and others argue is a central feature of a IT, then our answer from the model presented above is a negative one. Our central conclusion is that long-term growth rate is unaltered by the network effects in technology.

The steps we took to derive this central result may be summarized as follows. Section one above established that network effect together with the aging of machines determines the Sigmoid diffusion curve. This is quite intuitive as network of the old

generation delays adoption, but network of the new generation encourages it. Although stronger and more persistent networks compress the diffusion pattern (Section two), in general per capita income rises stepwise between sequential innovations (Section three). The height of each step, in addition, is given by the size of the innovation. The rise in per capita income per period of time, therefore, is entirely and negatively determined by the arrival rate of new innovations. The choice of this arrival rate, as shown in Section four and subject to a competitive innovative process, completely eliminates any influence of network on the rate of innovation.

A crucial assumption we made is the constant and exogenous  $\lambda$  - the size of each innovation over the technology that it replaces. All is not lost however if  $\lambda$  is not constant. The analysis in Section four did not depend on this assumption; hence the conclusion of  $T = 1/r$  still stands if the constancy of  $\lambda$  is relaxed. The rate of growth will be described by  $(\lambda_i - 1)r$  where  $\lambda_i$  differs according to the specific innovation in question. The steady state can then be seen in terms of some ‘average’  $\lambda$ .

Some inventions have been achieved in the model presented above, and we believe they offer fresh insights to the problem at hand. We have managed to study diffusion as an integral part of long-term economy growth. This allows diffusion and innovation to take place alternately as they do in practice. The model bridges the gap between our desires to study *continuous* long-term steady state growth, and the difficulty that innovations are empirically *discontinuous* and separated by periods of diffusion.

Another contribution of this paper concerns the mechanics of the diffusion process itself. We have added IT and network effect to the three existing candidates – epidemic, probit and information cascades – that drive the Sigmoid diffusion curve.

The stronger and more persistent is network, the faster will new technology be diffused among its users.

There is a long list of unanswered questions and this paper has probably added a few more to this list. Just two of the more pressing issues will be mentioned here in closing. First, network is important in IT but IT means more than just networks. We should continue to modernize our description of the growth engine to incorporate other IT characteristics in order to derive fresh insights into the growth prospect under IT. We could for instance make  $\lambda$  endogenous and let inventors choose  $\lambda$  as well as the invention date. Second, in Section one above we concluded that network may lead to diffusion being too slow from a social point of view, but Section four did not find this to influence invention incentives nor the growth rate. The reason lays with the fact that the competitive zero profit assumption for inventive activities eliminates the influence from diffusion. An interesting extension would be to allow for monopoly power and monopoly profits in research.

Despite these unresolved issues, we conclude that network effect exerts significant influence on the diffusion of technology, but not on per capita income growth via this variable diffusion process. This conclusion adds a sense of caution to the optimism that the unprecedented spurt of growth performance we witnessed in the 1990s would continue unfettered.

## Appendix

If  $p$  is not too high, which will in any case be true if the new technology is to be adopted at all, equation (5) in the text

$$s = \frac{r[b^g pr + a(1 + \alpha - b^g \lambda)] + (a + b^g pr + a\alpha) \ln b}{a\alpha(r + b^g r\lambda + \ln b)}. \quad (5)$$

is downward sloping with respect to  $g$ , and its second derivative is first negative and then positive thus having a ‘modified’ Sigmoid-shaped diffusion curve shown in

Figure one. The first derivative is  $\frac{\partial s}{\partial g} = \frac{b^g r \ln b (r + \ln b) [p(r + \ln b) - a(2 + \alpha)\lambda]}{a\alpha(r + b^g r\lambda + \ln b)^2}$ ,

which is negative provided  $p < \frac{a(2 + \alpha)\lambda}{r + \ln b}$  holds. Further,

$\frac{\partial^2 s}{\partial g^2} = -\frac{\partial s}{\partial g} [r(b^g \lambda - 1) - \ln b] \ln b$  negative if  $g < \frac{\ln[(r + \ln b)/r\lambda]}{\ln b}$  but positive if the

inequality is reversed.

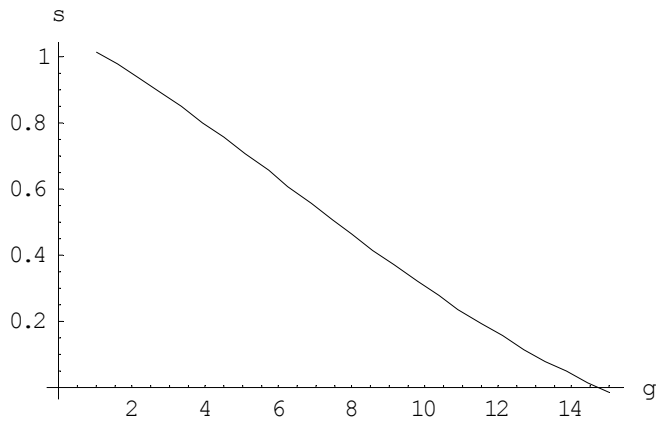


Figure 1. The Sigmoid diffusion curve given by equation 4 in the text assuming parameter values:  $(a, \alpha, r, b, p, \lambda) = (1, 2, 0.05, 1.2, 2, 1.4)$ .

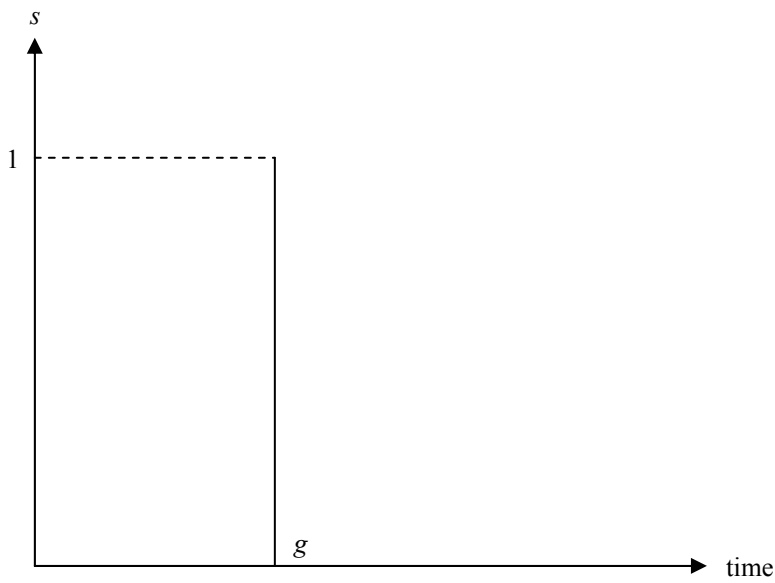


Figure 2. Diffusion curve being a straight-line when everyone adopts together at  $g$ .



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## Endnotes

<sup>1</sup> Under certain restrictions diffusion can be studied even when innovation is continuous. Chari and Hopenhayn (1991) does that in a vintage human capital model where old and new vintages are complements. With this complementarity the arrival of new a vintage raises the productivity of the old vintage, resulting in a pattern of peak usage versus non-peak usage, thus diffusion. But are vintages complements? Vintages of network seem more often than not substitutes. Examples are Beta Max versus VHS video recording, mainframe computers versus PCs, the QWERTY keyboard versus the Dvorak keyboard, and the MS Internet Explorer versus Netscape. Moreover, the Chari and Hopenhayn model assumes exogenous innovation rate, which is less than satisfactory.

<sup>2</sup> Another emerging literature, under the name of General Purpose Technology, also deals with diffusion and discontinuous innovation. The relation between the present model and GPT will be highlighted shortly.

<sup>3</sup> On the pervasiveness of network as externality see for example Katz and Shipiro (1985, 1986) and the critique by Liebowitz and Margolis (1994).

<sup>4</sup> See the excellent review by Geroski (2000).

<sup>5</sup> Lipsey, Bekar and Carlaw (1998, p.43) defined a GPT as “a technology that initially has much scope for improvement and eventually comes to be widely used, to have many users, and to have many Hicksian technological complementarities.”

<sup>6</sup> In the absence of network, diffusion is driven entirely by the aging of machines and our model is a pure ‘probit’ type. See Geroski (ibid.).

<sup>7</sup> For the moment innovation is exogenous, all labour  $L$  are employed in  $x$ -production. This will be modified when some labour is shifted to research employment in Section four and five.

<sup>8</sup> The seller of the new technology cannot practice intertemporal price discrimination.

<sup>9</sup> An upward shift of the curve, especially when accompanied by a counter-clockwise tilt, signifies a slower and longer diffusion process, and conversely. Simple numerical plotting reveals that a rise in  $a$  shifts the curve downwards but only slightly –  $a$  acts symmetrically on both  $G$  and  $G'$  and the effects cancel each other out; a rise in  $\alpha$  rotates the curve counter-clockwise – more powerful network of  $G$  delays early adoption significantly; a rise in  $r$  shifts the curve downwards and hastens diffusion – firms want to have  $G'$  earlier due to increased impatience; a rise in  $b$  shifts the curve clockwise and enhances the ‘Sigmoidness’ (curvature) of the curve – faster aging of machines hastens diffusion; a rise in  $p$  shifts the curve upwards – higher new machine price delays adoption; and finally a rise in  $\lambda$  shifts the curve downwards – more drastic innovations are adopted sooner. As we argued earlier the Sigmoid diffusion process is driven jointly by the aging of old machines, and it is assisted by the countervailing forces of the old and new networks. In general this conclusion is consistent with the findings of Jovanovic and Lach (1989).

<sup>10</sup> For examples of networks new and old see Shipiro and Varian (1999).

<sup>11</sup> An easy proof of this assertion is as follows. Suppose the time elapsed between the two adoption dates were  $t_1 - t_0 = \bar{g} < T$ . The situation cannot be repeated since the new generation will not be available in time for adoption. If  $t_1 - t_0 = \bar{g} > T$ , new generations will arrive progressively ‘too early’ and will, eventually and intermittently, be skipped. This implies that only  $t_1 - t_0 = \bar{g} = T$  can sustain steady state equilibrium, as claimed.

<sup>12</sup> This is not strictly true in equilibrium, as more workers are drawn to do research, less  $(L - L_r)$  will purchase it to produce the final good. However, the number of research workers is likely to be small in relation to the total workforce in the economy. A company engaging in the race to make the next generation IT innovation is not likely to be bothered with the impact of its research staff size  $L_r$  on the demand of its product by  $(L - L_r)$ . This assumption greatly simplifies the analysis that follows.