2-2003

Optimal Sequential Decision Architectures and the Robustness of Hierarchies and Polyarchies

Winston T. H. KOH
Singapore Management University, winstonthkoh@gmail.com

SMU ECONOMICS & STATISTICS
WORKING PAPER SERIES

Optimal Sequential Decision Architectures and The Robustness of Hierarchies and Polyarchies

Winston T.H. Koh

February 2003

Paper No. 5-2003

Optimal Sequential Decision Architectures and The Robustness of Hierarchies and Polyarchies*

Winston T.H. Koh+

School of Economics and Social Sciences

Singapore Management University

First Version: February 2003

Revised Version: June 2003

Forthcoming in *Social Choice and Welfare*

ABSTRACT

This paper studies collective decision making in the context of a project selection model. We derive the optimal decision architecture when marginal decision costs are present, and investigate the circumstances under which the hierarchy and polyarchy exist as optimal sequential architectures. Our analysis extends previous results on optimal committee decision-making to a sequential setting, and further demonstrates the fragility of the hierarchy and polyarchy as optimal architectures.

JEL Classification number: D81

Keywords: optimal decision architecture, project selection, hierarchy, polyarchy

* I would like to thank Ruth Ben-Yashar, Shmuel Nitzan and Raaj Sah for suggestions and advice on earlier versions of the paper. Two anonymous referees of this journal also provided many helpful comments and suggestions. The results in this paper were presented at the *Economic Architecture Conference*, 21-22 June 2003, at Odense, Denmark. I would like to thank the University of Southern Denmark and the Danish Social Science Research Council for their hospitality, and the participants at the Workshop for their constructive comments. Research support from the Singapore Management University is also gratefully acknowledged. The usual disclaimer applies here.

+ School of Economics and Social Sciences, Singapore Management University, 469 Bukit Timah Road, Singapore 259756. Tel: (65) 68220853; Email:winstonkoh@smu.edu.sg

1. Introduction

In many economic organizations, matters of strategic importance are often decided collectively by a team of fallible decision-makers, either in a committee setting where a pre-determined majority rule applies, or sequentially in a hierarchy where full consensus is required for a decision to be implemented. In some cases, a proposal or project is accepted as soon as it receives the support of one decision-maker. Such a decision architecture is sometimes referred to as a polyarchy. Even when decision-makers are well-intentioned and share the same objectives, mistakes are made if they are limited by the information they have access to, or if they are limited in their ability to process and evaluate information (Stiglitz, 2002).

Over the past two decades, a large literature has studied various aspects of collective decision-making in a large variety of contexts; these studies include Klevorick and Rothschild (1979), Nitzan and Paroush (1980, 1982, 1984, 1985), Klevorick, Rothschild and Winship (1984), Sah and Stiglitz (1985, 1986, 1988), Gradstein, Nitzan and Paroush (1990), Sah (1990, 1991), Koh (1992a, 1992b, 1993, 1994a, 1994b), Pete, Pattipati, Kleinman (1993), Austen-Smith and Banks (1996), Berg and Paroush (1998), Ben-Yashar and Nitzan (1997, 1998, 2001) and Ben-Yashar and Paroush (2001). Recently, the strategic aspects of collective decision-making in the committee setting have been studied by Feddersen and Pesedorfer (1998), Dekel and Piccione (2000), Li, Rosen and Suen (2001), Persico (2002), and others.

In this paper, I study the collective-decision problem in the context of a project selection model, where a team of decision-makers have to decide whether to accept or reject projects. My objective is two-fold. First, I solve for the optimal decision architecture when marginal decision costs are present and decision-makers are homogeneous in their expertise. I show that the optimal sequential architecture is a pair of sequential majority rules, which demarcates the ranges within which projects are accepted, rejected, or where an additional evaluation is called for. The analysis, presented in Section 3, extends the main result of Ben-Yashar and Nitzan (1997) – on the optimal majority rule in committee decision making – to a sequential setting where marginal decision costs are positive.

With positive marginal decision costs, there are many possible decision architectures, besides the hierarchy and polyarchy. These two architectures have attracted considerable attention in the literature (see Sah and Stiglitz (1985, 1986, 1988), Sah (1991), Koh (1992a, 1992b, 1994) and Ben-Yashar and Nitzan (2001)). My second objective in this paper is to investigate the circumstances under which the hierarchy and the polyarchy emerge as optimal sequential architectures. In a hierarchy, a project is rejected and evaluation ends if one decision-maker rejects the project. A project is only accepted if every decision-maker approves of the project. Decision-making authority is thus centralized in a hierarchy. By contrast, in a polyarchy, a team of decision-makers can undertake projects independently of each other. A project will be given further chances within the organization if it is turned down, and will be accepted as soon as it receives the support of one decision-maker. In this sense, decision-making authority is more decentralized in a polyarchy.

Although the hierarchy and the polyarchy are optimal sequential architectures for specific configurations of the quality of the investment environment and expertise of the decision makers, we show that the robustness of these two specific architectures is sensitive to perturbations in the environment. Furthermore, in the context of the optimal sequential decision architecture, the feasible optimal size of the hierarchy and polyarchy also turns out to be the minimum size of the organization. Therefore, under the general setting investigated in this paper, the application of either the hierarchical and polyarchical architecture to organizational decision-making will usually be sub-optimal. Our results, presented in Section 4, complement the analysis in Ben-Yashar and Nitzan (2001), which also examined the robustness of these two architectures.

The focus on marginal decision costs in this paper is motivated by the observation that most organizations operate at full managerial capacity, in the sense all the available managerial resources are fully deployed in making production and investment decisions at each point in time. With a fixed pool of managerial expertise within an organization, managerial tasks often have to be prioritized and for each task taken up by a group of decision-makers, another task will only be attended to later. In order to utilize managerial time and expertise optimally, the allocation of managerial expertise at each point in time should recognize the marginal benefit of further deliberation on a decision versus the opportunity costs to the organization of doing so. Besides the additional resources that would be deployed to continue the evaluation before a decision is made on acceptance or rejection, the opportunity costs also include the impact of potential delay in managerial attention on other projects, as well as potential monetary loss to the organization, as in the case when first-mover advantage matters in making investment decisions (if the project under review is also being considered for adoption by competitors). Other papers that have also considered marginal decision costs in collective decision-making include Nitzan and Paroush (1985) and Gradstein, Nitzan and Paroush (1990). Clearly, if marginal decision costs are absent, there is no necessity to engage in sequential review. The optimal decision architecture is simply a committee where all decision-makers evaluate a project simultaneously. The decision to accept or reject a project will be based on a super-majority rule, as shown in Sah (1990), and generalized in Ben-Yashar and Nitzan (1997).

The rest of the paper is organized as follows. Section 2 introduces the project selection model. The analysis of the optimal sequential architecture is presented in Section 3. Next, in Section 4, we examine the robustness of the hierarchy and polyarchy as optimal sequential architectures. Section 5 concludes the paper.

2. The Model

Consider an economic organization of n members whose objective is to maximize a common utility function associated with the selection of projects for investment. There are two types of projects; good (1) projects and bad (0) projects. Let s denote the state of a project; $s = 1$ and $s = 0$ are the two possible states of nature. For each project, there are two possibilities of making a correct decision: (1|1) invest in a good project, and (0|0) reject a bad project. There are also two possibilities of making an incorrect decision: (1|0) invest in a bad project, and (0|1) reject a good project.

The expected payoff associated with the approval (1) of a good project is $B(1|1)$ while the expected payoff associated with the rejection (0) of a bad project is $B(0|0)$. Similarly, the expected payoff associated with the rejection of a good project is $B(0|1)$, while the expected payoff of accepting a bad project is $B(1|0)$.[1] We require $B(1|1) > B(0|0)$, $B(1|1) > B(1|0)$ and $B(0|0) \geq B(0|1)$, so that there is an optimal action associated with each type of project. Let $B(1) = B(1|1) - B(0|1)$ be the net expected payoff when $s_i = 1$, and $B(0) = B(0|0) - B(1|0)$ be the net expected payoff when $s_i = 0$. For simplicity, suppose that the *a priori* probability that a project is good is known and is fixed at α, where $0 < \alpha < 1$. The expected project payoffs are summarized in Tables 1 below.

Table 1: The Expected Payoffs of Projects

		Project Quality	
		Good	**Bad**
Action	**Accept**	$B(1\|1)$	$B(1\|0)$
	Reject	$B(0\|1)$	$B(0\|0)$

The decision-makers can discriminate between good projects and bad ones, but only imperfectly. They vote independently to approve or reject projects, and are homogeneous in their decision-making expertise.[2] Specifically, the expertise of decision makers are represented as follows: p_1 is the probability that he will approve a good project, and p_0 that he will reject a bad project. Therefore, the probabilities $(1 - p_1)$ and $(1 - p_0)$ are the Type-I (reject a good project) and Type-II (accept a bad project) errors committed in the decision process. Decision-making expertise is imperfect in the sense that $p_1 < 1$, $p_0 > 0$ and $p_1 > (1 - p_0)$, i.e. a manager is

[1] In this formulation of the decision problem, the actual utility payoff from a project is still uncertain, although the expected payoffs are known, contingent on the nature of the project (good or bad) and the action taken by the organization (accept or reject).

[2] In a more general setting, we could allow for decision-makers to differ in their decision-making expertise. This is studied in Koh (2003).

more likely to accept a good project than a mediocre project. The expertise of the decision makers are summarized in Table 2 below.

Table 2: The Expertise of Decision-Makers

		Project Quality	
		Good	**Bad**
Action	**Accept**	p_1	$(1 - p_0)$
	Reject	$(1 - p_1)$	p_0

Project evaluation takes place sequentially and each evaluation incurs a constant marginal decision cost of C. When marginal decision costs are absent, as may be the case when managerial expertise are not fully deployed within the organization or when the organization can access additional resources at no cost, it is easy to see that the optimal decision rule takes the form of a super-majority rule with participation by all the decision-makers. When marginal decision costs are positive, the optimal sequential architecture includes the option to make the decision earlier, as the expected benefits of further deliberation may be out-weighed by the costs of doing so.

3. The Optimal Sequential Decision Architecture

The objective of the economic organization is to determine the optimal sequential decision architecture. Let $k\ (\leq n)$ denote the number of evaluations that the project has undergone. Denote z_i = 0 (rejection) or 1 (acceptance) as the recommendation of the ith decision-maker. We say that the decision process is in state X_k at stage k if there are X_k votes to accept the project (and $k - X_k$ votes to reject). The decision at stage k to accept the project, reject the project, or to proceed for another review, depends on X_k. The optimal decision rule is to end the evaluation process if the expected gain in payoff from another evaluation is less than the cost of doing so.

Let $P_k(X_k)$ denote the posterior probability that a project under review is a good project after k evaluations with X_k, favorable reviews, where $X_k = \sum_{i=1}^{k} z_i$. Therefore, z_i is a Bernoulli random variable, where p_1 is the probability that $z_i = 1$ when the project is good, and p_0 is the probability that $z_i = 0$ when the project is bad. The Bayesian updating of $P_k(X_k)$ is given by:

$$P_{k+1}(X_k+1) = \frac{P_k(X_k)p_1}{P_k(X_k)p_1 + \left[1-P_k(X_k)\right](1-p_0)} \tag{1}$$

$$P_{k+1}(X_k) = \frac{P_k(X_k)(1-p_1)}{P_k(X_k)(1-p_1) + \left[1-P_k(X_k)\right]p_0}$$

so that through recursion, we obtain

$$P_k(X_k) = \frac{\alpha p_1^{X_k}(1-p_1)^{k-X_k}}{\alpha p_1^{X_k}(1-p_1)^{k-X_k} + (1-\alpha)(1-p_0)^{X_k} p_0^{k-X_k}} \tag{2}$$

Let $R_a(P_k(X_k))$, $R_r(P_k(X_k))$ and $R_c(P_k(X_k))$ denote, respectively, the conditional expected project payoff if, after k evaluations with X_k positive reviews, the organization's decision is to accept the project, reject the project, or proceed for another review. We have:

$$R_a(P_k(X_k)) = P_k(X_k)B(1|1) + \left[1-P_k(X_k)\right]B(1|0) \tag{3}$$

$$R_r(P_k(X_k)) = P_k(X_k)B(0|1) + \left[1-P_k(X_k)\right]B(0|0)$$

Define the value function $V(P_k(X_k),k)$ where

$$V(P_k(X_k),k) \equiv \text{Max}\left\{R_a(P_k(X_k)), R_r(P_k(X_k)), R_c(P_k(X_k),k)\right\} \tag{4}$$

The value function $V(P_k(X_k),k)$ describes the maximum conditional expected project payoff of the project after k evaluations with X_k positive reviews. There are two possible events at the $(k+1)$th review; namely, manager (k+1) disapproves of the project (so that $z_{k+1} = 0$) or approves the project (so that $z_{k+1} = 1$). Since $P_k(X_k)$ provides the Bayesian-updated probability that the project under consideration is a good project, for a given history of reviews up to stage

k, the conditional probabilities of observing $z_{k+1} = 0$ or 1 at the $(k+1)$th review are given by, respectively,

$$H(1 \mid P_k(X_k)) = P_k(X_k)p_1 + \left[1 - P_k(X_k)\right](1 - p_0) \qquad (5)$$
$$H(0 \mid P_k(X_k)) = P_k(X_k)(1 - p_1) + \left[1 - P_k(X_k)\right]p_0$$

We can obtain a sequence of recursive equations for $V(P_k(X_k), k)$ as follows: (6)

$$R_c(P_k(X_k), k) \equiv V(P_{k+1}(X_k + 1), k+1)H(1 \mid P_k(X_k)) + V(P_{k+1}(X_k), k+1)H(0 \mid P_k(X_k)) - C$$

We first prove the following results on the optimal decision rule for $k \leq n-1$. The optimal decision rule for $k = n$ is a supermajority rule, and will be discussed in the next section.

Proposition 1[3]: The optimal evaluation policy consists of a pair of probability thresholds $\left\{q_k^L, q_k^U\right\}$, $k = 1, 2 \ldots, n$, where q_k^L is increasing in k, and q_k^U is decreasing in k.

[a] If $q_k^L < P_k(X_k) < q_k^U$; the evaluation process continues;

[b] If $P_k(X_k) < q_k^L$, the project should be rejected and evaluation ends;

[c] If $P_k(X_k) > q_k^U$, the project should be accepted and evaluation ends.

Corresponding to the optimal evaluation policy in Proposition 1 is an equivalent optimal decision architecture.

Proposition 2: The optimal decision architecture consists of a pair of sequential majority rules $\left\{X_k^L, X_k^U\right\}$, $k = 1, 2 \ldots, n-1$.

[a] If $X_k^L < X_k < X_k^U$; an additional evaluation is requested;

[b] if $X_k < X_k^L$, the project should be rejected and evaluation ends;

[c] If $X_k > X_k^U$, the project should be accepted and evaluation ends.

---

[3] The proofs are provided in the Appendix.

The optimal evaluation policy and the optimal decision architecture are related as follows. Since $P_k(X_k^L) \cong q_k^L$ and $P_k(X_k^U) \cong q_k^U$ (the approximation is due to the integer nature of X_k^L and X_k^U), it is straightforward to derive, for $k \leq n-1$,

$$X_k^U = \text{Min}\left[\frac{1}{\gamma+\delta}\left[\delta k - \mu + \ln\frac{q_k^U}{1-q_k^U}\right],\ k\right] \tag{7}$$

$$X_k^L = \text{Max}\left[\frac{1}{\gamma+\delta}\left[\delta k - \mu + \ln\frac{q_k^L}{1-q_k^L}\right],\ 0\right]$$

where

$$\mu \equiv \ln\frac{\alpha}{1-\alpha},\quad \gamma \equiv \ln\frac{p_1}{1-p_0} \text{ and } \delta \equiv \ln\frac{p_0}{1-p_1} \tag{8}$$ [4]

From (7), it is straightforward to show that there exists k^U and k^L such that $X_k^U = k$ for $k \leq k^U$, and $X_k^L = 0$ for $k \leq k^L$:

$$k^U = \frac{1}{\gamma}\left[-\mu + \ln\frac{q_{k^U}^U}{1-q_{k^U}^U}\right],\quad k^L = \frac{1}{\delta}\left[\mu - \ln\frac{q_{k^L}^L}{1-q_{k^L}^L}\right] \tag{9}$$

Hence, for $k < k^U$, the decision choices for the project under evaluation are either to proceed for further evaluation or to reject the project. Similarly, for $k < k^L$, the decision choices are to proceed for further evaluation or accept the project. Thus, for an organization of size n, $k^* \equiv \text{Min}\{k^L, k^U\}$ denotes the minimum number of evaluations that must be undertaken. Since no decision on acceptance or rejection should be made when $k < k^*$, the first k^* evaluations forms an initial review of the project, and it does not matter if the first k^* evaluations are carried out simultaneously or sequentially. In fact, if delay in selecting a project is costly, so that marginal decision costs is increasing with time taken to evaluate a project, it would be preferable for the first-stage of the evaluation process to be structured as a committee review by

[4] Note that $\gamma > 0$, $\delta > 0$, and $\mu > (<) 0$ depending on whether $\alpha > (<) 0.5$.

k^* decision-makers (simultaneously). From the second stage of the evaluation onwards, the evaluation of the project will be conducted sequentially.

Although closed-form solutions are generally not available for the sequential majority rules $\left\{X_k^L, X_k^U\right\}$, they can derived numerically by applying the following set of recursive rules.

Proposition 3: [a] For $k > k^L$, $X_k^L + \beta < X_{k+1}^L < X_k^L + 1$;

[b] For $k > k^U$, $X_k^U < X_{k+1}^U < X_k^U + \beta$ where $\beta \equiv \dfrac{\delta}{\delta + \gamma} < 1$.

Figure 1 below provides an illustration of the optimal sequential decision architecture.

INSERT FIGURE 1

The magnitude of the marginal decision cost has an important impact on the set of probability thresholds $\left\{q_k^L, q_k^U\right\}$. Firstly, if the marginal decision cost C becomes larger, the probability range, given by $\left(q_k^U - q_k^L\right)$, will become narrower. In other words, the scope for continuing with further project evaluation, as measured by the difference $\left(X_k^U - X_k^L\right)$, will be smaller, so that fewer evaluations will be undertaken for each project, on average. In the limit, when marginal decision costs are prohibitive, no evaluations will be undertaken. Conversely, if the marginal decision cost falls, the scope for further project evaluation is increased. In the limit, when marginal decision cost is zero, the optimal decision rule is to for the project to undergo *n* reviews, and then apply a super-majority rule to consider acceptance or rejection. In fact, in this case, the decision-makers should evaluate the project simultaneously, as there is no advantage to sequentially reviewing the project. Of course, even if there is no marginal decision cost, there are still fixed costs involved in engaging a team of *n* decision-makers.

Thus, the optimal size of the decision team will still be finite and it should be selected to maximize expected net payoff per project.

When marginal decision costs are positive, Proposition 2 indicates that the optimal decision architectures is a pair of sequential majority rules, which defines the range where projects are accepted, rejected or where an additional evaluation is called for. These results are related to the analysis in Nitzan and Paroush (1985), which also discusses the optimality of similar sequential decision rules in a different setting.

The analysis here also extends the main result of Ben-Yashar and Nitzan (1997) to a sequential setting for the case of homogenous decision-makers possessing identical expertise. In Ben-Yashar and Nitzan (1997), decisions are made in a fixed-size committee, and since there are no marginal decision costs, all the decision-makers participate in reviewing the project and voting simultaneously. By contrast, in the analysis presented in this paper, project evaluation is carried out sequentially, and it is only in the case when all the *n* managers have reviewed the project, that our results converge to those of Ben-Yashar and Nitzan (1997).

4. The Optimality of Hierarchy and Polyarchy Architectures

In this section, we investigate the conditions under which the optimal sequential decision architecture is either a hierarchy (where all decision-makers must approve the project before it is accepted) or a polyarchy (where a project is accepted as soon as one decision-maker approves it, and is only rejected if all the decision-makers turn it down).

First, let us derive the optimal decision rule when a project has reached the maximum number of evaluations, i.e. when all the *n* decision-makers have reviewed the project. The decision to accept or reject the project is based on whether $R_a(X_n, n) > (<)\ R_r(X_n, n)$, which translates into the equivalent condition that $P_n(X_n) > (<)\ \phi \equiv \frac{B(0)}{B(0)+B(1)}$. Let X_n^* solve $P_n(X_n^*) = \phi$, so that if $X_n > (<)\ X_n^*$, the optimal decision rule is to accept (reject) the project. Using the definition of $P_k(X_k)$ in (2),

$$X_n^* = \frac{\delta n - \theta - \mu}{\gamma + \delta} \quad , \quad \theta \equiv \ln \frac{B(1)}{B(0)} \tag{10}$$

In order that the sequential decision architecture is not trivial, we require that $0 < X_n^* < n$. It is then straightforward to prove the following result:

Proposition 4: [a] When $\theta + \mu > 0$, the minimum size of the organization is $n_P \equiv \frac{\theta + \mu}{\delta}$ with $X_n^* = 0$; otherwise, it is preferable to always accept projects. [b] When $\theta + \mu < 0$, the minimum size of the organization is $n_H \equiv -\frac{\theta + \mu}{\gamma}$ with $X_n^* = n_H$; otherwise, it is preferable to always reject projects.

In the model, $(\theta + \mu)$ is a measure of the quality of the investment environment, while γ and δ are measures of the decision-makers' expertise to select good projects and bad projects, respectively. In the absence of any evaluation, the optimal decision is clearly to accept a project if $\theta + \mu > 0$, reject it if $\theta + \mu < 0$, and be indifferent between accepting and rejecting the project if $\theta + \mu = 0$. Therefore, $\theta + \mu > 0$ describes an above-average investment environment, while $\theta + \mu < 0$ describes a mediocre environment. It follows that if project evaluation is to generate information of sufficient value – in the sense that neither the "accept" nor "reject" decision is the preferred default choice – the decision team must be of a minimum size, as indicated by Proposition 4.

When the investment environment is above-average, the optimal decision architecture described in Proposition 4a is a polyarchy of size $(\theta + \mu)/\delta$. Similarly, when the investment environment is mediocre, the decision architecture described in Proposition 4b is a hierarchy of size $-(\theta + \mu)/\gamma$. The optimality of either decision architecture is clearly sensitive to perturbations in the investment environment and the expertise of the decision-makers. An improvement in decision-making expertise, due to an increase in p_1 or an increase in p_0 (or both) reduces the minimum organizational size, as γ and δ are raised. Similarly, variations in the quality of the investment environment, as measured by $(\theta + \mu)$, affect the minimum organizational size and therefore, the optimality of the hierarchy and polyarchy architectures.

Suppose that the size of the decision team is reduced below the minimum level required for informative evaluation (due to, say, budgetary constraints), it follows from Proposition 4 that the hierarchy architecture, if maintained, is dominated by not considering any investment at all. Similarly, the polyarchy architecture, if maintained, is dominated by simply accepting all projects. Conversely, if the decision team is enlarged beyond the minimum size, then the optimal sequential architecture is clearly no longer a hierarchy or polyarchy, but a pair of sequential majority rules, as described in Proposition 2. Clearly, then, the conditions under which the hierarchy and polyarchy can exist as optimal architectures are very stringent; both structures are optimal decision architectures only when the organizational size is fixed at the minimum level stated in Proposition 4. Formally, with marginal decision costs of C per evaluation, the expected net payoff of accepting a project in a hierarchy (after n_H evaluations) must be greater than simply rejecting it without any evaluation. In other words, we require that

$$P^H B(1|1) + \left[1 - P^H\right] B(1|0) - n_H C > \alpha B(0|1) + (1-\alpha) B(0|0) \tag{11}$$

where $P^H = \dfrac{\alpha p_1^{\,n_H}}{\alpha p_1^{\,n_H} + (1-\alpha)(1-p_0)^{n_H}}$. Similarly, for a polyarchy, the expected net payoff from rejecting a project (after n_P evaluations) must be greater than simply accepting it without any review:

$$P^P B(0|1) + \left[1 - P^P\right] B(0|0) - n_P C > \alpha B(1|1) + (1-\alpha) B(1|0) \tag{12}$$

where $P^p = \dfrac{\alpha p_1^{\,n_P}}{\alpha p_1^{\,n_P} + (1-\alpha)(1-p_0)^{n_P}}$. However, even if the conditions in (11) and (12) hold for the hierarchy and polyarchy, respectively, they can be easily dominated by a larger organization if these larger organizations generate a higher expected payoff per project.

The analysis presented in this section is related to that in Ben-Yashar and Nitzan (2001), which also examined the optimality of the hierarchy and polyarchy architectures. In their paper, Ben-Yashar and Nitzan proposed a size robustness measure, defined as the maximal permissible change in the size of the organization that does not alter the optimal architecture. Applying this measure to the hierarchy and polyarchy, they found that the size robustness measure is very small for both the hierarchy and polyarchy, and similarly concluded that these two architectures are, in general, sub-optimal organizational structures.

5. Summary and Discussion

This paper studies optimal collective decision-making in a sequential setting. Decision makers have a common objective to maximize the expected project payoffs, and must decide whether to accept or reject the project under consideration. The optimal sequential decision architecture, presented in Proposition 2, is a pair of sequential majority rules $\left\{X_k^L, X_k^U\right\}$. We show, in Proposition 4, that while the hierarchy or the polyarchy could exist as an optimal sequential architecture, the conditions under which this can occur are stringent.

While our analysis considers constant marginal decision costs, it can be extended to the case where the marginal decision cost C is not constant, but increases with the stage of evaluation. Such a situation may arise in the case where senior managers hold greater responsibilities and face heavier demands on their time, or when further delay in decision-making may adversely affect the organization's chances of investing in the project as well as the eventual project payoff (as would be the case when there is competition to invest in the project). It is easy to see that impact of increasing marginal decision costs as evaluation progresses will lead to a tighter range for requesting an additional evaluation; i.e. the difference $\left(X_k^U - X_k^L\right)$ will be smaller, $k \leq n-1$. Thus, the likelihood of proceeding for an additional evaluation is reduced, compared with the case when marginal decision costs are constant. In general, the impact of rising marginal decision cost is to reduce the optimal organizational size and the expected number of evaluations for a project. The overall profitability of the organization may suffer as a greater number of good projects are rejected, and bad projects accepted, when decisions on acceptance and rejection are made earlier.

With rising marginal decision costs, the minimum feasible size of an organization – either as a hierarchy or a polyarchy – will likely be reduced as well. Thus, under the rare circumstances when they emerge as optimal architectures, the hierarchy and polyarchy will apply only to smaller organizations. In general, large hierarchies and polyarchies can almost always improve organizational efficiency by altering their architecture and adopting less extreme sequential majority rules in the decision-making process.

Appendix

To derive the optimal evaluation policy in Proposition 1, we require the following lemmas.

Lemma 1: [a] $V(p,k) \geq V(p,k+1)$; [b] $R_c(p,k) \geq R_c(p,k+1)$ $k = 1, \ldots, n-1$ and $p \in [0, 1]$.

Lemma 2: $V(p,k)$ and $R_c(p,k)$ are convex in p; $k = 1, \ldots, n-1$ and $p \in [0, 1]$.

Lemma 1 and 2 are well-known results in the optimal stopping rule literature (see Astrom (1970), DeGroot (1970) or Bertekas (1987)). We present the proofs here for completeness.

Proof of Lemma 1: We first show if $V(p,k) \geq V(p,k+1)$ is true for some k, then $V(p,k-1) \geq V(p,k)$. Suppose $V(p,k) \geq V(p,k+1)$, then $R_c(p_{k-1},k-1) = E\left[V(p_k,k)\right]$ $> E\left[V(p_k,k+1)\right] = R_c(p_k,k)$. Thus, $V(p_{k-1},k-1) = \text{Max}\left\{R_a(p_{k-1}), R_r(p_{k-1}), R_c(p_{k-1},k-1)\right\}$ $\geq \text{Max}\left\{R_a(p_{k-1}), R_r(p_{k-1}), R_c(p_{k-1},k)\right\} = V(p_{k-1},k)$. Next, we show that $V(p,n-1) \geq$ $V(p,n)$. Since the maximum number of evaluations is n, $V(p_{n-1},n-1) =$ $\text{Max}\left\{R_a(p_{n-1}), R_r(p_{n-1}), R_c(p_{n-1},n-1)\right\} \geq \text{Max}\left\{R_a(p_{n-1}), R_r(p_{n-1})\right\} = V(p_{n-1},n)$. *Q*.E.D.

Proof of Lemma 2: First, we can write $R_c(p_k,k) = U_k(1\,|\,p_k) + U_k(0\,|\,p_k) - C$ where $U_k(1\,|\,p_k) \equiv V\left(\dfrac{p_k p_1}{H(1\,|\,p_k)}, k+1\right) H(1\,|\,p_k)$, $U_k(0\,|\,p_k) \equiv V\left(\dfrac{p_k(1-p_1)}{H(0\,|\,p_k)}, k+1\right) H(0\,|\,p_k)$. The expressions $H(1\,|\,p)$ and $H(0\,|\,p)$ are defined in (5). To show the convexity of $R_c(p,k)$ in p, $k = 1, 2\ldots, n-1$ and $p \in [0, 1]$, it is sufficient to show that $U_k(1\,|\,p)$ and $U_k(0\,|\,p)$ are convex in *p*. To prove the convexity of $U_k(1|p)$ in *p*, we must show for $\lambda \in [0, 1]$, and p_a and $p_b \in [0, 1]$, $\lambda U_k(1|p_a) + (1-\lambda)U_k(1|p_b) \geq U_k(1|\lambda p_a + (1-\lambda)p_b)$. For $U_k(1|p)$,

$$\left[\frac{\lambda H(1|p_a)}{\lambda H(1|p_a)+(1-\lambda)H(1|p_b)}\right]V\left(\frac{p_a p_1}{H(1|p_a)},k+1\right)$$

$$+\left[\frac{(1-\lambda)H(1|p_b)}{\lambda H(1|p_a)+(1-\lambda)H(1|p_b)}\right]V\left(\frac{p_b p_1}{H(1|p_b)},k+1\right) \geq V\left(\frac{(\lambda p_a+(1-\lambda)p_b)p_1}{\lambda H(1|p_a)+(1-\lambda)H(1|p_b)},k+1\right)$$

Hence, $U_k(1|p)$ is convex in *p* if $V(p,k+1)$ is convex in *p*. The convexity of $U_k(0|p)$ can be proven similarly. Since $R_c(p_k,k) = U_k(1|p_k) + U_k(0|p_k) - C$, this implies that $R_c(p,k)$ is convex in *p* if $V(p,k+1)$ is convex in *p*. Let's suppose $R_c(p,k)$ is convex in *p*; it follows then that since $V(p,k)$ is the maximum of three convex functions, $V(p,k)$ is also convex in *p*. Hence, the convexity of $V(p,k+1)$ in *p* implies the convexity of $V(p,k)$ in *p*, $k = 1, 2\ldots, n-1$. Finally, $V(p,n)$ is convex is *p* since it is the maximum of two linear functions, $R_a(p)$ and $R_r(p)$.Therefore, $V(p,n-1)$ is convex in *p*. *Q.E.D.*

Proof of Proposition 1: The proof utilizes Lemmas 1 and 2. First, as *p* tends to 1, $R_c(p,k)$ tends to $B(1|1) - C$; similarly, as *p* tends to 0, $R_c(p,k)$ tends to $B(0|0) - C$, $k = 1, \ldots, n$. From Lemma 2, $R_c(p,k-1) \geq R_c(p,k)$ so that if the evaluation reaches (*n*–1)th stage, $R_c(\phi,n-1) = R_a(\phi) = R_r(\phi)$, where $\phi \equiv B(0)/\left[B(0)+B(1)\right]$. Since $R_a(p)$ and $R_r(p)$ are linear, and $R_c(p,k)$ is convex in *p*, with $R_c(1,k) > R_a(\phi) = R_r(\phi)$ and $R_c(0,k) > R_a(\phi) = R_r(\phi)$, it is easy to verify that the function $R_c(p,n-1)$ intersects the function $\text{Max}\left\{R_a(p),R_r(p)\right\}$ at two points, $\left(q_{n-1}^U,R_a(q_{n-1}^U)\right)$ and $\left(q_{n-1}^L,R_r(q_{n-1}^L)\right)$ so that $R_c(q_{n-1}^U,n-1) = R_a(q_{n-1}^U)$ and $R_c(q_{n-1}^L,n-1) = R_r(q_{n-1}^L)$, and $q_{n-1}^L < \phi < q_{n-1}^U$. Next, utilizing Lemma 1, it follows that the probability range (q_k^L, q_k^U) narrows as *k* increases. Since $R_c(p,k-1) \geq R_c(p,k)$ for $p \neq 0$ or 1, it follows that q_k^L is increasing in *p* and q_k^U is decreasing in *p*. *Q.E.D.*

We illustrate the relationship between $V_k(p,k)$, $R_c(p,k)$, $R_a(p)$ and $R_r(p)$ in Figure 2.

INSERT FIGURE 2

Proof of Proposition 3:

[a] To prove that $X_k^L+\beta<X_{k+1}^L<X_k^L+1$ for k = 1, …, n–1, we first note from Proposition 1 that $q_k^L<q_{k+1}^L$; hence, $P_k(X_k^L)<P_{k+1}(X_{k+1}^L)$. Using the definition of $P_k(X_k)$, it is routine to show that $X_k^L+\beta<X_{k+1}^L$. Next, to show that $X_{k+1}^L<X_k^L+1$, suppose that $X_k+1\le X_{k+1}^L$. This implies that $V(P_{k+1}(X_k+1),k+1)$ = $R_r(P_{k+1}(X_k+1))$ and $V(P_{k+1}(X_k),k+1)$ = $R_r(P_{k+1}(X_k))$. In turn, this implies, using (5) and (6) to take conditional expectation, that $R_c(P_k(X_k),k)$ = $R_r(P_k(X_k))$. Therefore, $V(P_k(X_k),k))$ = $R_r(P_k(X_k))$. Since this is true for $X_k<X_k^L$ it follows that since we assume $X_k+1\ \le X_{k+1}^L$ that $X_k+1\ \le X_k^L+1<X_{k+1}^L+1$.

[b] Next, to prove that for k = 1, …, n–1, $X_k^U<X_{k+1}^U<X_k^U+\beta$. Again, we note from Proposition 1 that q_k^U is decreasing, so that $q_k^U>q_{k+1}^U$; this implies $P_k(X_k^U)>P_{k+1}(X_{k+1}^U)$ Similarly, using the definition of $P_k(X_k)$, it is routine to show that $X_{k+1}^U<X_k^U+\beta$. Next, to show that $X_k^U\ <\ X_{k+1}^U$. Suppose $X_k\ge X_{k+1}^U$. This implies that $V(P_{k+1}(X_k+1),k+1)$ = $R_a(P_{k+1}(X_k+1))$ and $V(P_{k+1}(X_k),k+1)=R_a(P_{k+1}(X_k))$. This in turn implies, using (5) and (6) to take conditional expectation, $R_c(P_k(X_k),k)$ = $R_a(P_k(X_k))$. Therefore, we have $V(P_k(X_k),k)$ = $R_a(P_k(X_k))$. Since this is true for $X_k\ >\ X_k^U$, it follows from our assumption that $X_k\ \ge\ X_{k+1}^U$ that $X_k\ \ge\ X_{k+1}^U>X_k^U$. *Q.E.D.*

References

Astrom KJ (1970) Introduction to Stochastic Control Theory, Academic Press, New York

Austen-Smith D, Banks JS (1996) Information aggregation, rationality and the Condorcet jury theorem. American Political Science Review 90: 34-45

Ben-Yashar R, Nitzan S (1997) The optimal decision rule for fixed size committees in dichotomous choice situations: The general result. International Economic Review 38(1): 175-187

Ben-Yashar R, Nitzan S (1998) Quality and structure of organizational decision making. Journal of Economic Behavior and Organization 36: 521-534

Ben-Yashar R, Nitzan S (2001) The robustness of optimal organizational architectures: A note on hierarchies and polyarchies. Social Choice and Welfare 18: 155-163

Ben-Yashar R, Paroush J (2001) Optimal decision rules for fixed-size committees in polychotomous choice situations. Social Choice and Welfare 18: 737-746

Berg S, Paroush J, (1998) Collective decision making in hierarchies. Mathematical Social Sciences 35: 233-244

Bertsekas DP (1987) Dynamic Programming: Deterministic and Stochastic Models, Prentice-Hall, Inc., Englewood Cliffs, New Jersey

DeGroot MH (1970) Optimal Statistical Decisions, McGraw Hill, New York, USA

Dekel E, and Piccione M (2000) Sequential voting procedures in symmetric binary elections. Journal of Political Economy 108:34-55

Feddersen T, and Pesedorfer W (1998) Convicting the innocent: The inferiority of unanimous jury verdicts under strategic voting. American Political Science Review 92: 23-35

Gradstein M, Nitzan S, Paroush, J (1990) Collective decision making and the limits on the organization size. Public Choice 66: 279-291

Klevorick AK, Rothschild M (1979) A model of jury decision process. Journal of Legal Studies 8: 141-164

Klevorick AK, Rothschild M, Winship C (1984) Information processing and jury decisionmaking. Journal of Public Economics 23: 245-278

Koh WTH (1992a) Variable evaluation costs and the design of fallible hierarchies and polyarchies. Economic Letters 38: 313-318

Koh WTH (1992b) Human fallibility and sequential decision-making: Hierarchy versus polyarchy. Journal of Economic Behavior and Organization 18: 317-345

Koh WTH (1993) First-mover advantage and organizational structure. Economic Letters 43: 47-52

Koh WTH (1994a) Making decisions in committees: A human fallibility approach. Journal of Economic Behavior and Organization 23:195-214

Koh WTH (1994b) Fallibility and sequential decision making. Journal of Institutional and Theoretical Economics 150 (2): 362-374

Koh WTH (2003) Heterogeneous expertise and collective decision-making, Working Paper, Singapore Management University

Li H, Rosen S, Suen W (2001) Conflicts and common interests in committees. American Economic Review 21:1478-1497

Nitzan S, Paroush J (1980) Investment in human capital and social self protection. International Economic Review 21: 547-557

Nitzan S, Paroush J (1982) Optimal decision rules in uncertain dichotomous choice situations. International Economic Review 23: 289-97

Nitzan S, Paroush J (1984) The significance of independent decisions in uncertain dichotomous choice situations. Theory and Decision 17 (1): 47-60

Nitzan S, Paroush J (1985) Collective Decision-Making: An Economic Outlook, Cambridge University Press, London

Persico N (2002) Committee design with endogenous information. Review of Economic Studies, forthcoming

Pete A, Pattipati KR, Kleinman DL (1993) Optimal team and individual decision rules in uncertain dichotomous situations. Public Choice 75: 205-230

Sah R (1990) An explicit closed-form formula for profit-maximizing *k*-out-of-*n* systems subject to two kinds of failures. Microelectronics and Reliability 30:1123-1130

Sah R (1991) Fallibility in human organizations and political systems. Journal of Economic Perspectives 5: 67-88

Sah R, Stiglitz J (1985) The theory of economic systems: Human fallibility and economic organization. American Economic Review Papers and Proceedings 75: 292-297

Sah R, Stiglitz J (1986) The architecture of economic systems: Hierarchies and polyarchies. American Economic Review 76: 716-727

Sah R, Stiglitz J (1988) Committees, hierarchies and polyarchies, Economic Journal 98: 451 – 470

Stiglitz J (2002) Information and the change in the paradigm in economics, American Economic Review 92 (3): 460-501

Figure 1: The optimal sequential decision architecture

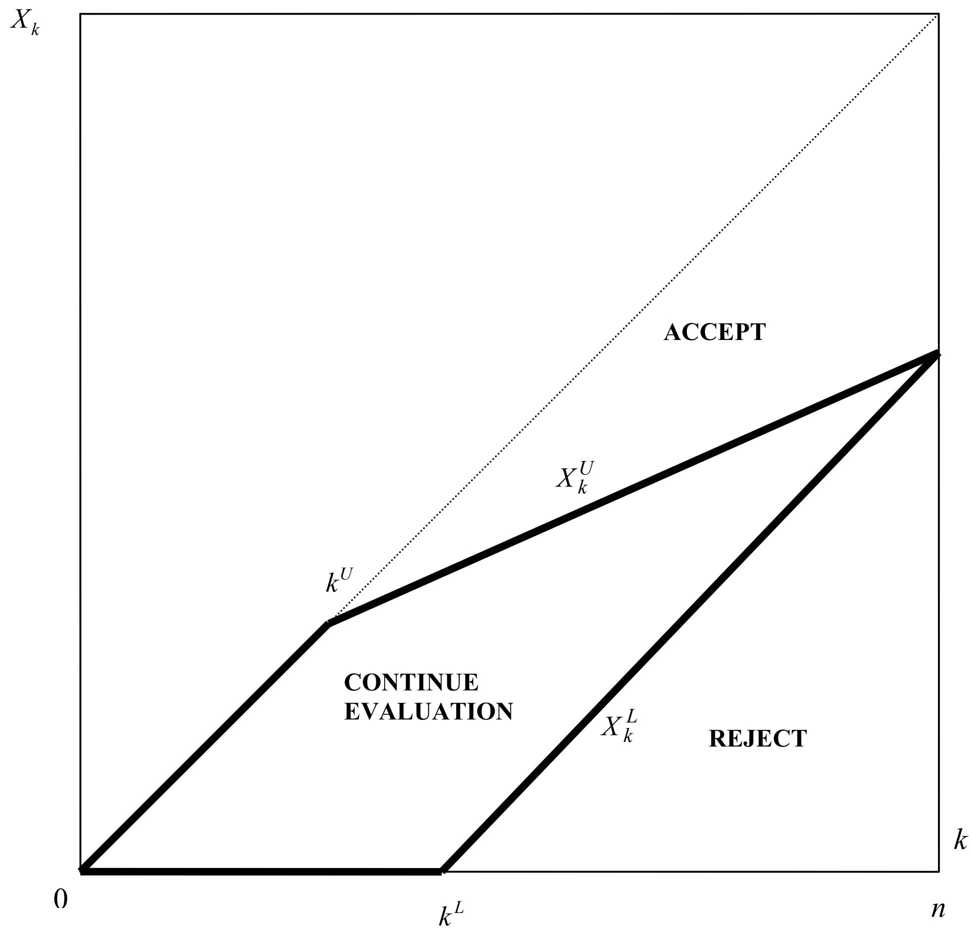

Figure 2: An illustration of $V_k(p,k) \equiv \text{Max}\{R_a(p), R_r(p), R_c(p,k)\}$