Controlling a Population*

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Abstract

We introduce a new setting where a population of agents, each modelled by a finite-state system, are controlled uniformly: the controller applies the same action to every agent. The framework is largely inspired by the control of a biological system, namely a population of yeasts, where the controller may only change the environment common to all cells. We study a synchronisation problem for such populations: no matter how individual agents react to the actions of the controller, the controller aims at driving all agents synchronously to a target state. The agents are naturally represented by a non-deterministic finite state automaton (NFA), the same for every agent, and the whole system is encoded as a 2-player game. The first player (Controller) chooses actions, and the second player (Agents) resolves non-determinism for each agent. The game with m agents is called the m-population game. This gives rise to a parameterized control problem (where control refers to 2 player games), namely the *population control problem*: can Controller control the m-population game for all $m \in \mathbb{N}$ whatever Agents does?

In this paper, we prove that the population control problem is decidable, and it is a EXPTIME-complete problem. As far as we know, this is one of the first results on parameterized control. Our algorithm, not based on cut-off techniques, produces winning strategies which are symbolic, that is, they do not need to count precisely how the population is spread between states. We also show that if there is no winning strategy, then there is a population size M such that Controller wins the m-population game if and only if $m \leq M$. Surprisingly, M can be doubly exponential in the number of states of the NFA, with tight upper and lower bounds.

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1 Introduction

Finite-state controllers, implemented by software, find applications in many different domains: telecommunication, planes, etc. There have been many theoretical studies from the model checking community to show that finite-state controllers are sufficient to control systems in idealised settings. Usually, the problem would be modeled as a game: some players model the controller, and some players model the system [5], the game settings (number of players, their power, their observation) depending on the context.

Lately, finite-state controllers have been used to control living organisms, such as a population of yeasts [23]. In this application, microscopy is used to monitor the fluorescence

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level of a population of yeasts, reflecting the concentration of some molecule, which differs from cell to cell. Finite-state systems can model a discretisation of the population of yeasts [23, 3]. A controller decide the frequency and duration of injections of a sorbitol solution, uniform over the yeast population. However, the response of each cell to the osmotic stress induced by sorbitol varies, influencing the concentration of the fluorescent molecule. The objective is to control the population to drive it through a sequence of predetermined fluorescence states.

In this paper, we model this system of yeasts in an *idealised* setting: we require the (perfectly-informed) controller to surely lead synchronously all agents of a population to a state (one of the predetermined fluorescence states). Such a population control problem does not fit in traditional frameworks from the model checking community. We thus introduce the *m-population game*, where a population of m identical agents is controlled uniformly. Each agent is modeled as a nondeterministic finite-state automaton (NFA), the same for each agent. The first player, called Controller, applies the same action, a letter from the NFA alphabet, to every agent. Its opponent, called Agents, chooses the reaction of each individual agent. These reactions can be different due to non determinism. The objective for Controller is to gather all agents synchronously in the target state (which can be a sink state w.l.o.g.), and Agents seeks the opposite objective. While this idealised setting may not be entirely satisfactory, it constitutes a simple setting, as a first step towards more complex settings.

Dealing with large populations *explicitly* is in general intractable due to the state-space explosion problem. We thus consider the associated *symbolic parameterized control problem*, asking to reach the goal independently of the population size. We prove that this problem is decidable. While *parameterized verification* received recently quite some attention (see related work), our results are one of the first on *parameterized control*, as far as we know.

Our results. We first show that considering an infinite population is not equivalent to the parameterized control problem for all non zero integer m: there are cases where Controller cannot control an infinite population but can control every finite population. Solving the ∞-population game reduces to checking a reachability property on the support graph [21], which can be easily done in PSPACE. On the other hand, solving the parameterized control problem requires new proof techniques, data structures and algorithms.

We easily obtain that when the answer to the population control problem is negative, there exists a population size M, called the *cut-off*, such that Controller wins the m-population game if and only if $m \leq M$. Surprisingly, we obtain a lower-bound on the cut-off doubly exponential in the number of states of the NFA. Following usual cut-off techniques would thus yield an inefficient algorithm of complexity at least 2EXPTIME.

To obtain better complexity, we developped new proof techniques (*not* based on cut-off techniques). Using them, we prove that the population control problem is EXPTIME-complete. As a byproduct, we obtain a doubly exponential upper-bound for the cut-off, matching the lower-bound. Our techniques are based on a reduction to a parity game with exponentially many states and a polynomial number of priorities. The parity game gives some insight on the winning strategies of Controller in the m-population games. Controller selects actions based on a set of *transfer graphs*, giving for each current state the set of states at time i from which agent came from, for different values of i. We show that it suffices for Controller to remember at most a quadratic number of such transfer graphs, corresponding to a quadratic number of indices i. If Controller wins this parity game then he can uniformly apply his winning strategy to all m-population games, just keeping track of these transfer graphs, independently of the exact count in each state. If Agents wins the parity game then he also has a uniform winning strategy in m-population games, for m large enough, which consists

in splitting the agents evenly among all transitions of the transfer graphs. Missing proofs are available in the research report [6].

Related work. Parameterized verification of systems with many identical components started with the seminal work of German and Sistla in the early nineties [16], and received recently quite some attention. The decidability and complexity of these problems typically depend on the communication means, and on whether the system contains a leader (following a different template) as exposed in the recent survey [13]. This framework has been extended to timed automata templates [2, 1] and probabilistic systems with Markov decision processes templates [7, 8]. Another line of work considers population protocols [4, 15]. Close in spirit, are broadcast protocols [14], in which one action may move an arbitrary number of agents from one state to another. Our model can be modeled as a subclass of broadcast protocols, where broadcasts emissions are self loops at a unique state, and no other synchronisation allowed. The parameterized reachability question considered for broadcast protocols is trivial in our framework, while our parameterized control question would be undecidable for broadcast protocols. In these different works, components interact directly, while in our work, the interaction is indirect via the common action of the controller. Further, the problems considered in related work are pure verification questions, and do not tackle the difficult issue of synthesising a controller for all instances of a parameterized system, which we do.

There are very few contributions pertaining to parameterized games with more than one player. The most related is [20], which proves decidability of control of mutual exclusion-like protocols in the presence of an unbounded number of agents. Another contribution in that domain is the one of broadcast networks of identical parity games [8]. However, the game is used to solve a verification (reachability) question rather than a parametrized control problem as in our case. Also the roles of the two players are quite different.

The winning condition we are considering is close to *synchronising words*. The original synchronising word problem asks for the existence of a word w and a state q of a *deterministic* finite state automaton, such that no matter the initial state s, reading w from s would lead to state q (see [24] for a survey). Lately, synchronising words have been extended to NFAs [21]. Compared to our settings, the author assumes a possibly infinite population of agents, who could leak arbitrarily often from a state to another. The setting is thus not parametrized, and a usual support arena suffices to obtain a PSPACE algorithm. Synchronisation for probabilistic models [11, 12] have also been considered: the population of agents is not finite nor discrete, but rather continuous, represented as a distribution. The distribution evolves deterministically with the choice of the controller (the probability mass is split according to the probabilities of the transitions), while in our setting, each agent can non deterministically move. In [11], the controller needs to apply the same action whatever the state the agents are in (like our setting), and then the existence of a controller is undecidable. In [12], the controller can choose the action depending on the state each agent is in (unlike our setting), and the existence of a controller reaching uniformly a set of states is PSPACE-complete.

Last, our parameterized control problem can be encoded as a 2-player game on VASS [9], with one counter per state of the NFA: the opponent gets to choose the population size (a counter value), and the move of each agent corresponds to decrementing a counter and incrementing another. Such a reduction yields a symmetrical game on VASS in which both players are allowed to modify the counter values, in order to check that the other player did not cheat. Symmetrical games on VASS are undecidable [9], and their asymmetric variant (in which only one player is allowed to change the counter values) are decidable in 2EXPTIME [19], thus with higher complexity than our specific parameterized control problem.

2 The population control problem

2.1 The m-population game

A nondeterministic finite automaton (NFA for short) is a tuple $\mathcal{A} = (Q, \Sigma, q_0, \Delta)$ with Q a finite set of states, Σ a finite alphabet, $q_0 \in Q$ an initial state, and $\Delta \subseteq Q \times \Sigma \times Q$ the transition relation. We assume throughout the paper that NFAs are complete, that is, $\forall q \in Q, a \in \Sigma\ , \exists p \in Q :\ (q, a, p) \in \Delta$. In the following, incomplete NFAs, especially in figures, have to be understood as completed with a sink state.

For every integer m, we consider a system $\mathcal{A}^m$ with m identical agents $\mathcal{A}_1, \ldots, \mathcal{A}_m$ of the NFA $\mathcal{A}$. The system $\mathcal{A}^m$ is itself an NFA $(Q^m, \Sigma, q_0^m, \Delta^m)$ defined as follows. Formally, states of $\mathcal{A}^m$ are called configurations, and they are tuples $\mathbf{q} = (q_1, \ldots, q_m) \in Q^m$ describing the current state of each agent in the population. We use the shorthand $\mathbf{q}_0[m]$, or simply $\mathbf{q}_0$ when m is clear from context, to denote the initial configuration $(q_0, \ldots, q_0)$ of $\mathcal{A}^m$. Given a target state $f \in Q$, the f-synchronizing configuration is $f^m = (f, \ldots, f)$ in which each agent is in the target state.

The intuitive semantics of $\mathcal{A}^m$ is that at each step, the same action from Σ applies to all agents. The effect of the action however may not be uniform given the nondeterminism present in $\mathcal{A}$: we have $((q_1, \ldots, q_m), a, (q'_1, \ldots, q'_m)) \in \Delta^m$ iff $(q_j, a, q'_j) \in \Delta$ for all $j \leq m$. A (finite or infinite) play in $\mathcal{A}^m$ is an alternating sequence of configurations and actions, starting in the initial configuration: $\pi = \mathbf{q}_0 a_0 \mathbf{q}_1 a_1 \cdots$ such that $(\mathbf{q}_i, a_i, \mathbf{q}_{i+1}) \in \Delta^m$ for all i.

This is the *m-population game* between Controller and Agents, where Controller chooses the actions and Agents chooses how to resolve non-determinism. The objective for Controller is to gather all agents synchronously in f while Agents seeks the opposite objective.

Our parameterized control problem asks whether Controller can win the m-population game for every $m \in \mathbb{N}$. A strategy of Controller in the m-population game is a function mapping finite plays to actions, $\sigma : (Q^m \times \Sigma)^* \times Q^m \to \Sigma$. A play $\pi = \mathbf{q}_0 a_0 \mathbf{q}_1 a_1 \mathbf{q}_2 \cdots$ is said to *respect* σ, or is a *play under* σ, if it satisfies $a_i = \sigma(\mathbf{q}_0 a_0 \mathbf{q}_1 \cdots \mathbf{q}_i)$ for all $i \in \mathbb{N}$. A play $\pi = \mathbf{q}_0 a_0 \mathbf{q}_1 a_1 \mathbf{q}_2 \cdots$ is *winning* if it hits the f-synchronizing configuration, that is $\mathbf{q}_j = f^m$ for some $j \in \mathbb{N}$. Controller wins the m-population game if he has a strategy such that all plays under this strategy are winning. One can assume without loss of generality that f is a sink state. If not, it suffices to add a new action leading tokens from f to the new target sink state ☺ and tokens from other states to a losing sink state ☹. The goal of this paper is to study the following parameterized control problem:

Population control problem
Input: An NFA $\mathcal{A} = (Q, q_0, q_u, \Sigma, \Delta)$ and a target state $f \in Q$.
Output: Yes iff for every integer m Controller wins the m-population game.

For a fixed m, the winner of the m-population game can be determined by solving the underlying reachability game with $|Q|^m$ states, which is intractable for large values of m. On the other hand, the answer to the population control problem gives the winner of the m-population game for arbitrary large values of m. To obtain a decision procedure for this parameterised problem, new data structures and algorithmic tools need to be developed, much more elaborate than the standard algorithm solving reachability games.

▶ **Example 1.** We illustrate the population control problem with the example $\mathcal{A}_{\mathsf{split}}$ on alphabet $\{a, b, \delta\}$ in Figure 1. Here, to represent configurations we use a counting abstraction, and identify $\mathbf{q}$ with the vector (n_0, n_1, n_2, n_3), where n_0 is the number of agents in state q_0, and so on. Under these notations, there is a way to gather agents synchronously to f. We can

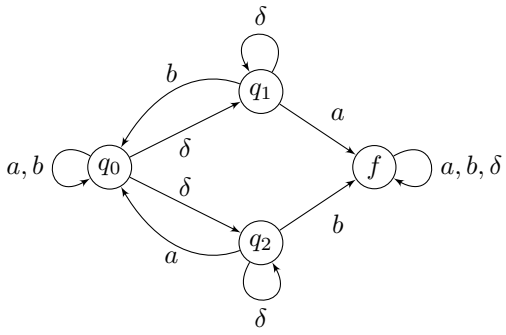

Figure 1 An exemple of NFA: The splitting gadget $\mathcal{A}_{\mathsf{split}}$.

give a symbolic representation of a memoryless winning strategy σ: $\forall k_0, k_1 > 0,\ \forall k_2, k_3 \geq 0,\ \sigma(k_0, 0, 0, k_3) = \delta,\ \sigma(0, k_1, k_2, k_3) = a,\ \sigma(0, 0, k_2, k_3) = b$. Indeed, the number of agents outside f decreases by at least one at every other step. The properties of this example will be detailed later and play a part in proving a lower bound (see Proposition 20).

2.2 Parameterized control and cut-off

A first observation for the population control problem is that $\mathbf{q}_0[m]$, f^m and Q^m are stable under a permutation of coordinates. A consequence is that the m-population game is also symmetric under permutation, and thus the set of winning configurations is symmetric and the winning strategy can be chosen uniformly from symmetric winning configurations. Therefore, if Controller wins the m-population game then he has a positional winning strategy which only counts the number of agents in each state of $\mathcal{A}$ (the counting abstraction used in Example 1).

▶ **Proposition 2.** *Let $m \in \mathbb{N}$. If Controller wins the m-population game, then he wins the m'-population game for every $m' \leq m$.*

The idea to define $\sigma_{m'}$ is to simulate the missing $m-m'$ agents arbitrarily and apply σ_m.

Hence, when the answer to the population control problem is negative, there exists a *cut-off*, that is a value $M \in \mathbb{N}$ such that for every $m < M$, Controller has a winning strategy in $\mathcal{A}^m$, and for every $m \geq M$, he has no winning strategy.

▶ **Example 3.** To illustrate the notion of cut-off, consider the NFA on alphabet $A \cup \{b\}$ from Figure 2. Unspecified transitions lead to a sink state ☹.

The cut-off is $M = |Q| - 2$ in this case. Indeed, we have the following two directions:

On the one hand, for $m < M$, there is a winning strategy σ_m in $\mathcal{A}^m$ to reach f^m, in just two steps. It first plays b, and because $m < M$, in the next configuration, there is at least one state q_i such that no agent is in q_i. It then suffices to play a_i to win.

Now, if $m \geq M$, there is no winning strategy to synchronize in f, since after the first b, agents can be spread so that there is at least one agent in each state q_i. From there, Controller can either play action b and restart the whole game, or play any action a_i, leading at least one agent to the sink state ☹.

2.3 Main results

Our main result is the decidability of the population control problem:

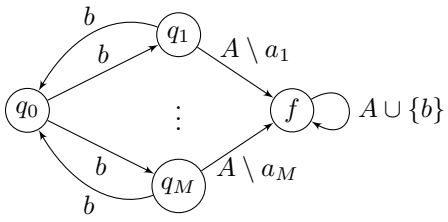

Figure 2 Illustration of the cut-off.

▶ **Theorem 4.** *The population control problem is* EXPTIME*-complete.*

When the answer to the population control problem is positive, there exists a symbolic strategy σ, applicable to all instances $\mathcal{A}^m$, that does not need to count the number of agents in each state. This symbolic strategy requires exponential memory. Otherwise, the cut-off is at most doubly exponential, which is asymptotically tight.

▶ **Theorem 5.** *In case the answer to the population control problem is negative, the cut-off is at most* $\leq 2^{2^{O(|Q|^4)}}$. *There is a family of NFA* $(\mathcal{A}_n)$ *of size* $O(n)$ *and whose cut-off is* 2^{2^n}.

3 The capacity game

The objective of this section is to show that the population control problem is equivalent to solving a game called the *capacity game*. To introduce useful notations, we first recall the population game with infinitely many agents, as studied in [21] (see also [22] p.81).

3.1 The ∞-population game

To study the ∞-population game, the behaviour of infinitely many agents is abstracted into *supports* which keep track of the set of states in which at least one agent is. We thus introduce the *support game*, which relies on the notion of *transfer graphs*. Formally, a transfer graph is a subset of $Q \times Q$ describing how agents are moved during one step. The domain of a transfer graph G is $\mathrm{Dom}(G) = \{q \in Q \mid \exists (q, r) \in G\}$ and its image is $\mathrm{Im}(G) = \{r \in Q \mid \exists (q, r) \in G\}$. Given an NFA $\mathcal{A} = (Q, \Sigma, q_0, \Delta)$ and $a \in \Sigma$, the transfer graph G is compatible with a if for every edge (q, r) of G, $(q, a, r) \in \Delta$. We write $\mathcal{G}$ for the set of transfer graphs.

The *support game* of an NFA $\mathcal{A}$ is a two-player reachability game played by Controller and Agents on the *support arena* as follows. States are supports, *i.e.*, non-empty subsets of Q and the play starts in $\{q_0\}$. The goal support is $\{f\}$. From a support S, first Controller chooses a letter $a \in \Sigma$, then Agents chooses a transfer graph G compatible with a and such that $\mathrm{Dom}(G) = S$, and the next support is $\mathrm{Im}(G)$. A play in the support arena is described by the sequence $\rho = S_0 \xrightarrow{a_1, G_1} S_1 \xrightarrow{a_2, G_2} \dots$ of supports and actions (letters and transfer graphs) of the players. Here, Agents best strategy is to play the maximal graph possible (this is not the case with discrete populations), and we obtain a PSPACE-complete algorithm [21]:

▶ **Proposition 6.** *Controller wins the* ∞*-population game iff he wins the support game.*

This result cannot be used for deciding the population control problem, because Controller might win every m-population game (with $m < \infty$) and at the same time lose the ∞-population game. For that, consider the example from Figure 1. As already shown, Controller wins any m-population game with $m < \infty$. However, Agents can win the ∞-population game by splitting agents from q_0 to both q_1 and q_2 each time Controller plays δ. This way, the sequence of supports is $\{q_0\}\{q_1, q_2\}(\{q_0, f\}\{q_1, q_2, f\})^*$, which never hits $\{f\}$.

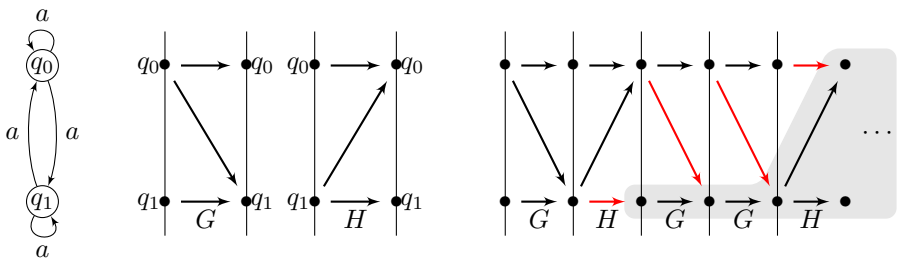

Figure 3 An NFA, two transfer graphs, and a play with finite yet unbounded capacity.

3.2 Realisable plays

Plays of the m-population game (for $m < \infty$) can be abstracted as plays in the support game, by forgetting the identity of agents and keeping only track of edges that are used by at least one agent. Formally, given a play $\pi = \mathbf{q}_0 a_0 \mathbf{q}_1 a_1 \mathbf{q}_2 \cdots$ of the m-population game, define for every integer n, $S_n = \{\mathbf{q}_n[i] \mid 1 \leq i \leq m\}$ and $G_{n+1} = \{(\mathbf{q}_n[i], \mathbf{q}_{n+1}[i]) \mid 1 \leq i \leq m\}$. We denote $\Phi_m(\pi)$ the play $S_0 \xrightarrow{a_1,G_1} S_1 \xrightarrow{a_2,G_2} \dots$ in the support arena, called the projection of π.

Not every play in the support arena can be obtained by projection. This is the reason for introducing the notion of realisable plays:

▶ **Definition 7** (Realisable plays)**.** A play of the support game is *realisable* if there exists $m < \infty$ such that it is the projection by Φ_m of a play in the m-population game.

To characterise realisability, we introduce entries of accumulators:

▶ **Definition 8.** Let $\rho = S_0 \xrightarrow{a_1,G_1} S_1 \xrightarrow{a_2,G_2} \dots$ be a play in the support arena. An *accumulator* of ρ is a sequence $T = (T_j)_{j\in\mathbb{N}}$ such that for every integer j, $T_j \subseteq S_j$, and which is *successor-closed i.e.*, for every $j \in \mathbb{N}$, $(s \in T_j \wedge (s,t) \in G_{j+1}) \implies t \in T_{j+1}$. For every $j \in \mathbb{N}$, an edge $(s,t) \in G_{j+1}$ is an *entry* to T if $s \notin T_j$ and $t \in T_{j+1}$.

▶ **Definition 9** (Plays with finite and bounded capacity)**.** A play has *finite capacity* if all its accumulators have finitely many entries, *infinite capacity* otherwise, and *bounded capacity* if the number of entries of its accumulators is bounded.

Realisability is actually equivalent to *bounded* capacity:

▶ **Lemma 10.** *A play is realisable iff it has bounded capacity.*

An example is given on Figure 3 which represents an NFA, two transfer graphs G and H, and a play $GHG^2HG^3\cdots$. Obviously, this play is not realisable because at least n agents are needed to realise n transfer graphs G in a row: at each G step, at least one agent moves from q_0 to q_1, and no new agent enters q_0. A simple analysis shows that there are only two kinds of non-trivial accumulators $(T_j)_{j\in\mathbb{N}}$ depending on whether their first non-empty T_j is $\{q_0\}$ or $\{q_1\}$. We call these top and bottom accumulators, respectively. All accumulators have finitely many entries, thus the play has finite capacity. However, for every $n \in \mathbb{N}$ there is a bottom accumulator with $2n$ entries. As an example, a bottom accumulator with 4 entries (in red) is depicted on the figure. Therefore, the capacity of this play is not bounded.

3.3 The capacity game

An idea to obtain a game on the support arena equivalent with the population control problem is to make Agents lose whenever the play is not realisable, *i.e.* whenever the play

has unbounded capacity. One issue with (un)bounded capacity is however that it is not a regular property for runs. Hence, it is not easy to use it as a winning condition. On the contrary, *finite* capacity is a regular property. We thus relax (un)bounded capacity using (in)finite capacity and define the corresponding abstraction of the population game:

▶ **Definition 11** (Capacity game)**.** The *capacity game* is the game played on the support arena, where Controller wins a play iff either the play reaches $\{f\}$ or the play has infinite capacity. A player *wins the capacity game* if he has a winning strategy in this game.

We show that this relaxation can be used to decide the population control problem.

▶ **Theorem 12.** *The answer to the population control problem is positive iff Controller wins the capacity game.*

This theorem is a direct corollary of the following proposition:

▶ **Proposition 13.** *Either Controller or Agents wins the capacity game, and the winner has a winning strategy with* finite memory*. In case Controller is the winner of the capacity game, he wins all m-population games, for every integer m. In case Agents wins the capacity game with a strategy with finite memory of size M, he wins the $|Q|^{1+|\mathsf{M}|\cdot 4^{|Q|}}$-population game.*

Proof of first and second assertions. We start with the first assertion. Whether a play has infinite capacity can be verified by a non-deterministic Büchi automaton of size $2^{|Q|}$ on the alphabet of transfer graphs, which guesses an accumulator on the fly and checks that it has infinitely many entries. This Büchi automaton can be determinised into a parity automaton (*e.g.* using Safra's construction) with state space M of size $\mathcal{O}\left(2^{2^{|Q|}}\right)$. The synchronized product of this deterministic parity automaton with the support game produces a parity game which is equivalent with the capacity game, in the sense that, up to unambigous synchronization with the deterministic automaton, plays and strategies in both games are the same and the synchronization preserves winning plays and strategies. Since parity games are determined and positional [25], either Controller or Agents has a positional winning strategy in the parity game, thus either Controller or Agents has a winning strategy with finite memory M in the capacity game.

Let us prove the second assertion. Assuming that Controller wins the capacity game with a strategy σ, he can win any m-population game, $m < \infty$, with the strategy $\sigma_m = \sigma \circ \Phi_m$. The projection $\Phi_m(\pi)$ of every infinite play π respecting σ_m is realisable, thus $\Phi_m(\pi)$ has bounded, hence finite, capacity (Lemma 10). Moreover $\Phi_m(\pi)$ respects σ, and since σ wins the capacity game, $\Phi_m(\pi)$ reaches $\{f\}$. Thus π reaches f^m and σ_m is winning.

The last assertion is proved in [6]. ◀

As consequence of Proposition 13, the population control problem can be decided by explicitely computing the parity game and solving it, in 2EXPTIME. In the next section we will improve this complexity bound to EXPTIME.

We conclude with an example showing that, in general, positional strategies are not sufficient to win the capacity game. Consider the example of Figure 4, where the only way for Controller to win is to reach a support without q_2 and play c. With a memoryless strategy, Controller cannot win the capacity game. There are only two memoryless strategies from support $S = \{q_1, q_2, q_3, q_4\}$. If Controller only plays a from S, the support remains S and the play has bounded capacity. If he only plays b's from S, then Agents can split tokens from q_3 to both q_2, q_4 and the play remains in support S, with bounded capacity. In both cases, the play has finite capacity and Controller loses.

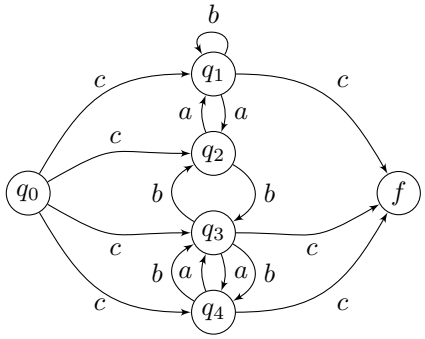

Figure 4 Population game where Controller needs memory to win the associated capacity game.

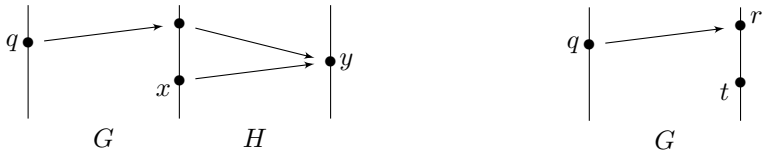

Figure 5 Left: G leaks at H; Right: G separates (r, t).

However, Controller can win the capacity game. His (finite-memory) winning strategy σ consists in first playing c, and then playing alternatively a and b, until the support does not contain $\{q_2\}$, in which case he plays c to win. Two consecutive steps ab send q_2 to q_1, q_1 to q_3, q_3 to q_3, and q_4 to either q_4 or q_2. To prevent Controller from playing c and win, Agents needs to spread from q_4 to both q_4 and q_2 every time ab is played. Consider the accumulator T defined by $T_{2i} = \{q_1, q_2, q_3\}$ and $T_{2i-1} = \{q_1, q_2, q_4\}$ for every $i > 0$. It has an infinite number of entries (from q_4 to T_{2i}). Hence Controller wins if this play is executed. Else, Agents eventually keeps all agents from q_4 in q_4 when ab is played, implying the next support does not contain q_2. Strategy σ is thus a winning strategy for Controller.

4 Solving the capacity game in EXPTIME

To solve efficiently the capacity game, we build an equivalent exponential size parity game with a polynomial number of parities. To do so, we enrich the support arena with a *tracking list* responsible of checking whether the play has finite capacity. The tracking list is a list of transfer graphs, which are used to detect certain patterns called *leaks*.

4.1 Leaking graphs

In order to detect whether a play $\rho = S_0 \xrightarrow{a_1,G_1} S_1 \xrightarrow{a_2,G_2} \dots$ has finite capacity, it is enough to detect *leaking* graphs (characterising entries of accumulators). Further, leaking graphs have special *separation* properties which will allow us to track a small number of graphs. For G, H two graphs, we denote $(a, b) \in G \cdot H$ iff there exists z with $(a, z) \in G$, and $(z, b) \in H$.

▶ **Definition 14** (Leaks and separations). Let G, H be two transfer graphs. We say that G *leaks at* H if there exist states q, x, y with $(q, y) \in G \cdot H$, $(x, y) \in H$ and $(q, x) \notin G$. We say that G *separates* a pair of states (r, t) if there exists $q \in Q$ with $(q, r) \in G$ and $(q, t) \notin G$.

The tracking list will be composed of concatenated graphs *tracking* i of the form $G[i, j] = G_{i+1} \cdots G_j$ relating S_i with S_j: $(s_i, s_j) \in G[i, j]$ if there exists $(s_k)_{i<k<j}$ with $(s_k, s_{k+1}) \in$

G_{k+1} for all $i \leq k \leq j$. Infinite capacity relates to leaks in the following way:

▶ **Lemma 15.** *A play has infinite capacity iff there exists an index i such that $G[i,j]$ leaks at G_{j+1} for infinitely many indices j.*

In this case, we say that index i *leaks infinitely often*. Note that if G separates (r,t), and r,t have a common successor by H, then G leaks at H. To link leaks with separations, we consider for each index k, the pairs of states that have a common successor, in possibly several steps, as expressed by the symmetric relation R_k: $(r,t) \in R_k$ iff there exists $j \geq k$ and $y \in Q$ such that $(r,y) \in G[k,j] \wedge (t,y) \in G[k,j]$.

▶ **Lemma 16.** *For $i < n$ two indices, the following three properties hold:*
1. *If $G[i,n]$ separates $(r,t) \in R_n$, then there exists $m \geq n$ such that $G[i,m]$ leaks at G_{m+1}.*
2. *If index i does not leak infinitely often, then the number of indices j such that $G[i,j]$ separates some $(r,t) \in R_j$ is finite.*
3. *If index i leaks infinitely often, then for all $j > i$, $G[i,j]$ separates some $(r,t) \in R_j$.*

4.2 The tracking list

The *tracking list* exploits the relationship between leaks and separations. It is a list of transfer graphs which altogether separate all possible pairs, and are sufficient to detect when leaks occur. Notice that telling at step j whether the pair (r,t) belongs to R_j cannot be performed by a deterministic automaton. We thus *a priori* have to consider every pair $(r,t) \in Q^2$ for separation. The tracking list $\mathcal{L}_n$ at step n is defined inductively as follows. $\mathcal{L}_0$ is the empty list, and for $n > 0$, the list $\mathcal{L}_n$ is computed in three stages:
1. first, every graph H in the list $\mathcal{L}_{n-1}$ is concatenated with G_n, yielding $H \cdot G_n$;
2. second, G_n is added at the end of the obtained list;
3. last, the list is filtered: a graph H is kept if and only if it separates a pair of states $(p,q) \in Q^2$ which is not separated by any graph that appears earlier in the list.

Because of the third item, there are at most $|Q|^2$ graphs in the tracking list. The list may become empty if no pair of states is separated by any graph, for example if all the graphs are complete. Let $\mathcal{L}_n = \{H_1, \cdots, H_\ell\}$ be the tracking list at step n. Then each transfer graph $H_r \in \mathcal{L}_n$ is of the form $H_r = G[t_r, n]$. We say that r is the *level* of H_r, and t_r the *index tracked* by H_r. Observe that the lower the level of a graph in the list, the smaller the index it tracks. When we consider the sequence of tracking lists $(\mathcal{L}_n)_{n\in\mathbb{N}}$, for every index i, either it eventually stops to be tracked or it is tracked forever from step i, *i.e.* for every $n \geq i$, $G[i,n]$ belongs to $\mathcal{L}_n$. In the latter case, i is said to be *remanent* (because it will never disappear).

Using Lemma 15 and the second and third statements of Lemma 16, we obtain:

▶ **Lemma 17.** *A play has infinite capacity iff there exists an index i such that i is remanent and leaks infinitely often.*

4.3 The parity game

We now describe a parity game $\mathcal{PG}$, which extends the support arena with on-the-fly computation of the tracking list.

Priorities. By convention, lowest priorities are the most important and the odd parity is good for Controller, so Controller wins iff the lim inf of the priorities is odd. With each level $1 \leq r \leq |Q|^2$ of the tracking list are associated two priorities $2r$ and $2r+1$, and on top of that are added priorities 1 and $2|Q|^2+2$, hence the set of all priorities is $\{1, \ldots, 2|Q|^2+2\}$.

When Agents chooses a transition labelled by a transfer graph G, the tracking list is updated with G and the priority of the transition is determined as the smallest among: priority 1 if the support $\{f\}$ has ever been visited, priority $2r+1$ for the smallest r such that H_r (from level r) leaks at G, priority $2r$ for the smallest level r where a graph was removed, and in all other cases priority $2|Q|^2+2$.

States and transitions. $\mathcal{G}^{\leq |Q|^2}$ denotes the set of list of at most $|Q|^2$ transfer graphs.

- States of $\mathcal{PG}$ form a subset of $\{0,1\} \times 2^Q \times \mathcal{G}^{\leq |Q|^2}$, each state being of the form $(b, S, H_1, \dots, H_\ell)$ with $b \in \{0,1\}$ a bit indicating whether a support in $\{f\}$ has been seen, S the current support and $(H_1, \dots, H_\ell)$ the tracking list. The initial state is $(0, \{q_0\}, \emptyset)$.
- Transitions in $\mathcal{PG}$ are all $(b, S, H_1, \dots, H_\ell) \xrightarrow{\mathbf{p},a,G} (b', S', H'_1, \dots, H'_{\ell'})$ where $\mathbf{p}$ is the priority, and such that $S \xrightarrow{a,G} S'$ is a transition of the support arena, and
  1. $(H'_1, \dots, H'_{\ell'})$ is the tracking list obtained by updating the tracking list $(H_1, \dots, H_\ell)$ with G, as explained in subsection 4.2;
  2. if $b = 1$ or if $S' \subseteq F$, then $\mathbf{p} = 1$ and $b' = 1$;
  3. otherwise $b' = 0$. In order to compute the priority $\mathbf{p}$, we let $\mathbf{p}'$ be the smallest level $1 \leq r \leq \ell$ such that H_r leaks at G and $\mathbf{p}' = \ell + 1$ if there is no such level, and we also let $\mathbf{p}''$ as the minimal level $1 \leq r \leq \ell$ such that $H'_r \neq H_r \cdot G$ and $\mathbf{p}'' = \ell + 1$ if there is no such level. Then $\mathbf{p} = \min(2\mathbf{p}' + 1, 2\mathbf{p}'')$.

We are ready to state the main result of this paper, which yields an EXPTIME complexity for the population control problem. This entails the first statement of Theorem 4, and together with Proposition 13, also the first statement of Theorem 5.

▶ **Theorem 18.** *Controller wins the game $\mathcal{PG}$ if and only if Controller wins the capacity game. Solving these games can be done in time $O(2^{(1+|Q|+|Q|^4)(2|Q|^2+2)})$. Strategies with $2^{|Q|^4}$ memory states are sufficient to both Controller and Agents.*

Proof. The state space of parity game $\mathcal{PG}$ is the product of the set of supports with a deterministic automaton computing the tracking list. There is a natural correspondence between plays and strategies in the parity game $\mathcal{PG}$ and in the capacity game.

Controller can win the parity game $\mathcal{PG}$ in two ways: either the play visits the support $\{f\}$, or the priority of the play is $2r+1$ for some level $1 \leq r \leq |Q|^2$. By design of $\mathcal{PG}$, this second possibility occurs iff r is remanent and leaks infinitely often. According to Lemma 17, this occurs if and only if the corresponding play of the capacity game has infinite capacity. Thus Controller wins $\mathcal{PG}$ iff he wins the capacity game.

In the parity game $\mathcal{PG}$, there are at most $2^{1+|Q|}\left(2^{|Q|^2}\right)^{|Q|^2} = 2^{1+|Q|+|Q|^4}$ states and $2|Q|^2+2$ priorities, implying the complexity bound using state-of-the-art algorithms [18]. Actually the complexity is even quasi-polynomial according to the algorithms in [10]. Notice however that this has little impact on the complexity of the population control problem, as the number of priorities is logarithmic in the number of states of our parity game.

Further, it is well known that the winner of a parity game has a positional winning strategy [18]. A *positional* winning strategy σ in the game $\mathcal{PG}$ corresponds to a *finite-memory* winning strategy σ' in the capacity game, whose memory states are the states of $\mathcal{PG}$. Actually in order to play σ', it is enough to remember the tracking list, *i.e.* the third component of the state space of $\mathcal{PG}$. Indeed, the second component, in 2^Q, is redundant with the actual state of the capacity game and the bit in the first component is set to 1 when the play visits $\{f\}$ but in this case the capacity game is won by Controller whatever is played afterwards. Since there at most $2^{|Q|^4}$ different tracking lists, we get the upper bound on the memory. ◀

5 Lower bounds

The proofs of Theorems 4 and 5 are concluded by the proofs of lower bounds.

▶ **Theorem 19.** *The population control problem is* EXPTIME*-hard.*

Proof. We first prove PSPACE-hardness of the population control problem, reducing from the halting problem for polynomial space Turing machines. We then extend the result to obtain the EXPTIME-hardness, by reducing from the halting problem for polynomial space *alternating* Turing machines. Let $\mathcal{M} = (S, \Gamma, T, s_0, s_f)$ be a Turing machine with $\Gamma = \{0, 1\}$ as tape alphabet. By assumption, there exists a polynomial P such that, on initial configuration $x \in \{0,1\}^n$, $\mathcal{M}$ uses at most $P(n)$ tape cells. A transition $t \in T$ is of the form $t = (s, s', b, b', d)$, where s and s' are, respectively, the source and the target control states, b and b' are, respectively, the symbols read from and written on the tape, and $d \in \{\leftarrow, \rightarrow, -\}$ indicates the move of the tape head. From $\mathcal{M}$ and x, we build an NFA $\mathcal{A} = (Q, \Sigma, q_0, \Delta)$ with a distinguished state ☺ such that, $\mathcal{M}$ terminates in s_f on input x if and only if $(\mathcal{A}, ☺)$ is a positive instance of the population control problem.

The high-level description of $\mathcal{A}$ is as follows. States in Q are of several types: contents of the $P(n)$ cells (one state (b, p) per content and per position), position of the tape head (one state p per possible position), control state of the Turing machine (one state s per control state), and three special states, namely an initial state q_0, a sink winning state ☺, and a sink losing state ☹. With each transition $t = (s, s', b, b', d)$ in the Turing machine and each position p of the tape, we associate an action $a_{t,p}$ in $\mathcal{A}$, which simulates the effect of transition t when the head position is p. Thus, on action $a_{t,p}$ there is a transition from the source state s to the target state s', another from the tape head position p to its update according to d, and also from (b, p) to (b', p). Moreover, from head position $q \neq p$, $a_{t,p}$ leads to ☹, so that in any population game, Controller only plays actions associated with the current head position. Similarly from states (b'', p) with $b'' \neq b$, states $s'' \neq s$, action $a_{t,p}$ leads to ☹. Initially, an `init` action is available from q_0 and leads to s_0, to position 0 for the tape head, and to cells (b, p) that encode the initial tape contents on input x. The NFA also has winning actions, that allow one to check that there are no agents in a subset of states, and send the remaining ones to the target ☺. One such action should be played when agents encoding the state of the Turing machine lie in s_f, indicating that $\mathcal{M}$ accepted. Another winning action `win` is played whenever there are not enough agents to encode the initial configuration: Agents needs m to be at least $P(n) + 2$ to fill states corresponding to the initial tape contents ($P(n)$ tokens), the initial control state s_0 and the initial head position. The sink losing state ☹ is used to pinpoint an error in the simulation of $\mathcal{M}$.

Now, in order to encode an alternating Turing machine, we assume that the control states of $\mathcal{M}$ alternate between states of Controller and states of Agents. The NFA $\mathcal{A}$ is extended with a state C, for Controller, and an additional transition labelled `init` from q_0 to C. Assume first, that C contains at most an agent; we will later explain how to impose this. Beyond C, the NFA also contains on state t per transition of $\mathcal{M}$, which will represent that Agents chooses to play transition t. To do so, from state C, for any action $a_{t,p}$, there are transitions to all states t'. From state t, actions of the form $a_{t,p}$ are allowed, leading back to C. That is, actions $a_{t',p}$ with $t' \neq t$ lead from t to the sink losing state ☹. This encodes that Controller must follow the transition t chosen by Agents. To punish Agents in case the current tape contents is not the one expected by the transition $t = (s, s', b, b', d)$ he chooses, there are trashing actions $\texttt{trash}_s$ and $\texttt{trash}_{p,b}$ enabled from state t. Action $\texttt{trash}_s$ leads from t to ☺, and also from s to ☹. Similarly, $\texttt{trash}_{p,b}$ leads from t to ☺ and from any position state $q \neq p$ to ☹, and from (b, p) to ☹. In this way, Agents will not move the token

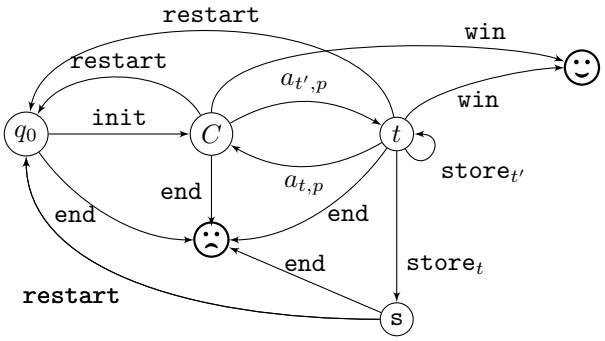

Figure 6 Gadget simulating a single agent in C.

from C to an undesired t. Last, there are transitions on action `end` from state ☺, C and any of the t's to the target state ☺. Moreover, action `end` from any other state (in particular the ones encoding the Turing machine configuration) leads to ☹. This whole construction encodes, assuming that there is a single agent in C after the first transition, that Controller can choose the transition from a Controller state of $\mathcal{M}$, and Agents can choose the transition from an Agents state.

Let us now explain how to deal with the case where Agents places several agents in state C on the initial action `init`, enabling the possibility to later send agents to several t's simultaneously. For that, consider the gadget from Figure 6. We use an extra state `s`, actions $\texttt{store}_t$ for each transition t, and action `restart`. Action $\texttt{store}_t$ leads from t to *store*, and loops on every other state. From all states except ☺ and ☹, action `restart` leads to q_0. Last, the effects of `win` and `end` are modified as follow: `win` leads from (non winning control states) $s \neq s_f$ to ☹ and loops on every other Turing machine , including s_f. It also leads from C and from any t to ☺; `end` goes from q_0, C, the t's and `s` to ☹ (it can be played only if all tokens from Figure 6 are in ☺), and leads all tokens from the Turing machine configuration to ☺.

Assume that input x is not accepted by the alternating Turing machine $\mathcal{M}$, and let m be at least $P(n) + 3$. In the m-population game, Agents has a winning strategy placing initially a single agent in state C. If Controller plays $\texttt{store}_t$ (for some t), either no agents are stored, or the unique agent in C is moved to `s`. Thus Controller cannot play `end` and has no way to lead the agents encoding the Turing machine configuration to ☺, until he plays `restart`, which moves all the agents back to q_0. This shows that $\texttt{store}_t$ is useless to Controller and thus Agents wins.

Conversely, if Controller has a strategy in $\mathcal{M}$ witnessing the acceptance of x, in order to win the m-population game, Agents would need to cheat in the simulation of $\mathcal{M}$ and place at least two agents in C to eventually split them to $t_1, \ldots, t_n$. Then, Controller can play the corresponding actions $\texttt{store}_{t_2}, \ldots, \texttt{store}_{t_n}$ moving all agents (but the ones in t_1) in `s`, after which he plays his winning strategy from t_1 resulting in sending some agents to ☺. Then, Controller plays `restart` and proceeds inductively with strictly less agents from q_0, and eventually plays `end` to win. ◀

Surprisingly, the cut-off can be as high as doubly exponential in the size of the NFA.

▶ **Proposition 20.** *There exists a family of NFA $(\mathcal{A}_n)_{n\in\mathbb{N}}$ such that $|\mathcal{A}_n| = 2n + 7$, and for $M = 2^{2^n+1} + n$, there is no winning strategy in $\mathcal{A}_n^M$ and there is one in $\mathcal{A}_n^{M-1}$.*

Proof. Let $n \in \mathbb{N}$. The NFA $\mathcal{A}_n$ we build is the disjoin union of *two* NFAs with different properties, namely $\mathcal{A}_{\mathsf{split}}, \mathcal{A}_{\mathsf{count},n}$. On the one hand, for $\mathcal{A}_{\mathsf{split}}$, winning the game with m

agents requires $\Theta(\log m)$ steps. On the other hand, $\mathcal{A}_{\mathsf{count},n}$ implements a usual counter over n bits (as used in many different publications), such that Controller can avoid to lose during $O(2^n)$ steps. The combination of these two gadgets ensures a cut-off for $\mathcal{A}_n$ of 2^{2^n}.

Recall Figure 1, which presents the splitting gadget that has the following properties. In $\mathcal{A}^m_{\mathsf{split}}$ with $m \in \mathbb{N}$ agents, $(s1)$ there is a winning strategy ensuring to win in $2\lfloor \log_2 m \rfloor + 2$ steps; $(s2)$ no strategy can ensure to win in less than $2\lfloor \log_2 m \rfloor + 1$ steps.

The counting gadget that implements a counter with states ℓ_i (meaning bit i is 0) and h_i (for bit i is 1) enjoys the following properties: $(c1)$ there is a strategy in $\mathcal{A}_{\mathsf{count},n}$ to ensure avoiding ☹ during 2^n steps, by playing α_i whenever the counter suffix from bit i is $01\cdots1$; $(c2)$ for $m \geq n$, no strategy of $\mathcal{A}^m_{\mathsf{count},n}$ avoid ☹ for 2^n steps.

The two gadgets (splitting and counting) are combined by a new initial state leading by two transitions labeled *init* to the initial states of both NFAs. Actions consist of pairs of actions, one for each gadget: $\Sigma = \{a, b, \delta\} \times \{\alpha_i \mid 1 \leq i \leq n\}$. We add an action $*$ which can be played from any state of $\mathcal{A}_{\mathsf{count},n}$ but ☹, and only from f in $\mathcal{A}_{\mathsf{split}}$, leading to the global target state ☺.

Let $M = 2^{2^n+1} + n$. We deduce that the cut-off is $M - 1$ as follows:

- For M agents, a winning strategy for Agents is to first split n tokens from the initial state to the q_0 of $\mathcal{A}_{\mathsf{count},n}$, in order to fill each l_i with 1 token, and 2^{2^n+1} tokens to the q_0 of $\mathcal{A}_{\mathsf{split}}$. Then Agents splits evenly tokens between q_1, q_2 in $\mathcal{A}_{\mathsf{split}}$. In this way, Controller needs at least $2^n + 1$ steps to reach the final state of $\mathcal{A}_{\mathsf{split}}$ $(s2)$, but Controller reachs ☹ after these $2^n + 1$ steps in $\mathcal{A}_{\mathsf{count},n}$ $(c2)$.
- For $M - 1$ agents, Agents needs to use at least n tokens from the initial state to the q_0 of $\mathcal{A}_{\mathsf{count},n}$, else Controller can win easily. But then there are less than 2^{2^n+1} tokens in the q_0 of $\mathcal{A}_{\mathsf{split}}$. And thus by $(s1)$, Controller can reach f within 2^n steps, after which he still avoids ☹ in $\mathcal{A}_{\mathsf{count},n}$ $(c1)$. And then Controller sends all agents to ☺ using $*$.

Thus, the family $(\mathcal{A}_n)$ of NFA exhibits a doubly exponential cut-off. ◀

6 Discussion

Obtaining an EXPTIME algorithm for the control problem of a population of agents was challenging. We also managed to prove a matching lower-bound. Further, the surprising doubly exponential matching upper and lower bounds on the cut-off imply that the alternative technique, checking that Controller wins all m-population game for m up to the cut-off, is far from being efficient.

The idealised formalism we describe in this paper is not entirely satisfactory: for instance, while each agent can move in a non-deterministic way, unrealistic behaviours can happen, *e.g.* all agents synchronously taking infinitely often the same choice. An almost-sure control problem in a probabilistic formalism should be studied, ruling out such extreme behaviours. As the population is discrete, we may avoid the undecidability that holds for distributions [11] and is inherited from the equivalence with probabilistic automata [17]. Abstracting continuous distributions by a discrete population of arbitrary size could thus be seen as an approximation technique for undecidable formalisms such as probabilistic automata.

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