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# On the Integration of Production and Financial Hedging Decisions in Global Markets

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We study the integrated operational and financial hedging decisions faced by a global firm who sells to both home and foreign markets. Production occurs either at a single facility located in one of the markets or at two facilities, one in each market. The company has to invest in capacity before the selling season starts when the demand in both markets and the currency exchange rate are uncertain. The currency exchange rate risk can be hedged by delaying allocation of the capacity to specific markets until both the currency and demand uncertainties are resolved and/or by buying financial option contracts on the currency exchange rate when capacity commitment is made. A mean-variance utility function is used to model the firm's risk aversion in decision making. We derive the joint optimal capacity and financial option decision, and analyze the impact of the delayed allocation option and the financial options on capacity commitment and the firm's performance. We show that the firm's financial hedging strategy ties closely to, and can have both quantitative and qualitative impact on, the firm's operational strategy. The use, or lack of use of financial hedges, can go beyond affecting the magnitude of capacity levels by altering global supply chain structural choices, such as the desired location and number of production facilities to be employed to meet global demand.

*Subject classifications:* inventory/production: capacity, allocation, stochastic; finance: hedging, currency exchange rate; utility: mean-variance.

*Area of review:* Manufacturing, Service, and Supply Chain Operations.

*History:* Received July 2004; revision received August 2005; accepted April 2006.

## 1. Introduction

### 1.1. Problem Motivation

As firms locate activities of their supply chain all over the world, and products flow across national boundaries, managers face the uncertainties and complexities of the global environment. Exchange rates and price uncertainties in production inputs are two of the complicating factors in the global supply chain environment. Exposure to exchange rates, in particular, affects the underlying economics of any firm dealing with foreign buyers, suppliers, or competitors through its impact on input costs, sales prices, and volume. Such currency fluctuations can be significant (fluctuations of 1% in a day or 20% in a year are not unheard of) with drastic impact on production and sourcing costs (Dornier et al. 1998).

Companies have employed different risk-management approaches to cope with exchange rate and input price uncertainties. The typical way is to use financial markets, whenever possible, to hedge against such risks. Currency options are the most frequently used tools for hedging currency exposure (O'Brien 1996). Options are financial instruments that allow a firm to buy the right, but not the obligation, to sell or buy currencies at set prices. A sometimes overlooked option, but an effective one, is for firms

to use operational strategies as effective hedges against exchange rate and input price uncertainties. Operational hedging strategies, as clearly defined and illustrated in Cohen and Huchzermeier (1999), Cohen and Mallik (1997), and Kouvelis (1999), can be viewed as real compound options that are exercised in response to the demand, price, and exchange-rate contingencies faced by firms in a global supply chain context. Such real options include postponement of assembly and distribution logistics decisions, delaying final commitment of capacity and process technology investments, and/or switching production locations or sourcing partners contingent on demand and/or exchange-rate scenarios. (We are going to restrict our attention to operational hedges with the real option to postpone the deployment of some of the firm resources in response to demand and exchange-rate/price scenarios. For the reader interested in switching options, see the work of Kogut and Kulatilaka 1994, Triantis and Hodder 1990, Li and Kouvelis 1999.)

Even though substantial literature has been developed on both the financial hedging (see O'Brien 1996, and references therein) and the operational hedging practices of price and currency risks (Cohen and Mallik 1997, Kouvelis 1999, and references therein), very little effort has been spent in developing an all-encompassing risk-management approach that effectively integrates financial

and operational hedges. This weakness of the literature is clearly pointed out, and outlined as a future research direction, in Cohen and Huchzermeier (1999), while the potential for its effectiveness is anecdotally exposed via examples in Dornier et al. (1998, Chapter 9). Our research takes a few steps in addressing this gap in the literature. For more detailed positioning of our research within the relevant literature, see §1.2: *Literature Review*.

We study the integration of operational and financial hedging policies of risk-averse global firms within a stylized, but representative, modeling setting. We consider a firm selling to both home and foreign markets. In a two-stage decision framework, early capacity/production commitments and financial hedging are decided in the presence of demand and exchange-rate (price) uncertainty in the first stage, while in the second stage, and after observing demand and exchange-rate realizations, the firm exercises its production “allocation” option in supplying the domestic and foreign market demand (i.e., how many units to “localize” and distribute to the two markets). The emphasis of our analysis is on clearly establishing the value of the joint use of the operational hedge (“allocation” option) and the financial hedge, and understanding their effects on a risk-averse firm’s capacity decisions and performance. Furthermore, interesting insights are obtained on the nature of optimal, or whenever possible perfect, financial hedges for our modeled environment.

## 1.2. Literature Review

The literature on operational hedging practices of price and exchange rate uncertainty is recent, with the work of Huchzermeier and Cohen (1996) the most influential from a modeling perspective. Their results demonstrate the benefits of operational hedging practices via excess capacity and production switching options in environments of volatile exchange rates. The work of Kogut and Kulatilaka (1994) is along the same direction with the emphasis on explicitly valuing the option of shifting production between two plants located in different countries as exchange rates fluctuate. Kazaz et al. (2005) illustrate that, for an expected profit-maximizing firm, production hedging policies of excess capacity and postponed allocation are features of robust optimal performance under exchange rate uncertainty. Li and Kouvelis (1999) explicitly study flexible and risk-sharing supply contracts under price uncertainty. Their discussion clearly illustrates how operational flexibility, supplier selection, and risk sharing, when carefully exercised, can effectively reduce the sourcing cost in environments of price uncertainty. For a thorough coverage of the vast global supply chain literature, the detailed positioning of the operational hedging research stream within it, and other work peripherally related to it, see Cohen and Mallik (1997), Cohen and Huchzermeier (1999), Van Mieghem (2004), and Boyabatlı and Toktay (2004).

There is a vast literature on the use of financial hedging instruments to better manage price and exchange rate

uncertainties. One could start from the classical papers on option prices of Black and Scholes (1973), Merton (1973), to the more currency option specific papers of Cornell and Reinganum (1981), Biger and Hull (1983), Jorion (2001), Shastri and Tandon (1986), and Bodurtha and Courtadon (1986). This literature develops creative financial instruments and values them to hedge uncertain magnitude cash flows, due to price/exchange rate uncertainties, but without consideration of how production decisions and operational hedging schemes might be interacting with the magnitude and variance of such cash flows.

The international finance literature, in particular the research stream dealing with defining and measuring different types of exchange rate exposure (Hodder 1982, Flood and Lessard 1986), clearly recognizes the need for a combination of financial and operational options in effectively managing operating exposure to exchange rate movements. However, beyond some intuitive and anecdotal level, discussions on the relationships between operational flexibility, financial hedging, and exchange risk (see Lessard and Lightstone 1986, or a textbook-level exposition in Shapiro 1988), no structural models or quantitative tools are provided to aid the integrated operational-financial hedging decision making in uncertain price/exchange rate environments. Our research addresses this issue by clearly relating capacity/production plan choices to financial hedging strategies, and showing that for the risk-averse firm the production plans it chooses are functions of both the financial hedging strategies it employs and the opportunity to exercise a real option (“allocation” option) contingent on price and market demand scenarios.

The work of Mello et al. (1995) is the one closest in spirit to our research. They present an integrated model of a multinational firm with flexibility in sourcing its production (i.e., a switching option in sourcing from different countries) and with the use of financial markets to hedge exchange-rate risk. The emphasis of Mello et al. (1995) is on valuing dynamically the operating exposure to exchange-rate movements via a state-contingent model in continuous time, with the model explicitly accounting for the strategic exercise of the switching option (sourcing flexibility) of the firm. Numerical results illustrate the interdependency of sourcing flexibility and hedging strategy in increasing the value of the firm. Chowdhry and Howe (1999) examine the capacity allocation and financial hedging decisions in a setting where the total production capacity is fixed. Our research focus is different, with an emphasis on understanding the implication of the use of the operational hedge (via an “allocation” option) and the financial hedge (via currency option contracts) on the capacity investment decision and firm’s mean-variance (MV) performance. Our model allows us to obtain closed-form formulas for both production plans and currency contract parameters, and thus to quantify the magnitude of the interaction of operational and financial hedges in an uncertain currency exchange rate and demand environment.

Risk-neutral newsvendor models with cost (price) uncertainty as implied in uncertain over-and-under stocking costs are examined in the work of Lowe et al. (1988). Kouvelis and Gutierrez (1997) consider price uncertainty, due to uncertain exchange rates, in a newsvendor problem in a global market under risk neutrality and two ordering opportunities. Gurnani and Tang (1999) consider the issue of demand and forecast updating and uncertain unit costs in a two-ordering instants setting for a risk-neutral newsvendor. Uncertain prices have been studied also in the setting of spot market as an alternative source of buying and selling, with the focus of finding the optimal balance of long-term contract and spot sourcing (see Seifert et al. 2004, Yi and Scheller-Wolf 2003 for risk-neutral settings, and Wu and Kleindorfer 2005, Dong and Liu 2007 for risk-averse settings). Our work differs from the above models in its focus on the risk-averse global newsvendor with a single-ordering (capacity building) opportunity and exploiting the combined use of an “allocation” option and financial hedges for MV-utility optimization.

Risk aversion issues in inventory and capacity management have been captured in the work of Bouakiz and Sobel (1992), Eeckhoudt et al. (1995), Agrawal and Seshadri (2000), and Chen et al. (2004). Most of these models are variations of the classical newsvendor problem under different risk-averse objective functions. While our work obviously has similarities with the above work in terms of incorporating risk aversion and MV trade-offs, within a newsvendor network setting, our decision emphasis is on integrated operational (capacity/inventory) and financial hedging decisions, which leads to completely different optimization problems and managerial insights.

Research efforts to hedge operational risk via financial instruments that exploit demand correlation with tradable market assets appeared early in Anvari (1987) and Chung (1990), with the study of newsvendor models within a capital asset pricing model (CAPM) framework. More recently, Gaur and Seshadri (2005) demonstrate the effectiveness of financial hedging when one can discover tradable market assets partially correlated with market demand, while Caldentey and Haugh (2006) provide a modeling framework that allows continuous trading in the financial market. Our work differs from the above models because it does not emphasize the financial hedging of demand uncertainty via traded market assets, but instead searches for integrated risk-management approaches that combine operational hedging (via postponed production allocation) with financial hedging instruments to mitigate the price risk associated with the currency exchange-rate volatility.

The structure of this paper is as follows: In §2, we provide a modeling framework for joint production and financial hedging decisions. In §3, we present the optimal financial hedging strategy for a given capacity global supply chain. In §4, we focus on a special case of a risk-averse firm selling exclusively to a foreign market with stochastic demand and clearly explain the role of operational and

financial hedges in such a context. Building on the intuition of these results, in §5 we proceed to analyze the same problem for a risk-averse firm selling to both domestic and foreign markets with stochastic demand. We show that some of the intuitive findings for the one-market case in §4 no longer hold for the two-market case. Section 6 illustrates that the use of financial hedge can affect supply chain structural decisions such as location and the number of production facilities to meet global demand. We conclude with a summary of main insights and discussions of future research in §7.

## 2. Model

We analyze the production and financial hedging decisions of a global firm selling to two markets: market 1, its “domestic” market (defined for our purpose as the market trading in the “home country” currency, i.e., the currency the firm uses to report its consolidated financial statements), and market 2, a “foreign country” market with an uncertain currency exchange rate.

We use a two-stage stochastic program to model the firm’s decisions. In the first stage, a “capacity-production” plan for the production facility is developed, and appropriate financial hedging contracts on the foreign currency are purchased in the presence of uncertainty in market demands, exchange rate, or both. During the first stage, the firm invests in needed technology, equipment, and factory space, or modifies existing facilities, in anticipation of market needs. With capacities in place, commitment of production resources as part of a first-stage production plan may occur, frequently in an effort to provide quick response to foreign market demand by executing, prior to demand realization, long lead time production activities (such as acquisition of raw materials, production of complex components and subassemblies, or even nonmarket specific “vanilla”-middle products). In the remainder of this paper, we will use “capacity” to refer to such a “capacity-production” plan. In the second stage, after observing the demand and exchange-rate realization, the firm makes production “allocation” decisions (e.g., how many units to appropriately configure—“localize”—and distribute in each market) with the necessary distribution and logistic costs to optimize its profits. The allocation option represents the firm’s flexibility in choosing ex post between domestic and foreign markets to sell its products. Postponement capability of manufacturing/assembly activities and the access to the worldwide distribution channel are required for such flexibility.

### 2.1. Real Options (“Postponed Allocation”) Modeling

We use the following notation:

$s$ : foreign market currency exchange rate in stage 2, that is, the value of one unit of the foreign currency measured in the domestic currency;

$\mathbf{X}$ : capacity (vector) reserved in stage 1.

$\mathbf{c}$ : unit capacity reservation cost (vector) in stage 1 (in home currency).

$p_i$ : price per unit in market  $i$  in stage 2 (in market  $i$  currency),  $i = 1, 2$ .

$\mathbf{d} = (d_i)_{i=1}^2$ : random variable (vector) that represents the demand in two markets.

$\pi^{\text{op}}(\mathbf{X}, s, \mathbf{d})$ : for a given capacity reservation  $\mathbf{X}$ , the optimal operations profit in stage 2 after the realization of demand  $\mathbf{d}$  and exchange rate  $s$ .

$e(\cdot)$ : probability density function (PDF) of the currency exchange-rate distribution.

$g(\cdot, \cdot)$ : PDF of the joint distribution of demand in two markets.

$g_i(\cdot)$ : PDF of demand distribution for market  $i$ ,  $i = 1, 2$ .

$G_i(\cdot)$ ,  $\bar{G}_i(\cdot)$ : cumulative distribution function (CDF) and the complementary CDF of demand in market  $i$ , respectively,  $i = 1, 2$ .

$f(\cdot, \cdot)$ : density function of joint distribution of exchange rate and demand.

$\text{T}$ : superscript that represents the transpose operator.

$*$ : superscript that represents optimal decisions and corresponding outcomes.

$x^+ = \max(x, 0)$ .

Usually firms do not adjust selling prices immediately at the swing of the currency exchange rate due to its potential long-term impact on firms' market shares. Thus, for the short- to medium-term planning problem studied in this paper, and especially for the second-stage production allocation decisions, we assume that the selling price in the foreign market,  $p_2$ , is fixed. Further motivation for this assumption is a perfectly competitive foreign market with a host of local and multinational firms contributing to the equilibrium price, with our firm having no control over such prices.

The functional form of  $\pi^{\text{op}}(\mathbf{X}, s, \mathbf{d})$  can vary depending on where the global firm locates its production facilities, and where the configuration adjustment ("localization") of the vanilla products to the market-specific final products is conducted. We find that the following functional form of  $\pi^{\text{op}}(\mathbf{X}, s, \mathbf{d})$  holds for a variety of assumptions on localization activities of stage 2 (as we will explain in detail later):

$$\begin{aligned} \pi^{\text{op}}(\mathbf{X}, s, \mathbf{d}) &= \mathbf{a}(\mathbf{X}, \mathbf{d})((s - \mathbf{S}^C)^+)^{\text{T}} \\ &\quad + \mathbf{b}(\mathbf{X}, \mathbf{d})((\mathbf{S}^P - s)^+)^{\text{T}} + c(\mathbf{X}, \mathbf{d}) \\ &= \sum_{i=0}^n a_i(\mathbf{X}, \mathbf{d})(s - S_i^C)^+ \\ &\quad + \sum_{i=1}^m b_i(\mathbf{X}, \mathbf{d})(S_i^P - s)^+ + c(\mathbf{X}, \mathbf{d}), \end{aligned} \quad (1)$$

where  $\mathbf{a}(\mathbf{X}, \mathbf{d}) = (a_i(\mathbf{X}, \mathbf{d}))_{i=0}^n$ ,  $\mathbf{b}(\mathbf{X}, \mathbf{d}) = (b_i(\mathbf{X}, \mathbf{d}))_{i=1}^m$ ,  $\mathbf{S}^C = (S_i^C)_{i=0}^n$ , and  $\mathbf{S}^P = (S_i^P)_{i=1}^m$ , with  $S_0^C = 0$ ,  $0 < S_i^C < S_j^C$  and  $0 < S_i^P < S_j^P$  for  $1 \leq i < j$ , and  $n$  and  $m$  are integers. (1) suggests that a firm's second-stage operations profit can

be viewed as a portfolio of real options on the exchange rate, some of them of a call-option nature (see the first term in (1)) and some of a put-option nature (see the second term in (1)). Following the obvious analogy to financial options, we often refer to  $\mathbf{S}^C$  and  $\mathbf{S}^P$  as the "real-option exercise prices." The more concrete interpretation and the motivation for the use of the various terms in (1) will be given in the discussion of the specific cases/examples that follow. Let us start with

**2.1.1. Case 1: One Facility and Localization at the Source.** The firm has a single production facility located in market 1 (i.e., the domestic market), and the stage 2 localization operation is also conducted at this facility. Define

$\tau_i$ : relevant unit localization costs for market  $i$  shipped in stage 2 (in market 1 currency),  $i = 1, 2$ .

$r_1 = p_1 - \tau_1 > 0$ : incremental profit per unit sold in stage 2 in market 1.

$r_2(s) = sp_2 - \tau_2$ : incremental profit per unit sold in stage 2 in market 2.

Thus,

$$\begin{aligned} \pi^{\text{op}}(X, s, \mathbf{d}) &= \begin{cases} r_1 \min((X - d_2)^+, d_1) + r_2(s) \min(X, d_2) & \text{if } r_2(s) \geq r_1, \\ r_1 \min(X, d_1) + r_2(s) \min((X - d_1)^+, d_2) & \text{if } r_1 > r_2(s) \geq 0, \\ r_1 \min(X, d_1) & \text{if } r_2(s) < 0. \end{cases} \end{aligned} \quad (2)$$

As (2) suggests, when  $r_2(s) \geq r_1$ , we first allocate capacity to meet demand in market 2, and the remaining units, if any, are localized for the needs of market 1. The allocation priorities are reversed when  $r_1 > r_2(s) \geq 0$ . For the case  $r_2(s) < 0$ , we only allocate the capacity for market 1's needs. (2) can be rewritten as

$$\begin{aligned} \pi^{\text{op}}(X, s, \mathbf{d}) &= r_2^+(s) \min((X - d_1)^+, d_2) + (r_2(s) - r_1)^+ \\ &\quad \cdot (\min(X, d_2) - \min((X - d_1)^+, d_2)) \\ &\quad + r_1 \min(X, d_1), \end{aligned} \quad (3)$$

that is,  $a_0 = 0$ ,  $a_1 = p_2 \min((X - d_1)^+, d_2)$ ,  $S_1^C = \tau_2/p_2$ ,  $a_2 = p_2(\min(X, d_2) - \min((X - d_1)^+, d_2))$ ,  $S_2^C = \tau_2/p_2 + (p_1 - \tau_1)/p_2$ , and  $c = r_1 \min(X, d_1)$ .

Similarly, a single production facility in the foreign market with localization conducted there results in

$$\begin{aligned} \pi^{\text{op}}(X, s, \mathbf{d}) &= r_2(s) \min(X, d_2) + (r_1(s) - r_2(s))^+ \\ &\quad \cdot (\min(X, d_1) - \min((X - d_2)^+, d_1)) \\ &\quad + r_1^+(s) \min((X - d_2)^+, d_1), \end{aligned}$$

where  $r_1(s) = p_1 - s\tau_1$ ,  $r_2(s) = s(p_2 - \tau_2)$ , and  $\tau_i$  is the unit localization cost for market  $i$  shipped in stage 2 (in market 2 currency),  $i = 1, 2$ . That is,  $a_0 = (p_2 - \tau_2) \min(X, d_2)$ ,  $b_1 = (p_2 + \tau_1 - \tau_2)(\min(X, d_1) - \min((X - d_2)^+, d_1))$ ,  $S_1^P = p_1/(p_2 + \tau_1 - \tau_2)$ ,  $b_2 = \tau_1 \min((X - d_2)^+, d_1)$ ,  $S_2^P = p_1/\tau_1$ , and  $c = 0$ .

**2.1.2. Case 2: One or Two Facilities and Localization in the Target Markets.** The firm has two production facilities with facility  $i$  located in market  $i$ ,  $i = 1, 2$ . Both facilities can produce vanilla products in the first stage, and facility  $i$  can configure the vanilla products to market  $j$  specific products,  $i, j = 1, 2$ . We assume that there are no capacity constraints for the localization operation in stage 2 (e.g., local content is added at ample capacity assembly and distribution facilities). Define:

$\tau_{ij}$ : unit localization cost of configuring facility  $i$  vanilla products to satisfy demand in market  $j$  (measured in market  $j$  currency),  $i = 1, 2$  and  $j = 1, 2$ .

$r_{ij}$ : unit profit of selling facility  $i$  product to market  $j$  (measured in market 1 currency). Specifically,  $r_{11} = p_1 - \tau_{11}$ ,  $r_{12}(s) = s(p_2 - \tau_{12})$ ,  $r_{21} = p_1 - \tau_{21}$ , and  $r_{22}(s) = s(p_2 - \tau_{22})$ .

It is reasonable to assume, without loss of generality, that  $\tau_{11} \leq \tau_{21}$  and  $\tau_{22} \leq \tau_{12}$ . That is, configuring facility  $i$ 's vanilla products into market  $j$  ( $\neq i$ ) products is more expensive than configuring facility  $j$ 's to satisfy market  $j$  demand. It follows that  $r_{11} \geq r_{21}$  and  $r_{22}(s) \geq r_{12}(s)$ . We can also assume that  $r_{ij} \geq 0$ . Then, we have

$$\begin{aligned} \pi^{\text{op}}(\mathbf{X}, s, \mathbf{d}) &= r_{12}(s) \min((d_2 - X_2)^+, (X_1 - d_1)^+) \\ &\quad + r_{22}(s) \min(X_2, d_2) + (r_{12}(s) - r_{11})^+ \\ &\quad \cdot (\min(X_1, d_1) - \min((X_1 - (d_2 - X_2)^+)^+, d_1)) \\ &\quad + (r_{21} - r_{22}(s))^+ \\ &\quad \cdot (\min(X_2, d_2) - \min((X_2 - (d_1 - X_1)^+)^+, d_2)) \\ &\quad + r_{11} \min(X_1, d_1) + r_{21} \min((X_2 - d_2)^+, (d_1 - X_1)^+). \end{aligned}$$

That is,  $a_0 = (p_2 - \tau_{12}) \min((d_2 - X_2)^+, (X_1 - d_1)^+) + (p_2 - \tau_{22}) \min(X_2, d_2)$ ,  $a_1 = (p_2 - \tau_{12})(\min(X_1, d_1) - \min((X_1 - (d_2 - X_2)^+)^+, d_1))$ ,  $S_1^C = (p_1 - \tau_{11}) / (p_2 - \tau_{12})$ ,  $b_1 = (p_2 - \tau_{22})(\min(X_2, d_2) - \min((X_2 - (d_1 - X_1)^+)^+, d_2))$ ,  $S_1^P = (p_1 - \tau_{21}) / (p_2 - \tau_{22})$ , and  $c = r_{11} \min(X_1, d_1) + r_{21} \min((X_2 - d_2)^+, (d_1 - X_1)^+)$ . The derivation of the above expression is given in the online appendix that can be found at <http://or.journal.informs.org/>.

The special case of a single production facility (located in either a domestic or foreign market) with the localization of shipped products performed in the target market is given by

$$\begin{aligned} \pi^{\text{op}}(X, s, \mathbf{d}) &= r_2(s) \min((X - d_1)^+, d_2) + (r_2(s) - r_1)^+ \\ &\quad \cdot (\min(X, d_2) - \min((X - d_1)^+, d_2)) + r_1 \min(X, d_1), \end{aligned}$$

where  $r_1 = p_1 - \tau_1$ ,  $r_2(s) = s(p_2 - \tau_2)$ , and  $\tau_i$  is the unit localization cost for market  $i$  shipped in stage 2 (in market  $i$  currency),  $i = 1, 2$ . That is,

$$\begin{aligned} a_0 &= (p_2 - \tau_2) \min((X - d_1)^+, d_2), \\ a_1 &= (p_2 - \tau_2) (\min(X, d_2) - \min((X - d_1)^+, d_2)), \\ S_1^C &= (p_1 - \tau_1) / (p_2 - \tau_2), \\ \text{and } c &= r_1 \min(X, d_1). \end{aligned}$$

**2.1.3. Case 3: Two Facilities Without Postponement.**

The firm has two production facilities with facility  $i$  located in market  $i$ ,  $i = 1, 2$ . Both facilities produce market-specific products and ship to the target market in the first stage. Define  $X_{ij}$  as facility  $i$ 's stage 1 production shipped to market  $j$ ,  $i, j = 1, 2$ . In this case, the stage 2 operations profit can be written as

$$\pi^{\text{op}} = p_1 \min(X_{11} + X_{21}, d_1) + s p_2 \min(X_{12} + X_{22}, d_2).$$

That is,  $a_0 = p_2 \min(X_{12} + X_{22}, d_2)$  and  $c = p_1 \min(X_{11} + X_{21}, d_1)$ .

Other possible variations of those cases along the same lines (e.g., two facilities localization at the source) satisfy (1) but are omitted for brevity.

**2.2. Financial Options**

We now describe the type of financial contracts we are considering, and how they can be valued. We consider currency option contracts. We will conduct most of our analysis with call- and put-option currency contracts. Consider a portfolio of call- and put-currency contracts  $h = ((S_C, S_P), (Q_C, Q_P))$ , where  $S_C = (S_{C_i})_{i=0}^{n_C}$  and  $S_P = (S_{P_i})_{i=1}^{n_P}$  are the row vectors of exercise prices of the call options and put options, respectively, and  $Q_C = (Q_{C_i})_{i=1}^{n_C}$  and  $Q_P = (Q_{P_i})_{i=1}^{n_P}$  are the row vectors of the corresponding contract sizes. The payoff of portfolio  $h$  in stage 2 is

$$\begin{aligned} R_h(s) &= (s - S_C)^+ Q_C^T + (S_P - s)^+ Q_P^T \\ &= \sum_{i=0}^{n_C} (s - S_{C_i})^+ Q_{C_i} + \sum_{i=1}^{n_P} (S_{P_i} - s)^+ Q_{P_i}. \end{aligned} \tag{4}$$

Let  $C(S)$  denote the price of a unit call or put option with exercise price  $S$  in stage 1, which is decided in the currency exchange rate market or determined by the option pricing theory. For example, if the currency exchange rate in stage 2,  $s$ , follows a lognormal distribution, then  $C(S)$  can be determined by using Black-Scholes' valuation (see Black and Scholes 1973). Let  $H(h)$  be the cost of acquiring financial contract  $h$  incurred in stage 1. We have

$$H(h) = \sum_{i=0}^{n_C} C(S_{C_i}) Q_{C_i} + \sum_{i=1}^{n_P} C(S_{P_i}) Q_{P_i}.$$

The risk premium of a unit call (respectively, put) option  $(S, Q)$  is defined as

$$\begin{aligned} \Delta C(S) &= C(S) e^{\gamma T} - E[(s - S)^+] \\ &\quad (\text{respectively, } \Delta C(S) = C(S) e^{\gamma T} - E[(S - s)^+]), \end{aligned} \tag{5}$$

where  $\gamma$  is the risk-free interest rate in home currency, and  $T$  is the time-to-maturity of the option. The risk premium of a unit portfolio  $h$  is defined as a row vector

$$\Delta C(S_C, S_P) = ((\Delta C(S_{C_i}))_{i=0}^{n_C}, (\Delta C(S_{P_i}))_{i=1}^{n_P}). \tag{6}$$

Forward contracts are commonly used to hedge foreign currency risk and can be viewed as special cases of call options. Consider that the firm buys a forward contract  $h1 = (f, Q)$  in stage 1, where  $f$  is the forward exchange rate to be used for the delivery of  $Q$  in stage 2. Then, the payoff of  $h1$  in stage 2 is

$$R_{h1}(s) = (s - f)Q = -(f - s_L)Qe^{-\gamma T}e^{\gamma T} + (s - s_L)Q,$$

where  $s_L = \inf\{s\}$ . Observe that  $R_{h1}(s)$  can be viewed as the sum of the stage 2 payoff of call option  $h2 = (s_L, Q)$  and the stage 2 value of a fixed payment of  $(f - s_L)Qe^{-\gamma T}$  made in stage 1. The portfolio replication argument requires that the cost of the call option  $h2$  in stage 1 should be  $H(h2) = (f - s_L)Qe^{-\gamma T}$ .<sup>1</sup> Thus, the stage 2 payoff of forward contract  $h1$  is the same as the net payoff of call option  $h2$  in stage 2. Similarly, we can establish that a forward contract can be viewed as a special case of a put option with exercise price  $s_u = \sup\{s\}$ . By considering only call and put options, we include forward contracts in the feasible set of contracts.

### 2.3. Integrated Risk-Management Problem Formulation

We can write the firm's profit in stage 2 as

$$\pi(\mathbf{X}, h, s, \mathbf{d}) = (-c\mathbf{X} - H(h))e^{\gamma T} + \pi^{\text{op}}(\mathbf{X}, s, \mathbf{d}) + R_h(s), \quad (7)$$

where  $T$  is the time duration between stage 1 and stage 2. We assume throughout, unless otherwise explicitly noted, that the firm is risk averse. Furthermore, we assume that

- (a) the firm's objective is to maximize the expected utility for the stage 2 profit as expressed by (7), and
- (b) the firm's expected utility is represented by

$$U(\pi) = E[\pi] - \lambda V[\pi],$$

where  $E[\cdot]$  and  $V[\cdot]$  are the expectation and variance operators taken on the joint distribution of exchange rate and demand, respectively, and  $\lambda \geq 0$  represents the rate at which the firm will substitute variance for expected value, also referred to as the MV ratio. It is well known (see Philipatos and Gressis 1975 and Jucker and Carlson 1976) that the use of the MV criterion is consistent with the principle of maximizing expected utility if

- (i) the firm's utility function can be represented by a quadratic function of time payoff, or
- (ii) the probability distribution of time payoff is a two-parameter distribution (e.g., normal distribution).

Van Mieghem (2004) discusses the applicability of the MV objective in the real-options context. The MV objective offers a second-order approximation of the true objective for a general utility function, and it provides clear insights to decision makers. Van Mieghem (2004) also conducts numerical studies to investigate the appropriateness of the

MV approximation to utility functions of constant absolute risk aversion (CARA) for the single-newsvendor problem and the sequential newsvendor network, and finds that the difference between utility-optimal paths are small, and the MV optimal resource levels typically underestimate the risk adjustments, but the difference is small when risk adjustment is small. Justification for the use of total risk measure (such as  $V[\cdot]$ ) instead of measures of systematic risk is provided in Hodder and Dincer (1986) with the main argument being that managers are typically concerned about total risk. This can be attributed to a concern with the probability of financial distress, or bankruptcy, as well as to agency considerations (see Jensen and Meckling 1976 and Markus 1982). Therefore, the firm's production and financial hedging problem is

$$\max_{\mathbf{x} \geq 0, h \in \Omega} \{E[\pi(\mathbf{X}, h, s, \mathbf{d})] - \lambda V[\pi(\mathbf{X}, h, s, \mathbf{d})]\}, \quad (8)$$

where  $\Omega$  is a given feasible financial hedging set (in this paper, portfolios of call and put options). In the remainder of this paper, we will use the MV objective when referring to the firm's utility.<sup>2</sup>

### 3. Optimal Financial Hedging Strategy

In this section, we present a general result on the optimal hedging sizes for a portfolio of call- and put-option currency contracts for a given production capacity  $\mathbf{X}$ . Let  $\mathbf{S} = (\mathbf{S}_C, \mathbf{S}_P)$  be the exercise price vector of the portfolio of call and put options. Define  $\text{cov}(\mathbf{x}, \mathbf{y})$  as the covariance matrix of random vectors  $\mathbf{x}$  and  $\mathbf{y}$ , and  $V[\mathbf{x}]$  as the variance-covariance matrix of random vector  $\mathbf{x}$ .

**PROPOSITION 1.** *Given capacity  $\mathbf{X}$  and a portfolio of call- and put-option currency contracts with prespecified exercise price vector  $\mathbf{S} = (\mathbf{S}_C, \mathbf{S}_P)$  in which  $\mathbf{S}_C$  and  $\mathbf{S}_P$  do not have common elements, the firm's MV objective is concave in the hedging size  $\mathbf{Q}$ , and the optimal hedge size vector  $\mathbf{Q}^*(\mathbf{X}, \mathbf{S})$  is the unique solution to the system of linear equations*

$$\begin{aligned} -(\Delta C(\mathbf{S}))^T &= 2\lambda(V[((s - \mathbf{S}_C)^+, (\mathbf{S}_P - s)^+)]\mathbf{Q}^{*T} \\ &+ \text{cov}(\pi^{\text{op}}(\mathbf{X}, s, \mathbf{d}), ((s - \mathbf{S}_C)^+, (\mathbf{S}_P - s)^+))). \end{aligned} \quad (9)$$

Proposition 1 reflects the marginal trade-off of adding a unit of option portfolio at  $\mathbf{Q}$ . First, buying one more unit of the portfolio will affect the firm's expected profit through the risk-premium term,  $\Delta C(\mathbf{S})$ , while the profit from the operations decision in the second stage  $\pi^{\text{op}}$  will not be affected. Second, the total variance of the profit will be affected in two ways—the variance of the portfolio payoffs  $R_h$  will increase by  $V[((s - \mathbf{S}_C)^+, (\mathbf{S}_P - s)^+)]\mathbf{Q}^T$ , and the covariance between the operations profit  $\pi^{\text{op}}$  and the portfolio profit  $R_h$  will increase by  $\text{cov}(\pi^{\text{op}}(\mathbf{X}, s, \mathbf{d}), ((s - \mathbf{S}_C)^+, (\mathbf{S}_P - s)^+))$ . The optimal portfolio size  $\mathbf{Q}^*(\mathbf{X}, \mathbf{S})$  should

balance the marginal impact on the expected profit and the total variance of the profit. Proposition 1 provides the optimal hedge size for a given capacity and a portfolio of call and put options with prespecified exercise prices. It applies to all functional forms of  $\pi^{op}$ .

We now proceed to ask several questions of interest: How many call options do we need for optimal hedging? What exercise prices of the financial options are good choices for a given capacity decision? Applying Proposition 1 to (1) yields some interesting results. First, observe that if demand  $\mathbf{d}$  is deterministic, then the firm’s operations profit in stage 2 is identical to the payoff of a portfolio of call and put options with exercise prices  $\mathbf{S}^C$  and  $\mathbf{S}^P$  (i.e., use the “real-option exercise prices” as the contracted exercise prices for the financial options) and contract sizes  $\mathbf{a}(\mathbf{X}, \mathbf{d})$  and  $\mathbf{b}(\mathbf{X}, \mathbf{d})$ , respectively. Thus, selling call and put options in the first stage will completely hedge the risk from the currency exchange rate, i.e., zero-variance (“perfect”) hedge can be achieved. When the market demand is stochastic, zero-variance hedges cannot be achieved. However, we can show that under reasonable assumptions the exercise prices  $\mathbf{S}^C$  and  $\mathbf{S}^P$  in the firm’s real options are indeed the best choices of exercise prices for financial call and put options.

First, without loss of generality, we assume that  $\mathbf{S}^C$  and  $\mathbf{S}^P$  do not have common elements. We can arrange the elements of  $\mathbf{S}^C$  and  $\mathbf{S}^P$  in (1) in an increasing order to obtain a new vector  $\mathbf{S}^{op} = (S_i^{op})_{i=0}^{m+n}$ . Correspondingly, we can define vector  $\mathbf{ab}(\mathbf{X}, \mathbf{d}) = (\mathbf{ab}_i(\mathbf{X}, \mathbf{d}))_{i=0}^{m+n}$  as

$$\mathbf{ab}_i(\mathbf{X}, \mathbf{d}) = \begin{cases} \text{the element of } \mathbf{a}(\mathbf{X}, \mathbf{d}) \text{ that is the coefficient} \\ \text{of } (s - S_i^{op})^+ \text{ in } \pi^{op} \\ \text{if } S_i^{op} \text{ is an element of } \mathbf{S}^C, \\ \text{the element of } \mathbf{b}(\mathbf{X}, \mathbf{d}) \text{ that is the coefficient} \\ \text{of } (S_i^{op} - s)^+ \text{ in } \pi^{op} \\ \text{if } S_i^{op} \text{ is an element of } \mathbf{S}^P. \end{cases}$$

Define a random vector  $\mathbf{F}(s, \mathbf{S}^{op}) = (\mathbf{F}_i(s, \mathbf{S}^{op}))_{i=0}^{m+n}$ , where

$$\mathbf{F}_i(s, \mathbf{S}^{op}) = \begin{cases} (s - S_i^{op})^+ & \text{if } S_i^{op} \text{ is an element of } \mathbf{S}^C, \\ (S_i^{op} - s)^+ & \text{if } S_i^{op} \text{ is an element of } \mathbf{S}^P. \end{cases}$$

**PROPOSITION 2.** (1) *Given capacity  $\mathbf{X}$  and exercise prices  $\mathbf{S}^C$  for call options and exercise prices  $\mathbf{S}^P$  for put options,*

(a) *The optimal hedge sizes are*

$$\mathbf{Q}^{opT} = -V^{-1}(\mathbf{F}(s, \mathbf{S}^{op})) \cdot (\text{cov}(\pi^{op}, \mathbf{F}^T(s, \mathbf{S}^{op})) + (\Delta C(\mathbf{S}^{op}))^T / 2\lambda).$$

(b) *If the demand is independent of the currency exchange rate, referred to as the independent case, then the optimal hedge sizes are*

$$\mathbf{Q}^{opT} = -E[\mathbf{ab}^T(\mathbf{X}, \mathbf{d})] - V^{-1}(\mathbf{F}(s, \mathbf{S}^{op}))(\Delta C(\mathbf{S}^{op}))^T / 2\lambda.$$

(2) *For the independent case, if zero risk premium is assumed, then the optimal hedging policy among all call and put options is a portfolio of call options ( $\mathbf{S}^C, -E[\mathbf{a}(\mathbf{X}, \mathbf{d})]$ ) and put options ( $\mathbf{S}^P, -E[\mathbf{b}(\mathbf{X}, \mathbf{d})]$ ).*

When the firm only considers trading currency options with exercise prices that are the same as the embedded real-option exercise prices,  $\mathbf{S}^C$  and  $\mathbf{S}^P$ , Proposition 2 provides the optimal hedging quantity. The optimal hedging quantity can be viewed as consisting of two parts (see Proposition 2(1)(a) and (b)). One part,  $-V^{-1}(\mathbf{F}(s, \mathbf{S}^{op}))\text{cov}(\pi^{op}, \mathbf{F}^T(s, \mathbf{S}^{op}))$ , aims to reduce the variance of operations profit through the counterbalancing effect between cash flows from operations and from exercising the currency exchange options; the other part,  $-V^{-1}(\mathbf{F}(s, \mathbf{S}^{op}))(\Delta C(\mathbf{S}^{op}))^T / 2\lambda$ , referred to as the *hedge size deviation*, reflects the trade-off between the gain/loss in risk premium and the gain in the variance of financial trading. In particular, when the foreign market demand is independent of the currency exchange rate, which implies that  $\mathbf{ab}^T(\mathbf{X}, \mathbf{d})$  is independent of the unit payoff of the real options  $\mathbf{F}^T(s, \mathbf{S}^{op})$ , the financial hedge can help to reduce the operations profit variance caused by the currency fluctuation but not that by the demand uncertainty. Moreover, when the risk premium is zero (Proposition 2(2)), i.e., financial trading does not affect the firm’s expected profit, the optimal hedge is the minimum-variance hedge. Selling a portfolio of call and put options with  $\mathbf{S}^C$  and  $\mathbf{S}^P$  as exercise prices and with  $E[\mathbf{a}(\mathbf{X}, \mathbf{d})]$  and  $E[\mathbf{b}(\mathbf{X}, \mathbf{d})]$  as hedging quantities will reduce the variance of operations profit to the minimum. Proposition 2(2) also highlights that, to some extent, the effective financial hedging contracts are related to (and affected by) the operational strategies that a firm employs. That is, there is a natural linkage between the firm’s real options and the financial options that the firm should employ to efficiently hedge its risk. For example, a firm that has two production facilities, one in each market, and conducts localization in target markets (case (2) in §2.1) will need a forward, a call option, and a put option to manage the exchange risk; a firm that has two facilities but with no postponement capability (case (3) in §2.1) will only need a forward contract to manage the exchange risk. We note that the optimal financial hedging strategies in Propositions 1 and 2 are specific to the MV objective because they are the result of balancing marginal expected profit and profit variance.

We will further specialize our results for an operational setting that was described previously (case (1) in §2.1): The firm has one production facility located in the domestic market, and stage 2 localization is also conducted there. This particular operational setting is instrumental for our presentation of results for the joint production capacity and hedging decision in §§4 and 5. Such practice is quite common. For example, in the automotive industry, most of the European brand autos are manufactured in Europe and exported to the United States. For this setting, the



stage 2 operations profit function  $\pi^{\text{op}}(X, s, \mathbf{d})$  is given by (3). In §4, we study in more detail its special case in which the domestic facility serves exclusively the demand in the foreign market (market 2). That is,

$$\begin{aligned}\pi^{\text{op}}(X, s, d_2) &= r_2^+(s) \min(X, d_2) \\ &= p_2(s - \tau_2/p_2)^+ \min(X, d_2).\end{aligned}$$

Section 5 focuses on the general case of the domestic facility supplying two markets (i.e.,  $\pi^{\text{op}}$  is described by (3)). The following two corollaries of Proposition 2 illustrate the optimal financial hedging policies for these two cases.

**COROLLARY 1.** (1) *For a domestic production facility supplying a foreign market, given capacity  $X$  and a call-option currency contract with prespecified exercise price  $S = \tau_2/p_2$ ,*

(a) *The optimal hedge size  $Q^*(X, \tau_2/p_2)$  is given by*

$$\begin{aligned}Q^*(X, \tau_2/p_2) &= -\frac{p_2 \text{cov}((s - \tau_2/p_2)^+ \min(X, d_2), (s - \tau_2/p_2)^+)}{\text{V}[(s - \tau_2/p_2)^+]} \\ &\quad - \frac{\Delta C(\tau_2/p_2)}{2\lambda \text{V}[(s - \tau_2/p_2)^+]}.\end{aligned}$$

(b) *If the foreign market demand is independent of the currency exchange rate, referred to as the independent case, the optimal hedge size is*

$$Q^*(X, \tau_2/p_2) = -p_2 \text{E}[\min(X, d_2)] - \frac{\Delta C(\tau_2/p_2)}{2\lambda \text{V}[(s - \tau_2/p_2)^+]}.$$

(2)<sup>3</sup> *For the independent case, if zero risk premium is assumed, then the optimal policy among all call and put options is a single call option with*

$$S = \tau_2/p_2 \quad \text{and} \quad Q^*(X, \tau_2/p_2) = -p_2 \text{E}[\min(X, d_2)].$$

**COROLLARY 2.** (1) *For a domestic production facility supplying both foreign and domestic markets, given capacity  $X$  and two call-option currency contracts at prespecified exercise price  $S_1 = \tau_2/p_2$  and  $S_2 = \tau_2/p_2 + (p_1 - \tau_1)/p_2$ ,*

(a) *The optimal hedge sizes are*

$$\begin{aligned}Q_1^*(X, S_1, S_2) &= -(\text{cov}(\pi^{\text{op}}, (s - S_1)^+) \text{V}[(s - S_2)^+] \\ &\quad - \text{cov}(\pi^{\text{op}}, (s - S_2)^+) \text{cov}((s - S_1)^+, (s - S_2)^+)) / \det V \\ &\quad - (\Delta C(S_1) \text{V}[(s - S_2)^+] \\ &\quad - \Delta C(S_2) \text{cov}((s - S_1)^+, (s - S_2)^+)) / (2\lambda \det V),\end{aligned}$$

$$\begin{aligned}Q_2^*(X, S_1, S_2) &= -(\text{cov}(\pi^{\text{op}}, (s - S_2)^+) \text{V}[(s - S_1)^+] \\ &\quad - \text{cov}(\pi^{\text{op}}, (s - S_1)^+) \text{cov}((s - S_1)^+, (s - S_2)^+)) / \det V \\ &\quad - (\Delta C(S_2) \text{V}[(s - S_1)^+] \\ &\quad - \Delta C(S_1) \text{cov}((s - S_1)^+, (s - S_2)^+)) / (2\lambda \det V),\end{aligned}$$

where  $\det V = \text{V}[(s - S_1)^+] \text{V}[(s - S_2)^+] - \text{cov}^2((s - S_1)^+, (s - S_2)^+)$ .

(b) *If the joint demand is independent of the currency exchange rate, referred to as the independent case, the optimal hedge sizes are*

$$\begin{aligned}Q_1^*(X, S_1, S_2) &= -p_2 \text{E}[\min((X - d_1)^+, d_2)] - (\Delta C(S_1) \text{V}[(s - S_2)^+] \\ &\quad - \Delta C(S_2) \text{cov}((s - S_1)^+, (s - S_2)^+)) / (2\lambda \det V), \\ Q_2^*(X, S_1, S_2) &= -p_2 \text{E}[\min(X, d_2) - \min((X - d_1)^+, d_2)] \\ &\quad - (\Delta C(S_2) \text{V}[(s - S_1)^+] \\ &\quad - \Delta C(S_1) \text{cov}((s - S_1)^+, (s - S_2)^+)) / (2\lambda \det V).\end{aligned}$$

(2) *For the independent case, if zero risk premium is assumed, then the optimal policy among all call and put options is a portfolio of two call options with exercise prices  $S_1 = \tau_2/p_2$  and  $S_2 = \tau_2/p_2 + (p_1 - \tau_1)/p_2$ , and respective contract sizes*

$$Q_1^*(X, S_1, S_2) = -p_2 \text{E}[\min((X - d_1)^+, d_2)],$$

$$Q_2^*(X, S_1, S_2) = -p_2 \text{E}[\min(X, d_2) - \min((X - d_1)^+, d_2)].$$

Some of the above results are based on the assumption that the demand and currency exchange rate are independent. This is a reasonable assumption for the short-term production allocation planning problem. Furthermore, this assumption leads to clean results and helpful intuition on the nature of the optimal financial hedging policy. When demand and currency exchange rate are indeed partially correlated, the interrelatedness of the operational decision (e.g., capacity) and the financial hedge is stronger because a natural hedge of the demand risk might arise by using the appropriate financial hedge. A firm may need a more complex optimal financial hedging strategy to achieve the minimum variance hedge than what is suggested in Proposition 2(2), and the form of the optimal financial hedging strategy will depend on the specific assumption on the correlation between demand and exchange rate. For example, for the one-market case in §4, if the foreign demand  $d_2 = (a - bs)^+$ , where  $a$  and  $b$  are constants, we can show that the firm will need infinitely many call options to achieve a zero-variance hedge.<sup>4</sup>

A few observations about the effect of the risk premium: In the exchange-rate market, the risk premium of options is typically positive and very small (see Biger and Hull 1983). The firm cannot make a lot of money from selling a call option, and the hedge-size deviation is small when the MV ratio is in a reasonable range. From an operations manager perspective, the main goal of participating in the currency trading market is hedging the risk in the operations profit due to the currency exchange-rate uncertainty, not gaining profit from the currency exchange market. Such profits often disappear when relevant transaction costs are

applied. Furthermore, from a mathematical tractability perspective, assuming zero risk premium offers clean results and clear insights for the problem. Thus, for the above reasons, we will present most of the results in this paper under a zero risk-premium assumption.

#### 4. Domestic Production Facility Supplying a Foreign Market

We will now study a simplified model in which the production facility is located in market 1 (domestic market) and serves exclusively demand in market 2 (foreign market). This model applies to the situation where either market 1 demand is zero or the firm uses a separate production facility to supply market 1. Again, the firm's stage 2 operations profit is

$$\begin{aligned}\pi^{\text{op}}(X, s, d_2) &= r_2^+(s) \min(X, d_2) \\ &= p_2(s - \tau_2/p_2)^+ \min(X, d_2).\end{aligned}$$

##### 4.1. Basic Results

**PROPOSITION 3.** *For a domestic production facility supplying a foreign market where the foreign market demand is independent of the currency exchange rate, given a call-option currency contract with prespecified exercise price  $S = \tau_2/p_2$ , the firm's MV objective is optimized by a pair of unique values  $(X^*, Q^*)$  that solves the following equations:*

$$\begin{aligned}(\mathbb{E}[r_2^+(s)] + p_2 \Delta C(\tau_2/p_2) \\ - 2\lambda \mathbb{E}[r_2^{+2}(s)] \mathbb{E}[(X^* - d_2)^+]) \bar{G}_2(X^*) = ce^{\gamma T},\end{aligned}$$

and

$$Q^* = -p_2 \mathbb{E}[\min(X^*, d_2)] - \frac{\Delta C(\tau_2/p_2)}{2\lambda \mathbb{V}[(s - \tau_2/p_2)^+]}$$

Moreover, if zero risk premium is assumed, then the optimal  $(X^*, Q^*)$  solves

$$\begin{aligned}(\mathbb{E}[r_2^+(s)] - 2\lambda \mathbb{E}[r_2^{+2}(s)] \mathbb{E}[(X^* - d_2)^+]) \\ \cdot \bar{G}_2(X^*) = ce^{\gamma T}\end{aligned}\quad (10)$$

and

$$Q^* = -p_2 \mathbb{E}[\min(X^*, d_2)].$$

When the MV ratio  $\lambda = 0$ , the model reduces to the traditional risk-neutral problem. Note that if the risk premium is not zero, the risk-neutral firm will be able to make infinite expected profit by trading an infinite number of options in the currency exchange market. To exclude this pathological case, we always assume zero risk premium when discussing the risk-neutral problem. Corollary 3 provides the optimal production capacity decision for a risk-neutral firm.

**COROLLARY 3.** *The optimal risk-neutral capacity  $X_N^*$  is the solution to*

$$\int_{\tau_2/p_2}^{\infty} \int_{X_N^*}^{\infty} r_2^+(s) f(s, d_2) dd_2 ds = ce^{\gamma T}.$$

Moreover, for the independent case,  $X_N^*$  is the solution to

$$\mathbb{E}[r_2^+(s)] \bar{G}_2(X_N^*) = ce^{\gamma T}. \quad (11)$$

For a risk-neutral firm, the optimal capacity is expressed by a newsvendor-type formula (11). For a risk-averse firm, the newsvendor formula is modified by a risk-aversion term (see (10)). Comparing (10) and (11), we have the following corollary.

**COROLLARY 4.** *For a domestic production facility supplying a foreign market, if the foreign market demand is independent of the currency exchange rate and the risk premium is zero, then the optimal capacity of a risk-averse firm is less than that of a risk-neutral firm; i.e.,  $X^* \leq X_N^*$ .*

##### 4.2. Analysis of Operational and Financial Hedges

We will now discuss how operational and financial hedges can help mitigate the firm's exposure to the exchange rate and demand uncertainties. Operationally, the firm might have committed to the foreign market demand and does not have the option to allocate the capacity contingent upon the currency exchange rate, i.e.,  $\pi^{\text{op}} = r_2(s) \min(X, d_2)$ .<sup>5</sup> We refer to this case as "without allocation option," as opposed to the case with allocation option where the firm is not bound to satisfy the foreign market demand and  $\pi^{\text{op}} = r_2^+(s) \min(X, d_2)$  (as we have analyzed in §4.1). Financially, a firm can choose between using or not using currency exchange options. Thus, we compare four cases: (a) without allocation option and without financial hedge (−A, −H), (b) with allocation option and without financial hedge (A, −H), (c) without allocation option and with financial hedge (−A, H), and (d) with allocation option and with financial hedge (A, H). (A graphical representation of comparisons among the four cases is given in Figure 1.)

Proposition 4 shows the effect of the allocation option on the firm's optimal capacity, expected profit, and MV objective.

**PROPOSITION 4.** *Assuming a domestic production facility supplying a foreign market when the demand and exchange rate are independent:*

(a) *By implementing an allocation option, a risk-averse firm not hedging financially increases its optimal capacity, expected profit, and MV objective. That is,  $X_{-A, -H}^* \leq X_{A, -H}^*$ ,  $E_{-A, -H}^* \leq E_{A, -H}^*$ , and  $U_{-A, -H}^* \leq U_{A, -H}^*$ .*

(b) *If the risk premium is zero, then by implementing an allocation option, a risk-averse firm using the optimal financial hedge increases its optimal capacity, expected profit, and MV objective. That is,  $X_{-A, H}^* \leq X_{A, H}^*$ ,  $E_{-A, H}^* \leq E_{A, H}^*$ , and  $U_{-A, H}^* \leq U_{A, H}^*$ .*

**Figure 1.** Summary of the effects of operational and financial hedges.

	Without allocation option (-A)	With allocation option (A)
Without financial hedge (-H)	-A, -H	A, -H
With financial hedge (H)	-A, H	A, H

Proposition 4(a) (top arrow), Proposition 4(b)\* (bottom arrow), Proposition 5(b) (left arrow), Proposition 5(a) (right arrow)

\*Represents the case of zero risk premium.

For a given capacity, the allocation option allows the firm to avoid unfavorable currency exchange-rate realizations and to capitalize on the positive part of the unit profit,  $r_2^+(s)$ . This leads to a twofold benefit: the increase of the unit expected operations profit and the decrease of the unit profit variance. The benefits of such value enhancement and risk reduction entice the firm to invest more capacity in the first stage than without allocation option and consequently bring a higher expected profit. However, a larger capacity also leads to a higher profit variance. Thus, the improved MV objective might be at the cost of higher profit variance (the numerical example in §4.3 will illustrate such a case).

Corollary 5 and Proposition 5 summarize the effect of the financial hedge.

**COROLLARY 5.** For a domestic production facility supplying a foreign market without allocation option, given capacity  $X$ , if the foreign market demand is independent of the currency exchange rate and zero risk premium is assumed, then the optimal policy among all call and put options is a single forward contract.

**PROPOSITION 5.** For a domestic production facility supplying a foreign market with the demand and exchange rate being independent, if the risk premium of call options and forward contracts are nonnegative, then

(a) With an allocation option, the use of the call option with exercise price  $\tau_2/p_2$  increases the firm's optimal capacity, expected profit, and MV objective. That is,  $X_{A,-H}^* \leq X_{A,H}^*$ ,  $E_{A,-H}^* \leq E_{A,H}^*$ , and  $U_{A,-H}^* \leq U_{A,H}^*$ ;

(b) Without an allocation option, if  $E[r_2(s)] > 0$ , then the use of the forward contract increases the firm's optimal capacity, expected profit, and MV objective. That is,  $X_{-A,-H}^* \leq X_{-A,H}^*$ ,  $E_{-A,-H}^* \leq E_{-A,H}^*$ , and  $U_{-A,-H}^* \leq U_{-A,H}^*$ .

The main impact of a financial hedge is to introduce a financial transaction that counterbalances the cash flow

from operations transactions to reduce the total variance of the profit. At a given capacity, the optimal financial contract always reduces the profit variance and thus creates incentives for the firm to invest in more capacity. As a result, with the financial hedge, the firm always invests more than without the financial hedge. Thus, although the financial hedge does not increase profit directly, indirectly through the reduction of the profit variance and the increase of the capacity investment, the firm's expected profit increases. Because the profit variance increases in capacity investment, the higher MV objective might be at the cost of higher profit variance. This insight applies to both cases of with and without allocation option.

The above analysis reveals intricate relationships among operational and financial hedges, capacity, and currency exchange and demand risks. At a given capacity, the currency exchange rate risk is reduced by using either the operational or the financial hedge; the expected profit can be increased only by the operational hedge. However, the reduction of the currency exchange risk allows the firm to be more aggressive on capacity investment and allows the firm to take on more of the demand risk, and thus increases the firm's expected profit. As a result, the firm that uses both hedging instruments is better off in expected profit and MV objective than the firm that uses none or only one instrument, and achieves closer-to-maximum-expected-profit performance with a lower risk than a risk-neutral firm. The study of Mello et al. (1995) on switching options in a dynamic setting offers similar insights on the interaction of the operational hedge (flexibility) and financial hedge. That is, the increased operational flexibility can increase the firm's first-best value, and an efficient financial hedge increases the firm's value, but only by creating an incentive to choose the optimal operational policy.

### 4.3. Numerical Examples

In this subsection, we will first give an example to illustrate the results of Propositions 4 and 5. Then, we will study the effects of the currency exchange-rate volatility, demand volatility, and MV ratio on the firm's performance. The following example (Example 1) data will be extensively used in this section.

**EXAMPLE 1.** Assume that the exchange rate  $s$  follows a lognormal distribution with  $\ln s \sim N(1, 1)$ . The foreign market demand is independent of the exchange rate and follows a normal distribution of  $d_2 \sim N(100, 30)$ . Let  $ce^{\gamma T} = 2$ ,  $p_2 = 1$ ,  $\tau_2 = 1.5$ , and  $\lambda = 0.0002$ . Assume zero risk premium.

Table 1 verifies the results of Propositions 4 and 5. We can see that both the allocation option and the financial hedge increase the optimal capacity, the expected profit, and the MV objective. In this particular example, the allocation option also increases the profit variance while the financial hedge decreases the profit variance significantly. As we discussed in §4.2, the allocation option provides the benefits of value enhancement and risk reduction, and the

**Table 1.** An example of the effect of allocation option and financial hedge.

	Without allocation option (–A)	With allocation option (A)	$((A, \cdot) - (-A, \cdot)) / (-A, \cdot)$
<b>Without financial hedge (–H)</b>			
$(X^*, U^*)$	(58.203, 31.323)	(65.039, 42.277)	(11.75%, 34.97%)
$(E^*, \lambda V^*)$	(53.818, 22.496)	(68.957, 26.679)	(28.13%, 18.59%)
<b>With financial hedge (H)</b>			
$(X^*, U^*)$	(84.961, 64.457)	(87.695, 77.71)	(3.22%, 20.56%)
$(E^*, \lambda V^*)$	(65.758, 1.301)	(79.209, 1.499)	(20.46%, 15.22%)
			$((A, H) - (-A, -H)) / (-A, -H)$
$((\cdot, H) - (\cdot, -H)) / (\cdot, -H)$			
$(X^*, U^*)$	(45.97%, 105.78%)	(34.83%, 83.81%)	(50.67%, 148.09%)
$(E^*, \lambda V^*)$	(22.19%, –94.22%)	(14.87%, –94.38%)	(47.18%, –93.34%)

financial hedge provides the benefit of risk reduction. When only one of the two instruments is considered by the firm, the relative effectiveness of the two instruments depends on the relative magnitudes of the value enhancement and the risk reduction which, in turn, are determined by the magnitude of the localization cost and the degree of the firm’s risk aversion. However, the joint use of both instruments improves the firms’s MV objective significantly, compared to the use of a single instrument. This observation is consistent with the empirical findings of Allayannis et al. (2001), who, however, use different measures of operational hedging strategies.

We then study how changes in the currency exchange-rate volatility affect the optimal capacities and the firm’s corresponding performance for the four cases. When adjusting the currency exchange-rate volatility, we let the standard deviation of  $\ln s$  vary from 1 to 2 while keeping the expectation of  $s$  at a constant level of  $e^{1.5}$  by adjusting the mean of  $\ln s$  accordingly, where  $e$  is the base of the natural logarithm. Figures 2(a)–(d) illustrate the effect of an increasing volatility of the currency exchange rate. Figure 2(a) shows that the optimal capacities decrease as the currency exchange rate becomes more volatile. For firms without financial hedge, capacity is the only instrument for variance control, and the optimal capacities decrease quickly as the exchange-rate volatility increases. Firms with the optimal financial hedge in place, on the other hand, are affected much less by the exchange-rate volatility because the risk associated with the exchange rate is reduced significantly through the financial hedge. Figure 2(b) shows that the expected profit decreases in the exchange-rate volatility except for the firm using both the allocation option and financial hedge. This is because the allocation option is equivalent to a call option (on the exchange rate) whose value increases as the exchange-rate volatility increases, and with financial hedge, the optimal capacity is not sensitive to the increase of the exchange-rate volatility. The increase of the expected unit profit can have a more dominating effect than the slight decrease of optimal capacity and thus leads to an increasing expected profit. Figure 2(c) shows that the relatively fast decrease of the optimal capacity leads to a fast decrease of profit variance for firms

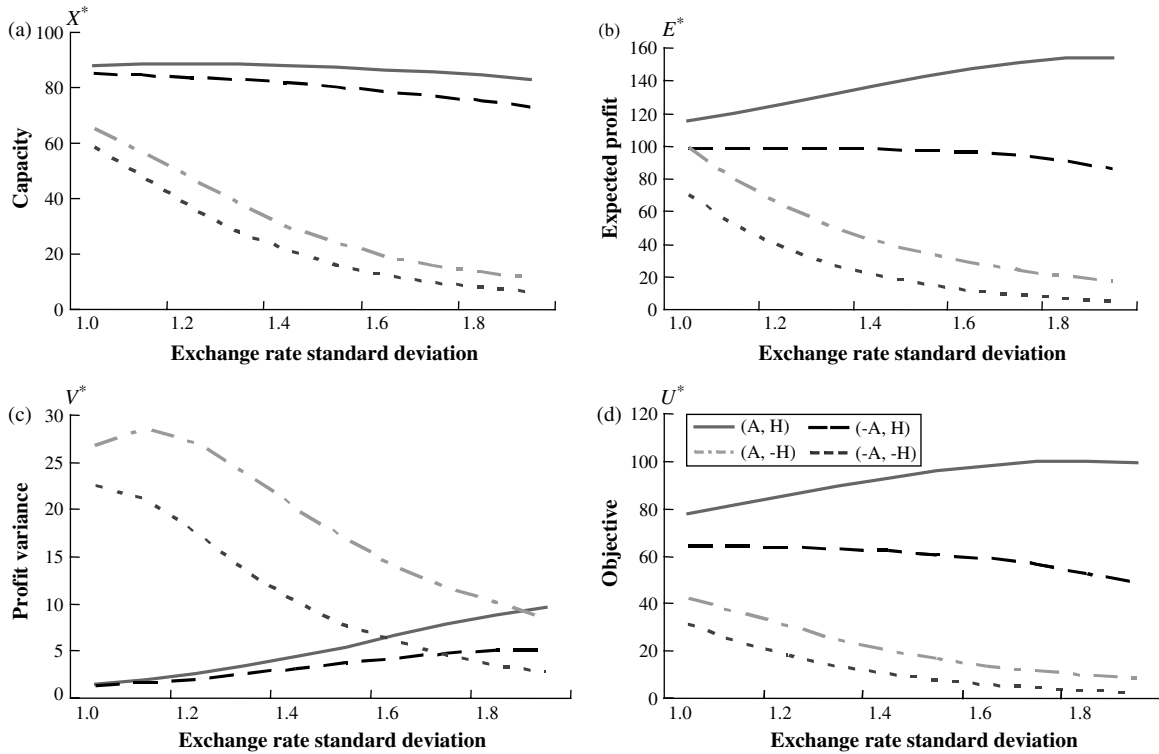
without financial hedge, while the insensitivity of optimal capacity leads to a slight increase of the profit variance for firms using financial hedge. Thus, the MV objectives of firms using financial hedge decrease slower than firms not using financial hedge as shown in Figure 2(d). For a firm that uses both allocation and financial options, the advantage of the increased expected profit outweighs the disadvantage of the increased variance; the MV objective might increase with an increasing exchange-rate volatility.

To study the effect of demand volatility, we let the standard deviation of the demand vary from 30 to 50, i.e., the coefficient of variation changes from 30% to 50%. Figure 3(a) shows that the optimal capacities decrease as the volatility of the foreign demand increases. Interestingly, firms using financial hedge are slightly less sensitive to the change in demand volatility. This is because a financial hedge reduces the risk caused by the currency exchange rate, i.e., the risk associated with the firm’s unit profit. Therefore, the demand volatility will have less of an effect on the firm’s total profit risk. Figure 3(b) shows that the expected profits decrease due to the decrease of optimal capacities. Similar to the observation in Figure 2(c), the insensitivity of optimal capacities to the demand volatility for firms using financial hedge leads to a slight increase of the total profit variance. The net effect, as Figure 3(d) shows, is that the MV objectives decrease as demand volatility increases. Firms using financial hedge see a faster decrease in the MV objective than firms not using financial hedge.

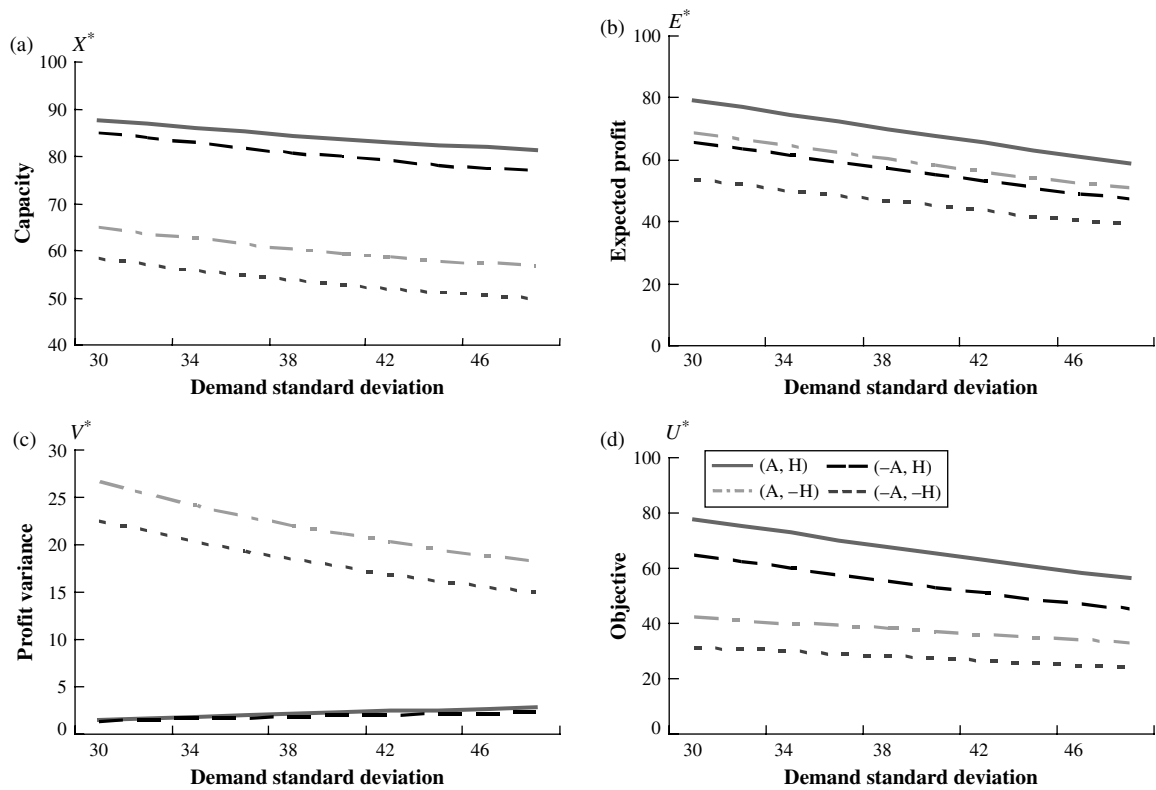
Figure 4 shows the effect of the MV ratio. For this particular set of parameters, firms using financial hedge reduce the profit variance more significantly than firms not using financial hedge. Thus, Figure 4(a) shows that as firms become risk averse, firms not using financial hedge have to reduce capacity faster than firms using financial hedge. Reflected on the corresponding expected profit, variance, and MV objective, as shown in Figures 4(b), (c), and (d), respectively, financial hedge makes firms less sensitive to the degree of their risk aversion.

As we have shown in Corollary 4, the optimal capacity for case (A, H) of the risk-averse firm is less than that of the risk-neutral firm. This result holds for the other three cases as well and can be observed in Figure 4(a)

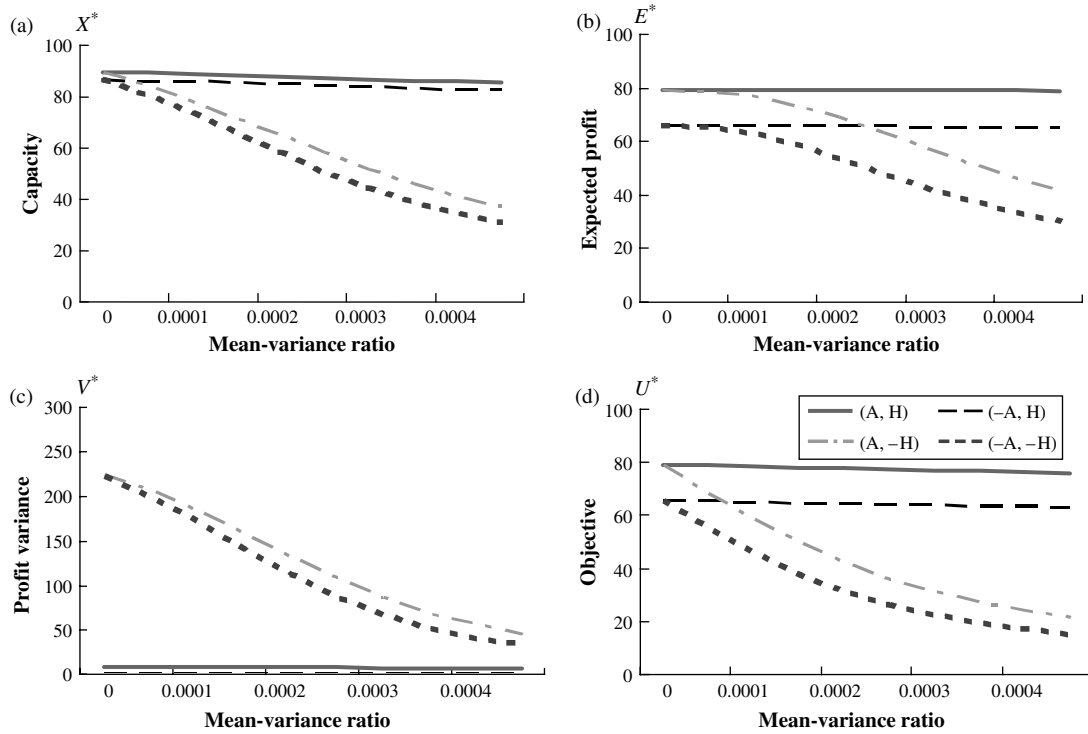
**Figure 2.** Effect of the currency exchange-rate volatility on (a) optimal capacity, (b) expected profit, (c) profit variance, and (d) MV objective.



**Figure 3.** Effect of the demand standard deviation on (a) optimal capacity, (b) expected profit, (c) profit variance, and (d) MV objective.



**Figure 4.** Effect of the MV ratio on (a) optimal capacity, (b) expected profit, (c) profit variance, and (d) MV objective.



by comparing the values of  $X^*$  at any  $\lambda > 0$  with those at  $\lambda = 0$ . In particular, the optimal capacity associated with risk-neutral firms that use the allocation option, i.e., cases (A, ·) at  $\lambda = 0$ , maximizes the firm’s expected profit and can be used as a benchmark. Taking any  $\lambda > 0$ , we start from case (–A, –H), which has the smallest optimal capacity, expected profit, and MV objective among the four cases. Adding the allocation option, case (A, –H) brings the optimal capacity and expected profit up closer to those of the benchmark. Then, adding the financial hedge, case (A, H) brings the optimal capacity and expected profit further up to be very close to those of the benchmark. A similar pattern can be observed for the sequence (–A, –H), (–A, H), and then (A, H). Thus, the allocation option and financial hedge are instruments through which the risk-averse firm can achieve the close-to-optimal expected profit at lower risk.

### 5. Domestic Production Facility Supplying Both Foreign and Domestic Markets

#### 5.1. Basic Results

Recall that for the two-market case, the firm’s operations profit in stage 2 is defined by

$$\begin{aligned} \pi^{\text{op}}(X, s, \mathbf{d}) &= r_2^+(s) \min((X - d_1)^+, d_2) + (r_2(s) - r_1)^+ \\ &\quad \cdot (\min(X, d_2) - \min((X - d_1)^+, d_2)) \\ &\quad + r_1 \min(X, d_1). \end{aligned} \tag{12}$$

The allocation to the foreign market starts when  $s \geq \tau_2/p_2$  (i.e.,  $r_2(s) \geq 0$ ), and the allocation priority is given to the foreign market when  $s \geq \tau_2/p_2 + (p_1 - \tau_1)/p_2$  (i.e.,  $r_2(s) > r_1$ ).

The counterpart of Proposition 3, i.e., the unimodality of the objective function in  $X$ , does not hold anymore. We will use  $\mathbf{h}^*$  to represent the portfolio of the two call options specified in Corollary 2.

**PROPOSITION 6.** *For the independent case, if zero risk premium is assumed, then*

- (1)  $E[\pi^{\text{op}}(X, h, s, \mathbf{d})]$  increases concavely in  $X$ .
- (2) If  $E[(X - d_i)1_{\{d_1 + d_2 > X, d_i \leq X\}}] \leq E[(X - d_i)^+] \Pr(d_1 + d_2 > X)$  for  $i = 1, 2$ , then  $V[\pi(X, \mathbf{h}^*, s, \mathbf{d})]$  increases in  $X$ .

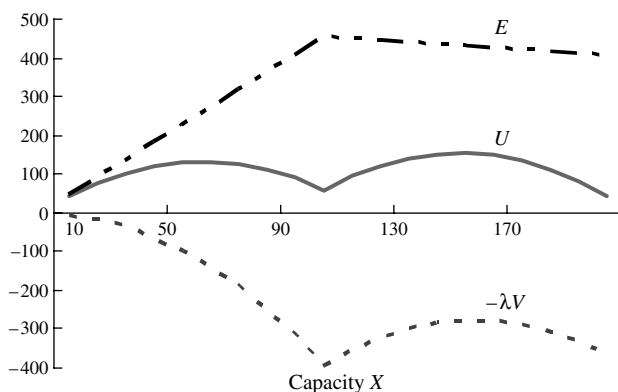
Proposition 6 states that, although the firm’s operations profit always increases concavely in capacity, the profit variance is not always monotone in capacity. An extra unit of capacity increases the expected sales in both markets and in general increases the profit variance in both markets. However, the covariance between the sales of the two markets tends to be reduced by the extra unit of capacity for the following reason. Consider a scenario of given exchange rate. Without loss of generality, assume that market 1 is the first-priority market (i.e.,  $r_1 \geq r_2(s)$ ). An extra unit of capacity (a) can satisfy one more unit of demand in the first-priority market when the first-priority market demand was not completely satisfied yet, i.e.,  $d_1 > X$ , or (b) can satisfy one more unit of demand in the second-priority market when the first-priority market demand was completely satisfied already, i.e.,  $d_1 \leq X$  and  $d_1 + d_2 > X$ ,

or (c) is useless if demand in both markets was completely satisfied already, i.e.,  $d_1 + d_2 \leq X$ . The covariance between sales in the two markets becomes more negative in case (b), but is not affected in cases (a) and (c) by the extra unit of capacity. Whether the total profit variance will decrease in capacity is determined by whether the negative part in the covariance of the two market sales is significant enough. Proposition 6(2) gives the sufficient condition of the profit variance increasing in capacity, which roughly represents the condition under which the effect of case (b) is outweighed by the effects of cases (a) and (c). When this condition fails, the profit variance may decrease in capacity. The following example shows that as a result of the non-monotonicity of profit variance in capacity, the firm's MV objective might be multimodal in capacity.

EXAMPLE 2. Assume that  $ce^{\gamma T} = 3$ ,  $\tau_1 = \tau_2 = 0$ ,  $p_1 = 10$ ,  $p_2 = 1$ ,  $\ln s \sim N(1.5, 0.5)$ , and the risk premium is zero,  $\lambda = 0.005$ . The two-market demand follows a joint two-point distribution,  $(d_1, d_2) \sim (100\delta, 200(1 - \delta))$  and  $\Pr(\delta = 0) = \Pr(\delta = 1) = 0.5$ . We can think of  $\delta$  as a location random variable that indicates a particular event will take place in either home or foreign country and will create demand in the corresponding market. Note that the total demand of the two markets is between 100 and 200. Thus, when  $X < 100$ , the firm is always in case (a) regardless of the exchange rate. As shown in Figure 5, the profit variance first increases for  $X \leq 100$ . When  $X \in [100, 200]$ , all three cases could happen, and the profit variance decreases for  $X \in [100, 150]$  then increases again for  $X \in [150, 200]$ . This is because the effect of case (b) first dominates but is then dominated by the effects of cases (a) and (c) (also reflected by the first-increase-then-decrease of case (b) probability in  $X$ , i.e.,  $\Pr(d_1 \leq X, d_1 + d_2 > X)$ ). As a result, the profit variance is multimodal in  $X$  and there exist multiple local optimal capacity decisions.

LEMMA 1. Assume that the risk premium is zero. If  $\partial V / \partial X \geq 0$  for  $X \geq 0$ , then  $X^* \leq X_N^*$ .

Figure 5. An example of multiple local optimal capacity decisions.



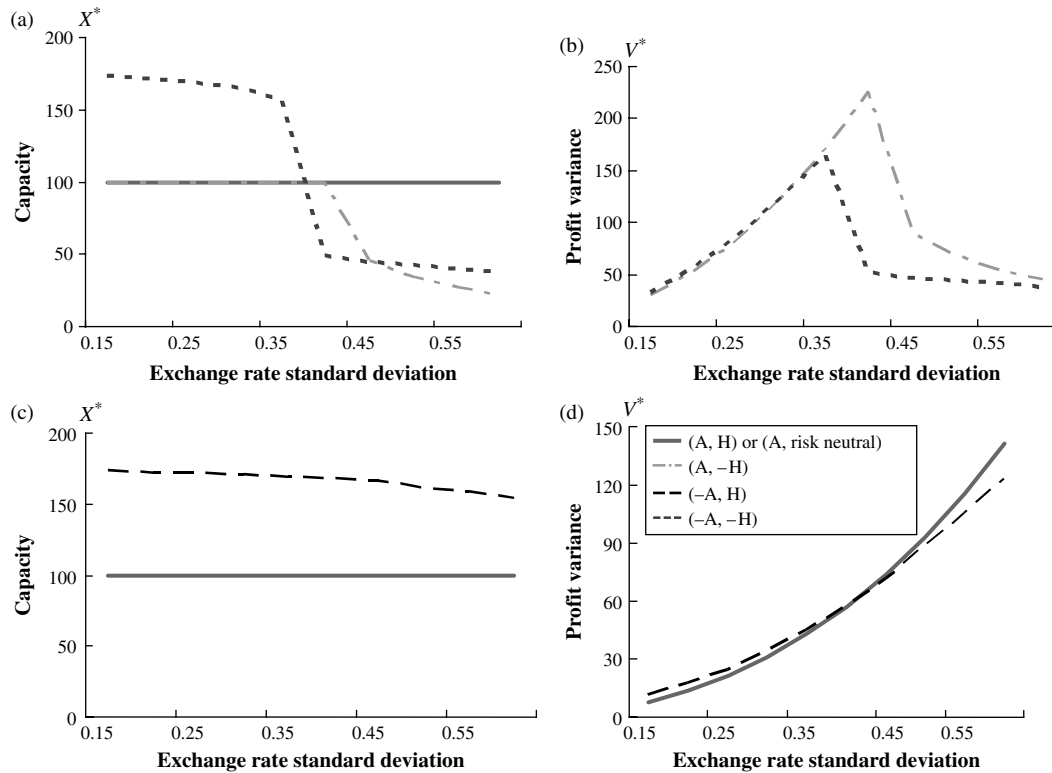
In the two-market case, as the monotonicity of the variance in capacity no longer holds, the comparison of optimal capacities between the risk-averse and the risk-neutral firms becomes complicated. In general, the straight dominance of the risk-neutral capacity over the risk-averse capacity in the one-market case is no longer true in the two-market case, as we shall see in numerical examples in §5.2. For results on the optimal risk-neutral capacity for the two-market case, we refer interested readers to Ding et al. (2004) (our previous working paper) and Kazaz et al. (2005).

## 5.2. Analysis of Operational and Financial Hedges

Recall that the main findings from the analysis of operational and financial hedges in the one-market case are: (1) The concerns about risks lead risk-averse firms to invest less in capacity than risk-neutral firms. However, risk-averse firms can use allocation option or financial hedge to increase capacity and expected profit. Using both instruments will achieve the biggest increase of capacity and expected profit. (2) Using the financial hedge makes a risk-averse firm's capacity investment less sensitive to volatilities in exchange rate and foreign demand and less sensitive to its degree of risk aversion.

Finding (1) is no longer true in the two-market case. Specifically, it is because the addition of the domestic market introduces the covariance between the sales in two markets. Consider the case without allocation option. An increase of capacity for one market could increase or decrease the covariance between the sales in two markets (i.e.,  $\text{cov}(\min(X_1, d_1), \min(X_2, d_2))$ ), depending on the demand correlation between the two markets. The relative sizes of the two markets also play a role in determining whether this marginal impact on covariance has a dominating effect on the change in total profit variance. Thus, unlike the one-market case in which the profit variance always increases in capacity, profit variance can be nonmonotone in capacity. Consider the case with allocation option, as we discussed in §5.1, that the marginal effect of an extra unit of capacity will have an even stronger and more complicated effect on the covariance. Thus, a risk-averse firm might invest more than a risk-neutral firm in seeking the benefit of a lower profit variance. Moreover, adopting allocation does not necessarily lead to an increase of the total capacity, even for a risk-neutral firm. The reason is similar to that for the general risk-pooling cases. An allocation option allows the firm to address the combined demand of two markets by using a common capacity. The combined demand can be much more skewed than the individual market demand (Yang and Schrage 2006), which can lead to a higher or lower total capacity investment. For a risk-averse firm, the impact of the allocation option on profit variance further complicates the impact on capacity investment. Example 3 shows, in the two-market case, how relationships between no-allocation and allocation, and

**Figure 6.** Effect of the allocation option in the two-market case in Example 3. (a) Capacity comparison and (b) variance comparison between (A, -H) and (-A, -H). (c) Capacity comparison and (d) variance comparison between (A, H) and (-A, H).



between risk-averse and risk-neutral, can be different from those in the one-market case.

**EXAMPLE 3.** Assume that  $ce^{\gamma T} = 1$ ,  $\tau_1 = \tau_2 = 0$ ,  $p_1 = e^{1.5}$ ,  $p_2 = 1$ , and the risk premium is zero. The joint demand follows a joint uniform distribution:  $d_1 = 100 - d_2$  and  $d_2 \sim U[0, 100]$ . Assume that  $\ln s \sim N(\mu, \sigma)$  and  $\lambda = 0.02$ . We increase  $\sigma$  from 0.15 to 0.6 and adjust  $\mu$  so that  $E[s]$  is kept at a constant level of  $e^{1.5}$ . Figure 6(a) compares capacities among risk averse (A, -H), (-A, -H), and risk neutral with allocation. We can see that  $X_{-A, -H}^* \geq X_{A, -H}^* \geq X_{A, \text{risk neutral}}^*$  for  $\sigma \in [0.15, 0.4]$ . Figure 6(b) shows that the allocation option could increase or decrease the total profit variance. Similar observations can be made for (A, H) and (-A, H) (Figures 6(c) and (d)).

The impact of the financial hedge on capacity is quite interesting in the two-market case. Note that under the same capacity, the financial hedge only reduces the foreign market profit variance and has no effect on the domestic market profit variance or the covariance between two-market profits. For a firm that already uses the allocation option, this benefit will motivate the firm to invest more in overall capacity. A firm that does not have the allocation option will be motivated to invest more dedicated capacity for the foreign market. However, as we discussed, the increase of the dedicated foreign market capacity might have a non-monotonic effect on the covariance between sales in the two

markets, and the domestic market capacity might need to be adjusted to control that effect on the total profit variance. Thus, for firms not having an allocation option, the capacity investment in the domestic market might increase or decrease when financial hedge is used. Corollary 6 and Proposition 7 are the counterparts to Corollary 5 and Proposition 5.

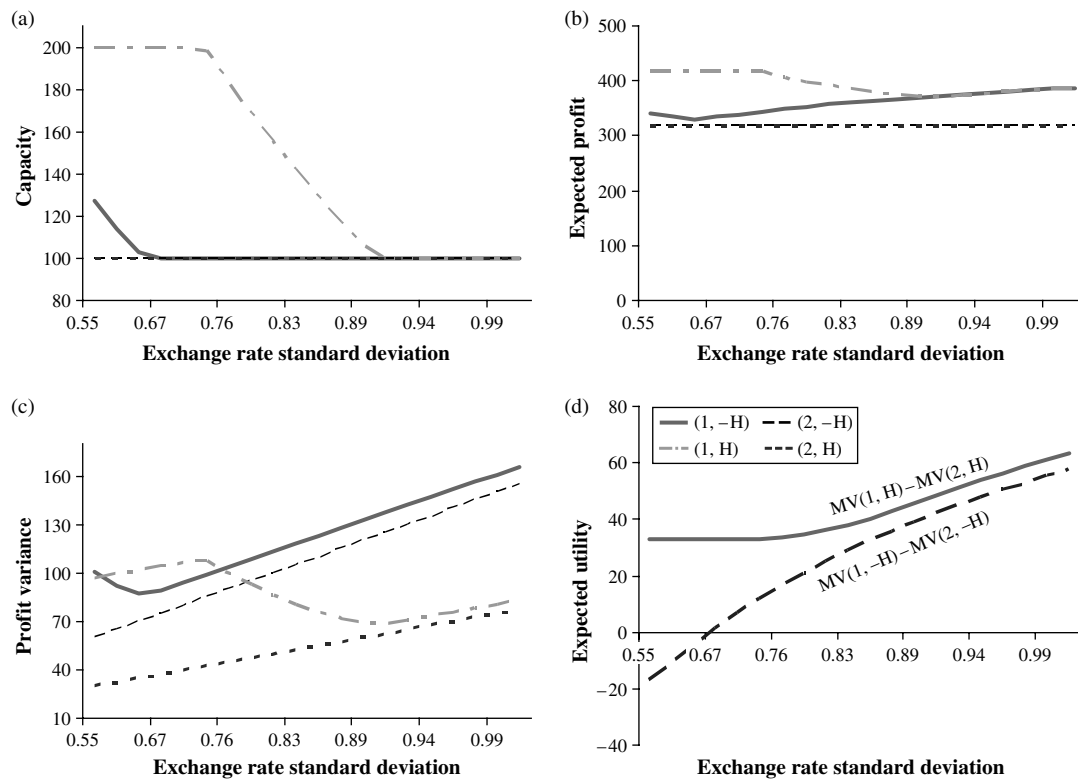
**COROLLARY 6.** For a domestic production facility supplying both foreign and domestic markets without the allocation option, given capacity  $X$ , if the demand and exchange rate are independent and the risk premium is assumed zero, then the optimal financial hedge among all call and put options is a single forward contract.

**PROPOSITION 7.** For a domestic production facility supplying both foreign and domestic markets with the demand and exchange rate being independent and the risk premium assumed zero:

- (1) With the allocation option, the use of the optimal financial hedging strategy (specified in Corollary 2) increases the optimal capacity and the MV objective. That is,  $X_{A, -H}^* \leq X_{A, H}^*$  and  $U_{A, -H}^* \leq U_{A, H}^*$ .
- (2) Without the allocation option, the use of the optimal forward contract increases the optimal capacity dedicated to the foreign market and the MV objective. That is,  $X_{-A, -H, 2}^* \leq X_{-A, H, 2}^*$  and  $U_{-A, -H}^* \leq U_{-A, H}^*$ , where  $X_{-A, -H, 2}^*$



**Figure 7.** An example illustrating a firm’s facility location decision can be affected by employing a financial hedge.



and  $X_{-A,H,2}^*$  are the optimal capacities dedicated to the foreign market in  $(-A, -H)$  and  $(A, -H)$ , respectively.

Extensive numerical studies of a similar nature were also conducted for the two-market case (with one or two facilities), and finding (2) remains valid in those studies as well.

## 6. Effect of Financial Hedging on Location and Supply Chain Structure

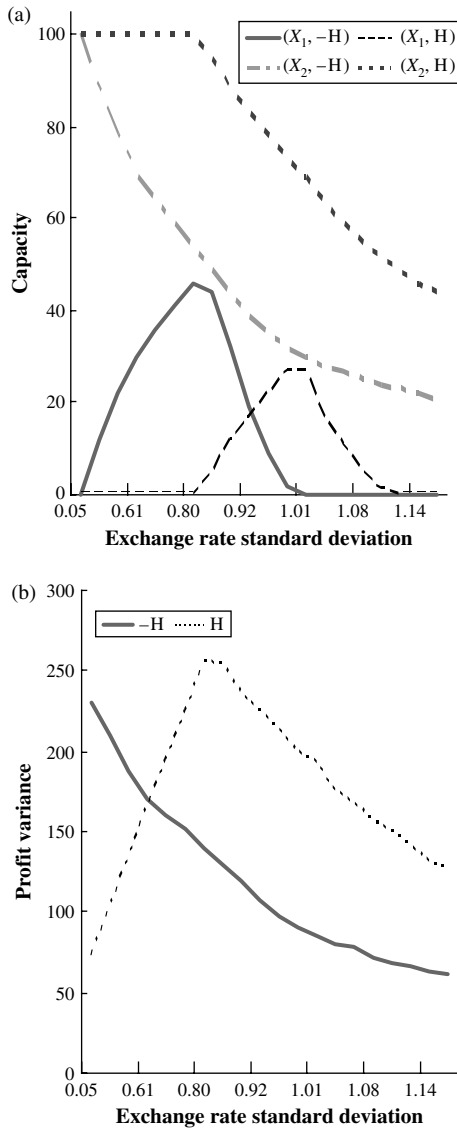
We will now show through numerical examples that in other operational settings, the profit variance reduction effect of the financial hedge can have an impact on a firm’s strategic decisions such as the location and the number of its production facilities.

**EXAMPLE 4.** We consider a firm that has the choice of setting up one facility in either the domestic or the foreign market and the initial production and localization are conducted at the same facility. We assume that  $p_1 = 10$ ,  $p_2 = 1$ ,  $d_1 = d_2 = 100\delta$  with  $\Pr(\delta = 0) = \Pr(\delta = 1) = 0.5$ ,  $\lambda = 0.0002$ . The exchange rate  $s$  follows a lognormal distribution with  $\ln s \sim N(\mu, \sigma)$  with  $\mu = 2$  and  $\sigma = 0.5$ . For the domestic facility, we have  $c_1 e^{\gamma T} = 2.5$ ,  $\tau_{11} = \tau_{12} = 0$ , and for the foreign facility,  $c_2 e^{\gamma T} = 1$ ,  $\tau_{21} = \infty$ ,  $\tau_{22} = 0$ . Assuming the allocation option, we use  $(i, -H)$  and  $(i, H)$ ,  $i = 1, 2$ , to denote the cases where the production facility is located in market  $i$ , without and with financial hedge, respectively. Figure 7 provides comparisons on capacity, expected profit, variance, and MV utility

as the exchange rate standard deviation  $\sigma$  changes from 0.55 to 1 while keeping the mean exchange rate constant. In particular, Figure 7(d) plots the MV utility difference between the domestic location and the foreign location for the without-hedging and with-hedging cases. That is, a positive (respectively, negative) value indicates that the firm would prefer the domestic (respectively, foreign) location. We can see that when  $\sigma$  is less than 0.67, without financial hedge the firm will choose the foreign location, but with financial hedge the firm will choose the domestic location. The domestic location has the obvious advantage of being able to serve demands in both markets. However, the fact that demands are perfectly correlated between two markets makes the domestic location riskier than the foreign location, especially when the domestic facility has capacity higher than 100. Thus, without financial hedge, the compound demand and exchange-rate risk makes the domestic location less attractive than the foreign location. As exchange-rate volatility increases, the domestic capacity is reduced to and stays at 100, the benefit from the allocation option outweighs exposure to the exchange-rate risk, and thus the domestic location is preferred. With financial hedge, the exchange-rate risk is significantly reduced, the firm always prefers the domestic location, and it can afford to invest full capacity of 200 for a large range of exchange-rate volatility.

**EXAMPLE 5.** We consider a firm that can set up two production facilities, one in the domestic market and the other

**Figure 8.** An example illustrating a firm’s decision on the number of production facilities can be affected by employing a financial hedge.



in the foreign market. We assume that  $p_1 = 8$ ,  $p_2 = 1$ ,  $(d_1, d_2) \sim (100\delta, 100(1 - \delta))$ ,  $\Pr(\delta = 0) = \Pr(\delta = 1) = 0.5$ , and  $\lambda = 0.0002$ . The exchange rate  $s$  follows a log-normal distribution with  $\ln s \sim N(\mu, \sigma)$  with  $\mu = 1.5$  and  $\sigma = 0.4$ . For the domestic facility, we have  $c_1 e^{\gamma T} = 4.5$ ,  $\tau_{11} = 0$ ,  $\tau_{12} = \infty$ , and for the foreign facility,  $c_2 e^{\gamma T} = 0.5$ ,  $\tau_{21} = \tau_{22} = 0$ . Assuming the allocation option, we compare the without-financial-hedge case with the with-financial-hedge case. Figure 8 compares capacity and profit variance for the two cases as the exchange rate standard deviation  $\sigma$  changes from 0.05 to 1.16, while keeping the mean exchange rate constant. Figure 8(a) shows that without financial hedge, the firm uses both the domestic and foreign facilities for  $\sigma \in (0.05, 1.01)$ , but uses only the foreign facility for  $\sigma$  outside that range. On the other hand,

with financial hedge, the firm uses both facilities when  $\sigma \in (0.85, 1.11)$ , and uses only the foreign facility for  $\sigma$  outside that range. In this example, the use of financial hedge affects the number of production facilities employed by the global firm to meet demand. To understand the changes in the firm’s capacity decision, we start from the case without hedging. Because the demand in two markets is perfectly negatively correlated (with a total demand of 100) and only the foreign facility can serve both markets, the firm prefers foreign capacity. For small exchange-rate volatility, the firm invests capacity 100 at the foreign facility and zero at the domestic facility. As the exchange rate becomes more volatile, the firm starts to decrease the foreign capacity and increase domestic capacity. When the exchange rate standard deviation exceeds 0.85, the need for controlling the total profit variance makes the firm also start to cut the capacity at the domestic facility and to stop using the domestic capacity when  $\sigma$  exceeds 1.01. The use of financial hedge allows the firm to keep the foreign capacity at full 100 for a larger range of exchange-rate volatility and use the domestic facility for a smaller range of exchange rate volatility. Finally, the solutions with the financial hedge dominate across the whole range of exchange rates, both in terms of profit and MV utility (and have decreased profit variance for all exchange rates above 0.85; see Figure 8(b)).

### 7. Summary

Global firms selling to multiple countries face demand uncertainties in different countries and currency exchange rate uncertainties. From the operations perspective, a firm could exploit the capacity allocation option by delaying the commitment of capacity to specific markets until demand and exchange rate uncertainties are realized. From the finance perspective, a firm could use financial instruments such as option contracts in the currency exchange market to hedge the exchange rate uncertainty. Often, the allocation option is viewed as an instrument for profit improvement and the financial hedge is viewed as a tool for profit variance control, while in fact both have an impact on the firm’s optimal capacity decision, expected profit, and profit variance. In this paper, we studied the impact of the simultaneous use of these two instruments in a stylized MV model in which a firm sells to markets in their home country as well as a foreign country through a production facility in the home country.

Our analysis started from developing the optimal financial hedging policy for the operations profit of general functional form. When a portfolio of call and put options is considered, we derived the financial hedging quantities for given exercise prices. This result illustrates the tight linkage between the firm’s operational strategy and financial strategy. To study the optimal joint capacity and financial hedge decisions, we focus on a simpler case of the home-country production facility supplying the foreign market, in which case the optimal capacity can be obtained from

the first-order condition. We then decomposed the effect of the joint use of the allocation option and financial hedge. As expected, given capacity, the allocation option enables the firm to avoid unprofitable exchange-rate scenarios and thus improves expected profit and reduces profit variance; financial hedge reduces the impact of exchange rates on its profit variance. Hence, the firm will increase capacity and improve the expected profit and MV objective by adopting either risk-hedging instrument. Naturally, the firm achieves its best performance when both instruments are utilized. Although the risk-averse firm's capacity and expected profit are always below the risk-neutral firm, the allocation option and financial hedge help firms get control of the profit variance and thus allows them to reach a capacity level that is closer to (however, not exceeding) the capacity level of a risk-neutral firm. Numerical studies showed that firms using financial hedge are less sensitive to demand and exchange-rate volatilities and their own risk attitude than firms not using financial hedge.

As the analysis extends to the case of the home-country production facility supplying both domestic and foreign markets, some insights from the one-market case remain true, while others no longer hold. This is mainly because the addition of the domestic market introduces covariance between sales in the two markets, and the effect of the covariance on the profit variance is not necessarily monotone in capacity. The main differences in results are: (1) the firm's MV objective can be multimodal in capacity, i.e., the firm can have multiple local optimal capacities. This is a direct result of the nonmonotonicity of covariance in capacity. (2) A risk-averse firm might invest in more capacity than the risk-neutral firm. The possible negative covariance between sales in the two markets may drive risk-averse firms to overinvest in capacity than a risk-neutral firm to seek a lower profit variance. (3) Adopting an allocation option might decrease the firm's capacity. (4) The firm that does not use the allocation option will always increase the dedicated capacity for the foreign market after the adoption of the financial hedge but could decrease or increase the dedicated capacity for the domestic market. Results that remain valid in the two-market case are: (1) The firm that uses the allocation option will increase the overall capacity after adopting financial hedge. (2) Firms using financial hedge are less sensitive to volatilities and risk attitude than firms not using financial hedge.

In our paper, we also dealt with how the presence of multiple production facilities and multiple markets can affect the nature of operational and financial hedging practices. While it is intuitively expected that the integrated use of operational and financial hedges will have an effect on the magnitude of capacity investments and production output, it is less intuitive to anticipate that the use, or lack of use, of financial hedges can affect global supply chain structural choices, such as the desired location and number of production facilities to be employed to meet global demand.

But this is exactly what we found in our numerical experiments with two-facility-two-market models in the presence of demand and exchange-rate uncertainty (see §6).

Our research makes an important first step in closing an apparent gap in the international operation and finance literature on quantifying the simultaneous setting of operational and financial hedging policy parameters and explaining the nature of implications of such practices for capacity decisions of global firms. Further research that understands the competitive implication in the presence of multiple firms serving global markets from the use, or lack of use, of integrated risk-management approaches will be fruitful. In particular, a concrete understanding of the effectiveness of operational and financial hedges in mitigating the risk of competitive exposure to exchange-rate fluctuation will be a valuable next step. We have started exploring some of these issues in our ongoing research efforts.

## 8. Electronic Companion

An electronic companion to this paper is available as part of the online version that can be found at <http://or.journal.informs.org/>.

## Endnotes

1. If  $H(h2) < (f - s_L)Qe^{-\gamma T}$ , then one can borrow  $H(h2)$  from the bank at a risk-free rate of  $\gamma$ , buy call option  $h2$ , and sell forward  $h1$  at stage 1. The stage 2 payoff would be  $(s - s_L)Q$  from the call option,  $-H(h2)e^{\gamma T}$  for paying off the loan,  $(f - s)Q$  from the forward, and the net payoff is  $(f - s_L)Q - H(h2)e^{\gamma T} > 0$ , arbitrage! If  $H(h2) > (f - s_L)Qe^{-\gamma T}$ , then one can sell call option  $h2$ , deposit  $H(h2)$  in the bank, and buy forward  $h1$  at stage 1. The stage 2 payoff would be  $-(s - s_L)Q$  from the call option,  $H(h2)e^{\gamma T}$  from the bank,  $(s - f)Q$  from the forward, and the net payoff is  $H(h2)e^{\gamma T} + (s_L - f)Q > 0$ , arbitrage!
2. One widely used measure of financial risk is value of risk (VaR), i.e., the quantile of the projected distribution of gains and losses by holding a portfolio over the target horizon (Jorion 2001). The computation of VaR can be challenging in operational settings where the distributions of profits are often functions of truncated demand/price distributions. Caldentey and Haugh (2005) show that a more tractable mean and standard deviation (MStd) constraint provides a lower bound to the VaR constraint, making it more tractable to study the problem of maximizing a firm's expected value subject to the downside risk constraint.
3. Wong and Yick (2004) show the optimal call-option size of a given call option for a general utility function under the assumption of deterministic demand.
4. Finite many call options can reduce the profit variance significantly and thus constitute a more practical financial hedging strategy. See Ding et al. (2004) (our previous working paper, which is available from the authors upon request).

5. Wong (2003 and references therein) uses “export flexibility” to refer to this allocation option, and gives examples of multinationals that may or may not have it.

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