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A Renewable-Resource Approach to Database Valuation

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Abstract

We demonstrate in this paper that firms should view customers in a database as a *renewable resource* when valuating them. Indeed, customer names flow in and out of the firm's databases, and the goal of the firm is to optimize the overall customer acquisition/cultivation/attrition process.

The renewable resource approach to the problem of maximizing the profits generated by a flow of customer names is more appropriate than the traditional Customer Lifetime Value (*CLV*) approach. We show that *CLV* underestimates the true value of names (by more than 400% in some cases) and leads to sub-optimal customer relationship management and acquisition strategies. For instance, while a firm should discount future profits when valuating its database, it should not discount these same profits when setting its acquisition policy.

We derive the first-order condition, and perform comparative statics, for a firm trying to optimize its contact policy. Perhaps the most interesting result stemming from the comparative statics is the implication that start-up companies should contact their customers less frequently than established companies. Start-ups should resist the temptation of boosting short-term revenues in favor of accumulating names by spacing communications.

We finish the paper with an analysis of customer heterogeneity.

Keywords: Database Marketing, Customer Lifetime Value, Internet Marketing, Direct Marketing, Customer Acquisition

1 Introduction

There have been two important developments in direct marketing in the past few years. First, the growth of the Internet as a communication medium has significantly decreased both the production and delivery costs of direct marketing communications (Gauthronet and Drouard 2001). This has led to an increase in communication frequency and in the number of firms engaged in direct marketing. Second, there has been a strong backlash from consumers to the pervasiveness of direct contact from firms. This is most noticeable on the Internet where consumers appear to have a very low threshold for inconvenience (Forrester 2001b).

This change in consumer attitude, as well as new legislation (e.g., the Anti-Spamming Act of 2001 in the US; the E-Privacy Directive in Europe), has impelled firms to engage in *Permission-Based Marketing* (*PBM*). In a *PBM* setting, a firm must obtain permission from consumers before it can contact them through a direct marketing medium (Barwise and Strong 2002). Further, with every communication sent, the firm must provide consumers with an opportunity to rescind the permission. Once consumers have withdrawn their permission, the firm may not send them further communications.

The importance of *PBM* from a research standpoint stems from its strong emphasis on the customer *acquisition* and *defection* processes. Indeed, when studying *Customer Lifetime Value* (*CLV*), researchers have often taken these two processes as being exogenous (Gupta and Lehmann 2003; Gupta, Lehmann, and Stuart 2003; Berger and Nasr 1998; Blattberg and Deighton 1991). This is understandable since, in a traditional direct marketing setting, firms often acquire names without the knowledge of the acquired individuals, and customer defection is rarely explicit (Schmittlein, Morrison, and Colombo 1987, Jackson 1985). This leads to an *always-a-share* (i.e., probabilistic) view of customer retention. This view is not appropriate, however, in the context of *PBM*, as acquisition and defection are now observed and should be made endogenous to the marketer's decision making.

We propose, in this paper, a model for the valuation of a database of potential customers that incorporates the acquisition decision and the impact of the marketer's action on customer retention. We show how extending the concept of customer lifetime value to a concept of database value necessitates a switch from a consumer revenue approach (i.e., focusing on the future revenues to be expected from current customers) to a customer flow approach (i.e., capturing the customer acquisition –in-flow– and attrition –out-flow– processes). To find the optimal trade-off between revenue extraction and defection, our model takes into account the effects that the marketing communications of the firm have on customers. This is accomplished by modeling the impact of communication frequency on communication content and customer acquisition as well as modeling the impact of communication content and frequency on attrition. In doing so, we adopt the perspective that a firm should look at its customers as a renewable resource rather than a perishable resource. This change in perspective alters the customer lifetime value maximization problem in three ways: First, we show that the firm's database will grow (or shrink) to a finite steady state size. This steady state size can be used to compute the long-term value of the database as a whole. Second, the database as a whole is more valuable than the sum of its parts. This is a critical finding because it demonstrates how *CLV* maximization *under-values* customers. Third, we show that *CLV* maximization leads to *sub-optimal* customer acquisition strategies (under-investment in acquisition) as well as sub-optimal revenue extraction strategies (under-utilization of names).

Within this framework, we derive the first-order condition for maximizing a firm's database. We show that acquisition costs do not affect marketing activities except as a go/no-go threshold. We also use static comparisons to study the impact of changes in retention rate, revenue per customer, and other environmental variables on the optimal communication interval. Finally, we discuss the issue of customer heterogeneity, and how it affects the firm's valuation and communication decisions.

2 Background and Motivation

We define Permission-Based Marketing as the management of exchange-based relationships between firms and groups of customers, where the firms initiate potential exchanges (e.g., by sending a marketing communication to potential customers) but where the customer is responsible for activating and ceasing the relationship (i.e., give or remove her permission to the firm of contacting her). Fundamentally, *PBM* involves the balance between the retention of the customer and the ability of the marketer to continue to extract rents from its database of customers. These two aspects have, to date, been treated independently. Some researchers have studied the issue of customer retention or attrition (e.g., Thomas 2001; Schmittlein and Peterson 1994); others have worked on maximizing the revenue stream generated by marketing communications while either ignoring attrition (e.g., Gönül, Kim, and Shi 2000; Gönül and Shi 1998; Bult and Wansbeek 1995) or having a constant attrition rule (e.g., Pfeifer and Carraway 2000; Bitran and Mondschein 1996). However, we know of no work that has merged the two issues in a cohesive theoretical framework (this lack of work on the topic is lamented by Jain and Singh 2002).

A prototypical example of Permission-Based Marketing, and the context adopted in this paper to facilitate exposition, is email marketing. The number of active email users has grown in excess of 600% over the years 1996-2000, to reach over 900 million users (Forrester 2001b). Budget wise, email is already the second largest marketing vehicle, barely trailing direct marketing (eMarketer 2002). Forrester Research predicts that this medium will continue to grow, forecasting industry-wide expenditures of around US$6.8 billion by the year 2006 (Forrester 2001a); others forecast $3.52 billion by the year 2005 (Internet Retailer 2003). However, as with most new communication media, marketers are still trying to learn the rules of the game, and are attempting to gauge what works with consumers and what does not. This search for balance has left many consumers unhappy. Observers are wary that permission-based marketers are "fouling their own nests" by sending email communications that are too long, too frequent, or are poorly targeted in their content (Forrester 2002b). This leads to discontented consumers and high customer attrition rates (Forrester 2001b, eMarketer 2002).

The impact of email content on customer attrition is straightforward. It is easy to see that poor content would lead to lower retention rates than good content. The effect of the periodicity of communication on attrition is more pernicious. To see why, one can conduct the following thought experiment: let us consider the two extremes in contact periodicity. At one extreme, a firm might contact its clients so often that the relationship becomes too onerous for the clients to maintain and thus they sever their links to the company, rendering their names worthless. At the other extreme, the firm never contacts its clients, and although the names have a *potential* value, this value is never realized. Thus, periodicity affects the value of names in two ways. On the one hand, more frequent contact leads to more opportunities to earn money. On the other hand, more frequent contacts provide customers with more opportunities to defect. The latter can quickly lead to high long-term attrition. Imagine the case of a company that has a retention rate of 97% from one campaign to another. This might look like outstanding loyalty. However, if the firm were to send an email every week, it would lose 80% of its database within a year (ignoring the acquisition of new names)! Clearly, there must be an intermediary situation where one maximizes the realized value from a name by optimally trading off the extraction of value in the short-term against the loss of future value due to customer defection.

Although we will use a single overarching example (email newsletters) as the background for our model, it can be applied to most situations where the firm has an ongoing concern and where it has a direct relationship with its customer base. Our model does not apply to firms that have a finite horizon or do not know the identity of their customers. For instance, our model would be of little use to Kellogg's® because it does not actually know who the end customers of its cereals are (expect in general terms) nor would it be useful to an America's Cup team that is managing a list of possible donors as these teams work with 4-5 year time horizons (the next America's Cup race is in 2007) where the goal is to maximize the donations collected before the event with little regard to what happens next.

With these two caveats in mind, our model (or portions of it) can be applied to a large set of firms such as direct marketers (both online and offline), credit card companies, telephone companies, and cable or satellite TV operators. All these companies have direct contact with their customers, receive notification from the customers when they wish to discontinue the service, and can influence the amounts spent by customers through direct communications (e.g., monthly communications from the cable operator regarding new pay-per-view movies).

2.1 Permission-Based Marketing in practice

When building a model, one must often make simplifying assumptions. Of course, such assumptions must preserve the phenomenon studied. Hence, in this section, we present some *PBM* numbers relevant to our model-building efforts. We will use the generic term of *name* to represent a unique customer identifier that can be used by a firm to directly contact its customers. Depending on the context, name can refer to an email address, a mailing address, a phone number, or any such kind of unique identifier (this is what Blattberg and Deighton (1991) refer to as an address). These names are collectively organized in a database, and used for direct marketing purposes. The flow of names in and out of the database is represented, in very general terms, in Figure 1.[1] We assume that there exists a population of potential customers (e.g., individuals, households, firms) whose existence is governed by its own birth and death process (e.g., marriage-separation, start-up-bankruptcy) that is outside the scope of this paper. As a result of the firm's acquisition efforts, names enter the database at a rate of g. Names are removed from the database through customer defection as a result of the firm's action, in terms of what messages it sends (content), and the timing of the contacts (periodicity), as well as a result of their own death process.

Firms have two options when building such a list of names to be used for *PBM* purposes: they can acquire such names from a professional list provider, or they can try to enlist consumers directly. There are over fifty companies currently in the business of renting names for permission-based email marketing purposes (Gauthronet and Drouard 2001). A typical cost of basic email information is $200 per thousand names (*CPM*); higher rates apply when segmentation criteria (e.g., age, income) are included. These companies typically have databases of 15 to 20 million names (much smaller than direct mail databases) and have the traditional limitations of name uses (e.g., number of uses, email content).

If the firm prefers to acquire names directly, it can do so using many different vehicles, such as registration cards or sweepstakes. The acquisition cost, the number of names collected, and the quality and quantity of the supplementary information collected all vary according to the medium used (see Table 1 for some examples from a U.S. division of a large multinational corporation). For instance, a survey may provide rich psycho-demographic information while a name collected via a sweepstake entry typically only consists of basic address information.

Owning names is only a starting point for permission-based marketers. There must also be an infrastructure in place to manage customer names and send emails with a high throughput (e.g., 250,000 emails an hour). Forrester (2002a) estimates the database set-up costs at $500,000 for a firm that wants to manage their own list (not including sending costs). They warn, however, that it takes more than a database to successfully deploy direct marketing campaigns and advise against doing it in-house. If the firm prefers to outsource, campaign set-up costs (creative and programming) will run in the $5,000 to $20,000 range, with an additional $2-$20 *CPM* for delivery (Forrester 2002c, eMarketer 2002). This means that a firm sending a weekly newsletter to one million recipients is likely to face yearly delivery costs of half a million dollars.

As far as content is concerned, the Internet has long been promising one-to-one marketing. Consumers are to receive content that is customized to their needs when they need it. To test whether the current *PBM* practices live up to this promise, we conducted a controlled experiment in which we subscribed to 196 newsletters and monitored their activity for six months. These newsletters belonged to a wide range of companies (see Table 2). Whenever the registration process asked for gender information we registered different male and female accounts; when age information was asked, we registered both a 25 and a 45 year old. In addition, for each newsletter-age-gender combination, we registered three different accounts: one for which the incoming emails would be left untouched; one for which the emails would be opened; and one for which the emails would be opened and, if they contained links, those links would be clicked on to show interest in the content. Hence, for a web site that asks for both gender and age information, 12 accounts would be created (2x2x3). If the web site did not ask for any demographic information, only three accounts would be created.

Of the 196 newsletters, 16% asked for either gender or age information (split evenly between the two), another 16% asked for both types of information. Only 107 (55%) newsletters actually contacted our accounts with enough frequency to be considered as engaging in Permission-Based Marketing (we set the cut-off at three or more emails per account during the six-month period) for a total of 12,946 emails received. The level of customization among the active newsletters was low (see table 3). Four percent of the newsletters customized based on gender information (10% of the active newsletters that asked for gender information), five percent customized based on age (22% of the active newsletters that asked for age information), and five percent customized based on consumer actions (i.e., open or click). Finally, 61% of the newsletters used a fixed contact periodicity; weekly newsletters (63%) being the preferred contact interval (see table 4).

In short, our exploration of the practical aspects of Permission-Based Marketing highlights the facts that (1) email marketing is not free (which may explain why so many of the early players have gone bankrupt); (2) there is only limited customization; and (3) although the contact periodicity varies widely from one firm to another, the majority of firms have a fixed contact interval. We will use these observations in the next section when we build our model.

3 Model

The problem of extracting the highest possible profits from a database of customer names is similar to the problems faced by economists in their study of the optimal management of resources. Economists (Sweeny 1992) consider three types of resources: depletable, renewable, and expendable. The distinction is made based on the time scale of the replenishment process.

Depletable resources, such as crude oil reserves, are those whose replenishment process is so slow that one can model them as being available once and only once. Renewable resources, such as the stock of fish in a lake or pond, adjust more rapidly so that they are self-renewing within the time horizon studied. But any action in a given time period that alters the stock of the resource will have an impact on the subsequent periods. Finally, expendable resources, such as solar energy, renew themselves at such a speed that their use in one period has little or no impact on subsequent periods.

Depending on which type of resources a firm is managing, it faces a different objective. In the case of a depletable resource, the firm is interested in the optimal use of the resource in its path to depletion. With a renewable resource, the firm is interested in long-term equilibrium strategies. Faced with an expendable resource, the firm is interested in its best short-term allocation.

In the context of direct marketing, we can view consumers as any of these types of resource, depending on the lens one uses for the problem. In the cohort view of consumers (the basis for *CLV*), they are a depletable resource. New cohorts are recruited over time, but each cohort will eventually become extinct (see Wang and Spiegel 1994). Attrition is measured by the difference in cohort size from one year to another. In a SPAM environment, where customers cannot prevent spammers from sending them emails, and where new email names can readily be found by "scrubbing" the Internet, marketers see consumers as an expendable resource and thus can take a short-term approach to profit maximization.

For a firm that has a long-term horizon, a depletable view of customers is not viable in a B2C context as it implies that it will lose all its customers at some point. Thus a customer acquisition process must be in place for the company to survive. Further, if the firm has a permission management process in place and incurs acquisition costs so that replacing lost customers is costly, then it cannot treat its customers as expendable. Hence, we argue that the appropriate way to look at the profit maximization problem is to view customers as a renewable resource. Therefore, the firm should look at long-term equilibrium strategies that lead to maximum database values.

3.1 Database Value model

In this paper, we consider the case of a firm that is trying to generate revenues by contacting members of its database. Customers are recruited through some acquisition policy and are contacted at fixed intervals. The problem of the firm is to find the acquisition spending and contact periodicity that will maximize its expected revenues, taking into account the various costs it faces (acquisition, communication) and the reaction of customers to the firm's policy (defection, purchase).

The lifetime value of the firm's database can be computed in the same way marketers compute the lifetime value of customers: by looking at the net present value of all expected future revenues and expenses associated with the growth and cultivation of the database (e.g. see Gupta *et al* 2003; Berger and Nasr 1998). On the revenue side, the firm enjoys an income stream generated by any communication sent to the database members. On the cost side, the firm must incur the expense of acquiring new names, and the fixed and variable expenses of communicating with the database members.

To apply the correct discount rate to expenses and revenues, one must pinpoint when each occurs. In this paper, we assume that acquisition expenses are an ongoing expenditure that the firm recognizes on a periodic basis (e.g., monthly) while communications expenses and revenues are recognized at the time of the communication. This assumption is consistent with our observations of *PBM* in practice, where the acquisition costs are decoupled from the marketing activities once the names are acquired. This is also a recommendation of Blattberg and Deighton (1996), highlighting the different roles of acquisition and retention efforts.

Based on these initial assumptions, we can specify the general database lifetime value as:[2]

$$DBV(\tau) = \sum_{i=0}^{\infty} e^{-ir\tau} \left(R_i(\tau) S_i(\tau) - FC_i \right) - \sum_{j=0}^{\infty} e^{-jr} AQ_j \ . \tag{1}$$

Where:

i	is an index of communications,
j	is an index of time periods,
e^{-r}	is the per time period discount rate[3],
τ	is the periodicity of contact,
$R_i(\tau)$	is the expected gross profit (revenues net of cost of goods sold and variable communication costs) per customer for communication i,
$S_i(\tau)$	is the number of people in the database when communication i was sent,
FC_i	is the fixed cost associated with communication i,
AQ_j	is the acquisition costs incurred in time period j.

Further, we define the customer profit ($R_i(\tau)$) and production ($S_i(\tau)$) functions as:

$$R_i(\tau) = \left(A_i(\tau) - VC_i \right) \tag{2}$$

$$S_i(\tau) = S_{i-1}(\tau) P_i(\tau) + g_i(\tau) \tag{3}$$

Where:

$A_i(\tau)$	is the expected per-customer revenue for communication i,

VC_i is the variable cost of sending communication i,

$S_0(\tau) = 0$,

$P_i(\tau)$ is the retention rate for communication i,

$g_i(\tau)$ is the number of names acquired between campaigns $i-1$ and i.

The model laid out above is a general model that can be used to valuate any database (in particular, it is a general case of the model used by Gupta *et al* 2003). However, we make the following simplifications in order to make the maximization problem more tractable. First, we assume that the acquisition efforts are constant over time and produce a steady stream of names (i.e., $g_i(\tau) = \tau . g$, $AQ_j = AQ(g)$). There is no free production of names and the cost of acquisition increases with the size of the name stream such that: $AQ(0) = 0$, $AQ(g)' > 0$.

Second, we assume that the fixed and variable communications costs are constant (i.e., $VC_i = VC$, $FC_i = FC$). Third, we assume that the communications are identical in nature, if not in actual content, and that customers' reaction to the communications depends only on their frequency such that: $A_i(\tau) = A(\tau)$[4] and $P_i(\tau) = P(\tau)$. Further, we assume that $A(\tau)$ is monotonically increasing in τ, with $A(0) = 0$ and $A(\infty)$ finite (i.e., the more time the firm has to come up with a new offer, the more attractive it can make it –up to a point), and that $P(\tau)$ is inverted-U shaped (i.e., retention is lowest when the firm sends incessant messages or when it never contacts its customers and there is a unique optimal communication periodicity for retention purposes). We justify these assumptions in Appendix A.

The assumptions given above allow us to simplify the database value equation as follows:

$$DBV(\tau) = \sum_{i=0}^{\infty} e^{-ir\tau} \left(R(\tau) S_i(\tau) - FC \right) - \sum_{j=0}^{\infty} e^{-jr} AQ(g) \qquad (4)$$

$$S_i(\tau) = S_{i-1}(\tau)P(\tau) + \tau.g \tag{5}$$

$$R(\tau) = \left(A(\tau) - VC\right). \tag{6}$$

We show (Lemma 1) that, given (5), in the long run, the database reaches an equilibrium size, regardless of the number of names (S_0) the firm is endowed with at time 0:

Lemma 1: *For any given constant marketing actions there exists a steady state such that the database is constant in size. The steady state size is* $\overline{S} = \dfrac{\tau.g}{1-P(\tau)}$.

Proof: *See Appendix B.*

The intuition behind Lemma 1 is straightforward given the database production function (5). For any given τ the firm acquires a constant number of new names ($\tau.g$) between every communication, but it loses names in proportion to its database size. Consequently, the database will be at equilibrium when the firm, between one communication and the next, acquires as many names as it loses due to the communication. Or:

$$\tau.g = S_i\left(1 - P(\tau)\right)$$

$$\Rightarrow \overline{S} = \frac{\tau.g}{1-P(\tau)}. \tag{7}$$

Lemma 1 can be viewed as Little's Law ($L = \lambda W$, Little 1961) applied to an inventory of names, where $L = \overline{S}$ is the expected number of *units* in the system, $\lambda = \tau.g$ is the *arrival rate* of new units, and $W = 1/(1-P(\tau))$ is the *expected time spent* by a unit in the system. Little's Law yields a finite inventory size, as long as λ and W are both finite and stationary. This law has been the subject of numerous papers and has been shown to hold under very general assumptions. This means that Lemma 1 will hold (in expected value) for any stochastic acquisition and retention process as long as they are stationary in the long run, that τ and g are finite, and $P(\tau) < 1$. Note that this relationship holds even if:

- there is seasonality in retention or acquisition (aggregated to the year level, these processes become stationary – e.g., *g=E[Yearly acquisition rate]*),
- the firm improves its ability to attract new customers (as long as $g = \lim_{t\to\infty} g(t)$ is finite, where *t* is the length of time the firm has been in business),
- there is heterogeneity in customer retention (see Section 6),
- or customer retention probability increases –possibly due to habit formation or inertia– as customers stay in the database longer as long as $\lim_{t\to\infty} P(\tau,t) < 1$, where *t* is the length of time during which a customer has been active.

In section 6.2 we use a simulation to illustrate the stationarity property of Lemma 1.

The value of Lemma 1 lies in the following: First, it allows managers to *ex ante* predict the long-term database size. All they need to know is their long-term attrition rate and acquisition effectiveness. This is particularly important when valuating young companies (e.g., Internet start-ups) as it provides a monitoring tool and a base for predicting long-term value. Second, it allows us to further simplify the database value formulation. Indeed, we can write the *steady state* database value as:

$$DBV(\tau) = \sum_{i=0}^{\infty} e^{-ir\tau}\left(R(\tau)\bar{S}(\tau) - FC\right) - \sum_{j=0}^{\infty} e^{-jr} AQ(g),$$

or, taking the limit of sums:

$$DBV(\tau) = \left(R(\tau)\bar{S}(\tau) - FC\right)\frac{e^{r\tau}}{e^{r\tau}-1} - AQ(g)\frac{e^{r}}{e^{r}-1} \qquad (8)$$

Third, the elements of (8) are quite easy to compute for direct marketers. Having a database of names they contact, they can measure $\bar{S}$ (it is the number of live names in the database), they can compute *R* as the average per-customer profit per campaign, the various costs can be optioned from their accounting department (as is *r*). Finally, they set τ .

Fourth, the optimal marketing actions and the resulting steady state database size is a function of the *rate* of acquisition, not its *cost*. We formalize this point in Lemma 2.

Lemma 2: *Given an acquisition stream, the marketing actions that maximize the DBV depend on the rate of acquisition, but are independent from the cost of acquisition.*

Proof: The proof is straightforward. One only needs to recognize that the database value can be split in two terms. The first one depends on *g* and τ, the second one depends on *g* only. That is:

$$DBV(\tau) = \underbrace{\left(R(\tau)\bar{S}(\tau) - FC\right)\frac{e^{r\tau}}{e^{r\tau}-1}}_{f(\tau,g)} - \underbrace{AQ(g)\frac{e^{r}}{e^{r}-1}}_{k(g)}.$$

Further, to maximize *DBV* with respect to τ one computes $\partial DBV(\tau)/\partial\tau$ such that:

$$\frac{\partial DBV(\tau)}{\partial\tau} = \frac{\partial}{\partial\tau}f\left(\tau,g\right) - \frac{\partial}{\partial\tau}k(g) = \frac{\partial}{\partial\tau}f\left(\tau,g\right) = 0$$

QED.

The essence of the proof stems from the observation that the acquisition expenditures precede and are independent from the frequency of contact or the message content. Once names have entered the database, it is the marketer's responsibility to maximize the expected revenue profits extracted from those names, with the consideration that the cost of acquisition is a sunk cost. That is, when optimizing the contact strategy, the firm only needs to know how many new names are acquired every month, not how much these names cost. Everything else being constant, two firms having the same acquisition rate, but different acquisition costs will have identical inter-communication intervals (but different overall profitability).

The importance of Lemma 2 is that it allows us to study the optimization problem in two steps. First, we solve (Section 3.2) the problem of maximizing the database value given a predetermined acquisition stream (i.e., find $\tau^*|g$). Second, we optimize (Section 5) the acquisition spending given the characterization of τ^*. Further, this Lemma gives credence to Blattberg and Deighton's (1996) suggestion that, when maximizing customer equity, the acquisition and the "customer equity" management tasks are very different, and should be treated separately.

Lemma 2 belies the belief that one should be more careful with names that were expensive to acquire than with names that were acquired at low cost. This does not mean, however, that acquisition costs are irrelevant. The long-term profitability of the firm relies on the revenues generated from the database being larger than the acquisition costs.

3.2 Cohort vs. renewable customer

We now turn to the profit maximization problem given an acquisition stream of names (g). As we have shown in the previous section, if the acquisition expenditures are independent of τ, we can ignore them when optimizing the communication strategy. The problem to the firm then becomes:

$$\max_{\tau>0} \; V_{db}(\tau) = \left(R(\tau)\bar{S}(\tau) - FC\right)\frac{e^{r\tau}}{e^{r\tau}-1}. \tag{9}$$

where V represents the net present value of all future expected profits and the subscript db indicates that we take our database valuation approach, as opposed to the customer lifetime value approach that we will formulate shortly (using clv as a subscript).

It is important to note that this database value formulation is fundamentally different from what marketers have traditionally used as objective function in their *CLV* maximization. We show in Proposition 1 that the database is actually more valuable than the sum of its parts. And, since the incremental value generated from maximizing the database as a whole depends on the periodicity of communication, we also show that maximizing the *CLV* is sub-optimal with regard to profitability.

Proposition 1: *The value of a database is larger than the sum of the value of each of its members valued independently.*

Corollary: *CLV maximization is sub-optimal with regard to profitability.*

Proof: See Appendix B.

When computing a *CLV* one accounts for the fact that, due to attrition, customers have a decreasing probability of being active as time passes. Given our notation, one would write the *CLV* of an individual customer as:

$$\begin{aligned} CLV(\tau) &= \sum_{i=0}^{\infty} e^{-ir\tau} P(\tau)^i \left(R(\tau) - \frac{FC}{\bar{S}} \right) \\ &= \left(R(\tau) - \frac{FC}{\bar{S}} \right) \frac{e^{r\tau}}{e^{r\tau} - P(\tau)}. \end{aligned} \tag{10}$$

If we now compute the value of the database by multiplying the *CLV* by the number of customers currently in the database, we obtain:

$$V_{clv}(\tau) = \left(R(\tau)\bar{S} - FC \right) \frac{e^{r\tau}}{e^{r\tau} - P(\tau)}. \tag{11}$$

This database value (V_{clv}) is smaller than the value (V_{db}) obtained in (9) since $P(\tau)$ is smaller than one. This difference exists because, from a *CLV* perspective, customers are perishable, while the database perspective views customers as a renewable resource. The benefit of treating the names as renewable can be computed as follows:

$$V_{db}(\tau) = V_{clv}(\tau)\frac{e^{r\tau} - P(\tau)}{e^{r\tau} - 1}. \tag{12}$$

In this equation, the ratio $\frac{e^{r\tau} - P(\tau)}{e^{r\tau} - 1} > 0$ represents the premium one puts on the value of each name when taking the renewable resource approach. As one can readily see, the lower the retention rate, the higher the premium. Also, the premium increases as the discount rate (*r*) or the communication interval (τ) decreases.

One should note that when computing the *CLV* (10) we allocated a portion of the campaign costs (*FC*) to each name. This allocation does not affect the substance of our findings. Indeed, if we were to ignore the fixed costs at the name level, we would compute the *CLV* as:

$$CLV(\tau) = R(\tau)\frac{e^{r\tau}}{e^{r\tau} - P(\tau)}. \tag{13}$$

Thus, we would have:

$$V_{db} = \left[CLV(\tau)\bar{S}(\tau)\right]\frac{e^{r\tau} - P(\tau)}{e^{r\tau} - 1} - FC\frac{e^{r\tau}}{e^{r\tau} - 1}.$$

The value of the database as a whole is the value of its names minus the costs associated with extracting profits from these names. In valuating each name we find the same multiplier $\left(\frac{e^{r\tau} - P(\tau)}{e^{r\tau} - 1}\right)$ as we had when we incorporated the costs directly in the name value.

To better understand where the difference in valuation comes from, let us look at a simple case. Imagine a firm has a database of 100 names. It has a yearly churn of 30% (i.e., it loses and acquires thirty people each year) and makes a profit of \$1 per active name. Using a *CLV* approach, one would see the list of names as shrinking over time; going from 100 in year one to 70 in year two, 49 in year three and so on (see Figure 2a). The Net Present Value (*NPV*) of the profits generated by the list would be $100 for the profits in year one, $63.64 for year two profits (i.e., $70 discounted at 10%), $40.50 for year three profits, and so on, for a *NPV* of total lifetime profits of $275 (see Figure 2b).

Using our approach, we recognize that, every year, thirty new names come to replace the names lost during the year. Thus, the database stays constant in size at 100 names. The composition of the database, however, changes over time (see Figure 3a) and is skewed toward recently acquired names (by year 10, only 4% of the original names are still active). Thus, instead of discounting future profits for both attrition and cost of money, one only discounts them for the cost of money (see figure 3b). This yields a *NPV* of total lifetime profits of $1,100, four times larger than the *CLV* of $275. If one were to factor in acquisition costs (at say $1 per name), one would compute the *CLV* at $175 ($275 - $100 for the acquisition of 100 names) and the database value at $770 ($1,100 - $330 for the acquisition of thirty names a year for the rest of eternity), or 4.4 times more. In other words, the *CLV* approach values the database at about 20% of its true value!

The essence of the proof of the corollary is as follows (see Appendix B for details). Following equation (12), the database value is equal to the *CLV* multiplied by a correction factor. Maximizing the *CLV* will thus lead to the maximum database value only if, at the maximum *CLV*, the derivative of the multiplier with respect to τ is equal to 0. We show in Appendix B that this can only occur if $\tau^* = 0$. But, as we will show in the next section, τ^* is always strictly greater than zero. Therefore, *CLV* maximization is always sub-optimal for the firm!

Proposition 1 and its corollary are important because they show that the traditional customer lifetime value framework underestimates the value of names. Even more important is the result that since the multiplier $\left(\frac{e^{r\tau}-P(\tau)}{e^{r\tau}-1}\bar{S}(\tau)\right)$ depends on τ, maximizing V_{clv} will not lead to the same optimum as maximizing V_{db}. And thus, per the corollary, maximizing the *CLV* is sub-optimal from a database value perspective. To illustrate this sub-optimality, we plot V_{db} and V_{clv} (see Figure 4) for a hypothetical situation where the parameters described in Appendix A have been set to: $\mu = 0.7$, c = 0.2, g = 100, discount rate = 10%, $R(\tau) = 10 * I(\tau)$, $P(\tau)$ is a logit function. Further, to ensure that our results are not driven by how costs are accounted for, we set both variable and fixed cost to zero (i.e., VF=FC=0). In this case, the optimal τ using V_{db} is 3 versus 5 for V_{clv}. Further, V_{db}^* is 3.7 times larger than V_{clv}^* (860 vs. 322). One should also note how asymmetric the database value functions are. It implies that in case of uncertainty, one should err towards longer τ's rather than shorter ones.

3.3 Finding the optimal periodicity (τ)*

In order to optimize τ for any given acquisition strategy, we calculate the first-order condition for optimality by differentiating (9) with respect to τ. Without further specifying the general functions that constitute the database value, it is not possible to generate an explicit closed-form solution for optimal τ. However, for our purposes, we can make some inferences using comparative static tools. We start by describing the first-order condition, expressed as a function of the elasticities of the retention and profit functions with respect to changes in the inter-communication interval. Then, we study how the first-order condition changes with respect to retention rates, profit functions, acquisition rates, and discount rates.

Proposition 2: *The first-order condition for a firm that seeks to maximize its database value by optimizing the inter-communication interval is:*

$$\eta_R + \eta_{\bar{S}} + \eta_D.GM = 0. \tag{14}$$

Where:

$\eta_R = \dfrac{\partial R(\tau)}{\partial \tau}\dfrac{\tau}{R(\tau)} = \dfrac{\partial A(\tau)}{\partial \tau}\dfrac{\tau}{A(\tau)-VC}$ is the elasticity of $R(\tau)$ with respect to τ,

$\eta_{\bar{S}} = 1 + \dfrac{\partial P(\tau)}{\partial \tau}\dfrac{\tau}{1-P(\tau)}$ is the elasticity of $\bar{S}$ with respect to τ,

$\eta_D = \dfrac{-r\tau}{e^{r\tau}-1}$ is the elasticity of the discount multiplier $\left(D(\tau) = \dfrac{e^{r\tau}}{e^{r\tau}-1} \right)$ with respect to τ,

$GM = \dfrac{R(\tau).\bar{S}(\tau) - FC}{R(\tau).\bar{S}(\tau)}$ is the gross margin yielded by each communication.

Proof: *See Appendix C.*

The proof of Proposition 2 is an algebraic exercise that leads to a simple expression of the first-order condition: a linear combination of elasticities. The optimal inter-communication interval (τ^*) is found when the sum of elasticities is equal to 0. When the sum is positive, the firm would increase its profits by increasing τ. When the sum is negative, the firm would be better off decreasing its τ. As we show in the Technical Appendix, if there exists a τ such that $V_{db}(\tau)$ is positive, then there exists a unique τ^* that is finite and strictly positive. This is not a restrictive condition, as it only assumes that it is possible for the firm to make *some* profits. If it were not the case, the firm should not pursue *PBM* and the search for an optimal τ becomes meaningless. Further, if $P(\tau)$ is not too convex then the maximum is unique. This, again, is not restrictive, as $P(\tau)$ will typically be concave over the domain of interest.

4 Static Comparison on the *FOC*

Armed with the first-order condition, we now perform a series of static comparisons to further our understanding of the effect of the various levers of the database value. What happens to τ^*, or the database value, when the discount rate of money increases, or when the firm spends more on acquisition? We look at each lever in turn.

4.1 Change in Retention rate ($P(\tau)$)

A change in retention rate response function can come about in two different ways. First, one might see an intercept or level shift that increases or decreases $P(\tau)$ overall without changing the sensitivity of $P(\tau)$ to changes in τ (i.e., the gradient $\partial P(\tau)/\partial\tau$ is unaffected). Second, one might see a change in the sensitivity of $P(\tau)$ to changes in τ (i.e., a change of $\partial P(\tau)/\partial\tau$ at τ^* while $P(\tau^*)$ is constant). One might, of course, observe a combination of these two changes. In such a case, the total impact of the changes will be the sum of the changes due to the level-shift and the sensitivity-shift. The impact of both changes is given in the following proposition:

Proposition 3$_a$: *An increase in retention sensitivity ($\partial P(\tau)/\partial\tau$) leads to an increase in τ^*. A level-shift increase in retention ($P(\tau)$) leads to an increase in V_{db}. It also leads to an increase in τ^* when τ^* is small, and a decrease in τ^* when it is large. The cut-off level is: $\tau^*: \eta_{\bar{S}(\tau^*)} > \left|\eta_{D(\tau^*)}\right| \frac{FC}{R(\tau^*).\bar{S}(\tau^*)} + 1$.*

Proof: ***See Appendix C.***

The retention probability affects the *FOC* through both $\eta_{\bar{S}}$ and *GM*. Thus, when looking at changes in *P* we need to consider the combined impact of both changes. In the case of a change in $\partial P(\tau)/\partial\tau$, the situation is straightforward, as *GM* is unaffected and thus the increase in

$\partial P(\tau)/\partial\tau$ leads to an increase in the *FOC* through $\eta_{\bar{S}}$. Hence, the firm would react by increasing its τ. In other words, an increase in $\partial P(\tau)/\partial\tau$ means that the firm has more to gain by waiting a little longer between communications, and since the system was at equilibrium before, it now leans in favor of a larger τ^*.

In terms of database value, V_{db} is not directly affected by changes in $\partial P(\tau)/\partial\tau$ and thus, strictly speaking, a change in $\partial P(\tau)/\partial\tau$ will not affect the database value. However, a change in $\partial P(\tau)/\partial\tau$ cannot reasonably arise without some change in $P(\tau)$. And thus, V_{db} will be affected through the change in $P(\tau)$. It is straightforward to show that a level-shift increase in $P(\tau)$ increases V_{db}. The *envelope* theorem tells us that although τ^* is affected by changes in *P*, when looking at the net impact on V_{db}, we can ignore the impact of changes in *P* on τ^*, and simply look at the sign of $\partial V_{db}/\partial P$. Here, we have $\partial V_{db}/\partial P > 0$ and thus V_{db} is increasing in *P*.

In the case of a level-shift, the situation is made complex in that *GM* always increases when $P(\tau)$ increases (which leads to a decrease in the *FOC* as η_D is negative), but the effect on $\eta_{\bar{S}}$ depends on $\partial P(\tau)/\partial\tau$. If we assume, as explained in Appendix A, that $P(\tau)$ is increasing at a decreasing rate for small τ (i.e., $\partial P(\tau)/\partial\tau > 0$, $\partial^2 P(\tau)/\partial\tau^2 < 0$, $\forall\,\tau < \tau_p$), then decreasing after some threshold ($\partial P(\tau)/\partial\tau < 0$, $\forall\,\tau > \tau_p$), then we find that for small τ, an intercept-shift increase in $P(\tau)$ leads to an increase in τ^*, and for large τ it leads to a decrease in τ^*. The difference in behavior comes from the fact that when τ is small, an increase in $P(\tau)$ has a large impact on the database size and spurs the company to seek an even larger database while, when τ is large, an increase in retention allows the firm to milk the database to a greater extent.

4.2 Change in Expected Profit per Contact ($R(\tau)$)

The expected profit per contact has two components: the expected gross revenue ($A(\tau)$) and the variable costs (*VC*). We look at costs in the next section and focus here solely on $A(\tau)$. Similar to $P(\tau)$, $A(\tau)$ can be changed through an intercept-shift or through a change in sensitivity to τ. The situation is, however, a bit more complex for $A(\tau)$ than for $P(\tau)$ because $P(\tau)$ depends on $A(\tau)$. Indeed, recall from Appendix A that we defined $P(\tau)$ to be a monotonic transformation on the rate at which a customer receives information (i.e., $P(\tau) = f\left(A(\tau)\right), f' > 0$). Thus, we must account not only for the direct effect of a change in $A(\tau)$ on the *FOC*, but also for indirect effects through changes in $P(\tau)$.

Proposition 3$_b$: *An increase in revenue per contact sensitivity ($\partial A(\tau)/\partial\tau$) leads to an increase in τ^*. An intercept-shift in revenue per contact ($A(\tau)$) leads to an increase in V_{db}. It also leads to a decrease in τ^* when τ^* is small to moderate, and an increase in τ^* when it is large. A lower-bound to the cut-off level is: $\tau^* : \eta_{\bar{S}} = \tau^2 + 1$.*

Proof: *See Appendix C.*

The impact of $A(\tau)$ on V_{db} follows from the envelope theorem. In terms of the impact of $A(\tau)$ and $\partial A(\tau)/\partial\tau$ on τ^*, the proof in Appendix C is longer than for the previous proposition because one must take into consideration the effects of *A* on *P*. The essence of the proposition is, however, straightforward. When the sensitivity of revenue to longer τ increases, there is a pressure towards longer τ^*. In case of a positive intercept-shift, there is a tendency for the firm to take advantage of the shift by milking the database.

4.3 Change in Acquisition Rate (g)

Proposition 3$_c$: *An increase in acquisition rate leads to a decrease in τ^* and an increase in V_{db}.*

Proof: The proof is similar to the proof of Proposition 3$_a$. Since $\overline{S}$ is a linear function of *g*, the elasticity $\eta_{\overline{S}}$ does not depend on *g*. The only term in the *FOC* that depends on the acquisition rate is the gross margin (through $\overline{S}$). Thus all we are interested in is the sign of:

$$\begin{aligned}\frac{\partial GM}{\partial g} &= \frac{\partial}{\partial g}\frac{R(\tau).\overline{S}(\tau) - FC}{R(\tau).\overline{S}(\tau)} \\ &= \frac{FC}{R(\tau).\overline{S}(\tau)^2}\frac{\partial \overline{S}(\tau)}{\partial g} \quad .\end{aligned}$$

Hence, $\frac{\partial GM}{\partial g} > 0$ since $\frac{\partial \overline{S}(\tau)}{\partial g} = \frac{\tau}{1 - P(\tau)} > 0$. Thus, given that *GM* acts as a multiplier to η_D, which is negative, an increase in acquisition rate – through increased acquisition spending or increased acquisition effectiveness – will lead to a decrease in τ^*. Finally, the increase in V_{db} is proved by the envelope theorem and the fact that $\partial V_{db} / \partial g$ is positive.

4.4 Change in Discount Rate (r)

Proposition 3$_d$: *An increase in discount rate leads to an increase in τ^* and a decrease in V_{db}.*

Proof: Although counterintuitive with regards to τ^*, Proposition 3$_d$ is straightforward to prove. To show that τ^* increases in *r*, we first note that η_R, $\eta_{\overline{S}}$, and *GM* are independent from *r*. Hence, a change in discount rate will only affect η_D. The change will be as follows:

$$\frac{\partial \eta_D}{\partial r} = \frac{\partial}{\partial r}\frac{-r\tau}{e^{r\tau}-1}$$
$$= \frac{\tau}{e^{r\tau}-1}\left(\frac{r\tau e^{r\tau}}{e^{r\tau}-1}-1\right) > 0,\ \forall r>0, \tau>0.$$

The derivative of η_D with respect to *r* is positive for all positive *r* and τ.[5] This implies that the optimal reaction for a firm faced with an increase in discount rate is to increase its inter-communication interval. This may sound counter-intuitive at first, as one might believe that increasing *r* makes future revenues less attractive and, thus, one might want to decrease τ so as to realize more profits in the short-term.

What actually happens is that an increase in *r* leads to a decrease in the discount multiplier (*D*). This decrease means that, holding everything else constant, the value of each name decreases, and thus the value of the database decreases. This decrease must be counteracted by either an increase in database size ($\overline{S}$) or an increase in expected profit per name per communication (*R*). This is accomplished by increasing τ .

It is straightforward to show that as *r* increases, the value of the database decreases. Applying the envelope theorem one more time we have:

$$\frac{\partial V_{db}}{\partial r} = \left(R\overline{S} - FC\right)\frac{\partial}{\partial r}\frac{e^{r\tau}}{e^{r\tau}-1}$$
$$= \left(R\overline{S} - FC\right)\frac{-\tau e^{r\tau}}{\left(e^{r\tau}-1\right)^2} < 0.$$

Hence, an increase in *r* leads to both an increase in optimal inter-communication time (τ^*) and a decrease in database value (V_{db}).

The impact of *r* on τ^* could be used to study how a firm should vary its communication time as interest rates go up and down. However, a more interesting study is how τ^* changes as a firm grows from a start-up to a large legitimate company. Indeed, a start-up is a risky venture and its implicit discount rate will be high. As the firm grows, its future becomes less uncertain and its discount rate diminishes. Proposition 3_d states that in such case, the firm will decrease its inter-communication time as it grows. This decrease in τ effectively shifts the firm from a :database growth" regime to a "profit generation" regime as it now tries to extract more revenues from its database and leaves itself less time between communication to replace the names lost.

4.5 Changes in Costs

The firm's reaction to any increase in cost, whether fixed or variable, is to increase τ^*. The reason for this is straightforward. An increase in cost reduces per-campaign profits. The firm must then increase τ^* to try to boost its profits per campaign.

Proposition 3$_e$: *An increase in fixed or variable costs leads to a decrease in V_{db} and an increase in τ^*.*

Proof: *Comes from* $\frac{\partial V_{db}}{\partial FC} < 0$, $\frac{\partial V_{db}}{\partial VC} < 0$, $\frac{\partial \eta_R}{\partial VC} > 0$, *and* $\frac{\partial GM}{\partial VC} < 0$.

5 Acquisition Policy

We have so far worked under the assumption that acquisition expenditures were fixed. We were allowed to do so because Lemma 2 states that the actual spending on name acquisition is irrelevant to optimize τ, and that only the acquisition rate matters. Thus, one can first figure out how to optimize τ given an acquisition rate, and then optimize the acquisition spending given that one knows how to optimize τ (Section 3). Section 4 ignored acquisition except for the finding that an increase in acquisition rate leads to a decrease in τ^* (Proposition 3_c).

Having explored the problem of optimizing τ, we can now tackle the larger problem of optimizing *g*. When doing so, we assume that the firm optimizes τ according to Proposition 2 and that τ^* is known for all acquisition spending. Further, following the envelope theorem, small changes in *g* lead to negligible changes in τ^* and thus, we can consider τ^* to be constant for the time being (we will look more closely at the secondary effect in Proposition 5).

Proposition 4: *If the periodicity at which the acquisition expenditures are recognized is equal to the periodicity of the communications (i.e., $\tau^* = 1$), then optimal acquisition rate occurs when the marginal acquisition cost per name is equal to the* <u>*non-discounted*</u> *expected profits from the name.*

Proof: The firm will increase its acquisition spending until the marginal profit generated by the last acquired name is equal to the marginal costs of acquiring names. This point can be found by studying the first-order condition for acquisition cost:

$$\frac{\partial DBV}{\partial g} = \underbrace{R(\tau^*)\frac{\tau^*}{1-P(\tau^*)}\frac{e^{r\tau^*}}{e^{r\tau^*}-1}}_{MR>0} - \underbrace{\frac{e^r}{e^r-1}\frac{\partial AQ}{\partial g}}_{MC>0} = 0\,. \tag{15}$$

Rearranging (15) to isolate the maximum amount one should be willing to pay to acquire one more customer, we obtain:

$$\frac{\partial AQ}{\partial g} = \underbrace{\frac{e^{r\tau^*}}{e^r}\frac{e^r-1}{e^{r\tau^*}-1}}_{I}\underbrace{\frac{\tau^*.R(\tau^*)}{1-P(\tau^*)}}_{II}\,. \tag{16}$$

There are two components, labeled *I* and *II*, to this equation. The first term is a discount adjustment factor that corrects the expected profits for any discrepancies between the interval at which expenses are recognized and profits are realized. If expenses and profits are recognized at different rates, say monthly expenses and quarterly newsletters, then the firm needs to adjust its acquisition expenditure to reflect the fact that profits will lag or precede expenditures.[6] However, when $\tau^* = 1$ (i.e., acquisition expenses are recognized at the same interval as the revenue) this term is equal to 1. In this case, we can simplify (16) to show that marginal cost must equal expected marginal profits:

$$\frac{\partial AQ}{\partial g} = \frac{R(\tau^*)}{1 - P(\tau^*)}. \tag{17}$$

The importance of Proposition 4 is that the profits are *not* discounted. In traditional *CLV* calculation (see equation (13)) one discounts future profits, thus the marginal acquisition costs should not be greater than the *CLV* or:

$$\frac{\partial AQ}{\partial g} = R(\tau^*)\frac{e^{r\tau}}{e^{r\tau} - P(\tau^*)} = \frac{R(\tau^*)}{1 - \frac{P(\tau^*)}{e^{r\tau^*}}}. \tag{18}$$

The optimal spending from a database point of view (17) is greater than from a *CLV* point of view (18). This parallels the corollary to Proposition 1 such that the *CLV* framework not only leads to sub-optimal τ^*'s, but also sub-optimal acquisition expenditures.

The reason for using non-discounted profits is that this model considers acquisition and profit extraction to be a steady state ongoing phenomenon. Hence, in any period, the firm will spend money acquiring new names, and at the same time, will extract profits from its current database. At any point in time, the database will contain names that were acquired one period ago, two periods ago, etc. Profitability at stationarity requires that the acquisition costs in any period are more than offset by the profits generated during that period (and hence non-discounted) from the names acquired in all previous periods, which have not defected.

Proposition 5: *If the marginal returns to acquisition spending are constant (i.e., $\partial^2 AQ / \partial g^2 = 0$) then the value of the database is convex in acquisition spending.*

Proof: *see Appendix D.*

The proof relies on the fact that the value of names increases as the database grows. Thus, doubling the number of names in the database more than doubles the value of the expected profits generated from these names. If acquisition costs are linear in the number of names, the firm becomes relatively more profitable as it increases its acquisition efforts. This leads to a double jeopardy type environment where “the rich get richer,” where the bigger firms can afford to pay more per name than the smaller firms, and produces a “winner takes all” situation. Fortunately, acquisition costs are unlikely to be linear. It is more realistic to have convex costs where the marginal cost of names increases as one increases the acquisition rate ($\partial^2 AQ / \partial g^2 > 0$). In such a case, the increase in name cost will more than offset the increase in name value, leading to a finite optimal spending level, and a finite optimal database size.

6 Heterogeneity

No analysis of the value of a database would be complete without a discussion of customer heterogeneity. We have, so far, ignored the topic and assumed that the database value could be expressed as a function of an “average” consumer. Although this is a simplified view, we show below that this approach has broad applicability.

There are two issues with heterogeneity. First, how does unobserved heterogeneity affect the results of Sections 4 and 5 (i.e., what happens when one treats everybody as identical when they are not)? Second, at what point is it profitable to address heterogeneity head-on and start sending different communications to different groups of customers, or send communications at different rates to different customers?

The first issue is particularly important in light of the evidence shown in Section 2.1 where we find that a small minority of the firms currently engaged in *PBM* via email actually customize either their content or their periodicity. In addition, firms that rely on their customers or on database-driven rules to address heterogeneity issues for them can be considered to treat everybody identically. For example, American Airlines (*AA*) sends a weekly email to each of its registered users in which it lists current 'Web Specials' that the user might be interested in. Different customers receive different content; however, there are no customization costs for newsletter periodicity (no additional fixed campaign costs incurred), since AA uses the same contact interval (weekly) for everybody, nor are there customization costs for content as it is database-driven based on what consumers have revealed they would like to see,[7] or based on past travel patterns. Hence, from the firm's perspective, the content can be considered to be identical for all users, too. Therefore, the benefits for *AA* of taking heterogeneity into account through a database-driven email generator can be easily predicted by Proposition 3_b. Indeed, by mass-customizing their email content, *AA* simply induces an intercept-shift on $A(\tau)$, yielding larger V_{db} and *P*, and a shorter τ^*. Thus, we only consider here the effect of unobserved heterogeneity.

6.1 Unobserved heterogeneity

Customers can be heterogeneous in their retention probability (*P*) and/or their expected return from each communication (*A*). If consumers have identical retention probability, but are heterogeneous in their expected return, then, if one treats all consumers as being identical, one can use the average expected return ($\bar{A}$) for the purpose of profit maximization. Indeed, let *f(A)* be the probability density function of *A*. We can compute the expected database value by integrating out across all customers:

$$\begin{aligned} E[DBV] &= \int\left(\left(A - VC\right)\overline{S} - FC\right)\frac{e^{r\tau}}{e^{r\tau}-1}f(A)dA \\ &= \left(\int Af(A)dA\right)\overline{S}\frac{e^{r\tau}}{e^{r\tau}-1} - \left(VC\overline{S} + FC\right)\frac{e^{r\tau}}{e^{r\tau}-1} \\ &= \overline{A}\overline{S}\frac{e^{r\tau}}{e^{r\tau}-1} - \left(VC\overline{S} + FC\right)\frac{e^{r\tau}}{e^{r\tau}-1} \\ &= \left(\left(\overline{A} - VC\right)\overline{S} - FC\right)\frac{e^{r\tau}}{e^{r\tau}-1}. \end{aligned}$$

QED.

Taking the same approach is a little more complicated for heterogeneity in the retention rate. Indeed, *P* appears on the denominator of *DBV* through $\overline{S}$. Thus, going back to Little's Law, we need to compute:

$$\begin{aligned} \overline{S} &= \tau.g.E[\frac{1}{1-P}] = \tau.g\int\frac{1}{1-P}f(P)dP \\ &\neq \frac{\tau.g}{1-\int Pf(P)dP}. \end{aligned}$$

In this case, customers with higher retention rates stay in the database longer than customers with lower retention rates, and thus, over time, the database becomes a "cleaned-up" version of the customers coming in through the acquisition stream. For instance, if we assume that the acquisition efforts yield a stream of customers whose retention rate has a Beta distribution, $\mathrm{B}(\alpha,\beta)$ with $\beta > 1$ (to ensure that $f(1)=0$[8]), then we have:

$$\begin{aligned} \overline{S} &= \tau g\int_0^1\frac{f(P)}{1-P}dP \\ &= \tau g\int_0^1\frac{1}{1-P}\frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)}P^{\alpha-1}(1-P)^{\beta-1}dP \\ &= \tau g\int_0^1\frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)}P^{\alpha-1}(1-P)^{\beta-2}dP. \end{aligned}$$

Since $\Gamma(n+1) = n\Gamma(n)$, we have:

$$
\begin{aligned}
\overline{S} &= \tau g \int_0^1 \frac{(\alpha+\beta-1)\Gamma(\alpha+\beta-1)}{\Gamma(\alpha)(\beta-1)\Gamma(\beta-1)} P^{\alpha-1}(1-P)^{\beta-2}\,dP \\
&= \tau g \frac{(\alpha+\beta-1)}{(\beta-1)} \underbrace{\int_0^1 \frac{\Gamma(\alpha+\beta-1)}{\Gamma(\alpha)\Gamma(\beta-1)} P^{\alpha-1}(1-P)^{\beta-2}\,dP}_{=1} \\
&= \frac{\tau . g}{1-\dfrac{\alpha}{(\alpha+\beta-1)}}.
\end{aligned}
$$

where we recognize the term $\frac{\alpha}{(\alpha+\beta-1)}$ as the expected value of a $\mathrm{B}(\alpha, \beta-1)$. Hence, if the heterogeneity in retention rate in the acquisition stream is characterized by a $\mathrm{B}(\alpha, \beta)$, then the firm should optimize its database using the expected value of a $\mathrm{B}(\alpha, \beta-1)$ as its average *P*.

6.2 Simulation

To illustrate our theory and to show that it is robust to a stochastic environment, we conducted a simulation where the acquisition, retention, and purchase processes were stochastic. We simulated a process by which a company acquires names on a continuous basis. Once per period, the firm sends a communication to the names that are still active. These names respond to the offers in a stochastic manner and have an individual level retention rate. In line with the queuing theory literature, we assumed an exponential arrival rate and modeled it through a per-period acquisition process that is Poisson. For each acquired individual, we drew a retention rate from a Beta(3,3) distribution; in each period, the active names made a purchase drawn from a Uniform(0,50) distribution using a different draw from the distribution for each individual in each period. Further, to illustrate what happens when the acquisition policy changes, we ran the simulation for 250 periods with the acquisition stream set as a Poisson(7,500) (i.e., the mean number of names acquired per period is 7,500), then increased the arrival rate to produce an average of 10,000 names per period.

Our theory predicts that if the retention probabilities in the incoming name stream are distributed Beta(3,3), then, in the long run, the retention probabilities of the members of the database will be distributed Beta(3,2). To check this, we plotted (Figure 5a) the distribution of the retention probabilities of all the names acquired in our simulation (in grey) alongside the distribution of the retention probabilities of the names in the database at the end of the 500-period simulation (in black). As one can see, the simulation results match our expectation.

A Beta(3,2) has a mean of 3/5. Thus, Lemma 1 predicts that the size of the database will be 7,500/(3/5)=18,750 during the first 250 periods of the simulation. It will then increase to 25,000 during the next 250 periods. Further, given that the purchases are distributed Uniform(0,50), the expected revenue during the first 250 periods will be 468,750; increasing to 625,000 in the next 250 periods. We show the period-by-period database size and revenues in Figures 5b and 5c respectively. Again, our model outcomes are confirmed. Once the database reaches its steady-state size it does not deviate much from the expected values. Given our process, it seems to take about 20 periods for the database to grow from 0 names to the steady-state. When the acquisition rate increases to 10,000 names per period, it takes about 15 periods for the database to adjust its size.

7 Managerial Implications and Conclusion

We demonstrate in this paper that the perspective a firm adopts for its (potential) customers has a profound impact on how it relates to them, both in terms of acquisition and profit extraction. For example, spammers view names as expendable resources. In their view, since new names are easy to come by, they are used for short-term gains without regard for long-term consequences. In contrast, traditional direct marketers view customers as depletable resources. Every customer name acquired has a limited time horizon, and care must be taken to extract the maximum value during this finite life. The perspective adopted in this paper is that firms should view customers in a database as a renewable resource. Customer names are acquired and lost, and the purpose of the firm is to optimize the overall process of customer acquisition/cultivation/attrition.

We claim that this renewable resource approach to the problem of maximizing the profits generated by a flow of names is more appropriate than the traditional customer lifetime value approach. We show in Lemma 1 that our model can be used to predict the long-term steady state of the firm's database. We also show that if a *CLV* approach were used to optimize marketing actions over a database of names, it would lead to sub-optimal customer relationship management (Proposition 1) and acquisition strategies (Proposition 4). This leads to the surprising result that although the firm should discount future profits when valuating its database (equation (8)), it should not discount these same profits when setting its acquisition policy (equation (16)).

Lemma 2 and Proposition 4, taken together, provide a useful insight for organizing the acquisition and communication function. They show that the two functions can be coordinated quite easily. All that is needed to optimize the communication strategy is the size of the acquisition stream. All that is needed to figure out how much to spend on acquisition is the adjusted expected profits per customer.

The strength of the first-order condition derived in Proposition 2 is that it is easy to optimize it empirically. The firm can run a series of tests to estimate the various elasticities and infer from them if it should increase or decrease its communication periodicity. This is simplified by the fact that the database value has a unique maximum. The first-order condition also lends itself well to static comparisons (Propositions 3_a to 3_d). Perhaps the most interesting result stemming from the comparative static analysis is the implication that start-up companies should contact their customers less frequently than established companies. That is, start-ups should resist the temptation of boosting short-term profits through incessant communications, in favor of the accumulation of names in their databases, by spacing communications, and allowing acquisition streams to build up the database size.

We finish the paper with a discussion of customer heterogeneity. We show that our model is robust to unobserved heterogeneity. The learning point there is that ignoring heterogeneity in customers' valuation of the communication is less of an issue than ignoring customer heterogeneity in retention rate.

Although our study is conducted in the context of Permission-Based Marketing, it applies to a broad set of problems. Our model and conclusions can readily be transposed to most direct marketing problems (e.g., catalogs or telemarketing). Our findings in terms of database value can be applied to any situation where a firm tries to valuate its customer base (e.g., frequent shopper programs).

This paper is, of course, not the last word in terms of customer or database value. It provides a cohesive framework to study the problems inherent with Permission-Based Marketing. To achieve this, we made some assumptions regarding the relationship between customer retention and the firm's communication strategy, both in terms of timing (through τ) and in content (through the relationship between A and P). We have dissociated the issue of heterogeneity from our main findings. Finally, we have only concerned ourselves with the steady state regime without concern for the ramp-up phase. More can certainly be done in those fronts.

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Appendix A: Assumptions about Consumer and Firm Behavior

Although our model is quite general, we need to make a few assumptions about the shape of $A(\tau)$ and $P(\tau)$ in order to say something meaningful about the behavior of the firm and its customers, and how they relate to optimum policies. Specifically, we assume that $\partial A(\tau)/\partial\tau > 0$, $\partial^2 A(\tau)/\partial\tau^2 < 0$ (i.e., $A(\tau)$ is increasing but concave in τ) and that $P(\tau) = f\left(\frac{A(\tau)-c}{\tau}\right)$ with $\partial f/\partial\tau > 0$. We justify these assumptions in the following two sections.

A.1 Firm Behavior

$A(\tau)$ is the expected gross revenue realized by the firm for each person to whom it has sent a communication. This revenue can take on many different forms depending on which type of business is conducted by the firm. To understand how τ affects A let us look at a concrete example. *CDNow* uses its email newsletter to inform its customer of new music *CD* arrivals in the hope that they will purchase them. What happens to *CDNow* if it spaces out the newsletters? On the one hand, if more time elapses, then *CDNow* can expect that more new *CDs* will be released, and thus it is more likely that a *CD* with broad appeal is released (which would lead to an *increase* in A). On the other hand, information about *CD*s is likely to be time sensitive in that the longer *CDNow* waits, the more likely its customers will have already purchased the *CD* promoted by the newsletter (which leads to a *decrease* in A).

In more general terms, let us assume that the firm uses *PBM* to provide its customers information that might be valuable to them (e.g., the release of Britney Spear's new album for *CDNow*; the release of a new laser printer for *Hewlett Packard*). This information arrives to the firm on a continuous basis, but varies in its value to the customer. The arrival process is exponential and the value of the information is distributed Uniform[0,1]. Once this information is in the possession of the firm, its value to the customer will decay at an exponential rate until it is sent. Finally, let us assume that for each communication the firm will pick the one piece of information, among all the information that it received since the last communication, that it deems most valuable to its customers.

These assumptions mean the following: (1) the information is generated by a memory-less process that is not under the control of the marketer; (2) not every piece of information is as valuable to the customer; (3) the information is time-sensitive; and (4) firms are limited in the length of their communications. This last assumption may seem arbitrary. Its purpose is to ensure that there are decreasing marginal returns from waiting even when the information is not time sensitive. It can be justified theoretically by the research on information overload (see Bettman 1979 or Wright 1973), which shows that consumers can be overwhelmed when they are presented with too much information. It can also be justified empirically by the fact that three out of the top five reasons people give for un-subscribing to a newsletter are: the content is irrelevant (55%), poor content (32%), and too long (26%) (Forrester Research 2001a).

The revenue that the firm will be able to extract from the consumer is a function of how valuable the information is to the consumer. It is natural to assume that the more valuable the information is to the consumer, the more revenue the firm can generate out of it (i.e., $A(\tau) = f\left(I(\tau)\right)$, and $\frac{\partial f(I)}{\partial I} > 0$). In other words, $A(\tau) = f\left(I(\tau)\right)$ is the production function of the firm that maps the value of the information to consumers ($I(\tau)$) into revenue to the firm ($A(\tau)$) via a demand curve.

$I(\tau)$ is the expected value of the most valuable piece of information arriving in τ periods of time. If the arrival rate for the information is Poisson of mean μ and the value of the information is Uniform[0,1], then in the absence of information decay, $I(\tau)$ is the expected value of a Poisson mixture with the order statistics of series of Uniform draws and is given by $I(\tau) = \dfrac{\tau}{\tau + \frac{2}{\mu}}$ (for details, see the Technical Appendix, available upon request from the authors).

This is a concave function that starts at 0 (I(0)=0) and asymptotes to 1. In particular:

$$\frac{\partial I}{\partial \tau} = \frac{2}{\mu\left(\tau + \frac{2}{\mu}\right)^2} > 0$$

$$\frac{\partial^2 I}{\partial \tau^2} = \frac{-2\tau}{\mu\left(\tau + \frac{2}{\mu}\right)^4} < 0$$

$$\eta_I = \frac{\partial I}{\partial \tau}\frac{\tau}{I} = \frac{2}{2+\mu\tau}. \tag{A-1}$$

Hence, we also have $\partial A / \partial \tau > 0$, $\partial^2 A / \partial \tau^2 < 0$, $\eta_A > 0$.

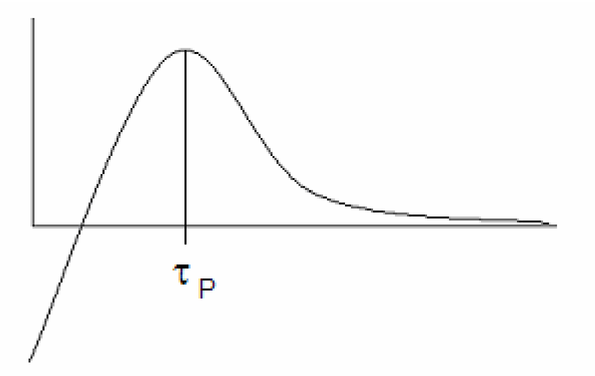
In the presence of information decay, the shape of $I(\tau)$ is similar except that it asymptotes to a number smaller than one. The sign of the first and second-order derivatives are preserved.

A.2 Consumer Behavior

Consumers will give the firm permission to contact them if they expect that the return they will derive from the relationship exceeds a certain threshold. The firm then retains this permission as long as their expected value remains above the threshold. We are going to assume that consumers evaluate their relationship with the firm as a function of the expected rate at which firms provide them information. That is, they will provide the firm with their permission if they expect that the rate at which the firm will send them valuable information is high enough. The advantage of talking in terms of rate is that it accounts for both the quality and the quantity of the information provided to the consumer.

If we consider the cost of processing each communication, we can say that the value of the relationship is increasing in $\dfrac{I(\tau)-c}{\tau}$, where $I(\tau)$ is the value of the information to the consumer as discussed in the previous section, $c > 0$ is the cost to the customer of processing that information, and $1/\tau$ is the information arrival rate set by the inter-communication time. Since we have assumed in the previous section that $A(\tau) = I(\tau)$, we can write that $P(\tau) = f\left(\dfrac{A(\tau)-c}{\tau}\right)$ with $\partial f / \partial \tau > 0$.

The shape of $\frac{A(\tau)-c}{\tau}$ is shown below.



Appendix B: Proof of Lemma 1 and Proposition 1

B.1 Proof of Lemma 1.

Let $P(\tau)$ be the proportion of the database that is retained from one campaign to the next, given that τ is the fixed inter-campaign time interval. Let the acquisition stream be $g(\tau)=\tau.g$. If the firm begins with a database size at $S_0 \neq \overline{S}$, we show that the long-term stationary value of the database is still $\overline{S}$. To find this value, solve the Law of Motion for the size of the database as:

$$\begin{aligned}\overline{S} &= \overline{S}.P(\tau)+\tau.g\\ &= \frac{\tau.g}{1-P(\tau)}.\end{aligned}$$

What if the database size is not at the stationary value? Then, if τ is constant, the state variable converges to the stationary value. To see this, pick any arbitrary value for the starting size of the database e.g. pick any $\varepsilon \neq 0$ such that $S_i < \overline{S}$ or $S_i > \overline{S}$ e.g.:

$$S_i = \frac{\tau.g}{1-P(\tau)}+\varepsilon.$$

so that for the next period:

$$\begin{aligned}S_{i+1} &= S_i P(\tau)+\tau.g\\ &= \left(\frac{\tau.g}{1-P(\tau)}+\varepsilon\right)P(\tau)+\tau.g\\ &= \frac{\tau.g}{1-P(\tau)}+\varepsilon.P(\tau)\end{aligned}$$

and for any n>0,

$$S_{i+n} = \frac{\tau.g}{1-P(\tau,k)}+\varepsilon P(\tau)^n$$

and $\lim_{n\to\infty} S_{i+n} = \overline{S}$, since $P(\tau)\in(0,1)$ and therefore $\varepsilon P(\tau)^n \to 0$ as $n\to\infty$.

B.2 Maximization of Customer Value versus Database Value.

Assume that we look at the equilibrium conditions so that $S_i = \overline{S}$. The value of an individual customer name is defined as:

$$\begin{aligned} V_{clv} &= \sum_{i=0}^{\infty} e^{-ir\tau} P(\tau)^i \left(R(\tau) - \frac{FC}{\overline{S}} \right) \\ &= \left(R(\tau) - \frac{FC}{\overline{S}} \right) \frac{e^{r\tau} P(\tau)^{-1}}{e^{r\tau} P(\tau)^{-1} - 1} \\ &= \left(R(\tau) - \frac{FC}{\overline{S}} \right) \frac{e^{r\tau}}{e^{r\tau} - P(\tau)}. \end{aligned}$$

The database value, is defined as:

$$\begin{aligned} V_{db} &= \sum_{i=0}^{\infty} e^{-ir\tau} \left(R(\tau)\overline{S} - FC \right) \\ &= \left(R(\tau)\overline{S} - FC \right) \frac{e^{r\tau}}{e^{r\tau} - 1}. \end{aligned}$$

Hence, we have the database value as a function of the customer value:

$$V_{db} = V_{clv}\, \overline{S}\, \frac{e^{r\tau} - P(\tau)}{e^{r\tau} - 1}.$$

To maximize, we differentiate with respect to τ :

$$\frac{\partial V_{db}}{\partial \tau} = \overline{S}\, \frac{e^{r\tau} - P(\tau)}{e^{r\tau} - 1} \frac{\partial V_{clv}}{\partial \tau} + V_{clv} \frac{\partial}{\partial \tau} \overline{S}\, \frac{e^{r\tau} - P(\tau)}{e^{r\tau} - 1}.$$

Hence, maximizing the *CLV* and the *DBV* will be identical iff:

$$\underbrace{\frac{\partial V_{db}}{\partial \tau}}_{0?} = \underbrace{\overline{S}\, \frac{e^{r\tau} - P(\tau)}{e^{r\tau} - 1}}_{\neq 0} \underbrace{\frac{\partial V_{clv}}{\partial \tau}}_{=0} + \underbrace{V_{clv}}_{\neq 0} \underbrace{\frac{\partial}{\partial \tau} \overline{S}\, \frac{e^{r\tau} - P(\tau)}{e^{r\tau} - 1}}_{=0?} = 0.$$

The fourth term $\left(\frac{\partial}{\partial \tau} \overline{S}\, \frac{e^{r\tau} - P(\tau)}{e^{r\tau} - 1} \right)$ will be equal to 0 iff:

$$\eta_{\overline{S}} + \eta_{\frac{e^{r\tau} - P(\tau)}{e^{r\tau} - 1}} = 0$$

or

$$\eta_{\overline{S}} = -\eta_{\frac{e^{r\tau} - P(\tau)}{e^{r\tau} - 1}}.$$

We know that $\eta_{\overline{S}} = 1 + \frac{\partial P(\tau)}{\partial \tau} \frac{\tau}{1 - P(\tau)}$, we now need to compute $\eta_{\frac{e^{r\tau} - P(\tau)}{e^{r\tau} - 1}}$:

$$\frac{\partial}{\partial\tau}\frac{e^{r\tau}-P(\tau)}{e^{r\tau}-1}=\frac{re^{r\tau}-\frac{\partial P(\tau)}{\partial\tau}}{e^{r\tau}-1}-\frac{\left(e^{r\tau}-P(\tau)\right)re^{r\tau}}{\left(e^{r\tau}-1\right)^2}$$

$$=\frac{1}{e^{r\tau}-1}\left(re^{r\tau}-\frac{\partial P(\tau)}{\partial\tau}-\frac{\left(e^{r\tau}-P(\tau)\right)re^{r\tau}}{e^{r\tau}-1}\right).$$

Thus:

$$\eta_{\frac{e^{r\tau}-P(\tau)}{e^{r\tau}-1}}=\frac{1}{e^{r\tau}-1}\left(re^{r\tau}-\frac{\partial P(\tau)}{\partial\tau}-\frac{\left(e^{r\tau}-P(\tau)\right)re^{r\tau}}{e^{r\tau}-1}\right)\frac{\tau}{\frac{e^{r\tau}-P(\tau)}{e^{r\tau}-1}}$$

$$=\frac{\tau}{e^{r\tau}-P(\tau)}\left(re^{r\tau}-\frac{\partial P(\tau)}{\partial\tau}-\frac{\left(e^{r\tau}-P(\tau)\right)re^{r\tau}}{e^{r\tau}-1}\right)$$

$$=-\left[\tau re^{r\tau}\left(\frac{1}{e^{r\tau}-1}-\frac{1}{e^{r\tau}-P(\tau)}\right)+\frac{\partial P(\tau)}{\partial\tau}\frac{\tau}{e^{r\tau}-P(\tau)}\right]$$

$$=-\left[\tau re^{r\tau}\left(\frac{1-P(\tau)}{\left(e^{r\tau}-1\right)\left(e^{r\tau}-P(\tau)\right)}\right)+\frac{\partial P(\tau)}{\partial\tau}\frac{\tau}{e^{r\tau}-P(\tau)}\right].$$

This means that $\eta_{\bar{S}}$ and $-\eta_{\frac{e^{r\tau}-P(\tau)}{e^{r\tau}-1}}$ are both affine transformations of $\frac{\partial P(\tau)}{\partial\tau}$, and thus will be equal for all $\frac{\partial P(\tau)}{\partial\tau}$ iff both their intercepts and their slopes are equal, or:

$$1=\tau re^{r\tau}\left(\frac{1-P(\tau)}{\left(e^{r\tau}-1\right)\left(e^{r\tau}-P(\tau)\right)}\right)$$

and

$$\frac{\tau}{1-P(\tau)}=\frac{\tau}{e^{r\tau}-P(\tau)}.$$

The second condition gives us that they will be equal only when $\tau=0$. Applying l'Hospital Rule to the first condition we find that, at the limit for $\tau\to 0$, the first condition is also satisfied.

Hence, this shows that it will only be for $\tau=0$ that $\frac{\partial}{\partial\tau}\bar{S}\frac{e^{r\tau}-p(\tau)}{e^{r\tau}-1}$. And thus, maximizing the *CLV* and the *DBV* lead to the same optimal only when $\tau^*=0$, which cannot happen since at $\tau=0$ the database value is negative ($DBV(0)=-\infty$).

QED

Appendix C: Maximum Derivation

We handle the maximization of the database value in three steps. First, we derive the first-order condition that needs to be satisfied for a τ to be optimal. Second we show that such a τ exists. And third, we provide conditions under which the maximum is known to be unique.

C.1 First-order Condition

To derive the first-order condition related to the maximization of the database value with respect to the inter-communication time, we seek the point at which the derivative of (8) with respect to τ is null. We do so in the following steps:

$$DBV(\tau) = \left(R(\tau).\bar{S}(\tau) - FC\right)\frac{e^{r\tau}}{e^{r\tau}-1} - AQ\frac{e^{r}}{e^{r}-1}. \tag{C-1}$$

Let $\Omega(\tau) = R(\tau).S(\tau)$, hence:

$$\begin{aligned}\frac{\partial DBV(\tau)}{\partial\tau} &= \frac{\partial}{\partial\tau}\left(\Omega(\tau) - FC\right)\frac{e^{r\tau}}{e^{r\tau}-1}\\ &= \frac{e^{r\tau}}{e^{r\tau}-1}\frac{\partial\Omega(\tau)}{\partial\tau} + \left(\Omega(\tau)-FC\right)\left[\frac{re^{r\tau}}{e^{r\tau}-1} - \frac{re^{2r\tau}}{\left(e^{r\tau}-1\right)^2}\right]\\ &= \frac{e^{r\tau}}{e^{r\tau}-1}\left[\frac{\partial\Omega(\tau)}{\partial\tau} + r\left(\Omega(\tau)-FC\right)\left(1-\frac{e^{r\tau}}{e^{r\tau}-1}\right)\right]\\ &= \frac{e^{r\tau}}{e^{r\tau}-1}\frac{\Omega(\tau)}{\tau}\left[\eta_\Omega + \eta_D\frac{\Omega(\tau)-FC}{\Omega(\tau)}\right].\end{aligned}$$

Further, since $\Omega(\tau) = R(\tau).S(\tau)$, then $\eta_\Omega = \eta_R + \eta_{\bar{S}}$. Hence:

$$\frac{\partial DBV(\tau)}{\partial\tau} = \frac{e^{r\tau}}{e^{r\tau}-1}\frac{R(\tau).\bar{S}(\tau)}{\tau}\left[\eta_R + \eta_{\bar{S}} + \eta_D\frac{R(\tau).\bar{S}(\tau)-FC}{R(\tau).\bar{S}(\tau)}\right]. \tag{C-2}$$

If we restrict ourselves to cases where the optimal database value is positive (otherwise the firm would not engage in PBM) then we have $R(\tau) > 0$ and $R(\tau)\bar{S}(\tau) - FC > 0$ and thus, at the maximum, the following first-order condition needs to be satisfied:

$$\eta_R + \eta_{\bar{S}} + \eta_D GM = 0 \tag{C-3}$$

Where $GM = \dfrac{R(\tau).\bar{S}(\tau)-FC}{R(\tau).\bar{S}(\tau)}$ is the gross margin generated by each communication.

The technical appendix contains the proof of the existence and uniqueness of the maximum.

C.2 Change in Retention Probabilities

i. Change in $\partial P(\tau)/\partial\tau$

The retention sensitivity ($\partial P(\tau)/\partial\tau$) only affects the FOC through $\eta_{\bar{S}}$. An increase in retention sensitivity will lead to an increase in $\eta_{\bar{S}}$ as:

$$\frac{\partial \eta_{\bar{S}}}{\frac{\partial P(\tau)}{\partial(\tau)}} = \frac{\partial}{\frac{\partial P(\tau)}{\partial(\tau)}}\left[1+\frac{\partial P(\tau)}{\partial(\tau)}\frac{\tau}{1-P(\tau)}\right]$$

$$= \frac{\tau}{1-P(\tau)} > 0.$$

This increase in $\eta_{\bar{S}}$ will lead the firm to increase its τ to reach maximum profits.

ii. Intercept-shift in $P(\tau)$

We look here at the change in *FOC* resulting from an intercept-shift increase in retention probabilities. That is:

$$P_1(\tau) = P(\tau) + p_0$$

$$\frac{\partial P_1(\tau)}{\partial \tau} = \frac{\partial P(\tau)}{\partial \tau}.$$

Since $R(\tau)$ and D are both independent from $P(\tau)$, we have $\frac{\partial \eta_R}{\partial p_0} = 0$ and $\frac{\partial \eta_D}{\partial p_0} = 0$. Further:

$$\frac{\partial \eta_{\bar{S}}}{\partial p_0} = \frac{\partial}{\partial p_0}\left[1+\frac{\partial P_1(\tau)}{\partial \tau}\frac{\tau}{1-P_1(\tau)}\right]$$

$$= \frac{\partial P(\tau)\tau}{\partial \tau}\frac{\partial}{\partial p_0}\left[\frac{1}{1-P(\tau)-p_0}\right]$$

$$= \frac{\tau}{\left(1-P(\tau)-p_0\right)^2}\frac{\partial P(\tau)}{\partial \tau}. \qquad \text{(C-4)}$$

Hence, $\partial \eta_{\bar{S}} / \partial p_0$ has the same sign as $\partial P(\tau)/\partial \tau$. For small τ, where $\partial P(\tau)/\partial \tau$ is positive, the intercept-shift will have a positive impact on $\eta_{\bar{S}}$. For large τ, where $\partial P(\tau)/\partial \tau$ is negative, the impact will be negative.

For *GM* we have:

$$\frac{\partial GM}{\partial p_0} = \frac{\partial}{\partial p_0}\frac{R(\tau).\bar{S}(\tau) - FC}{R(\tau).\bar{S}(\tau)}$$

$$= -\frac{\partial}{\partial p_0}\frac{FC}{R(\tau).\bar{S}(\tau)} \qquad \text{(C-5)}$$

$$= \underbrace{\frac{FC}{R(\tau)\bar{S}(\tau)^2}}_{>0}\frac{\partial \bar{S}(\tau)}{\partial p_0}.$$

And:

$$\frac{\partial \bar{S}(\tau)}{\partial p_0} = \frac{\partial}{\partial p_0}\frac{\tau.g}{1-P(\tau)-p_0}$$

$$= \frac{\tau.g}{\left(1-P(\tau)-p_0\right)^2} > 0. \qquad \text{(C-6)}$$

Hence, an intercept-shift increase in $P(\tau)$ leads to an increase in *GM* that leads to a decrease in FOC. Putting (C-4), (C-5), and (C-6) back into the FOC, we have that an intercept-shift increase in $P(\tau)$ will lead to higher τ^* if:

$$\frac{\tau}{\left(1-P(\tau)-p_o\right)^2}\frac{\partial P(\tau)}{\partial \tau} > |\eta_D| \frac{\tau.g}{\left(1-P(\tau)-p_o\right)^2}\frac{FC}{R(\tau)\bar{S}(\tau)^2}$$

$$\frac{\partial P(\tau)}{\partial \tau} > |\eta_D| \frac{g}{\bar{S}(\tau)}\frac{FC}{R(\tau).\bar{S}(\tau)}$$

$$\frac{\partial P(\tau)}{\partial \tau}\frac{\tau}{1-P} > |\eta_D| \frac{FC}{R(\tau).\bar{S}(\tau)}$$

$$\eta_{\bar{S}} > |\eta_D| \frac{FC}{R(\tau).\bar{S}(\tau)} + 1. \tag{C-7}$$

That is, if we assume that $P(\tau)$ is inverted-U shape, τ^* increases for small τ^* when there is an intercept-shift in $P(\tau)$, and decreases for large τ^*.

C.3 Change in Revenue per Contact

i. Change in $\partial A(\tau)/\partial\tau$

The revenue sensitivity affects the FOC through both η_R and $\eta_{\bar{S}}$. We have:

$$\frac{\partial \eta_R}{\partial \frac{\partial A(\tau)}{\partial \tau}} = \frac{\partial}{\partial \frac{\partial A(\tau)}{\partial \tau}} \frac{\partial A(\tau)}{\partial \tau}\frac{\tau}{A(\tau)-VC}$$

$$= \frac{\tau}{A(\tau)-VC} > 0.$$

and

$$\frac{\partial \eta_{\bar{S}}}{\partial \frac{\partial A(\tau)}{\partial \tau}} = \frac{\partial}{\partial \frac{\partial A(\tau)}{\partial \tau}}\left[1+\frac{\partial P(\tau)}{\partial \tau}\frac{\tau}{1-P(\tau)}\right]$$

$$= \frac{\partial}{\partial \frac{\partial A(\tau)}{\partial \tau}}\left[1+\frac{\tau}{1-P(\tau)}\frac{\partial}{\partial \tau} f\left(\frac{A(\tau)-c}{\tau}\right)\right]$$

$$\frac{\partial \eta_{\bar{S}}}{\partial \frac{\partial A(\tau)}{\partial \tau}} = \frac{\partial}{\partial \frac{\partial A(\tau)}{\partial \tau}}\left[1+\frac{\tau}{1-P(\tau)}\frac{\partial f(x)}{\partial x}\frac{\partial}{\partial \tau}\left[\frac{A(\tau)-c}{\tau}\right]\right]$$

$$= \frac{\partial}{\partial \frac{\partial A(\tau)}{\partial \tau}}\left[1+\frac{\tau}{1-P(\tau)}\frac{\partial f(x)}{\partial x}\left(\frac{\frac{\partial A(\tau)}{\partial \tau}}{\tau}-\frac{A(\tau)-c}{\tau^2}\right)\right]$$

$$= \frac{1}{1-P(\tau)}\frac{\partial f(x)}{\partial x} > 0.$$

Hence, the database sensitivity to τ increases when the sensitivity of the revenue increases, creating a compounding effect that leads the firm to increase its optimal sending rate (τ^*).

ii. *Intercept-shift in* $A(\tau)$

An intercept-shift in $A(\tau)$ will be felt through η_R, $\eta_{\bar{S}}$, and *GM*. Thus, we compute the following:

$$A_1(\tau) = A(\tau) + a_0$$

$$\frac{\partial A_1(\tau)}{\partial \tau} = \frac{\partial A(\tau)}{\partial \tau}$$

$$\frac{\partial \eta_R}{\partial a_0} = \frac{\partial}{\partial a_0}\frac{\partial A(\tau)}{\partial \tau}\frac{\tau}{A(\tau) - VC}$$

$$= \frac{\partial A(\tau)}{\partial \tau}\frac{-\tau}{\left(A(\tau) - VC\right)^2}$$

$$= \frac{-\eta_R}{R(\tau)} < 0$$

$$\frac{\partial P(\tau)}{\partial a_0} = \frac{\partial}{\partial a_0} f\left(\frac{A(\tau) - c}{\tau}\right)$$

$$= \frac{1}{\tau}\frac{\partial f(x)}{\partial x} > 0$$

$$\frac{\partial \frac{\partial P(\tau)}{\partial \tau}}{\partial a_0} = \frac{\partial}{\partial a_0}\left[\frac{\partial f(x)}{\partial x}\left(\frac{\partial A(\tau)}{\partial \tau}\frac{1}{\tau} - \frac{A(\tau) - c}{\tau^2}\right)\right]$$

$$= -\frac{1}{\tau^2}\frac{\partial f(x)}{\partial x} < 0$$

$$\frac{\partial \bar{S}}{\partial a_0} = \frac{\partial}{\partial a_0}\left[\frac{\tau . g}{1 - P(\tau)}\right]$$

$$= \tau . g \frac{\frac{\partial P(\tau)}{\partial a_0}}{\left(1 - P(\tau)\right)^2}$$

$$= \frac{\bar{S}}{1 - P(\tau)}\frac{\partial P(\tau)}{\partial a_0}$$

$$= \frac{\bar{S}}{1 - P(\tau)}\frac{1}{\tau}\frac{\partial f(x)}{\partial x} > 0$$

$$\frac{\partial GM}{\partial a_0} = -\frac{\partial}{\partial a_0}\frac{FC}{R(\tau).\bar{S}(\tau)}$$

$$= FC\left(\frac{\frac{\partial R(\tau)}{\partial a_0}.\bar{S}(\tau) + R(\tau).\frac{\partial \bar{S}(\tau)}{\partial a_0}}{\left(R(\tau).\bar{S}(\tau)\right)^2}\right)$$

$$= FC\left(\frac{\bar{S}(\tau) + \frac{R(\tau).\bar{S}(\tau)}{1-P(\tau)}\frac{1}{\tau}\frac{\partial f(x)}{\partial x}}{\left(R(\tau).\bar{S}(\tau)\right)^2}\right)$$

$$= \frac{FC}{R(\tau).\bar{S}(\tau)}\left(\frac{1}{R(\tau)} + \frac{1}{1-P(\tau)}\frac{1}{\tau}\frac{\partial f(x)}{\partial x}\right) > 0$$

$$\frac{\partial \eta_{\bar{S}}}{\partial a_0} = \frac{\partial}{\partial a_0}\left[1 + \frac{\partial P(\tau)}{\partial \tau}\frac{\tau}{1-P(\tau)}\right]$$

$$= \frac{\tau}{1-P(\tau)}\frac{\partial \frac{\partial P(\tau)}{\partial \tau}}{\partial a_0} + \tau\frac{\partial P(\tau)}{\partial \tau}\frac{\frac{\partial P(\tau)}{\partial a_0}}{\left(1-P(\tau)\right)^2}$$

$$= \frac{\tau}{1-P(\tau)}\frac{-1}{\tau^2}\frac{\partial f(x)}{\partial x} + \tau\frac{\partial P(\tau)}{\partial \tau}\frac{\frac{-1}{\tau^2}\frac{\partial f(x)}{\partial x}}{\left(1-P(\tau)\right)^2}$$

$$= \frac{-1}{\tau(1-P(\tau))^2}\frac{\partial f(x)}{\partial x}\left[\left(1-P(\tau)\right) + \frac{\partial P(\tau)}{\partial \tau}\right].$$

Hence, $\partial \eta_{\bar{S}} / \partial a_0$ is negative for small to moderate levels of τ^* (i.e., $\tau^* : 1 - P(\tau^*) > -\partial P(\tau^*)/\partial \tau$) and positive for larger τ^*. Thus, for small to moderate τ^*, the negative impacts on η_R, $\eta_{\bar{S}}$ and the positive impact on *GM* will both yield a smaller τ^*. For large τ^*, the net impact might be positive. The point at which the effect reverses itself is given by:

$$\tau : \frac{-\eta_R}{R(\tau)} + \frac{-1}{\tau\left(1-P(\tau)\right)^2}\left(\frac{\partial P(\tau)}{\partial \tau} - \tau\left(1-P(\tau)\right)\right)\frac{\partial f(x)}{\partial x} + \eta_D\frac{FC}{R(\tau).\bar{S}(\tau)}\left(\frac{1}{R(\tau)} + \frac{1}{1-P(\tau)}\frac{1}{\tau}\frac{\partial f(x)}{\partial x}\right) = 0.$$

This expression is not tractable, but a lower-bound on τ^* is given by:

$$\tau^* : \frac{\partial P(\tau)}{\partial \tau} = \tau\left(1-P(\tau)\right)$$

or $\tau^* : \eta_S = \tau^2 + 1$.

Appendix D: Return from Acquisition Costs

We have shown in Section 6 that the returns to acquisition are positive (i.e., $\partial DBV / \partial g > 0$ per (15)). We now want to look at the second derivate ($\partial^2 DBV / \partial g^2$) to see if the returns are increasing or decreasing in *AQ*. Increasing returns would mean that the rich get richer (a double jeopardy scenario for database size). Decreasing returns would imply that there is a natural limit to the firm's database size.

When computing the second derivative of *DBV*, we will assume that the returns from acquisition are linear (i.e., $\partial^2 AQ / \partial g^2 = 0$) so as to clearly isolate the database growth mechanism. When doing so, we cannot invoke the envelope theorem anymore since, as there are no more first-order effects, all the action is in the second-order effects. Starting with (15) we show that $\partial^2 DBV / \partial g^2 > 0$ when $\partial^2 AQ / \partial g^2 = 0$:

$$\begin{aligned}
\frac{\partial^2 DBV}{\partial g^2} &= \frac{\partial}{\partial g}\left(R(\tau^*)\frac{\tau^*}{1-P(\tau^*)}\frac{e^{r\tau^*}}{e^{r\tau^*}-1} - \frac{e^r}{e^r-1}\frac{\partial AQ}{\partial g} \right) \\
&= \frac{\partial R(\tau^*)}{\partial g}\frac{\tau^*}{1-P(\tau^*)}\frac{e^{r\tau^*}}{e^{r\tau^*}-1} + \frac{\partial \tau^*}{\partial g}\frac{R(\tau^*)}{1-P(\tau^*)}\frac{e^{r\tau^*}}{e^{r\tau^*}-1} \\
&\quad + \frac{\partial P(\tau^*)}{\partial g} R(\tau^*)\frac{\tau^*}{\left(1-P(\tau^*)\right)^2}\frac{e^{r\tau^*}}{e^{r\tau^*}-1} + \frac{\partial \frac{e^{r\tau^*}}{e^{r\tau^*}-1}}{\partial g} R(\tau^*)\frac{\tau^*}{1-P(\tau^*)} \\
&= \frac{e^{r\tau^*}}{e^{r\tau^*}-1}\frac{R(\tau^*).\tau^*}{g\left(1-P(\tau^*)\right)}\left[\gamma_R + \gamma_\tau - \gamma_{1-P} + \gamma_D\right].
\end{aligned} \tag{D-8}$$

Where γ_i is the elasticity of *i* with respect to *g*.

In (D-8), the term in front of the brackets is always positive, and thus, the sign of $\partial^2 DBV / \partial g^2$ will be the same as the sign of the term in brackets. *DBV* will be convex in *g* if the term in brackets is positive, and concave otherwise.

Using the property of elasticities that $\gamma_i = \eta_i \gamma_\tau$, we can rewrite the term in brackets as:

$$\begin{aligned}
\gamma_R + \gamma_\tau - \gamma_{1-P} + \gamma_D &= \eta_R \gamma_\tau + \gamma_\tau - \eta_{1-P}\gamma_\tau + \eta_D \gamma_\tau \\
&= \gamma_\tau\left(\eta_R + 1 - \eta_{1-P} + \eta_D\right) \\
&= \gamma_\tau\left(\eta_R + \eta_{\overline{S}} + \eta_D\right).
\end{aligned} \tag{D-9}$$

We know from Proposition 5_b that $\partial \tau / \partial g$ is negative and thus γ_τ is negative too. We also know from Proposition 4 that at τ^*, $\eta_R + \eta_{\overline{S}} + \eta_D.GM = 0$. Thus, since 0<*GM*<1 and $\eta_D < 0$, $\eta_R + \eta_{\overline{S}} + \eta_D < 0$. This implies that (D-9) is positive (the product of two negative numbers), and thus (D-8) is positive also.

QED

Table 1: Sample Acquisition Costs

Source (number of campaigns)	Avg. # Names	Avg. Stream Cost	Cost per 1,000 names ($)
Registration card (n=27)	42,832	1,606.48	38
Sweepstakes (n=34)	16,340	3,347.00	205
Surveys (n=3)	1,048	2,619.00	2,499
El. Product registration (n=13)	4,342	625.00	144
List Providers	15-20 Million		200

Table 2: Newsletters List

1-800 Flowers.com	Chef's Catalog	Jacobson's	Penn National Gaming
1st CyberStore	ChemicalOnline	JC Wyatt	PETsMART.com
Abercrombie & Fitch	Chico's FAS	Jocolo Online Superstore	Pier 1 Imports
Acuity Brands	CKBProducts.com	Johnson Outdoors	Pillsbury.com
AirTran Holdings	Clinique Laboratories	Jos. A Bank Clothiers	Pizza Hut
AJ Prindle & Co.	Club Wholesale	Kenneth Cole	Polo Ralph Lauren
Alloy, Inc.	Coca Cola	Kmart	Procter & Gamble
Amazon	Coldwater Creek	Kodak	Quicksilver
America West	Colgate-Palmolive	Kohl's	Rags Shops
American Airlines	Continental Airlines	Kraft Foods	Rawlings Sporting Goods
American Ballet Theatre	CVS Corporation	L.L. Bean	Reebok
American Canadian Caribbean Line Inc.	Dave & Buster's	Lance	Rich's Department Stores
American Eagles Outfitters, Inc.	dealofday.com	Landry's Restaurants	Rivertown Trading Company
American Family Life Assurance Company	dELiA*s Corp.	Lands' End	RollerUSA
American Frame Corporation	Dell Computer	Levi's	Ross-Simons
American Medical Systems	Delta Air Lines	Lifedrive Online	S & K Famous Brands
American Trans Air	Dillard's	Longs Drug Stores	SaleSafari
American West Steamboat Company	Dooney & Bourke	Macy's	San Francisco Music Box Company
Ann Taylor	Dormitory (The)	Mail-Well	Sands Regent (The)
Apple Computer	drugstore.com	Marsh Supermarkets	Sara Lee
Arctic Cat	E Com Ventures	Marshall Fields	Seventh Avenue
Atherbys	Eckerd Corporation	Marvel Enterpries	Six Flags
Avon Products	e-Luxury	Mattel	Solutions
B.D. Jeffries	eZiba	Maurice's BBQ Online	Starbucks
Banana Republic	F.A.O.	Max & Erma's Restaurants	Stride Rite (The)
Bass Pro Shops	Family Christian Stores	McAfee	Sun Microsystems
Bebe stores	Finish Line (The)	Meijer	Sundance Catalog
Beckman Coulter	Fisher Price	Michaels Stores	Texas Instruments
BestBuysOnTheNet.com	Foley's	Midwest Express Holdings	Timken Co.
BeWild.com	Fossil	Morton's Restaurant Group	Tommy Hilfiger
Big Lots	Frontgate	MotherNature.com	Toys at 24hour-mail.com
BizRate	Gateway	Motorola	UAL Corporation
Blair Corporation	General Electric	mySimon	Uncommon Goods
Bloomingdale's	Go2orlando.com	Nasco	Undergear
Boeing	Gottschalks	Nautica	Urban Outfitters
Boscov's	GreenField Flower Shop	Neiman Marcus	USA Complete Sales Center
Bose	Gump's	Nike	UTLA.com
Brookstone	Hallmark	OfficeMax	ValueVision International
Brylane Home	Hammacher Schlemmer	O'Reilly Automotive	Vans
Buckle (The)	Harley-Davidson	Orlando Predators Entertainment (The)	Vermont Teddy Bear Co (The)
Budweiser	Hastings Entertainment	Orvis	Vickerey
BuyBidWin.com	Haverty Furniture Companies	Outback Steakhouse	Victoria's Secret
BuyUnionNow.com	Hewlett-Packard	Overstock.com	Walgreen Company
Cadbury Schweppes	Hibbett Sporting Goods	Pacific Sunwear of Carlifornia	Wal-mart
California Pizza Kitchen	IBM	Palm.com	West Marine
Camera World	Intel	Pampers.com	Wet Seal (The)
Campbell Soup	J. M. Smucker	Pathmark Stores	Whispering Pines
Canon	J. Marco Galleries	PC Connection	Wild Oats Market
Carson Pirie Scott & Co.	Jack Daniel's	PC Mall	Wilsons The Leather Experts

Table 3: Description of Newsletters

	Subscribed		Received more than 3 emails		Customize	
N	196		107			
Gender information	48	24%	41	38%	4	10%
Age information	47	24%	23	21%	5	22%

Table 4: Contact Frequency of Newsletters

Periodicity	N	Percent
Day	1	2%
Bi-Weekly	3	5%
Weekly	41	63%
Bi-Monthly	14	22%
Monthly	6	9%
Total	65	

Table 5: Notation

Variable	Construct
$A(\tau)$	Expected gross revenue from each communication ($A'>0, A''<0, A(0)=0, A(\infty)=k$)
$AQ(g)$	Acquisition spending required to acquire g names per period (AQ'>0)
D_i=$D(i)$	Discount multiplier $\left(D(i)=\frac{e^{ri}}{e^{ri}-1} \right)$
$DBV(\tau)$	Database value
FC	Fixed costs associated with each communication
g	Number of names acquired in a time period
GM	Gross margin yielded by each communication
$P(\tau)$	Retention rate ($P(\tau)=f\left(\frac{A(\tau)-c}{\tau}\right), f'>0$)
r	Discount rate
$R(\tau)$	Expected Profits, revenues net of goods sold and variable communication costs per customer per communication (i.e., $R(\tau)=A(\tau)-VC$)
$S(\tau)$	Database size
$\overline{S}(\tau)$	Steady state database size
τ	Inter-communication interval
V_c	Net present value of the income stream using a *CLV* approach
V_{db}	Net present value of the income stream using a *DBV* approach
VC	Variable cost of communication

[1] We would like to thank Ward Hanson for suggesting this representation.

[2] For the reader's convenience, all notation used in this paper are consolidated in Table 5.

[3] NPV calculations traditionally uses $1/(1+d)$ as the discount factor. To make our derivations simpler, we use e^{-r} instead. The two formulations are equivalent if we set $r = \ln(1+d)$.

[4] There is conflicting evidence regarding whether revenues per customer increase or decrease as a function of the length of the relationship with the firm. Gupta *et al* (2003) fail to find any systematic pattern in per customer revenues for the companies they studied.

[5] To see this, note that at $\tau = 0$, $r\tau e^{r\tau} = e^{r\tau} - 1 = 0$, and for all $\tau > 0$,

$$\frac{\partial}{\partial \tau} r\tau e^{r\tau} = re^{r\tau} + r^2 \tau e^{r\tau} > \frac{\partial}{\partial \tau} e^{r\tau} - 1 = re^{r\tau} .$$

[6] This is similar in spirit to the concept of positive or negative float in a retailing environment.

[7] Upon signing up for the newsletter, users indicate which cities they fly to often. The web specials are partly selected based on these cities.

[8] If $f(1)>0$ then some people will stay in the system forever, regardless of the firm's actions. This would lead to degenerate solutions.