View metadata, citation and similar papers at core.ac.uk

brought to you by CORE

provided by Institutional Knowledge at Singapore Management University

Singapore Management University
Institutional Knowledge at Singapore Management University

Research Collection Lee Kong Chian School Of Business

Lee Kong Chian School of Business

1-2007

Risky Debt-Maturity Choice under Information Asymmetry

Chunchi WU
Singapore Management University, wuchu@missouri.edu

Sheen LIU
Youngstown State University

DOI: https://doi.org/10.1142/9789812772824_0004

Follow this and additional works at: https://ink.library.smu.edu.sg/lkcsb_research

Part of the Finance and Financial Management Commons, and the Portfolio and Security Analysis Commons

Citation

WU, Chunchi and LIU, Sheen. Risky Debt-Maturity Choice under Information Asymmetry. (2007). *Advances in Quantitative Finance and Accounting*. 4, 75-96. Research Collection Lee Kong Chian School Of Business.
Available at: https://ink.library.smu.edu.sg/lkcsb_research/1574

This Book Chapter is brought to you for free and open access by the Lee Kong Chian School of Business at Institutional Knowledge at Singapore Management University. It has been accepted for inclusion in Research Collection Lee Kong Chian School Of Business by an authorized administrator of Institutional Knowledge at Singapore Management University. For more information, please email libIR@smu.edu.sg.

Published in Advances in Quantitative Finance and Accounting. Volume 4. World Scientific, 2007, Pages 75-96
https://doi.org/10.1142/9789812772824_0004

Risky debt-maturity choice under information asymmetry

Sheen Liu
Youngstown State University, USA

Chunchi Wu*
Singapore Management University, Singapore and Syracuse University, USA

The traditional equilibrium models of signaling with debt-maturity require transaction costs by firms when raising new capital. In this paper, we propose a new model that has no such requirement. We demonstrate that a separating equilibrium of debt-maturity choice exists under a much more general condition, once accounting for the interactions between borrowers and lenders. The model is able to explain the observed complex financial structure. It is found that callable debt functions much like short-term debt, and serial debt similar to long-term debt. In equilibrium, high-quality firms issue short-term debt, and low-quality firms issue long-term debt.

Keywords: Bond maturity; information asymmetry; signaling; sequential games.

1. Introduction

Under information asymmetry, firm insiders with better information than outside investors will choose to issue those securities the market appears to value most. Knowing this, rational investors will try to infer insider information from firms' financing strategies. Signaling theory contends that under certain conditions firms' choice of risky debt-maturity can convey the insider information about firm quality.[1] Plausible signaling equilibria often require transaction costs by firms when raising capital (see, e.g., Bhattacharya, 1979; Flannery, 1986). In particular, for firms to signal their true quality to the market effectively, transaction costs of issuing or retiring debts must be high enough to deter low-quality firms from mimicking high-quality firms.[2] Conversely, when

*Corresponding author.
[1]See Ravid (1996) for a review of debt-maturity signaling literature.
[2]See Flannery (1986) and Wu (1993).

financial market transactions are costless and changes in firm value are independent over time, firms' debt-maturity structure may fail to provide a credible signal. Kale and Noe (1990) examine the decision of debt-maturity choice using precise equilibrium refinements. They demonstrate that in the absence of transaction costs, there is no separating Nash sequential equilibrium since low-quality firms always have an incentive to mimic high-quality firms. Under this condition, both short- and long-term debt poolings are Nash sequential equilibrium outcomes, but only the short-term debt pooling equilibrium is universally divine.[3] On the other hand, a separating equilibrium exists if there are transaction costs and investment outcomes are correlated. Diamond (1991, 1993) shows that liquidity risk may force low-quality firms to use short-term debt, leaving only intermediate-quality firms to issue long-term debt.[4] Using a different approach, Titman (1992) shows that a separating debt-market equilibrium can be obtained if swap agreements are allowed to resolve the problem of interest rate uncertainty.

The requirement of transaction costs for a separating equilibrium may be due to the underlying assumptions in signaling models, some of which are arguably refutable. For example, previous studies often contend that without transaction costs, there is only one plausible outcome for firms' debt-maturity choice (M): both "Good" (G) and "Bad" (B) firms choose to issue short-term debt, $M = \{S, S\}$. A critical assumption behind these models is that investors will price risky debt at the average quality of firms where the distribution of quality is prior knowledge. This assumption results in a pooling equilibrium in which the value of Bad firms increases at the expense of Good firms (see Rothschild and Stiglitz, 1976; Ross, 1977; Campbell and Kracaw, 1980). The separating equilibrium is not forthcoming because Bad firms can always mimic Good firms in the absence of transaction costs.

However, the outcome of pooling may not be incentive-compatible. It is not necessarily costless for Bad firms to mimic Good firms. When the time comes for Bad firms to refinance their debt, they will more likely be in a worse

[3]Note that when the assumption of independent changes in firm value is relaxed, Kale and Noe (1990) demonstrate that a separating equilibrium may exist, in which high-quality firms issue short-term debt and low-quality firms issue long-term debt.

[4]Diamond (1991) does not explicitly assume transaction costs. In his model, forced liquidation results in a loss of management's control rent. In a sense, lost control rent is an opportunity cost of signaling. It can be shown that an absence of the control rent would result in a pooling equilibrium. Guedes and Opler (1996) and Stohs and Mauer (1996) find results consistent with Diamond's predictions.

state and to pay a much higher premium to refinance short-term debt. Bad firms ought to consider this consequence when deciding whether they should mimic Good firms. Even without transaction costs, mimicking may not be the best strategy for Bad firms because they may be penalized upon refinancing their short debt.[5]

A simple example may help illustrate this point. Two applicants apply for the same job, and the employer offers two contracts, short- and long-term. The short-term contract offers a higher annual salary than does the long-term contract, and there are no other costs for renewing the contracts. Knowing her own productivity, the "Good" applicant does not worry about the renewal of her contract and so prefers the short-term contract. Although the "Bad" applicant can also get a higher salary by signing the short-term contract, she knows that she may not be able to renew her contract after it expires. Thus, mimicking the "Good" applicant is not costless. If the cost of mimicking is greater than the gain from the higher salary of the short-term contract, the "Bad" applicant will prefer the long-term contract. If the employer can somehow design the contract optimally to allow each candidate to differentiate herself, a separating equilibrium can arise.

One serious drawback of traditional debt-maturity models is that they assume investors are not actively involved in the signaling game. A direct consequence is that the pricing mechanism of debts is exogenously given, instead of being endogenously derived from investors' rational choices. In this setting, investors wait passively for the outcome of the game between Good and Bad firms. If both firms choose to issue short-term debt, investors will price this debt at the average quality of firms, resulting in a pooling equilibrium. Conversely, if Good firms borrow short and Bad firms borrow long, investors will price short debt at the quality of Good firms and long debt at the quality of Bad firms, resulting in a separate equilibrium. Either the pooling or the separating equilibrium is the outcome of the game solely between Good and Bad firms, and investors cannot influence their financing strategy. For example, Good firms will choose to issue short-term debt only if the added refinancing cost of a rollover strategy is smaller than their misinformation value in the pooling equilibrium. A separating equilibrium can occur when at the same time the gain that Bad firms achieve from issuing short-term debt is less than the flotation cost

[5]The recent events of Enron, Tyco, and WorldCom are excellent examples of how market discipline is enforced. After investors discover that truthful information was not disclosed, these firms can no longer have normal access to debt-markets.

incurred. When these conditions are not met, both firms will issue the same debt and a pooling equilibrium occurs. Firms optimally (or suboptimally) choose a debt-maturity structure based on market conditions, and investors play little role in this process.

The assumption that investors are inactive is rather unrealistic. In reality, investors (particularly institutional investors) in the debt-market often interact with the issuers or investment bankers to come to an agreement with the terms of debts. Market equilibrium is typically an outcome of interactions between suppliers and demanders. Investors can change their pricing strategy to affect the firm's debt choice and ultimately alter the equilibrium outcome. Like the aforementioned example of labor contracts, investors may set different terms for borrowers so that they will reveal their true credit quality.

In this paper, we propose an alternative model of debt-maturity choice that accounts for the interactions between borrowers and lenders. In this model, both firms and investors play an important role in the determination of a debt-market equilibrium. Good firms have an incentive to differentiate themselves from Bad firms to reduce their debt financing costs. Investors have an incentive to identify Bad firms to reduce their investment risk associated with adverse selection. Good firms use different debt instruments to signal their credit quality to the market. Investors actively search for an optimal pricing scheme to induce firms to differentiate among themselves by choosing different debt instruments. Including investors as active strategic players in the game produces an equilibrium outcome dramatically different from previous ones. We show that a separating equilibrium of debts with different maturities exists under a much more general condition. In particular, flotation costs are no longer required for the existence of a separating equilibrium.

The model is capable of explaining the complicated debt structure observed in the financial world. It is found that bond covenants are useful for resolving the problem of asymmetric information. For example, the call provision can reduce the misinformation value (dead-weight cost) or the cost of signaling in achieving the informational equilibrium. Similarly, the sinking-fund provision conveys the quality of the bond issuers. The sinking-fund call feature is shown to reinforce the effect of the amortization scheme in resolving the problem of asymmetric information faced by the issuers. In contrast, serial debt with no sinking-fund calls behaves much like long-term debt. Thus, bond covenants may either enhance the maturity effect or simply serve a function similar to debt-maturity in corporate financing decisions.

The remainder of this paper is organized as follows: Section 2 presents a pricing model of debts with asymmetric information. Section 3 discusses investors' pricing strategies and derives the equilibrium of a sequential game including the investor as a player. Section 4 provides numerical examples to illustrate the separating equilibrium with and without flotation costs. Finally, Section 5 concludes the paper.

2. The Model

This section sets up a valuation model of long- and short-term bonds under information asymmetry. The key assumptions underlying this model are summarized as follows:

(A.1) There are two periods in the model. Each firm invests in a single project at the beginning of period 1, t_0. The project is liquidated at the end of period 2, t_2, and the distribution of its liquidation value is common knowledge. The liquidation values are M_3, M_4, and M_5, where $M_3 > M_4 > M_5$. The probabilities of reaching different states and final liquidation values are displayed in Figure 1.[6] The firm does not default at any state except S_5. At state S_5, M_5 is zero; that is, there is no residual value, or the recovery rate of the debt is zero upon default.[7] At t_0, the firm must borrow an exogenous amount of debt D to finance the project, which generates no cash flow before its liquidation at t_2.

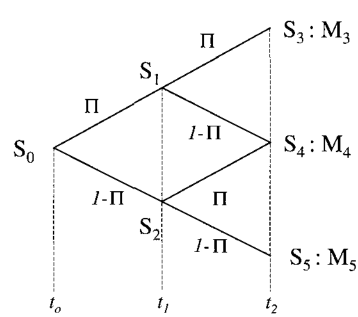

Figure 1. The two-period binomial tree of the firm's project.

[6]The setup of this probability structure is similar to Flannery (1986).

[7]To simplify the problem, $M_5 = 0$ is assumed. This assumption can be easily relaxed to consider the seniority of debts.

(A.2) Two types of debt instruments are considered for financing the project: long-term and short-term debts. Long-term debt lasts for two-periods, whereas short-term debt lasts only for one-period.

(A.3) When a short debt is retired at the end of the first period, t_1, it is refinanced with another short debt maturing at t_2. We refer to the combination of two short debts as the "short-term" financing strategy and the issuance of long debt as the "long-term" strategy.

(A.4) In the discrete case, we assume two types of firms: "Good" firms have projects with an "up" probability $p = p_G$, and "Bad" firms have projects with $p = p_B < p_G$. Investors know the fact that θ percent of firms (projects) are "Good," but they cannot identify a particular firm's quality. In the continuous case, the true "up" probability, p, for each firm is unobservable and distributed on $p \in L = (0, 1)$, according to a strictly increasing function $f(p) \in C^{\infty}$. We use the discrete case to illustrate the fundamental principle of choice between long and short debts. The discrete case is then extended to a continuous distribution of credit quality to generalize the results to multiple debt instrument choice.

(A.5) There is information asymmetry in the sense that the management's information set is different from outside investors'. Consequently, the management's perception of the "up" probability ($\Pi = p$) differs from investors' ($\Pi = \pi$). Investors have homogeneous expectations and adopt the same rule of valuation on risky claims.

(A.6) Firm managers and investors are risk-neutral, expected wealth maximizers.

Given the estimate of "up" probability, π, risk-neutral investors require an interest factor (one plus the coupon rate) on the long-term debt issued at t_0, R_L^{π}, such that the expected payoff on risky debt equals the principal amount lent:

$$F = \pi F R_L^{\pi} + (1 - \pi)\pi F R_L^{\pi}. \tag{1}$$

This equality yields an interest factor for the long-term debt

$$R_L^{\pi} = \frac{1}{2\pi - \pi^2}. \tag{2}$$

The risk-neutral manager's valuation of equity when pursuing a long-term borrowing strategy is

$$V_L = p\{p\lfloor M_3 - R_L^{\pi} F\rfloor + (1 - p)\lfloor M_4 - R_L^{\pi} F\rfloor\} + (1 - p)p\lfloor M_4 - R_L^{\pi} F\rfloor. \tag{3}$$

Substituting Equations (2) into (3) and rearranging yields

$$V_{\mathrm{L}} = V^{i} + V_{\mathrm{L}}^{\mathrm{mis}} \tag{4}$$

where the firm's value is composed of an intrinsic value

$$V^{i} = p^{2}M_{3} + 2p(1-p)M_{4} - F \tag{5}$$

and a misinformation value

$$V_{\mathrm{L}}^{\mathrm{mis}} = F\frac{2(\pi - p) + (p^{2} - \pi^{2})}{2\pi - \pi^{2}}, \tag{6}$$

which is caused by asymmetric information. The misinformation value is represented by the difference between the value viewed by the outside investor (reflected in his π estimate) and its fair value based on the insider's information (for p).

The firm issuing short-term debt retires it at t_1. By (A.1), no default occurs at t_1 and so the entire principal F is retired (the coupon rate is zero) and the same amount of short debt is reissued. At state S_1, investors require an interest factor $(R_1^{\Pi}|S_1)$ for short debt. Similarly, at state S_2, given investors' estimate of "up" probability $\Pi = \pi$, investors require an interest factor $(R_1^{\pi}|S_2)$ for short debt. Thus, for the short debt issued at t_0, risk-neutral investors will require one-period interest factors such that

$$F = \pi F(R_1^{\pi}|S_1) + (1-\pi)F\pi(R_1^{\pi}|S_2) \tag{7}$$

Lemma 1 establishes the values of the short-term interest factors $(R_1^{\Pi}|S_i)$, $i = 1, 2$ at different states.

Lemma 1
The short-term interest factor for refinancing at t_1 is given by

$$(R_1^{\pi}|S_1) = 1, \qquad (R_1^{\pi}|S_2) = \frac{1}{\pi}. \tag{8}$$

Proof. At state S_1, F amount of short debt is retired, and the same amount of short debt is re-issued. At this state, investors know that short debt is default-free, and thus, they charge an interest factor $(R_1^{\Pi}|S_1) = 1$. At state S_2, given the estimate of "up" probability Π, investors know that short debt has a default probability of $1 - \Pi$ and a recovery rate of zero. Thus, they require an interest factor

$$(R_1^{\pi}|S_2) = \frac{1}{\pi}. \tag{9}$$

Alternatively, using $(R_1^\Pi | S_1) = 1$ and Equation (7), we have

$$F = \pi F + (1 - \pi)\pi F(R_1^\pi | S_2)$$

by which we can solve for the one-period interest factor at state S_2

$$(R_1^\pi | S_2) = \frac{1}{\pi}$$

Note that we made no assumption before that investors know the true probability of the "up" state for each firm's project. The values of the interest factors in Equations (2) and (9) depend on investors' *estimate* of the "up" probability π. Setting $\Pi = \pi$, we can obtain the interest factors required by investors for both short and long debts. For ease of notation, we henceforth replace $(R_1^\Pi | S_1)$ with one, and $(R_1^\Pi | S_2)$ with R_1^Π.

The risk-neutral manager's valuation of equity under the short-term debt financing strategy is

$$V_S = p\{p[M_3 - F] + (1 - p)[M_4 - F]\} + (1 - p)p\lfloor M_4 - FR_1^\pi \rfloor. \quad (10)$$

Substituting Equation (9), with R_1^Π evaluated at $\Pi = \pi$, into Equation (10) and rearranging, we have

$$V_S = V^i + V_S^{\text{mis}}, \qquad V_S^{\text{mis}} = F(1 - p)\frac{\pi - p}{\pi}. \quad (11)$$

Previous studies (see, e.g., Flannery, 1986; Kale and Noe, 1990; Diamond, 1991) have implicitly assumed that investors take a passive role in the determination of the signaling equilibrium. We denote the pricing strategy when investors are inactive as *pricing strategy A*. Under this pricing strategy, the values of the "up" probability are determined according to firms' debt-maturity choices (M):

1. If $M = \{L, S\}$ or $M = \{S, L\}$, then $\pi_S = p_G$ and $\pi_L = p_B$.[8]
2. If $M = \{L, L\}$ or $M = \{S, S\}$, then π_S or π_L is chosen so that $\sum V_i^{\text{mis}}(q) = 0$,

where $i = S$ (short debt), L (long debt), and $q = G$ (good firm), B (bad firm). In the first case, Good and Bad firms choose different financing strategies, and so the probability of the "up" state is assigned according to the quality of each firm. In the second case, Good and Bad firms choose the same financing

[8]Previous studies (Flannery, 1986; Kale and Noe, 1990; Diamond, 1990) show that $M = \{L, S\}$ is not a viable separating equilibrium.

strategy. Since it is not possible to distinguish Good from Bad firms through their financing patterns, an average price is charged to all bonds such that the aggregate misinformation value is equal to zero. On the sell side of the market, Bad firms gain and Good firms lose. On the buy side, those who invest in Bad firms' bonds pay an excessive price.

There are two potential difficulties with this pricing strategy commonly adopted in the existing debt-maturity literature: First, the pricing strategy presumes that investors will accept whatever pricing rules that are given. However, if investors are rational, they should be able to choose a pricing rule that better serves their interests. Second, it assumes no investor learning. In reality, investors may receive a signal, $m \in M$, conveyed by firms or information agencies. They may then estimate π based on m and price the debts either under the separating equilibrium or under the pooling equilibrium. In either case, investors' pricing strategy would be based on their best assessment of π, rather than on a passive reaction to firms' debt choice or an exogenously given pricing rule. The pricing strategy chosen by investors should directly affect the firm's choice of debt or alternatively, the firm's financing decision should take into account the expected pricing strategy of the investors. In the following section, we discuss an alternative pricing strategy and a sequential game in which investors' pricing strategy is explicitly accounted for.

3. Debt-Market Equilibrium

Under information asymmetry, investors are uncertain about the quality of Good and Bad firms. This uncertainty could cause a mispricing of bonds with investors paying an excessive price for low-quality bonds. It is therefore in their interest to try to distinguish Good from Bad firms. For example, investors can offer different prices to the bonds issued by firms by assigning different values of π_{L} and π_{S} based on their best judgment. Given the values of π_{L} and π_{S}, the firm will compare its equity values under different financing strategies. If $V_{\mathrm{L}} > V_{\mathrm{S}}$, it will issue long-term debt; otherwise, it will issue short-term debt. Thus, the criterion for the firm's financing decision is the value difference:

$$\begin{aligned} \Delta V &= V_{\mathrm{L}} - V_{\mathrm{S}} + c \\ &= p\frac{(1-p)(2-\pi_{\mathrm{L}})\pi_{\mathrm{L}} + (2-p-(2-\pi_{\mathrm{L}})\pi_{\mathrm{L}})\pi_{\mathrm{S}}}{(2-\pi_{\mathrm{L}})\pi_{\mathrm{L}}\pi_{\mathrm{S}}} + c, \end{aligned} \tag{12}$$

where the flotation cost c is included. If the difference in Equation (12) is greater than zero, the firm chooses to issue long debt; otherwise, it issues short debt.

If investors could somehow find a combination of π_L and π_S so that Good and Bad firms choose to issue different debts, they would be able to discern the firm type and to price the debt more efficiently. Although investors may not know exactly the initial quality of each firm, they could assign a plausible set of π values to the debts issued by firms and observe their response. The response of Good and Bad firms to investors' π estimates, or their choice of long or short debt, sends a signal back to investors. Investors refine their estimate for the firm quality based on the feedback signal they receive. They would then change their offer based on their revised probability estimates and observe firms' response in the next round. This learning and adjustment process may continue until precise estimates of π are obtained and a market equilibrium is achieved. We define this strategy of actively searching for a better price or a better estimate of Π (firm quality) as *pricing strategy B*. We will show that, under pricing strategy B, an optimal combination of π_L and π_S exists even under zero flotation costs such that the separating equilibrium is always achievable.

3.1. *A separating equilibrium without flotation costs*

We first examine the case with $c = 0$, which represents a debt-market with zero flotation costs. It can be shown that a separating equilibrium of the debt-market exists in the absence of flotation costs. We summarize the equilibrium condition as follows.

Proposition 1

A separating equilibrium of the debt-market exists if π_L and π_S satisfy the condition that

$$0 \le \pi_L \le \pi_S \le \frac{(2 - \pi_L)\pi_L}{2 - (2 - \pi_L)\pi_L}. \tag{13}$$

Firms with a quality $p < p^*$ prefer long-term debt, whereas firms with a quality $p > p^*$ prefer short-term debt. Firms with a quality $p = p^*$ are indifferent to long- and short-term debts. The value of the cutoff quality p^* is given by

$$p^* = \frac{(2 - \pi_L)\pi_L - \pi_S[2 - (2 - \pi_L)\pi_L]}{(2 - \pi_L)\pi_L - \pi_S}. \tag{14}$$

Proof. See Appendix A.

The inequality in Equation (13) establishes the necessary condition for a separating equilibrium. The sufficient condition further requires that $p_B < p^*$

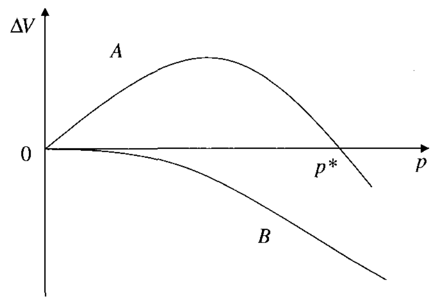

Figure 2. Changes in ΔV with quality p under zero c.

and $p_G > p^*$ where p_i, $i =$ B, G are the true probability of the "up" state for Bad and Good firms, respectively. Figure 2 gives a graphical presentation of the above results. Assuming that $p = 0$, from Equation (12) we have $\Delta V = 0$ given $c = 0$. But for a separating equilibrium, there must exist a positive p^* that makes $\Delta V = 0$. Curve A in Figure 2 shows one possible path for this condition to be held where the value of the criterion function goes up and then goes down to cross the horizontal axis. Firms prefer long debt when $p < p^*$ but prefer short debt when $p > p^*$. On the contrary, the path depicted by curve B does not cross the horizontal axis, and so no separating equilibrium exists in this case.

Curve A has a positive slope at $p = 0$ and a concave curvature so that the curve crosses $\Delta V = 0$ line at a strictly positive p. The comparative statistics of Equation (12) show that when $c = 0$,

$$\left.\frac{\partial \Delta V}{\partial p}\right|_{p=0} = \frac{(1+\pi_S)(2-\pi_L)\pi_L - 2\pi_S}{\pi_S(2-\pi_L)\pi_L}, \tag{15}$$

$$\frac{\partial^2 \Delta V}{\partial p^2} = -2\frac{(2-\pi_L)\pi_L - \pi_S}{\pi_S(2-\pi_L)\pi_L}. \tag{16}$$

It is straightforward to show that if

$$\left.\frac{\partial \Delta V}{\partial p}\right|_{p=0} > 0, \tag{17}$$

then the second-order derivative must be negative

$$\frac{\partial^2 \Delta V}{\partial p^2} < 0. \tag{18}$$

Using Equation (15), the condition in Equation (17) can be explicitly expressed as:

$$\pi_S \leq \frac{(2 - \pi_L)\pi_L}{2 - (2 - \pi_L)\pi_L}. \tag{17a}$$

Combining Equations (17a) with (A.5) in Appendix A and noting that $\pi_L > 0$, we can easily obtain the necessary condition in Equation (13). Thus, Equation (17) is a critical condition for a separating equilibrium. Curve B in Figure 2 has a negative value for both the first and second derivatives. Because the condition in Equation (13) is violated, there is no separating equilibrium. As shown, curve B does not cross the horizontal axis ($\Delta V = 0$) at any positive p.

Figure 3 gives a graphical representation of the necessary condition for the separating equilibrium. The horizontal axis measures the "up" probability of long-term debt π_L and the vertical axis measures that of short-term debt π_S. The dotted line represents $\pi_L = \pi_S$, and the upper left triangular region consists of $\pi_L \leq \pi_S$. The solid curve represents the boundary for Equation (17) and the dashed curve that for Equation (18). The region to the right of the solid curve satisfies the condition in Equation (17), while the region to the right of the dashed curve satisfies the condition in Equation (18). Hence, the region between the solid line and the dotted line indicates where the separating equilibrium exists under the condition of no flotation costs.

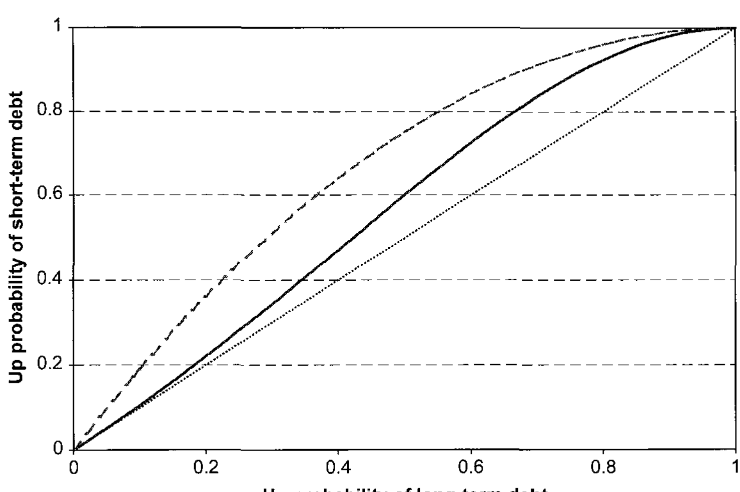

Figure 3. The separating equilibrium region for π_L and π_S.

3.2. *A separating equilibrium with flotation costs*

We next consider the case with flotation costs. Previous studies have often relied on a restrictive flotation cost structure to derive a signaling equilibrium. We show that a separating equilibrium always exists in the presence of flotation costs, but this equilibrium is a special case of the more general equilibrium that includes zero floatation costs.

Proposition 2

If $c > 0$, and π_L and π_S satisfy the condition that

$$0 \le \pi_L \le \pi_S \le (2 - \pi_L)\pi_L, \tag{19}$$

then a separating equilibrium exists. Firms with a quality $p < p^*$ prefer the long debt, whereas firms with a quality $p > p^*$ prefer the short debt. Firms with a quality $p = p^*$ are indifferent to long and short debts. The value of the cutoff quality p^* is given by

$$\begin{aligned} p^* = {} & \frac{(2-\pi_L)\pi_L - \pi_S[2-(2-\pi_L)\pi_L]}{2[(2-\pi_L)\pi_L - \pi_S]} \\ & + \frac{\sqrt{[\pi_S(2-(2-\pi_L)\pi_L) - (2-\pi_L)\pi_L]^2 + 4c\pi_S(2-\pi_L)\pi_L[(2-\pi_L)\pi_L - \pi_S]}}{2[(2-\pi_L)\pi_L - \pi_S]}. \end{aligned} \tag{20}$$

Proof. See Appendix A.

Figure 4 shows possible functions of ΔV with respect to p for the cases with (Curve B) and without flotation costs (Curve A). For a positive flotation cost, $c > 0$, ΔV is simply shifted up in parallel to the zero c curve (Curve A). Because Curve A has a negative first-order derivative of ΔV with respect to p,

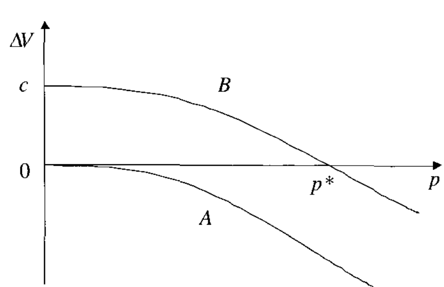

Figure 4. A shift in ΔV given a positive c.

$(\partial \Delta V/\partial p)|_{p=0} < 0$, and a negative second-order derivative, $(\partial^2 \Delta V/\partial p^2) < 0$, it does not cross the horizontal axis. However, with a positive value of c, ΔV does cross the horizontal axis (see Curve B), since $(\partial^2 \Delta V/\partial p^2) < 0$. Therefore, for $c > 0$, the condition for the first-order derivative in Equation (17) is no longer required. Combining $(\partial^2 \Delta V/\partial p^2) < 0$, (A.11) in Appendix A, and $\pi_L > 0$, we obtain the condition in Equation (19). As a result, the separating equilibrium region in Figure 3 is expanded since a positive first-order derivative is no longer required. The separating equilibrium region is now located between the dashed curve and the dotted line. Curve A in Figure 4 is drawn purposely to be similar to Curve B in Figure 2. It is shown that for some cases not having a separating equilibrium when flotation costs are zero, a separating equilibrium can be achieved when flotation costs become positive. Thus, the flotation cost differential between long and short debt strategies makes it easier to reach a separating equilibrium.

3.3. *Comparison of pricing strategies A and B*

When investors' pricing strategies are taken into consideration, the original game (under pricing strategy A) must be augmented to include investors as an additional player. The setting of the extended game incorporating investors' strategic behavior is shown in Figure 5. This game includes two reduced games: one is under pricing strategy A, and the other is under pricing strategy B. Previous studies (e.g., Flannery, 1986; Diamond, 1991; Kale and Noe, 1990)

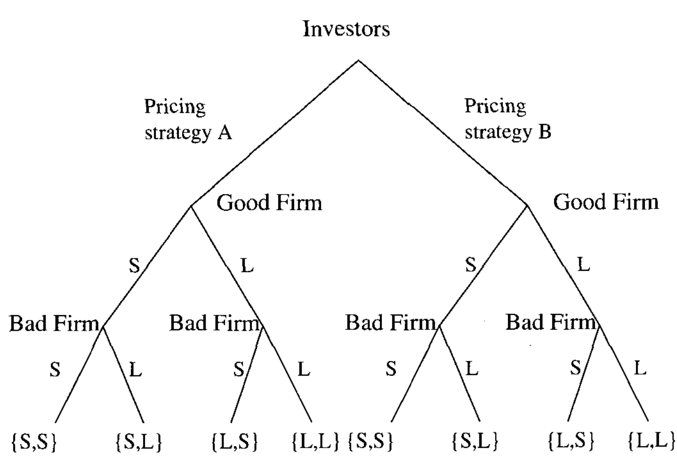

Figure 5. The game involving active investors.

consider only the first reduced game, in which pricing strategy A is the only possibility and investors are passive. The outcome of the first reduced game is either a pooling equilibrium or a separating equilibrium depending on the nature of the flotation cost function.

The second reduced game assumes that investors adopt pricing strategy B and interact with firms to determine equilibrium bond prices. Since investors know the distribution of firm quality, θ, they can select proper probability estimates satisfying $\pi_{\mathrm{S}} > \pi_{\mathrm{L}}$, and search a p^* value such that $p_{\mathrm{G}} > p^* > p_{\mathrm{B}}$. Under this pricing strategy, the sufficient condition of the separating equilibrium is satisfied. Good firms borrow short, whereas Bad firms borrow long, and both firms are better off than in any other choices given investors' estimates of π_{S} and π_{L}. Thus, this financing strategy is unequivocally the optimal choice for both types of firms. To resolve the adverse selection problem, investors minimize the total absolute misinformation value,

$$V^{\mathrm{mis}} = |V^{\mathrm{mis}}(G)| + |V^{\mathrm{mis}}(B)|. \tag{21}$$

It can be easily shown that the total absolute misinformation value is always smaller under strategy B than under strategy A.

Corollary 1

Under pricing strategy B, there will be a separating equilibrium, in which Good firms borrow short, while Bad firms borrow long.

This result does not depend on the magnitude of flotation cost. If the flotation cost satisfies the condition that $V_{\mathrm{L}}^{\mathrm{mis}}(G) < -c$ and $V_{\mathrm{S}}^{\mathrm{mis}}(B) \le c$, the two pricing strategies, A and B, lead to the same separating equilibrium. If the flotation cost is not high enough to satisfy this condition, these two pricing strategies lead to different equilibria. Pricing strategy A leads to a pooling equilibrium in short debt, whereas pricing strategy B still leads to a separating equilibrium. Investors' best interest is to minimize the total absolute misinformation value of the debt-market. Since pricing strategy B leads to a lower total absolute misinformation value than does pricing strategy A, it is preferred by investors. Comparing these two cases, we conclude that pricing strategy B dominates pricing strategy A.

Corollary 2

Investors always prefer pricing strategy B to strategy A. There is a separating equilibrium under pricing strategy B regardless of flotation costs.

4. Numerical Examples

In this section, we provide numerical examples to explain the intuition behind the model. Good and Bad firms' decisions to issue long versus short debt depend on the cost of each debt, which is in turn conditional on π_{L} and π_{S} set by investors. At the beginning of the game, investors do not know the exact value of p_{G} and p_{B} for each firm, but they possess a knowledge of the proportion of Good (θ) and Bad ($1-\theta$) firms and the average quality of firms, p_{avg}. At any stage of the game, investors will always balance their estimates of π_{S} and π_{L} such that their combination equals the average quality of firms:

$$p_{\mathrm{avg}} = \theta\pi_{\mathrm{S}} + (1-\theta)\pi_{\mathrm{L}}. \tag{22}$$

Hence, when investors raise the estimate for π_{S}, they must lower the estimate for π_{L}, given that p_{avg} is known.

Table 1 provides a numerical example for the adjustment process. Here, we assume that $p_{\mathrm{G}} = 0.97$, $p_{\mathrm{B}} = 0.93$, and $\theta = 0.5$. This gives an average "up" probability $p_{\mathrm{avg}} = 0.95$. Since initially investors do not know the exact "up" probability of each firm, they may try to price long and short debts based on the average probability $\pi_{\mathrm{L}} = \pi_{\mathrm{S}} = p_{\mathrm{avg}} = p^* = 0.95$, and observe the response of each firm. As shown in line one of Table 1, at these values of π_{L} and π_{S}, long debt yields a higher firm value for Bad firms than short debt does ($\Delta V_{\mathrm{B}} = 0.093$), while short debt provides a higher value for Good firms ($\Delta V_{\mathrm{G}} = -0.097$). Thus, Bad firms would prefer long debt, and Good firms would prefer short debt. The response of each type of firm tells investors that $p_{\mathrm{G}} > 0.95$ and $p_{\mathrm{B}} < 0.95$, and accordingly, they would reduce π_{L} and increase π_{S}. Investors may eventually reduce π_{L} to 0.944 and increase π_{S} to 0.956, and

Table 1. Numerical Example 1 ($c = 0$, $p_{\mathrm{G}} = 0.97$, $p_{\mathrm{B}} = 0.93$, and $\theta = 0.5$).

π_{L}	π_{S}	p^*	ΔV_{B}	ΔV_{G}	V^{mis}
0.9500	0.9500	0.9500	0.093	−0.097	0.152
0.9490	0.9510	0.9466	0.076	−0.111	0.145
0.9480	0.9520	0.9431	0.058	−0.124	0.138
0.9470	0.9530	0.9394	0.041	−0.138	0.131
0.9460	0.9540	0.9354	0.023	−0.152	0.125
0.9450	0.9550	0.9311	0.005	−0.166	0.118
0.9440	0.9560	0.9266	−0.013	−0.180	0.110

Note: p^* is calculated from Equation (14), ΔV from Equation (12), and V^{mis} from Equation (21).

discover that both firms no longer choose different debts but instead both prefer short debt. This outcome convinces investors that the best they can do is to set $\pi_L = 0.945$ and $\pi_S = 0.955$ to keep Bad firms from mimicking Good firms.

The aforementioned example assumes zero flotation costs, $c = 0$. Although there are no flotation costs to prevent Bad firms from mimicking Good firms, the resulting misinformation value serves for this function. The misinformation value in this separating equilibrium is less than that in the pooling equilibrium under pricing strategy A, which is represented in line one of Table 1. It is in investors' interests to prevent the pooling equilibrium from occurring. Investors minimize the sum of misinformation values by choosing $\pi_L = 0.945$ and $\pi_S = 0.955$.

The next example assumes a positive flotation cost $c = 0.001$. As shown in Table 2, at $\pi_L = \pi_S = p_{avg} = 0.95$, long debt yields a higher value than does short debt for both Bad and Good firms ($\Delta V_B = 0.193$, $\Delta V_G = 0.003$), given a positive flotation cost c. Thus, both firms would prefer long debt. The response of Good firms is different from the case with $c = 0$ (see Table 1) because flotation costs make short-term debt financing more costly. The existence of flotation costs allows investors to increase the difference between π_L and π_S to reach the separating equilibrium. For $c = 0.001$, investors can minimize the absolute misinformation value by choosing $\pi_L = 0.94$ and $\pi_S = 0.96$.

It can be shown that an additional increase in flotation costs will further reduce the misinformation value. At a certain level of flotation costs, the perfect revealing separating equilibrium may emerge. This case is illustrated in Table 3

Table 2. Numerical Example 2 ($c = 0.001$, $p_G = 0.97$, $p_B = 0.93$, and $\theta = 0.5$).

π_L	π_S	p^*	ΔV_B	ΔV_G	V^{mis}
0.95	0.95	0.9706	0.193	0.003	0.152
0.949	0.951	0.9678	0.176	−0.011	0.145
0.948	0.952	0.9649	0.158	−0.024	0.138
0.947	0.953	0.9618	0.141	−0.038	0.131
0.946	0.954	0.9585	0.123	−0.052	0.125
0.945	0.955	0.9549	0.105	−0.066	0.117
0.944	0.956	0.9512	0.087	−0.080	0.110
0.943	0.957	0.9471	0.068	−0.095	0.103
0.942	0.958	0.9428	0.050	−0.110	0.096
0.941	0.959	0.9382	0.031	−0.125	0.088
0.94	0.96	0.9332	0.012	−0.140	0.081
0.939	0.961	0.9279	−0.007	−0.155	0.073

Table 3. Numerical Example 3 ($c = 0.0029$, $p_G = 0.97$, $p_B = 0.93$, $\theta = 0.5$).

π_L	π_S	p^*	ΔV_B	ΔV_G	V^{mis}
0.95	0.95	1.0074	0.3832	0.1928	0.201
0.949	0.951	1.0056	0.3659	0.1794	0.201
0.948	0.952	1.0037	0.3484	0.1658	0.201
0.947	0.953	1.0017	0.3300	0.1520	0.201
0.946	0.954	0.9995	0.3129	0.1381	0.201
0.945	0.955	0.9971	0.2948	0.1240	0.201
0.944	0.956	0.9946	0.2766	0.1096	0.201
0.943	0.957	0.9919	0.2581	0.0951	0.201
0.942	0.958	0.9890	0.2395	0.0803	0.201
0.941	0.959	0.9859	0.2207	0.0654	0.201
0.94	0.96	0.9826	0.2017	0.0503	0.201
0.939	0.961	0.9790	0.1825	0.0349	0.201
0.938	0.962	0.9752	0.1632	0.0194	0.201
0.937	0.963	0.9710	0.1436	0.0037	0.201
0.936	0.964	0.9665	0.1238	−0.0122	0.050
0.935	0.965	0.9617	0.1039	−0.0284	0.042
0.934	0.966	0.9564	0.0838	−0.0447	0.034
0.933	0.967	0.9507	0.0634	−0.0612	0.025
0.932	0.968	0.9446	0.0429	−0.0779	0.017
0.931	0.969	0.9378	0.0222	−0.0949	0.009
0.93	0.97	0.9305	0.0013	−0.1120	0
0.929	0.971	0.9225	−0.0197	−0.1293	0.009

where we set $c = 0.0029$. At this level of flotation costs, both Good and Bad firms would prefer long debt up to $\pi_L = 0.937$ and $\pi_S = 0.963$. By increasing the difference between π_L and π_S further, investors would observe that Good and Bad firms start to choose different debts until the perfect revealing separating equilibrium is reached. Investors reduce the sum of absolute misinformation value to zero by choosing $\pi_L = 0.93$ and $\pi_S = 0.97$.

Obviously, higher flotation costs may allow investors to increase the differential between π_L and π_S further. However, it is not optimal for investors to do so. An increase in the differential between π_L and π_S beyond the optimal level would only increase the total absolute misinformation value since the perfect revealing condition is optimal for pricing long and short debts.

The preceding examples show that the perfect revealing separating equilibrium is a special case of the general separating equilibrium. As indicated, the perfect revealing separating equilibrium requires the flotation cost to be above a certain level. This result is consistent with previous findings. However, contrary to previous studies, we show that a separating equilibrium can still be achieved when the flotation cost is below this level, or even becomes zero. The separating equilibrium may not be able to eliminate the misinformation value entirely, but its magnitude is always smaller than that in a pooling equilibrium.

5. Conclusions

In a market where rational investors are active participants in the signaling game, their pricing strategy leads to a separating equilibrium of debts with different maturity arrangements in the absence of flotation costs. The interaction of borrowers' incentives and investors' inferences about firm quality results in an informational equilibrium under a much more general condition. Firms can effectively signal their true quality to the market even if financial market transactions are costless. Unlike previous studies, we show that a firm's debt-maturity structure can provide a credible signal in the absence of transaction costs.

Information asymmetry can create a rather complex maturity structure. Our analysis can be easily generalized to the case of multiple maturity structure. In equilibrium, higher-quality firms issue shorter-term debt, resulting in a pecking order of debt financing. Firms of the highest quality issue short-term debt, and firms of the lowest quality issue long-term debt or serial debt. Firms of intermediate quality issue debts of intermediate maturity. Thus, bond ratings should be related to the effective bond maturity *ceteris paribus*. Moreover, to the extent that industries are characterized by different degrees of information asymmetry, there should be cross-sectional variations in debt-maturity structure. Industries with higher information asymmetry will tend to use short-term debt. Conversely, industries with lower information asymmetry would be more likely to follow the asset-liability matching principle to determine the maturity structure of debt (see Demirgüc-Kunt and Maksimovic, 1999; Emery, 2001).

Appendix A

A.1. *Proof of Proposition 1*

Setting the criterion function for the firm's debt financing decision to be zero,

$$\Delta V = p\frac{(1-p)(2-\pi_{\mathrm{L}})\pi_{\mathrm{L}} + (2-p-(2-\pi_{\mathrm{L}})\pi_{\mathrm{L}})\pi_{\mathrm{S}}}{(2-\pi_{\mathrm{L}})\pi_{\mathrm{L}}\pi_{\mathrm{S}}} = 0 \qquad \text{(A.1)}$$

and solving for p, we can obtain the cutoff quality of firm (p^*):

$$p^* = 0, \qquad p^* = \frac{(2-\pi_{\mathrm{L}})\pi_{\mathrm{L}} - \pi_{\mathrm{S}}[2-(2-\pi_{\mathrm{L}})\pi_{\mathrm{L}}]}{(2-\pi_{\mathrm{L}})\pi_{\mathrm{L}} - \pi_{\mathrm{S}}}, \tag{A.2}$$

where $p^* = 0$ represents the case that the firm would go bankrupt for certain, and so it should be ruled out. The coexistence of short and long debts requires that

$$0 < p^* < 1. \tag{A.3}$$

Imposing the condition in Equation (A.3) on Equation (A.2), we have

$$0 < \pi_{\mathrm{S}} < \frac{(2-\pi_{\mathrm{L}})\pi_{\mathrm{L}}}{2-(2-\pi_{\mathrm{L}})\pi_{\mathrm{L}}}. \tag{A.4}$$

By definition,

$$\pi_{\mathrm{L}} \leq \pi_{\mathrm{S}}. \tag{A.5}$$

Combining Equations (A.4) and (A.5) gives

$$0 < \pi_{\mathrm{L}} \leq \pi_{\mathrm{S}} < \frac{(2-\pi_{\mathrm{L}})\pi_{\mathrm{L}}}{2-(2-\pi_{\mathrm{L}})\pi_{\mathrm{L}}}. \tag{A.6}$$

A.2. *Proof of Proposition 2*

Assume $c > 0$, and set the criterion function for the firm's debt decision to be zero:

$$\Delta V = p\frac{(1-p)(2-\pi_{\mathrm{L}})\pi_{\mathrm{L}} + (2-p-(2-\pi_{\mathrm{L}})\pi_{\mathrm{L}})\pi_{\mathrm{S}}}{(2-\pi_{\mathrm{L}})\pi_{\mathrm{L}}\pi_{\mathrm{S}}} + c = 0. \tag{A.7}$$

Solving for p, we can obtain the cutoff quality of firms:

$$p^* = 0,$$

$$p^* = \frac{(2-\pi_{\mathrm{L}})\pi_{\mathrm{L}} - \pi_{\mathrm{S}}[2-(2-\pi_{\mathrm{L}})\pi_{\mathrm{L}}]}{2[(2-\pi_{\mathrm{L}})\pi_{\mathrm{L}} - \pi_{\mathrm{S}}]} + \frac{\sqrt{[\pi_{\mathrm{S}}(2-(2-\pi_{\mathrm{L}})\pi_{\mathrm{L}}) - (2-\pi_{\mathrm{L}})\pi_{\mathrm{L}}]^2 + 4\mathrm{G}\pi_{\mathrm{S}}(2-\pi_{\mathrm{L}})\pi_{\mathrm{L}}[(2-\pi_{\mathrm{L}})\pi_{\mathrm{L}} - \pi_{\mathrm{S}}]}}{2[(2-\pi_{\mathrm{L}})\pi_{\mathrm{L}} - \pi_{\mathrm{S}}]}. \tag{A.8}$$

Again, the case of $p^* = 0$ is discarded. Imposing the condition in Equation (A.3) on Equation (A.8), we have

$$-\frac{2(2-\pi_L)\pi_L}{3-2(1-G)\pi_L+(1-G)\pi_L^2} < \pi_S < (2-\pi_L)\pi_L. \tag{A.9}$$

By definition,

$$\pi_L \leq \pi_S. \tag{A.10}$$

Combining Equations (A.9) and (A.10) gives

$$0 < \pi_L \leq \pi_S < (2-\pi_L)\pi_L. \tag{A.11}$$

Acknowledgments

We thank Richard Green, Kose John, and Lemma Senbet for helpful comments.

References

Bhattacharya, S., "Imperfect Information, Dividend Policy and the "Bird in the Hand" Fallacy." *Bell Journal of Economics* 10, 259–270 (1979).

Campell, T. S. and W. A. Kracaw, "Information Production, Market Signaling and the Theory of Financial Intermediation." *Journal of Finance* 35, 863–882 (1980).

Demirgüc-Kunt, A. and V. Maksimovic, "Institutions, Financial Markets, and Firm Debt Maturity." *Journal of Financial Economics* 54, 295–336 (1999).

Diamond, D. W., "Debt Maturity Structure and Liquidity Risk." *Quarterly Journal of Economics* 106, 709–737 (1991).

Diamond, D. W., "Seniority and Maturity of Debt Contracts." *Journal of Financial Economics* 33, 341–368 (1993).

Emery, G. W., "Cyclical Demand and the Choice of Debt Maturity." *Journal of Business* 74, 557–590 (2001).

Flannery, M. J., "Asymmetric Information and Risky Debt Maturity Choice." *Journal of Finance* 41, 19–37 (1986).

Guedes, J. and T. Opler, "The Determinants of the Maturity Structure of Corporate Debt Issues." *Journal of Finance* 51, 1809–1833 (1996).

Kale, J. R. and T. H. Noe, "Risky Debt Maturity Choice in a Sequential Game Equilibrium." *Journal of Financial Research* 2, 155–165 (1990).

Ravid, S. A., "Debt Maturity — A Survey." *Financial Markets, Institutions & Instruments* 5, 1–69 (1996).

Ross, S. A., "The Determination of Financial Structure: The Incentive-Signaling Approach." *Bell Journal of Economics* 8, 23–40 (1977).

Rothschild, M. and J. E. Stiglitz, "Equilibrium in Competitive Insurance Markets: An Essay on the Economics of Imperfect Information." *Quarterly Journal of Economics* 91, 629–649 (1976).

Stohs, M. and D. Mauer, "The Determinants of Corporate Debt Maturity Structure." *Journal of Business* 69, 279–312 (1996).

Titman, S., "Interest Rate Swaps and Corporate Financing Choices." *Journal of Finance* 47, 1503–1516 (1992).

Wu, C., "Information Asymmetry and the Sinking Fund Provision." *Journal of Financial and Quantitative Analysis* 28, 399–416 (1993).