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Abstract

It has been long recognized that endogenous default probabilities cannot explain spreads between corporate and the riskless bonds. Recently, this issue has been subjected to rigorous scrutiny. Previous studies have found that for investment-grade debt, structural models explain only 15-25% of the observed spreads. On the other hand, for the high-yield debt, the structural models exaggerate actual spreads 1.5-2 times. These findings are perplexing because, while one could argue that factors other than default risk, e.g. illiquidity, can influence the spread for investment-grade bonds, it is difficult to justify the findings for junk bonds. In this paper, we offer an explanation to these puzzling results. Specifically, we account for the differential tax treatment of regular income and capital gains. In addition, we consider the uncertainty of the residual assets due bondholders, as well as the tax liability of the residual asset. We argue that the uncertainty in the claims on the future assets of the company in the case of potential default drives the spreads of an investment-grade debt as much as, if not more than, the probability of default.

Calibration of the Structural Model of Corporate Bond Spreads

There are two approaches for modeling the term structure of defaultable bonds. The structural approach was pioneered by Black and Scholes (1973) and Merton (1974) and extended by Black and Cox (1976), Longstaff and Schwartz (1995) and Collin-Dufresne and Goldstein (2001). This approach provides an integrated framework that simultaneously addresses the issues related to capital structure, default probability and yield spreads. Structural models allow for the pricing of debt where no proxy exists, or the firm's capital structure changes as a result of debt issuance or changing tax regimes. On the other hand, the reduced-form approach (Jarrow and Turnbull (1995) and Duffie and Singleton (1999)) treats corporate bond default as an unpredictable stopping time involving a sudden loss in the market value of the firm. The advantages of the reduced-form approach are its analytical tractability and empirical flexibility to fit observed credit spreads. However, there is no clear link between firm value and default, and so the determinant of the hazard rate of default is unknown. In contrast, the structural approach is often able to generate important economic insights on the behavior of the hazard rate of default.

On the empirical front, neither the structural models nor the reduced-form models are able to explain the corporate bond spread satisfactorily (see Jones, Mason and Rosenfeld (1984), Huang and Huang (2002) and Collin-Dufresne et al. (2001)). Huang and Huang (2002) find that only a small fraction (15-25%) of the spread between Treasuries and investment-grade corporate bonds can be explained by default risk. For lower-quality bonds, the spread calculated from probabilities of default often significantly exceeds actual spreads. Hund (2003) and Ericsson and Remby (2004) attribute the credit puzzle to the statistical misspecification of the structural model. Recently, Liu and Wu (2004), using the reduced-form model, find that the problem of estimating the spread can be significantly reduced if one takes into account tax effects.

The structural approach has a distinct advantage over the reduced-form approach for modeling the tax effect; in particular it allows the tax factor to interact with financing and operating decisions. As such, one can identify the channels through which default risk and taxes affect corporate bond spreads relatively easily. In this paper we adopt the structural approach to incorporate the effects of taxes into the term structure model to analyze their effects on spreads.

Despite its flexibility, there are important insights currently missing from the structural models. Structural models take as their input the historical market data for the company and its financial statements. Yet, as it is the case with equity, the most important drivers for the future value are the market estimate of the value added by the current management (macroeconomists like to refer to this as Tobin's *q*) and the changes in regulatory environment. Both factors are absent in the traditional structural approach. We simulate the influence of these two factors by (1) the uncertainty in the terminal value of the firm in the case of default and (2) different personal income and corporate tax rates.

The influence of the tax structure on the corporate value has been recently affirmed by Graham (2003). In Table 1 of his paper, he displays main changes in the tax regime between 1970 and 2000. For our preliminary study we choose the 30-months period between mid-1994 and 1996. In this period, marginal personal income tax stayed at 36%, the consequence of the tax raise by Clinton, and the corporate tax was the same for income and capital gains at 35%. Capital gains tax for individuals was 28% in 94-95 but fell to 20% in 1996. In the simulations with the assumed tax rates, we obliquely account for the fact of this transition.

Our model incorporates many features of Longstaff-Schwartz (1995, hereafter LS), Anderson-Sundaresan-Tychon (1996, hereafter AST) and Mella-Barral and Perraudin (1996, hereafter MP) models, all of which sprang from the classical work of Merton (1974). As we will demonstrate later, these models have deep conceptual differences despite their formal affinities and almost identical formulaic expressions. Unlike these models, we incorporate taxes in the pricing of corporate bonds.

We find that our tax model performs much better than previous models. In our model, shorter-maturity investment-grade bond yields can be accurately fitted by the asset volatility not exceeding 15%. Applying the fitted parameters from the short-term bond series to the longer-maturity bonds of the same firm explains 30-40% of the spread by default risk, which is considerably higher than the estimates of the previous studies. If we fit the tax rate and recovery ratio for the long-term bonds directly, we can almost fully explain the average size of the spread. Longer-maturity junk debt can also be fitted quite successfully.

The remainder of this paper is organized as follows. In Section 1, we develop a simple formula for valuing corporate debt. The main feature of this formula, as in all endogenous formulas, is the existence of the "trigger value" for default. In Section 2, we provide the expression for the trigger value using the correspondence principle of the AST and MP theories. In Section 3, we conduct simulations to explain the behavior of spreads. In Section 4, we calibrate the model with AT&T and Revlon bonds. In Section 5, we outline applications of the structural model to credit-risk modeling. Finally, we summarize major findings and conclude the paper in Section 6.

1. Expected value of risky debt

We start from a different premise than previous studies using the structural approach. Previous studies typically begin with calculating (or postulating) the trigger value for default and then explore the properties of corporate debt. On the contrary, we propose a heuristic formula for valuation of corporate debt and then derive the default trigger value. We favor this approach because double taxation of corporation influences debt in several ways. First, taxes influence the potential value of the firm for creditors in the case of default. Second, they alter the net cash flow. Third, because the potential value of the firm to creditors is different from that in the no-tax case, the boundary for the corporate default is shifted as well.

In formulation of their model, AST (1996) propose a game-theoretical framework to derive the equation for debt pricing. Our framework is different due to the presence of taxes and additional stochastic risk factors with respect to asset volatility. Yet, it is constructed along a similar line. We show the two-stage (default-no-default) game in Figure 1.[2] According to the probability structure in Figure 1, we decompose the price of the risky debt D_0 at time t=0 into three components:

$$D_0 = F_T - CB(W, W^*; T) + S(W, W^*, T) \tag{1}$$

where F_T is the price of the riskless debt with time T to maturity (e.g., U.S. Government bonds), $CB(W,W^*,T)$ is the loss (cost-of-bankruptcy) to bondholders in case of default,

[2] We can compare this figure with AST's Figure 1.

and $S(W,W^*,T)$ are the gains resulting from the acquisition of assets of the impaired company by its creditors. In (1), W is the firm's asset value, W^* is the trigger value initiating the bankruptcy procedure and T is the time to maturity. We impose the following assumptions: first, we assume a flat term structure of U.S. Treasuries at an average value for the observation period r, and consider only one category of zero-coupon debt, and second, we consider both capital gains and ordinary income taxes. Assumption (1) will be replaced in subsequent sections with actual data from observed U.S. Treasury rates, which we treat as exogenous parameters of the model.

The present value of the riskless zero-coupon debt is

$$F_T = F_0 \exp(-r \bullet T) \tag{2}$$

where F_0 is equal to the face value of the firm's debt and T is time to maturity. Riskless debt value is not affected by taxes by assumption (2).[3]

To quantify two other terms we need to establish what exactly happens at the moment of corporate bankruptcy. The answer may seem self-evident but different theories provide divergent answers to this question.

The value-based AST theory conceptualizes a "strategic default", which is triggered by stockholders to extract concessions from bondholders in terms of cost of debt servicing.[4] Because of this possibility, bondholders require a premium, which manifests itself in a spread between risky and riskless debts. In the income-based MP theory, the concept of strategic default is also present but they introduce explicit parameters to describe the operation of the company after default. In this theory, there is a potential difference between liquidation bankruptcy and operating bankruptcy, which we shall ignore.[5] The practical trigger of the default in the MP model is the fall of its

[3]Correct formulation of Equation (2) would be $F_T = F_0 \exp(-rT)[(1-\tau_i)+\tau_c]$ where the first term reflects investor's income tax and the second term—contribution of tax shield from interest to corporate income tax but we propose $\tau_i=\tau_c$. Moreover, the implicit tax (Scholes and Wolfson (1992)) is incorporated in observable Federal interest rate, which we treat exogenously. This sterilizes taxation effect on riskless debt, as we expect it would.

[4] See AST (1996), p. 878.

[5] MP introduces two ad hoc factors to describe this situation: $\xi_0<1$—the reduction in output and $\xi_1>1$—the increase in costs. Contrary to naïve intuition, operating default occurs at a much higher level of debt, because at a higher level of debt, bondholders have more to lose in case of bankruptcy and thus are eager to negotiate better conditions for equityholders.

equity value to zero.[6]

We adopt an alternative static concept of default. It assumes neither the exact reason for default nor its precise timing. The firm, which issues the debt, simultaneously buys an implicit option to stop interest payments if its equity value V falls below some trigger value typically considered as zero. This reduces the value of risky debt. This view of credit spread is similar in spirit to the Longstaff-Schwartz (LS) model. However, the firm also sells the option to bondholder to acquire its assets with all its profitable uses under the same conditions in return for relinquishing its rights for receiving interest. From bondholders' the point of view, they acquire an option to swap their claim of ownership of corporate debt to the ownership of corporate assets net of bankruptcy costs in the event of default.

Thus, the value of the second term in (1) can be modeled as a F_T units of binary put[7] on assets $W = V + D_T$ with the exercise price equal to the trigger value W^* which we shall calculate in Section 2. In the case when the firm value falls below the trigger value W^*, the F_0 face amount of bonds in their hands becomes worthless, i.e., at the exercise of the option, bondholders lose F_T:

$$CB(W, W^*; T) = F_T P_B^{(1)}(W, W^*, T) \tag{3}$$

The term $P_B^{(1)}(W, W^*, T)$ has the intuitive meaning of the default probability. As it will be shown later, the value of W^* is affected by the capital gains tax rate.

The value of the third term in (1) is the price of another binary option,[8] with the same exercise price multiplied by the fraction (δ) of corporate assets, which the firm owes to bondholders:

$$S(W, W^*, T) = \delta P_B^{(2)}(W, W^*, T) \tag{4}$$

The "number of units" of this put is equal to the corporate value W. The third term constitutes the bondholder's gains on the acquisition of assets of the defaulted firm and is subject to the capital gains tax rate.

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[6]Duffie and Lando (2001) treat the firm's value in the spirit of the agent theory as resulting from an information asymmetry between stockholders (owners, insiders) and bondholders (outsiders). In their approach, bankruptcy eliminates this asymmetry because creditors become the insiders after default. Spreads are explained in terms of the uncertainty in accounting information. An intrinsic feature of their theory is the stochastic asset density and its unobservable distribution is driven by the so-called "accounting noise".

[7] Hull (1997) calls this binary *cash-or-nothing put*.

If we try to fit this naïve paradigm into an option-based valuation, we can readily ascribe appropriate tax rates to the terms in (1). Namely, the symbolic expression for the market value of corporate debt is the following (compared with Schönbucher (1998)):

$$D_t(1-\tau_c) = E_t^Q\left[F_0 e^{-\int_t^T r_s ds} 1_{t\geq T}\right](1-\tau_c) + E_t^Q\left[(-F_0+\theta W)^+ e^{-\int_t^T r_s ds} 1_{t<T}\right](1-\tau_{cg}) \qquad (5)$$

where τ_c is a corporate tax rate and τ_{cg} is a marginal personal tax rate on capital gains. We shall discuss this expression in more details in the Section 2, in relation with the AST and MP models.

Note that initially we assume no pre-set analytic formula for the prices of binary options. Nor do we propose that debt price D_t obeys any explicit differential equation. Valuation of binary calls can be influenced by a particular analytic model, and the condition of exercise (European, or more realistically American, with different covenants or indentures). Below we shall use conventional Black-Scholes option valuation but we emphasize that the BS-type formulas are invoked simply for the purpose of analytical tractability.

The analytical BS form for the cash-or-nothing put is given by (Hull (1997)):

$$P_{BS}^{(1)} = e^{-rT} N(-d_2) \qquad (6)$$

where $d_2 = (\log(W/W^*)+(r\text{-}Div\text{-}\sigma^2/2)T)/\sigma T^{1/2}$, for the tax-free case, Div is the dividend yield and σ is the asset volatility. We immediately notice that the value of the argument of the cumulative normal distribution coincides with the *distance-to-default*—a metric introduced heuristically by KMV corp.—under the risk-neutral measure (see Lando, (2004)).[9]

Similarly, the formula for the asset-or-nothing put can be written as

$$P_{BS}^{(2)} = We^{-rT} N(-d_1) \qquad (7)$$

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[8] Hull (1997) calls this binary *asset-to-nothing put*.
[9] KMV corp.—San Francisco-based provider of risk analytics.

where as usual, $d_1 = d_2 - \sigma T^{1/2}$. Equation (7) does not account for an important effect. Namely, in the traditional structural approach, the default boundary is determined by the market value of the company. However, in the event of default, the market value of the company far from default is meaningless. What matters here is the liquidation value.

In the tax-free world with $\theta = 1$ (complete transfer of firm's assets to bondholders), using Black-Scholes pricing formula, we can reduce Equation (5) to the familiar expression for zero-coupon debt (Longstaff and Schwartz (1995), Collin-Dufresne and Goldstein (2001), and Das (2001)):

$$D_t = F_{T-t} - p_{BS}(W, W^*, T-t) \tag{8}$$

where p_{BS} is a price for Black-Scholes plain vanilla put with the exercise price equal to the trigger value W*.

As indicated in Figure 1, (5) simply means that in the event of default, the bondholders become de-facto equityholders because of their primary claims on assets. However, this is not free. They lose the face value of the now-worthless debt, but acquire some uncertain θ portion of the asset and pay capital gains taxes (or receive a tax rebate) on the difference. Capital gains tax rate for individuals is lower than the ordinary income tax rate and corporate income tax rate. A natural assumption for the recovery process θ is that it is independent from the corporate asset process and obeys the following equality:

$$E^Q[\theta \mid I_0] = \delta$$

where I_t is the information available by time t and t^* is the (unspecified) time of default.

We provide the following example to emphasize the practical importance of the latter consideration. Imagine two companies with an identical capital structure, but while the first is prospering, the second is failing. Obviously, the former one has a good credit rating (and consequently, a low credit spread) and the latter has a bad credit rating (high spread). Yet in the event of default, they offer a comparable value to their creditors if we ignore the recovery factor δ, which has been already taken into account in (4). Why this happens? Intuitively, the residual assets are indifferent to the credit rating assigned in the past. In the quantitative finance paradigm, default of a "good" firm is a more informative event than a default of a "bad" firm and hence, reduces more corporate value.

In the consistent approach, we should price the second term in (5) as a compound option:

$$E_t^Q\left[(-F_0+\theta W)^+ e^{-\int_t^T r_s ds} 1_{t<T}\right] = E_\vartheta^Q\left[\left[E^Q(-F_0+\theta W)^+ e^{-\int_t^T r_s ds} 1_{t<T}\right] \middle| \theta=\vartheta\right]$$

Instead, we replace the price of the compound option with the price of the plain-vanilla call multiplied by an adjustable parameter β_V, which reflects the difference between liquidation and market values of the company:[10]

$$D_t = F_t - p_{BS}(W, W^*, T-t) + \beta_V c_{BS}(W, W^*, T-t)\frac{(\tau_c - \tau_{cg})}{1-\tau_c} \tag{9}$$

The meaning of (8) is as follows. Because personal capital gains tax rate is typically lower than corporate income tax rate, bondholders benefit from a tax rebate when bondholders, not the company, liquidate property. In Section 3, we shall use Equation (1) with terms provided by Equations (2, 5-9) to estimate the yield of the risky debt. For the demonstration in Section 3, we shall use $\beta_V = 1$. For empirical tests, we shall treat β_V as an adjustable parameter. However, we expect its value to be close to

$$\beta_V = \frac{\overline{BV}}{\overline{MV}}$$

where BV is an average book value and MV is an average market value for the period.

Our formulation of the problem still lacks a definition of W^* which we shall provide in the next section. As shown in Figure 1, the parameter β_V is equivalent to a linear approximation of transformation of the outcomes tree into a riskless frame. This choice is not unique.

2. Trigger value for corporate default

While in formulating the framework for corporate debt we follow the LS model, we shall use the structural approach of AST and MP to calculate the trigger. In Huang and Huang (2002), Anderson-Sundaresan-Tychon and Mella-Barral-Perraudin models are grouped together. In fact, despite the look-alike equations, these two models, as well as Longstaff-Schwartz model and their common predecessor—Merton's classical model (1974, hereafter M74) are different in very substantive aspects. There are two main

[10] We do not know anything about the character of the process governing evolution of θ and its specification would be an imposition.

conceptual distinctions among the AST, MP, LS and M74 models. Below we formulate a synthetic approach, which combines the features of both models.

As we have seen, the AST model is based on the Bellman equation for the debt price with the firm value as a dependent variable. This equation can be deduced from the following set of equations on W, the firm value, and $Y = B(W,t)$, the value of debt. Next, we introduce taxes into the Merton framework:

$$\begin{cases} dW = (\alpha W - C)(1-\tau_c)dt + \sigma(1-\tau_c)^{1/2} W dz \\ dY = (\alpha_Y - \beta_Y(1-\tau_i))Y dt + \sigma_y (1-\tau_{cg})^{1/2} Y dz_y \end{cases} \tag{10}$$

where α is the return on assets, C is the (taxable) payout to the stockholders and debtholders, α_Y and β_Y are the expected return and the dollar payout rate of the debt, respectively. Here, we add an individual income tax τ_i. Our goal is to determine W^* -- the value for which company would be unable to pay coupon on its debt. The ordinary income tax rate is applied to firm's profits and debt payouts.

Modeling of the corporate debt value by the second equation in (10) is inconsistent. In general, we do not know, whether debt value Y obeys any closed-form stochastic differential equation. Thus our definition of trigger will be obtained in a simplified framework more reminiscent of the reduced-form approach.

In Merton's model, α_Y and σ_Y are unobservable state parameters and they are calculated in an after-tax gauge. The standard deviation of the firm value is scaled as a $(1 - \tau_c)^{1/2}$ because we can express dz as $\varepsilon(dt)^{1/2}$ where ε is a normally distributed variable with a unit standard deviation and the time value of money is reduced by the factor $(1 - \tau_c)$. If we follow Merton's derivation, we obtain the following equation:

$$\frac{F_t}{1-\tau_c} + \left(\beta_y \frac{1-\tau_i}{1-\tau_c} - r\right)F + (rW - C)F_W + \frac{1}{2}\sigma^2\left(\frac{1-\tau_{cg}}{1-\tau_c}\right)W^2 F_{WW} = 0 \tag{11}$$

Here, we must caution that rescaling of the asset volatility for tax factor is redundant for empirical tests because the observed volatility already incorporates tax effects but it is absolutely necessary for consistency when simulating variable tax regimes. We seek the solution of the stationary Bellman equation in a conventional form:

$$F(W) \propto W^{\gamma} \tag{12}$$

If we use the AST approximation ($\beta_Y B = \beta F$, for $W > W^*$)[11], we arrive at the following algebraic expression for it roots:

$$\gamma_{1,2} = -\left(\frac{(r(1-\tau_c) - \beta(1-\tau_i))}{\sigma^2(1-\tau_{cg})} - \frac{1}{2}\right) \pm \sqrt{\left(\frac{(r(1-\tau_c) - \beta(1-\tau_i))}{\sigma^2(1-\tau_{cg})} - \frac{1}{2}\right)^2 + \frac{2r(1-\tau_c)}{\sigma^2(1-\tau_{cg})}} \tag{13}$$

To calculate the trigger value, we start with the equation of the MP model for the value

$$rV(p_t) = p_t - w - b + \frac{d}{d\Delta} E_t(V_t)\bigg|_{\Delta=0} \tag{14}$$

where p_t is the price of the firm's output, w is current non-interest expenses and b is the interest expense. We can interpret the left hand side as a risk-free return on equity (thus, needed to be multiplied by ($1-\tau_c$)) and the first three terms on the right-hand side as cash flow to equity. In a taxable framework, corporate tax rate is applied to net cash flow to equity, which is represented by first three terms in Equation (14). The expected value consists, by Ito's lemma, of two terms: the expected growth of the asset and the diffusion term. Utility of cash flows from dividends (coupons) for the investor is multiplied by the ordinary income tax rate τ_i, while the drift term is multiplied by capital gains tax rate τ_{cg}. The resulting equation is

$$r(1-\tau_c)V(p) = (p - w - b)(1-\tau_c) + \mu(1-\tau_i)pV'(p) + \frac{1}{2}\sigma^2(1-\tau_{cg})p^2V''(p) \tag{15}$$

where τ_{cg} is a capital gains tax rate. Equation (15) results in the same characteristic equation for the parameter γ, as does the W-dependent Equation (11) in the AST version. If we seek the solution of (15) in the form:

$$V(p) = A + Bp + Cp^{\gamma} \tag{16}$$

with A, B and C being indeterminate coefficients, we get the formula for the trigger value defined as $W^* = W(p_b)$, where p_b is a root of an equation $V(p_b) = 0$. When we use Table 1 for identification of parameters, we obtain

[11] Approximation that the current yield on the bond is equal to its yield to maturity.

$$W^* = \frac{1}{1-1/\gamma_2}\left(\frac{c(1-\tau_i)F_0}{r(1-\tau_c)} + K_2\right) \quad (17)$$

Together with (13) for $\gamma_2(\tau_c,\tau_{cg})$, the negative root of the Bellman equation corrected for the taxes and K_2, the fixed cost of bankruptcy, this expression formally solves the trigger value problem in the presence of taxes. We now proceed to the calculation of W^{**}, the terminal value of the firm in the presence of incomplete recovery ($\delta < 1$).[12]

In the case of a complete recovery, $W^{**} = W^*$. However, a typical recovery of corporate assets is incomplete and strongly depends on the seniority of the debt (see, for example, Fabozzi et al. (2001)). Instead of the AST formula,

$$W^{**} = W^* + (1-\delta)W^{**}$$

and consequently,

$$W^{**} = W^* / \delta \quad (18)$$

We have now the tax-modified expressions

$$W^{**} = W^* + (1-\delta(1-\tau_{cg}))W^{**}$$

and

$$W^{**} = W^* / \delta(1-\tau_{cg}) \quad (19)$$

The capital gains tax shield, available to the creditors in case of default, raises the strike price of the put option to default, which reduces the price of debt but it also increases the payout of the post-default collections. The expected value of the firm's debt is influenced by the capital gains tax rebate even when the firm is far from default. This influence, as we shall see, is typically negative for the good-quality credit and positive for the junk debt.

3. Qualitative simulations

Equation (1) with parameters defined by Equations (2, 4-7) and (18) provides an implicit algebraic equation (both r.h.s. and l.h.s. contain D_T) for the value of corporate debt. This equation can be solved through the iteration procedure for the deterministic

[12] The above-cited correspondence principle between MP and AST allows us to express K_2 as $K_2 = \left(\frac{\beta}{r} - \frac{(1-\gamma_2)\tau_c}{(1-\tau_c)\gamma_2}\right)$, but we use adjusted book value (see Appendix) as a proxy for the bankruptcy cost.

function $V(t)$—equity value of the corporation. In the simulation, we treat interest rates and volatility as constant. We solve the equation for certain values of model parameters. In most cases, we use the following values to price the options: r = 5%, Div = 2%, β = 3%, w = 1.5 (operational expenses; capitalized value of current expenses w/r = 30% of the face value of the long-term debt), δ (recovery factor) = 0.6. Tax rates are τ_i = 35% and $\tau_{cg} = \tau_c$ = 20%. The face value of the debt is 100 and equity value at t = 0 is 80. Stock price is a linear function of time growing at 13.4% (positive outlook) or 3.6% (negative outlook).

In Figure 2, we plot two pairs of yield curves: one for the positive and the other for the negative stock outlook with and without the capital gains tax. All the curves have the humped shape, a characteristic for the LS-type models, but yields drastically decrease, especially for shorter maturities, if we eliminate the capital gains tax. This happens because in the absence of taxes, there is little difference between debt and equity as components of the capital structure. The higher is the tax rate, the larger is the potential tax rebate in the case of default. Because, all else equal, this implies higher company value at default, the probability of the default increases. For longer maturities, this effect dominates the rescaled volatility (see Eq. (11)), and the slope of the curve without capital gains taxes becomes more gradual than for the taxable yield curve.

Figure 3 illustrates the effect of the quality of debt. The control parameter which we use to change the debt quality is the debt-equity ratio: low for investment grade bonds and high for junk bonds. We observe that our simple approach reproduces a qualitative shape of the curves as a function of maturity if we identify the first curve with investment-grade bonds, the second curve with speculative-grade bonds and the third curve with junk bonds. Namely, the junk-bond spread diminishes with time to maturity, the investment-grade bond spread increases and on the borderline between investment-grade and junk bonds, the spread as a function of time to maturity has a characteristic humped shape (Das (2001)).

In Figure 4, we demonstrate the effect of adding taxes on spreads in the structural model. For the speculative-grade debt, between 20% and 40% of the spread is explained by the tax effect. In Figure 5, we demonstrate the fact that the influence of taxes varies with debt quality. For high-grade debt, almost all of the spread is explained by the residual value of the corporation in the case of default. For intermediate-grade debt, the tax-related portion falls off. For junk bonds the advantage of capital loss tax rebate in the case of default outweighs the expected loss of face value and decreases the spread. The relative influence of taxes on spreads may change with maturity for intermediate-grade debt. This conclusion agrees very well with the results of a tax-free treatment of Huang and Huang (2002): the higher the grade of debt, the smaller portion of the spread could be explained by default probability. On the contrary, default-inferred yields of low-quality debt were higher than the actual by as much as -146% (see also Huang and Huang (2002), Table 3). This premium on a low debt quality seems irrational in a tax-free framework.

For taxable companies without much probability of default, taxes decrease the payout to investors. The implicit collection/rebate option is out of the money and does not influence the bond price. This increases the spread. For low-grade bonds, the option to take over the company is close to be in the money, which, in the case of low market value of the expected payments, increases the value of the debt for the bondholder.

4. Calibration

A. Investment-grade bonds

For model calibration, we use thirty monthly yields of AT&T corporate bonds between June 1994 and December 1997. AT&T has a long history of corporate bond issuance. AT&T bonds are relatively liquid and there are several series of bonds. We compare their yields with the actual interest rate curve for the T-bonds with the same time to maturity. To build the synthetic T-bond we use the GovPX data on constant maturity Treasuries rates of which we linearly interpolated between quoted maturities. In the present analysis, we choose not to model the riskfree rate so as to focus on the effect of taxes. The results are shown on Figures 6 and 7.

We attempt to fit the spread curve using two adjustable parameters: asset volatility and β_V (proxy for average market-to-book value, see Equation (9)). We extract corporate parameters as averages from the AT&T balance sheets and income statements for the same period (1994-1996). For an explanation of the parameter values, see Appendix B.

We first use average riskfree interest rate and average share value to fit the spread curve. We also use actual riskfree interest rate and average share value, reflecting the nonlinear response of the spread to the shocks of the riskfree rate. Fitted asset volatility is roughly 15%, which still seems too high. Yet, the fit without considering taxes would provide volatility of 20-30%. Our observation is, that while it is possible to fit the structural model to the actual spread, it requires excessive volatility as reported by Collin-Dufresne (2001). However, the inclusion of taxes allows us not only to reduce this volatility very significantly (by about 50%), but to provide a natural explanation of the anomalous behavior of low-grade debt documented by Huang and Huang (2002). The results of our simulations are displayed in Figure 8. The first approximation reproduces average behavior of the spread much better and the second approximation demonstrates some correlation with the peaks and troughs of the observed spreads for the periods longer than 8-9 months to expiration.

To evaluate the quality of our calibration, we use the parameters obtained from the 001957AC bond series to calculate the implied spreads on the bond series of 001957AD, 001957AH, 001957AE, 001957AF, and for test purposes, the BellSouth bond 079857AA in the same 30-month period. Of course, each bond series could be better fitted with its own set of parameters, but this would devalue the structural model because underlying all of these bond series except one is the same corporation. The mean square errors for σ and β_V, which were fitted from the bond 001957 AC are displayed in Table 2.

Table 2. Summary of fitting errors

Bond series	Mean square errors (%)	MSE/Average spread (%)	Notes
001957AC	0.110	35.6	Benchmark bond
001957AD	0.0130	35	
001957AH	0.0367	65.7	
001957AE	0.0316	66.3	
001957AF	0.0349	71.3	
079857AA	0.0082	22.2	BellSouth bond

We show the match of the average-interest rate approximation (the same as used in solid curve, Figure 8) in Figures 9 and 10. In Figure 9, we group together bonds with remaining maturities 0-17 months at the end of the period, which we shall name a "short-term" group. We observe that the average spread is reproduced more or less accurately, while the shape does not fit as well. One of the results is that our structural parameters fit BellSouth bond very well, which indicates that there is not enough distinction in the market between corporate bonds of similar risk and industry, because the balance sheet and income statement parameters for BellSouth were different from that of AT&T.

Longer maturity bonds (28-51 months to maturity at the end of the period) follow a different pattern in Figure 10. The central tendency of the spreads to increase gradually with decreasing time to maturity is retained but the numerical value of the spread is barely 40% of the real value.

We propose a very simple solution to a long-standing structural model spread puzzle. If we fit another series of bonds, 1957AF, with three adjustable parameters: σ—the asset volatility, β_V—the risk-adjustment parameter for asset collection and τ_c – the implied corporate tax rate, we get an overall agreement of the spreads with the following values of parameters: $\sigma = 13.1\%$, $\beta_V = 0.093$ and $\tau_c = 19.45\%$ (see Fig. 11). We note that the difference in new asset volatility is not significant and probably can be explained by volatility smile absent in our model. The implied corporate tax rate is close to the 20%. This agrees well with some observations that implied corporate tax rate is much lower than the maximum statutory tax rate.

Another adjustable parameter which is much different from the fitting of the 1957AC series of bonds is β_V, which is reduced from its value of 0.159 for the near-maturity bonds almost by half. Remembering the heuristic meaning of the parameter β_V from the end of Section 1, we can formulate our conclusion in the hypothetical form. Larger spreads than those predicted by default probability in structural models for the investment-grade bonds are due to the: (1) uncertainty in tax rate at the time of maturity of the corporate bonds and (2) uncertainty in assets that could be claimed by bondholders at default. The first consideration decreases the advantages implicit in the tax shield from debt. The uncertainty in the claimable assets is also larger for investment-grade bonds than for the junk debt because investment-grade bonds are likely to be issued by firms with a high M/B ratio. This ratio reflects information asymmetry between the value of corporate assets implied by market participants (market value) and the accounting residual (book value). In other words, for the "good" company, the market presumes that its managers add a lot of value, which will evaporate in the unlikely event of default while for the "bad" company management-added value is small or even negative.

B. High-yield bonds

As an example of a high-yield bond, we select Revlon Worldwide bond series 76154KAB for the same 30-month period (06/1994-12/1996). The situation with Revlon during this period was highly irregular: the company had negative book equity. Revlon's stock was publicly quoted beginning in 1996. Consequently, this calibration test is of a hypothetical nature because in theory the corporation with negative book equity should cease to exist, and certainly cannot have any market value. Otherwise, this would imply a negative market-to-book ratio.

We use the following non-adjustable structural parameters, which we infer from the balance sheet and income statements. If we assume average long-term debt for Revlon during 1995-1996 as 100%, the 1996 market cap of Revlon is 74% and its book value is -37%. Company paid no dividends and the payout ratio (0.75 for 1996 and 1.108 for 1995-1996) was irrelevant. We calculate the market cap from the numbers of shares outstanding 1994-1996 from the balance sheet (the income statement uses a different number of outstanding shares for year 1996, which closely agrees with the number of shares on the balance sheet). The provision for income taxes from Revlon income statement implied an abnormal 51% tax rate for 1996 and comparable dollar figures for years 1994 and 1995, despite the fact that Revlon lost money during these years! Certainly, the company was highly distressed and the market gave a junk bond status for its debt.

Nevertheless, for our simulations we apply standard 35% corporate tax rate for Revlon and 20% capital gains tax rate for the Revlon investors. During this period, we observe a significant spread (up to 1200 bp) between different categories of Revlon debt. Revlon Consumer Products and Revlon, Inc. had better credit quality than Revlon Worldwide and the spreads between two former groups could be positive as well as negative. We do not know which institutional reasons singled out Revlon Worldwide bond as especially risky. We assume the recovery rate δ = 20%, which is typical for junior subordinated debt (see Das (2001), Figure 6.10).

The best fit for asset volatility was σ = 44% and β_V = -5.80. Here, very high asset volatility is not so surprising because we deal with a company with a theoretical leverage higher than 100%. The large and negative value for β_V, in principle, correctly indicates (see the end of Section 1) that the bondholders expect that the asset value, to which they can lay claim after the bankruptcy, is negative[13], but this interpretation is purely speculative. If we stretch this interpretation to the limits, because of the numerical value $\beta_V\delta = -1.16 \approx -1$, investors could have expected that, before the default, the management may split the company with the Revlon Worldwide obligations being transferred to the "poorer sister" of the corporate twins.[14] We note here that in the case of junk bonds with substantial probability of default within a predictable horizon, there is much less uncertainty with respect to the corporate assets that can be collected as well as the accrued tax benefits.

5. Estimation of asset volatility of the real companies

Asset volatility of the real companies, despite the ubiquity of the concept, is rarely estimated though the methods to do it are taught now in every course of corporate finance. Namely, any corporation can be represented by a fictitious undiversified balanced mutual fund consisting of three types of investments: risk-free investments (cash and short-term assets and liabilities), bonds (its own debt) and stock (its own equity: stock and retained earnings). This statement has been rigorously proven by Hellwig (1981). The volatility and covariance of each type of these assets/liabilities can be calibrated with historical data and added up according to the conventional statistical formula for variance. The weights for these types of assets can be extracted from a balance sheet analysis. We can deduce the percentage weights of cash and cash equivalents, debt and debt equivalent and equity (retained earnings are ascribed the volatility of company's stock). If we perform these calculations, we estimate AT&T asset volatility as 9% and Revlon asset volatility as 50.3% (the parameters of our estimates are listed in Appendix C). With our analytic model, we need an implied volatility of 13-15% for AT&T and 44% for Revlon for the best fit. Thus, for investment-grade debt, our

[13] Because of negative capital, the bondholders were expected to inherit obligations to the owners of higher seniority debt and to other claimants rather than acquire capital assets from the bankruptcy.
[14] Marriott used this tactics in 1993 and the memory was still fresh in investor's minds in 1994-1996. For background on the Marriott case, see Parrino (1997) and the references therein.

simulations reduce the mismatch of implied asset volatility from 2.5-3 in the most previous applications of the structural model to 50%. For junk bonds, our implied volatility is even smaller than its empirical value. We found this fact as re-assuring for the future of structural models.

6. Conclusion

In this paper, we propose a solution to the spreads puzzle in a structural model. Previously, it was considered that realistic default probabilities are too low to explain the spread, especially for investment grade bonds. Our explanation is the following: we must take into account the uncertainty not only of the asset volatility, but also of an implied tax rebate from default and an expected value of collectible assets several years from now. The latter two causes of risk are absent for the near-maturity bonds as well as for the junk bonds for which the time of expected default is near.

Introducing taxes allows us to reduce the implied asset volatility of the structural model from unreasonable 25-30% to a somewhat exaggerated value of 13-15%. Moreover, tax and seniority effects explain the overshooting phenomenon for corporate bonds observed by Huang and Huang (2002). Huang and Huang observe that for low-quality bonds, default-based models overestimate the observed spread. Our model avoids this problem. We argue that this is a result of the implied probability of a takeover when the creditor can (1) acquire the assets of a distressed company in the case of bankruptcy and (2) receive tax rebates from writing down of the debt. Companies with poor credit quality (high yield, and low equity coverage of the trigger value) have a higher expected value of these residual assets for bondholders than companies with good credit.

For investment-grade bonds, we fit the parameters of the structural model to the shortest maturity bond. The mean square error of this fitting is 0.1% or 0.17% dependent on a particular approximation. With these parameters, we calculate the spread for other bonds. For bonds with shorter time to maturity, the MSE is between 0.08% and 0.13%.
The ratio between actual spread and MSE is 22-36%, which is somewhat indicative of unexplained spread variation. If we apply the parameters from the fitting of the short time-to-maturity bond (0-27 months), they reproduce only 30-40% of the spread for the longer-maturity investment-grade bonds (82-51 months). If we adjust corporate tax levels

and the uncertainty in value of assets, available for collection by the bondholders, for the long-term separately, we can completely explain the magnitude of the spread.

For high-yield bonds 70-80% of the yield can be explained without any adjustments for collection rate and the implicit tax rate. Even in our case of highly distressed nature of company's equity,[15] our structural model seems to be sufficiently robust to justify its application.

[15] During the period of study, Revlon had negative book equity which contradicts conventional wisdom of financial theory.

Appendix A

Both AST and MP models involve the same Bellman equation, which is frequently called Black-Scholes in the context of finance, but for different variables. In the AST model, following the original treatment in M74, the Bellman equation was written for a bond price $B(V,t)$ as a function of time and a corporate equity value. In the MP model, a similar equation is deduced for the value V as a function of the price of firm's output (of a single good) p. Default is triggered by $V(p_t) = 0$ and expressed in terms of output price. For practical purposes, output prices are unobservable. Even single-commodity producers, for which market prices are publicly available such as power plants, sell their output in a number of spot and derivative transactions and one must have the information on the entire portfolio to calculate the weighted average price.

The second difference is that the MP model is cashflow-based, while the AST model is value-based. Consequently, income tax is easier to be incorporated into the MP framework and capital gains tax is easier to be included in the AST model. Below is the table of correspondences between the AST and MP models.

Table 1. Correspondence between the parameters of AST and MP models

AST	MP	Notes
cF_0	b	Self-evident
K	$(r-\mu)w/r^2$	If debtholders trigger bankruptcy they receive expected return on the riskless value of its liabilities and accounts payable w/r
W^*(trigger value for $\delta=1$)	p_b/r	See the text below
W^{**} (trigger value for $\delta<1$)	N/A	See the end of the section for tax-induced corrections

Here, c is the coupon, F_0 is the face value of the debt (which they assume to be a perpetual bond),[16] b is the cost of debt service, μ is the expected growth, w is the operational expense and p_b is the trigger price for the firm's output below which it cannot perform its debt obligation. To introduce taxes into the structural model framework, we use the "correspondence principle" under which the AST and MP models must agree in the sense that the expression for taxes, which is introduced in AST framework, must be true also in the MP framework.

[16] There is a legitimate question: to what extent we can apply the AST and MP results obtained for perpetual bond for real-life studies. We can postulate that we approximate preferred stock of the company by a perpetual bond and assume that the company declares bankruptcy when it ceases to pay dividends on the preferred stock. The difference between this definition and the actual default can be subjected to empirical verification.

Appendix B

Our model demands as inputs: (1) dividend yield, (2) payout ratio, (3) tax rates, recovery ratio, (4) K_2 (liquidation value of assets) and (5) V_0—market value of equity. Inference of an average dividend yield and the payout ratio from the balance sheet and the income statement is relatively straightforward. For taxes, we use tax rates of 35% for the personal income tax rate and 20% for corporate taxes. Recovery ratio is not directly observable and we assume it equal to 42%. For comparison, according to the Moody's report, in the period of 1977-1998, average recovery rate for all public debt was 45.02%, for all senior categories of unsecured debt was 40.67% and for the average defaulted bonds during 1974-1995 was 45.26% (see Das (2001), Figures 6.10, 6.11).[17]

We use the book value of the firm net or gross of capital leases, which we identify with the category of "other (non-current) debt" on the AT&T balance sheet for the proxy of liquidation value of assets K_2. We use all non-current debts of corporation net of tax items, as a proxy for the face value of debt. This presumes that: (1) coupon bonds are currently traded at par, and (2) deferred taxes and investment tax credits cannot be inherited by new owners. We calculate this value net or gross of capital leases consistently with their inclusion in K_2. The difference in the values calculated by neglecting or including capitalized leases is not significant and we take an average of their values.

For the market value of equity, we use the average of end-of-month stock price ($77.42 per share for the whole period) multiplied by the average number of shares outstanding during the respective calendar year. For the value of a long-term debt, we use averages from 1994-1996 balance sheets, mainly bonds outstanding and capital leases.

We compute parameter $\beta_{V,}$ by minimizing the sum of squared errors of fitting the calculated to observed yields, as 0.242. In Section 1 we notice that β_V should be close to the book-to-market ratio for the firm. Average book-to-market ratio for the years 1994-1996 was 0.159, the agreement which we find acceptable.

[17] Loans secured by banks or collateralized by equipment have higher recovery rates, but this is not indicative for the pricing of publicly held corporate debt.

Appendix C

We express the parameters for the estimation of asset volatility in the following table:

Volatilities	ATT	Revlon	Correlations	ATT	Revlon	Weights	ATT	Revlon
σ_{Cash}	1.4%	1.4%	$\rho_{Cash\text{-}Stock}$	-0.44	0.47	w_{Cash}	0.176	0.026*
σ_{Stock}	14.4%	31.7%	$\rho_{Stock\text{-}Bond}$	-0.44	0.47*	w_{Stock}	0.617	-2.61*
σ_{Bond}	1.41%	17.2%	$\rho_{Bond\text{-}Cash}$	0.98	-0.47	w_{Bond}	0.207	3.58*

* The equity of the Revlon Corporation was negative during the period, which was stated in the balance sheet as corporate deficiency. Consequently, the leverage was higher than 100% and the sum of the weights exceeded 1, which is impossible if we are to interpret weights in the framework of probability theory. To account for this irregularity, we (1) normalized the sum of weights to one and (2) *reversed* the empirical sign of stock-bond correlations.

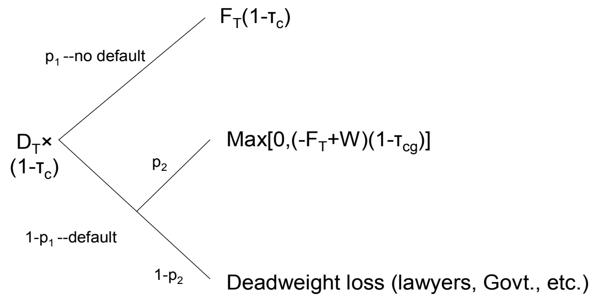

Fig. 1. The two-period game of corporate debt.

The probability p_1 is calculated by the Black-Scholes model. Note that this game is not risk-neutral. The return on D_T is equal to the bond yield and not to the risk-free rate. To return the debt valuation to a risk-free frame, we introduce an adjustable parameter β for the residual value (see Section 1), though this choice is not unique.

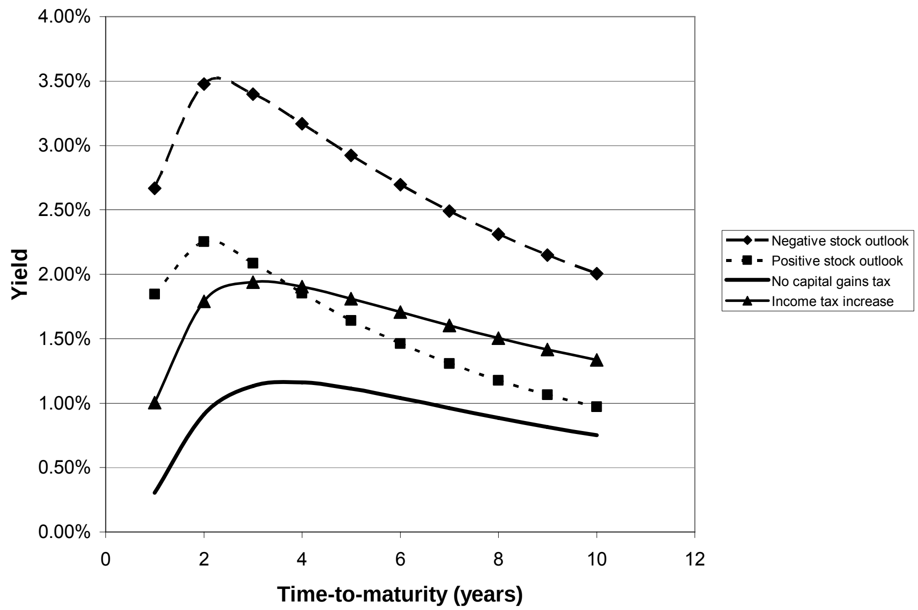

Fig. 2. Plots of bond yields as a function of time to maturity.

r=5%, Div=2%, β=3%, w=1.5 (operational expenses), and δ (recovery factor)=0.6. Tax rates are τ_c = 35% and τ_{cg} = 20% (both dashed curves), τ_c = 35%, τ_{cg} = 0% (no capital gains tax case) and τ_c = 50%, τ_{cg} = 20% (increased income tax case). The face value of debt is 100 and equity value at t=0 is 80. Stock price is a linear function of time growing at 13.4% (positive outlook for equity growth) or 3.6% (negative outlook for equity growth). Parameters are the same for Figs. 1-4 unless specifically mentioned. Boldfaced curve indicates case of zero capital gains with a positive outlook for corporate equity, triangular shapes mark the no-capital gains tax case and negative outlook, squares and diamonds indicate the taxable case with positive and negative outlook, respectively.

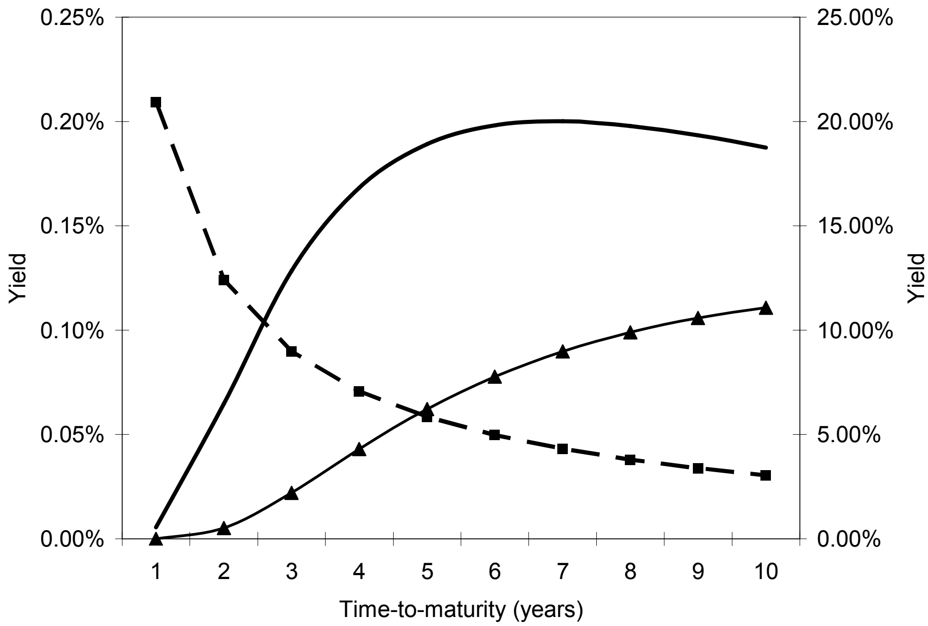

Fig. 3. Yield as a function of equity coverage and time-to-maturity.

High-quality debt (V_0=V/F_0=300, triangles), average-quality debt (V_0=180, boldface) and low-quality debt (V_0=30, squares, dashed line). The left axis applies to the two solid curves, and the right axis to the dashed curve.

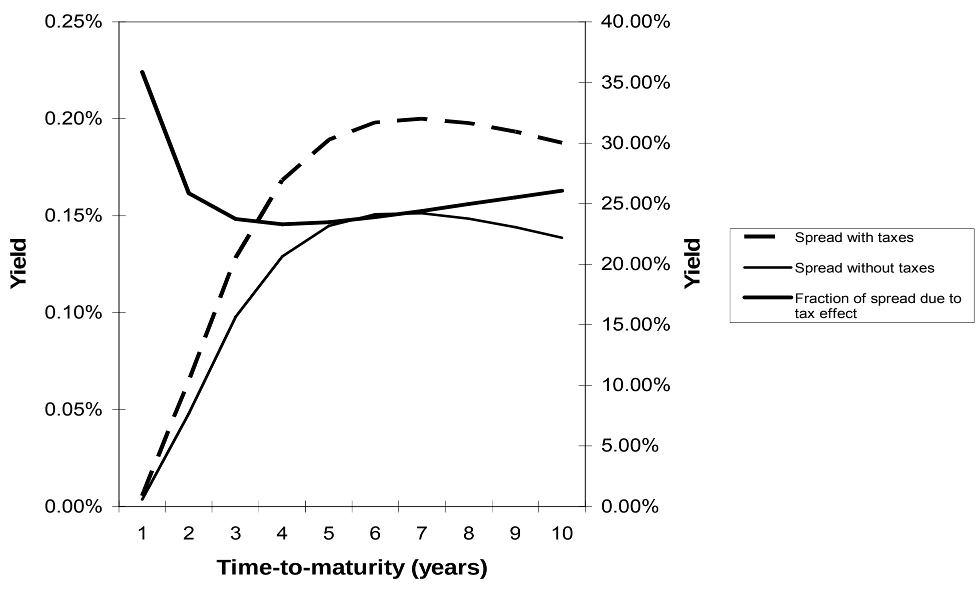

Fig. 4. Spread between taxable (dashed) and tax-free (regular line) debt.

For average debt quality, V_0=180. On the second axis, we plot the percentage of spread due to taxation (boldfaced line).

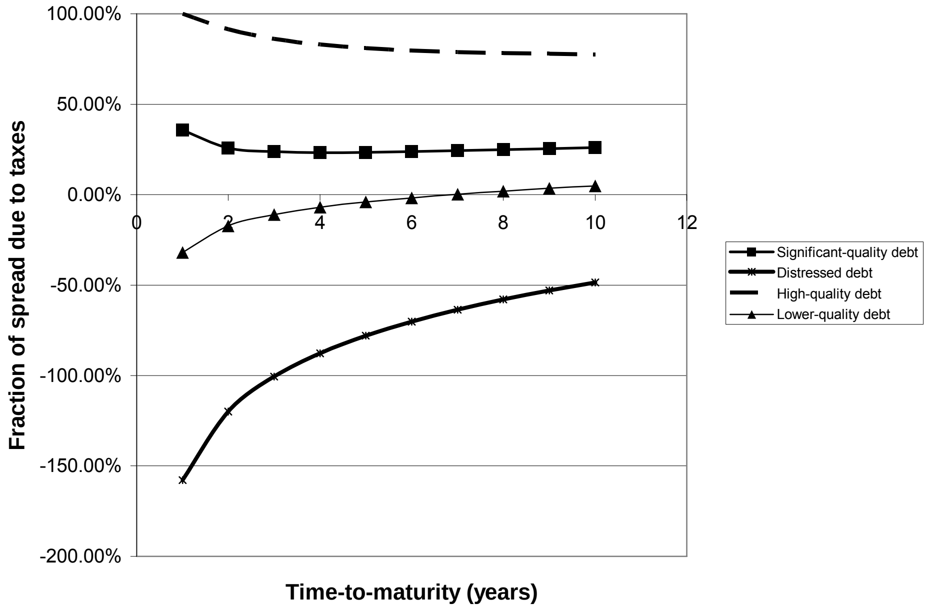

Fig.5 Ratio of bond spreads in taxless and taxable cases as a function of time-to-maturity.

A positive tax signifies increase and a negative fraction—decrease of the spread due to the introduction of taxes. High-quality debt, with initial equity debt coverage of V/D=3.0 (dashed line), significant-quality debt, V/D=1.8 (squares), lower-quality debt, V/D=0.8 (triangles) and highly distressed debt, V/D=0.3 (boldfaced line). For debt with poorer quality, the tax effect can change sign. For the debt with junk status the spread significantly decreases because of the implied possibility of taking over the assets of the defaulting firm.

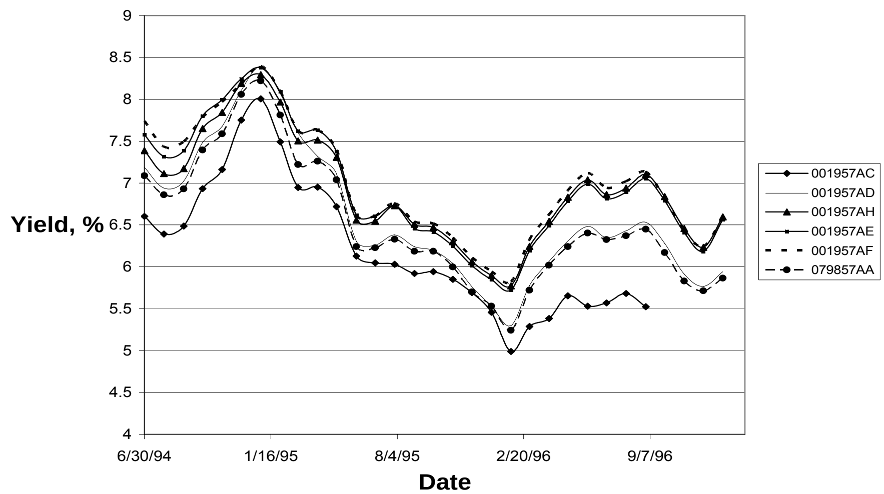

Fig. 6 Yields of corporate bonds.

We display AT&T bonds 001957 series AC, AD, AH, AE and AF and, for comparison, BellSouth bond 0079857AA. Independent of expiration time, they exhibit a significant degree of co-movement. Moreover, the difference between BellSouth bond and AT&T bond AE is less than the difference inside the AT&T bond group.

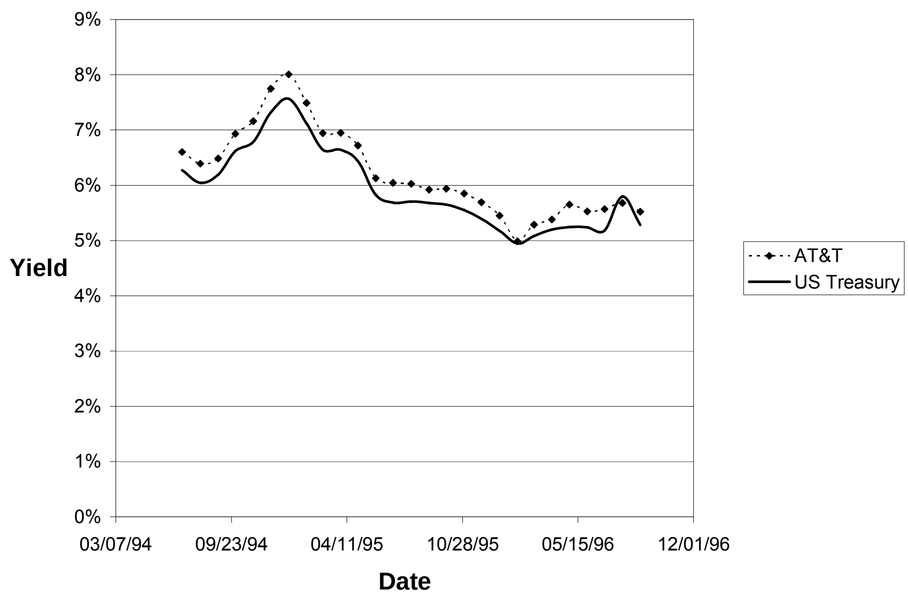

Fig. 7. Spreads between synthetic U.S. Treasury bond and AT&T bond 001957AC.
The negative spread for the month preceding the expiration is probably the artifact of the AT&T bond's illiquidity and the lack of reported price movement.

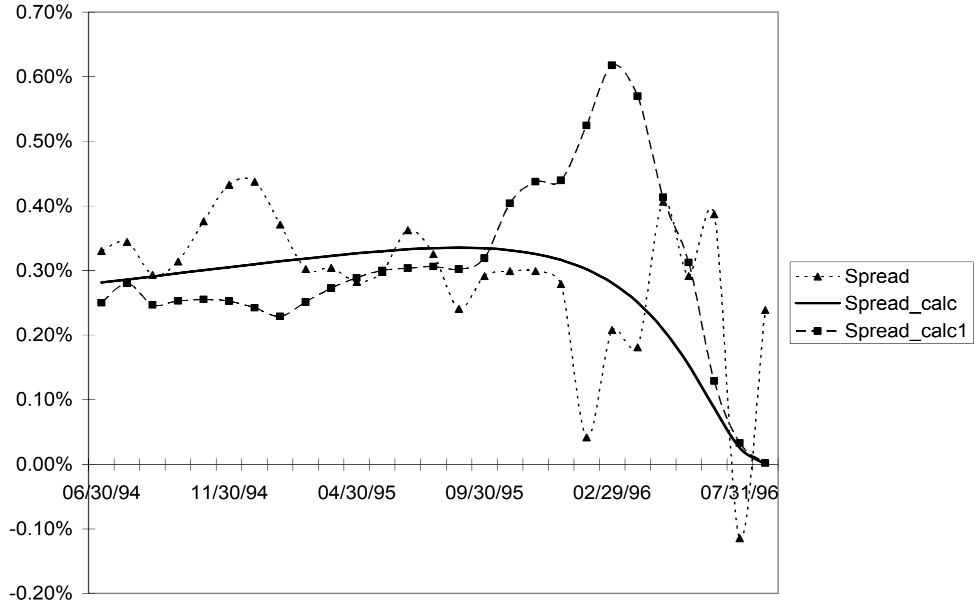

Fig. 8. Observed spreads between AT&T 001957AC and a synthetic T-Bond

Both bonds expire on 08/30/1996. Observed spread is shown in triangles, dotted line. The calculated spreads are given: (1) by the structural model with average interest rates (solid line) and (2) by the structural model with actual monthly interest rates (squares). Parameters used: dividend yield 2.54%, payout ratio 40.5%, τ_c=35%, τ_{cg}=20%, δ (recovery ratio) = 42%, K_2 (liquidation value of assets) = 163.7 (in units of face value of debt=100) and V_0 = 526, in the same units. The model with taxes reduces asset volatility necessary for the fit to 14.9% and β_V=0.159. Methodological notes on the numerical values of these parameters are provided in Appendix B.

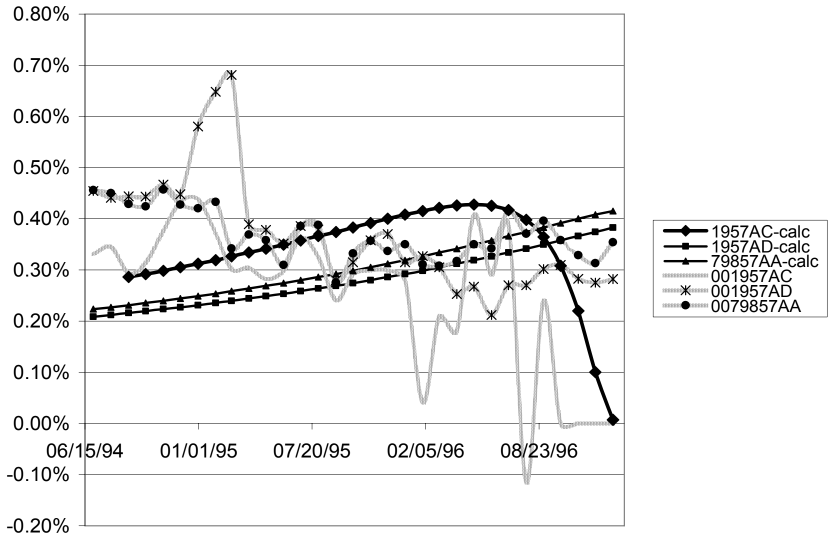

Fig. 9. Calculated and true spreads of the short-term group of bonds.

We observe that the curves correctly reproduce average spreads, while the general shape is different. There seems to be little difference between spreads of AT&T bonds and BellSouth bond 0079857AA. In fact, BellSouth spreads follow the pattern of the closest in maturity bond 001957AD more accurately than the other bonds of the AT&T series. This suggests a general lack of differentiation in the corporate bond market.

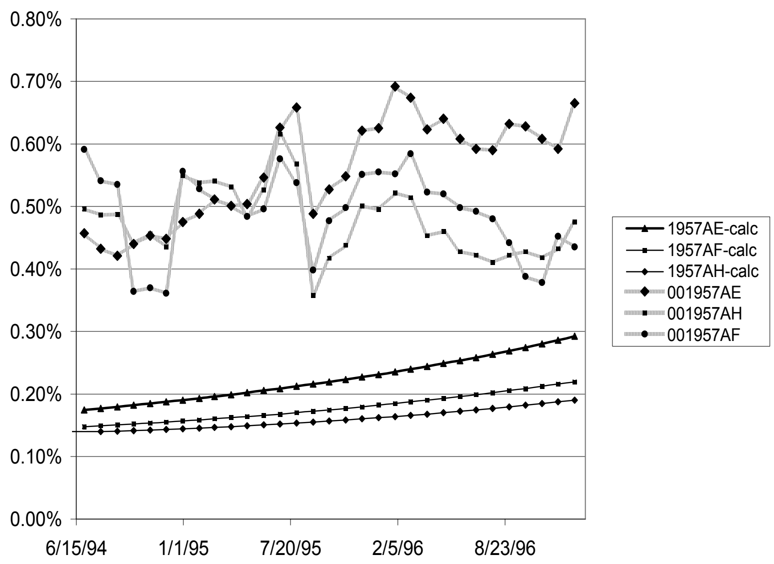

Fig. 10. Spread for longer time-to-maturity bonds.

The long-term bond series (28-51 months to maturity in the end of the period) demonstrate a general increase in spreads with a diminishing time to maturity but calculated spreads are roughly 40% of their observed values.

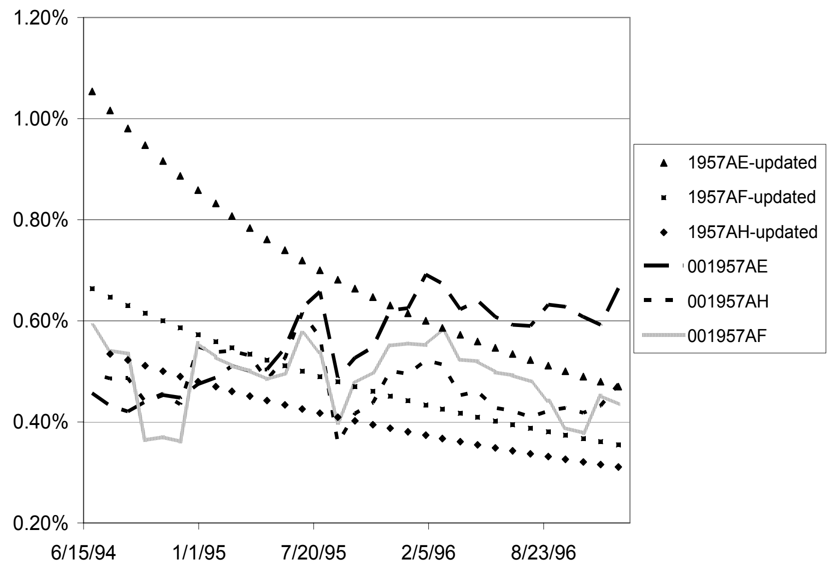

Fig. 11 Spreads for longer time-to-maturity bonds with updated parameters.
Here we have σ=13.1% (14.7%) , β_V=0.093 (0.159) and τ_c=19.45% (35%). Old parameters are listed in parentheses.

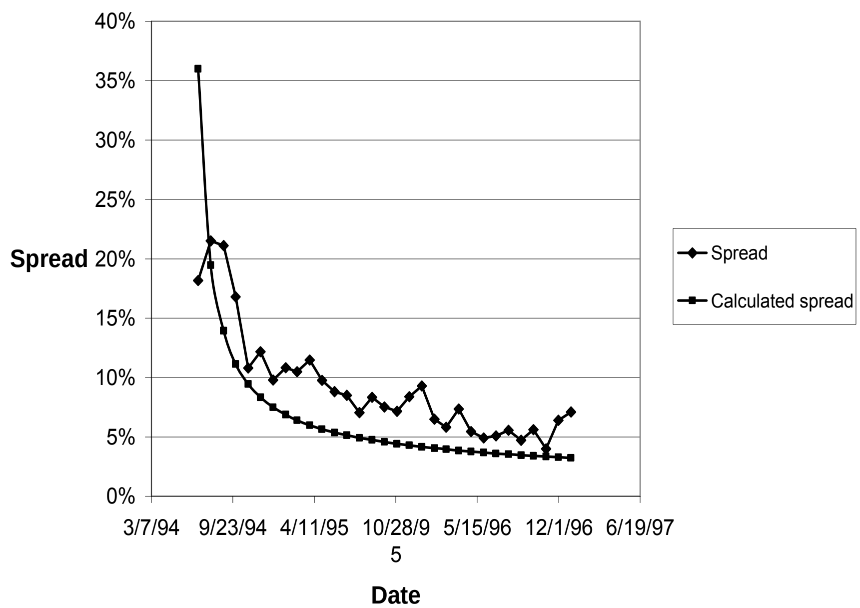

Fig. 12. Fitting of the spread for the distressed Revlon debt.

Spread of the high-yield bonds of Revlon Worldwide series 76154KAB for the 30-month period (06/94-12/96). Fitting parameters are outlined in the text. The shape is qualitatively correct and quantitatively agreeable despite a highly irregular situation: namely, negative book equity for Revlon for the period.

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