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Fallibility and Sequential Decision-Making

by

WINSTON T. H. KOH *

Individuals typically make errors in evaluating information. In this paper, we consider a project selection problem, where fallible managers screen projects sequentially, and provide independent opinions on its quality. We compare the relative performance of the hierarchy and polyarchy when n ($n \geq 2$) managers are employed to evaluate projects. In addition to generalizing the existing results on the optimal screening standards, I discuss and characterize the optimal organizational size, and how the relative desirability of the two sequential decision processes are affected by changing the project quality. Finally, I also discuss the effect of variable evaluation costs on the optimal evaluation standards. (JEL: D81)

1. Introduction

There are two basic reasons why collective decisions are desirable. Firstly, if individual interests diverge, making collective decisions is the only way to resolve conflicts and make compromises. Secondly, an individual's ability to make correct decisions or undertake complex tasks is limited. Collective decision-making is a means to overcome limited individual rationality. To take a concrete example: in big organizations, major decisions regarding production, marketing strategies and investment plans, etc, are often jointly made by a team of managers, whose goal is to improve the organization's performance in the market-place. Each manager does not have all the information necessary to make a good decision, so joint decision-making is advantageous. Different managers contribute different pieces of information, and, as the saying goes, "two heads are always better than one." If the limitation on individual decision-making is the lack of information, then by assigning individuals to collect different parts of the relevant information for which they have expertise, and structuring suitable exchanges of information, perfect decisions are possible.

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However, individuals are fallible and they make errors in collecting and communicating information, and making judgements.

Mistakes are made partly because it is too costly to collect and process all the information. Furthermore, communication is usually costly and time-consuming. What is often communicated is a message of lower dimension. For instance, the individual is required only to cast a vote in favor or against some project under consideration. In general, individuals communicate their information more fully, but better communication brings with it the possibilities that information may be purposely distorted, censored or withheld, generating incentives problems. The implications of fallibility are discussed in the work of SAH and STIGLITZ [1986] and KOH [1992]. An important implication is that different decision structures would amplify or mitigate the human errors and biases in different ways, and even if the decision environment is constant, changing the organizational structure can have significant impact on the performance of the management.

In their seminal work of fallibility, SAH and STIGLITZ [1986] were interested primarily in studying the desirability of different arbitrary organizational structures – hierarchies, polyarchies, and committees – using a two-manager binary-signal model in most of their analysis. KOH [1991] and [1992] extended some of the results, using a continuous-signal model. KOH [1992] also provided a preliminary analysis of incentive problems in the presence of fallibility. Other related work on the subject of fallibility include BULL and ORDOVER [1987], GEANAKOPLOS and MILGROM [1992], and MEYER [1991]. In BULL and ORDOVER [1987], the authors looked at how market competition affected the size and structure of organizations and derived various comparative statics results relating the optimal size of organizations to the market parameters. GEANAKOPLOS and MILGROM [1992] constructed a model of hierarchical decision-making based on the idea that managers are limited in their ability to evaluate because of a time constraint. Managers must divide their time between observing various parameters under their control. MEYER [1991] studied a model of learning by an organization when it only has coarse information available. Refining the information is not feasible due to fallibility. Instead, the organization adopts an information structure that maximizes its learning, given the coarseness of the information. In particular, MEYER [1991] shows that the organization can enhance its learning about a worker's ability by introducing bias into the interpretation of the binary signals it receives about the worker's performance.

In this paper, we present an analysis of the optimal decision-making process when management is fallible. Our analysis goes further than SAH and STIGLITZ [1986] and KOH [1992], both of which study the relative desirability of two sequential decision processes – hierarchy and polyarchy – using a two-manager model. In this paper, we extend the analysis to more than two managers. In section 2, I describe a variation of the project selection problem, which forms the framework of analysis in SAH and STIGLITZ [1986] and KOH [1992]. Section 3 compares the performance of the hierarchy and the polyarchy, when n ($n \geq 2$)

managers are employed to evaluate projects. We disregard incentive problems. In addition to generalizing the existing results on the optimal screening standards, I also discuss and characterize the optimal organizational size, and look at how the relative desirability of the two sequential decision processes are affected by changing the project quality. The results obtained on these two aspects are new, and form the paper's main contribution to the growing literature on organization theory. I also discuss the effect of variable evaluation costs on the optimal evaluation standards. Section 4 concludes the study.

2. *The Project Selection Model*

Projects arrive randomly at a firm, which must decide whether to undertake it or not. There are two types of projects: (G)ood projects which yield a payoff of π_{GA} if accepted, and π_{GR} if rejected; (B)ad projects which yield a payoff of π_{BA} if accepted, and π_{BR} otherwise. We assume $\pi_{GA} > \pi_{BR} > \pi_{GR} > \pi_{BA}$, so that the optimal decision is to accept good projects and reject bad projects, and that accepting good projects is more profitable than just rejecting bad projects. The fraction of good projects is α. A measure of the quality of the project pool is:

$$\beta \equiv \frac{\alpha\{\pi_{GA} - \pi_{GR}\}}{(1-\alpha)\{\pi_{BR} - \pi_{BA}\}}. \tag{1}$$

The expected payoff per project is $\alpha\pi_{GA} + (1-\alpha)\pi_{BA}$ when they are always accepted, and $\alpha\pi_{GR} + (1-\alpha)\pi_{BR}$ when they are always rejected. Hence, $\beta > (<) 1$ implies that always accepting (rejecting) yields a higher payoff. A project pool is said to be of high (low) quality if $\beta > (<) 1$.

Managers are hired to evaluate projects, and they have the expertise to evaluate project quality, but not perfectly. If he exerts a (fixed) effort, he observes a signal $\theta \in [\theta^-, \theta^+]$, which is imperfectly correlated with the project quality Q ($= G$ or B). The imprecision of the signal is one reason the firm may wish to have a second opinion. The signal θ indicates project quality in the sense: let $f(\theta|Q)$, be the density function of θ conditional on Q, where $f(\theta|Q) > 0$ for all θ. We assume that $f(\theta|Q)$ satisfies the *montone likelihood ratio condition* (MLRC), i.e. $f(\theta|B)/f(\theta|G)$ is decreasing in θ. As is well known, MLRC implies that $f(\theta|G)$ dominates $f(\theta|B)$ in the sense of first order stochastic dominance. Hence, $F(\theta|G) < F(\theta|B)$, which means that we are less likely to observe low-valued signals for good projects than for bad projects.

Managers have identical preferences represented by a utility function, additively separable into an income component $U(Y)$, and an effort component V, which is the disutility from exerting effort to observe θ. Managerial actions are unobservable. Communication is also costly and time-consuming; managers are only required to communicate a single summary statistic of their opinion:

"accept" or "reject." Project payoffs are, however, common knowledge, so that contractual arrangements can be made contingent on the project payoffs. We assume there are no incentive problems.

The strategy for a manager is to choose a cutoff point r such that he votes in favor of the project if he observes a signal θ greater than r, and votes against acceptance otherwise. The probability that a project of quality Q will be accepted is $P(\theta > r \mid Q) \equiv [1 - F(r \mid Q)]$. Clearly, $P(\theta > r \mid Q)$ is decreasing in r, and by MLRC, $P(\theta > r \mid G) > \text{Prob}(\theta > r \mid B)$.

2.1 The One-Manager Case

To understand the basic problem facing managers, consider the case when one manager is hired, and he sets a cutoff point r. This is the problem studied by LAMBERT [1986]. Denote the probability that an accepted project is of quality Q by $P(Q \mid \theta > r)$, so that by Bayes Rule, $P(G \mid \theta > r) = \alpha P(\theta > r \mid G)/P(\theta > r)$, where $P(\theta > r) \equiv \alpha P(\theta > r \mid G) + (1 - \alpha) P(\theta > r \mid B)$ is the unconditional probability of accepting a project. $P(B \mid \theta > r) = 1 - P(G \mid \theta > r)$. The probability that a rejected project is of quality Q, denoted $P(Q \mid \theta < r)$, is similarly defined.

The expected projects payoffs conditional on acceptance is $\Pi(\theta > r) \equiv P(G \mid \theta > r)\, \pi_{GA} + P(B \mid \theta > r)\, \pi_{BA}$, while expected payoffs conditional on rejection is $\Pi(\theta < r) \equiv P(G \mid \theta < r)\pi_{GR} + P(B \mid \theta < r)\pi_{BR}$. The unconditional expected payoff is

$$
\begin{aligned}
\Pi(r) &\equiv P(\theta > r)\, \Pi(\theta > r) + P(\theta < r)\, \Pi(\theta < r) \\
&= \alpha \{\pi_{GA} - F(r \mid G)\, [\pi_{GA} - \pi_{GR}]\} \\
&\quad + (1 - \alpha)\, \{\pi_{BA} + F(r \mid B)\, [\pi_{BR} - \pi_{BA}]\}
\end{aligned} \tag{2}
$$

Absent incentives problems, the manager chooses r to maximize Π(r).

$$
\frac{d\Pi(r)}{dr} = (1 - \alpha) f(r \mid G)\{\pi_{BR} - \pi_{BA}\}\left\{\frac{f(r \mid B)}{f(r \mid G)} - \beta\right\} \tag{3}
$$

Since $f(r \mid B)/f(r \mid G)$ decreases with r, $\Pi(r)$ is single-peaked. Therefore

$$
r^* \in (\theta^-, \theta^+) \quad if \ \frac{f(\theta^+ \mid B)}{f(\theta^+ \mid G)} < \beta < \frac{f(\theta^- \mid B)}{f(\theta^- \mid G)},
$$

where $r^* = \operatorname{argmax} \Pi(r)$. By MLRC, $f(\theta^- \mid B)/f(\theta^- \mid G) > 1 > f(\theta^+ \mid B)/f(\theta^+ \mid G)$. Corner solutions occur when $\beta > f(\theta^- \mid B)/f(\theta^- \mid G)$, portfolio quality is sufficiently good so that $r^* = \theta^-$ or when $\beta < f(\theta^+ \mid B)/f(\theta^+ \mid G)$, portfolio quality is sufficiently bad so that $r^* = \theta^+$. In either case, it is unnecessary to hire a manager to screen projects.

Therefore, having more information about individual projects is not always necessary, when the portfolio quality is either very bad or very good. It is only in an intermediate range that evaluation is beneficial. However, even if r^* is interior, it only pays to hire the manager when $\Pi(r^*) - Y > \max\{\Pi(\theta^-), \Pi(\theta^+)\}$, where Y, the amount paid the manager, satisfies $U(Y) - V = U$, and U is the reservation utility.

The result can be cast differently. Suppose $\beta > 1$, so that undertaking a project now yields a positive expected payoff. However, there is value to gathering more information about the individual project if β is less the threshold $f(\theta^- | B)/f(\theta^- | G)$. Delaying investment and waiting for further information is beneficial in this case. This is similar to the insight in the recent literature on investment under uncertainty (see DIXIT [1992]), although in the present framework, information must be generated at a cost.

3. *Sequential Decision-Making: Hierarchy Versus Polyarchy*

When two or more managers are employed to screen projects, it matters how their opinions are aggregated, whether one manager has veto power over others, and if there are variable costs (e.g. research expenses). SAH and STIGLITZ [1986], [1988] and KOH [1991], [1992] have attempted to analyze the properties and performance of simple decision processes such as the hierarchy, polyarchy and the committee. Here, I want to develop the analysis of sequential decision-making in fallible hierarchies and polyarchies. A number of new results are obtained.

Following SAH and STIGLITZ [1986], let us define a *polyarchy* as a sequential decision process where decision makers can recommend projects independently. Authority is decentralized in the sense that a project which fails one evaluation gets a second chance with the next reviewer. The process of submitting papers to academic journals fits this description of the polyarchy. By contrast, authority is more concentralized in a *hierarchy* (again, following SAH and STIGLITZ [1986]) in the sense that only one manager can recommend projects for acceptance while other managers down the chain of command provide (unanimous) decision support. The authority structure in big corporations fits this description of the hierarchy.

Suppose n managers are employed, and they are labelled 1 to n. Although evaluation is sequential, it is not necessarily the case manager i will be the ith reviewer for a project (if the project gets to him at all). This is true if the evaluation sequence can start with any manager. In what follows below, we shall assume that the evaluation sequence can start with any manager, and that managers do not know the evaluation history of any project that comes to him. This ensures that there exists a symmetric solution for the optimal evaluation standards.

In a hierarchy (H), a project is accepted if and only if every manager approves of it. In a polyarchy (P), a project is rejected if and only if it is rejected by every manager. For a project of quality Q, the probabilities that it will be accepted in a hierarchy and polyarchy are, respectively,

$$\prod_{i=1}^{n} \{1 - F(r_i|Q)\} \quad \text{and} \quad 1 - \prod_{i=1}^{n} F(r_i|Q),$$

where r_i is manager i's cutoff point. Assuming no incentives problems and no variable costs, the fixed evaluation costs per project is n times a manager's salary. Let $\boldsymbol{r} \equiv (r_1,\dots,r_n)$ and $\Pi^S(\boldsymbol{r}, n)$ denote the gross expected profits per project for the decision process S:

$$\begin{aligned}
\Pi^H(\boldsymbol{r}, n) &= \alpha\{\pi_{GA} + \prod_{i=1}^{n} \{1 - F(r_i|G)\}(\pi_{GA} - \pi_{GR})\} \\
&\quad + (1-\alpha)\{\pi_{BR} - \prod_{i=1}^{n} \{1 - F(r_i|B)\}(\pi_{BR} - \pi_{BA})\} \\
\Pi^P(\boldsymbol{r}, n) &= \alpha\{\pi_{GA} - \prod_{i=1}^{n} F(r_i|G)(\pi_{GA} - \pi_{GR})\} \\
&\quad + (1-\alpha)\{\pi_{BR} + \prod_{i=1}^{n} F(r_i|B)(\pi_{BR} - \pi_{BA})\}.
\end{aligned} \tag{5}$$

Let $\boldsymbol{r}_{-i} \equiv (r_1,\dots, r_{i-1}, r_{i+1},\dots, r_n)$. By MLRC, it is straightforward to show (as in the one-manager case) the $\Pi^S(\boldsymbol{r}, n)$ is single-peaked in r_i, given $\boldsymbol{r}_{-i}$. Manager i chooses r_i to maximize $\Pi^S(\boldsymbol{r}, n)$, given $\boldsymbol{r}_{-i}$. Let manager i's optimal cutoff point be r_i^S. Given our assumption that the evaluation sequence can begin with any manager, and that managers are not aware of the evaluation history, symmetric solutions to r_i^S exist, and since we are interested in comparing optimal cutoff points, we focus our attention on the symmetric optimal cutoff point r^S. If r^S is an interior solution, it is given by

$$\frac{f(r^H|B)\,[1 - F(r^H|B)]^{n-1}}{f(r^H|G)\,[1 - F(r^H|G)]^{n-1}} = \beta = \frac{f(r^P|B)\,F(r^P|B)^{n-1}}{f(r^P|G)\,F(r^P|G)^{n-1}}. \tag{6}$$

Lemma 1:

[a] $\dfrac{f(\theta|B)[1 - F(\theta|B)]^{n-1}}{f(\theta|G)\,[1 - F(\theta|G)]^{n-1}}$ is decreasing in θ and increasing in n;

[b] $\dfrac{f(\theta|B)F(\theta|B)^{n-1}}{f(\theta|G)F(\theta|G)^{n-1}}$ is decreasing in θ and n;

[c] $\dfrac{f(\theta|B)[1 - F(\theta|B)]^{n-1}}{f(\theta|B)[1 - F(\theta|G)]^{n-1}} < \dfrac{f(\theta|B)F(\theta|B)^{n-1}}{f(\theta|G)F(\theta|G)^{n-1}}$.

Proof: Using MLRC.

Lemma 1 and (6) imply that $r^H < r^P$, when at least one of them is an interior solution. The intuition is as follows: In a hierarchy, each manager knows that a project that he approves will either be rechecked by another manager, or has already been approved by another manager. In a polyarchy, a manager's approval would not be rechecked by another, and the project he receives may already have been rejected before by other managers. Intuitively, therefore, the optimal cutoff point in a polyarchy should be higher than in the hierarchy. This result generalizes the result in SAH and STIGLITZ [1986] and KOH [1992], for a two-manager setting.

It also follows from Lemma 1 that if project quality improves (i.e. β is now higher), r^S should be lowered. If there are more managers and project quality is unchanged, r^H should be lowered and r^P raised, since screening becomes more (less) stringent in a hierarchy (polyarchy) with more managers. The optimal cutoff point must be lowered (raised) to compensate for this.

3.1 Comparison of Comparative Advantage

Below, I derive the conditions for r^H and r^P to be interior solutions. Comparison of these conditions suggests that the polyarchy has a comparative advantage over the hierarchy as project quality improves. Lemma 1 and (6) implies that r^H is interior if

$$\frac{f(\theta^+|B)\,[1-F(\theta^+|B]^{n-1}}{f(\theta^+|G)\,[1-F(\theta^+|G)]^{n-1}} < \beta < \frac{f(\theta^-|B)\,[1-F(\theta^-|B)]^{n-1}}{f(\theta^-|G)\,[1-F(\theta^-|G)]^{n-1}}.$$

By L'Hôpital rule, the above condition simplifies to

$$\frac{f(\theta^+|B)^n}{f(\theta^+|G)^n} < \beta < \frac{f(\theta^-|B)}{f(\theta^-|G)}. \tag{7}$$

Similarly, r^P is an interior solution if

$$\frac{f(\theta^+|B)}{f(\theta^+|G)} < \beta < \frac{f(\theta^-|B)^n}{f(\theta^-|G)^n}. \tag{8}$$

The conditions in (7) and (8) imply that both r^H and r^P are interior if and only if

$$\frac{f(\theta^+|B)}{f(\theta^+|G)} < \beta < \frac{f(\theta^-|B)}{f(\theta^-|G)} \tag{9}$$

since by MLRC, $f(\theta^-|B)/f(\theta^-|G) < 1 < f(\theta^+|B)/f(\theta^+|G)$. The condition in (9) is exactly the same as the condition in (4) for the one-manager case. This implies that if expected gross payoffs can be improved by employing one manager, then the hierarchy and the polyarchy are both feasible decision processes, since r^S is interior.

Suppose project quality is sufficiently good so that screening is not required in the one-manager case; i.e. $\beta > f(\theta^-|B)/f(\theta^-|G)$. The conditions in (7) and

(8) imply that expected gross profits can only be improved by a polyarchy. Similarly, if project quality is such that $\beta < f(\theta^+|\text{B})/f(\theta^+|G)$, only a hierarchy can improve expected payoffs. This suggests that the polyarchy (hierarchy) is a better organizational structure when the project quality is high (low).

The intuition is that since $[1 - F^n(r|Q)] > [1 - F(r|Q)]^n$, it follows that if both structures adopt the same cutoff point, the polyarchy has a higher probability of accepting a project compared with the hierarchy. The Type-I error (of rejecting a good project) is lower for the polyarchy while the Type-II error (of accepting a bad project) is lower for the hierarchy. The comparative advantage of the hierarchy is therefore in evaluating low-quality projects, while that of the polyarchy is in evaluating high-quality projects. In section 3.2, we provide another example to show that the ranking of the hierarchy and polyarchy is not necessarily "monotonic" in project quality.

3.2 Optimal Size of the Organization

Let r_n^S denote the optimal cutoff point for n managers, and $\Pi^S(r_n^S, n)$ denote the expected gross profits (defined in (5)) when r_n^S is used. Treating n as continuous, and using the Envelope theorem, we differentiate $\Pi^H(r_n^H, n)$ with respect to n to obtain

$$\text{(10)} \quad \text{Sign}\left\{\frac{d\Pi^H(r_n^H, n)}{dn}\right\} = \text{Sign}\left\{\frac{\{1 - F(r_n^H|B)\}^n \ln\{1 - F(r_n^H|B)\}}{1 - F(r_n^H|G)\}^n \ln\{1 - F(r_n^H|G)\}} - \beta\right\}.$$

Using the equilibrium condition in (6), this implies that

$$\text{(11)} \quad \frac{d\Pi^H(r_n^H, n)}{dn} > (<) 0 \Leftrightarrow \frac{d\ln\{\ln[1 - F(r_n^H|G)\}}{d\ln\{\ln[1 - F(r_n^H|B)\}} > (<) 1,$$

where $\ln[1 - F(r|Q)]$ is the log-likelihood of observing θ *greater* than r given project quality Q.

Hiring more managers lowers the optimal cutoff point in the hierarchy, thereby increasing the probability of accepting both types of projects. The condition in (11) says that increasing the size of the hierarchy improves expected gross payoffs if and only if the proportional increase in the log-likelihood of accepting good projects is *greater* than the proportional increase in the log-likelihood of accepting bad projects. For the polyarchy, the equivalent conditions is:

$$\text{(12)} \quad \frac{d\Pi^P(r_n^P, n)}{dn} > (<) 0 \Leftrightarrow \frac{d\ln\{\ln F(r_n^P|G)\}}{d\ln\{\ln F(r_n^P|B)\}} < (>) 1.$$

The condition in (12) says that expanding the polyarchy improves gross payoffs if and only if the proportional increase in the log-likelihood of rejecting good projects is *less* than the proportional increase in the log-likelihood of rejecting bad projects, when the optimal cutoff point is raised.

Define $n^S = \text{argmax}\ \Pi^S(r_n^S, n)$, so that n^S provides the upperbound to the optimal organizational size, which will be smaller than n^S because the additional costs incurred in hiring more managers were not considered in the computation of n^S. The next result shows that n^S is finite, which in turn implies that the optimal organization size is finite. By definition,

$$\Pi^S(r_n^S, n^S) > Max\,\{\Pi(\theta^+), \Pi(\theta^-)\}\,, \tag{13}$$

where $\Pi(\theta^+)$ and $\Pi(\theta^-)$ denote, respectively, the expected payoffs from always rejecting and always accepting projects. If n^S is sufficiently large, then by (6) and Lemma 1, r^H is close to θ, and r^P close to θ^+. Therefore, the limit of $\Pi^S(r_n^S, n^S)$ is max $\{\Pi(\theta^+), \Pi(\theta^-)\}$. The same level of expected gross payoff can be replicated by either accepting or rejecting projects.

It is straightforward to show that the optimal organizational size is also sensitive to project quality. An improvement in quality, measured by β, increases expected gross profits even if the optimal cutoff points are unadjusted; the increase is greater when the optimal cutoff points are adjusted. However, it is ambiguous if better project quality implies hiring fewer managers. This is because the higher manpower costs can be offset by the higher profitability resulting from tighter evaluation.

3.3 Comparative Organizational Performance

The conditions (7) and (8) imply that, without restrictions on organizational size, the hierarchy dominates the polyarchy in terms of profitability if $0 < \beta < f(\theta^+|B)/f(\theta^+|G)$, and is dominated by the polyarchy if $\beta > f(\theta^-|B)/f(\theta^-|G)$. In the intermediate range, $f(\theta^+|B)/f(\theta^+|G) < \beta < f(\theta^-|B)/f(\theta^-|G)$, both structures are feasible. When optimal cutoff points are used, we can show that for n hired managers,

$$\Pi^H(r^H) > (<)\, \Pi^P(r^P) \Leftrightarrow \frac{1 - F(r^P|B)^n - \{1 - F(r^H|B)\}^n}{1 - F(r^P|G)^n - \{1 - F(r^H|G)\}^n} > (<)\, \beta\,. \tag{14}$$

To interpret the condition, note that the numerator in (14) is a measure of the comparative advantage of the hierarchy over the polyarchy in rejecting bad projects, while the denominator is a measure of the comparative advantage of the polyarchy over the hierarchy in accepting good projects. If the ratio of the comparative advantage is greater (smaller) than project quality, then the hierarchy (polyarchy) yields higher expected gross payoffs. We illustrate the comparison with two examples.

Example 1. $F(\theta|B)=\theta$ and $F(\theta|G)=\theta^2$; $\theta\in[0,1]$. Two managers. For $\beta\in(0.5,\infty)$, both r^H and r^P are interior solutions:

$$r^H=\left\{\frac{1}{4}+\frac{1}{2\beta}\right\}^{0.5}-\frac{1}{2}\qquad r^P=\left\{\frac{1}{2\beta}\right\}^{0.5}$$

$$\Pi^H(r^H)>(<)\,\Pi^P(r^P)\Leftrightarrow 2\beta^5+4\beta^4+16\beta^3-7\beta^2+6\beta-1>(<)\,0\,.$$

Clearly, the polyarchy dominates the hierarchy for $\beta\in(0.5,\infty)$.

Example 2. $F(\theta|B)=0.5(3-\theta)\theta$ and $F(\theta|G)=0.5(1+\theta)\theta$; $\theta\in[0,1]$. Two managers. For $\beta\in(1/3,3)$, both r^H and r^P are interior solutions:

$$r^H=\frac{7+5\beta(1+9\beta^2+134\beta)^{0.5}}{4(1-\beta)}\qquad r^P=\frac{3(3+\beta)-(\beta^2+9+134\beta)^{0.5}}{4(1-\beta)}\,.$$

We are able to show that

$$\begin{array}{ll}\Pi^H(r^H)>\Pi^P(r^P) & \text{if } \beta\in(1/2,1) \text{ or } \beta\in(2,3)\\ \Pi^H(r^H)<\Pi^P(r^P) & \text{if } \beta\in(1/3,1/2) \text{ or } \beta\in(1,2)\\ \Pi^H(r^H)=\Pi^P(r^P) & \text{if } \beta=1/2,1 \text{ or } 2.\end{array}$$

Hence, the two organizational types are profitable over different ranges of project quality.

3.4 Variable Evaluation Costs

If project evaluation involves a fixed sequence, i.e. manager i is always the ith manager to evaluate a project, and there are variable expenses (e.g. research fees) that are proportional to the number of evaluations, there are no symmetric optimal cutoff points. It is straightforward to show that the optimal cutoff points for the hierarchy should become lower for managers further up the evaluation sequence. For the polyarchy, the opposite is true. Suppose each evaluation incurs a cost of I. Denote expected variable costs, given $\boldsymbol{r}\equiv(r_1,\ldots,r_n)$, by $V^S(\boldsymbol{r},n)\cdot I$, where

$$(15)\quad \begin{aligned} V^H(\boldsymbol{r},n)&=1+\alpha\left\{\sum_{j=1}^{n-1}\prod_{i=1}^{j}[1-F(r_i|G)]\right\}+(1-\alpha)\left\{\sum_{j=1}^{n-1}\prod_{i=1}^{n}[1-F(r_i|B)]\right\}\\ V^P(\boldsymbol{r},n)&=1+\alpha\left\{\sum_{j=1}^{n-1}\prod_{i=1}^{n}F(r_i|G)\right\}+(1-\alpha)\left\{\sum_{j=1}^{n-1}\prod_{i=1}^{n}F(r_i|B)\right\}.\end{aligned}$$

Denote expected profits, net of evaluation costs, by $W^S(\boldsymbol{r},n)$:

$$(16)\qquad W^S(\boldsymbol{r},n)=\Pi^S(\boldsymbol{r},n)-V^S(\boldsymbol{r},n)\cdot I-nY,$$

where Y is the salary of one manager, and $\Pi^S(\boldsymbol{r}, n)$, the expected gross profits, is defined earlier in (5). Let $\boldsymbol{r}^S$ be the vector of optimal cutoff points. Assuming interior solutions for all, each optimal cutoff point r_k^S, given $\boldsymbol{r}_{-k}^S$, is characterized by the first-order condition:

$$\left.\frac{\partial W^S(\boldsymbol{r}, n)}{\partial r_k}\right|_{\boldsymbol{r}^S} = 0 \quad \forall k = 1, \ldots, n,$$

where

(17)
$$\frac{\partial \Pi^H(\boldsymbol{r}, n)}{\partial r_k} = -\alpha f(r_k|G)\Delta_G \prod_{j=1, j\neq k}^{n} [1 - F(r_j|G)] + (1-\alpha) f(r_k|B)\Delta_B \prod_{j=1, j\neq k}^{n} [1 - F(r_j|B)]$$

$$\frac{\partial V^H(\boldsymbol{r}, n)}{\partial r_k} = -\alpha \sum_{j=k-1}^{n-1} \prod_{i=1}^{j} [1 - F(r_i|G)] f(r_k|G) - (1-\alpha) \sum_{j=k-1}^{n-1} \prod_{i=1}^{j} [1 - F(r_i|B)] f(r_k|B)$$

$$\frac{\partial \Pi^P(\boldsymbol{r}, n)}{\partial r_k} = -\alpha f(r_k|G)\Delta_G \prod_{j=1, j\neq k}^{n} F(r_j|G) + (1-\alpha) f(r_k|B)\Delta_B \prod_{j=1, j\neq k}^{n} F(r_j|B)$$

$$\frac{\partial V^P(\boldsymbol{r}, n)}{\partial r_k} = \alpha \sum_{j=k-1}^{n-1} \prod_{i=1}^{j} F(r_i|G) f(r_k|G) + (1-\alpha) \sum_{j=k-1}^{n-1} \prod_{i=1}^{j} F(r_i|B)] f(r_k|B)$$

and $\Delta_G \equiv [\pi_{GA} - \pi_{GR}]$ and $\Delta_B \equiv [\pi_{BR} - \pi_{BA}]$.

There are no symmetric solutions to $\boldsymbol{r}^S$. To see this, suppose to the contrary, that $r_k^S = r_1^S$ for some $k > 1$. From (17),

$$\frac{\partial \Pi^S(\boldsymbol{r}, n)}{\partial r_k} = \frac{\partial \pi^S(\boldsymbol{r}, n)}{\partial r_1} \quad \text{but} \quad \frac{\partial V^S(\boldsymbol{r}, n)}{\partial r_k} \neq \frac{\partial V^S(\boldsymbol{r}, n)}{\partial r_1},$$

when the derivatives are evaluated at $\boldsymbol{r}^S$, contradicting the assumption that $r_k^S = r_1^S$. Furthermore, in the hierarchy, $r_{i+1}^H < r_i^H$. We prove the result by contradiction. Suppose $r_k^H > r_1^H$ for $k > 1$. Interchange the two cutoff points and let $\boldsymbol{r}^\sim$ denote the new vector of cutoff points. From (17),

$$W^H(\boldsymbol{r}^S, n) - W^H(\boldsymbol{r}^\sim, n) = V^H(\boldsymbol{r}^\sim, n) - V^H(\boldsymbol{r}^S, n) < 0$$

contradicting that $\boldsymbol{r}^S$ is optimal. Similar arguments can be used to show that in the polyarchy, $r_i^P < r_{i+1}^P$. The intuition is straightforward. Suppose cutoff points of individual managers are fixed. Expected gross payoff in either the hierarchy or the polyarchy does not depend on how the managers are arranged. However, variable evaluation costs are sensitive to the arrangement of managers. The optimal arrangement for each organizational structure must minimize the variable costs: in the hierarchy, the manager with the highest cutoff point screens projects first, followed by the manager with the second highest cutoff point, and so on. In the polyarchy, the arrangement is the opposite, the manager with the lowest cutoff point screens projects first.

The result would also hold in the absence of variable evaluation costs, but with a fixed evaluation sequence. In the hierarchy, the prior project quality of manager $i + 1$ is the posterior project quality of manger i. Since manager $i + 1$ gets to screen a project if it has been approved by manager i, the prior quality of manager $i + 1$ is higher than that of manager i. With a better prior, manager $i + 1$ should set a lower cutoff point. In the polyarchy, manager $i + 1$ only screens projects rejected by manager i. Hence, his prior is lower than that for manager i. Manager $i + 1$ should set a higher cutoff point.

The results have a similar flavor to the main finding in MEYER [1991], which shows that in deciding between two workers for promotion, when ability is unobservable and only rank-order information about output is available, the firm should bias the comparison – the gap between output – in favor of better-performed worker in later rounds of assessment. The same principle underlies both Meyer's analysis and the problem here: with a better prior in the later rounds, evaluation standards for the better project/worker should be less stringent.

There are no general results on the comparison of the relative profitability of the hierarchy and polyarchy when there are variable evaluation costs, and different optimal cutoff points for each manager. Restrictive assumptions have to be imposed to generate some comparative results, as in proposition 6 of KOH [1992].

4. Conclusion

The results reported here extend the work in SAH and STIGLITZ [1986] and KOH [1992]. In a hierarchy, a project is accepted if and only if every manager approves the project; inspection stops as soon as bad review is received. In a polyarchy, a project is rejected if and only if every manager rejects the project; inspection stops when a good review is received. In a committee, acceptance is based on the number of good reviews exceeding a minimum acceptance consensus, as discussed in SAH and STIGLITZ [1988] and KOH [1991]. The hierarchy and polyarchy are clearly suboptimal, while the committee may be preferable if there is a time constraint for screening projects, so that inspection can be carried out simultaneously by all the managers to expedite evaluation.

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