4-2008

Labor Hoarding Contracts and Coordination Fictions

Nicolas L. JACQUET

Singapore Management University, njacquet@smu.edu.sg

Serene TAN

National University of Singapore

Labor Hoarding Contracts with Coordination Frictions*

Nicolas Jacquet
Singapore Management University

Serene Tan
National University of Singapore

April 2008

Abstract

This paper considers a directed search model with risk-neutral firms and risk-averse workers. Although each firm has only one job to fill, firms can hire as many workers as they wish, and the wage a worker is paid can be contingent on the queue length at the firm and his position in the queue. We first show that, contrary to standard directed search models, the application subgame does not necessarily have a unique *symmetric* solution; although uniqueness is guaranteed if all firms post Flat-Wage Contracts (FWCs), i.e., contracts where firms commit to employ a fixed number of workers at a fixed wage. We then show that there is a unique equilibrium such that the expected utility of having applied to a firm is either decreasing or increasing everywhere in the number of applicants for all firms, and it is an equilibrium where all firms post FWCs such that employment is guaranteed to all workers. Compared to standard directed models where firms post one vacancy, workers are better off and firms worse off: although a firm can reduce its wage bill by insuring workers through guaranteeing employment, when all firms do so the additional number of vacancies posted increases competition among firms. In fact, in equilibrium workers are paid a perfectly competitive wage, even when the economy is finite, whereas this outcome cannot be achieved without labor hoarding contracts, even in a large economy.

JEL Codes: D40; J41; J60.

Keywords: Directed Search; Labor hoarding; Vacancies; Risk Sharing; Competition.

*We would like to acknowledge that this project originated from a discussion with John Kennes. We thank Randy Wright for his comments at an early stage of this project, Guillaume Rocheteau, and participants of the NUS Macro Brownbag and 2007 Singapore Economtic Theory Conference.
Contact: njacquet@smu.edu.sg; serenetan@nus.edu.sg.

1 Introduction

The directed search approach to modelling markets where trade is decentralized, like the labor market and some goods markets, has becoming popular in recent years.[1] One reason is that, although in such models the market structure is not perfectly competitive in that firms seeking to hire workers (sellers seeking to attract buyers) are not price takers, firms (sellers), contrary to random search models, compete directly with each other on price. This is because workers (buyers) observe all wages (prices) or price mechanisms offered, offers that firms (sellers) are committed to, and workers (buyers) can therefore direct their search towards the firm (seller) offering them the highest expected payoff.[2] But coordination frictions limit the extent of competition since workers (sellers) cannot be guaranteed employment (delivery of the good) at their preferred firm (seller), and therefore firms (sellers) do not face a perfectly elastic labor supply (demand for the good). Hence, prices play a better allocative role than that in the random search framework.[3]

In standard directed search models, in the labor market context, a firm can make itself more attractive to workers by increasing the wage it posts. A higher wage implies workers will apply to this firm with a greater probability, implying that the probability the firm receives no applicant decreases. But the probability a given worker is chosen for the job decreases, and hence, there is a trade-off between the wage and the probability of employment at a given firm for workers, while there is a trade-off between profit net of the wage and the probability of filling the job for firms. These trade-offs are unavoidable in standard directed search models because firms have only one tool at their disposal, the wage they post, since an applicant's probability of employment is implicitly determined by the wage.

There are two important assumptions that are maintained throughout these studies: firms cannot compensate the unlucky workers who have shown up but are not awarded the job; and all agents are risk-neutral. The aim of this paper is to

[1] A non-exhaustive of list of papers includes Montgomery (1991), Peters (1991, 1997, 2000), McAfee (1993), Burdett, Shi and Wright (2000), Julien et al. (2000, 2007), Shi (2001, 2005), and Shimer (2004).

[2] In random search models, if past offers can be recalled or on-the-job search is allowed, two or more firms (sellers) can compete directly with each other for a worker (buyer). However, this is true for only *some* of the meetings, and because search is still random prices do not play as important an allocative role as they do in directed search models.

[3] See for instance Acemoglu and Shimer (1999) for a comparison of the efficiency properties of the allocation under different mechanisms.

relax these two assumptions, which we believe is particularly relevant for the study of the labor market. Assuming that firms cannot compensate workers who have applied but are not chosen to fill the job is clearly restrictive.[4] In fact, even if a firm has only one job to fill, it could rationally decide to commit to hoard labor in some cases. There at least two reasons why a firm might want to do so. First, by posting a contract where it can choose both the wage(s) and the number of vacancies, the firm breaks, at least partially, the tight link between the wage posted and the probability with which a given worker will be employed. This is particularly relevant when workers are risk-averse since they are eager to avoid unemployment. By posting a contract where it commits to hoard labor, a firm should therefore be able to offer workers a lower expected wage than otherwise, and it might thereby be able to reduce its expected wage bill despite expecting to hire more than one worker with positive probability.

More specifically, we study a directed search model of the labor market which is standard in all but two respects: we assume that workers are risk-averse, and firms are free to post any employment contract they want. That is, a firm can hire as many workers as it wants, despite having one productive job, and the wage paid can be contingent both on the number of workers applying and the position of the worker in the queue. We first show that, contrary to standard models, there might not be a unique *symmetric* equilibrium of the application subgame. This multiplicity can arise when labor hoarding is allowed because workers' payoffs are no longer necessarily monotonic in the probability with which other workers apply to a firm. But if all firms post Flat-Wage Contracts, FWCs hereafter, that is, contracts where the wage is not contingent on the number of workers applying, then the equilibrium is unique.

When then show that there is a unique equilibrium such that the expected utility of having applied to a firm is either decreasing or increasing everywhere in the number of applicants for all firms, and it is a FWC-equilibrium where all firms guarantee employment to all workers. Although this result might seem intuitive with risk averse workers because of the wage bill argument of Akerlof and Miyazaki (1980), the result is actually not trivial in a directed search framework because of the existence of coordination frictions.

[4] It is also restrictive in a goods market since sellers could choose to pay a fee to all buyers who show up at their store. For instance, Faig and Huangfu (2007), in a competitive search model of money, allow marketmakers to pay some agents to come to their markets, and they show that, in equilibrium, marketmakers will indeed choose to pay a fee to some agents.

The welfare implications of labor hoarding contracts can seem surprising: when the number of firm is exogenous, it turns out that workers are strictly better off than if firms post standard wage contracts, whereas firms are strictly worse off. How can firms be worse off when offering labor hoarding contracts to risk-averse workers since such contracts should intuitively lower the expected wage bill each firm has to pay because of implicit risk-premium levied on workers' wages? The reason lies in the greater competition that arises among firms when labor hoarding contracts are posted. In fact, the insurance element enables a firm, taking as given the contract posted by other firms, to decrease its expected wage bill compared to the standard directed search model. But when all firms do the same, competition is strengthened, thereby reducing firms' expected profits.

When hoarding labor is allowed, the equilibrium wage paid by all firms is equal to the expected marginal productivity of a worker, and is therefore equal to the perfectly competitive wage. Hence, despite the fact that the market is decentralized, workers' utility level is the same as if the labor market was perfectly competitive, even for a finite economy. This contrasts with standard directed search models where agents' payoffs only converge to those obtained in a perfectly competitive economy as the economy becomes large, and only in the case of risk-neutral workers. Although firms are not price takers, they are too small to have any impact on the expected utility of applying to other firms, and therefore all firms then take as given the expected utility they have to offer to workers. In that case the markets are, in essence, complete, and the equilibrium is therefore efficient.[5]

When workers are risk-averse in a standard directed search model with wage posting, the expected wage is below the value of a worker's expected marginal productivity, even in a large economy. Further, because employment is not guaranteed, workers' expected utility is reduced further compared to the equilibrium with hoarding. The fact that the expected wage is below the perfectly competitive wage can be explained as follows: when workers are risk-averse the probability with which they apply to a firm is less responsive to a reduction in the wage the firm offers than for risk-neutral workers. This is because risk-averse workers are more willing to accept a reduction in wage in exchange for a greater probability of employment

[5] In directed search models with multiple applications (see Albrecht et al., 2007; and Galenianos and Kircher, 2007), the equilibrium is inefficient. This is because in these models vacancies that have made offers to workers who reject them, because they have another better offer, are not allowed to make subsequent offers to their other applicants. Kircher (2007) shows that efficiency can be obtained if a market for such unfilled vacancies exists.

than risk-neutral workers. Hence, although markets are incomplete[6] when workers are risk-averse, firms can complete the markets by offering labor hoarding contracts.

This paper is closely related to the literature on decentralized trade investigating the choice of mechanism by firms (sellers), either in a random search (Delacroix and Camera, 2004) or directed search framework (McAfee, 1993; Peters, 1997; Coles and Eeckhout, 2003; Michelacci and Suarez, 2006). But all these studies share the assumptions that all agents are risk-neutral and the firm (seller) cannot compensate the worker (buyers) who are not awarded the job (good).

A branch of the literature also closely related to our work is interested in whether the equilibrium outcome in decentralized exchange converges to the competitive outcome, again both in random (Rubinstein and Wolinsky, 1985, 1990; Gale, 1986, 1987; Binmore and Herrero, 1988) and directed search frameworks (Peters, 1991, 1997, 2000; McAfee, 1993). In these papers, if the outcome can be perfectly competitive it is only asymptotically - as agents become infinitely patient for random search models, and as the economy becomes large in the directed search ones, whereas in our model labor hoarding contracts result in a perfectly competitive outcome even for a finite economy.

The implicit contract literature (Baily, 1974; Azariadis, 1975; Polemarchakis, 1979; Akerlof and Miyazaki, 1980) is also interested in risk-neutral firms insuring risk-averse workers. And as highlighted by Holmstrom (1983) and Kihlstrom and Laffont (1983) the implicit contracts offered by firms to workers complete the markets, exactly as labor hoarding contracts do in our model. However, the implicit contract literature considers frictionless environments where the insurance firms provide workers with is an insurance related to a common risk: the state of the world, and therefore the productivity of each firm, is ex ante uncertain. In this literature, it is the introduction of implicit contracts that can lead to the existence of unemployment since with spot markets no worker is unemployed and the risk workers face is coming from the fluctuation in the wage. In our model firms insure workers against the risk of unemployment, which exists without labor hoarding contracts, and unemployment risk is faced only by workers.

The paper is organized as follows. The model is laid out in the next section, and the application subgame is considered in section 3. Section 4 characterizes

[6] Markets are incomplete without labor hoarding contracts because risk-averse workers care about more than just the expected wage they will receive, they also care about the probability of being employed or unemployed.

the equilibrium of the model. The welfare implications of labor hoarding contracts are studied in section 5, and section 6 deals with the efficiency properties of the equilibrium. Section 7 concludes.

2 The Model

There are M identical firms, all profit maximizing and risk neutral, indexed by $m \in \{1, ..., M\}$, and N identical risk-averse workers indexed by $n \in \{1, ..., N\}$. Each firm has one job, and if the job is filled, the product to be split is 1. If the job is unfilled, the firm is idle and nothing is produced. A firm m posts a contract $\mathbf{w}_m = (w_m^{r,k})_{r=1,k=1}^{N,N}$, where $w_m^{r,k}$ is the wage paid to the r^{th} worker (in the queue) when k workers have applied to firm m. Since all applications are received at the same time and all workers are identical to a firm, it is assumed that firms treat workers identically so that the position of a worker in the queue is determined randomly. Hence, when a worker applies to firm m and $k-1$ other workers have also applied to the same firm, he is assigned to each position in the line with equal probability $1/k$. We also assume, without loss of generality, that $w^{r,k} = 0$ for all $r > k$. We restrict ourselves to non-negative wages, and since in equilibrium a firm will never post a wage greater than the product of a match, for otherwise they would make negative expected profits, we know that wages are bounded above by 1. When firm m posts the wage contract $\mathbf{w}_m$ and $k \geq 1$ workers have applied, its profit is

$$\pi^k(\mathbf{w}_m) = 1 - W^k(\mathbf{w}_m), \tag{1}$$

where $W^k(\mathbf{w}_m) \equiv \sum_{r=1}^{k} w_m^{r,k}$ is the wage bill. It is assumed that firms are owned by entrepreneurs with deep pockets so that no restrictions are put on the size of the wage bill, and in particular firms are allowed to make negative profits ex-post.

Each worker makes one application. When a worker is employed at wage w his utility is $u(w)$ where u is strictly increasing, strictly concave, and twice continuously differentiable with $u(0) = 0$. We also assume that the utility of unemployment is zero. Combined, these two assumptions implicitly imply that there is no disutility from working.[7] We assume, for simplicity, that when facing the choice of being employed at a zero wage or being unemployed, two options that yield the same payoffs, a worker chooses to work.

[7] Section 6 considers the case where workers value leisure.

The game is a one-shot, two-stage game. In the first stage, firms simultaneously decide the contract they each want to post, and this announcement, which firms are committed to, is publicly observable. In the second stage, once workers have observed the posting of all firms, they choose their application strategy, and we denote by $\theta_{n,m} \in [0,1]$ the probability that worker n applies to firm m. As mentioned earlier, since all applications are received at the same time and all workers are identical to the firm, the position of the worker in the queue is determined randomly. Ultimately our goal is to study the existence of labor hoarding contracts in a large, but not necessarily limit, economy, and in this case the anonymity assumption seems natural. Moreover, as Peters (1997) argues,[8] a symmetric equilibrium has the desirable property that the probability with which a deviating firm is chosen depends smoothly on the contract its offers whereas in when coordination is allowed this is not necessarily the case. We therefore maintain this assumption throughout the paper.

If we denote by $\mathcal{W} = \{(w^{r,k})_{r=1,k=1}^{N,N} |\ w^{r,k} \in [0,1]$ and $w^{r,k} = 0$ for all $r > k\}$ the set of wage contracts, and we restrict ourselves to the case where firms use pure strategies, an equilibrium is defined as follows.

Definition 1 *A Nash equilibrium is a strategy profile* $((\mathbf{w}_m)_{m=1}^{M}, (\boldsymbol{\theta}_n)_{n=1}^{N})$, *where* $\mathbf{w}_m$ *is the posting strategy of firm* m *and* $\boldsymbol{\theta}_n = (\theta_{n,m})_{m=1}^{M}$ *is the application strategy of worker* n, *such that:*
(i) Given $\mathbf{w} \equiv (\mathbf{w}_m)_{m=1}^{M}$, $\boldsymbol{\theta} \equiv (\boldsymbol{\theta}_n)_{n=1}^{N}$ *is the Nash equilibrium in the application subgame; and*
(ii) Given $\boldsymbol{\theta} \equiv (\boldsymbol{\theta}_n)_{n=1}^{N}$ *in the subgame, with* $\boldsymbol{\theta}_n(\mathbf{w}) : \mathcal{W}^N \to S^M$, $\mathbf{w} \equiv (\mathbf{w}_m)_{m=1}^{M}$ *is the Nash equilibrium in the contract-posting game, where* S^M *is the* $M-$ *dimensional simplex.*

This problem is a two-stage game, and we solve it by backward induction. We first look at the application subgame in section 3, then we solve for equilibrium in section 4, while Section 5 considers welfare and efficiency. Section 6 concludes.

3 The Application Subgame

Let us consider the problem of worker n. When firms' posting strategy profile is $\mathbf{w}$, and given that all other workers' application strategy profile is $\boldsymbol{\theta}_{-n} \equiv (\boldsymbol{\theta}_i)_{i \neq n}$, the

[8] See pages 103-4.

expected utility of worker n in applying to firm m is independent of the probability with which other workers apply to other firms, and is given by

$$U_{n,m}\left(\mathbf{w}_m;\boldsymbol{\theta}_{-n,m}\right)=\sum_{k=0}^{N-1}p_{N-1}^{k}\left(\boldsymbol{\theta}_{-n,m}\right)u^{e}(\mathbf{w}_m^{k+1}), \tag{2}$$

where $\boldsymbol{\theta}_{-n,m}$ is the vector of application probabilities to firm m by all other workers; $p_{N-1}^{k}\left(\boldsymbol{\theta}_{-n,m}\right)$ is the probability that k other workers, out of the remaining $N-1$ workers, apply to firm m when other workers' application strategy profile is $\boldsymbol{\theta}_{-n}$; and

$$u^{e}(\mathbf{w}_m^{k+1})=\frac{1}{k+1}\sum_{r=1}^{k+1}u(w_m^{r,k+1}) \tag{3}$$

is the expected utility of having applied to firm m when k other workers have applied to that same firm, with $\mathbf{w}_m^{k+1}$ denoting the vector of wages paid by the firm when $k+1$ workers have shown up.

There is one type of contract which is of particular interest. They are such that $w^{r,k}=w$ for all (r,k), $r\leq k\leq v$, for some number $v\leq n$. That is, a firm commits to hire a number v of workers at wage w, and w is not contigent on the number of workers who have applied to the firm. We call such contracts flat-wage contracts**,** FWC(s) hereafter. Since in these contracts firms are de facto posting a fixed number v of vacancies with a fixed wage w, we denote a FWC by $\overline{\mathbf{w}}(v)=w\cdot 1_{N,N}\left(v\right)$, where $1_{N,N}\left(v\right)$ is the matrix of dimension (N,N) filled with ones in its $min\{k,v\}$ first lines for column k, $k=1,...,N$, and zeros elsewhere. If firm m posts a FWC $\overline{\mathbf{w}}(v)$, the expression for $U_{n,m}\left(\mathbf{w}_m;\boldsymbol{\theta}_{-n,m}\right)$ simplifies to

$$U_{n,m}\left(\overline{\mathbf{w}}(v);\boldsymbol{\theta}_{-n,m}\right)=\Omega\left(\boldsymbol{\theta}_{-n,m};v\right)u(w), \tag{4}$$

where

$$\Omega\left(\boldsymbol{\theta}_{-n,m};v\right)\equiv\sum_{k=0}^{N-1}p_{N-1}^{k}\left(\boldsymbol{\theta}_{-n,m}\right)\frac{\min\{k+1,v\}}{k+1} \tag{5}$$

is the probability to be hired by firm m, conditional on having applied, when it posts v vacancies and other workers' application strategy profile is $\boldsymbol{\theta}_{-n}$. Naturally, when a firm posts $v=N$ vacancies, all workers applying to it will end up being employed so that $\Omega\left(\boldsymbol{\theta}_{-n,m};N\right)=1$ and the expected utility of applying to that firm is equal to the utility of being employed at the wage it posted.

The equilibrium probability of application to firm m for worker n must be such that

$$\theta_{n,m}=\begin{cases} 0 & , \quad \text{if } U_{n,m}\left(\mathbf{w}_m;\boldsymbol{\theta}_{-n,m}\right)<Max_{j\neq m}\,U_{n,j}\left(\mathbf{w}_j;\boldsymbol{\theta}_{-n,j}\right); \\ 1 & , \quad \text{if } U_{n,m}\left(\mathbf{w}_m;\boldsymbol{\theta}_{-n,m}\right)>Max_{j\neq m}\,U_{n,j}\left(\mathbf{w}_j;\boldsymbol{\theta}_{-n,j}\right); \text{ and} \\ [0,1] & , \quad \text{if } U_{n,m}\left(\mathbf{w}_m;\boldsymbol{\theta}_{-n,m}\right)=Max_{j\neq m}\,U_{n,j}\left(\mathbf{w}_j;\boldsymbol{\theta}_{-n,j}\right), \end{cases}$$

and $\sum_{m=1}^{M}\theta_{n,m}=1$. We will be focusing on equilibria where workers use symmetric application strategies. In this case, $\boldsymbol{\theta}_i=\boldsymbol{\theta}_j$, for all pairs of workers (i,j), and to save on notation we denote by $\boldsymbol{\theta}=(\theta_1,...,\theta_M)$ their common application strategy where θ_m is the application probability to firm m. When workers use symmetric application strategies, $p_N^k(\theta)$ is simply given by the standard binomial expression

$$p_N^k(\theta)=C_N^k\theta^k(1-\theta)^{N-k}. \tag{6}$$

Furthermore, the probability of being employed by a firm, conditional on having applied, when all other workers apply to that same firm with probability θ is given by (5) where the vector of application strategies $\boldsymbol{\theta}_{-n,m}$ is replaced by θ. This way, (5) can be rewritten as

$$\Omega(\theta;v)=\frac{\Gamma(\theta;v)}{N\theta}, \tag{7}$$

where

$$\Gamma(\theta;v)\equiv\sum_{k=0}^{v}kp_N^k(\theta)+v\left(\sum_{k=v+1}^{N}p_N^k(\theta)\right) \tag{8}$$

is the expected number of workers that the firm will be hiring. This is intuitive: $\theta\Omega(\theta;v)$ is the unconditional probability for a worker to be employed by the firm, which implies that $N\theta\Omega(\theta;v)=\Gamma(\theta;v)$ is the expected number of hires for a firm who is applied to with probability θ; and $\Gamma(\theta;v)$ is given by (8) since if $k\leq v$ workers apply all get hired, whereas if $k>v$ apply only v will be employed. It is proved in the appendix that $\Gamma(\theta;v)$, and therefore $\Omega(\theta;v)$, increases with v.

Hence, the application profile $\boldsymbol{\theta}=(\theta_1,...,\theta_M)$ is a Nash equilibrium in symmetric visit strategies if and only if for all $m\in\{1,...M\}$,

$$\theta_m=\begin{cases} 0 & , \quad \text{if } U_m(\mathbf{w}_m;\theta_m)<Max_{j\neq m}\,U_j(\mathbf{w}_j;\theta_j); \\ 1 & , \quad \text{if } U_m(\mathbf{w}_m;\theta_m)>Max_{j\neq m}\,U_j(\mathbf{w}_j;\theta_j); \text{ and} \\ [0,1] & , \quad \text{if } U_m(\mathbf{w}_m;\theta_m)=Max_{j\neq m}\,U_j(\mathbf{w}_j;\theta_j), \end{cases} \tag{10}$$

and $\sum_{m=1}^{M}\theta_m=1$.

We further assume that when two firms m and j post contracts $\mathbf{w}_m$ and $\mathbf{w}_j$, respectively, such that $u^e(\mathbf{w}_m^k) = u^e(\mathbf{w}_j^k)$ for all k, i.e., the expected utility of having applied to both firms, conditional on the same number of workers having shown up, is the same for all possible queue lengths, then workers treat both firms symmetrically by applying to each of them with the same probability. Although this anonymity assumption is not in general innocuous, it turns out that it does not matter for the main results of the paper. Hence, since it simplifies the presentation and derivation of some of the results, we will maintain this assumption and we will point out the instances where it matters.

Proposition 1 *(i) For all wage posting strategy profiles* $\mathbf{w}$ *a Nash equilibrium in symmetric visit strategies exists.*

(ii) Furthermore, if for all m, $u^e(\mathbf{w}_m^k) \geq (\leq)\, u^e(\mathbf{w}_m^{k+1})$ *for all* k, *then the equilibrium is unique.*

Proof. In the appendix. ■

In standard directed search models wage contracts posted by all firms are FWCs with 1 vacancy, i.e., for all m $\overline{\mathbf{w}}_m(1) = w_m \cdot 1_{N,N}(1)$. Hence, $U(\overline{\mathbf{w}}_m(1);\theta_m) = \Omega(\theta_m; 1)\, u(w_m)$, and the expected utility of applying to firm m, is such that

$$\frac{\partial U(\overline{\mathbf{w}}_m(1);\theta_m)}{\partial \theta_m} = \frac{\partial \Omega(\theta_m; 1)}{\partial \theta_m} u(w_m) < 0,$$

which is intuitive since as θ increases the competition for the vacancy posted by the firm increases, and therefore $\Omega(\theta_m; 1)$, the probability of being employed conditional on having applied, decreases. Here is the intuition why the application subgame has a unique equilibrium: if we assume that both $\boldsymbol{\theta}$ and $\widetilde{\boldsymbol{\theta}}$ are equilibria of the application subgame of the standard directed search model, take any two firms m and j such that $\theta_m > \widetilde{\theta}_m \geq 0$ and $\widetilde{\theta}_j > \theta_j \geq 0$. Since $\partial\Omega(\theta; 1)/\partial\theta < 0$,

$$U(\overline{\mathbf{w}}_m(1);\widetilde{\theta}_m) > U(\overline{\mathbf{w}}_m(1);\theta_m) \geq U(\overline{\mathbf{w}}_j(1);\theta_j) > U(\overline{\mathbf{w}}_j(1);\widetilde{\theta}_j),$$

which contradicts the fact that $\widetilde{\theta}_j > 0$. It is clear that the uniqueness of equilibrium is guaranteed by the fact that the probability of employment at a each firm strictly, and therefore the expected utility offered by each firm, strictly decreases with the probability the firm is applied is applied to.

In this paper firms are allowed to post any contract, and in this case

$$\frac{\partial U(\mathbf{w}_m;\theta_m)}{\partial \theta_m} = \sum_{k=0}^{N-1} \frac{k p_{N-1}^k(\theta_m)}{\theta_m} \left[u^e(\mathbf{w}_m^{k+1}) - u^e(\mathbf{w}_m^k)\right].$$

When the contracts posted are such that for all firms m, $u^e(\mathbf{w}_m^k) \geq u^e(\mathbf{w}_m^{k+1})$ for all k, and $u^e(\mathbf{w}_m^k) > u^e(\mathbf{w}_m^{k+1})$ for some k, then the expected utility of applying to each firm is also strictly decreasing in the probability of application, exactly as in the one vacancy case. However, it turns out that if there is a unique firm, say firm i, for which $u^e(\mathbf{w}_i^k) = u^e(\mathbf{w}_i^{k+1})$ for all k, i.e., the payoffs of applying to firm i are independent of the number of workers applying, then the equilibrium is still unique. This reasoning can then be used to see, as proposition 1 establishes, that with anonymity, as long as the expected payoffs of applying to all firms are either all weakly decreasing or all weakly increasing in the number k of workers who show up, then the equilibrium is unique. This is because if two firms m and i offer contracts that are such that for all k $u^e(\mathbf{w}_m^k) = u^e(\mathbf{w}_m^{k+1}) = u^e(\mathbf{w}_m)$ and $u^e(\mathbf{w}_i^k) = u^e(\mathbf{w}_i^{k+1}) = u^e(\mathbf{w}_i)$, then if $u^e(\mathbf{w}_m^k) < (>) u^e(\mathbf{w}_i^k)$ no worker will ever apply to firm m (i), and if $u^e(\mathbf{w}_m^k) = u^e(\mathbf{w}_i^k)$, then by anonymity all workers will apply to both firms with the same probability (which might be zero). Therefore, anonymity implies that all firms posting contracts whose payoffs are independent of the number of workers applying, and which are applied to in equilibrium, can all be treated symmetrically.

Corollary 1 *If all firms post FWCs, then there exists a unique Nash equilibrium in symmetric application strategies.*

Proof. If all firms post 1 vacancy, then $u^e(\mathbf{w}_m^k) > u^e(\mathbf{w}_m^{k+1})$ for $k \geq 1$; if all firms post FWCs with $1 < v \leq N-1$ vacancies, then for all m, $u^e(\mathbf{w}_m^k) = u^e(\mathbf{w}_m^{k+1})$ for all $k \leq v-1$ and $u^e(\mathbf{w}_m^k) > u^e(\mathbf{w}_m^{k+1})$ for $k > v-1$, and if $v = N$ then $u^e(\mathbf{w}_m^k) = u^e(\mathbf{w}_m^{k+1})$ for all $k \leq N-1$, then the result then follows from proposition 1. ■

It is worth highlighting that without the anonymity assumption regarding the way workers treat firms for their application strategies, the conditions that guarantee uniqueness in proposition 1 and corollary 1 would be more stringent. If we consider corollary 1 (the same reasoning applies to proposition 1), the condition to be satisfied to guarantee uniqueness would be that there must be *at most* one firm posting a FWC with N vacancies, because otherwise there would be a continuum of equilibria. In fact, consider such a posting profile with two firms m and i posting FWCs with N vacancies and the same wage w. If $\boldsymbol{\theta}$, one equilibrium of the application subgame, is such that $\theta_m \geq 0$ and $\theta_i \geq 0$ with $\theta_m + \theta_i > 0$, then, defining $\overline{\theta}$ as the sum of θ_m and θ_i, it follows that any $\widetilde{\boldsymbol{\theta}}$ such that $\widetilde{\theta}_m + \widetilde{\theta}_i = \overline{\theta}$ and $\widetilde{\theta}_j = \theta_j$ for all $j \neq m, i$ is also an equilibrium. This is because the expected utility of applying to firms posting FWCs with N vacancies is independent of the probability with which other workers apply to these firms since $u^e(\overline{\mathbf{w}}_m^k(N)) = u(w_m)$ for all k. Hence, it appears that when more than one firm post FWCs with N vacancies anonymity conveniently rules out all but one equilibrium, which eliminates indeterminacy issues. Note, however, that all these equilibria are payoff-equivalent for workers.

Even with anonymity, uniqueness cannot be guaranteed when firms post non-FWCs. To illustrate this, let us consider a specific economy with two firms and three workers, and consider worker 1. When the two other workers apply to firm 1 with probability θ_1, the expected utility of workers in applying to firms 1 and 2 are respectively

$$\begin{aligned} U_1(\mathbf{w}_1;\theta_1) &= (1-\theta_1)^2 u^e\left(\mathbf{w}_1^1\right) + 2\theta_1(1-\theta_1) u^e\left(\mathbf{w}_1^2\right) + \theta_1^2 u^e\left(\mathbf{w}_1^3\right), \text{ and} \\ U_2(\mathbf{w}_1;\theta_1) &= \theta_1^2 u^e\left(\mathbf{w}_2^1\right) + 2\theta_1(1-\theta_1) u^e\left(\mathbf{w}_2^2\right) + (1-\theta_1)^2 u^e\left(\mathbf{w}_2^3\right). \end{aligned} \tag{11}$$

It is clear that if $u^e\left(\mathbf{w}_1^1\right) = u^e\left(\mathbf{w}_2^3\right)$, $u^e\left(\mathbf{w}_1^2\right) = u^e\left(\mathbf{w}_2^2\right)$ and $u^e\left(\mathbf{w}_1^3\right) = u^e\left(\mathbf{w}_2^1\right)$, then $U_1(\mathbf{w}_1;\theta_1) = U_2(\mathbf{w}_1;\theta_1)$ for all $\theta_1 \in [0,1]$. This is because in this case the wage contracts posted by the two firms are such that no matter what application strategy two of the three workers choose, the third worker is indifferent between applying to either firms: $u^e\left(\mathbf{w}_1^1\right) = u^e\left(\mathbf{w}_2^3\right)$ implies that if the two other workers apply to firm 2, then worker 1 is indifferent between applying to firm 1 or 2, and so on. In the case of such contract postings any $\theta_1 \in [0,1]$ is a symmetric equilibrium of the application subgame, and all equilibria are payoff-equivalent for the workers. This type of multiplicity does not exist in standard directed search models because in these models firms are restricted to posting FWCs with 1 vacancy. Thus, when both firms post FWCs with 1 vacancy we have that $u^e\left(\mathbf{w}_1^1\right) = u(w_1) > u^e\left(\mathbf{w}_1^2\right) = u(w_1)/2 > u^e\left(\mathbf{w}_1^3\right) = u(w_1)/3$ and $u^e\left(\mathbf{w}_2^3\right) = u(w_2)/3 < u^e\left(\mathbf{w}_2^2\right) = u(w_2)/2 < u^e\left(\mathbf{w}_2^1\right) = u(w_2)$, and therefore it is not possible to have $u^e\left(\mathbf{w}_1^1\right) = u^e\left(\mathbf{w}_2^3\right)$, $u^e\left(\mathbf{w}_1^2\right) = u^e\left(\mathbf{w}_2^2\right)$ and $u^e\left(\mathbf{w}_1^3\right) = u^e\left(\mathbf{w}_2^1\right)$.

There is another type of situation in which multiplicity of equilibria can be obtained with general contracts, but do not exist in standard directed search models. Focusing on interior solutions, workers apply to firms 1 and 2 with some positive probability if and only if $U_1(\mathbf{w}_1;\theta_1) = U_2(\mathbf{w}_1;\theta_1)$, that is, subtracting $U_2(\mathbf{w}_1;\theta_1)$ from $U_1(\mathbf{w}_1;\theta_1)$ as given in (11), if and only if

$$\theta_1^2 \times \left[\left(u^e\left(\mathbf{w}_1^1\right) + u^e\left(\mathbf{w}_1^3\right) - 2u^e\left(\mathbf{w}_1^2\right)\right) - \left(u^e\left(\mathbf{w}_2^1\right) + u^e\left(\mathbf{w}_2^3\right) - 2u^e\left(\mathbf{w}_2^2\right)\right)\right]$$

$$+\theta_1 \times 2\left[\left(u^e\left(\mathbf{w}_1^2\right)-u^e\left(\mathbf{w}_1^1\right)\right)-\left(u^e\left(\mathbf{w}_2^2\right)-u^e\left(\mathbf{w}_2^3\right)\right)\right]+\left[u^e\left(\mathbf{w}_1^1\right)-u^e\left(\mathbf{w}_2^3\right)\right]=0 \tag{12}$$

Given the posting profile $\mathbf{w}$, this is an equation of the second degree in θ_1, which can therefore have 0, 1 or 2 solutions. Note that since θ_1 is the probability with which workers apply to firm 1, the only relevant root(s) are the one(s) whose value is between 0 and 1. Assume, for simplicity, that $u^e\left(\mathbf{w}_1^1\right)=u^e\left(\mathbf{w}_2^3\right)$ so that (12) simplifies to

$$\theta_1\left\{\left[\left(u^e\left(\mathbf{w}_1^3\right)-2u^e\left(\mathbf{w}_1^2\right)\right)-\left(u^e\left(\mathbf{w}_2^1\right)-2u^e\left(\mathbf{w}_2^2\right)\right)\right]\times\theta_1+2\left[u^e\left(\mathbf{w}_1^2\right)-u^e\left(\mathbf{w}_2^2\right)\right]\right\}=0.$$

In this case the two solutions to this equation are $\theta_1=0$ and

$$\widetilde{\theta}_1=\frac{2\left[u^e\left(\mathbf{w}_2^2\right)-u^e\left(\mathbf{w}_1^2\right)\right]}{\left(u^e\left(\mathbf{w}_1^3\right)-u^e\left(\mathbf{w}_2^1\right)\right)+2\left(u^e\left(\mathbf{w}_2^2\right)-u^e\left(\mathbf{w}_1^2\right)\right)}.$$

It is clear that if $u^e\left(\mathbf{w}_2^2\right)>u^e\left(\mathbf{w}_1^2\right)$ and $u^e\left(\mathbf{w}_1^3\right)>u^e\left(\mathbf{w}_1^2\right)$, then $\widetilde{\theta}_1\in(0,1)$, which shows that with non-FWCs it is possible to have multiple equilibria for the application subgame.

When all firms post FWCs with 1 vacancy, equation (12) can be rewritten as

$$\theta_1^2\times\left(\frac{u(w_1)-u(w_2)}{3}\right)-\theta_1\times\left(\frac{3u(w_1)+u(w_2)}{3}\right)+\left(u(w_1)-u(w_2)\right)=0 \tag{13}$$

In the appendix it is shown that although this equation in θ_1 can have two solutions, there is at most one of them whose value is between zero and one, which confirms the result that in standard directed search models there is a unique symmetric solution to the application subgame. With more general contracts, firms can post wages that are contingent on the number of workers who applied, and the additional degrees of freedom that this gives to firms compared to the one-vacancy model make it possible to find more than one solution to equation (2) with values between zero and one.[9]

[9] More generally, when there are M firms and N workers, the expected utility of applying to firm m is

$$U_m\left(\mathbf{w}_m;\theta_m\right)=\sum_{k=0}^{N-1}p_{N-1}^k\left(\theta_m\right)u^e\left(\mathbf{w}_m^{k+1}\right).$$

Hence, assuming an interior solution, the expected utility of applying to any two firms m and i is the same so that we have that $\boldsymbol{\theta}$ is such that $U_m\left(\mathbf{w}_m;\theta_m\right)=U_i\left(\mathbf{w}_i;\theta_i\right)$ for all (m,i), implying that for all (m,i)

$$\sum_{k=0}^{N-1}p_{N-1}^k\left(\theta_m\right)u^e\left(\mathbf{w}_m^{k+1}\right)=\sum_{k=0}^{N-1}p_{N-1}^k\left(\theta_i\right)u^e\left(\mathbf{w}_i^{k+1}\right).$$

Hence, taking firm M as the reference, we have that $\boldsymbol{\theta}=\left(\theta_1,\theta_2,...,\mathbf{1}-\sum_{m=1}^{M-1}\theta_m\right)$ and therefore the vector of dimension $M-1$ $(\theta_1,\theta_2,...,\theta_{M-1})$ is the solution to the system of $(M-1)$ polynomial equations of degree N.

4 Equilibrium

In this section we first look into the optimal contract posting decision of a firm before showing the existence and uniqueness of what we call a FWC-equilibrium.

4.1 A Firm's Posting Decision

Firm m's expected profit when it posts $\mathbf{w}_m$ and it is applied to with probability θ is

$$\Pi(\mathbf{w}_m;\theta) = \sum_{k=1}^{N} p_N^k(\theta)\pi^k(\mathbf{w}_m), \qquad (14)$$

where $\pi^k(\mathbf{w}_m)$ is given by (1). It is clear that, conditional on k workers applying, the composition of the firm's wage bill $W_m^k(\mathbf{w}_m)$ does not matter for the firm. But intuitively, it does matter for workers because they are risk averse.

Lemma 1 *For all $\mathbf{w}_{-m}$ the best-response of firm m is to post a contract $\mathbf{w}_m$ such that $w_m^{r,k} = w_m^k$ for all $r \leq k$.*

Proof. Consider a contract $\mathbf{w}_m$ such that for some k $w_m^{r,k} \neq w_m^{p,k}$, for $r, p \leq k$. If instead firm m were to post the contract $\widetilde{\mathbf{w}}_m$ such that for some k, $\widetilde{w}_{jm}^{r,k} = \widetilde{w}_m^k$ for all $r \leq k$, and such that $u(\widetilde{w}_m^k) = u^e(\mathbf{w}_m^k)$, then the firm offers to the workers the same conditional expected utility of applying for the two contracts. And by offering $\widetilde{\mathbf{w}}_m$ instead of $\mathbf{w}_m$ it reduces its wage bill when k workers show up to $W_m^k(\widetilde{\mathbf{w}}_m) = k\widetilde{w}_m^k$, and therefore $\pi^k(\widetilde{\mathbf{w}}_m) > \pi^k(\mathbf{w}_m)$. In fact, the strict concavity of u yields that

$$W_m^k(\mathbf{w}_m) - W_m^k(\widetilde{\mathbf{w}}_m) = \sum_{r=1}^{k} w_m^{r,k} - ku^{-1}\left[\frac{1}{k}\sum_{r=1}^{k} u(w_m^{r,k})\right] > 0.$$

Furthermore, for all k $u^e(\widetilde{\mathbf{w}}_m) = u^e(\mathbf{w}_m^k)$, and therefore the equilibrium application strategy profiles of the application subgame implied by $\mathbf{w} = (\mathbf{w}_m, \mathbf{w}_{-m})$ and $\widetilde{\mathbf{w}} = (\widetilde{\mathbf{w}}_m, \mathbf{w}_{-m})$ are the same, and we denote them by $\boldsymbol{\theta}^1, ..., \boldsymbol{\theta}^L$, $L \geq 1$.[10] Hence, since for all $l = 1, ..., L$, $\Pi(\widetilde{\mathbf{w}}_m; \theta_m^l) > \Pi(\mathbf{w}_m; \theta_m^l)$ it is a best-response to other firm posting $\mathbf{w}_{-m}$ for firm m to post a contract such that $w_m^{r,k} = w_m^k$ for all $r \leq k$. ■

The intuition for the result is that since workers are risk averse, when a firm lowers the variability of the payoffs associated with the employment contract it is posting, risk-averse workers value this decreased variability. This then enables the

[10] This is true because we assume anonymity.

firm to reduce its expected wage bill by reducing the expected wage promised in its contract for each possible number of workers showing up, k. But since for each possible number of workers applying the firm offers the same expected utility, workers apply to the firm with the same probability, which means the firm's expected profit has increased for each possible equilibrium probability of the application subgame.

With this intuition in mind it is tempting to draw as a conclusion that a firm maximizes its profit if and only if it posts a FWC with N vacancies. However, there are two subtleties, one related to the possibility of multiple solutions to the application subgame and the other related to the issue of how firms posting the same contracts are treated, which imply that it is not necessarily the case that posting a FWC with N vacancies is the best-reponse to other firms' postings. The following lemma gives sufficient conditions under which posting a FWC with N vacancies is indeed the best-reponse.

Lemma 2 *If $\mathbf{w}_{-m}$ is such that for all firms $i \neq m$, $u^e(\mathbf{w}_i^k) \geq (\leq)\, u^e(\mathbf{w}_i^{k+1})$ for all k, with $u^e(\mathbf{w}_i^k) > (<)\, u^e(\mathbf{w}_i^{k+1})$ for some k, then the best-response for firm m is to post a FWC with N vacancies.*

Proof. In the Appendix. ■

The reason why it might not be a best-reponse, even weak, for a firm to post a FWC if other firms' contract posting profile $\mathbf{w}_{-m}$ is such that for some i, $u^e(\mathbf{w}_i^k) > u^e(\mathbf{w}_i^{k+1})$ for some k and $u^e(\mathbf{w}_i^{k'}) < u^e(\mathbf{w}_i^{k'+1})$ for some k' is as follows. In this situation there might be more than one equilibrium strategy profile associated with a wage contract posting $\mathbf{w}_m$ for firm m, no matter whether firm m posts a FWC. Let us suppose, without loss of generality, that when firm m posts the contract $\mathbf{w}_m$ which is not a FWC with N vacancies, it induces two possible application strategy profiles $\boldsymbol{\theta}$ and $\widetilde{\boldsymbol{\theta}}$ such that $\theta_m \neq \widetilde{\theta}_m$. If firm m could choose which of the two application probabilities workers will use, it is true that for each of these application profiles the firm could do better by posting a flat-wage contract: either $\overline{\mathbf{w}}_m$ or $\widetilde{\overline{\mathbf{w}}}_m$ such that $U\left(\overline{\mathbf{w}}_m;\theta_m\right) = U\left(\mathbf{w}_m;\theta_m\right)$ and $U(\widetilde{\overline{\mathbf{w}}}_m;\widetilde{\theta}_m) = U(\mathbf{w}_m;\widetilde{\theta}_m)$, depending on whether it wishes to receive applications with probability θ_m or $\widetilde{\theta}_m$. In fact, for each of the two application strategy profiles, firm m can reduce the variability in payoffs of applying to it by posting FWCs. And since workers are risk averse, FWCs that deliver the same expected utility as with $\mathbf{w}_m$ imply lower expected wage bills for the firm, and therefore $\Pi(\overline{\mathbf{w}}_m \mid \theta_m) > \Pi(\mathbf{w}_m \mid \theta_m)$ and $\Pi(\widetilde{\overline{\mathbf{w}}}_m \mid \widetilde{\theta}_m) > \Pi(\mathbf{w}_m \mid \widetilde{\theta}_m)$. However, given that $\mathbf{w}_{-m}$ is such that for some i, $u^e(\mathbf{w}_i^k) > u^e(\mathbf{w}_i^{k+1})$ for some

k and $u^e(\mathbf{w}_i^{k'}) < u^e(\mathbf{w}_i^{k'+1})$ for some k', there can be more than one equilibrium application strategy profile associated with firm m posting $\overline{\mathbf{w}}_m$. If there are two equilibrium application profiles $\boldsymbol{\theta}$ and $\widehat{\boldsymbol{\theta}}$ in this case, although we know for sure that $\Pi(\overline{\mathbf{w}}_m \mid \boldsymbol{\theta}) > \Pi(\mathbf{w}_m \mid \boldsymbol{\theta})$, it is not possible to rule out that $\Pi(\mathbf{w}_m \mid \boldsymbol{\theta}) > \Pi(\overline{\mathbf{w}}_m \mid \widehat{\boldsymbol{\theta}})$. Hence, if firm m were to change its contract posting from $\mathbf{w}_m$ to $\overline{\mathbf{w}}_m$, it would gain if workers were to apply according to $\boldsymbol{\theta}$, but it would be worse off if they were to apply according to $\widehat{\boldsymbol{\theta}}$. And the same applies for the contract posting $\widetilde{\overline{\mathbf{w}}}_m$. Hence, in this case it is not necessarily the case the switching to a FWC is profitable, which means that posting a FWC is not necessarily a best-response to other firms' postings.

If instead $\mathbf{w}_{-m}$ is such that there exists at least one firm i for which $u^e(\mathbf{w}_i^k) = u^e(\mathbf{w}_i^{k+1})$ for all k, whether or not it is a FWC, and all other firms j different from i and m post contracts such that $u^e(\mathbf{w}_j^k) \geq u^e(\mathbf{w}_j^{k+1})$ for all k, with $u^e(\mathbf{w}_j^k) > u^e(\mathbf{w}_j^{k+1})$ for some k, then posting a FWC with N vacancies is not necessarily the best-response for firm m either. Assume, without loss of generality, that there is a unique firm i which posts a contract $\mathbf{w}_i$ such that $u^e(\mathbf{w}_i^k) = u^e(\mathbf{w}_i^{k+1}) = u(\mathbf{w}_i)$ for all k. Then $U(\mathbf{w}_i; \theta_i)$ is independent of the probability with which other workers apply to firm i, and let us denote it by $\overline{U}_i$. Hence, if firm m posts a FWC with N vacancies and wage w such that $u(w) < \overline{U}_i$, then no worker applies to firm m, in which case its expected profit is zero. If instead, it chooses a wage w such that $u(w) \geq \overline{U}_i$, then workers might apply. In particular, if $u(w) = \overline{U}_i$, then workers are indifferent between applying to firms m and i, and in this case, by anonymity, workers apply to both firms with the same probability. Hence, given $\mathbf{w}_{-m}$ and that firm m posts a FWC $\overline{\mathbf{w}}_m$(N)=u$^{-1}\left(\overline{U}_i\right) \cdot 1_{N,N}(N), firm m will be applied to with some probability, say \widetilde{\theta}$. Assume that $\widetilde{\theta} > 0$. Then, if $\overline{\mathbf{w}}_m$ is such that $u(w_m) = \overline{U}_i + \epsilon$, $\epsilon > 0$ arbitrarily small, by continuity of workers' application strategies in firms' wage postings, firm m is applied to with probability $\overline{\theta} = 2\widetilde{\theta} + \eta$, $\eta > 0$ small.

We can represent graphically how firm m's expected profit varies with the wage posted. Firm m's expected profit is zero for all $w_m < u^{-1}\left(\overline{U}_i\right)$, and if $w_m = u^{-1}\left(\overline{U}_i\right)$ then expected profits are given by $\Pi(\overline{\mathbf{w}}_m(N); \widetilde{\theta})$. If firm m instead posts a FWC such that $u(w_m) = \overline{U}_i + \epsilon$, $\epsilon > 0$ arbitrarily small, then there are two cases to consider. In the first case, depicted in figure 1, when firm m posts a FWC with a wage that exceeds what firm i offers its expected profit jumps to a level greater than $\Pi(\overline{\mathbf{w}}_m(N); \widetilde{\theta})$, it continues to increase as w_m increases up to $\widetilde{w}$, beyond which expected profits decline.[11] It is also possible to represent how the expected profit of firm m varies with θ, as is done in panel b of figure 1: the firm is applied to with probability

$$\theta_m = \begin{cases} 0 & , \text{ if } w_m < u^{-1}\left(\overline{U}_i\right); \\ \widetilde{\theta} & , \text{ if } w_m = u^{-1}\left(\overline{U}_i\right); \\ \theta_m\left(\overline{\mathbf{w}}_m(N); \mathbf{w}_{-m}\right) > 2\widetilde{\theta} & , \text{ if } w_m > u^{-1}\left(\overline{U}_i\right), \end{cases}$$

where $\theta_m\left(\overline{\mathbf{w}}_m(N); \mathbf{w}_{-m}\right)$ is the equilibrium application probability for firm m for the unique equilibrium application profile $\boldsymbol{\theta}\left(\overline{\mathbf{w}}_m(N); \mathbf{w}_{-m}\right)$ for $w_m > u^{-1}\big(\overline{U}_i\big)$. Note that $\theta_m\left(\overline{\mathbf{w}}_m(N); \mathbf{w}_{-m}\right)$ increases with w_m,[12] and reaches 1 for some wage less than 1. In this first case, posting a FWC with N vacancies with wage $\widetilde{w}$ is for firm m a best-response to other firms posting $\mathbf{w}_{-m}$. In the second case, which is depicted in figure 2, when firm m posts a FWC with a wage that exceeds what firm i offers, its expected profits jump to a level lower than $\Pi(\overline{\mathbf{w}}_m(N); \widetilde{\theta})$, and then it decrease as w_m increases.[13] On panel b of figure 2 we can see how in this case expected profits for firm m vary with θ_m.

From the graphical representation of firm m's profits as a function of the application probability θ_m, one can see that a firm's profit maximization problem can either be thought of as choosing the wage contract to post, or as first choosing the best contract to post for each possible probability of application, and then the firm chooses the probability with which it wants workers to apply. This second way of thinking about a firm's maximization problem makes it clear what are the potential problems for firm m to choose a FWC with N vacancies: firm m cannot freely choose the application probability. In fact, θ_m is limited to be either 0, $\widetilde{\theta}$ or a value between $(2\widetilde{\theta}, 1]$. For the first case discussed above, this is not a problem since the application probability choice for a FWC with N vacancies is in the range of values that firm m can choose from. But for the second case this is not true anymore. Now suppose that instead of posting a FWC with N vacancies, firm m were to post a FWC with $(N-1)$ vacancies. With this posting the application probability θ_m is

[11] Profits when $w_m = 1$ will be negative because this is the highest possible wage a firm can offer, and firm m guarantees employment, whereas other firms do not, implying the contract firm m offers strictly dominates all other firms' offers. Hence, firm m's expected profits are in this case $1 - n \times 1 = -(n-1) < 0$.

[12] As firm m increases its wage, it becomes more attractive to workers and therefore they will apply to firm m with a (weakly) greater probability.

[13] If Profits are lower than $\Pi(\overline{\mathbf{w}}_m(N); \widetilde{\theta})$ the same result can be obtained. What is important is that profits fall for all $\theta < \widehat{\theta}$ for some $\widehat{\theta} < \overline{\theta}$.

such that

$$\theta_m = \begin{cases} 0, & \text{if } w_m < \widehat{w}\left(\mathbf{w}_{-m}\right); \\ \theta_m\left(\overline{\mathbf{w}}_m(N-1); \mathbf{w}_{-m}\right), & \text{if } w_m > \widehat{w}\left(\mathbf{w}_{-m}\right), \end{cases}$$

where $\widehat{w}\left(\mathbf{w}_{-m}\right) > u^{-1}\left(\overline{U}_i\right)$ is the minimum wage firm m must post if it wants workers to apply given that other firms' postings are given by $\mathbf{w}_{-m}$. Clearly firm m is not minimizing its wage bill for $\theta_m \in (2\widetilde{\theta}, 1]$ as figure 3 suggests: the expected profit function for $v = N - 1$ is in red whereas the blue line still represents the expected profit for $v = N$. But by posting a FWC with $N-1$ vacancies it can indirectly choose the probability with which workers apply. In the first case discussed above, since a FWC with $N - 1$ vacancies delivers lower expected profits than the best FWC with N vacancies for all values of θ - see figure 3, this is true at $\theta_m\left(\widetilde{w} \cdot 1_{N,N}(N); \mathbf{w}_{-m}\right)$ and as mentioned above this is not a problem in this case that firm m is restricted in its choice of application probability. But in the second case (see figure 4), it is possible that the maximum expected profits that firm m can achieve by posting a FWC with $N - 1$ vacancies exceeds what it can obtain when posting a FWC with N vacancies because in this former case it can freely choose the probability of application workers will use.[14]

Corollary 2 *If all $i \neq m$, $\mathbf{w}_i = w_i \cdot 1_{N,N}(v_i)$ with $v_i < N$, then the best-response for firm m is to post a FWC with N vacancies.*

4.2 FWC-Equilibrium

In this section we characterize equilibria where all firms post FWCs, which we call FWC-equilibria. From corollary 2 in the previous section we know that if all firms except one firm, say firm m, post FWCs with less than N vacancies, then firm m's best-response is to post a FWC with N vacancies. This implies that in a FWC-equilibrium there must be at least one firm posting N vacancies. It turns out that the unique FWC-equilibrium is such that all firms post N vacancies.

Proposition 2 *In an economy with N workers and M firms, there is a unique FWC-equilibrium. It is such that all firms post N vacancies at wage*

$$w^*_{N,M}[N] = \left(1 - \frac{1}{M}\right)^{N-1}, \tag{15}$$

[14] In this case anonymity is where the problem is originating from for FWC with N vacancies. However, it can be shown that when anonymity is assumed away, another (bigger) problem arises: a firm still cannot (indirectly) choose the application probability which results in indeterminacy.

and workers apply to each firm with probability $1/M$*.*

Proof. In the Appendix. ■

The intuition for the result is as follows. First, it follows from lemma 2 that a FWC-equilibrium must be such that at least one firm posts N vacancies, since otherwise firms would have an incentive to deviate and post a FWC with N vacancies. And if all other firms post the FWC $\overline{\mathbf{w}}(N) = w \cdot 1_{N,N}(N)$, then when firm m posts a FWC at wage w_m it will be applied to with probability 0 if $w_m < w$, $1/M$ if $w_m = w$ and 1 if $w_m > w$. But if firm m instead chooses to post an almost-FWC that approximates the FWC $\overline{\mathbf{w}}(N) = w \cdot 1_{N,N}(N)$, it can choose the probability with which workers will apply. In fact, by posting a contract for which there is some variability in workers' payoffs, the firm makes the expected utility of applying dependent on the probability with which other workers apply. For instance, a firm can post a contract that pays $\overline{w}$ to all workers who have applied when at most $N-1$ did apply and such that $u(\overline{w}) = u(w) + \epsilon$, for $\epsilon > 0$ arbitrarily small, and that pays $\underline{w}$ to all workers when all N workers have applied and such that $u(\underline{w}) = u(w) - \delta$, for $\delta > 0$ arbitrarily small. When other workers apply to this firm with probability θ the probability that the $N-1$ other workers have applied is θ^{N-1}, and the probability that at most $N-2$ other workers have applied is $1-\theta^{N-1}$, and therefore the expected utility of applying to a firm posting such a contract is $\theta^{N-1}u(\underline{w}) + (1-\theta^{N-1})u(\overline{w})$. It is clear that for each pair $(\underline{w}, \overline{w})$ there is a unique $\theta \in [0, 1]$ such that

$$\theta^{N-1}u(\underline{w}) + (1-\theta^{N-1})u(\overline{w}) = u(w), \tag{16}$$

and therefore for all $\theta \in [0, 1]$ a firm can induce workers to apply to it with this probability by posting a pair $(\underline{w}, \overline{w})$ satisfying (16) for the desired θ. In particular, if we denote by $\theta^*(w)$ the probability with which a firm would like workers to apply given that all other firms post the FWC $\overline{\mathbf{w}}(N) = w \cdot 1_{N,N}(N)$, a firm can indeed induce workers to apply with probability $\theta^*(w)$ by posting a contract $\widetilde{\overline{\mathbf{w}}}(N)$ such that the two wages satisfy (16) for $\theta = \theta^*(w)$. Naturally, risk-averse workers must be compensated for any additional variability in payoffs arising from the existence of two possible wages compared to the contract offering $\overline{\mathbf{w}}(N)$. Hence, the expected wage bill associated with the contract posting $\widetilde{\overline{\mathbf{w}}}(N)$ exceeds the expected wage bill for $\overline{\mathbf{w}}(N)$. However, for a small enough variation in the two wages $\underline{w}$ and $\overline{w}$ the extra cost compared to the full FWC in terms of the wage bill is small, so that posting an almost FWC is more profitable than posting $\overline{\mathbf{w}}(N)$. From here one can intuit that if the probability $\theta^*(w)$ that maximizes a firm's expected profit when posting the

FWC with N vacancies and with wage w is equal to $1/M$, then no firm wants to deviate and post an almost-FWC. Therefore, a firm's best-response in this case is to post the same FWC as other firms. And for any other candidate FWC-equilibrium there is at least one firm which has a profitable deviation by posting either a FWC or an almost-FWC.

Hence, in equilibrium all workers are employed. This is reminiscent of the implicit contract literature (see for instance Azariadis, 1975). However, in the implicit contract literature firms are insuring workers against wage fluctuations due to fluctuations in the level of economic activity whereas in our model firms insure workers against the risk of unemployment in an environment without shocks.[15]

Payoffs in the Finite Economy Case. Workers expected utility in this case is simply

$$U^*_{N,M}[N] = u\left[\left(1 - \frac{1}{M}\right)^{N-1}\right]. \tag{17}$$

A firm's expected profit is given by (14) for $\mathbf{w}_m = \overline{\mathbf{w}}^*_{N,M}[N] = w^*_{N,M}[N] \cdot 1_{N,N}(N)$, with

$$\pi^k(\overline{\mathbf{w}}^*_{N,M}[N]) = 1 - kw^*_{N,M}.$$

Replacing $w^*_{N,M}[N]$ by its expression in (15), we have that

$$\Pi^*_{N,M}[N] = \sum_{k=1}^{N} p^k_N \left(\frac{1}{M}\right)\left(1 - k\left(1 - \frac{1}{M}\right)^{N-1}\right),$$

which simplifies to[16]

$$\Pi^*_{N,M}[N] = 1 - \left(1 - \frac{1}{M}\right)^N - \frac{N}{M}\left(1 - \frac{1}{M}\right)^{N-1}. \tag{18}$$

[15] The result in the implicit literature that delivers employment fluctuations is due to some restrictive assumptions (see Akerlof and Miyazaki, 1980).

[16] This because

$$\begin{aligned} & \sum_{k=1}^{N} p^k_N \left(\frac{1}{M}\right)\left(1 - k\left(1 - \frac{1}{M}\right)^{N-1}\right) \\ = & \sum_{k=1}^{N} p^k_N \left(\frac{1}{M}\right) - \left(1 - \frac{1}{M}\right)^{N-1} \sum_{k=1}^{N} kp^k_N \left(\frac{1}{M}\right) \\ = & 1 - \left(1 - \frac{1}{M}\right)^N - \left(1 - \frac{1}{M}\right)^{N-1} \times \frac{N}{M}, \end{aligned}$$

the last line following from the expression for the mean of a binomial distribution: $\sum_{k=1}^{N} kp^k_N (1/M) = N/M$.

Since each firm has only one job, not all matches are productive: when more than one worker apply to one firm, only one unit of the good is produced, exactly as if only one worker showed up. Since each firm is applied to with probability $1/M$, the probability that a firm receives at least one application is $1-(1-1/M)^N$, and and therefore the expected number of productive matches is given by

$$P(N,M) = M\left[1-\left(1-\frac{1}{M}\right)^N\right],$$

which corresponds to the number of matches in the standard directed search model without labor hoarding.[17]

Payoffs in the Limiting Case. If we denote by b the ratio of workers to firm, i.e., $b \equiv N/M$, and we let N go to infinity, the contract posted $\overline{\mathbf{w}}^*_{\infty,b}$ in this case has all firms offering the wage

$$w^*_{\infty,b} \equiv \lim_{N\to\infty}\left(1-\frac{b}{N}\right)^{N-1} = e^{-b}.$$

Hence, the expected utility enjoyed by workers and the level of expected profit for firms are then

$$U^*_{\infty,b} = u(e^{-b}) \text{ and } \Pi^*_{\infty,b} = 1-e^{-b}-be^{-b}.$$

Free-Entry. Since $\Pi^*_{N,M} > \Pi^*_{N,M+1}$ for all $M \geq 2$,[18] it is straightforward to endogenize the number of active firms by assuming that there is a large number of inactive firms and that there is a fixed cost c for a firm to become active. Then, assuming free-entry, $M(N;c)$, the number of active firms when there are $N<\infty$ workers in the economy and the cost of entry is c, is such that

$$\Pi^*_{N,M} \geq c \text{ and } \Pi^*_{N,M+1} < c. \tag{19}$$

It is straightforward, although a bit tedious, to show that $M(N;c)$ is weakly decreasing with the set-up cost c and weakly increasing in the number of workers N. In the limiting case, the free-entry condition determines the ratio of workers to firm $b(\infty;c)$, which solves

$$1-e^{-b}-be^{-b} = c. \tag{20}$$

It is clear that as the cost of becoming active for a firm increases the ratio of workers to firm increases.

[17] See for instance Burdett et al. (2001).

[18] If $M=1$ the equilibrium wage is determined differently from the above anlaysis: the unique firm in the economy does not face any competition and will therefore offer 0 as a wage.

5 Welfare Impact of Labor Hoarding Contracts

5.1 Exogenous Number of Firms

Comparison of Payoffs. We have shown that when firms are allowed to post general contracts, and we focus on FWC, they post contracts that fully insure risk-averse workers against unemployment risk. These contracts enable each firm to minimize its wage bill, given the level of utility that is offered by other firms. One could be tempted to conclude that without restriction on contract postings firms are therefore better off than in the standard directed model with one vacancy, and that workers are no worse off. Although it is true that workers are no worse off, they are in effect strictly better off, firms are actually worse off.

In the standard directed search model where firms are restricted to post one vacancy, in the symmetric equilibrium, $w^*_{N,M}[1]$, the wage posted by all firms is[19]

$$w^*_{N,M}[1] = \frac{(1-1/M)^{N-1}}{\Omega_{N,M}(1/M;1) + \Lambda_{N,M}(1)}, \tag{21}$$

where

$$\Lambda_{N,M}(v) = -\left[\frac{\gamma(w^*_{N,M}[v])}{M-1} - \frac{1}{M}\right] \frac{\partial \Omega_{N,M}(\theta; v)}{\partial \theta}\bigg|_{\theta=1/M} > 0, \tag{22}$$

with[20]

$$\gamma(w) \equiv \frac{u(w)}{w \times u'(w)} \geq 1, \text{with strict inequality for } u \text{ strictly concave.} \tag{23}$$

---

[19] The proof can be found in the appendix. Although it is not possible in general to obtain a closed-form expression for the wage, if u is of the CRRA kind with $u(w) = w^{1-\sigma}/(1-\sigma)$, we obtain that the expression for the equilibrium wage in the text simplifies to:

$$w^*_{N,M}[1] = \frac{(1-1/M)^{N-1}}{\Omega_{N,M}(1/M;1) - \left(\frac{\sigma(M-1)+1}{M-1}\right) \theta \frac{\partial \Omega_{N,M}(\theta;v)}{\partial \theta}\Big|_{\theta=1/M}}.$$

[20] This is because for all $x \geq 0$

$$u(x) = u(0) + \int_0^x u'(y)dy.$$

Since $u(0) = 0$ and for u concave $u'(x) \leq u'(y)$ for all $y < x$, with strict inequality if u strictly concave, we have

$$u(x) \geq \int_0^x u'(x)dy = u'(x)x,$$

again with strict inequality if u strictly concave.

Thus, the expected utility for workers in the one-vacancy directed search model is

$$U^*_{N,M}[1] = \Omega_{N,M}(1/M;1) \times u\left[\frac{(1-1/M)^{N-1}}{\Omega_{N,M}(1/M;1)+\Lambda_{N,M}(1)}\right], \tag{24}$$

whereas in the symmetric FWC-equilibrium described above workers (expected) utility is given by (17). It is clear that $U^*_{N,M}[N] > U^*_{N,M}[1]$. Moreover, firms' expected profits in the standard directed search equilibrium is given by

$$\Pi^*_{N,M}[1] = 1 - \left(1-\frac{1}{M}\right)^N - \frac{\Omega_{N,M}(1/M;1)}{\Omega_{N,M}(1/M;1)+\Lambda_{N,M}(1)}\frac{N}{M}\left(1-\frac{1}{M}\right)^{N-1}, \tag{25}$$

whereas in the symmetric FWC-equilibrium expected profits are given by (18), which is strictly less than $\Pi^*_{N,M}[1]$.

Analysis. It might seem surprising that firms are worse off when offering labor hoarding contracts to risk-averse workers since such contracts can presumably lower the expected wage bill a firm has to pay because of the risk-premium firms are implicitly levying on workers' wages. However, there is another force at play here, which is the greater competition among firms when labor hoarding contracts are posted. The insurance element enables a firm, taking as given the contract posted by other firms, to decrease its wage bill compared to the standard directed search model by insuring risk-averse workers. But when all firms do the same, workers are guaranteed employment by all firms, which stiffens competition, thereby reducing firms' expected profits. From our above calculations it is clear that in the present case the competition effect dominates the insurance effect.

We can actually be more precise. In the FWC-equilibrium workers are getting paid their expected marginal product, and in that sense the equilibrium outcome is the outcome of a perfectly competitive environment. In fact, when a worker applies to a firm, with probability $1-(1-1/M)^{N-1}$ at least one of the $N-1$ other workers has applied to that same firm, which means the firm is able to produce without the extra worker, and therefore his marginal productivity is 0. But with probability $(1-1/M)^{N-1}$ no other worker has applied to the firm, in which case the worker's marginal product is 1. In total we therefore obtain that, given that all workers apply to each firm with probability $1/M$, a worker's expected marginal product is

$$\left[1-\left(1-\frac{1}{M}\right)^{N-1}\right] \times 0 + \left(1-\frac{1}{M}\right)^{N-1} \times 1 = w^*_{N,M}[N].$$

However, when firms do not post hoarding contracts the outcome then fails to be perfectly competitive. To understand better the forces at work, consider the following: Assume that all firms are restricted to post FWCs with the same number of vacancy v. It can be shown (see lemma B1 in the appendix) that for any $v \in \{1, ..., N\}$, the unique symmetric equilibrium is such that all firms post the FWC with wage

$$w^*_{N,M}[v] = \frac{(1 - 1/M)^{N-1}}{\Omega_{N,M}(1/M; v) + \Lambda_{N,M}(v)}, \tag{26}$$

where $\Omega_{N,M}$ and $\Lambda_{N,M}$ are defined as before, and the expressions for expected utility and expected profits are still valid but with $v \in \{1, ..., N\}$. If we define by $W^*_{N,M}[v]$ the expected wage, i.e., $W^*_{N,M}[v] \equiv \Omega_{N,M}(1/M; v)w^*_{N,M}[v]$, we have that[21]

$$\Delta W^*_{N,M}[v] = -\left[1 + (\gamma(w) - 1)\, M\right] \varepsilon(\widetilde{U}/\theta), \tag{27}$$

where

$$\Delta W^*_{N,M}[v] \equiv \frac{W^*_{N,M}[N] - W^*_{N,M}[v]}{W^*_{N,M}[v]}, \text{ and } \varepsilon(\widetilde{U}/\theta) = -\frac{\partial \widetilde{U}}{\partial \theta}\frac{\theta}{\widetilde{U}}$$

is the elasticity of the expected utility of applying to a firm j with respect to the probability θ with which workers apply to another firm m.

When workers are risk-neutral, in which case workers are indifferent between receiving the wage $W^*_{N,M}[v]$ for sure or receiving the wage $w^*_{N,M}[v]$ with probability $\Omega_{N,M}(1/M; v)$, since $\gamma(w) = 1$, (27) simplifies to

$$\Delta W^*_{N,M}[v] = -\varepsilon(\widetilde{U}/\theta), \tag{27b}$$

which is positive since $\partial\widetilde{U}/\partial\theta$ is positive. As was highlighted earlier, one can think of a firm's posting decision problem when it posts a FWC with a given number of vacancies as either choosing the wage to post or choosing the probability with which it wants workers to apply. Hence, (27b) indicates that the greater $\varepsilon(\widetilde{U}/\theta)$ is (in absolute value), that is the greater the impact of a firm's posting decision on the expected utility of applying to other firms, the more the equilibrium expected wage will deviate from the perfectly competitive wage. This is actually quite intuitive: when a firm posts a FWC with $v < N$ vacancies, and it decreases the wage it pays, the firm becomes less attractive to workers, which implies that θ must decrease. This in turn implies that workers apply to other firms with a greater probability, thereby

[21] This is because $\varepsilon(\widetilde{U}/\theta) = -\varepsilon(U/\theta)/(M - 1)$ in an interior solution.

decreasing the expected utility of applying to them. Hence, not only do we have that because of the coordination frictions a firm has some pricing (or monopoly) power in setting its wage, we also have that when a firm decreases its wage the fall in θ, which we can interpret as a fall in labor supply, is limited by the fact that it becomes harder to be employed at other firms.

There are two instances in which firms do not have any pricing power, when all firms post N vacancies and when the economy is large. In the first case, all firms guarantee employment to all workers, so that if a firm decides to change its wage, this has no impact on the expected utility of applying to other firms since it does not depend on the probability with which workers apply. When the economy is large, a firm is too small to have any impact on the queue length at other firms, and therefore a firm cannot have any impact on the expected utility of applying to other firms either. In fact, we have that, fixing the ratio of firms to workers to b, (22) can be re-expressed as

$$\Lambda_{N,M}(v) = \frac{N\left[\gamma(w^*_{N,M}[v]) - 1\right] + b}{(N-b)\,b} \times v \sum_{k=v+1}^{N} p^k_N(1/M), \tag{22b}$$

and since $\gamma(w) = 1$ for all w when workers are risk-neutral,

$$\lim_{N\to\infty} \Lambda_{N,M}(v) = \lim_{N\to\infty} \frac{1}{(N-b)} \times \sum_{k=v+1}^{N} \mathcal{P}^k(b) = 0,$$

where $\mathcal{P}^k(b)$ is the Poisson probability $e^{-b}\left(b^k/k!\right)$. Hence, we have that when workers are risk-neutral their expected wage $W^*_{\theta,b}[v] = \Omega_{\theta,b}(b;v)w^*_{\theta,b}[v]$ is equal to their expected marginal product e^{-b}.

When workers are risk-averse although it is still true that the wage paid is the competitive wage in the FWC-equilibrium characterized in proposition 2, it is no longer true that this is also the case in a large economy when firms do not post FWCs with N vacancies. Assuming that $\gamma(w)$ is bounded for all w,[22] we obtain from (22b) that

$$\Lambda_{\infty,b}(v) = \lim_{N\to\infty} \Lambda_{N,M}(v) = \frac{\gamma(w^*_{\infty,b}[v]) - 1}{b} \times \sum_{k=v+1}^{N} \mathcal{P}^k(b) > 0,$$

[22] This is true for CRRA utility functions: if $u(w) = w^{1-\sigma}/(1-\sigma)$, then $\gamma(w) = 1/(1-\sigma)$ for all w. For a CARA utility function $u(w) = 1 - e^{-\sigma w}$, $\gamma(w) = \left(1 - e^{-\sigma w}\right)/\sigma w e^{-\sigma w}$; and from expression (?) giving the equilibrium value for the wage, it must be that $\gamma(w)$ is finite. In fact, as $\gamma(w) \to \infty$, $w \to 0$. But $\left(1 - e^{-\sigma w}\right)/\sigma w e^{-\sigma w} \to 1$ as $w \to 0$.

which implies that

$$W^*_{\infty,b}[v] = \frac{\Omega_{\infty,b}(b;v)e^{-b}}{\Omega_{\infty,b}(b;v) + \Lambda_{\infty,b}(v)} < e^{-b}, \text{ for all } v < \infty.$$

The reason for this difference between the perfectly competitive expected wage and the expected marginal product comes from the fact that, although a firm does not have any impact on the expected utility of applying to other firms, when it decreases its wage in a FWC with $v < N$ vacancies, it becomes less attractive to workers, which in turn decreases its queue length. The shorter queue length means that the probability of being employed, conditional on having applied to the firm, increases. When workers are risk-averse, they are willing to accept a larger decrease in their wage than risk-neutral workers would, because they value the probability of being employed relative to the wage if employed more than risk-neutral workers do. And since the weight workers put on the probability of being employed relative to the wage paid if employed is greater than that of a risk-neutral firm, firms can offer workers contracts that offer them an expected wage lower than their expected marginal product. Although we have not been able to show it analytically, numerical results show that $W^*_{\infty,b}[v]$ increases with v, suggesting that as the number of vacancies increases, since worker are employed in equilibrium with a greater probability, the pricing power firms decreases. Once again this is intuitive: As the probability of being employed increases workers are more sensitive to changes in wages.

Note that even if we had $\Lambda_{\infty,b}(v) = 0$ for all v, so that $W^*_{\infty,b}[v] = e^{-b}$ for all v, the expected utility of workers would then be

$$U^*_{\infty,b}[v] = \Omega_{\infty,b}(b;v)u\left(\frac{e^{-b}}{\Omega_{\infty,b}(b;v)}\right) < U^*_{\infty,b}[\infty], \text{ for all } v < \infty.$$

This is because when $v < N$, although the expected wage offered by firms is equal to workers' expected marginal product, workers are not guaranteed employment. Hence, the existence of the risk of being unemployed reduces the level of welfare for workers compared to the case where $v = N$. As the number v of vacancies posted by firms increases, $\Omega_{\infty,b}(b;v)$ increases, so that workers' expected utility would increase as well.

5.2 Endogenous Number of Firms

In the finite economy case, although the welfare gains for workers are clear when there is a fixed number of firms in the economy, when the number of active firms is determined endogenously, it is in general not possible to say whether workers benefit from firms posting contracts that insure them completely against the risk of unemployment. The reason lies in the fact that for a given number of firms M, when firms are not restricted in the type of contracts they can post, their expected profits are lower than with standard one-vacancy contracts. Hence, when the number of active firms is determined endogenously through free-entry, less firms will be active with hoarding contracts than with one-vacancy contracts. In fact, $M(N;v)$, the number of active firms when there are N workers in the economy and when all firms post FWC with v vacancies, is such that

$$\Pi^*_{N,M}[v] \geq c \text{ and } \Pi^*_{N,M+1}[v] < c,$$

where

$$\Pi^*_{N,M}[v] = 1 - \left(1 - \frac{1}{M}\right)^N - \frac{\Omega_{N,M}(1/M;v)}{\Omega_{N,M}(1/M;v) + \Lambda_{N,M}(v)} \frac{N}{M} \left(1 - \frac{1}{M}\right)^{N-1}.$$

We know that since $\Lambda_{N,M}(v) > 0$ for all $v < N$ and $\Lambda_{N,M}(N) = 0$, $\Pi^*_{N,M}[N] < \Pi^*_{N,M}[1]$ for all M, and therefore $M(N;N) \leq M(N;1)$. This implies a firm is less likely to fail to fill its vacancy with standard one-vacancy contracts than with hoarding contracts.

The lower number of active firms reduces competition, for the same number of vacancy, and therefore the wage offered by firms is such that

$$w^*_{N,M(N;N)}[N] = \left(1 - \frac{1}{M(N;N)}\right)^{N-1} < \left(1 - \frac{1}{M(N;1)}\right)^{N-1}.$$

And therefore it is not possible to say whether the expected wage $W^*_{N,M(N;N)}[N] = w^*_{N,M(N;N)}[N]$ is greater than the expected wage when firms post standard one-vacancy contracts

$$W^*_{N,M(N;1)}[1] = \frac{\Omega_{N,M}(1/M;1)}{\Omega_{N,M}(1/M;1) + \Lambda_{N,M}(1)} \left(1 - \frac{1}{M(N;1)}\right)^{N-1},$$

or how the expected utility levels compare.

6 Efficiency Properties of the Equilibrium

7 Conclusion

We have shown that in a directed search model of the labor market with risk-averse workers, when firms are free to offer the contract they want, and in particular are allowed to hoard labor, firms will indeed choose to offer labor hoarding contracts. This implies that the simple wage posting contract assumed in the current literature is not a contract firms would choose to post, and therefore the equilibrium of standard models is not robust to the introduction of risk-aversion.

It should be noted that the case considered in this paper is extreme in that workers do not value leisure and there is no unemployment insurance payout, implying that workers are indifferent between working at a zero wage and not working. This in turns puts no lower bound on the wage firms can offer. This is crucial to obtain the full-insurance result of this paper: It is clear that if we were to modify the model in such a way that the reservation wage of workers were to be strictly positive, and exceed the wage offered by firms in the full insurance contracts, then firms could no longer offer contracts that fully insure workers against the risk of unemployment, although they would still offer labor hoarding contracts. There are several, non-mutually exclusive, instances in which the reservation wage of workers can be strictly positive: workers value leisure; unemployed workers receive Unemployment Insurance (UI) benefits; the game is a repeated game so that workers who do not secure a job at a given point in time can look for a job later on; workers are allowed to send multiple applications.[23]

There are a number interesting applications of the directed search model with labor hoarding contracts when firms do not offer contracts that fully insure workers against unemployment. One of them is its implications for the design of an optimal unemployment benefit scheme. Our brief analysis above clearly indicates that the level of UI payments, and more generally the design of a UI scheme, has an impact on the contracts firms offer to workers. And in particular it suggests that with more generous UI payments firms would insure workers against unemployment to a lesser extent. Hence, when designing a UI scheme it is important to take into account the impact a UI scheme will have on firms' contract offering, and especially the impact on the level of insurance firms offer to workers. In fact, one wants to make sure that a UI scheme does not merely serve as a substitute to the insurance offered by firms through labor hoarding contracts.

[23] In the first two cases the reservation wage is exogenous, whereas it is endogenous in the two latter.

Appendix A

Proof of Proposition 1

(i) The proof of the first part of the proposition is a standard fixed-point problem. Let $z_m(\boldsymbol{\theta}) \equiv U_m(\mathbf{w};\theta_m) - \sum_{j=1}^{M} \theta_j U_j(\mathbf{w};\theta_j)$ as the excess expected utility of applying to firm m over the mixed strategy application strategy $\boldsymbol{\theta}$, and let

$$T_m(\boldsymbol{\theta}) \equiv \frac{\theta_m + \max\{0; z_m(\boldsymbol{\theta})\}}{1 + \sum_{j=1}^{M} \max\{0; z_j(\boldsymbol{\theta})\}}.$$

It is clear that because $U_m(\mathbf{w};\theta_m)$ is continuous in θ_m, for all m, $z_m(\boldsymbol{\theta})$ is also a continuous function of $\boldsymbol{\theta}$, and therefore $T_m(\boldsymbol{\theta})$ is continuous in $\boldsymbol{\theta}$. The function T which transform $\boldsymbol{\theta}$ into $(T_m(\boldsymbol{\theta}))_{m=1}^{M}$ is from S^M into S^M. Hence, by Brouwer's fixed point theorem there exists a vector $\boldsymbol{\theta}$ such that $T(\boldsymbol{\theta}) = \boldsymbol{\theta}$, i.e., $T_m(\boldsymbol{\theta})=\theta_m$ for all m.

If $\theta_m = 0$, it must be that $T_m(\boldsymbol{\theta}) = 0$, and therefore that $z_m(\boldsymbol{\theta}) \leq 0$.

If $\theta_m > 0$, there are two cases to consider, whether the denominator is equal to or greater than 1. If it is equal to 1, it follows that $z_m(\boldsymbol{\theta}) \leq 0$. If, however, the denominator is equal to $\alpha > 1$, we then have that $\theta_m = \alpha\theta_m + \alpha \max\{0; z_m(\boldsymbol{\theta})\}$, which is equivalent to $(1-\alpha)\theta_m z_m(\boldsymbol{\theta}) = \alpha \max\{0; z_m(\boldsymbol{\theta})\} z_m(\boldsymbol{\theta})$. However, $\sum_{m=1}^{M} \theta_m z_m(\boldsymbol{\theta}) = 0$. Therefore $\sum_{m=1}^{M} (1-\alpha)\theta_m z_m(\boldsymbol{\theta}) = \sum_{m=1}^{M} \alpha \max\{0; z_m(\boldsymbol{\theta})\} z_m(\boldsymbol{\theta}) = 0$, implying that $z_m(\boldsymbol{\theta}) \leq 0$ for all m. But since $\sum_{m=1}^{M} \theta_m z_m(\boldsymbol{\theta}) = 0$, we have that the fixed-point $\boldsymbol{\theta}$ has the following properties: if $\theta_m > 0$, then $z_m(\boldsymbol{\theta}) = 0$ and if $\theta_m = 0$, then $z_m(\boldsymbol{\theta}) \leq 0$, which is consistent with an equilibrium application strategy. Therefore, there exists an Nash equilibrium in symmetric strategies.

(ii) First, suppose that for all firms m, $u^e(\mathbf{w}_m^k) \geq u^e(\mathbf{w}_m^{k+1})$ for all k, with $u^e(\mathbf{w}_m^k) > u^e(\mathbf{w}_m^{k+1})$ for some k. Suppose that there exist two equilibrium application strategy profiles $\boldsymbol{\theta}$ and $\widetilde{\boldsymbol{\theta}}$. Then there exist m and j such that $\widetilde{\theta}_m > \theta_m \geq 0$ and $0 \leq \widetilde{\theta}_j < \theta_j$. However, since in this case $U(\boldsymbol{\theta}; v)$ is strictly decreasing in θ for all firms, this implies that $U_j(\mathbf{w};\widetilde{\boldsymbol{\theta}}) > U_j(\mathbf{w};\boldsymbol{\theta})$. But since $\theta_j > 0$, it must be true that $U_j(\mathbf{w};\boldsymbol{\theta}) \geq U_m(\mathbf{w};\boldsymbol{\theta})$. And $\widetilde{\theta}_m > \theta_m$ implies that $U_m(\mathbf{w};\boldsymbol{\theta}) > U_m(\mathbf{w};\widetilde{\boldsymbol{\theta}})$. In total we obtain that $U_j(\mathbf{w};\widetilde{\boldsymbol{\theta}}) > U_m(\mathbf{w};\widetilde{\boldsymbol{\theta}})$, which contradicts the fact that $\widetilde{\theta}_m > 0$.

Now suppose that for all firms $m \neq j$ for some j, $u^e(\mathbf{w}_m^k) \geq u^e(\mathbf{w}_m^{k+1})$ for all k, with $u^e(\mathbf{w}_m^k) > u^e(\mathbf{w}_m^{k+1})$ for some k, and $u^e(\mathbf{w}_j^k) = u^e(\mathbf{w}_j^{k+1})$ for all k, and suppose that there exist two equilibrium application strategy profiles $\boldsymbol{\theta}$ and $\widetilde{\boldsymbol{\theta}}$. If $\widetilde{\theta}_j = \theta_j = 0$, then we can use the argument above to show it must be that $\boldsymbol{\theta} = \widetilde{\boldsymbol{\theta}}$. If either $\widetilde{\theta}_j$ or θ_j is strictly positive, we have two cases to consider. (a) If $\boldsymbol{\theta}$ and $\widetilde{\boldsymbol{\theta}}$ are such that $0 \leq \widetilde{\theta}_m < \theta_m$ and $\widetilde{\theta}_i > \theta_i \geq 0$ for some m and i: If $\theta_j > 0$, one must have $U_m(\mathbf{w};\widetilde{\boldsymbol{\theta}}) > U_m(\mathbf{w};\boldsymbol{\theta}) = U_j(\mathbf{w};\boldsymbol{\theta}) \geq U_i(\mathbf{w};\boldsymbol{\theta}) > U_i(\mathbf{w};\widetilde{\boldsymbol{\theta}})$, contradicting the fact that $\widetilde{\theta}_i > 0$; and if $\widetilde{\theta}_j > 0$, one must have $U_i(\mathbf{w};\boldsymbol{\theta}) > U_i(\mathbf{w};\widetilde{\boldsymbol{\theta}}) = U_j(\mathbf{w};\widetilde{\boldsymbol{\theta}}) \geq U_m(\mathbf{w};\widetilde{\boldsymbol{\theta}}) > U_m(\mathbf{w};\boldsymbol{\theta})$, contradicting the fact that $\theta_m > 0$. (b) If instead $\boldsymbol{\theta}$ and $\widetilde{\boldsymbol{\theta}}$ are such that $0 \leq \widetilde{\theta}_m < \theta_m$ for some m and that $\widetilde{\theta}_i = \theta_i \geq 0$ for all $i \neq j, m$, the fact that $0 \leq \widetilde{\theta}_m < \theta_m$ for some m and that $\widetilde{\theta}_i = \theta_i \geq 0$ for all $i \neq j, m$ implies that $\widetilde{\theta}_j > \theta_j \geq 0$. Since $\theta_m > 0$, it must be that $U_j(\mathbf{w};\boldsymbol{\theta}) \leq U_m(\mathbf{w};\boldsymbol{\theta})$. Assume thus that $U_j(\mathbf{w};\boldsymbol{\theta}) = U_m(\mathbf{w};\boldsymbol{\theta})$. Then $U_j(\mathbf{w};\boldsymbol{\theta}) < U_m(\mathbf{w};\widetilde{\boldsymbol{\theta}})$, contradicting the fact that $\widetilde{\theta}_j > 0$. If instead $U_j(\mathbf{w};\boldsymbol{\theta}) < U_m(\mathbf{w};\boldsymbol{\theta})$, then it must be that $\theta_m = 0$, contradicting the fact that $\theta_m > \widetilde{\theta}_m \geq 0$.

If more than one firm post contracts such that $u^e(w_m^k) = u^e(w_m^{k+1}) = u^e(w_m)$ for all k, then only the firms offering the highest $u^e(w_m)$ are applied to in equilibrium, and therefore all other firms with postings such that for all k $u^e(w_m^k) = u^e(w_m^{k+1}) < \max_j u^e(w_j)$ can be ignored. By anonymity all firms posting these contracts, and that are applied to with positive probability are applied to with the same probability, and one can therefore follow the above argument to show uniqueness of equilibrium.■

Proof of Lemma 2

Since $\mathbf{w}_{-m}$ is such that for all firms $j \neq m$, $u^e(\mathbf{w}_j^k) \geq (\leq)\, u^e(\mathbf{w}_j^{k+1})$ for all k, with $u^e(\mathbf{w}_j^k) > (<)\, u^e(\mathbf{w}_j^{k+1})$ for some k, if there is multiplicity of equilibria of the application subgame, we know from proposition 1 the multiplicity is due to the contract posted by firm m. If we first assume that for a given contract profile $\mathbf{w}_{-m}$ there is a unique symmetric equilibrium application profile when firm m posts $\mathbf{w}_m$, then we can use the same line of argument as used in the proof of lemma 1 to show that the firm can reduce its wage bill for a given level of expected utility promised to the workers by posting instead a FWC $\overline{\mathbf{w}}_m = \overline{w}_m \cdot 1_{N,N}(N)$ such that $u_m(\overline{w}_m) = \sum_{k=0}^{N-1} p_{N-1}^k(\theta_m) u_m^e(\mathbf{w}_m^k)$. In fact, we know from lemma 1 that a firm's optimal contract is such that all workers who have applied to the firm must be paid the same wage, i.e., $w_m^{r,k} = w_m^k$ for all $r \leq k$, all k. Hence, the expected wage bill in this case is

$$W(\overline{\mathbf{w}}_m) = \sum_{k=1}^{N} p_N^k(\theta_m) k \overline{w}_m = \sum_{k=1}^{N} p_N^k(\theta_m) k u^{-1} \left[\sum_{k=1}^{N} p_N^k(\theta_m) u_m^e(\mathbf{w}_m^k) \right],$$

where $u_m^e(\mathbf{w}_m^k) = u(w_m^k)$. However,

$$\sum_{k=1}^{N} p_N^k(\theta_m) k u^{-1} \left[\sum_{k=1}^{N} p_N^k(\theta_m) u(w_m^k) \right] < \sum_{k=1}^{N} p_N^k(\theta_m) k w_m^k$$

for all $\mathbf{w}_m$ such that $w_m^k \neq w_m^{k'}$ for some k, k'. Hence, for all $\mathbf{w}_m$ such that $w_m^k \neq w_m^{k'}$ for some k, k' and $\overline{\mathbf{w}}_m = \overline{w}_m \cdot 1_{N,N}(N)$ such that $u_m(\overline{w}_m) = \sum_{k=0}^{N-1} p_{N-1}^k(\theta_m) u_m^e(\mathbf{w}_m^k)$, $W(\overline{\mathbf{w}}_m) < W(\mathbf{w}_m)$ implying that $\Pi(\overline{\mathbf{w}}_m; \theta_m) > \Pi(\mathbf{w}_m; \theta_m)$.

If there are more than one equilibrium application strategy profile when firm m posts a non-flat wage $\mathbf{w}_m$, let us denote by $\boldsymbol{\theta}^1, ..., \boldsymbol{\theta}^L$ the $L > 1$ profiles. In this case, firm m can choose to post the contract $\overline{\mathbf{w}}_m = \overline{w}_m \cdot 1_{N,N}(N)$ such that $\overline{w}_m = \arg\max_{s=1,...L}.\Pi(\overline{\mathbf{w}}_m^s; \theta_m^s)$, where for all s $\overline{\mathbf{w}}_m^s = \overline{w}_m^s \cdot 1_{N,N}(N)$ such that $u_m(\overline{w}_m^s) = \sum_{k=0}^{N-1} p_{N-1}^k(\theta_m^s) u_m^e(\mathbf{w}_m^k)$. In fact, given the properties of $\mathbf{w}_{-m}$, by posting a FWC firm m induces a unique equilibrium of the application subgame among the L profiles $\boldsymbol{\theta}^1, ..., \boldsymbol{\theta}^L$. And for each FWC posting associated with each application probability θ_m^s, we have that $\sum_{k=1}^N p^k(\theta_m^s) k \overline{w}_m^s < \sum_{k=1}^N p^k(\theta_m^s) k w_m^k$. Hence, for all $\mathbf{w}_m$ such that $w_m^k \neq w_m^{k'}$ for some k, k' and $\overline{\mathbf{w}}_m = \overline{w}_m \cdot 1_{N,N}(N)$ such that $u_m(\overline{w}_m) = \sum_{k=0}^{N-1} p_{N-1}^k(\theta_m) u_m^e(\mathbf{w}_m^k)$, $\Pi(\overline{\mathbf{w}}_m) > \Pi(\mathbf{w}_m)$.■

Proof of Proposition 2:

(*i-a*) Assume there exists an equilibrium such that all firms post the same FWC $\overline{\mathbf{w}}(N) = w \cdot 1_{N,N}(N)$ with $w \neq w_{N,M}^*[N]$. A firm's expected profits if it posts the contract $\overline{\mathbf{w}}(N)$, and if it could choose the probability with which workers will apply, is

$$\Pi(\overline{\mathbf{w}}(N); \theta) = 1 - (1-\theta)^N - N\theta w.$$

We have that

$$\frac{\partial \Pi(\overline{\mathbf{w}}(N); \theta)}{\partial \theta} = N(1-\theta)^{N-1} - Nw, \text{ and } \frac{\partial^2 \Pi(\overline{\mathbf{w}}(N); \theta)}{\partial \theta^2} = -N(N-1)(1-\theta)^{N-2} < 0 \text{ for } N > 2.$$

Hence, given what other firms post, if a firm could choose the probability with which workers apply, it would choose

$$\theta^*(w) = 1 - w^{1/(N-1)}. \tag{A1}$$

In fact, if the firm chooses $\theta = 0$ then it makes zero expected profits; if it chooses $\theta = 1$ then it needs to post a wage strictly greater than w, which then implies its profits are strictly less than $1 - Nw$, whereas if it chooses $\theta^*(w)$ then its profit is

$$\Pi(\overline{\mathbf{w}}(N); \theta^*(w)) = 1 - Nw + (N-1) w^{N/(N-1)} > 1 - Nw.$$

If $\theta^*(w) \neq 1/M$, it means that a firm has a profitable deviation by posting an almost-FWC that replicates the contract $\overline{\mathbf{w}}(N)$:

$$w = \begin{cases} \overline{w}, & \text{such that } u(\overline{w}) = (w) + \epsilon \quad \text{if } k < N \text{ workers apply;} \\ \underline{w}, & \text{such that } u(\underline{w}) = u(w) - \delta \quad \text{otherwise,} \end{cases} \tag{A2}$$

for $\epsilon > 0$ and $\delta > 0$ arbitrarily small. ϵ and δ must be such that

$$(1-\theta^*(w))^{N-1}\left[u(w)+\epsilon\right]+\left(1-(1-\theta^*(w))^{N-1}\right)\left[u(w)-\delta\right]=u(w),$$

and therefore one must have

$$\delta\left(\epsilon\right)=\frac{(1-\theta^*(w))^{N-1}}{1-(1-\theta^*(w))^{N-1}}\epsilon. \tag{A3}$$

This implies that the extra cost of this almost-FWC compared to the FWC with N vacancies with wage w is

$$\begin{aligned} C(\theta^*(w);\epsilon) &= N\theta^*(w)(1-\theta^*(w))^{N-1}\left\{u^{-1}[u(w)+\epsilon]-w\right\}+ \\ &\quad N\theta^*(w)(1-(1-\theta^*(w))^{N-1})\left\{u^{-1}\left[u(w)-\delta\left(\epsilon\right)\right]-w\right\}. \end{aligned} \tag{A4}$$

For any $\Delta > 0$ one can find an $\epsilon(\theta^*(w);\Delta) > 0$ such that $C(\theta^*(w);\Delta) < \Delta$: choose

$$\epsilon(\theta^*(w);\Delta)=u\left[u(w)+\frac{\Delta}{N\theta^*(w)(1-\theta^*(w))^{N-1}}\right]-u(w). \tag{A5}$$

We then have

$$C(\theta^*(w);\epsilon)=\Delta+N\theta^*(w)(1-(1-\theta^*(w))^{N-1})\left\{u^{-1}\left[u(w)-\delta\left(\epsilon\right)\right]-w\right\}<\Delta,$$

since

$$u^{-1}\left[u(w)-\delta\left(\epsilon\right)\right]-w]<0.$$

Hence, if initially firms' expected profit is $\Pi(\overline{\mathbf{w}}(N);1/M)$, then we can find $\epsilon(\theta^*(w);\Delta) > 0$ as given in (A5) such that

$$\Pi(\overline{\mathbf{w}}(N);\theta^*(w))-C(\theta^*(w);\epsilon)>\Pi(\overline{\mathbf{w}}(N);1/M)\geq 0,$$

where $C(\theta^*(w);\epsilon)$ is given by (A4); that is, the expected profit of posting the almost-FWC as given by (A2) is greater than posting the FWC $\overline{\mathbf{w}}(N)$. But $\theta^*(w) = 1/M$ if and only if

$$1-w^{1/(N-1)}=1/M,$$

which means that $\theta^*(w) = 1/M$ if and only if

$$w=w^*_{N,M}[N].$$

Hence, firms have an profitable deviation and all firms posting the FWC $\overline{\mathbf{w}}(N) \neq w^*_{N,M}[N]$ cannot an equilibrium.

(i-b) If all firms post a FWC with N vacancies but with different wages, only the firms posting the highest wage can receive applications. Assume that $J \geq 1$ of the firms posting N vacancies can receive applications. These J firms must be posting the same wage, and if this is a candidate equilibrium none of these J firms must have a profitable deviation. This means that they are all applied to with probability $1/J$ and the common wage posted is

$$w^*_{N,J}[N] = \left(1 - \frac{1}{J}\right)^{N-1}.$$

But if a firm that will not be applied to changes its posting to an almost-FWC that replicates $w^*_{N,J}[N]$, then it can make strictly positive expected profits, which means it has a profitable deviation. A firm's expected profits if it posts the contract $w^*_{N,J}[N]$, and if it could choose the probability with which workers will apply, is

$$\Pi(w^*_{N,J}[N];\theta) = 1 - (1-\theta)^N - N\theta\left(1 - \frac{1}{J}\right)^{N-1}.$$

Hence, the choice of θ is given by (A1) with $w = w^*_{N,J}[N]$, which yields that the deviant firm's expected profit if it post an almost-FWC of the (A2) type is

$$\begin{aligned} & \Pi(w^*_{N,J}[N];\theta^*(w^*_{N,J}[N])) - C(\theta^*(w^*_{N,J}[N]);\epsilon) \\ = \; & 1 - N\left(1 - \frac{1}{J}\right)^{N-1} + (N-1)\left(1 - \frac{1}{J}\right)^{N} - C(\theta^*(w^*_{N,J}[N]);\epsilon), \end{aligned}$$

which simplifies to

$$\begin{aligned} & \Pi(w^*_{N,J}[N];\theta^*(w^*_{N,J}[N])) - C(\theta^*(w^*_{N,J}[N]);\epsilon) \\ = \; & 1 - \left(1 - \frac{1}{J}\right)^{N} - \frac{N}{Q}\left(1 - \frac{1}{J}\right)^{N-1} > 0. \end{aligned}$$

Therefore the deviant firm would make strictly positive expected profits, and therefore the deviation to posting an almost–FWC is indeed a profitable deviation.

(ii) Assume that all firms post FWCs and that $\exists j$ and k such that $v_j = N$ and $v_k < N$.

(ii-a) Assume $\theta_j > 0$ for all j such that $v_j = N$. If $\theta_k > 0$ then firm k' initial expected profits are non-negative and it therefore has a profitable deviation that yield strictly positive expected profit by posting either a FWC or an almost-FWC depending on whether its desired application probability requires one or the other. If initially $\theta_k = 0$ for all k such that $v_k < N$, then we can follow the reasoning from

(*i-b*) to show that a firm not applied to initially has a profitable deviation.
(*ii-b*) Assume $\theta_j = 0$ for all j such that $v_j = N$. Then we can apply corollary 2 to the truncated economy containing only firms that are applied to with strictly positive probability to show that this cannot be an equilibrium because the best-response of a firm m is to post a FWC with N vacancies.

(*iii*) If no firm post N vacancies, then we can again appeal to corollary 2 to show that this cannot be an equilibrium.

(*iv*) Finally, from analysis in (*i*) it is clear that all firms posting the FWC with N vacancies at wage $w^*_{N,M}[N]$ and all workers applying to each firm with probability $1/M$ is an equilibrium. Since all other possible types of FWC-equilibria have been ruled out in (*i*)-(*iii*), the above equilibrium is the unique FWC-equilibrium.■

Appendix B

Proof that $\Gamma(\theta; v)$ increases with v

$\Gamma(\theta; v+1) - \Gamma(\theta; v) = 1 + (v+1)p_N^{v+1}(\theta) - (v+1)\sum_{k=0}^{v+1} p_N^k(\theta) + v\sum_{k=0}^{v} p_N^k(\theta)$, and which simplifies to

$$\Gamma(\theta; v+1) - \Gamma(\theta; v) = 1 - \sum_{k=0}^{v} p_N^k(\theta) > 0,$$

for all $v < N$.

Proof that there is at most one solution when firms post FWCs with one vacancy with $m = 2$ and $n = 3$.

Define $a \equiv (u(w_1) - u(w_2))/3$, $b \equiv (3u(w_1) + u(w_2))/3$, $c \equiv (u(w_1) - u(w_2))$ and $\Delta \equiv b^2 - 4ac$. The two possible solutions for the application probability are $\theta_1^* = (-b + \sqrt{\Delta})/2a$ and $\widetilde{\theta}_1^* = (-b - \sqrt{\Delta})/2a$. However, in this case $c = 3a$ so that $\Delta = b^2 - 12a^2$. Hence, there are two solutions if only if $b^2 - 12a^2 > 0$, which is equivalent to having either $-b/2a > \sqrt{3}$ or $-b/2a < -\sqrt{3}$. If $-b/2a > \sqrt{3}$, then we have that $\theta_1^* = (-b + \sqrt{\Delta})/2a = (-b/2a) + \sqrt{(-b/2a)^2 - 3} > \sqrt{3} > 1$. If $-b/2a < -\sqrt{3}$, then $\widetilde{\theta}_1^* = -b/2a - \sqrt{(-b/2a)^2 - 3} < 0$. In either case there is at most one solution whose value is between zero and one.■

Lemma 3 *If firms are restricted to post FWCs with v vacancies, then there exists a unique symmetric equilibrium and it is such that the wage posted by all firms is* *given by* $w^*_{N,M}[v]$ *solving*

$$\begin{aligned}\left(1-\frac{1}{M}\right)^{N-1} &= \left(\frac{1}{M}\left.\frac{\partial\Omega_{N,M}(\theta;v)}{\partial\theta}\right|_{\theta=1/M}+\Omega_{N,M}\left(\frac{1}{M};v\right)\right)w^*_{N,M}[v] \\ &\quad -\frac{1}{M-1}\left.\frac{\partial\Omega_{N,M}(\theta;v)}{\partial\theta}\right|_{\theta=1/M}\frac{u(w^*_{N,M}[v])}{u'(w^*_{N,M}[v])}.\end{aligned}$$

Proof. Taking the FOC of the profit function $\Pi(\widetilde{w},v)$ with respect to $\widetilde{w}$ yields

$$\frac{\partial\widetilde{\theta}}{\partial\widetilde{w}}\left[N(1-\widetilde{\theta})^{N-1}-N\left(\widetilde{\theta}\frac{\partial\Omega_{N,M}(\widetilde{\theta};v)}{\partial\widetilde{\theta}}+\Omega_{N,M}(\widetilde{\theta};v)\right)\widetilde{w}\right]=N\widetilde{\theta}\Omega_{N,M}(\widetilde{\theta};v).$$

In a symmetric equilibrium in the second stage game, workers must be indifferent between visiting firm m or any other firm, which implies that $F=\Omega_{N,M}(\widetilde{\theta};v)u(\widetilde{w})-\Omega_{N,M}(\theta;v)\,u(w)=0$. The Implicit Function Theorem then yields that

$$\frac{\partial\widetilde{\theta}}{\partial\widetilde{w}}=-\frac{\Omega_{N,M}(\widetilde{\theta};v)u'(\widetilde{w})}{\frac{\partial\Omega_{N,M}(\widetilde{\theta};v)}{\partial\theta}u(\widetilde{w})-\frac{\partial\Omega_{N,M}(\theta;v)}{\partial\theta}u(w)}.$$

Since workers apply to the other firms with probability $\theta=(1-\widetilde{\theta})/(M-1)$, we have that $\frac{\partial\Omega_{N,M}(\theta;v)}{\partial\theta}=-\frac{\partial\Omega_{N,M}(\widetilde{\theta};v)}{\partial\theta}\frac{1}{(M-1)}$. This yields that the FOC can be rewritten as

$$\begin{aligned}(1-\widetilde{\theta})^{N-1} &= \left(\widetilde{\theta}\frac{\partial\Omega_{N,M}(\widetilde{\theta};v)}{\partial\widetilde{\theta}}+\Omega_{N,M}(\widetilde{\theta};v)\right)\widetilde{w} \\ &\quad -\frac{\widetilde{\theta}}{u'(\widetilde{w})}\left[\frac{\partial\Omega_{N,M}(\widetilde{\theta};v)}{\partial\theta}u(\widetilde{w})+\frac{\partial\Omega_{N,M}(\widetilde{\theta};v)}{\partial\theta}\frac{1}{(M-1)}u(w)\right],\end{aligned}$$

which evaluated at $\widetilde{w}=w$ and $\widetilde{\theta}=\theta=1/M$ yields $\Psi(w(v))=0$ where

$$\begin{aligned}\Psi(w(v)) &\equiv \left(1-\frac{1}{M}\right)^{N-1}-\left(\frac{1}{M}\left.\frac{\partial\Omega_{N,M}(\theta;v)}{\partial\theta}\right|_{\theta=1/M}+\Omega_{N,M}\left(\frac{1}{M};v\right)\right)w(v) \\ &\quad +\frac{1}{M-1}\left.\frac{\partial\Omega_{N,M}(\theta;v)}{\partial\theta}\right|_{\theta=1/M}\frac{u(w(v))}{u'(w(v))}.\end{aligned}$$

$\Psi(0)=(1-1\backslash M)^{N-1}>0$ and $\Psi(w(v))$ is strictly decreasing since

$$\begin{aligned}\frac{\partial\Psi(w)}{\partial w} &= -\left(\frac{1}{M}\left.\frac{\partial\Omega_{N,M}(\theta;v)}{\partial\theta}\right|_{\theta=1/M}+\Omega_{N,M}\left(\frac{1}{M};v\right)\right) \\ &\quad +\frac{1}{M-1}\left.\frac{\partial\Omega_{N,M}(\theta;v)}{\partial\theta}\right|_{\theta=1/M}\frac{(u'(w))^2-u(w)u''(w)}{(u'(w))^2},\end{aligned}$$

is strictly negative since

$$\begin{aligned}
\frac{\partial \Omega_{N,M}(\theta;v)}{\partial \theta} &= \frac{-v(1-\sum_{k=0}^{v} p_{N-1}^{k}(\theta))}{N\theta^2} < 0, \\
\frac{(u'(w))^2 - u(w)u''(w)}{(u'(w))^2} &> 0, \text{ and} \\
\theta \frac{\partial \Omega_{N,M}(\theta;v)}{\partial \theta} + \Omega_{N,M}(\theta;v) &= \frac{\sum_{k=0}^{v} k p_{N-1}^{k}(\theta)}{N\theta} > 0.
\end{aligned}$$

In addition,

$$\begin{aligned}
\Psi(1) &= \left(1-\frac{1}{M}\right)^{N-1} - \left(\left(1-\frac{1}{M}\right)^{N-1} + \frac{\sum_{k=2}^{v} k p_{N-1}^{k}(\theta)}{N\theta}\right) \\
&\quad + \frac{1}{M-1}\left(\frac{-v(1-\sum_{k=0}^{v} p_{N-1}^{k}(\theta))}{N\theta^2}\right)\frac{u(w(v))}{u'(w(v))},
\end{aligned}$$

which is strictly negative. Therefore there exists a unique $w(v) \in (0,1)$ solving $\Psi(w(v)) = 0$. And given that there is a unique value of θ that makes the workers indifferent between the different firms, this the unique possible $w(v)$. ■

References

[1] Acemoglu, D., and R. Shimer. 1999. "Hold-up and Efficiency with Search Frictions," *International Economic Review*, 40, 827-850.

[2] Albrecht, J. P.A. Gautier and S. Vroman. 2007. "Equilibrium Directed Search with Multiple Applications," *Review of Economic Studies*, 73, 869-

[3] Akerlof, G.A. and H. Miyazaki. 1980. "The Implicit Contract Theory of Unemployment meets the Wage Bill Argument," *Review of Economic Studies*, XLVII, 321-338.

[4] Azariadis, C. 1975. "Implicit Contracts and Underemployment Equilibria," *Journal of Political Economy*, 83(6), 1183-1202.

[5] Baily, M.N. 1974. "Wages and Employment under Uncertain Demand," *Review of Economic Studies*, 41, 37-50.

[6] Binmore, K.G. and M.J. Herrero. 1988. "Matching and Bargaining in Dynamic Markets," *Review of Economic Studies*, 55, 17-31.

[7] Burdett, K., S. Shi, and R. Wright. 2001. "Pricing and Matching with Frictions." *Journal of Political Economy*, 109(5), 1060-1085.

[8] Camera, G. and A. Delacroix. 2004. "Trade Mechanism Selection in Markets with Frictions," *Review of Economic Dynamics*, 7, 851-868.

[9] Cole, M.G., and J. Eeckhout. 2003. "Indeterminacy and Directed Search," *Journal of Economic Theory*, 111, 265-276.

[10] Faig, M. and X. Huangfu. 2007. "Competitive-Search Equilibrium in Monetary Economies," *Journal of Economic Theory*, 136, 709-718.

[11] Gale, D. 1986. "Bargaining and Competition Part I: Characterization," *Econometrica*, 54,785-806.

[12] Gale, D. 1987. "Limit Theorems for Markets with Bargaining," *Journal of Economic Theory*, 43, 20-54.

[13] Galenianos, M. and P. Kircher. 2006. "Directed Search with Mutiple Job Applicants," mimeo, Univ. of Pennsylvania.

[14] Holmstrom, B. 1983. "Equilibrium Long-Term Labor Contracts," *Quarterly Journal of Economics*, 98 (Suppl.), 24-54.

[15] Julien, B., J. Kennes and I. King. 2000. "Bidding for Labor," *Review of Economic Dynamics*, 3, 619-649.

[16] Julien, B., J. Kennes and I. King. 2006. "Bidding for Money," *Journal of Economic Theory* (forthcoming).

[17] Kihlstrom, R.E., and J.J. Laffont. 1983. "Implicit Labor Contracts and Free Entry," *Quarterly Journal of Economics*, 98 (Suppl.), 55-105.

[18] Kircher, P. 2007. "Efficiency of Simultaneous Search," mineo, Univ. of Pennsylvania.

[19] McAfee, R.P. 1993. "Mechanism Design by Competing Sellers," *Econometrica*, 61(6), 1281-1312.

[20] Michelacci, M. and J. Suarez. 2006. "Incomplete Wage Posting," *Journal of Political Economy*, 114(6), 1098-1123.

[21] Montgomery, J.D. 1991. "Equilibrium Wage Dispersion and Interindustry Wage Differential," *Quarterly Journal of Economics*, 106, 163-179.

[22] Peters, M. 1991. "Ex Ante Price Offers in Matching Games: Non-Steady-States," *Econometrica*, 59(5), 1425-1454.

[23] Peters, M. 1997. "On the Equivalence of Walrasian and Non-Walrasian Equilibria in Contract Markets: The Case of Complete Contracts," *Review of Economic Studies*, 64, 241-264.

[24] Peters, M. 2000. "Limits of Exact Equilibria For Capacity Constrained Sellers with Costly Search," *Journal of Economic Theory*, 95, 139-168.

[25] Polemarchakis, H.M. 1979. "Implicit Contracts and Employment Theory," *Review of Economic Studies*, XLVI, 97-108.

[26] Rubinstein A. and A. Wolinsky. 1985. "Equilibrium in a Market with Sequential Bargaining," *Econometrica*, 53, 1133-1550.

[27] Rubinstein A. and A. Wolinsky. 1990. "Decentralized Trade, Strategic Behavior, and the Walrasian Outcome," *Review of Economic Studies*, 57.

[28] Shi, S. 2001. "Frictional Assignment. I. Efficiency," *Journal of Economic Theory*, 98, 232-260.

[29] Shi, S. 2005. "Frictional Assignment. II. Infinite Horizon and Inequality," *Review of Economic Dynamics*, 8, 106-137.

[30] Shi, S. 2006. "Search Theory; Current Perspectives," U of Toronto mimeo.

[31] Shimer, R. 2005. "The Assignment of Workers to Jobs in an Economy with Coordination Frictions," *Journal of Political Economy*, 113, 993-1025.

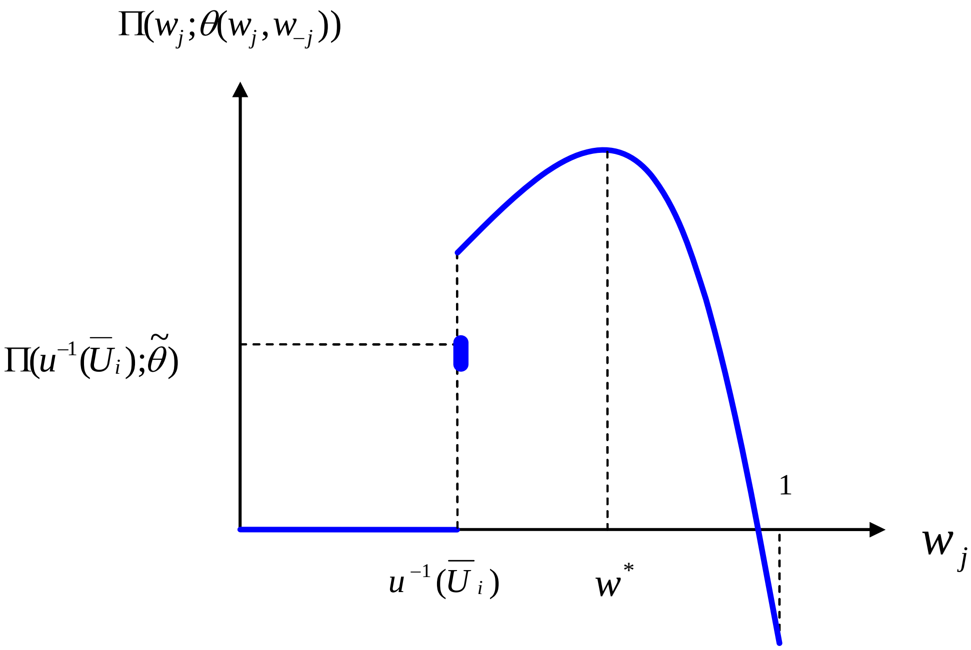

Figure 1 - Panel a

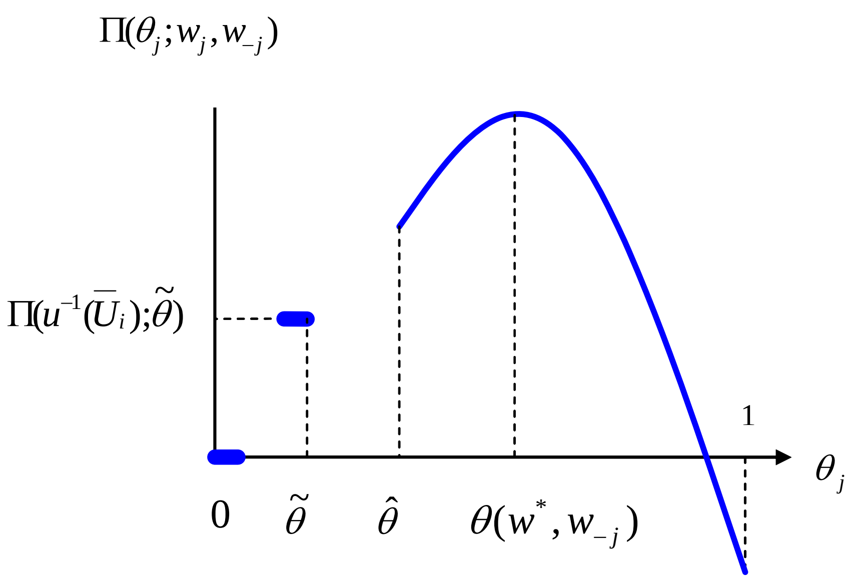

Figure 1 - Panel b

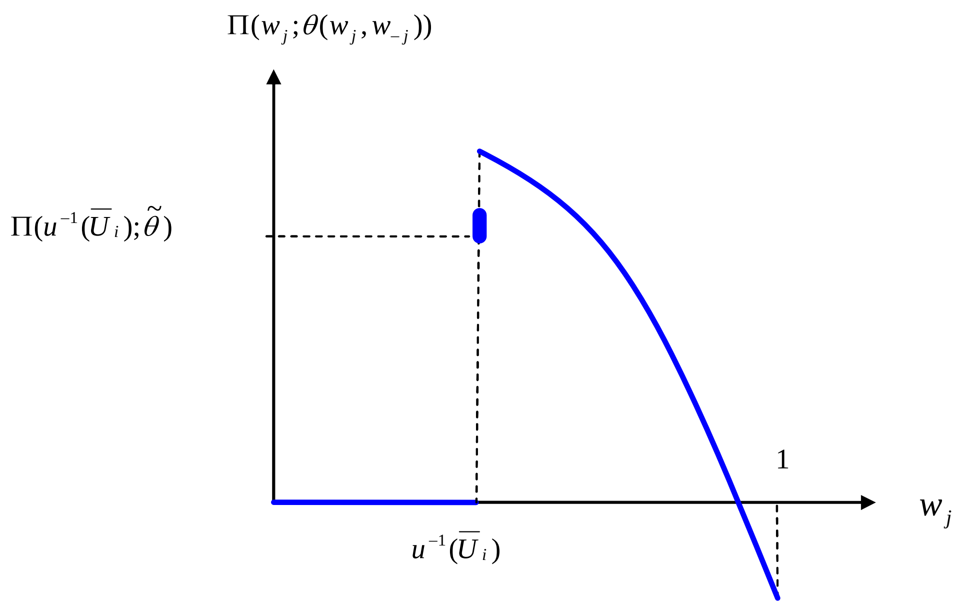

Figure 2 - Panel a

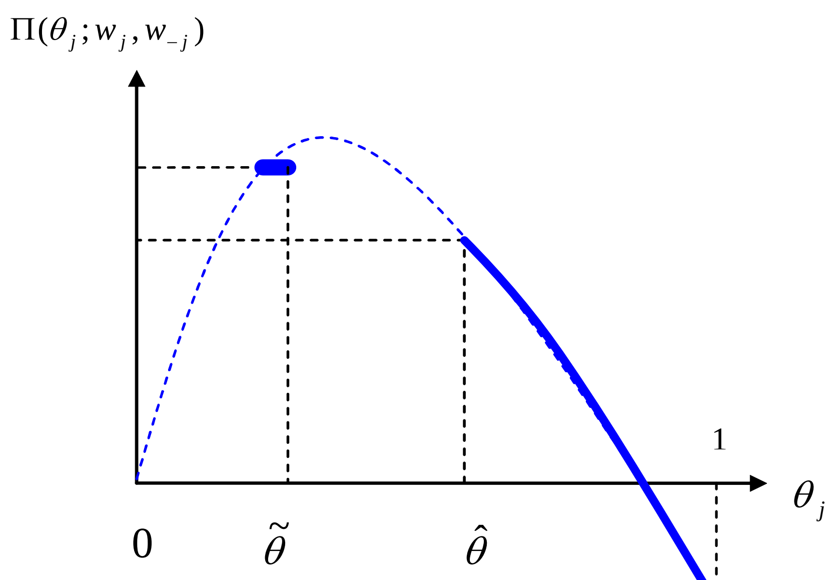

Figure 2 - Panel b

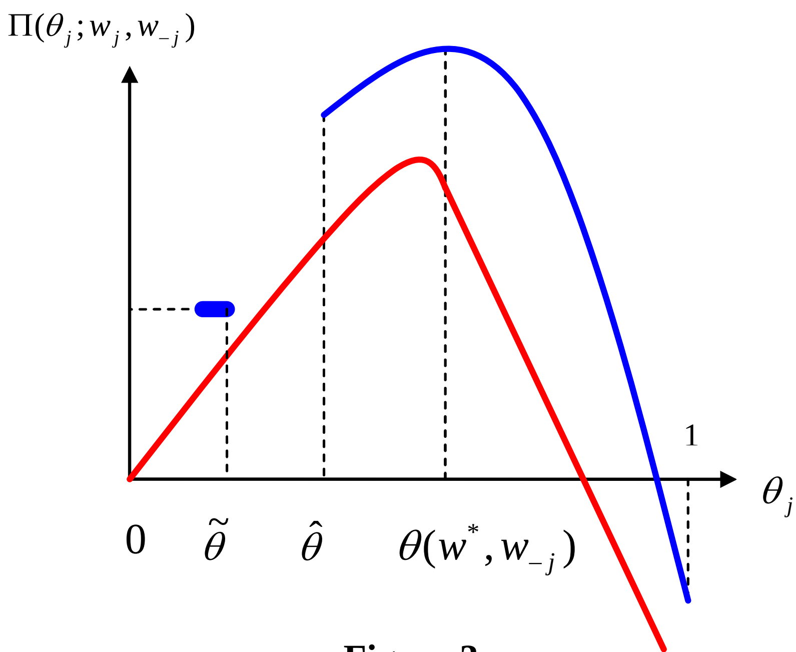

Figure 3

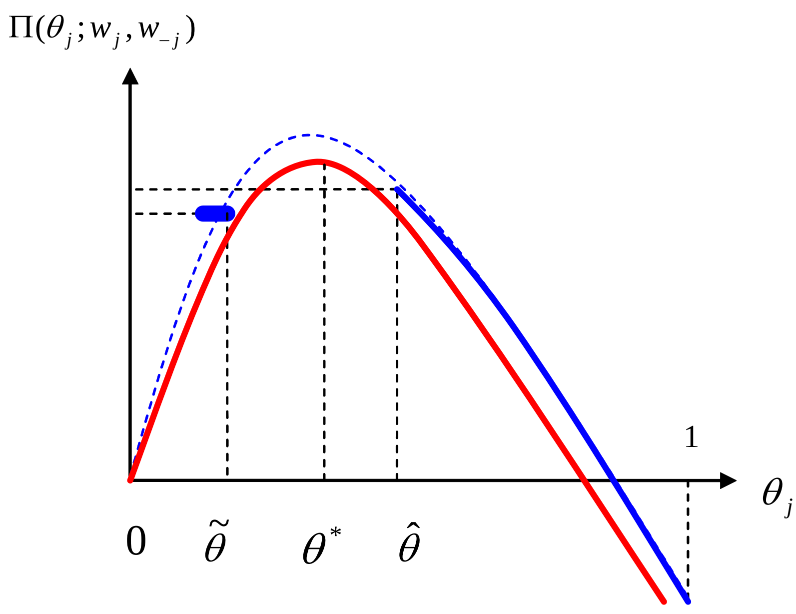

Figure 4