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Abstract

We argue that the degree of substitutability between skilled and unskilled workers in production increases long-term income growth rate. Growth rate reaches its maximum when such factors are perfect substitutes, but falls to zero when they are perfect complements. This model brings together the diverging relative wage and the human-capital-growth literature. Easier substitution absorbs more workers into the skilled profession, and their training fuels human capital accumulation and growth. Our result implies, among other things, that growth is positively related to between- and within-group inequality.

Key words: Growth, factor substitution, skilled/unskilled wage

JEL Classification: 015, 041.

1. Introduction

The study of long-term economic growth has so far discovered a small set of variables that qualify as basic exogenous determinants of the rate at which per-capital income and output grow. In Lucas' (1988) framework it is narrowed down to two - "growth increases with the effectiveness (δ) of investment in human capital and declines with increases in the discount rate ρ" (p. 23, ibid). In the models of Arrow (1962), Romer (1987, 1990), Aghion and Howitt (1992) and Grossman and Helpman (1991), growth depends on society's incentive to inventive activities, which is in turn dependent on the society's competitive environment and interest rate. It follows that government action to maintain law and order, to protect free trade, and to provide an efficient financial infrastructure can affect growth. Rebelo (1991) points out that taxation policy can generate growth in the absence of increasing returns.

The thesis advanced in this paper argues that the degree of substitutability between certain factors in the production function, namely the skilled and the unskilled, is another fundamental exogenous variable affecting growth.[1] In particular, growth rate increases with the substitutability between the skilled and the unskilled, rising to its maximum under perfect substitution, falling to a standstill under perfect complementarity. We argue that factor substitutability works in conjunction with Lucas' δ to determine growth rate. In fact Lucas' model, where homogenous labor implies perfect substitutability, is a special case of the model developed below. We show that growth rate is in general slower when skilled workers are imperfect substitutes for the unskilled.

The mechanism underlying the model presented below is a simple one and it can be stated informally as follows. If *everyone* spends a fraction of time-endowment

$0 < u < 1$ working, then a society with N workers spends an aggregate resource amounting to $N(1-u)$ in human capital accumulation that sustains its growth via the effectiveness of human capital accumulation captured by Lucas' δ. But if production requires two factors, the skilled and the unskilled say, *only the skilled* devote time to training and to accumulate human capital, then we need to add an endogenous career choice mechanism to the model. Career choice depends not only on individual's aptitude to receive education, but also on how easily skilled workers are absorbed into the production process. If the skilled is a good substitute to the unskilled, more will engaged in education, human capital accumulation will be faster, and so will growth.

Empirical evidence on factor substitution, scanty as they are, focuses on capital - skill complementarity (which of course directly implies substitutability) in the production function rather than that between the skilled and the unskilled. Griliches (1969) finds evidence from two United States cross-sectional data sets (1954 and 1963) that capital is complementary to the skilled, but much less so (more substitutable) to the unskilled labor. We are not aware of any good evidence on the precise substitution relationship between skilled and unskilled labor.

There is, however, evidence that the relations between factors have changed over time. Goldin and Katz (1998) make an interesting observation that automobile production began in the 1900's in artisanal shops where capital and labor were highly complementary. This changed substantially with the arrival of robotized assembly lines where machines became much better substitutes for and reduced the demand for unskilled labor. In many production processes besides automobile the division of labor were evidently more rigidly defined than, say, under the modern management

[1] The substitutability (or complementarity) between labor and physical capital is an interesting topic but it will be left for future work.

concepts of multi-tasking and job-rotation. We may take this as evidence that the skilled have become better substitutes to the unskilled.

Further supports for increased substitutability between the skilled and the unskilled are found in the growing literature bent on explaining the divergence of relative wages in the United States in recent decades. A common theme in papers such as Bound and Johnson (1992), Katz and Murphy (1992), Levy and Murnane (1992), Juhn, Murphy, and Pierce (1993), Krueger (1993), and Autor, Katz and Krueger (1998) is that late-twentieth century technological change is often 'skilled-biased', in the sense of increasing the complementarity between capital (such as computers in which the new technology is embodied) and the skilled, and increasing the substitutability between capital (including the human capital embodied in the skilled) and the unskilled. If this increases, as it probably does, the substitutability between the skilled and the unskilled, then our thesis predicts an increase in growth rate to coincide with skilled-biased technological change. This is consistent with the observation, as Barro and Sala-i-Martin (1995) wrote, "the growth rate for 1970-90 is high in relation to the long-term history" (p.6, ibid.).

In terms of modeling our paper brings together two dynamic mechanisms from the recent theoretical literature. The first one, as mentioned earlier, is the Uzawa (1965) and Rosen (1976) model of human capital accumulation, simplified and adopted by Lucas (1988) as the centerpiece of the mechanism of endogenous growth. The second is the dynamic mechanism of career choice between skilled and unskilled, featuring with minor variations in many recent papers on wage inequality (Galor and Zeira, 1993; Eicher, 1996; Galor, Oded and Tsiddon, 1997; Acemoglu, 1998; Galor and Moav, 2000; Eicher and Garcia-Penalosa, 2001). As we argued earlier these two mechanisms are clearly related. Our contribution is to enrich the Uzawa - Rosen -

Lucas economy by allowing two factors into the production function, and by incorporating the endogenous career choice mechanism as part of the dynamic engine of growth.

The simple model developed below turns out to be capable of extending in a number of ways with interesting contributions. First it predicts that within- (skilled) group inequality increases with growth rate. This is consistent with the empirical finding of Forbes (1998), though criticized by Aghion, Caroli and Garcia-Penalosa (1999). We believe, and explain below, that our finding complements rather than contradicts that of Aghion, Caroli and Garcia-Penalosa (ibid.). Second, human capital accumulation is suboptimal if left to the market but the degree of suboptimality falls with factor substitutability. Third and consistent with Rebelo (1991), governments can increase growth rate by reducing impediments to factor substitutability by class, castes, sex or race. Finally, by noting the increase in factor substitutability from the time when a new technology is first introduced to the time when it is gradually familiarized, we can explain fluctuations in growth rates (similar process are observed and modeled by Eicher (1996) and Acemoglu (1998) but they do not draw the connection between wage inequality and growth rate).

The remainder of this paper is organized as follows. The next section connects the two dynamic mechanisms, namely the human capital accumulation and the endogenous career choice. Section three formally establishes the results under perfect substitutability and perfect complementarity. Section four shows that the result holds generally with varying degrees of substitution. In general, long-term endogenous growth rate rises with factor substitutability. Section five extends the model briefly to the four areas mentioned in the last paragraph. Section six summarizes and concludes the essay.

2. The Dynamics of Growth and Career Choice

Lucas (1988) adopts a simplified linear version of the Uzawa (1965) and Rosen (1976) mechanism linking the rate of human capital accumulation to its level. The Lucas engine of growth (in discrete time periods) has the representation,

$$h(t+1)-h(t)=h(t)\delta[1-u(t)] \tag{2.1}$$

where $h(t)$ is current level human capital, δ is the constant "effectiveness of investment in human capital", and $[1-u(t)]$ is the proportion of time a *representative individual* spends off work training.

In order to study substitution between skilled (human capital) and unskilled labor, we depart from Lucas and abandon the *representative individual* assumption. Ignoring population growth and denoting the constant population size by L, equation (2.1) becomes

$$h(t+1)-h(t)=h(t)\delta\int_{i=1}^{L}[1-u_i(t)]di\,. \tag{2.2}$$

The size of the integral term $\int_{i=1}^{L}[1-u_i(t)]di$ depends critically on the endogenous process of career choice in the economy. Moreover, since $\int_{i=1}^{L}[1-u_i(t)]di$ increases with (a) the proportion of L who chooses to train in order to join the skilled profession, and (b) the amount of time each individual spends on training, so do the speed of human capital accumulation and the speed of growth.

Let each discrete time period have unit length. Suppose in any period t the economy has one final product $x(t)$ produced with a constant returns to scale technology using two inputs, namely (*effective units of*) skilled labor $s(t)$ and ('raw' units of) unskilled labor $n(t)$. The particular representations of the production technology will be specified later but the general representation is

$$x(t) = f[s(t), n(t)]. \tag{2.3}$$

At the beginning of each period t a generation of size L is born. Each individual lives for a single period in which he/she trains, works, consumes and at the end passes away without material bequest. Much of the career choice dynamics hinges on the description and composition of $s(t)$.

Following Lucas (1988), human capital accumulation is a *social activity*. In the present context the external effect of this activity takes the form of knowledge augmented and passed on from one generation to the next. At birth a generation t inherits knowledge level $h(t)$ from their forefathers, augmenting it through training (education) in the form of (2.2), and passes it on to their offspring in t+1. Let the aggregate effective skilled labor units at period t be specified as

$$s(t) = h(t)\int_{i=1}^{z(t)} u_i(t)\,di \tag{2.4}$$

where $h(t)$ as specified earlier is inherited human capital level, $z(t)$ is the total 'headcount' of the skilled workforce. The ith skilled worker devotes a portion of his/her unit time endowment $0 \le u_i(t) \le 1$ to working instead of training; $i \in [1, z(t)]$. In so doing, the ith skilled worker offers $s_i(t) = h(t)u_i(t)$ units of effective skilled labor power for productive employment. Without loss of generality the constant population size is henceforth set to unity. It follows that the two professions must have the relation $z(t) + n(t) = L = 1$.

Let $a_i(t)$ denote individual i's intrinsic (cognitive) ability at birth prior to training (if any). Assume further $a_i(t)$ is uniformly and independently (and time-invariantly) distributed along the unit segment: $a_i(t) \in [0, 1]$. Unskilled labors use instead raw physical strength, which is assumed constant across the population.

Unskilled wage (or earnings per head at *t*), denoted $w^n(t)$, is therefore identical across the unskilled workforce. The earnings of a skilled worker, by contrast, depends on the skilled wage, the time he/she spends at work, and the inherited human capital level; viz.

$$w^z(t) = w^s(t)\, u_i(t)\, h(t) \tag{2.5}$$

where $w^s(t)$ is salary *per effective skilled unit* of labor which is of course uniform across the skilled workforce.

Cognitive ability $a_i(t)$ reduces the time (denoted $\tau_i(t) \equiv 1 - u_i(t)$) individual i needs to spend training in order to gain entry into the skilled profession. A general representation of the training technology may be $\tau_i(t) = G[a_i(t)],\, dG/da_i < 0$. To simplify the algebra with little loss of generality we adopt the following formulation

$$\tau_i(t) = 1 - a_i(t)\,. \tag{2.6}$$

Since it readily follows that $u_i(t) = a_i(t)$, this formulation has a convenient interpretation that each qualified skilled worker devotes to work strictly according to his/her intrinsic cognitive ability.

Equilibrium career choice is defined by a particular individual whose ability is $\hat{a}(t)$, who devotes a fraction $\hat{u}(t)$ working and $[1 - \hat{u}(t)]$ training, and who in so doing earns the same amount from either the skilled or the unskilled profession. Thus we have

$$w^s(t)\, \hat{u}(t)\, h(t) = w^s(t)\, \hat{a}(t)\, h(t) = w^n(t)\,.$$

Rearranging,

$$\hat{a}(t) = \frac{w^n(t)}{w^s(t)\, h(t)}\,. \tag{2.7}$$

The equilibrium value of $\hat{a}(t) \in [0, 1]$ plays a pivotal role in this model and it has the following interpretation. First, relative market wages $w^n(t) \big/ w^s(t)$ reflects relative derived demand for factors, which in turn reflects factor-substitutability of a particular production technology.[2] Second, as mentioned earlier, social human capital $h(t)$ is inherited from the past and is treated as an exogenous parameter at *t*. Hence the right-hand side of the equation $\hat{a}(t)$ fully reflects relative demand for factors at *t*. Third, from the uniform distribution of ability, the supply (denoted by superscript '*s*') of unskilled labor is $n^s(t) = \hat{a}(t)$, and the supply of skilled *headcounts* is $[1 - \hat{a}(t)]$ (recall total population size is assumed to be unity). But given the simple training technology (2.4), the value of $\hat{a}(t)$ also completely determines the supply of effective skilled-labor units since $s^s(t) = \int_{\hat{a}(t)}^{1} u_i(t)\, h(t)\, di$; substituting $u_i(t) = a_i(t)$ and evaluating the definite integral,

$$s^s(t) = \frac{h(t)\,[1 - \hat{a}(t)^2\,]}{2}. \tag{2.8}$$

In an equilibrium to be defined shortly, $\hat{a}(t) \in [0, 1]$ captures fully the relative supply of factors at *t*, and via equation (2.7) determines equilibrium career choice contingent on factor substitution as well as other characters in the production technology for $x(t)$.

Furthermore, $\hat{a}(t)$ completely describes the dynamic link via social human capital accumulation activities since we can rewrite equation (2.2) as

$$h(t+1) - h(t) = h(t)\delta \int_{\hat{a}(t)}^{1} [1 - u_i(t)]di .$$

[2] It also conveys messages concerning the returns to scale and other characteristics of the production technology.

Substituting $u_i(t) = a_i(t)$ and evaluating the definite integral, the speed of technical progress via social human capital accumulation is expressed only in terms of $\hat{a}(t)$ and δ

$$h(t+1) - h(t) = h(t)\delta\,[\frac{1}{2} - \hat{a}(t) + \frac{\hat{a}(t)^2}{2}]\,. \tag{2.9}$$

The intuition of $\hat{a}(t)$ determining the speed of technical progress and human capital accumulation is simple, and is revealed in its starkest form in two limiting cases $\hat{a}(t) = 0$ and $\hat{a}(t) = 1$. If $\hat{a}(t) = 0$ every individual in the society chooses to acquire skill and the unskilled profession is empty. Social human capital accumulation proceeds at its maximum speed $(\delta / 2)$. If $\hat{a}(t) = 1$ no individual in the society choose to acquire skill and the skilled profession is empty. Social human capital accumulation grinds to a complete standstill. The next section shows that these limiting cases arise from perfect substitutability and perfect complementarity in the production function.

3. Two Limiting Cases: Perfect Substitutes and Perfect Complements in Production

This section is a preliminary step linking dynamic social human capital accumulation and career choice. It serves as a preamble to the more general model developed in section IV below.

We will show that the two limiting cases $\hat{a}(t) = 0$ and $\hat{a}(t) = 1$, discussed at the end of the last section, coincide with the two limiting cases in production technology - perfect substitution and perfect complementarity. We now deal with each of these in turn.

Perfect Substitutability and Maximum-Speed Growth

Assume for the moment *an effective unit* of skilled labor is a perfect substitute to a unit of unskilled labor. The (constant returns to scale) production function has the form

$$x(t) = f[s(t), n(t)] = \alpha s(t) + \beta n(t) \tag{3.1}$$

where α and β are positive constants. The minimized cost function takes the form $c[w^s(t), w^n(t), x(t)] = \min[w^s(t)/\alpha + w^n(t)/\beta]x(t)$. The (Kuhn-Tucker) solution for the firm's cost-minimization is

$$\begin{cases} w^s(t) = w^n(t) \Leftrightarrow s(t) > 0, n(t) > 0; \\ w^s(t) > w^n(t) \Leftrightarrow s(t) = 0, n(t) > 0; \\ w^s(t) < w^n(t) \Leftrightarrow s(t) > 0, n(t) = 0. \end{cases} \tag{3.2}$$

Assume initial human capital stock $h(0) > 1$ at $t = 0$ when we begin our inquiry.[3] We will return to examine this assumption shortly. Since a skilled worker possesses $h(t)u_i(t)$ effective units of labor power, at least some able newborn at $t = 0$ would find it worthwhile to train and join the skilled profession provided $h(0)$ is sufficiently large. This will be substantiated shortly.

There are three cases from (3.2) to consider. In the first case, suppose $w^s(0) = w^n(0)$, and $s(t) > 0, n(t) > 0$ hold. From the career choice equation (2.7) we have $0 < \hat{a}(0) = 1/h(0) < 1$. This confirms the conjecture in the last paragraph, namely that *the skilled profession will be non-empty* for as long as $h(0) > 1$ since the most able individuals hardly need to train to acquire the skill. We saw in section 2 above

[3] The case of $h(t) < 1$ can be dismissed out of hand for it is inconsistent with the definition of a skilled labor. To see this recall from equation (2.6) that the most able individual joins the skilled profession without training and offers $u(t)h(t) = h(t)$ units of effective skilled labor power for employment. $h(t) < 1$ would have implied that even the most able skilled worker is *less* productive as skilled labor than as unskilled.

that $\hat{a}$ links up production substitutability with the dynamic process in the model. Now for as long as $\hat{a}(t) < 1$, the education process is active and the social human capital accumulation process proceeds by equation (2.9) and $h(1) > h(0)$ holds. By similar argument $h(t+1) > h(t)$ for all *t*. But this feeds back into equation (2.7), the skilled profession grows $[\hat{a}(t+1) < \hat{a}(t)]$ until $\hat{a}(t) = 0$ for some finite $t > 0$ and the unskilled profession is empty. From then on everyone trains for skill; human capital accumulates at its maximum speed $\dfrac{h(t+1) - h(t)}{h(t)} = \dfrac{\delta}{2}$. National output, since equation (3.1) takes the form $x(t) = as(t)$ when $n(t) = 0$, grows also at its maximum speed $\delta / 2$.

The third case in equation (3.2) is just a special case of that discussed in the last paragraph. With the unskilled profession already empty from the first period under consideration, i.e. $\hat{a}(t) = 0$ for $t \geq 0$, both social human capital accumulation and national income grows at a maximum speed $\delta / 2$. A low-ability individual chooses the skilled profession by spending a large fraction of his/her time training, even though $w^s(t) < w^n(t)$, when his/her *earning* is greater than staying unskilled. Using the skilled earning per head equation (2.5), this implies $w^z(t) = w^s(t)\, u_i(t)\, h(t) > w^n(t)$, i.e., $u_i(t)\, h(t) > 1$ which will be true when $h(t)$ is sufficiently large.

The only remaining case is the second in equation (10), namely $w^s(0) > w^n(0)$, and $s(0) = 0, n(0) > 0$. This implies $\hat{a}(0) = 1$. Since the skilled profession is empty, no one gets trained and the social human capital accumulation process is at a standstill. Reflecting on the skilled earning per head equation (2.5), we know $w^z(0) = w^s(0)\, u_i(0)\, h(0) < w^n(0)$. But this inequality must be true even for the most able individual who has $a_i(0) = u_i(0) = 1$. We infer therefore $h(0) < 1$ *strictly* under $w^s(0) > w^n(0)$. We dismiss this case as intuitively meaningless using the argument in footnote two.[4]

A quick intuitive summary for this subsection is in order. The definition of skill restricts our attention to $h(t) \geq 1$. Perfect substitutability implies $0 < \hat{a}(t) \leq 1$ at some arbitrary initial time *t*. What we have shown is that the process of human capital accumulation once started will lead the economy to the corner solution $\hat{a}(t) = 1$. Everyone trains; both human capital and national income grow at its maximum speed $\delta / 2$.

Perfect Complementarity and Growth Stagnation

Assume instead *an effective unit* of skilled labor is a perfect complement to a unit of unskilled labor. The production function has the form

$$x(t) = f[s(t), n(t)] = \min\{\alpha s(t), \beta n(t)\} \tag{3.3}$$

where α and β are positive constants. Production must use $s(t)/\alpha$ units of skilled labor *and* $n(t)/\beta$ units of unskilled labor whatever are $w^s(t)$ and $w^n(t)$. The derived relative demand between effective skilled labor units and unskilled labor is $\frac{s(t)}{n(t)} = \frac{\beta}{\alpha}$.

[4] The text omits the only remaining case concerning initial human capital stock - $h(0) = 1$. The analysis and result are identical to the case of $h(0) > 1$ so we relegate it to a footnote. The skilled earning per head equation becomes $w^z(0) = w^s(0)\, u_i(0)$. Taking this into account, growth rates for human capital as well as national income in the three cases in equation (10) are

$$\begin{cases} w^s(0) = w^n(0) \Leftrightarrow s(0) > 0,\, n(0) > 0;\, 0 < \hat{a}(0) < 1,\, \hat{a}(t) = 0 \; as \; t \to \infty;\; g = \delta / 2 \\ w^s(0) > w^n(0) \Leftrightarrow s(0) = 0,\, n(0) > 0; \text{dismissed since it implies } h(0) < 1, \text{see foot note 2} \\ w^s(0) < w^n(0) \Leftrightarrow s(0) > 0,\, n(0) = 0; \hat{a}(t) = 0 \text{ for all } t \geq 0,\; g = \delta / 2. \end{cases}$$

This reinforces the result derived in the text.

Again let $h(0) > 1$ at initial time period $t = 0$. The supply for skilled units is given by equation (2.8) and the supply for unskilled labor is simply $\hat{a}(0)$. Equating relative demand and supply yields the equilibrium condition $\frac{h(0)\,[1-\hat{a}(0)^2]}{2\,\hat{a}(0)} = \frac{\beta}{\alpha}$.

This quadratic equation in $\hat{a}(0)$ has two solutions $\hat{a}(0) = [-\beta \pm \sqrt{h(0)^2\alpha^2 + \beta^2}\,]/\alpha$.

Only the positive solution is economically meaningful, thus

$$\hat{a}(0) = \frac{-\beta + \sqrt{h(0)^2\alpha^2 + \beta^2}}{\alpha} > 0. \tag{3.4}$$

The inequality follows from the intuitive restriction $h(t) \geq 1$ (see footnote 2).

Now invoke the dynamic social human capital accumulation process equation (2.9). Equation (3.4) feeding into (2.9) implies $h(t+1) > h(t)$ if $\hat{a}(t) < 1$. Again using this in (12) implies $d\hat{a}(t)/dt > 0$. Long-term equilibrium has $\hat{a}(t)$ approaching unity, more precisely the skilled profession eventually contains a single most able individual, who devotes his/her entire time endowment working. Human capital accumulation and national income growth grind to a complete standstill.

The two limiting cases together show that growth rate achieve its maximum under perfect substitution, but it is totally dissipated under perfect complementarity in production. To complete our inquiry we shift our attention to study the interior solutions.

4. Substitutability and Growth: Interior Solutions

This section has two simple objectives: to prove the existence of interior-solution equilibrium in the dynamic growth system where the skilled and unskilled professions are non-empty, and to show that in this equilibrium easier substitution leads to faster long-term growth.

Existence of Interior-Solution Equilibrium

There are different ways to represent the degree of substitutability in the constant returns to scale production function $x(t) = f[s(t), n(t)]$. Let $f[s(t), n(t)] = \text{constant}$ define the system of the constant product curve (isoquant) on the *0ns* plane. For our purpose it suffices to work with the curvature ($d^2 s(t) / dn(t)^2 > 0$) of the isoquant.[5] A production function exhibiting easier substitution between s and n would be less curved, becoming a straight line with constant slope in the limiting case of perfect substitutes. The intuitive property of easy substitution is that when a factor's (effective skilled units) supply increases, it is more easily absorbed into the production process, thus necessitating a relatively small perturbation in the marginal rate of substitution and relative wage rates.

Following the last section our focus remains firmly on $\hat{a}(t)$ which links endogenous career choice to social human capital accumulation. Long-term interior-solution equilibrium is defined by $\hat{a}(t) = \bar{a} = \text{constant}$ for all $t = 0,1,2,\ldots,\ 0 < \bar{a} < 1$ strictly. It is "interior" in the sense that the two professions are strictly non-empty. From equation (8) the long-term rate of human capital accumulation is $[h(t+1) - h(t)] / h(t) = \delta\, (\frac{1}{2} - \bar{a} + \frac{\bar{a}^2}{2}) = g > 0$. Writing

[5] The elasticity of substitution of the linearly homogenous production function $x = f(s, n)$ can be written as $\sigma = \frac{r}{sn} \frac{nr + s}{c}$ where $r = -\frac{ds}{dn}$ is the marginal rate of substitution, and $c = \frac{d^2 s}{dn^2} = r \frac{dr}{ds} - \frac{dr}{dn}$ is the curvature of the isoquant. The degree of substitutability between s and n is thus inversely proportional to the curvature of the isoquant (cf. Allen (1938), p.342).

$x(t) = f[s(t), n(t)] = s(t)\phi[n(t)/s(t)]$, since $s(t) = \int_{\bar{a}}^{1} h(t)a_i di = h(t)(1-\bar{a}^2)/2$, it is easily seen that in the long-term equilibrium (if one exists) $[s(t+1)-s(t)]/s(t) = [h(t+1)-h(t)]/h(t) > 0$.

Consider a system of isoquants having a finite curvature $\infty > d^2 s(t)/dn(t)^2 > 0$, but we are allowed to compare 'neighboring' systems with different curvatures.[6] Suppose at time τ human capital accumulation has reached an arbitrary level $h(\tau)$ and endogenous career choice is $\hat{a}(\tau) = \bar{a}$. This configuration would be a long-term equilibrium if $\hat{a}(\tau) = \bar{a}$ for all $t > \tau$. If it exists, in this equilibrium $n(t) = \bar{a}$, $z(t) = 1 - \bar{a}$ and $s(t) = h(t)(1-\bar{a}^2)/2$ for all $t > \tau$. Although s grows at a constant rate g, they are absorbed completely into the skilled profession. The increasing supply of s continuously reduces wage w^s per effective unit of s, yet no skilled worker finds it worthwhile to shift to the unskilled profession since the fall in w^s is exactly offset by the rise in h.

To prove the existence of such an equilibrium, suppose in the system just described $\hat{a}(\tau+1) = \bar{a} + \varepsilon$ where ε is an arbitrarily small constant. The growth of efficient units of skilled labors during τ must have so depressed relative wages w^s/w^n in $\tau+1$ that a larger number of citizens than in τ find it worthwhile to choose the unskilled profession. Now if we allow the system to be replaced by ones with continuously reducing curvatures, the fall in w^s/w^n between τ and $\tau+1$ approaches zero as $d^2 s(t)/dn(t)^2$ approaches zero. The existence result follows from

[6] Imagine we denote the *i*th production system in terms of its curvature c_i (see footnote 4 above), where $c_i \in [0, \infty)$, $i = 1, 2, \ldots$. Imagine also technology is sufficiently compact such that c_i is

the finiteness of ε and the fact that the curvature asymptotically approaches zero (linear isoquants when *s* and *n* are perfectly substitutes).

Characterization of The Interior-Solution Equilibrium

If the production systems are sufficiently compact such that curvature c_i is continuous and differentiable (see footnote 5), it follows from the argument just presented that there exists a continuum of such equilibria each identified by its degree of substitutability between *s* and *n* (the curvature of its isoquants). Each of these equilibria will be characterized by a different career configuration $\overline{a}$ and human capital and income will grow at a different rate.

Again we rank such equilibria in terms of the curvature of their isoquants c_i such that $c_i < c_j$ for $i < j$. Consider two neighboring equilibria *i* and *j* where $c_i < c_j$. The greater substitutability of c_i allows a larger fraction of the population as skilled labor (by analogous reasoning presented in subsection IV*A*). In other words $\overline{a}_i > \overline{a}_j$ holds. It follows immediately from equation (2.9) that social human capital grows faster under equilibrium *i* compared to *j*. This argument is readily extended to all equilibria, and it allows us to conclude that long-term human capital and income grow at a higher rate under easier substitution between effective skilled and unskilled labors.

5. Extensions and Applications

This section extends and applies the model developed so far to tackle four interesting issues. First, we show that growth rate is positively and endogenously linked to between-group as well as within-group inequalities via substitutability between the

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continuously differentiable, and we rank them such that $c_i < c_j$ for $i < j$. The two limiting cases are perfect substitutability - $c_i = 0$, and perfect complements - $c_i = \infty$.

skilled and the unskilled. This finding is consistent with the empirical result of Forbes (1998), which is however debated by Aghion, Caroli and Garvia-Penalosa (1999). Second, in subsection V*B* we follow Lucas (1988) and investigate a socially (and intertemporally) optimal solution that captures the positive externality arising from social human capital accumulation. Third, we connect our results to developing countries human capital growth (or the lack of growth, see Eicher, and Garcia-Penalosa (2001)). We argue that the removal of impediments to factor substitution promotes development. Finally, we look briefly at the implication our model has, via changing substitution possibilities in the course of technological change, on growth cycles.

Between-group and Within-group inequality

It is important to recognize that the income of skilled worker *i*th is $w^s(t)\,a_i(t)\,h(t)$, not $w^s(t)$. Owing to the uniform distribution of $a_i \in [\bar{a}, 1]$, we measure inequality (denoted λ_{sn}) by the ratio of the mean skilled earning to the unskilled wage $w^n(t)$. Equilibrium mean skilled earning is $(w^s h - w^s h\bar{a})/\bar{a}$. The between-group inequality measure has the representation $\lambda_{sn} = h \cdot \frac{w^s}{w^n} \cdot \frac{(1-\bar{a})}{\bar{a}}$. Using equation (6) we have

$$\lambda_{sn} = \frac{(1-\bar{a})}{\bar{a}^2}. \tag{3.5}$$

We conclude that between-group earnings inequality is positively related to long-term growth rate. The intuition is as follows. Easier substitution between effective skilled units and the unskilled swells the number of skilled workers and shrinks the number of unskilled. This is accomplished by raising the w^s / w^n ratio, worsening earnings inequality. In addition, the greater mass of skilled workforce hastens social human capital accumulation, which further aggravates earnings inequality. This conclusion is consistent with the empirical finding of Forbes (1998), although it differs from that of Aghion, Caroli and Garvia-Penalosa (1999). It is important to emphasize that our result arises from an entirely different economic process than Aghion et. al. (ibid). Aghion et. al. (ibid) argues that inequality is bad for growth when there is imperfection in the capital market. Our model, on the other hand, contains no such imperfection but instead depends on factor substitution dynamics. Our results therefore complements rather than contradicts Aghion et. al. (ibid).

Now we turn to inequality within the skilled group. This is the only type of within-group inequality since unskilled wage and earnings are uniform. Owing to the uniform distribution of $a_i \in [\overline{a}, 1]$, within-group inequality (denoted λ_s) can be represented by the earning gap between the top and the bottom skilled earners

$$\lambda_s = w^s h - w^s h\overline{a} = w^s h(1-\overline{a})\,. \tag{3.6}$$

We conclude that within-group inequality is also positively related to long-term growth rate. Intuitively, as a larger number of workers are drawn to the skilled profession via easier substitution between the factors, the basic ability of the skilled group becomes more diverged. Empirical work typically glosses over within-group ability differences and concentrates on observed pay differences. This may be misleading. Two skilled workers, earning the same observed wage, one taking twenty years to qualify while the other ten, have very different lifetime earnings and welfare. To sum up, our result shows that both between-group and within-group inequality rise with growth rate via the mechanics of factor substitution.

Socially Optimal Human Capital Accumulation and Growth

Free market maximizes output $x(t)$ taking $h(t)$ as given in each period *t*. To distinguish it from the social planner's choice we denote the market allocation by $\{a_m(t)\}$. It ignores the external benefit each generation has on future generations in terms of human capital accumulation. The social planner instead maximizes $y = \int_0^T x[a(t)]dt$ where $T < \infty$ is the planning horizon. Three inferences can be made immediately regarding the social planner's choice of career allocation, denoted $\{a_s(t)\}$.

First, $a_s(t) \le a_m(t)$ in order to internalize the positive externality; this implies also that $a_m(t) - a_s(t)$ rises with the extent of the externality (which is represented by the parameter δ in equation (1)).[7]

Second, it follows from the last observation that human capital and income growth rates are higher under the social planner than under free market career allocation.

Third, $a_m(t) - a_s(t)$ falls with the degree of substitutability between *s* and *n* in the production function. To see this note $a \in [0, 1]$ and $a_m \to 0$ as *s* and *n* become perfect substitutes, implying $a_m(t) = a_s(t)$. Subsection IV*B* above establishes that a_m falls when the isoquants become more curved - increased substitutability. The desired result follows immediately.

[7] Since $a_m(\tau) = \arg\max[x(a(\tau))]$, $a_s(\tau) \ne a_m(\tau)$ implies $x[a_s(\tau)] < x[a_m(\tau)]$. I.e., for $a_s(\tau) < a_m(\tau)$ to hold, the rise in aggregate output for $t \in [\tau + 1, T]$ from social planning must be larger than the shortfall of $x[a_m(\tau)] - x[a_s(\tau)]$. Otherwise $a_s(\tau) = a_m(\tau)$ holds.

Factor Substitution and Economic Development

The foregoing has established that factor substitutability is positively associated with the speed of human capital accumulation and the long-term income growth rate. This has two important implications on economic development.

First, if there are social, political, or other impediments to factor substitutability, the removal of these impediments promote long-term endogenous growth. Societies in the past as well as the present discriminate people by class, caste, sex and wealth. Some overtly ban groups of individuals (such as women) from joining certain professions (or from working at all!). Financial or other market failures prevent people from attend school of colleges to advance their career development. It is of course common knowledge that improvements in these areas benefit the society at large, but our contribution is to present a mechanism by which they increase long-term growth rate.

Second, the factor substitutability - growth relation provides a fresh perspective to understand the experience of many developing countries. Suppose developed countries, producing predominantly technology-intensive commodities, have a high degree of s-n substitution. Suppose also less-developed countries, producing predominantly primary produce such as foodstuff and natural resources, require more rigidly s-n ratio by comparison. Our model predicts that these suppositions work against growth convergence.

Factor Substitution and Growth Cycles

Several recent papers have noted cyclical behavior related to wage inequality and human-capital-led growth. Katz and Murphy (1992) find empirical evidence that college wage premium in the United States rose from 1963-71, fell from 1971-79, and rose sharply in 1979-87. Eicher (1996) argues that the ‘absorption’ of new technology takes two periods. In period one new technology is accessible to the skilled workers, only in period two it can be used by the unskilled. This creates fluctuations in the demand for skilled and unskilled, leading to fluctuations in relative wages just noted. Acemoglu (1998) also explains similar cyclical behavior by postulating that increases in the supply of skills reduces the skilled premium initially, inducing skilled-biased technical change that raises the skilled premium above its initial level.

Both Eicher's (ibid.) and Acemoglu's (ibid.) models can be framed in terms of changes in substitution possibilities between the skilled and the unskilled, that is, they are more complementary when a technology is new, becoming more substitutable when the technology becomes more familiar. The fresh insight from our model is that in addition to the changes in relative labor demand discussed by Eicher and Acemoglu, the curvature of isoquants becomes more curved on the arrival of a new technology, and it becomes flatter when the technology is more familiar. This gives rise to an additional mechanism previously unobserved that tends to slow down human capital accumulation upon the arrival of a new technology, but it speeds up human capital accumulation when the technology becomes more familiar. The adoption of a new technology, by making factors more complementary to each other, may so reduce human capital accumulation that it lessens the likelihood of further innovations. When the adopted technology becomes more familiar, by making factors more substitutable, may so increase human capital accumulation that it increases the likelihood of further innovations. The momentum of an endogenously mechanism of innovation-adoption-innovation is thus in place. Growth rate as well as relative wages fluctuate as this mechanism unfolds.

6. Summary and Conclusions

The model presented in this paper brings together two dynamic mechanisms in the literature to show that factor substitutability increases long-term growth rate. The first is the Rosen – Uzawa – Lucas equation linking human capital accumulation with individual choice of investment in education. The second is the mechanism of individual career choice, commonly found in models of skill-biased technological change and wage inequality. These mechanisms feed into each other and elevate factor substitutability to the position of an integral element of the growth engine.

Factor substitutability, framed in our model in terms of substitutability between skilled and unskilled labor, captures a fundamental property of a production process. It interacts with Lucas' (1988) effectiveness (δ) of investment in human capital to offer a more complete picture of endogenous growth.

While factor substitutability is deep-seated and stable given a production technology, it probably changes when there are fundamental technological shifts such as the type of energy used, the method of organizing production, and the way information is transmitted. Goldin and Katz (1998), discussed in our Introduction section above, documents how electric power and car assembly lines change the way skilled and unskilled workers are combined in production. We believe such fundamental technology shifts are capable of altering factor substitutability, and via our model changes the rate of growth. It is tempting to conjecture that Goldin and Katz's assembly line, and more recently computers, scientific management technique and the Internet are increasing the substitutability between the college-educated and the unskilled, and this helps to explain the faster growth observed during the 1980s and the 1990s than during the early 1900s. Empirical efforts to examine this hypothesis would be an interesting direction for future research.

The model of this paper has direct implications on other interesting issues and four of these are examined in section V above. First, growth is positively related to within-group inequality. Second, increase in factor substitutability reduces the distortion inherent in the social human capital accumulation process. Third, government policies that reduce impediments to factor substitutability by class, castes, sex or race also increases growth rate. Finally, technological progress, by changing the substitutability of factors, also changes growth rate along the path of invention and diffusion of new production techniques.

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